Beyond Shell Model: radii, halo orbits and shell structure

Andrés P. Zuker

IPHC Institut Pluridisciplinaire Hubert Curien Strasbourg

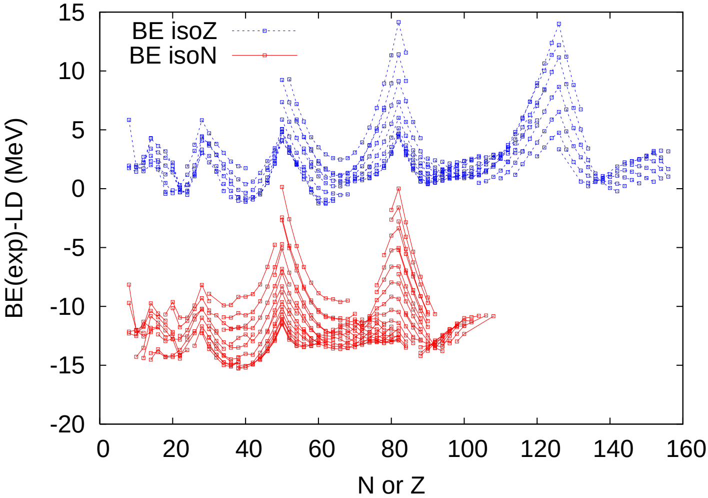
Orsay-Strasbourg June 2017

Preliminaries

The Shell Model is about solving a Schrödinger equation with suitable interactions. So far, Nuclear radii, binding energies and basic shell structure cannot be explained by the Shell Model. To "go beyond" means addressing the problem phenomenologically in ways to make it possible to discover basic mechanisms at play. Our starting point is the observation that nuclear radii and masses exhibit strong shell effects associated with "magic numbers" 6,14,28,50,82 and 126. Their origin is an open problem, difficult to pinpoint in the case of masses but subsumed by a single phenomenological term that brings down root mean square deviations with observed radii to 0.01 fermi. The origin of this term is unambiguously traced to the existence of huge "halo" orbits...

Skeleton: Frames(F) and Biblio[..]

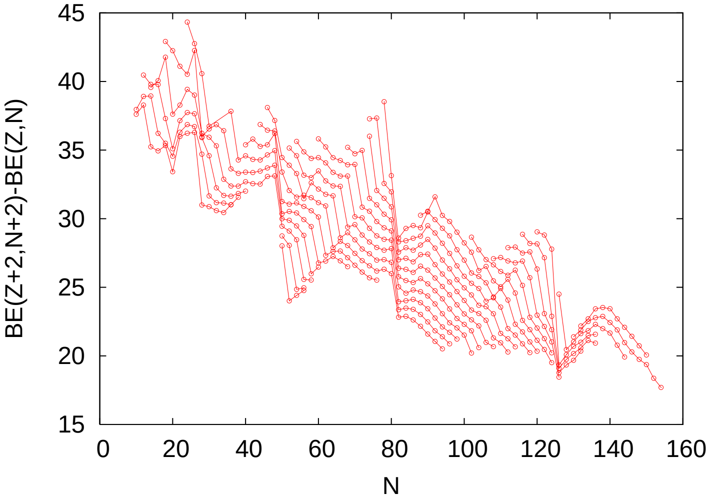
BE Shell Effects



$$\mathrm{LD} = 15.5A - 17.8A^{2/3} - 28.6\frac{4T(T+1)}{A} + 40.2\frac{4T(T+1)}{A^{4/3}} - \frac{.7Z(Z-1)}{A^{1/3}}.$$

