Supersymmetric UV safety

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Introduction and motivation

In most SUSY SO(10) matter fields of 1 generation typically live in

$$16 = \underbrace{Q + u^c + e^c}_{10} + \underbrace{d^c + L}_{\bar{5}} + \underbrace{\nu^c}_{1}$$

The (almost) always present Yukawa

$$W_{Yukawa} = 16\, 10_H\, Y_{10}\, 16$$

not enough for fitting masses (and no mixing).

Models differ mainly by the Higgs sector and additional Yukawa structure:

- small Higgs representations: $16_H, 45_H \ldots$, non-renormalizable Yukawa

$$\delta W_{Yukawa} = 16 \left(\frac{10_H \, 45_H}{M} Y'_{120} + \frac{{16_H}^2}{M} Y'_{126} + \ldots \right) 16$$

- large Higgs representation: $10_H, 126_H, 210_H \ldots$, renormalizable Yukawa

$$\delta W_{Yukawa} = 16 \left(126_H \, Y_{126} + \ldots \right) 16$$

My personal (biased) point of view is that models with large representations are more appealing because

- able to predict automatic R-parity conservation at low energies:

$$R = (-1)^{3(B-L)} \quad , \quad (B-L)(\langle 126_H \rangle) = 2$$

  while

$$(B-L)(\langle 16_H \rangle) = 1$$

- the whole model can be made renormalizable and thus simpler, minimal

The minimal such model is susy SO(10) with

$3 \times 16 + 10_H + 126_H + \overline{126}_H + 210_H$

$$
\begin{aligned}
W &= m_{126}\overline{126}_H 126_H + m_{210}210_H^2 + \lambda 210_H^3 + \eta \overline{126}_H 210_H 126_H \\
&+ m_{10}10_H^2 + \alpha 10_H 210_H 126_H + \bar{\alpha} 10_H 210_H \overline{126}_H \\
&+ \sum_{a,b=1}^{3} 16_a \left(10_H Y_{10}^{ab} + 126_H Y_{126}^{ab}\right) 16_b
\end{aligned}
$$

- $\langle 210_H, 126_H, \overline{126}_H \rangle \sim \mathcal{O}(M_{GUT})$
- MSSM Higgses in doublets of $10_H, 126_H, \overline{126}_H, 210_H$
- Doublet-triplet by explicit fine-tuning

Assuming a split susy scenario with $M_S \sim 10^{14}$ GeV and $M_\lambda \sim 10^5$ GeV the model could fit the data except for

- θ_{13} (at the time there was only an upper limit)
- Higgs mass (at the time has not been measured yet)

I believe that both issues can be resolved:

- by including θ_{13} in the fit instead of just assuming an upper bound
- by allowing more general soft susy terms

After all with proper soft susy terms a much more difficult case of minimal SU(5) has been made to work (without neutrinos)

If this is not enough, one could add for example an extra 54_H

Rest of phenomenology consistent:

- proton decay only $d = 6$ and close to exp limit (to be found in next round of detectors):

$$
\begin{aligned}
BR(p \to \pi^+ \bar{\nu}) &= 49\% \\
BR(p \to \pi^0 e^+) &= 44\%
\end{aligned}
$$

- no dangerous FCNC (split susy)
- LSP candidate for dark matter

But all these should be checked again in the new solution

Fans of small representation have a strong objection though:

The minimal large representation supersymmetric renormalizable model has the following chiral superfields

$$3 \times 16 + (10_H + 126_H + \overline{126}_H + 210_H)$$

and thus the 1-loop β function is

$$\beta_1 \equiv 3T(G) - \sum_i T(R_i) = 3 \times 8 - (3 \times 2 + 1 + 35 + 35 + 56) = -109$$

i.e. large and negative and so a Landau pole appears in the SO(10) gauge coupling g

$$\mu \frac{dg}{d\mu} = -\frac{\beta_1}{16\pi^2} g^3 \quad \to \quad g^2(\mu) = -\frac{8\pi^2}{\beta_1 \log(\Lambda/\mu)}$$

