Fyzikální ústav
Akademie věd ČR, v. v. i.

Πανεπιστήμιο Κύπρου

The gravitational side of the Dark Matter problem

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Operational Programme Research, Development and Education

MINISTRY OF EDUCATION, YOUTH AND SPORTS

Motivation

- General Relativity and beyond
- IR modifications of gravity: requirements
- Modified Newtonian Dynamics and variants
- Other Dark Matter motivated gravitational theories
- Outlook

Lovelock's theorem

(Lovelock, 1967)

The only

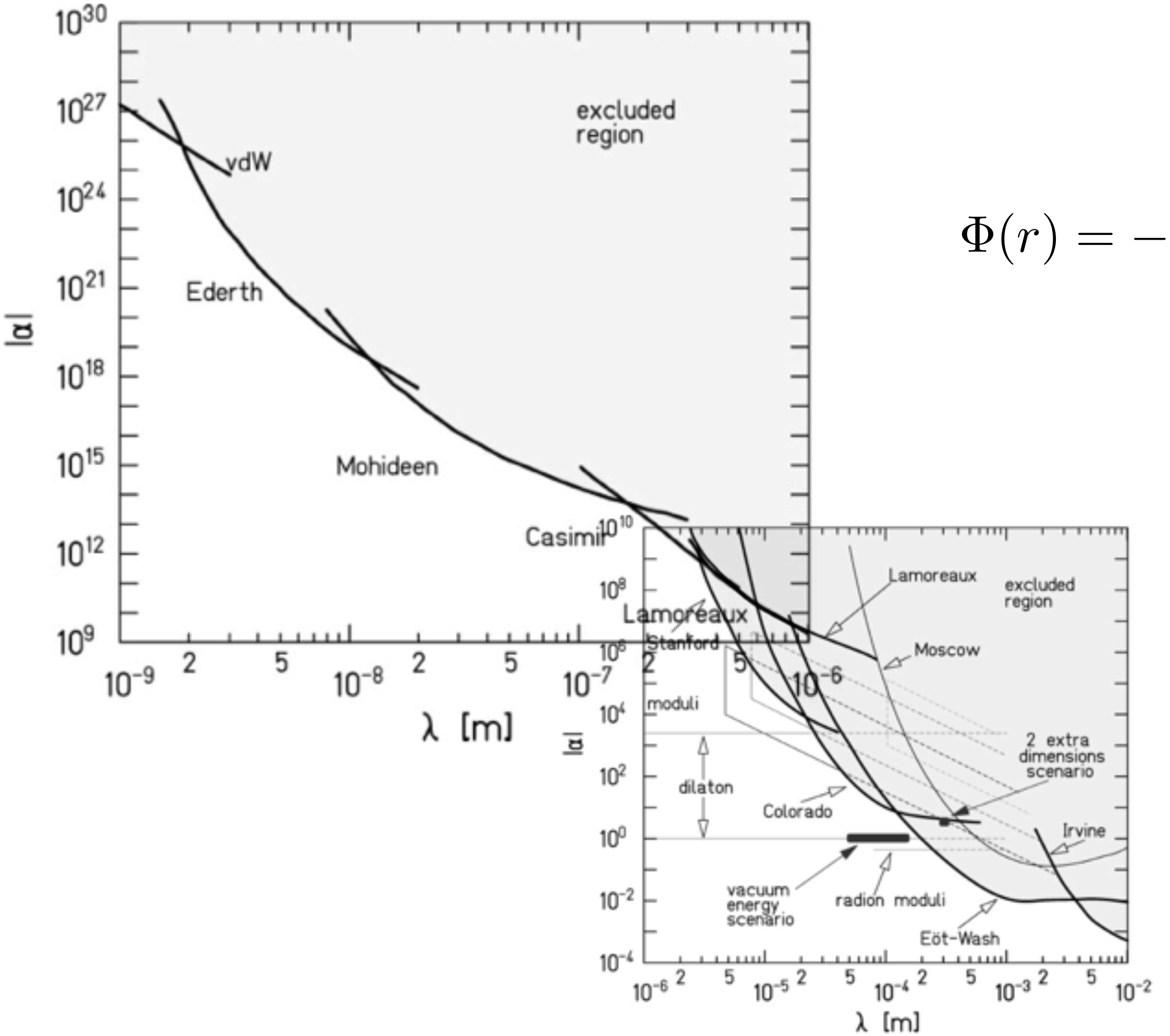
- local,
- diffeomorphism invariant action,
- which leads to 2nd order field equations
- and which depends only on a metric

in 4D

is a linear combination of the Einstein-Hilbert action with a cosmological constant up to a total derivative.

GR: $$S = \frac{1}{16\pi G} \int d^4x \, [R - 2\Lambda] + S_m[g, \psi^A]$$

Tests of Newton's inverse square law



$$\Phi(r) = -\frac{GM}{r}\left(1 + \alpha e^{-r/\lambda}\right)$$

Testing GR with astrophysical systems

Parameterized Post-Newtonian (PPN) expansion (C. Will, Liv. Reviews. Rel.)

Perturbation order: $v^2 \sim U \sim \rho$

$$
\begin{aligned}
g_{00} &= -1 + 2U - 2\beta\, U^2 - 2\xi\Phi_W + (2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\
&\quad + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3\Phi_3 + 2(3\gamma + 3\zeta_4)\Phi_4 \\
&\quad - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2\Psi - \alpha_2 w^i w^j U_{ij} \\
&\quad + (2\alpha_3 - \alpha_1)w^i V_i
\end{aligned}
$$

$$
\begin{aligned}
g_{0i} &= -\frac{1}{2}(3 + 4\gamma + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)\, W_i \\
&\quad - \frac{1}{2}(\alpha_1 - 2\alpha_2)\, w^i U
\end{aligned}
$$

$$
g_{ij} = (1 + 2\gamma U)\delta_{ij}
$$

10 Parameters

Parameter	What it measures relative to GR	Value in GR	Value in semi-conservative theories	Value in fully conservative theories
γ	How much space-curvature produced by unit rest mass?	1	γ	γ
β	How much "nonlinearity" in the superposition law for gravity?	1	β	β
ξ	Preferred-location effects?	0	ξ	ξ
α_1	Preferred-frame effects?	0	α_1	0
α_2		0	α_2	0
α_3		0	0	0
α_3	Violation of conservation of total momentum?	0	0	0
ζ_1		0	0	0
ζ_2		0	0	0
ζ_3		0	0	0
ζ_4		0	0	0

10 Potentials

$$
U(\vec{x}) = \int d^3x' \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|}
$$

$$
\Phi_1 = \int d^3x \frac{\rho v^2}{|\vec{x} - \vec{x}'|}
$$

etc

Key assumption: asymptotic flatness

Constraints on PPN parameters

Solar system	$\gamma - 1$	space curvature	2.3×10^{-5}	Bertotti et al (2003), Cassini tracking
			$(-0.3 \pm 2.5) \times 10^{-5}$	Verma et al (2013), Messenger tracking
	$\beta - 1$	non-linearity	$(0.2 \pm 2.5) \times 10^{-5}$	Verma et al (2013), Messenger tracking
	$\lvert\xi\rvert$	preferred location	$< 10^{-3}$	Earth tides
	α_1	preferred frame	$(-0.7 \pm 1.8) \times 10^{-4}$	Müller et al (2008), lunar ranging
	$\lvert\alpha_2\rvert$	preferred frame	$< 2.4 \times 10^{-7}$	Nordtvedt (1987), solar alignment with the ecliptic
Binary pulsars	$\lvert\xi\rvert$		$< 3.9 \times 10^{-9}$	Shao & Wex (2013), millisecond pulsars
	α_1		$-0.4^{+3.7}_{-3.1} \times 10^{-5}$	Shao & Wex (2012), pulsar-WD binaries
	$\lvert\alpha_2\rvert$		$< 1.6 \times 10^{-9}$	Shao et al (2013), millisecond pulsars

Conservative theories: $\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$

Equivalence principle

Dicke 1960-1965

WEP: Weak Equivalence Principle

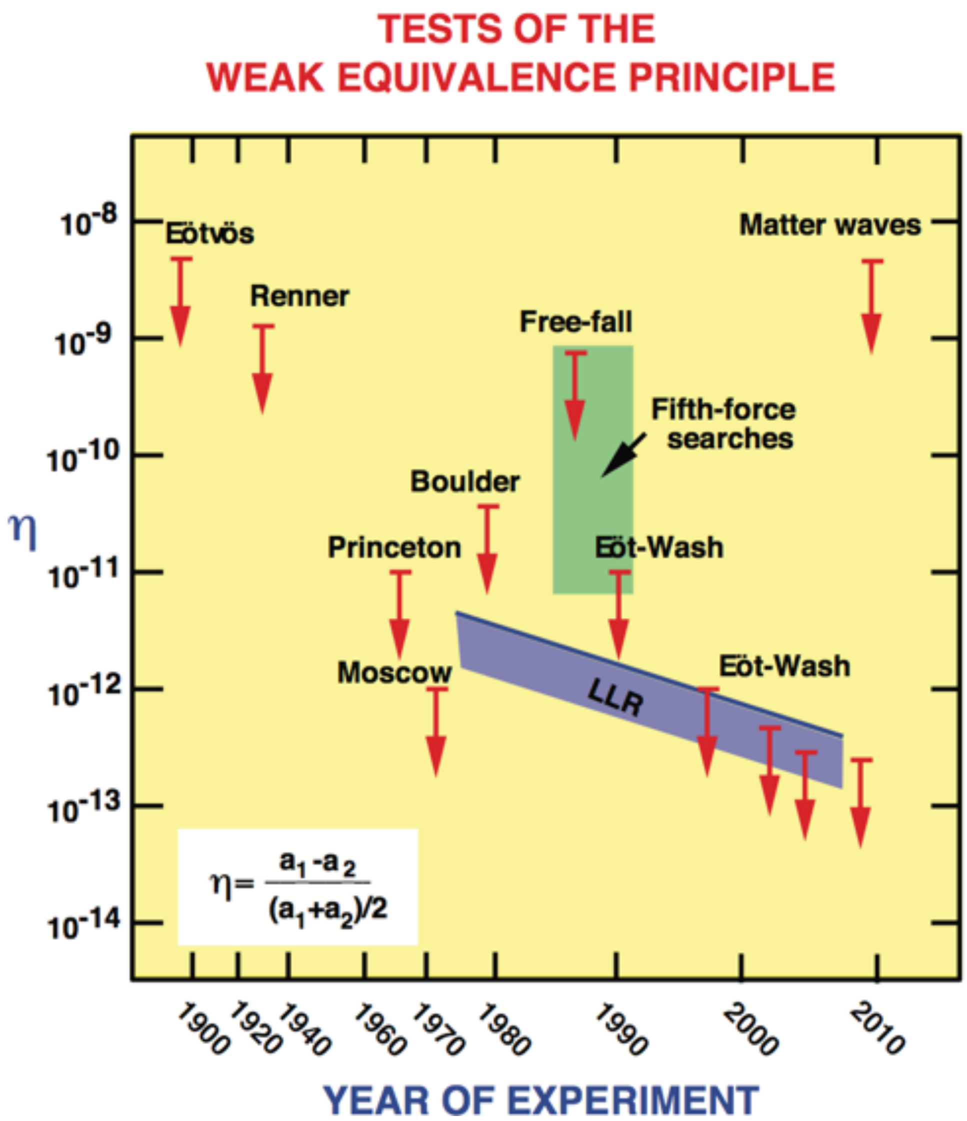
The trajectory of a freely falling "test" body is independent of its internal structure and composition.

EEP: Einstein Equivalence Principle

- WEP is valid
- The outcome of any local **non-gravitational** experiment is independent of the velocity of the freely-falling reference frame in which it is presented.
- The outcome of any local **non-gravitational** experiment is independent of where and when in the Universe it is performed.

C. Will, Liv. Rev. Rel.

Experimental tests of WEP



Microscope (launched Apr 2016), exp. sensitivity $\sim 10^{-15}$

Standard lore

Newton: $$\Phi = -\frac{GM}{r}$$

GR: $$\Phi = -\frac{GM}{r} + \left(\frac{GM}{r}\right)^2 + \ldots + C_n \left(\frac{GM}{r}\right)^n + \ldots$$

Quantum gravity: $$\Phi = -\frac{GM}{r} + \left(\frac{GM}{r}\right)^2 + \frac{31 G^2 M \hbar}{15 r^3} + \ldots$$

Bjerrum-Bohr, Donoghue, Holstein (2003)

$r \to \infty \quad \Rightarrow \quad$ Asymptotic Minkowski

Understanding tests of gravity

T. Baker, D. Psaltis and C.S., Astrophys. J. 802, 63 (2015)

Quantifying gravitational fields

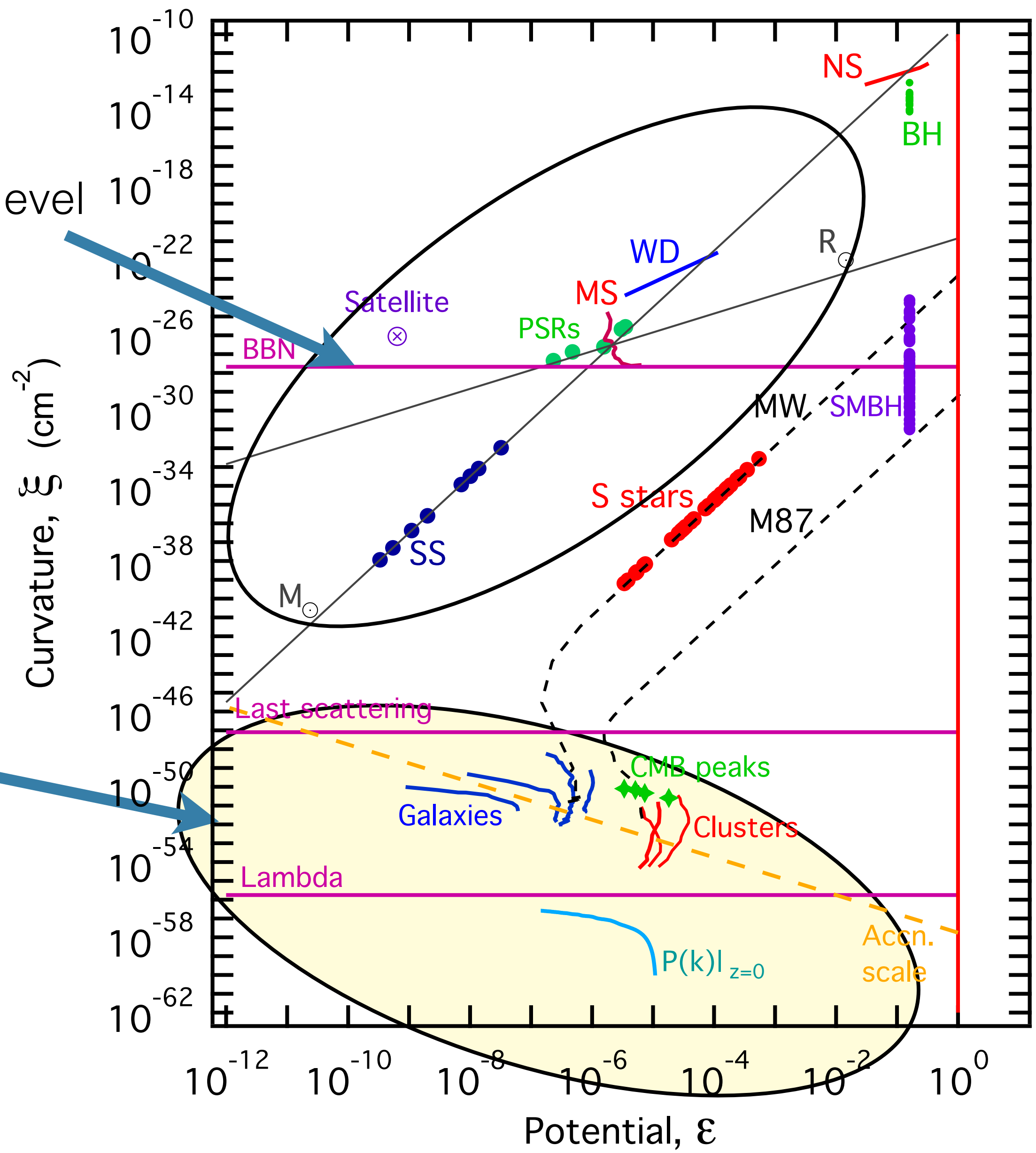
$$g_{\mu\nu} \approx \bar{g}_{\mu\nu} + h_{\mu\nu} \longrightarrow \text{"field proxy"} \quad \epsilon = \frac{\Phi}{c^2} \sim \frac{GM}{c^2 r}$$

$$R^{\alpha}{}_{\beta\mu\nu} \longrightarrow \text{"curvature proxy"} \quad \xi = \sqrt{R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu}} \sim \frac{GM}{c^2 r^3}$$