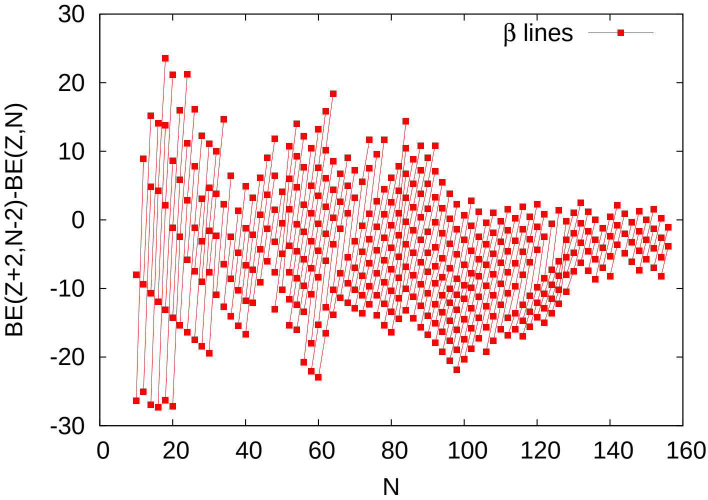
Alpha lines

Refine view of shell effects.
First at constant $t = N - Z$. Much stucture.



Beta lines

Now at constant A. No structure.



AM table

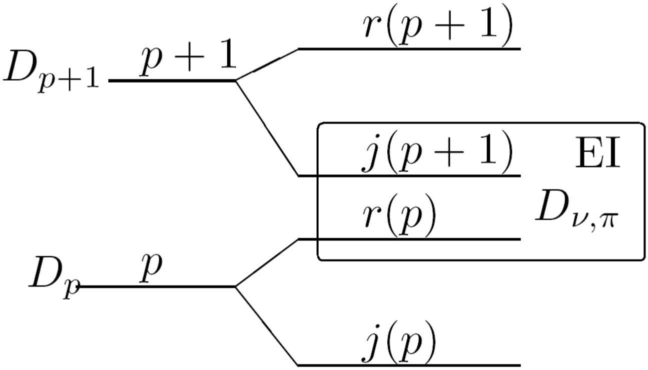
Get acquainted with Angeli Marinova table.

Table 1 (*continued*)

Z	el.	A	N	$\delta\langle r^2\rangle$ (fm^2)	$\Delta\delta\langle r^2\rangle$ (fm^2)	R (fm)	$\Delta_{tot}R$ (fm)	ΔR_{rel}
		68	38	0.131	0.002	3.9658	0.0014	0.0003
		70	40	0.286	0.003	3.9845	0.0019	0.0004
31	Ga	69	38			3.9973	0.0017	
		71	40			4.0118	0.0018	
32	Ge	70	38			4.0414	0.0012	
		72	40			4.0576	0.0013	
		73	41			4.0632	0.0014	
		74	42			4.0742	0.0012	
		76	44			4.0811	0.0012	
33	As	75	42			4.0968	0.0020	
34	Se	74	40			4.0700	0.0200	
		76	42			4.1395	0.0016	
		77	43			4.1395	0.0018	
		78	44			4.1406	0.0017	
		80	46			4.1400	0.0018	
		82	48			4.1400	0.0019	
35	Br	79	44			4.1629	0.0021	
		81	46			4.1599	0.0021	
36	Kr	72	36	−0.168	0.018	4.1635	0.0060	0.0022
		74	38	0.030	0.005	4.1870	0.0041	0.0006
		75	39	0.221	0.007	4.2097	0.0041	0.0008
		76	40	0.156	0.004	4.2020	0.0036	0.0005
		77	41	0.209	0.005	4.2082	0.0037	0.0006
		78	42	0.172	0.003	4.2038	0.0033	0.0004
		79	43	0.168	0.004	4.2034	0.0032	0.0005
		80	44	0.114	0.007	4.1970	0.0029	0.0008
		81	45	0.099	0.004	4.1952	0.0026	0.0005
		82	46	0.071	0.003	4.1919	0.0025	0.0004
		83	47	0.031	0.003	4.1871	0.0023	0.0004
		84	48	0.042	0.001	4.1884	0.0022	0.0001
		85	49	0.009	0.004	4.1846	0.0022	0.0004
		86	**50**	**0**	**0**	**4.1835**	**0.0021**	**0**
		87	51	0.125	0.003	4.1984	0.0027	0.0004
		88	52	0.282	0.004	4.2171	0.0043	0.0005
		89	53	0.379	0.004	4.2286	0.0054	0.0005
		90	54	0.495	0.010	4.2423	0.0069	0.0012
		91	55	0.597	0.006	4.2543	0.0081	0.0007
		92	56	0.751	0.005	4.2724	0.0099	0.0006
		93	57	0.811	0.004	4.2794	0.0107	0.0005
		94	58	0.989	0.004	4.3002	0.0129	0.0005