$\Lambda =$ Landau pole $\lesssim 10 M_{GUT}$ $\qquad (g(\Lambda) = \infty)$

Can we save somehow these theories? they seem UV sick. Various possibilities:

- incorporate this SO(10) theory into a larger gauge group (for example E_6): does not work, on the contrary, it makes the problem worse ($\beta_1(E_6) = -159$);
- make gravity with a lower effective M_{Planck}, this also predicted because of large number of degrees of freedom present; but it is a kind of sweeping the problem under the carpet: magic gravity will somehow solve all problems, but we have no control over it;
- try to make sense of the field theory with asymptotic safety.

Asymptotic safety

We got a Landau pole at 1-loop

What about higher loops?

$$\mu \frac{dg}{d\mu} = -\left(\frac{\beta_1}{16\pi^2} g^3 + \frac{\beta_2}{(16\pi^2)^2} g^5 + \dots \right)$$

This important only if

$$\frac{g^2}{16\pi^2} \gtrsim \mathcal{O}(1)$$

destroying perturbativity.

The only hope is that non-perturbatively the Landau pole is avoided and the gauge coupling (and eventually other couplings) flow to a finite (but large, non-perturbative) value.

Hard to work with non-perturbativity.

However if a solution of the Landau pole exists, then the theory in the UV is asymptotically conformal (no running). We lost perturbativity but gained the conformal symmetry

This we will use (in connection with supersymmetry)

Constraints on conformal field theories

Imagine we have a field theory in $d = 4$

Trace anomaly of stress-energy tensor $T^{\mu\nu}$:

$$T^\mu{}_\mu = -\frac{a}{16\pi^2} E_4 + \frac{c}{16\pi^2} Weyl^2 + \ldots$$

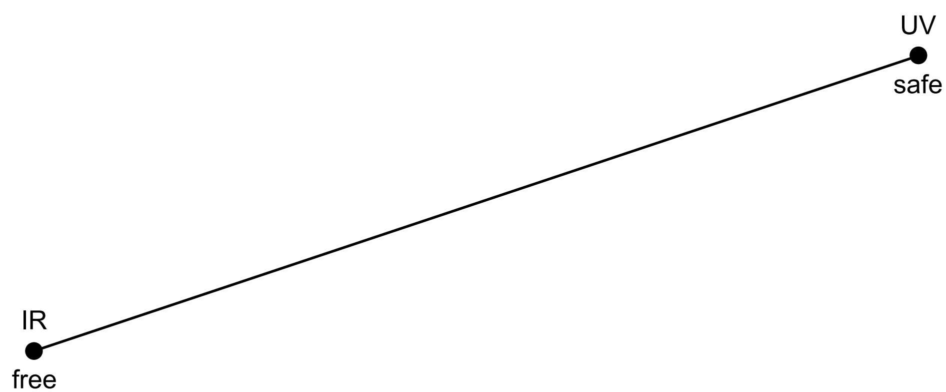
where

$$\begin{aligned} Weyl^2 &\equiv R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 2R^{\alpha\beta} R_{\alpha\beta} + \frac{1}{3} R^2 \\ E_4 &\equiv R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} - 4R^{\alpha\beta} R_{\alpha\beta} + R^2 \end{aligned}$$

are quadratic diffeomorphism invariants.

Our set-up is a $d = 4$ supersymmetric theory

- free in the IR
- with hypothetical UV interacting fixed point= asymptotically safe theory



If this true in the UV the theory is an interacting CFT

One can prove that:

1. $a_{UV} > a_{IR}$
2. $c_{UV} > 0$
3. $\frac{1}{6} \leq \frac{a_{UV}}{c_{UV}} \leq \frac{1}{2}$
4. no gauge invariant operator with $R < 2/3$

1. is the famous a-theorem (4d version of the 2d c-theorem):

in a theory with spontaneously broken conformal symmetry the dilaton is the Numbu-Goldstone boson; calculate dilaton-dilaton scattering:

$$amplitude \propto \frac{\Delta a}{f^4} s^2$$

$$\text{unitarity} \;\rightarrow\; \Delta a \equiv a_{UV} - a_{IR} > 0$$

because of it

- RG flow is irreversible
- a provides a measure for # of d.o.f.