Gravity and the Dark Sector

- GR experimentally tested
- Deviations at $10^{-3} - 10^{-20}$ level

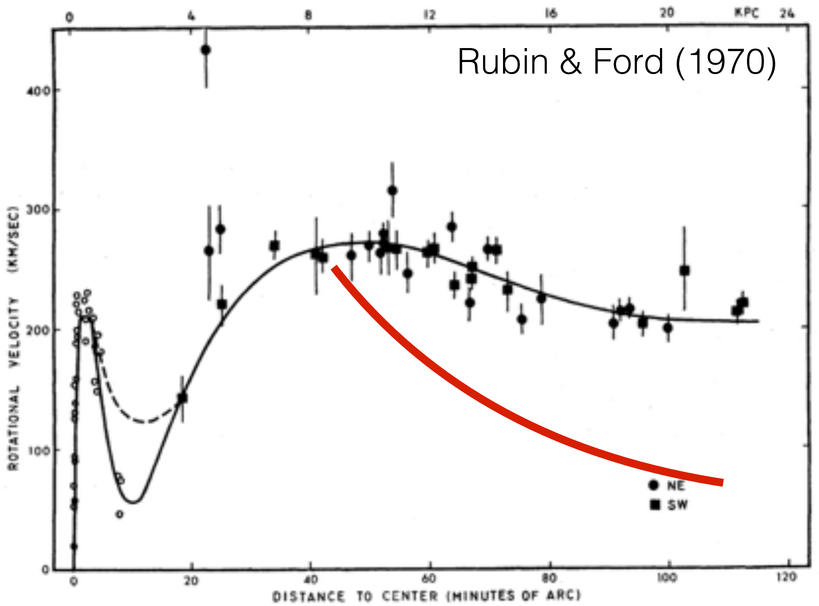
- Need DM and DE
- GR not tested



Constantinos Skordis, Institute of Physics, Prague

Galaxies: Gravity must be stronger in the IR

Galaxies: rotation curves do not decay with increasing distance



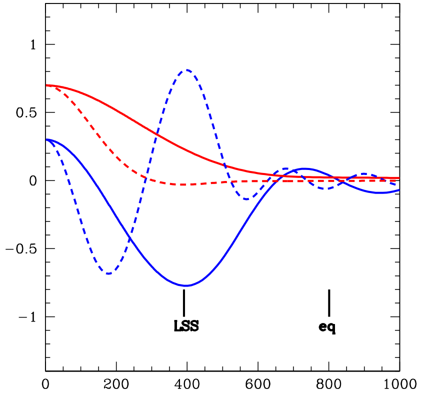
Clusters: stronger gravitational lensing

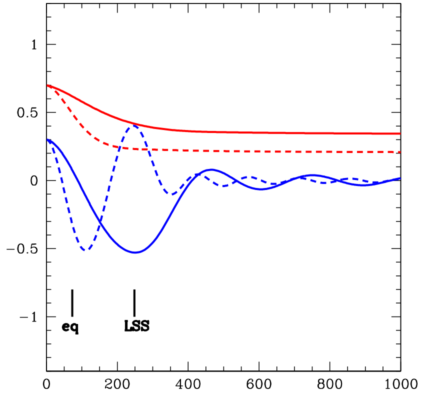
Cosmology: Gravity must be stronger in the IR

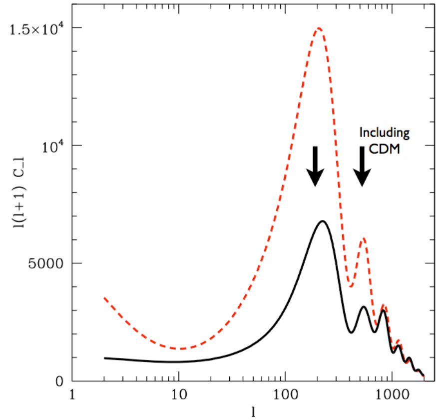
$$H_{obs} > H_{baryon}$$

$$\Phi_{obs} > \Phi_{baryon}$$

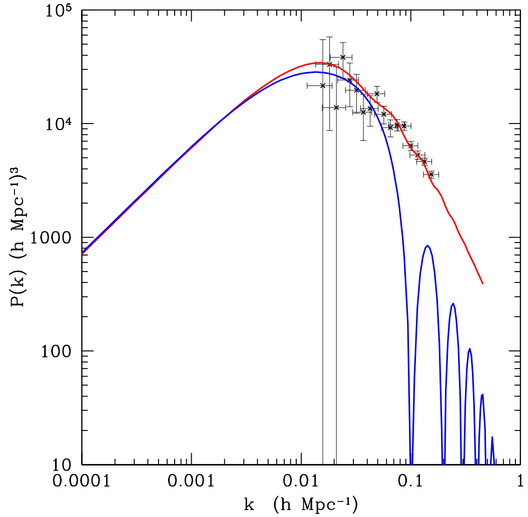
CMB: weaker driven oscillations, weaker early Integrated Sachs-Wolfe



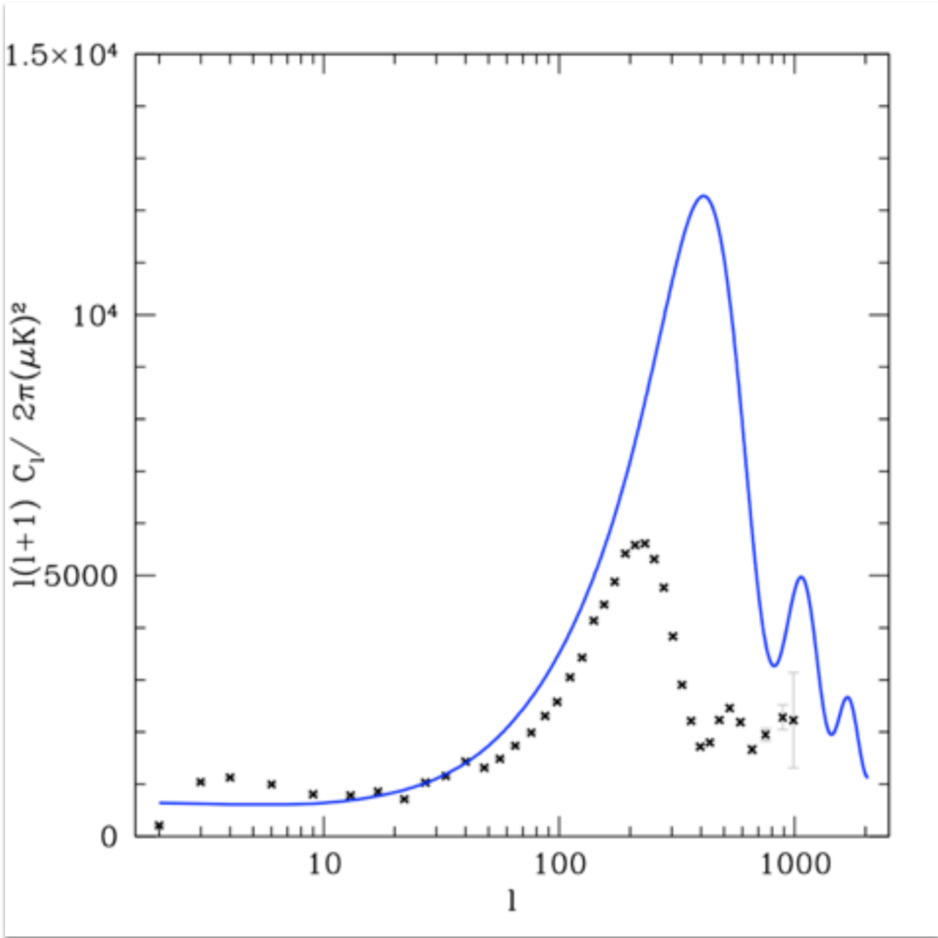


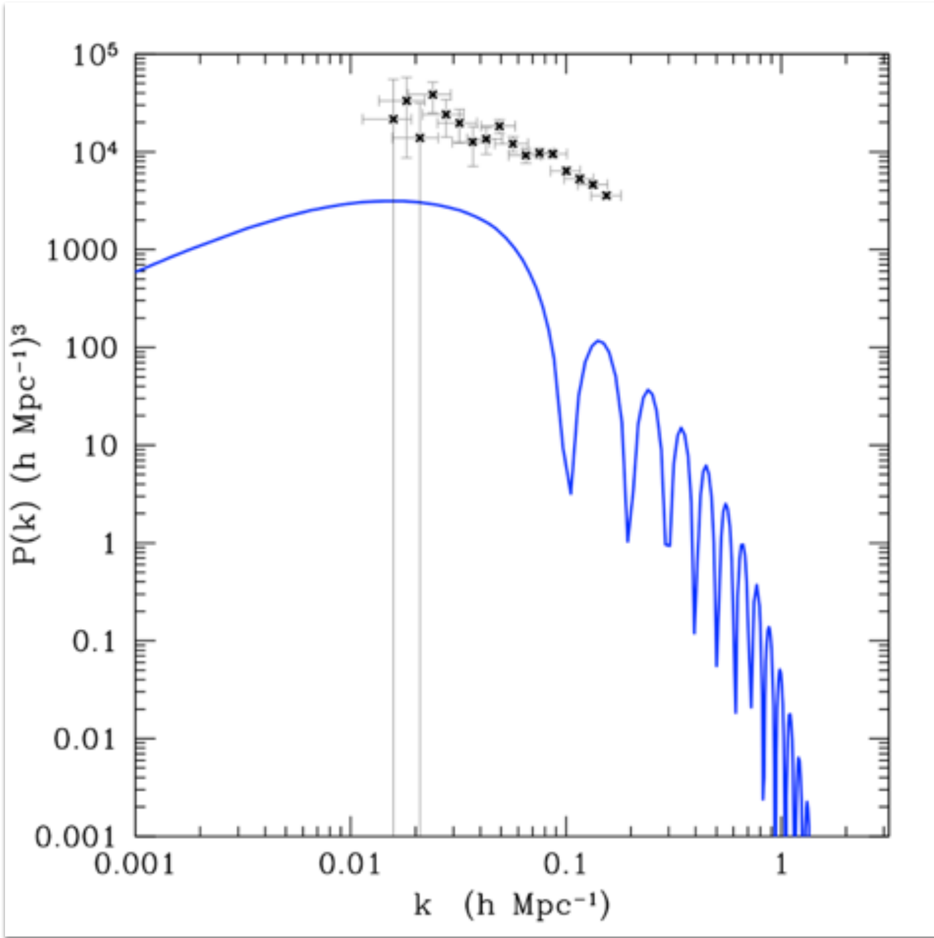


Large scales: stronger gravitational clustering



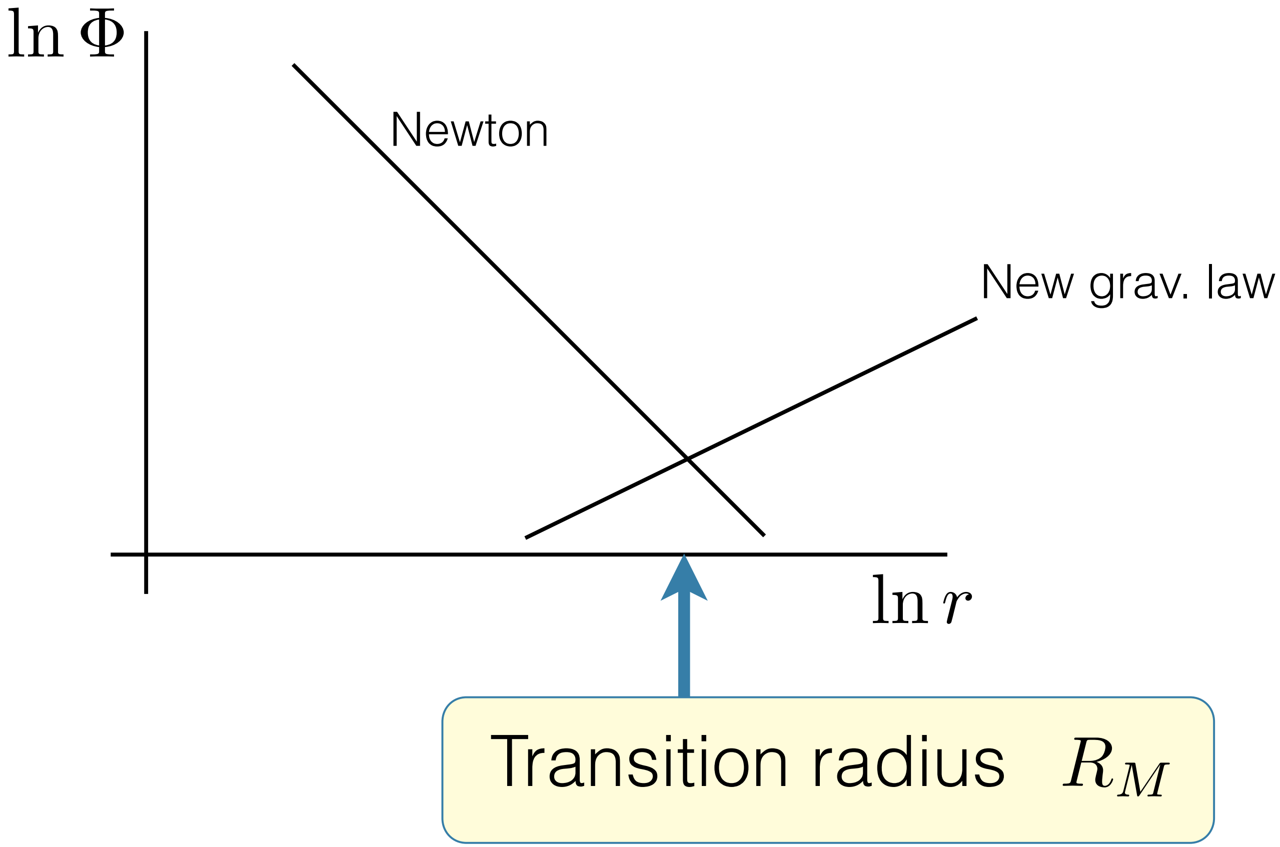
GR cosmology without DM



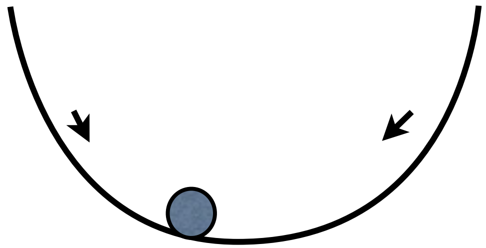


Requirements for IR corrections to GR

- Gravity must become stronger than GR in the IR
- Lovelock: new degrees of freedom