DZ and DZ* radii: HO and EI spaces

Refresh ideas about spaces



$D_{HO} = (p+1)(p+2) \quad D_r = p(p+1)$
$D_{EI} = (p+1)(p+2) + 2 = D_{\nu\pi}$
HO magic at N, Z=2, 8, 20, 40, 70, 112, 168
EI magic at N, Z=2, 6, 14, 28, 50, 82, 126

DZ and DZ*

$D_{HO} = (p+1)(p+2) \quad D_r = p(p+1) \quad D_{EI} = (p+1)(p+2)+2$
$S_x = m_x(D_x - m_x)/D_x^2, \;\; Q_x = m_x(D_{rx} - m_x/)D_x^2, \quad x = \nu$ or π

$$\mathrm{DZ} = \rho = \rho_0 + \mathcal{D} \tag{1}$$

$$\rho_0 = A^{1/3}\left(\rho_0 + \frac{\upsilon}{2}(\frac{t}{A})^2 + \frac{\zeta}{2}\frac{t_z}{A^{4/3}}\right)e^{(g/A)} \tag{2}$$

$$\mathcal{D} = (\lambda S_\pi S_\nu + \mu Q_\pi Q_\nu)A^{-1/3} \tag{3}$$

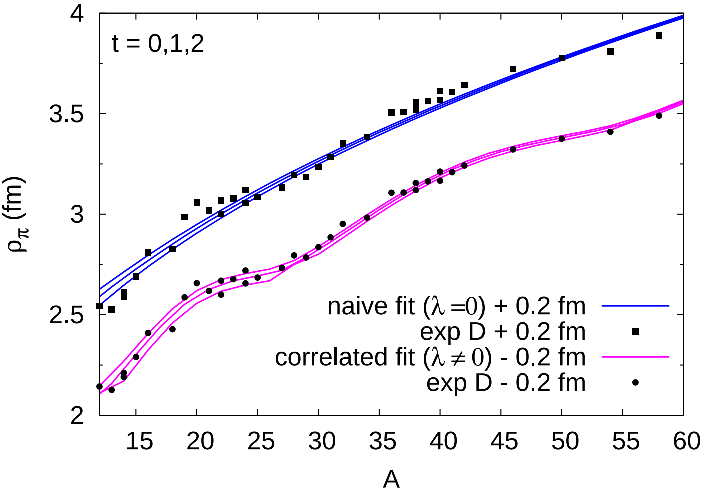
$$\mathrm{DZ}* = \rho_*^2 = \rho_{0*}^2 + \mathcal{D}* \tag{4}$$

$$\rho_{0*}^2 = A^{2/3}\left(\rho_0 + \frac{\tau}{2}\frac{t}{A^{4/3}} + \frac{\zeta}{2}\frac{t_z}{A^{4/3}}\right)e^{(g/A)} \tag{5}$$

$$\mathcal{D}* = \lambda(S_\pi + S_\nu)^2(1 + \xi\frac{e^{(g/A)}t_z}{2A^{4/3}}) + \mu(Q_\pi + Q_\nu)^2 \tag{6}$$

The ρ_0 and ρ_{0*}^2 are the equivalent of LD. Ignore ζ, ξ for proton radii.