2. follows from

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0)\rangle = c\,\Pi_{\mu\nu\alpha\beta}(\partial)\frac{1}{x^4}$$

$$\rightarrow c > 0$$

3. is the ”conformal collider bound”, follows from positivity of measured energy, in any conformal theory

$$\frac{1}{9} \leq \frac{a}{c} \leq \frac{31}{54}$$

in supersymmetry this reduces to

$$\frac{1}{6} \leq \frac{a}{c} \leq \frac{1}{2}$$

4. is due to unitarity: in any conformal field theory the dimension of a gauge invariant primary (no derivatives) operator $\mathcal{O}$ is

$$D(\mathcal{O}) \geq 1$$

From superconformal algebra

$$R = \frac{2}{3} D$$

$$\rightarrow \; R > 2/3$$

Calculation of central charges

In a generic field theory a and c can be calculated perturbatively.

In our case this not useful because fixed point non-perturbative

Fortunately in supersymmetry central charges can be got exactly

$$(R_i, n_i) \quad \ldots \quad (\mathrm{R-charge}, \#\ \mathrm{d.o.f.})\ \text{of chiral field}\ i$$

$$|G| \quad \ldots \quad \text{dimension of gauge group}\ G = \#\ \text{of gauge fields}$$

$$a = 2|G| + \sum_i n_i a_1(R_i) \quad , \quad a_1(R) = 3(R-1)^3 - (R-1)$$

$$c = \underbrace{4|G|}_{gaugino} + \underbrace{\sum_i n_i c_1(R_i)}_{chiral\ fields} \quad , \quad c_1(R) = 9(R-1)^3 - 5(R-1)$$

These exact relations are due to the fact that

$T_{\mu\nu}$ and j_R^μ are different components of the same supermultiplet

$\rightarrow$ relations between $T^\mu{}_\mu$ and $\partial_\mu j_R^\mu$:

$$
\begin{aligned}
T^\mu{}_\mu &= -a\, E_4 + c\, Weyl^2 + \ldots \\
\partial_\mu j_R^\mu &= \underbrace{\left[Tr\, U(1)_R\right]}_{\propto \sum_i n_i (R_i - 1)} R_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta} + \underbrace{\left[Tr\, U(1)_R^3\right]}_{\propto \sum_i n_i (R_i - 1)^3} F_{R\mu\nu}\tilde{F}_R^{\mu\nu}
\end{aligned}
$$

$U(1)_R$ symmetry unavoidable in supersymmetric fixed points (conformal theories): R charge part of the superconformal algebra

Since

$$R(\text{chiral superfield}) = \frac{2}{3} D(\text{chiral superfield})$$

for a free theory ($D(\phi_{free}) = 1$)

$$R(\phi_{free}) = 2/3$$

Gaugino has by definition always

$$R(\text{gaugino}) = 1$$

If we know the R-charges, we know the central charges a, c

How do we get the R-charges R_i?

In SCFT the β functions must vanish:

- NSVZ β function is proportional to

$$T(G) + \sum_i T(r_i)(R_i - 1) = 0$$

T ...Dynkin index

- β function for superpotential coupling λ_a of

$$W = \lambda_a \prod_i \phi_i^{q_{ia}}$$

is proportional to

$$\sum_i q_{ia} R_i - 2 = 0$$

Three possibilities:

1. # of constraints above bigger than number of chiral fields
   $\rightarrow$ no SCFT

2. # of constraints above equal to number of chiral fields
   $\rightarrow$ the solution to above equations unique and represents a possible candidate for CFT; to check consistency with inequalities mentioned above

3. # of constraints above smaller than number of chiral fields
   $\rightarrow$ one uses the above equations to express some R-charges with the others; then applies the a-maximization to calculate the remaining R-charges:

a-maximization:

$$\frac{\partial a}{\partial R_i} = 0$$

This gives same number of equations than unknowns R_i.