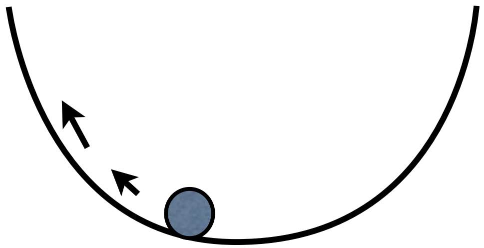
Potential pitfalls

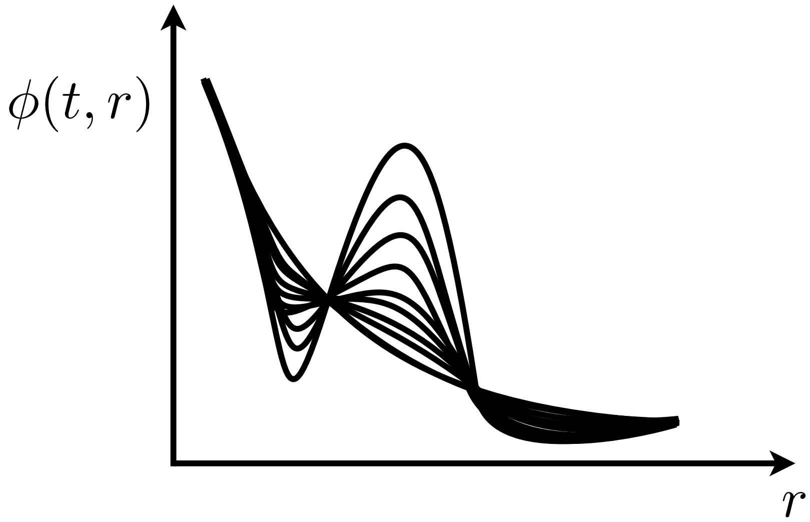


$$\rho = \frac{1}{2}\dot{\phi}^2 + m^2\phi^2$$

Ghost

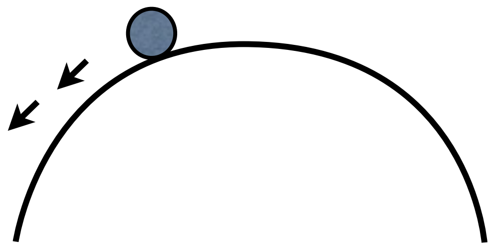
$\dot{\phi}^2 < 0$





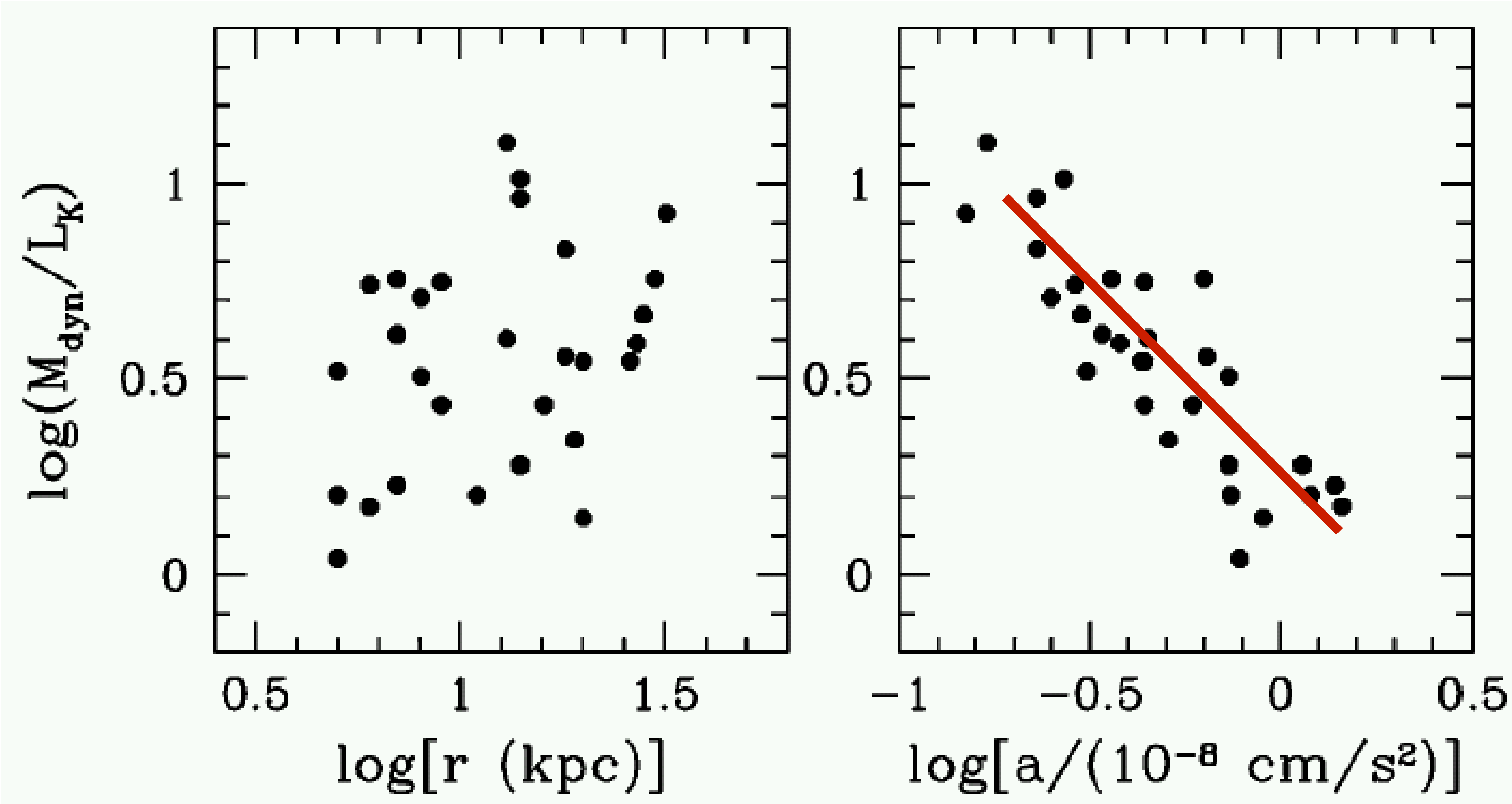
Tachyon

$m^2 < 0$



See Anca's talk

Galaxies



No fixed transition scale

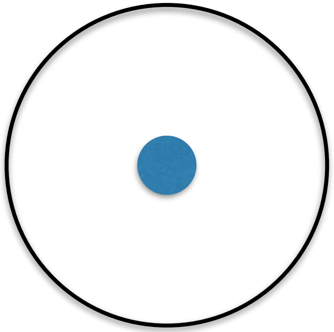
acceleration scale

Milgrom (1983)

$$a_0 \sim 1.2 \times 10^{-10} m/s^2$$

Modified Newtonian Dynamics (MOND)

Milgrom (1983)



circular orbits $a = \frac{v^2}{r}$

Newton: $$a = |\nabla\Phi| = \frac{GM}{r^2} \Rightarrow \quad v \propto r^{-1/2}$$

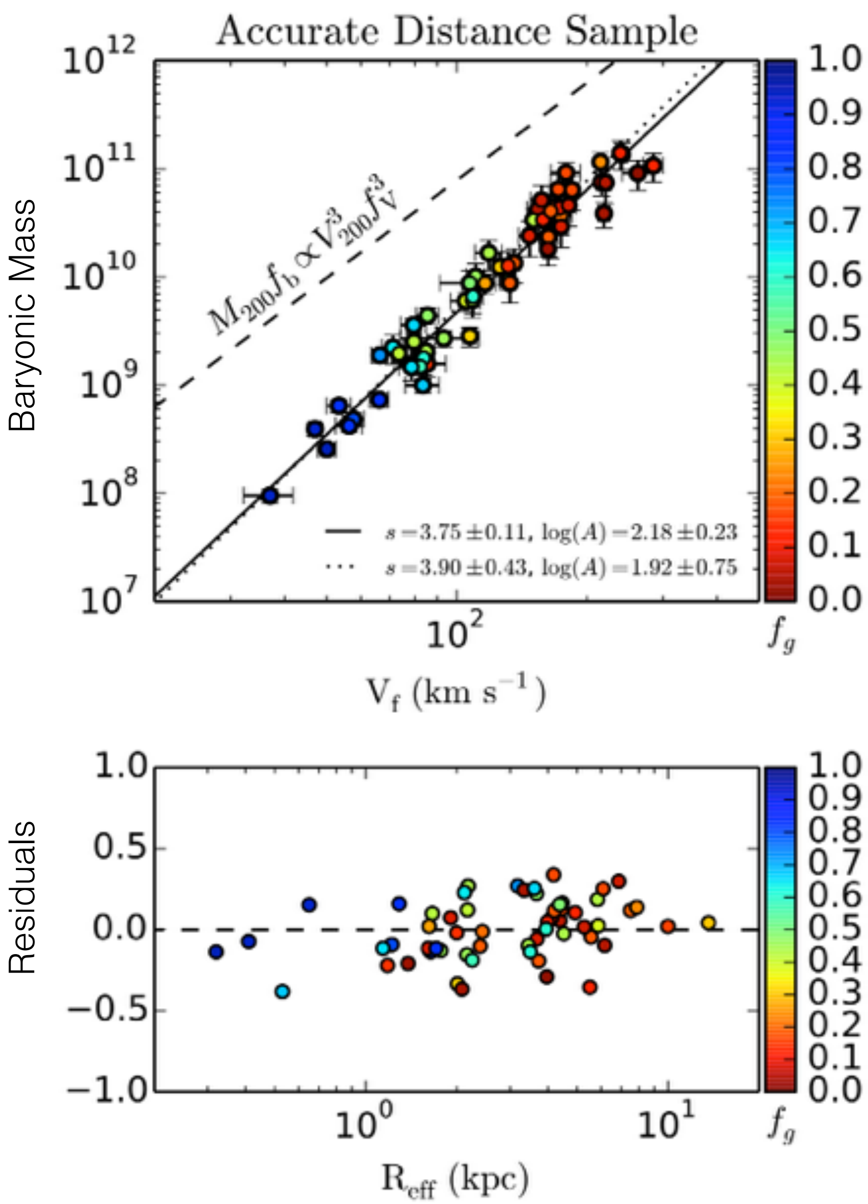
Potential obeys: $\nabla^2\Phi = 4\pi G\rho$

MOND: $$\frac{a^2}{a_0} = |\nabla\Phi| = \frac{GM}{r^2} \Rightarrow \quad v \propto constant$$

Violation of conservation laws

Prediction: Baryonic Tully-Fisher

Tully & Fisher, 1977
McGaugh 2004



MOND $v^4 = G a_0 M_b$

Slope ~ 4

Correct normalisation $\frac{1}{G a_0}$

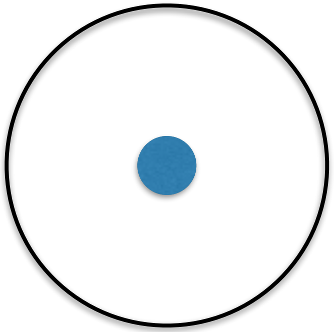
No dependence on surface brightness

Scatter around the fit is too small.

Lelli, McGaugh & Schombert (2015)

Aquadratic Lagrangian Gravity

Bekenstein & Milgrom (1984)



Newton: $a = |\nabla\Phi| = \frac{GM}{r^2}$

MOND: $\frac{a^2}{a_0} = |\nabla\Phi| = \frac{GM}{r^2}$

Define $a = -\nabla\Phi_N$

$$\Rightarrow \nabla\Phi = \frac{|\nabla\Phi_N|}{a_0}\nabla\Phi_N$$

Potential obeys: $\nabla\cdot\left(\frac{|\nabla\Phi_N|}{a_0}\nabla\Phi_N\right) = 4\pi G\rho$

Derivable from a Lagrangian: Conservation laws obeyed

Aquadratic Lagrangian Gravity

Bekenstein & Milgrom (1984)

Geodesic motion $a = -\nabla\Phi_N$

Newton $\nabla^2\Phi_N = 4\pi G\rho$

Transition

MOND $\nabla \cdot \left(\frac{|\nabla\Phi_N|}{a_0} \nabla\Phi_N \right) = 4\pi G\rho$

Transition radius

Bekenstein & Milgrom (1984)

$$\vec{a} = -\nabla\Phi$$

$$\rho = M\delta^{(3)}(\vec{r}) = M\frac{\delta(r)}{r^2}$$

Newton $\nabla^2\Phi = 4\pi G\rho$ → $\Phi = -\frac{GM}{r}$

MOND $\nabla\cdot\left(\frac{|\nabla\Phi|}{a_0}\nabla\Phi\right) = 4\pi G\rho$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{a_0}\left|\frac{d\Phi}{dr}\right|\frac{d\Phi}{dr}\right) = \frac{GM}{r^2}\delta(r)$$

$$\Rightarrow \quad \left|\frac{d\Phi}{dr}\right|\frac{d\Phi}{dr} = \frac{GMa_0}{r^2} \quad \Rightarrow \quad \Phi = \sqrt{GMa_0}\ln r$$

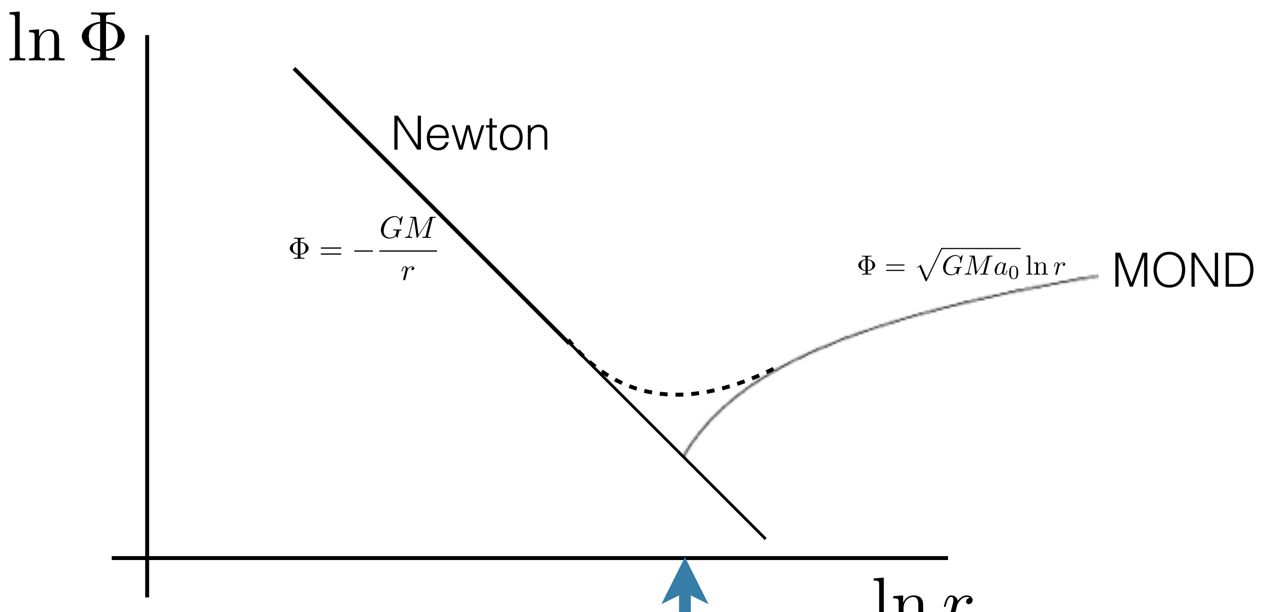
MOND potential

Equate forces: $\left.\frac{d\Phi}{dr}\right|_{R_M} = \frac{\sqrt{GMa_0}}{R_M} = \frac{GM}{R_M^2}$

$$R_M = \sqrt{\frac{GM}{a_0}}$$

Requirements for IR corrections to GR

- Gravity must become stronger than GR in the IR
- Lovelock: new degrees of freedom
- MOND: acceleration scale dictates potential



Transition radius R_M

Coincidences

- Relation to Hubble constant $$a_0 \sim \frac{1}{6} c H_0$$

- Characteristic length $$R_M = \sqrt{\frac{GM}{a_0}} \sim 0.29 \sqrt{R_s R_h}$$

Universe: $M = \frac{4}{3}\pi R_h^3 \rho$

$\rho = \frac{3H_0^2}{8\pi G}$

$\rightarrow R_M \sim 0.29 R_h \sim 1300 Mpc$

DE?