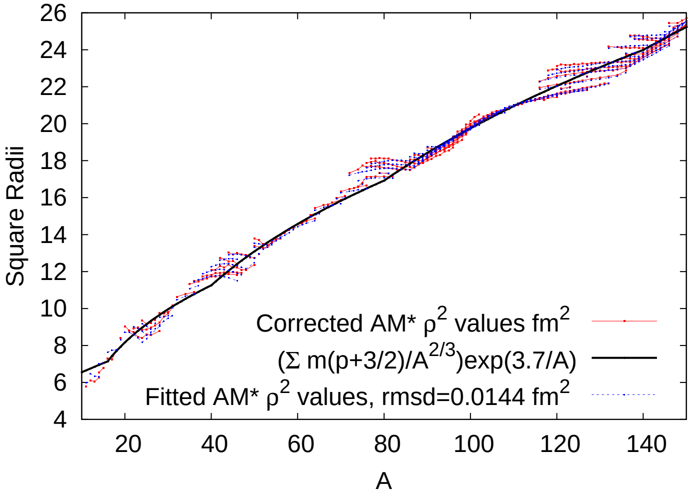
Influence of Duflo term



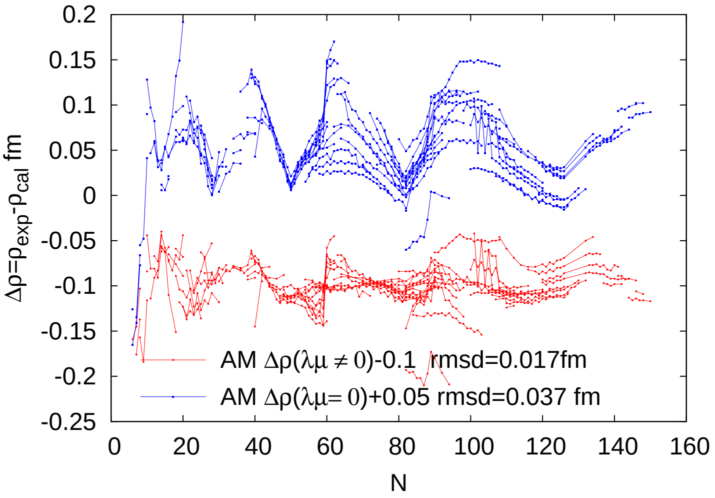
Global view of squared radii ρ^2

$\rho^2 = \frac{41.47}{\hbar\omega} \sum_i m_i(p_i + 3/2)/A \rightarrow (\sum_i m_i(p_i + 3/2)/A^{2/3}) \exp(3.7/A)$

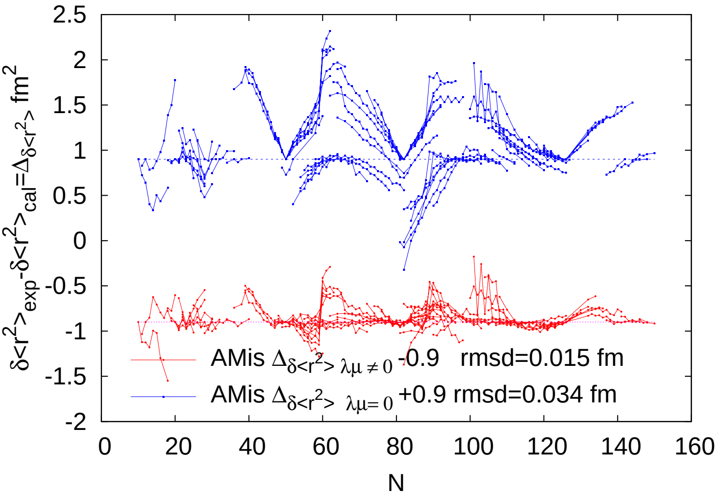
i.e., $\boldsymbol{\hbar\omega = 41.47/A^{1/3}}$ and $\exp(3.7/A)$ correction added.