Equations are quadratic so there can be several real solutions. One should choose the one with

$$\frac{\partial^2 a}{\partial R_i \partial R_j} \qquad \text{all negative eigenvalues}$$

SUSY UV safety at large N_f

Imagine we have a gauge group G with

n_1 generations of r_1

n_2 generations of r_2

One type of representation only will not work. NSVZ:

$$T_G + n_1 T_1 (R_1 - 1) = 0 \quad \rightarrow \quad R_1 = 1 - \frac{T_G}{n_1 T_1} < 1$$

In the IR $R_1 = 2/3$ and $a_{IR} > a_{UV}$ (a-theorem violated) with 2 different representations in principle possible: R_2 calculated from NSVZ, R_1 from a-maximization. If we want to satisfy the a-theorem one of the two needs to have $R > 5/3$.

We can vary G, r_1, r_2, n_1, n_2.

Not possible to get acceptable solutions for any choice of r_1, r_2.

We need that

1. $\Delta a > 0$
2. $c > 0$
3. $1/6 \leq (a/c) \leq 1/2$
4. no GIO with $R < 2/3$

To be concrete consider two examples.

SO(10) at large N_f

Scan over all possible r_1 and $r_2 > r_1$ among

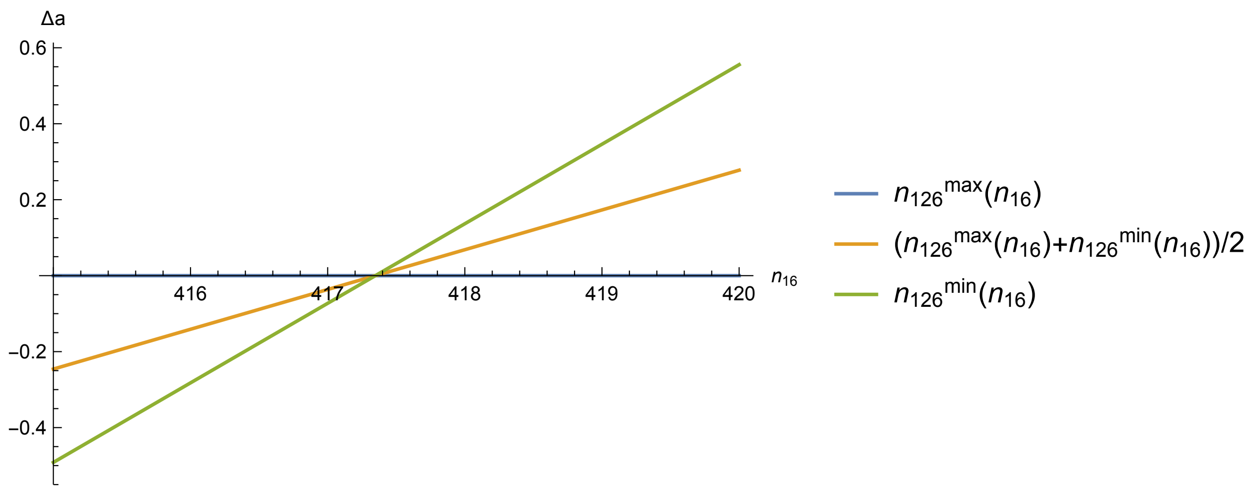
$10, 16, 45, 54, 120, 126, 144, 210$

There are solutions satisfying all checks only if

$(r_1, r_2) = (10, 126)$ or $(16, 126)$

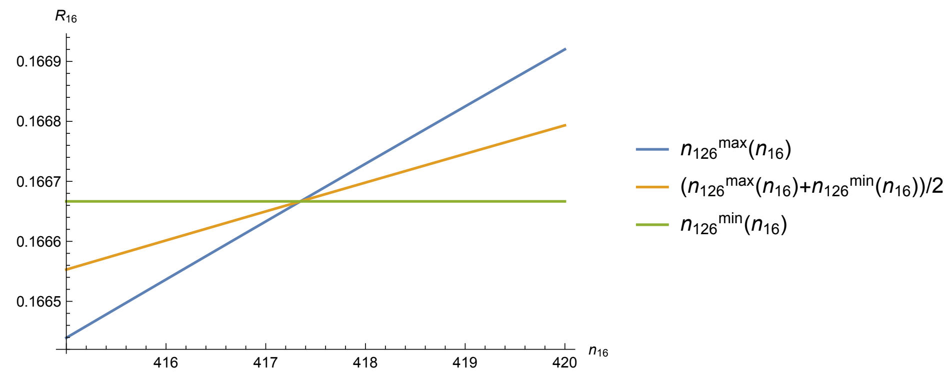
The number of generations involved are naively very large, at least few hundreds ($n_{10} \geq 554$ and $n_{16} \geq 418$)

Analogous to the non-supersymmetric example (see talk Antipin tomorrow)



This is because for $n_{16} < 418$ $\rightarrow$ $R(16^4) < 2/3$

GIO becomes free



But in this case formula for Δa changes so in principle lower number of generations can be possible

Number of such solutions is ∞ (not bounded from above)

For $n_{16} \to \infty$ the solution exists providing

$$\frac{1}{7}\sqrt{\frac{3}{38}} < \frac{n_{126}}{n_{16}} < \frac{2(\sqrt{301}-11)}{315}$$

In numbers

$$0.0401394 < \frac{n_{126}}{n_{16}} < 0.0403133$$