Connecting MOND to Newton

- AQUAL equation $$\nabla \cdot \left[\mu \left(\frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right] = 4\pi G \rho$$

Interpolating function

$\mu(x) \to 1$	$x >> 1$	Newton
$\mu(x) \to x$	$x << 1$	MOND

e.g. $\mu = \frac{x}{1+x}$ $\mu = \frac{x}{\sqrt{1+x^2}}$

Non-analytic

- Screening mechanisms

Galileon k-mouflage Babichev, Deffayet, Esposito-Farese (2011)

Types of MOND theories: AQUAL

Type-1 $$\Phi = \Phi_P + \phi$$ ← New scalar field

$$\nabla^2 \Phi_P = 4\pi G\rho$$

$$\nabla \cdot \left[\mu_I\left(\frac{|\nabla\phi|}{a_0}\right)\nabla\phi\right] = 4\pi G\rho$$

Original AQUAL theory

Type-2

$$\nabla^2 \phi = \nabla \cdot \left[\mu_{II}\left(\frac{|\nabla\Phi_P|}{a_0}\right)\nabla\Phi_P\right]$$

Quasi-linear MOND

Type-3

$$\nabla \cdot \left[\mu_{III}\left(\frac{|\nabla\Phi|}{a_0}\right)\nabla\Phi\right] = 4\pi G\rho$$

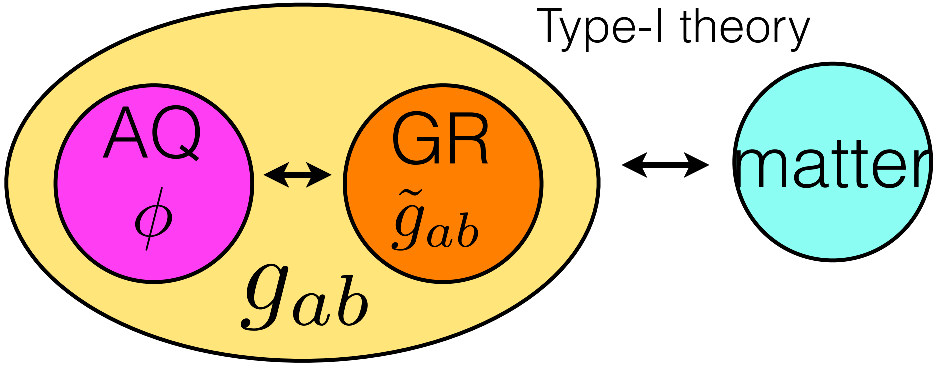
No known examples

Type-S: use of screening mechanism

Relativistic AQUAL

Bekenstein & Milgrom (1984)

Type-I theory



$$g_{ab} = e^{2\phi}\tilde{g}_{ab}$$

$$\Phi = \Phi_P + \phi$$

Define $X = \frac{1}{C}(\tilde{\nabla}\phi)^2$

$$S[g,\phi] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[\phi R - f(X)] + S_m$$

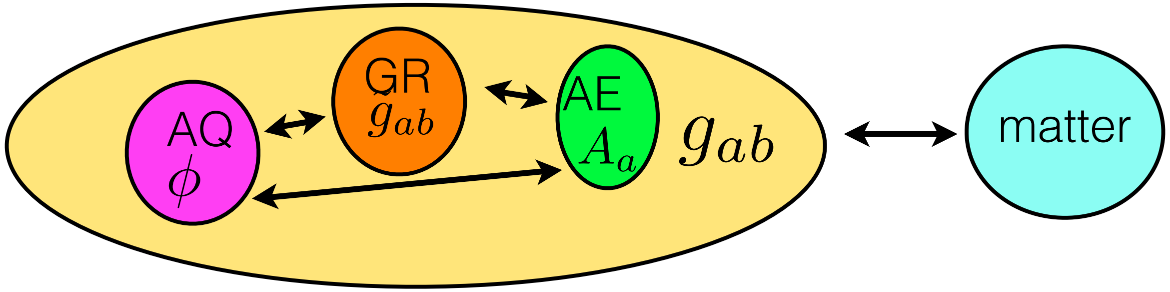
(subset of Horndeski)

(non-analytic) Kinetic term: $f(X)$ → X Brans-Dicke; → $X^{3/2}$ "MOND"

MOND function: $\mu \sim \frac{df}{dX}$

Ruled out: light insensitive to scalar

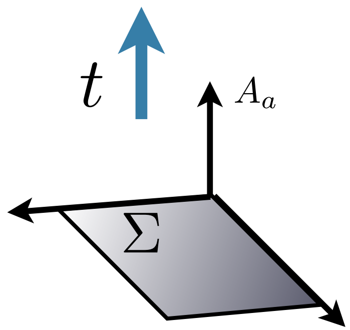
Tensor-Vector-Scalar (TeVeS) theory



Bekenstein (2004)

generalisation: CS (2007)

Preferred frame: $\tilde{g}^{ab} A_a A_b = -1$ Sanders (1997)



$$g_{ab} = -e^{2\phi} A_a A_b + e^{-2\phi} \left(\tilde{g}_{ab} + A_a A_b\right)$$

temporally stretched | spatially squashed

$$S_T[\tilde{g}, \phi, A] = \quad S_A[\tilde{g}, \phi] - \frac{1}{16\pi G} \int d^4x \sqrt{-\tilde{g}} \left[K^{abcd} \tilde{\nabla}_a A_b \, \tilde{\nabla}_c A_d - \lambda (A^a A_a + 1) \right]$$

RAQUAL + Einstein-Aether

Scalar: 1 free function

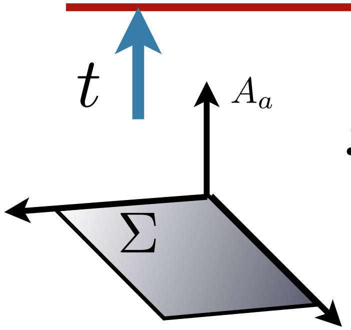
μ_0 ℓ_B

Vector: K

- Type-1 MOND in weak-field limit
- Lensing as if DM was present
- Large acceleration limit not GR but Einstein-Aether

TeVeS and caustics

Contaldi, Wiseman & Withers



$$g_{\mu\nu} = -A_\mu A_\nu + h_{\mu\nu}$$

$$B_{\mu\nu} = \nabla_\mu A_\nu$$

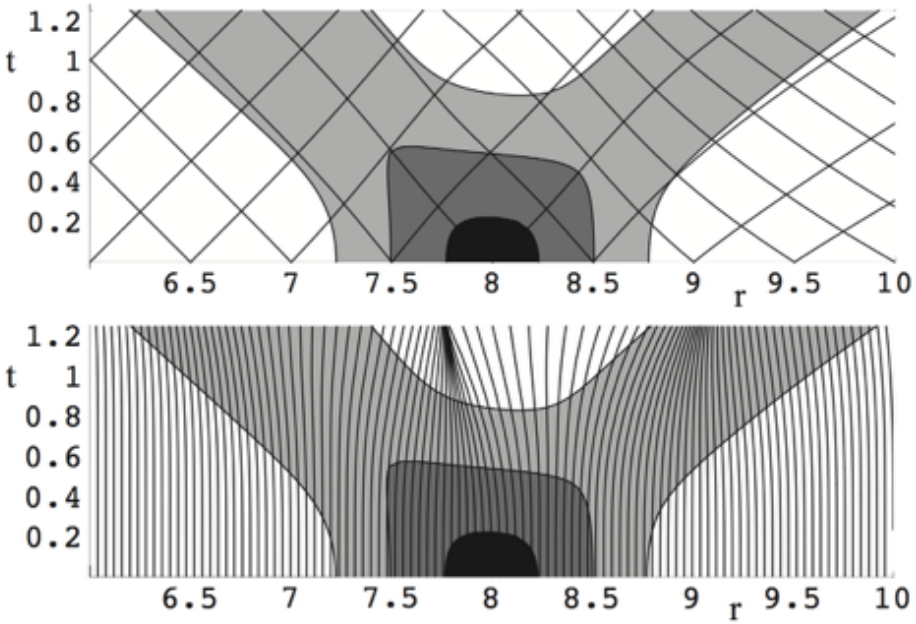
Expansion $\theta = h^{\mu\nu} B_{\mu\nu}$

Shear $\sigma_{\mu\nu} = \frac{1}{2} B_{(\mu\nu)} - \frac{1}{3}\theta h_{\mu\nu}$

Rotation $\omega_{\mu\nu} = \frac{1}{2} B_{[\mu\nu]}$

Raychaudhury $\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} - \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}A^\mu A^\nu$

Caustic formation $\frac{d\theta}{d\tau} \leq -\frac{1}{3}\theta^2$



Avoided: Generalised TeVeS

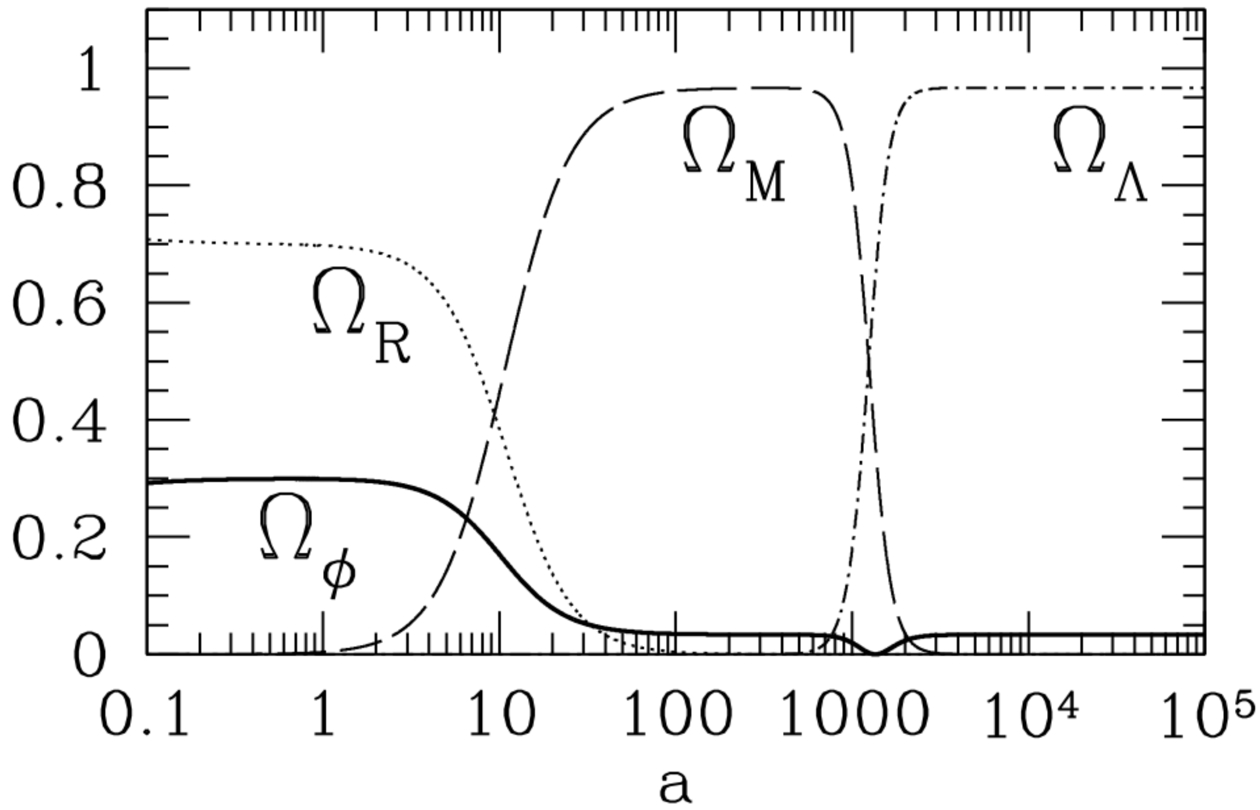
$$\omega_{\mu\nu} \neq 0$$

TeVeS cosmology

CS, Mota, Ferreira, Boehm (2005)

Scalar tracks dominant matter with equation of state w

$$\Omega_\phi = \frac{(1+3w)^2}{6\mu_0(1-w)^2}$$



Typically $\Omega_\phi < 10^{-3}$

Cosmological perturbations

CS, Mota, Ferreira, Boehm (2005)

CS (2005)