Radial Shell effects for ρ

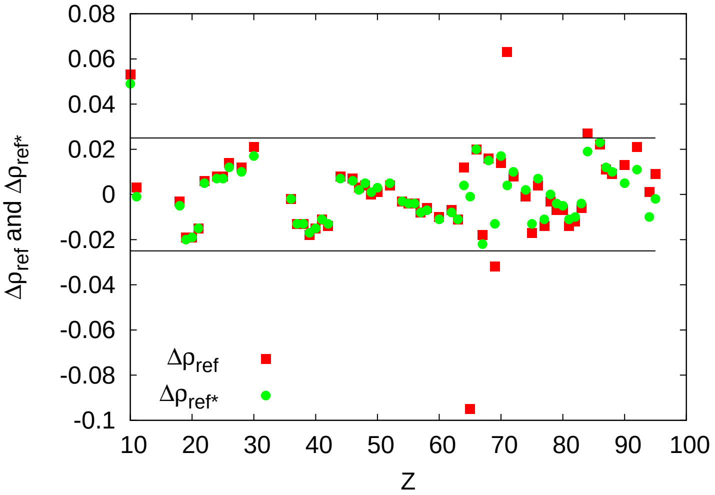


Radial Shell effects for δ_{r^2}



Toileting AM I

Fit reference states. rmsd=0.0142 ⟶ 0.0106 fm

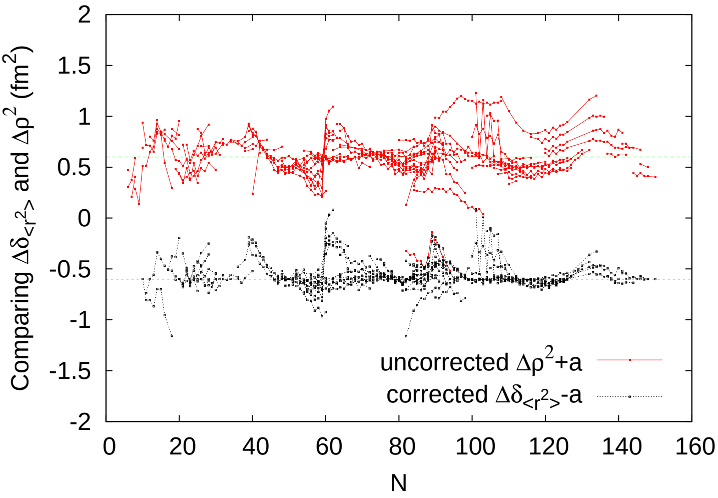


Toileting AM II

Table : Results of fits for sets AMref=Dr, AMref*=Dr*, 27=AM=D, AM*=D*, AMis=Dis, AMis*=Dis*). The reference values of the starred sets(*) have been corrected according to 65+0.1(0.15), 69+0.02(0.035), 71-0.06(0.034), 84 -0.010(0.176) where the Z number is followed by the correction in fm. In parenthesis the error attributed to the reference isotope in AM. Z_M is the maximum Z allowed in the fit, nd the number of species involved and iu the data set used.

i	ρ_0	g	λ	μ	τ	nd	iu	Z_M	rmsdf	rmsdv
1	0.90	3.70	2.80	8.00	-1.50	*	fixed coefficients			
2	0.90	4.01	2.46	5.46	-1.44	61	Dr	96	0.015	**0.0142**
3	0.90	4.13	2.41	8.37	-1.44	61	Dr*	96	0.013	**0.0106**
4	0.90	3.61	2.91	6.81	-1.56	876	D	96	0.017	0.0166
5	0.90	3.76	2.72	9.28	-1.63	876	D*	96	0.015	0.0144
6	0.90	3.61	2.93	6.76	-1.54	827	Dis	96	0.015	0.0154
7	0.90	3.85	2.68	9.45	-1.61	827	Dis*	96	0.013	0.0127
8	0.91	3.53	2.37	0.00	-1.30	107	D	30	*	**0.025**
9	0.89	4.69	1.89	0.00	-1.24	80	Dis	30	*	**0.015**