This is a large $\underbrace{N_{f1}/N_c}_{n_{126}/10}$ and $\underbrace{N_{f2}/N_c}_{n_{16}/10}$ case with bounded N_{f1}/N_{f2}

SU(5) at large N_f

We can easily repeat the above exercise; taking as possible representations any two among

$$5, 10, 15, 24, 35, 40, 45, 50, 70, 70', 75$$

it is easy to show that only these pairs have solutions:

$$(r_1, r_2) = (5, 35), (5, 40), (5, 70), (5, 70'), (5, 75), (10, 70'), (24, 70')$$

The difference with respect to SO(10) is that we need special care to cancel gauge anomalies.

We allow to have also complex conjugate representations. Some examples with all constraints satisfied:

r_1	n_1	$n_{\bar{1}}$	$R_1 = R_{\bar{1}}$	r_2	n_2	$n_{\bar{2}}$	$R_2 = R_{\bar{2}}$	Δa	c_{UV}	$(a/c)_{UV}$
5	15	147	0.43695	35	0	3	1.96684	2.15	1422.	0.178
5	61	119	0.36651	70	2	0	2.06152	6.38	1652.	0.173
5	9	165	0.54917	70'	0	1	1.81481	0.75	1395.	0.185
5	90	90	0.35869	75	2	-	2.05436	0.99	1637.	0.172
10	51	51	0.43853	70'	1	1	1.96316	13.70	1786.	0.179

Both chiral and vectorlike solutions.

Non-zero superpotential

The first supersymmetric UV fixed point found by Martin, Wells:

2 adjoints X, Y plus $N_f \times (Q + \tilde{Q})$ with

$$W = y_1 \tilde{Q} X Q + y_2 Tr\, X^3$$

Automatically $R(Q) = R(\tilde{Q}) = R(X) = 2/3$ and

$$T_G + T_X\,(R(X) - 1) + T_Y\,(R(Y) - 1) + 2N_f T_Q\,(R(Q) - 1) = 0$$

$\rightarrow \qquad \Delta a > 0 \qquad$ if $\qquad N_f > 4N_c$

The idea here is to make the superpotential terms determine R-charges of all fields except 1, the last one being determined by the vanishing NSVZ.

In Martin-Wells example, all fields (X, Q, $\tilde{Q}$) have $R = 2/3$ except one (Y) which has $R > 5/3$.

Possible to generalize. For example take $N_c/N_f = 0.46$, $N_c \to \infty$, and

$$W = y_1 \tilde{Q} X^4 Q + y_2 Tr\ X^6$$

leads to UV fixed point with all constraints satisfied.