Metric

$$g_{00} = -a^2(1 + 2\Psi)$$

$$g_{ij} = a^2(1 - 2\Phi)\gamma_{ij}$$

Scalar

$$\phi = \bar{\phi} + \delta\phi$$

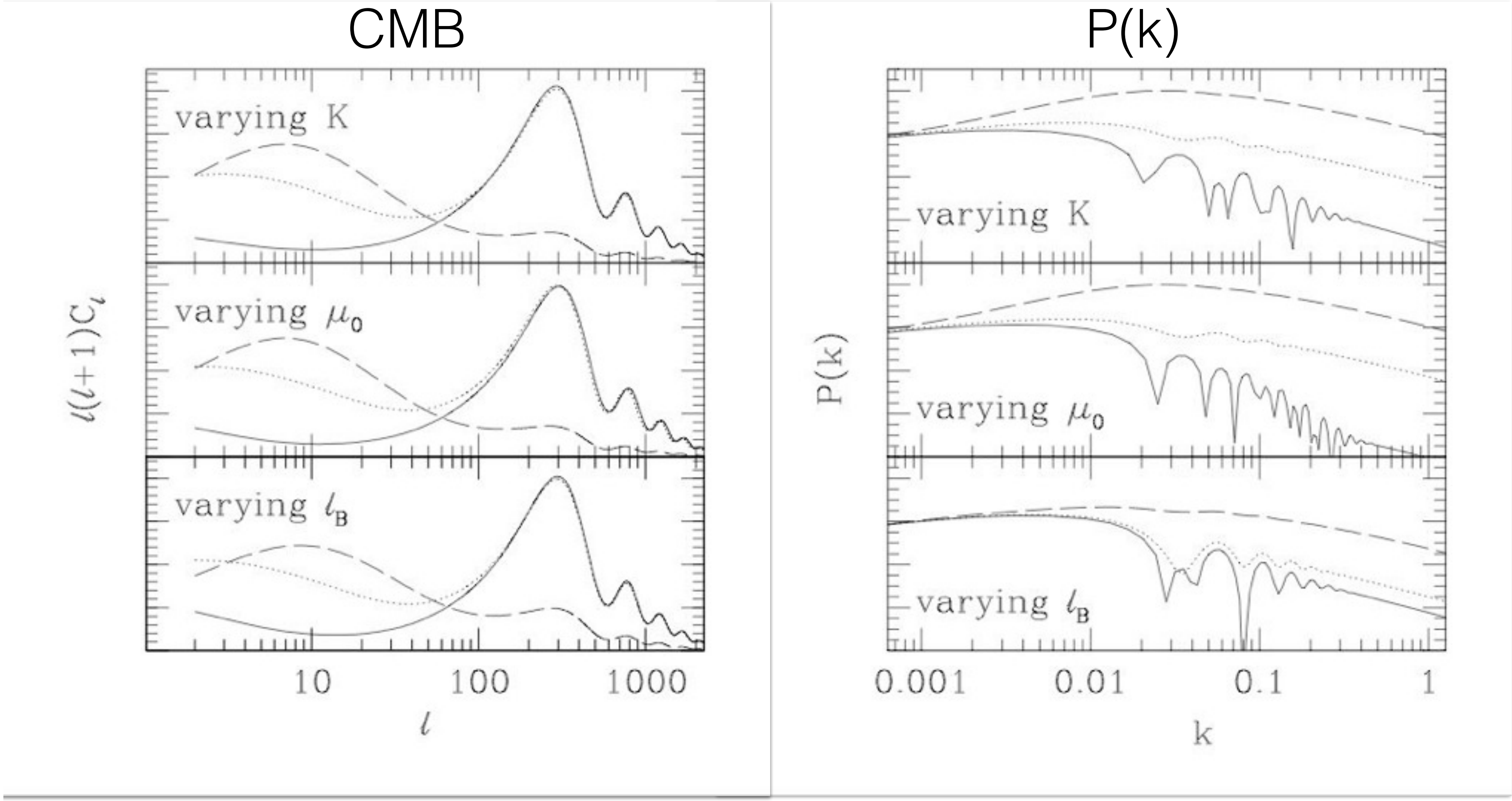
Vector

$$A_\mu = \left(1 + \Psi + \delta\phi, \vec{\nabla}_i \alpha\right)$$

} extra d.o.f $\delta\phi$ α

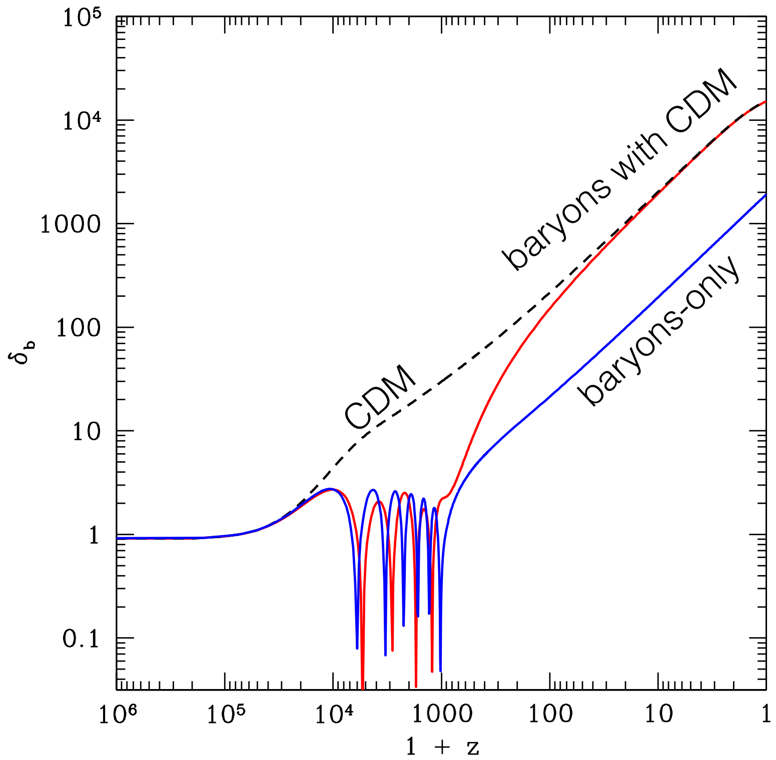
Observables

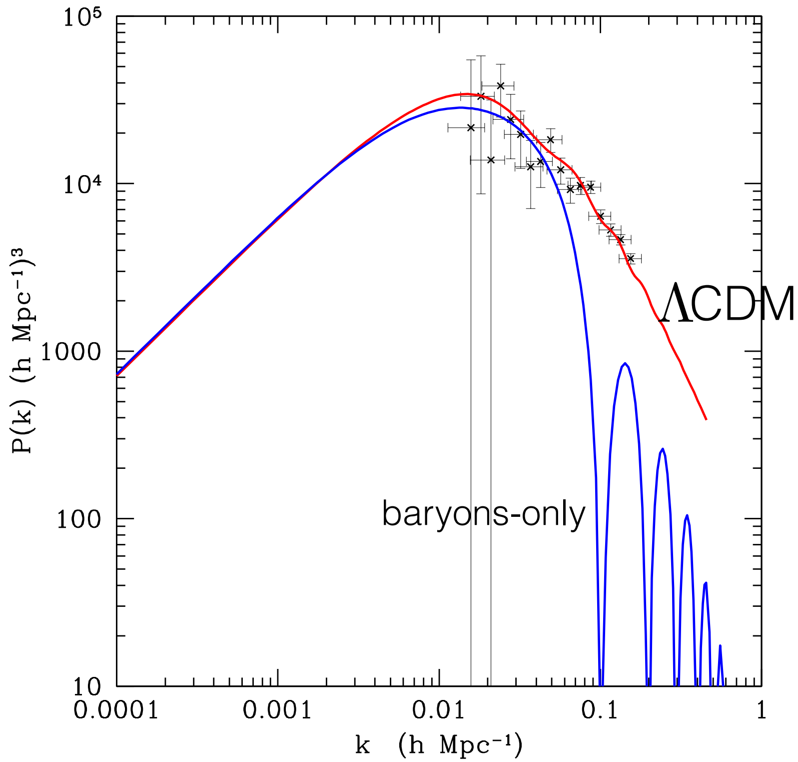
CS, Mota, Ferreira, Boehm (2005)



Large scale structure

Power spectrum $\langle\delta(\vec{k})\delta(\vec{k}')\rangle = (2\pi)^3 P(k)\delta^3(\vec{k}-\vec{k}')$

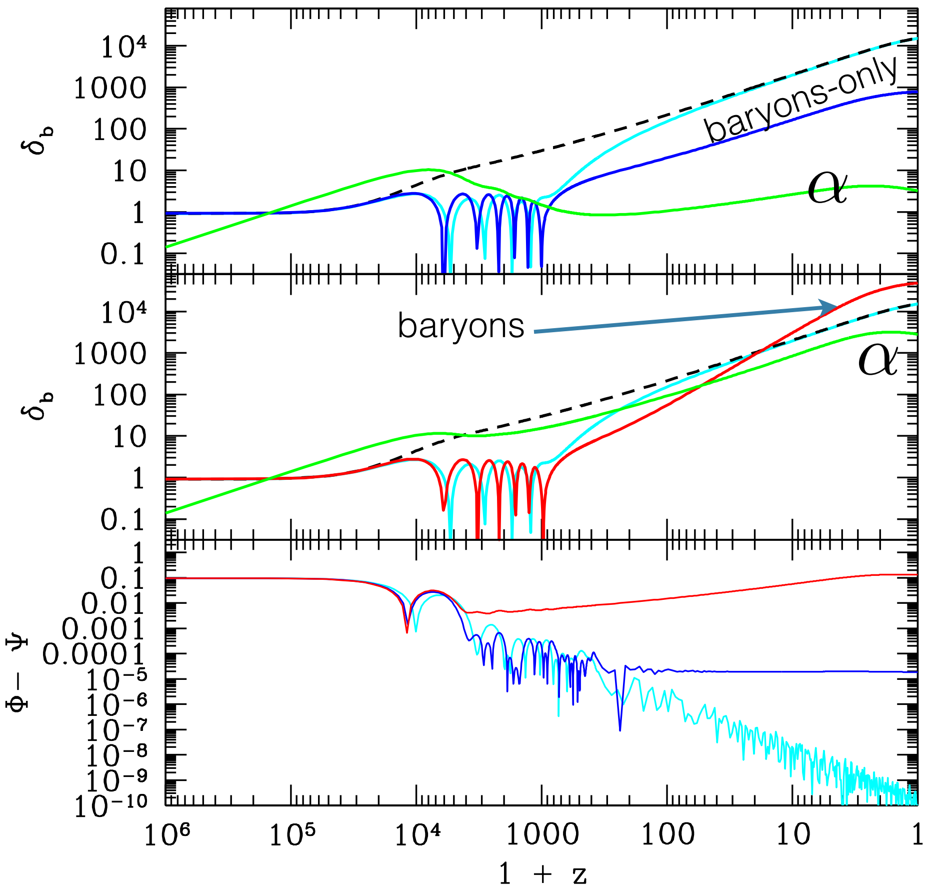


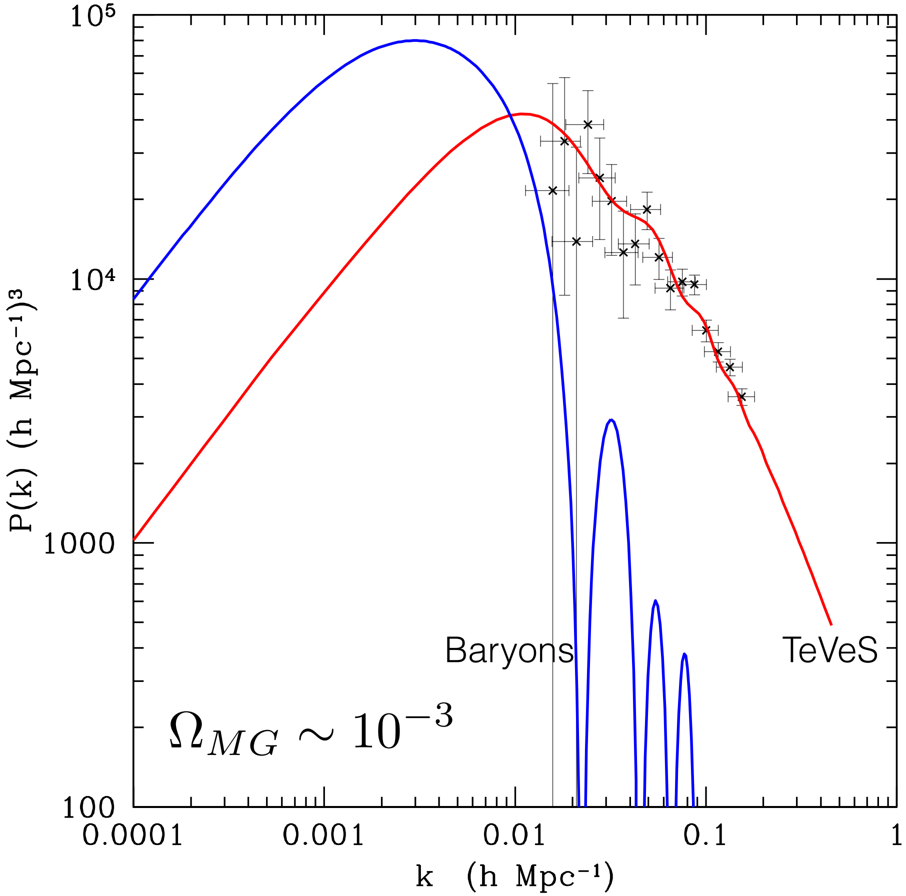


Large scale structure in TeVeS

(Skordis, Mota, Ferreira, Boehm, 2005)

(Dodelson & Liguori, 2006)





growing mode in α sources both δ_b and $\Phi - \Psi$

Growing mode in TeVeS

Scalar field perturbations irrelevant (Dodelson & Liguori, 2006)

Vector field perturbations: $A_\mu = \Big(1 + \Psi + \delta\phi, \vec{\nabla}_i \alpha\Big)$

$$\ddot{\alpha} + \frac{4}{\tau}\dot{\alpha} + \frac{2(1-\epsilon)}{\tau^2} = S[\Phi, \Psi]$$

where $\epsilon \sim 12 \ln\left[\frac{a}{5\times 10^{-5}}\right]\frac{1}{\mu_0 K}$

Soln: $\alpha \propto \tau^n \quad \Longrightarrow \quad n = \frac{1}{2}\left[-3 \pm \sqrt{1+8\epsilon}\right]$

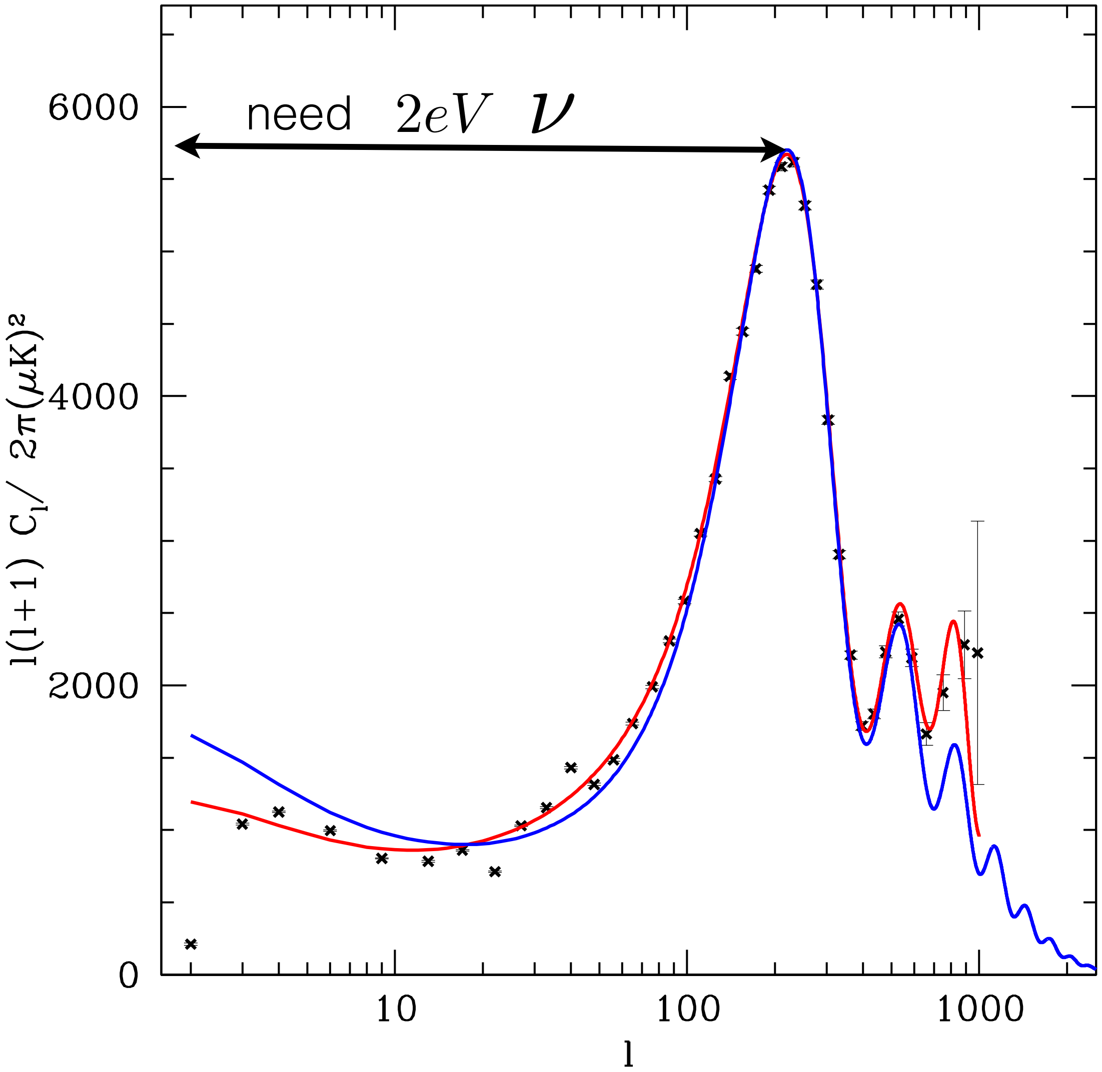
$n > 0$ for $\mu_0 K < 100$

Perturbed Einstein eq. (ignore scalar)

$$2\vec{\nabla}^2\Phi = 3\frac{\dot{a}^2}{a^2}\Delta_b^{(com)} - e^{-4\phi_i} K \vec{\nabla}^2\left[\dot{\alpha} + \frac{\dot{a}}{a}\alpha - \Psi\right]$$

Cosmic Microwave Background

(Skordis, Mota, Ferreira, Boehm, 2005)



within current tritium-β limits

will be probed by Katrin exp.