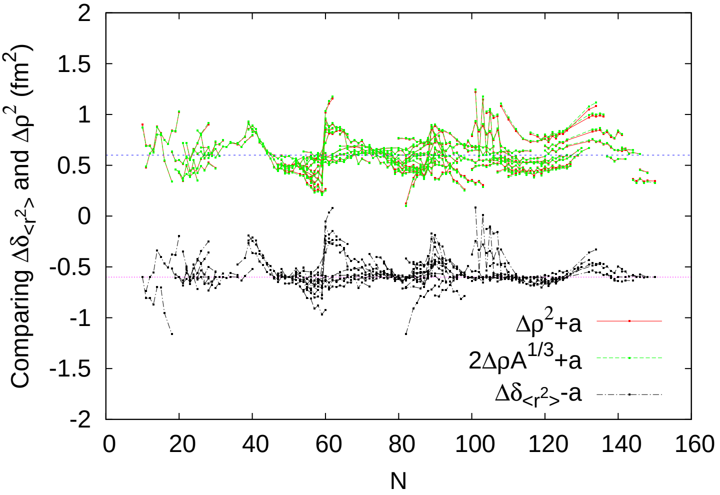
Worse and Best



rmsd=0.0166 and **0.0127** fm

Best and "Best"

$\rho = \rho_0 A^{1/3} + \delta$ so $\rho^2 \approx \rho_0^2 A^{2/3} + 2\delta\rho_0 A^{1/3} \approx \rho_0^2 A^{2/3} + 2\delta\rho A^{1/3}$



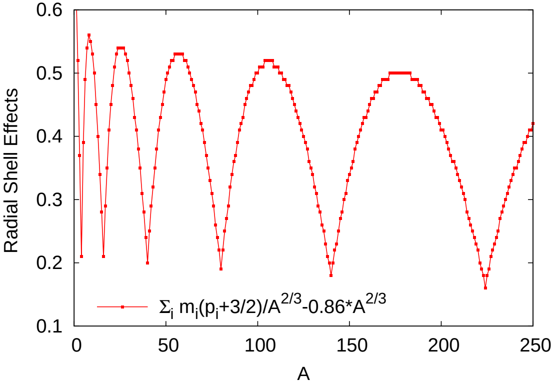
rmsd=0.0127 fm in BOTH cases, but refs absent in $\Delta\delta_{<r^2>}$:

Basis for claiming rmsd$\approx$ 0.01 fm

Radial Shell effects “theory”

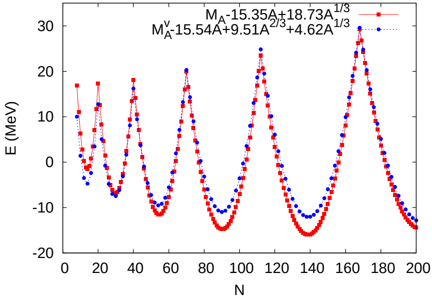
$$\rho^2_{\nu,\pi} = \frac{41.47}{\hbar\omega_{\nu,\pi}} \sum_i (n_i, z_i)(p_i + 3/2 + \delta_i)/(Z, N) \quad (7)$$

$$\textbf{OR} \quad \rho^2 = \frac{41.47}{\hbar\omega} \sum_i m_i(p_i + 3/2 + \delta_i)/A \quad (8)$$



Master shell effects. $A^{1/3}$ scaling

BE is dominated by a master term with possible asymptotics
$M_A \asymp 17.05A - 20.87A^{1/3}$
$M_A^v \asymp 17.27A - 10.57A^{2/3} - 5.13A^{1/3}$