We will see later on a phenomenologically interesting example of this type.

SO(10) with W=0

Easy to analyze, to get a flavor of the procedure

$$a = 2|G| + \sum_i |r_i| a_1(R_i) + \lambda_G \underbrace{\left(T(G) + \sum_i T(r_i)(R_i - 1) \right)}_{NSVZ}$$

$|G| \ldots$ dimension of gauge group ($= 45$ in SO(10))

$|r_i| \ldots$ dimension of representation r_i

$i \ldots$ runs over chiral superfields

$\lambda_G \ldots$ Lagrange multiplier for vanishing of NSVZ β-function

Maximizing a we get

$$\frac{\partial a}{\partial R_i} = |r_i| \left(9(R_i - 1)^2 - 1\right) + \lambda_G T(r_i) = 0$$

$$\rightarrow \quad R_i(\lambda_G) = 1 - \frac{\epsilon_i}{3}\sqrt{1 - \frac{T(r_i)}{|r_i|}\lambda_G} \quad \epsilon_i = \pm 1$$

One can imagine that λ_G is changing along the flow (a function of the gauge coupling g^2):

$\lambda_G = 0$ $\qquad\qquad$ $\lambda_G = \lambda_G^*$

IR ———————————— UV

$\mu = 0$ $\qquad\qquad$ $\mu = \infty$

- In the IR $\lambda_G = 0$, all $\epsilon_i = +1$ and so $R_i = 2/3$ (free!)
- For small λ_G the theory is perturbative and one finds the 1-loop relation

$$\lambda_G = \frac{g^2}{2\pi^2} + \mathcal{O}(g^4)$$

- one can repeat the calculation up to 3-loops getting agreement for the scheme independent part of the perturbative calculation of the anomalous dimensions
- if there is a UV CFT, it happens at some λ_G^* such that NSVZ vanishes:

$$T(G) + \sum_i T(r_i)\left(R_i(\lambda_G^*) - 1\right) = 0$$

This last step is possible only if $\sqrt{\text{positive number}}$:

$$1 - \frac{T(r_i)}{|r_i|}\lambda_G^* \geq 0$$

i.e. if

$$\lambda_G^* \leq \lambda_G^{max} \equiv min_i \left(\frac{|r_i|}{T(r_i)} \right)$$

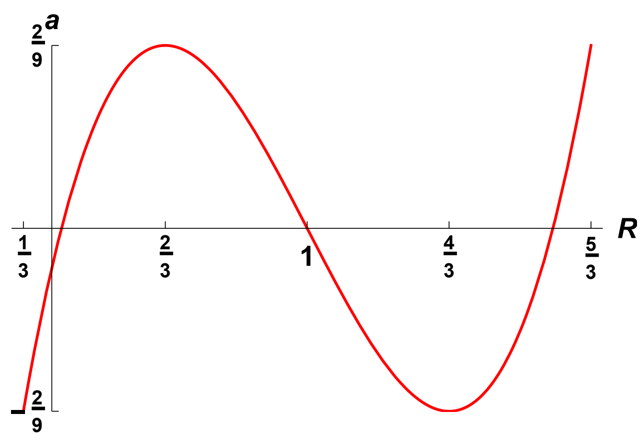
The minimal SO(10) model has $10 + 2 \times 126 + 210 + 3 \times 16$

$$\lambda_G^{max} \equiv min_i \left(\frac{|r_i|}{T(r_i)} \right) = \frac{|126|}{T(126)} = \frac{126}{35}$$

On the other side a possible fixed point with all $\epsilon_i = +1$ will not satisfy the $\Delta a > 0$ theorem.