Not done:

- Full MCMC
- General initial conditions

see CMB talk tomorrow

TeVeS in a single frame

Zlosnik, Ferreira & Starkman (2006)

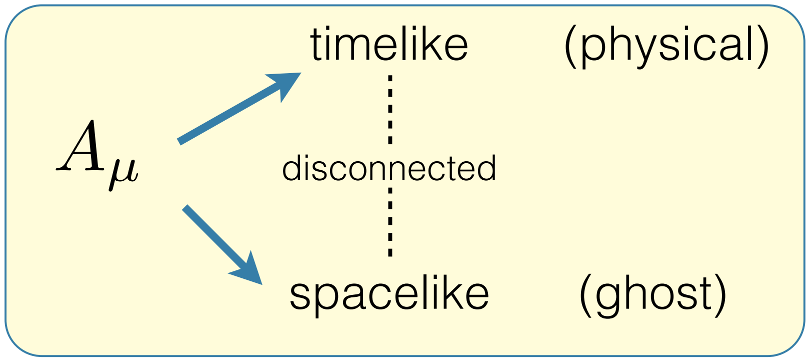
Express scalar field in terms of matter frame metric and vector field (use the disformal transformation)

$$e^{-2\phi} = -\tilde{g}^{ab} A_a A_b = -A^2$$

Eliminate both Einstein frame metric and scalar field from the action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R + K^{\mu\nu\alpha\beta} \nabla_\mu A_\alpha \nabla_\nu A_\beta \right) + S_m[g]$$

$$K^{\mu\nu\alpha\beta} \sim \frac{1}{A^2} \quad \text{or} \quad \frac{1}{A^4} \quad \text{etc}$$



Generalized Einstein-Aether

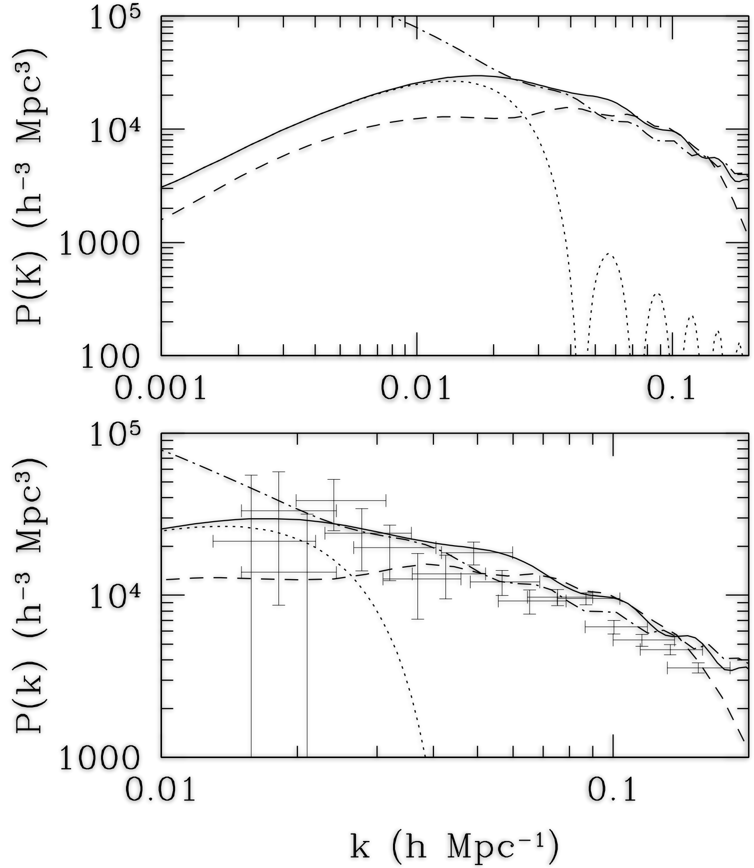
Zlosnik, Ferreira & Starkman (2006,2007)

Remove 1 dof

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\left[R + \mathcal{F}(K) - 2\lambda\left(A^\mu A_\mu + 1\right)\right] + S_m[g]$$

$$K = K^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta$$

- Similar phenomenology to TeVeS (e.g. MOND, LSS)
- F: may also provide for acceleration
- CMB, stability issues unknown



Strong-field MOND

Skordis & Zlosnik (2011)

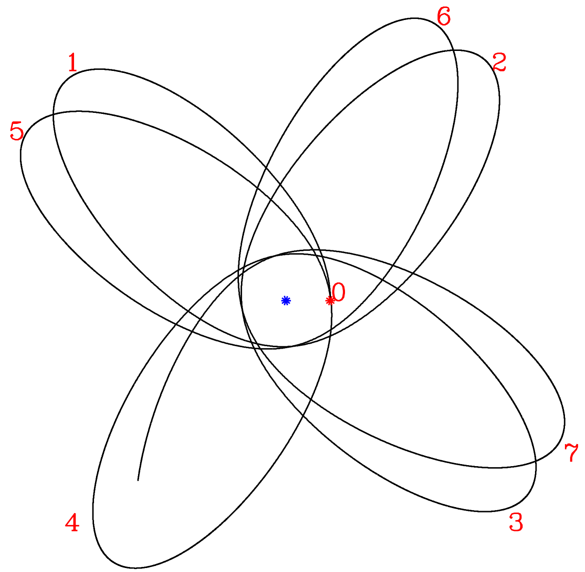
$+2c$ for RAQUAL

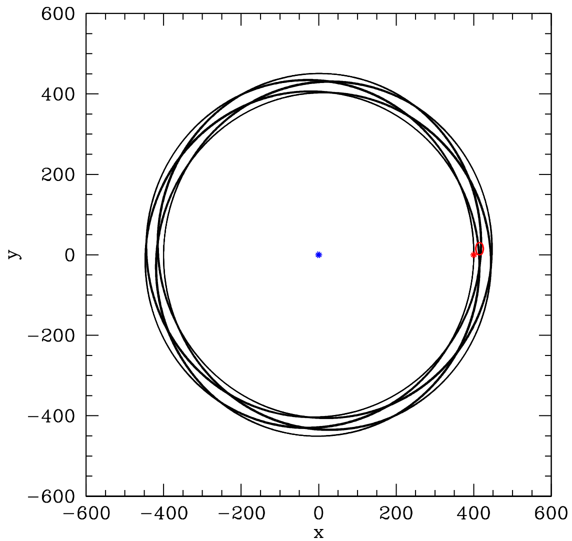
TeVeS $$ds^2 = -\left(\frac{r}{r_0}\right)^{2c}\left(1+\frac{(2+c)\Phi_1}{r}+\dots\right)dt^2 + \left(\frac{r}{r_0}\right)^{-2c}\left(1-\frac{(2+c)\Phi_1}{r}+\dots\right)dL^2$$

$$c = \sqrt{GMa_0}$$

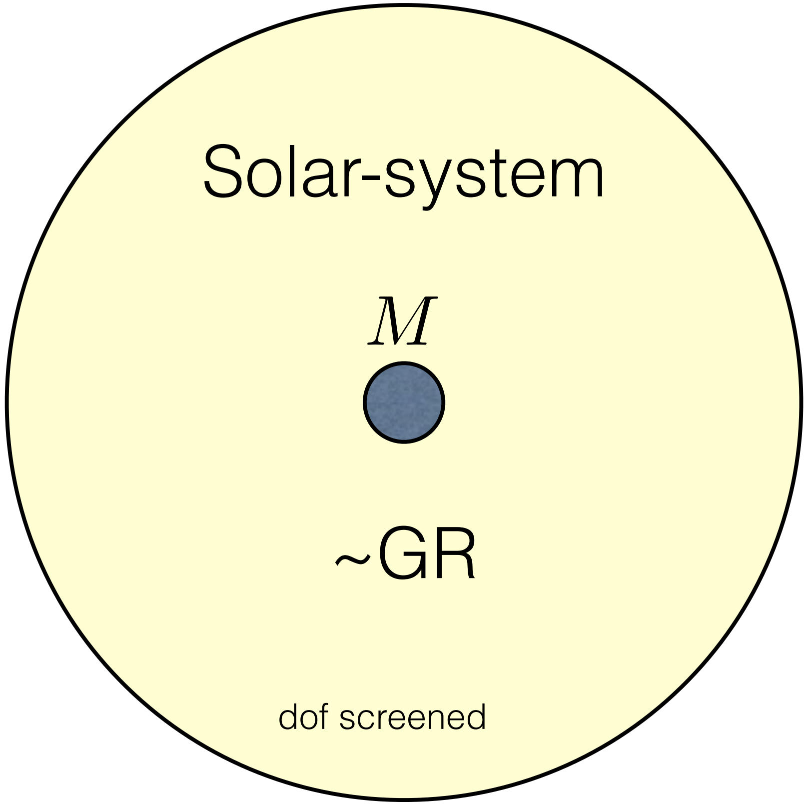
- Non-singular: all curvature invariants vanish at infinity
- Deficit solid angle: $1-c$, like a Barriola-Vilenkin global monopole
- Total confinement:
  - -Escape velocity is the speed of light
  - -Photons can only reach infinity at the expense of loosing all their energy

No particle, massless or massive, may escape to infinity with non-zero energy





Screening mechanisms



- Chameleon
- Symmetron
- Kinetic

Cosmology

non-GR

Kinetic screening

Realization of the Vainshtein mechanism in a number of theories

Derivatives of ϕ become large near massive sources

$$L_\chi \sim -\frac{1}{2}(\tilde{\nabla}\chi)^2 - K[\chi, \tilde{\nabla}\chi] \;\; L_{NL}[\chi, \tilde{\nabla}\chi, \tilde{\nabla}\tilde{\nabla}\chi]$$

standard ⟷ non-linear

relative strength

Galileons in 4D.

cubic: $L^{(3)}_{NL} = \tilde{\Box}\chi$

quartic: $L^{(4)}_{NL} = (\tilde{\Box}\chi)^2 - \tilde{\nabla}^\mu\tilde{\nabla}_\nu\chi\tilde{\nabla}^\nu\tilde{\nabla}_\mu\chi$

quintic: $L^{(5)}_{NL} = (\tilde{\Box}\chi)L^{(4)}_{NL} - 2(\tilde{\Box}\chi)\tilde{\nabla}^\mu\tilde{\nabla}_\nu\chi\tilde{\nabla}^\nu\tilde{\nabla}_\mu\chi + 2\tilde{\nabla}^\rho\tilde{\nabla}_\nu\chi\tilde{\nabla}^\nu\tilde{\nabla}_\mu\chi\tilde{\nabla}^\mu\tilde{\nabla}_\rho\chi$

$KL_{NL} >> X = -\frac{1}{2}(\nabla\phi)^2 \quad \Rightarrow$ screening

gTeVeS: TeVeS with Galileon k-mouflage

Babichev, Deffayet, Esposito-Farese (2011)

$$S_\phi = -\frac{1}{8\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\epsilon_c X + \frac{2}{3\tilde{a}_0} X\sqrt{|X|} + \frac{2k_{\rm gal}}{3}\tilde{\epsilon}^{\alpha\beta\gamma\delta}\tilde{\epsilon}^{\mu\nu\rho\sigma}\tilde{\nabla}_\alpha\phi\tilde{\nabla}_\mu\phi\tilde{\nabla}_\nu\tilde{\nabla}_\beta\phi\tilde{R}_{\gamma\delta\rho\sigma}\right)$$

Canonical — MOND (RAQUAL) — Galileon

Spherical symmetry: $\frac{d\phi}{dr} = \left(\sqrt{\frac{8k_{\rm gal}}{r^2} + \frac{r^2}{G_N M a_0} + \left(\frac{\epsilon_c r^2}{2G_N M}\right)^2} + \frac{\epsilon_c r^2}{2G_N M}\right)^{-1}$

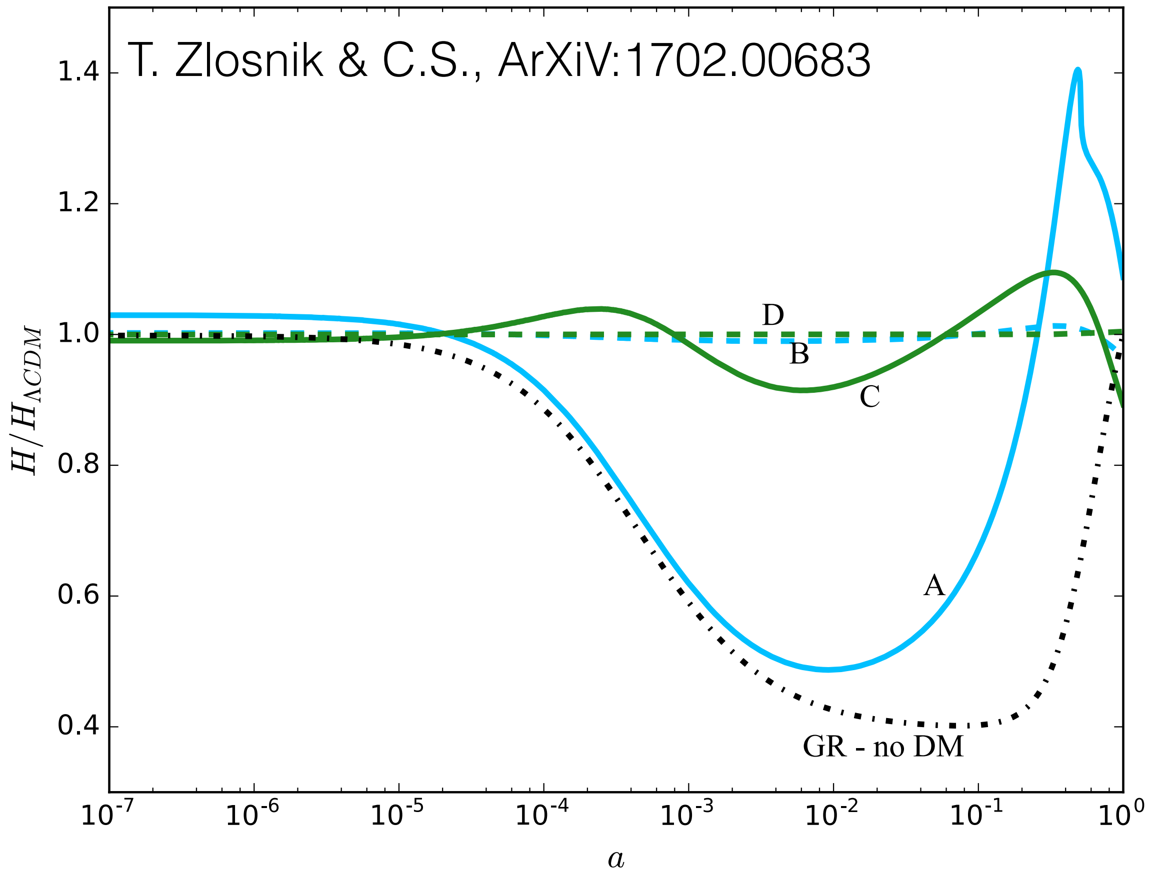
$r_V = (8k_{\rm gal} G M a_0)^{1/4}$ $\qquad$ $r_M = \frac{2}{\epsilon_c}\sqrt{\frac{GM}{a_0}}$

GR	MOND	Newton
(strongly-coupled galileon)		(with rescaled G)
$\phi \sim \frac{1}{\sqrt{k_{\rm gal}}} r^2$	$\phi \sim \sqrt{GMa_0}\ln r$	$\phi \sim -\frac{GM}{\epsilon_c r}$

FRW cosmology: proximity to ΛCDM

Minimize

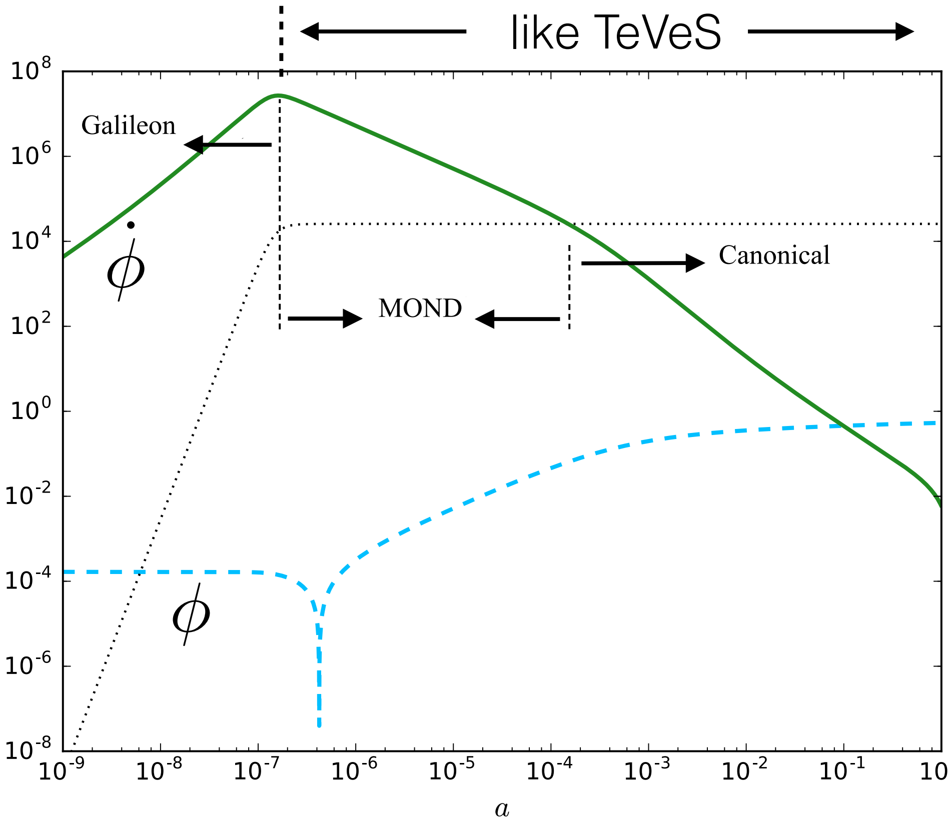
$$\mathcal{S} \equiv \frac{\int_{a_i}^{a_n} \left(\frac{H - H_{\Lambda\rm CDM}}{H_{\Lambda\rm CDM}} \right)^2 d\ln a}{\int_{a_i}^{a_n} d\ln a}$$