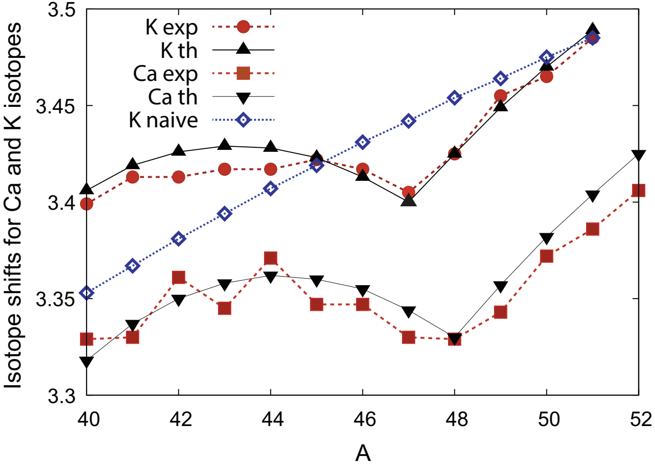


Master shell effects produced by M_A and M_A^v for $t = N - Z = 0$.
They are parabolic segments bounded by HO closures at $N = 8$, 20, 40, 70, 112 and 168, that scale asymptotically as $A^{1/3}$

Looking for hints

Vindication of the Duflo term.

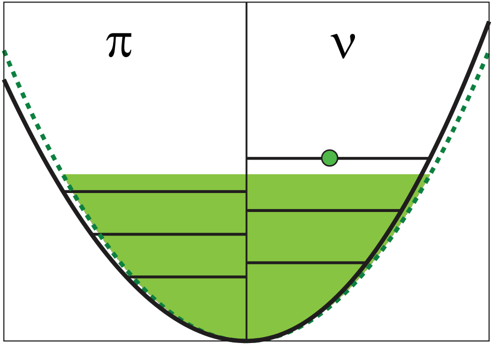
It says that p orbits are large.



Why? How?

Nolen Schiffer (NS) and Thomas Ehrman (TE)

NS $BE(^{41}\mathrm{Ca}\text{-}^{41}\mathrm{Sc})$=7.28 MeV(exp) *vs* 6.69 MeV(naive) **$f_{7/2}$**
TE $BE(^{41}\mathrm{Ca}\text{-}^{41}\mathrm{Sc})$=7.05 MeV(exp) *vs* 7 MeV(naive) **$p_{3/2}$**
NS solved by isovector monopole polarizability.



And TE ?? . Treat both on same footing. Use
$C_{Zx} = 0.67Z(Z-1)/\rho_x$ and
$\mathrm{MDE} = C_{Z+1\pi>} - C_{Z\pi<} = C_{Z+1\nu<} - C_{Z\pi<}$
Think of $^{17}\mathrm{F}\text{-}^{17}\mathrm{O}$. Implementation comes next.

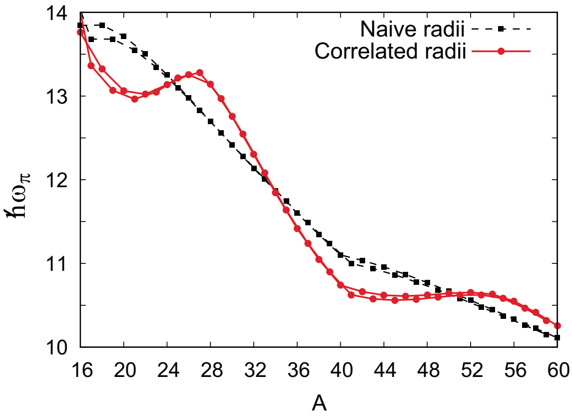
Do some SM

The proton radius of ^{17}O we know from DZ
The proton radius of ^{17}F is the neutron radius of ^{17}O
To do SM we need $\hbar\omega$ for neutrons and protons, obtained by inverting Eq.(7) for $N > Z$

$$\rho^2 = \langle r^2 \rangle = \frac{41.47}{\hbar\omega} \sum_i m_i(p_i + 3/2 + \delta_i)/A$$
$$\hbar\omega_\pi = \frac{41.47}{\langle r_\pi^2 \rangle} \sum_i z_i(p_i + 3/2)/Z,$$ **The inversion trick**