$$R_{\epsilon=+1} \leq 1$$

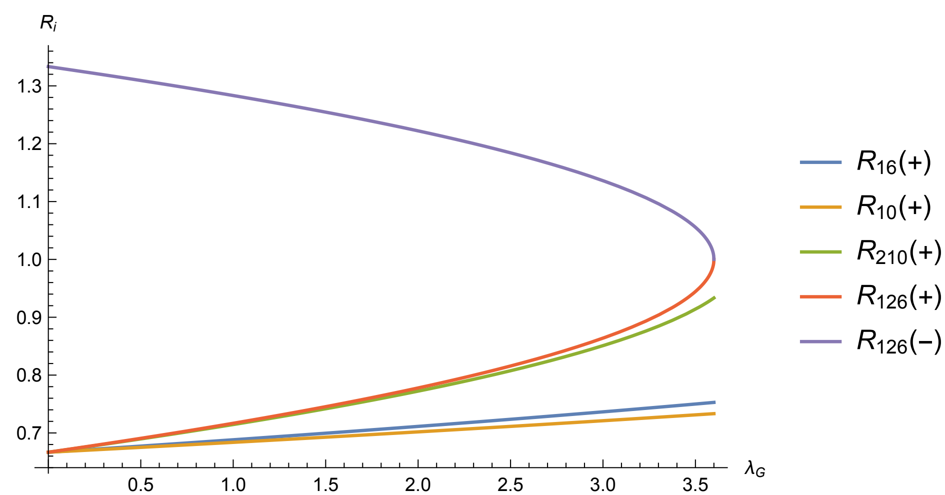
But for these values $a_1(R_{\epsilon=+1}) < a_1(2/3)$ and so $a_{UV} < a_{IR}$.

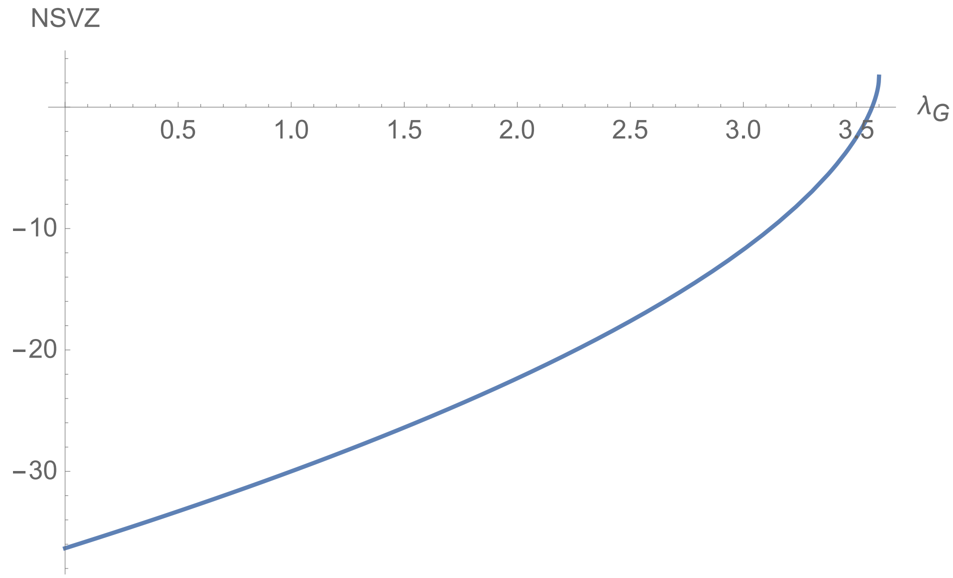


At least one chiral field must have $R_i > 5/3 \;\rightarrow\; \epsilon_i = -1$

The only possibility is

1. with λ_G running from 0 reach λ_G^{max} without satisfying NSVZ with all $\epsilon_i = +1$ at any point $0 \leq \lambda_G \leq \lambda_G^{max}$
2. at $\lambda_G = \lambda_G^{max}$ we can change sign of ϵ_{126} and/or $\epsilon_{\overline{126}}$
3. returning back with λ_G towards 0 finding a point λ_G^* where NSVZ vanishes with these new ϵ's.