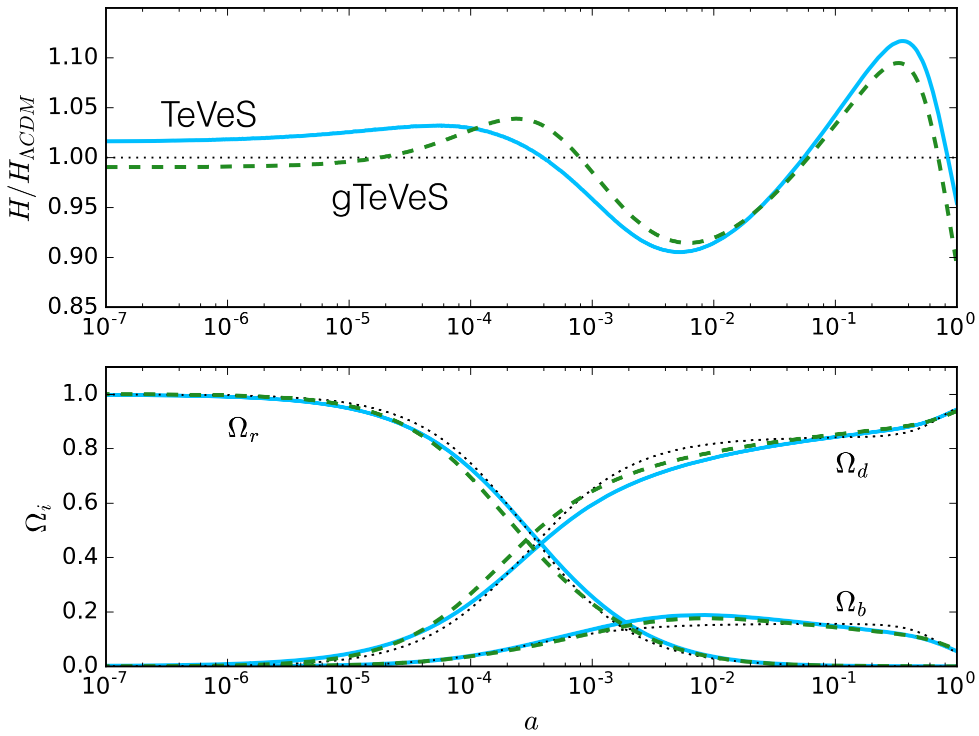


		$\mathcal{S}$	ϵ_c	$1/\tilde{a}_0$	$8k_{\rm gal}$	Λ	ρ_c/ρ_b	ϕ_i	y_i	$G_C/G-1$
restricted, no DM	A	7.0×10^{-2}	4.7×10^{-2}	5.4	1.6×10^{-24}	2.3	0.0	6.0×10^{-2}	1.2×10^{-8}	6.0×10^{-2}
restricted, with DM	B	3.9×10^{-5}	1.6×10^{-13}	5.4	1.0×10^{-25}	1.6	5.2	1.4	5.6×10^{-9}	4.8×10^{-3}
unrestricted, no DM	C	1.7×10^{-3}	2.7×10^{1}	1.7×10^{-4}	-7.2×10^{-39}	1.3×10^{-1}	0.0	8.9×10^{-3}	-3.1×10^{-5}	-1.9×10^{-2}
unrestricted, with DM	D	9.9×10^{-7}	2.3×10^{13}	8.0×10^{-1}	-6.7×10^{-35}	2.1	5.4	2.4×10^{-5}	-1.1×10^{-8}	-2.1×10^{-4}

Cosmological evolution

T. Zlosnik & C.S., ArXiV:1702.00683





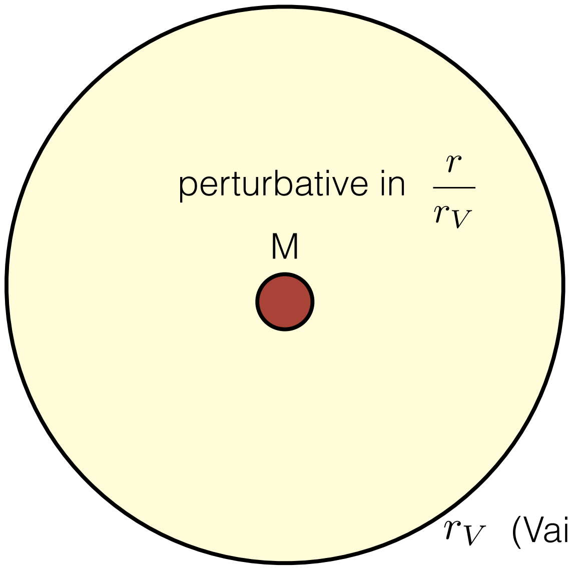
Parametrised Post-Newtonian-Vainshteinian formalism

A. Avilez, A. Padilla, P. Saffin and C.S., JCAP (2015)

New dof give corrections to the PPN potentials, but…two scales -> two regimes

two-order expansion:

$$h_{00} = \sum_{n=1}^{\infty} \sum_{m=0}^{\pm\infty} h_{00}^{(2n,m)}$$



Primary order: v (as in PPN)

Secondary order: α The (only) combination of the Schwarzschild radius and the Vainshtein radius which is independent of the source mass

- powers outside + powers inside

Cubic Galileon and the Vainshtein mechanism

$$S[g,\phi] = S_{BD} + \frac{M_p}{8\Lambda}\int d^4x\sqrt{-g}\,\frac{(\nabla\phi)^2}{\phi^3}\Box\phi + S_m(g)$$ (part of Horndeski)

↑

Strong coupling scale

Galileon parameter $\alpha = \frac{M_p}{\Lambda^3}$

Vainshtein radius $r_V = \frac{1}{\Lambda}\left(\frac{M}{M_p}\right)^{1/3}$

$\Big\}$ $\alpha = \frac{r_V^3}{r_s}$

cosmological regime $\alpha \to 0$

local regime $\alpha \to \infty$

Vainshtein mechanism for Cubic Galileon

Conformal relation $g_{\mu\nu} = e^{2\chi}\tilde{g}_{\mu\nu}$ → $h_{00} = \tilde{h}_{00} - 2\chi = \frac{2GM}{r} - 2\chi$

Point source on Minkowski $\frac{2\omega+3}{r^2}\frac{d}{dr}\left[r^2\chi'\right] + \frac{\alpha}{r^2}\frac{d}{dr}\left[r{\chi'}^2\right] = GM\frac{\delta(r)}{r^2}$

↓

$$\chi' = \frac{2\omega+3}{2\alpha}r\left[-1+\sqrt{1+\frac{4GM\alpha}{(2\omega+3)r^3}}\right]$$

$\alpha \to 0$

$$\chi = -\frac{GM}{(2\omega+3)r} + \frac{(GM)^2}{4(2\omega+3)^3}\frac{\alpha}{r^4} + \ldots$$

↓

$$h_{00} = \frac{\tilde{r}_s}{r}\left[1 - \frac{1}{64\pi(2\omega+3)^2(2+\omega)}\left(\frac{r_V}{r}\right)^3 + \ldots\right]$$

two limits

$\alpha \to \infty$

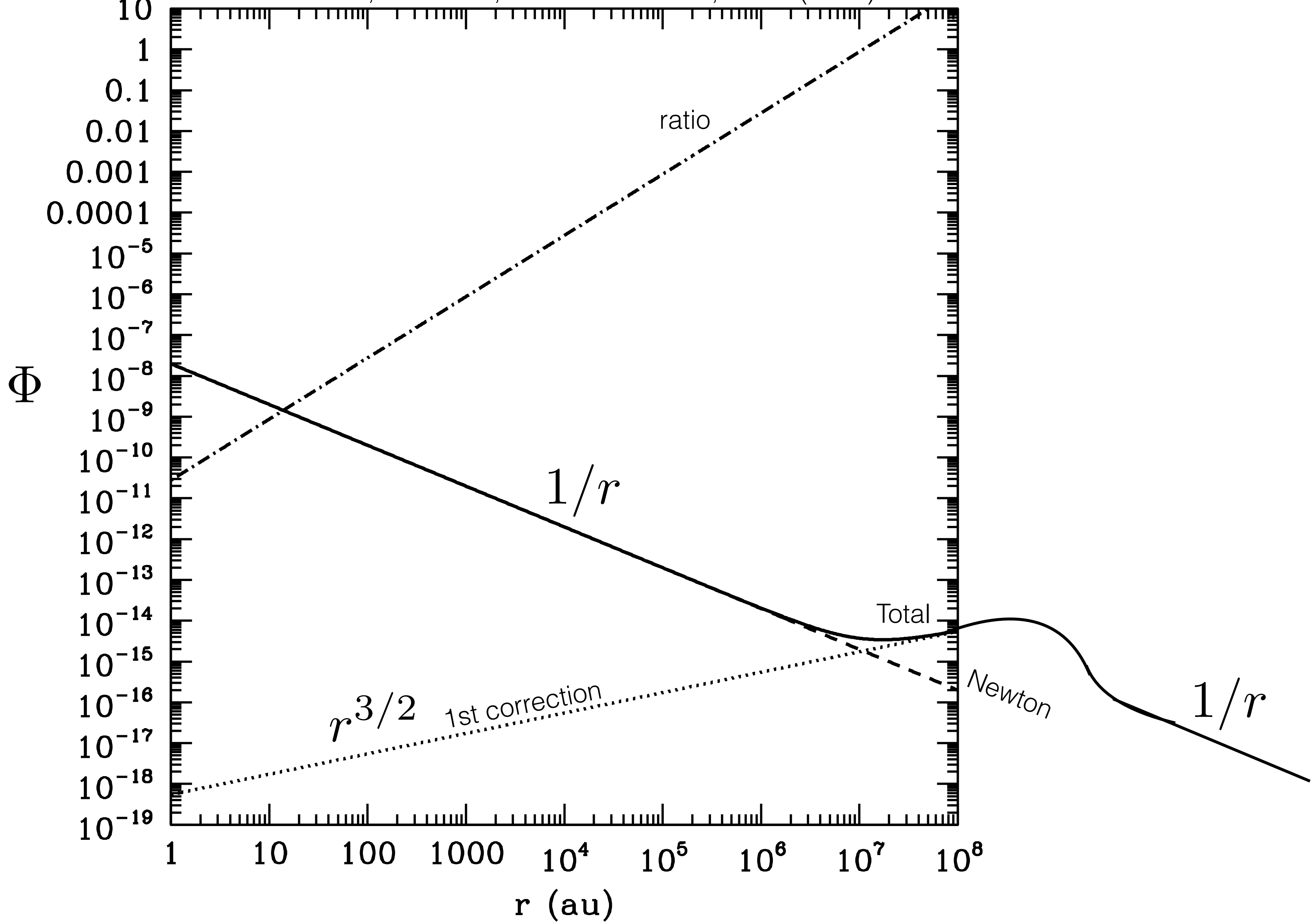
$$\chi = 2\frac{GM}{\alpha}r^{1/2} + \ldots$$

↓

$$h_{00} = \frac{r_s}{r}\left[1 - 4\sqrt{2\pi}\left(\frac{r}{r_V}\right)^{3/2} + \ldots\right]$$

Vainshtein corrections to Newton

A. Avilez, A. Padilla, P. Saffin and C.S., JCAP (2015)



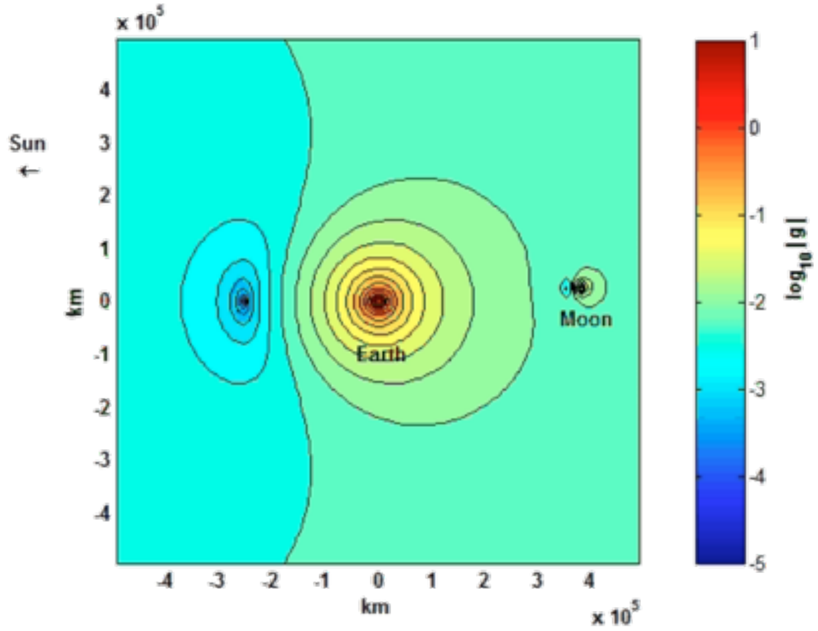
Modified gravity bubbles in the Solar System

Bekenstein & Magueijo (2006)

Newtonian Saddle Point: tidal stress vanishes

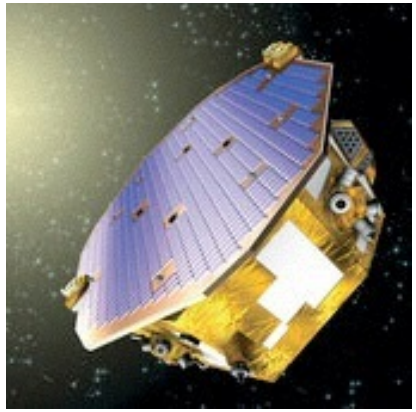
MONDian Saddle Point: $S_{ij} = \frac{\partial^2 \phi}{\partial_i \partial_j} \propto \frac{1}{\sqrt{r}}$

size of MONDian bubble $\sim 400km$



Lisa Pathfinder

- 2 test masses in near perfect gravitational free fall
- picometer sensitivity
- Primary mission: test technology for GW detection
- Secondary mission (proposed): test modified gravity (Trenkel et al.)

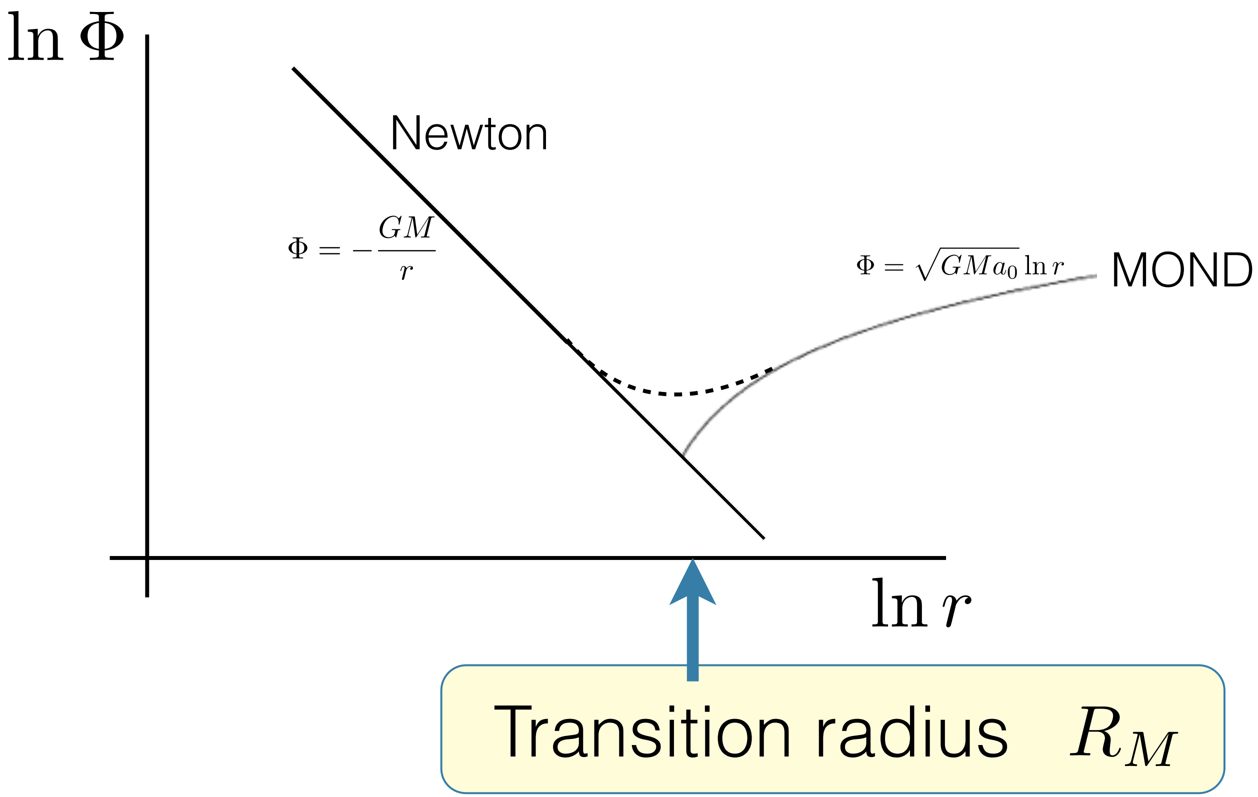


MOND signal expected at 25σ

Non-detection will rule out type I (e.g. TeVeS)
and most type II MONDs

Requirements for IR corrections to GR

- Gravity must become stronger than GR in the IR
- Lovelock: new degrees of freedom
- MOND: acceleration scale dictates potential
- Lorentz Violation: Lensing, cosmology, MOND Lagrangian
- Screening



Sponteneous metri-zation

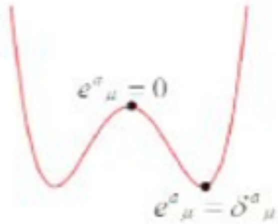
(Bañados 2007)

First-order GR

$$T^a = de^a + \omega^a_{bc} e^c$$
$$\epsilon_{abcd} R^{ab} e^c = 0$$
$$\epsilon_{abcd} T^a e^b = 0$$

} Trivially solved by $e^a = 0$

- Unbroken state of GR — Witten 1988
- Topology change — Horowitz 1990
- Sponteneous symmetry breaking — Giddings 1991



$$e^a = 0 \longrightarrow \begin{matrix} \omega^a_{bc} \\ R_{ab} \end{matrix}$$

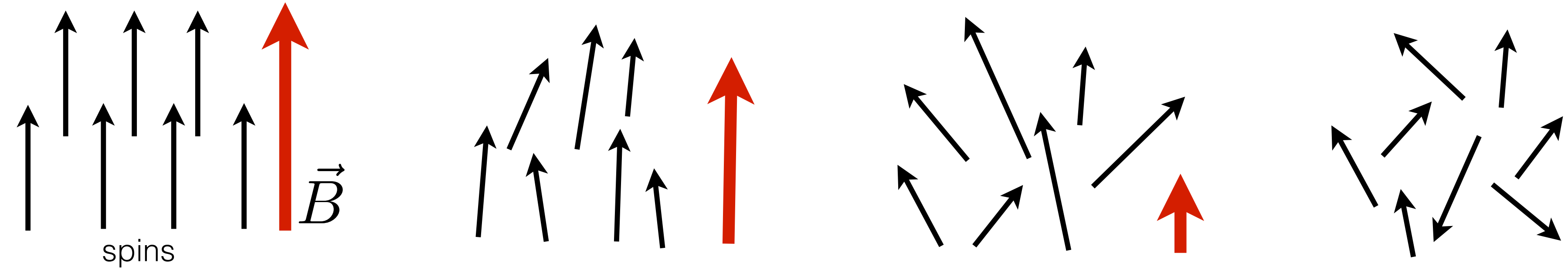
completely unconstrained

(and thus random!)

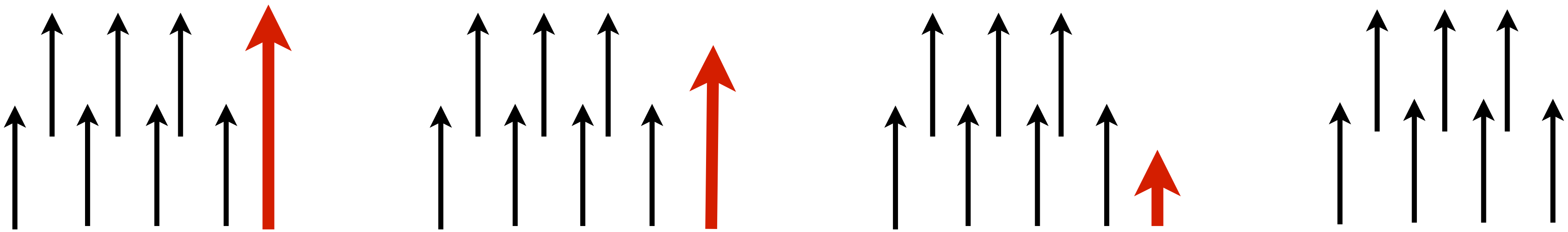
Spins in an external magnetic field

(Bañados 2007)

$T > T_c$: random



$T < T_c$ Remain ordered



Can we modify GR so that ω^c_{ab} does not become random when e^a is removed?

Parametric vanishing of the metric

$$g_{\mu\nu}[\alpha^I] \rightarrow 0 \qquad \text{as} \qquad \alpha^I \rightarrow 0$$

"metric-less connection" $$C^c_{ab} = \lim_{g \longrightarrow 0} \Gamma^c_{ab}$$

Curvature of C^c_{ab} $$R^{(0)}_{ab} = \partial_{[c} C^c_{a]b} - C^d_{c[d} C^c_{b]a} = \lim_{g \longrightarrow 0} R_{ab}$$

$$G^{(0)}_{ab} = \lim_{g \longrightarrow 0; T \longrightarrow 0} G_{ab}$$

Modified Einstein equations:

$$G_{ab} = 8\pi G T_{ab} + G^{(0)}_{ab}$$

Static-spherical symmetry

$$ds^2 = -A^2(r)dt^2 + B^2(r)dr^2 + C^2(r)d\Omega^2$$

Take limits:

$$A \to 0 \qquad \Rightarrow \frac{1}{B}\frac{dA}{dr} \to 0 \qquad \frac{1}{A}\frac{dA}{dr} \to \text{finite}$$

$$R^{(0)}_{tt} = 0 \qquad R^{(0)}_{rr} = q(r) \qquad R^{(0)}_{\theta\theta} = 1$$

Solve Einstein equations :

$$G_{tt} = 0 \qquad G_{rr} = q(r) \qquad G_{\theta\theta} = 1$$

$$g_{00} \approx -1 + 2\Phi = -1 + \frac{2Gm}{r} + C_0 \ln r$$

↑ MOND potential!

Cosmology

$$ds^2 = -N^2 dt^2 + a^2 d\vec{x}^2$$

$$a \rightarrow 0 \qquad N \rightarrow 0 \qquad \Rightarrow \qquad \frac{\dot{a}}{N} \rightarrow 0$$

but $\frac{\dot{a}}{a}$ $\frac{\dot{N}}{N}$ finite

$$R^{(0)}_{tt} = q(t) \qquad R^{(0)}_{ij} = 0$$

Solve Einstein equations: $3H^2 = q = \frac{C_0}{a^3}$

Dark matter!

Eddington-Born-Infeld gravity

(Banados 2009)

$$I[g_{\mu\nu}, C^{\alpha}_{\mu\nu}] = \frac{1}{16\pi G}\int d^4x\left[\sqrt{-g}R + \frac{2}{\alpha\ell^2}\sqrt{|g_{\mu\nu} - \ell^2 K_{\mu\nu}|}\right]$$

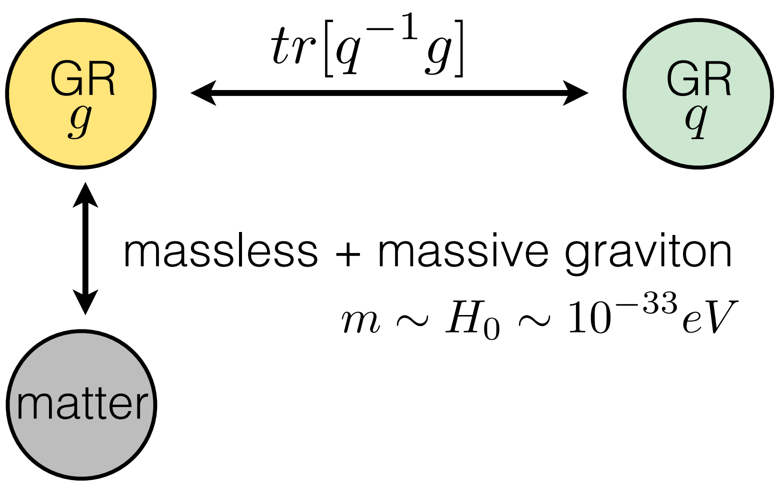
2nd connection

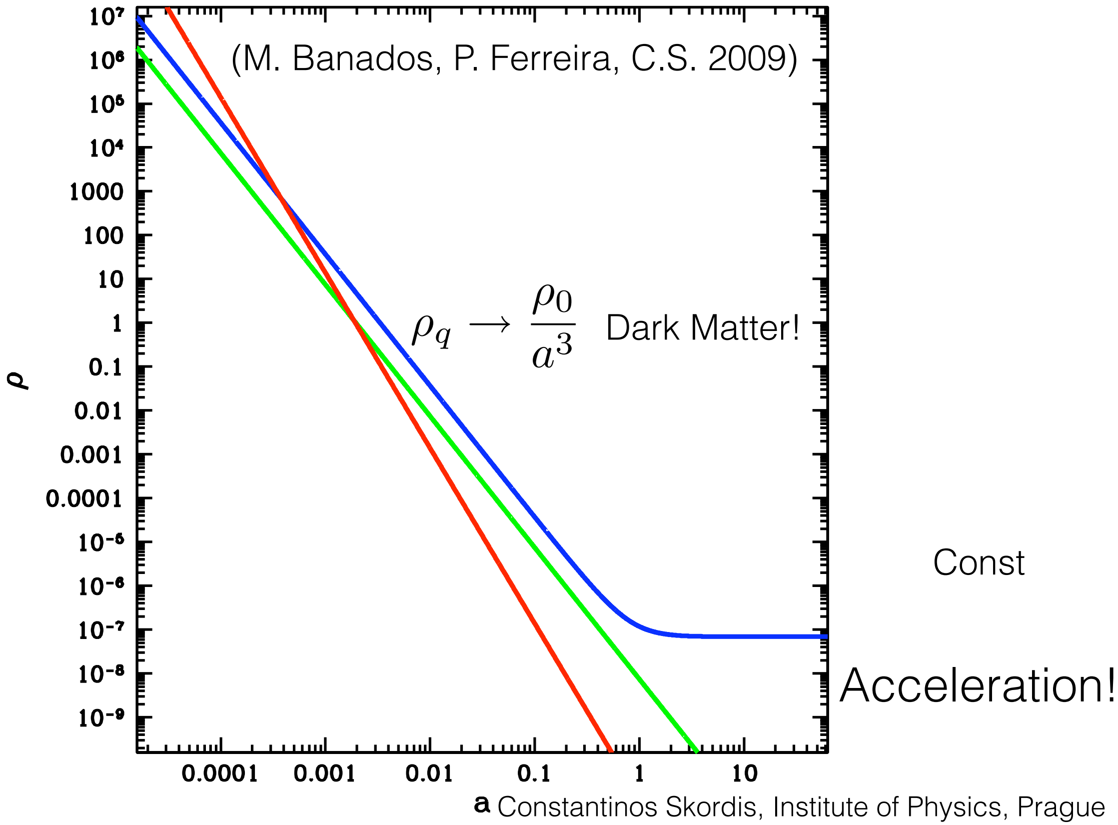
2 parameters

curvature of $C^{\alpha}_{\mu\nu}$

$C^{\alpha}_{\mu\nu}$ → $q_{\mu\nu}$ → bigravity (Isham, Salam, Strathdee 1971)

2nd metric

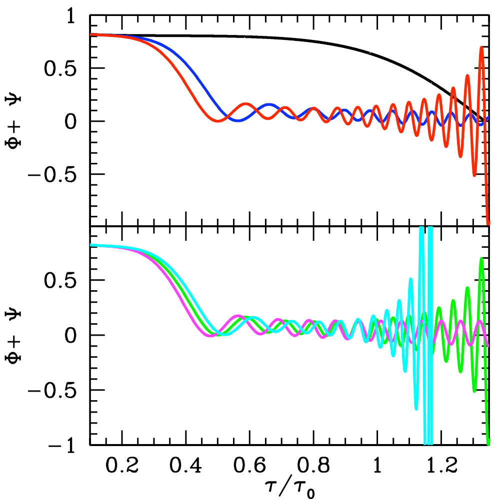




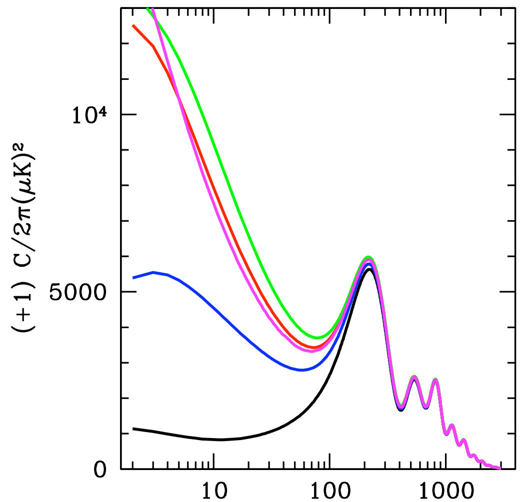
Perturbations

(M. Banados, P. Ferreira, C.S. 2009)

q metric has 4 new dof $\longrightarrow$ $\delta, \theta, \delta P, \sigma$



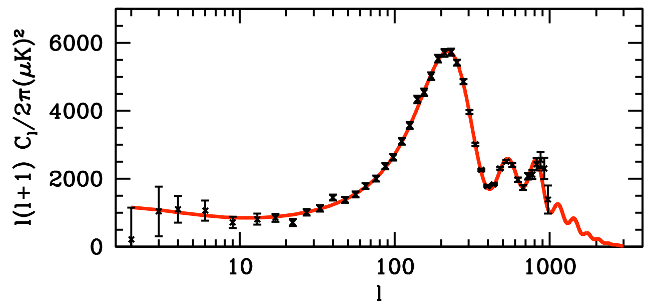
Bouleware-Deser ghost



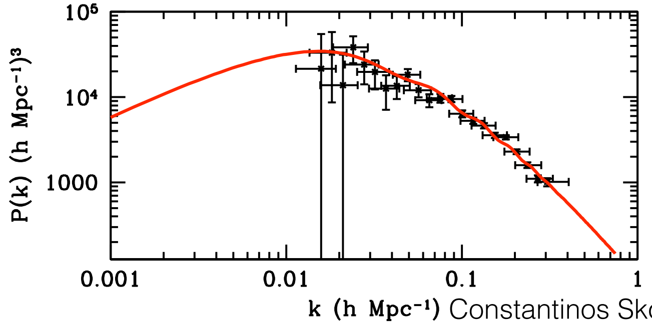
CDM-like

CDM

Instability removed by re-introducing bare



Indistinguishable from (but no particle DM)



Outlook

- General Relativity strongly constrained in large-curvature regime.

  Not so at low-curvatures: Dark Sector

- Relativistic Modified Newtonian Dynamics and variants: Structure formation OK. CMB still unclear (and not expected to work).

- Relativistic MOND: screening mechanisms + Lorentz violation.

- Other Dark Matter motivated gravitational theories — interesting but do not explain MOND relations.