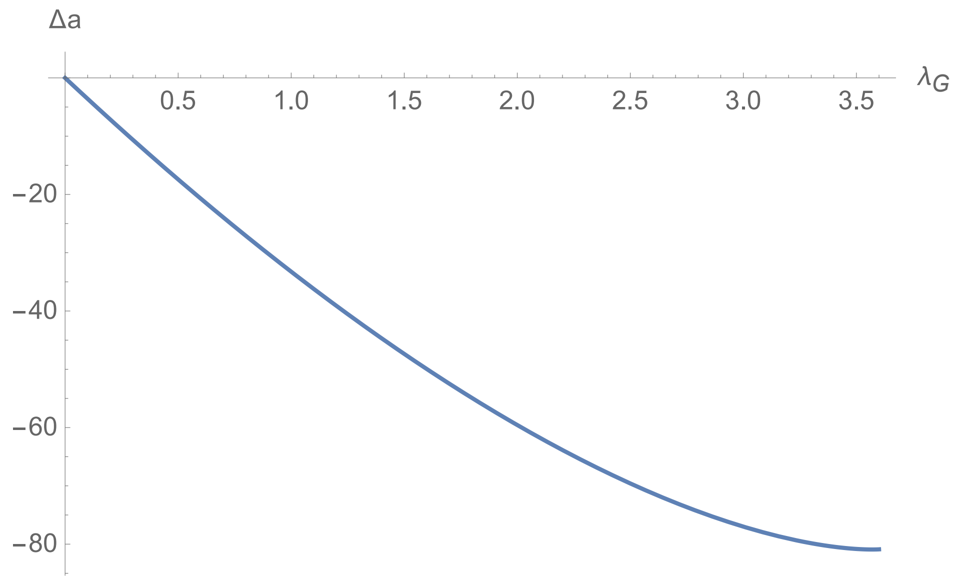




In our case with all $\epsilon_i = +1$ we get

$$\begin{aligned} \beta_{NSVZ}(0) &< 0 \\ \beta_{NSVZ}(\lambda_G^{max}) &> 0 \end{aligned}$$

and thus a zero is somewhere in between but with $a_{UV} < a_{IR}$.



→ no consistent UV fixed points in minimal SO(10) with $W = 0$

SO(10) with $W \neq 0$

We tried various trilinear terms in the superpotential

The only solution we found was with the superpotential

$$
\begin{aligned}
W &= y_1\, 210^3 + y_2\, 210\, 126\, \overline{126} + y_3\, 210\, 126\, 10 + y_4\, 210\, \overline{126}\, 10 \\
&+ \sum_{a,b=2,3} 16_a\, 16_b\, \left(y_{5,ab}\, 10 + y_{6,ab}\, \overline{126} \right)
\end{aligned}
$$

i.e. all the most general trilinear couplings except that 16_1 never appearing in W

The constraints (all β-functions vanishing) fix

$$R(16_1) = \frac{113}{6}$$

and all other $R = 2/3$.

Comments:

- the solution found describes one massless generation
- there are more Lagrange multipliers than equations of motion: our solution is a manifold of fixed points
- if some gauge invariant operators have $R < 2/3$ the correct interpretation is that these composites become free (with $R = 2/3$) but the expressions for the central charges must be changed in a known way:

$$a_{new} = a_{old} + \sum_{R(\mathcal{O})<2/3} (a_1(2/3) - a_1(R(\mathcal{O})))$$

We tried to look in some of these cases but with no success (no consistent UV fixed point found)

- we avoided such cases when $R_i < 0$; although in principle such cases can be studied, the calculation is complicated (finding out all the gauge invariant operators of the chiral ring)

Conclusion

- In GUTs problem of Landau pole due to supersymmetry (no such problem in non-susy below Planck scale)
- But supersymmetry can help analyzing the non-perturbative problem: inequalities on central charges a, c used
- Theory: two types of supersymmetric asymptotically safe theories presented
  1. large N_f
  2. Martin-Wells' type
- Phenomenology: in minimal renormalizable SO(10) GUT a quasi-realistic possibility for a UV safe theory found: one generation of matter fields decoupled from the superpotential