For $N < Z$ use $\hbar\omega_\nu$ with ζ adjusted to reproduce the experimental MDE or MED.
Remember $\Delta r_{\nu\pi} = \rho_\nu - \rho_\pi = \frac{\zeta t}{A} e^{g/A}$

Do some SM. Repeat inversion trick



$$\rho^2 = \langle r^2 \rangle = \frac{41.47}{\hbar\omega} \sum_i m_i (p_i + 3/2 + \delta_i)/A \qquad (9)$$

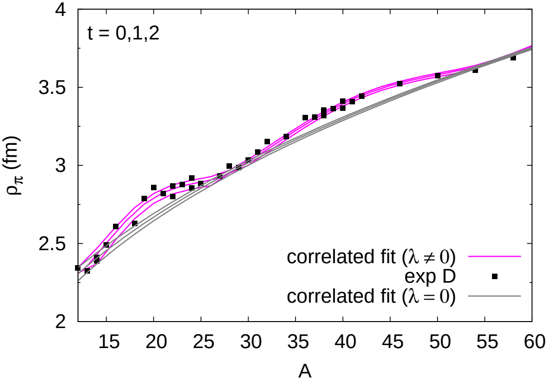
$$\hbar\omega_\pi = \frac{41.47}{\langle r_\pi^2 \rangle} \sum_i z_i (p_i + 3/2)/Z. \qquad (10)$$

TABLE

Table : MDE and MED ΔE for $T = 1/2$ mirror nuclei of mass A, $\hbar\omega_{\pi,\nu}$ in MeV and the corresponding skin parameters and radii in fm. Note that the radii correspond to the $N > Z$ nuclei, they are interchanged for the mirror partners. Experimental and calculated ΔE values coincide by construction. Interaction N3LO with cutoff $\Lambda = 2\text{fm}^{-1}$.

A	J^π	ΔE	$\hbar\omega_\nu$	$\hbar\omega_\pi$	ζ	$\Delta r_{\nu\pi}$	r_π	r_ν
15	$1/2^-$	3.537	14.55	14.62	0.358	0.025	2.507	2.532
	$3/2^-$	3.389	14.39	14.66	0.609	0.043	2.503	2.547
17	$5/2^+$	3.543	13.62	13.38	0.906	0.056	2.641	2.697
	$1/2^+$	3.167	12.86	13.51	**2.367**	0.147	2.628	2.776
39	$3/2^+$	7.307	10.97	10.91	0.258	0.007	3.361	3.368
	$1/2^+$	7.253	10.90	10.89	0.523	0.014	3.365	3.379
41	$7/2^-$	7.278	10.78	10.63	0.610	0.015	3.422	3.437
	$3/2^-$	7.052	10.61	10.59	1.513	0.038	3.427	3.465
	$1/2^-$	7.129	10.61	10.59	1.482	0.037	3.428	3.465
	$5/2^-$	7.351	10.75	10.61	0.702	0.018	3.424	3.442
	$5/2^-$	7.338	10.75	10.61	0.725	0.018	3.427	3.442

SM calculation of radii



$$\langle r_\pi^2 \rangle = \frac{41.47}{\hbar\omega_\pi} \sum_i m_i(p_i + 3/2 + \delta_i)/A \quad (11)$$

$$\langle r_\pi^2 \rangle = \rho_\pi^2(\lambda = 0) + \frac{41.47}{\hbar\omega} \frac{m_{s_{1/2}}\delta}{A} \quad (12)$$

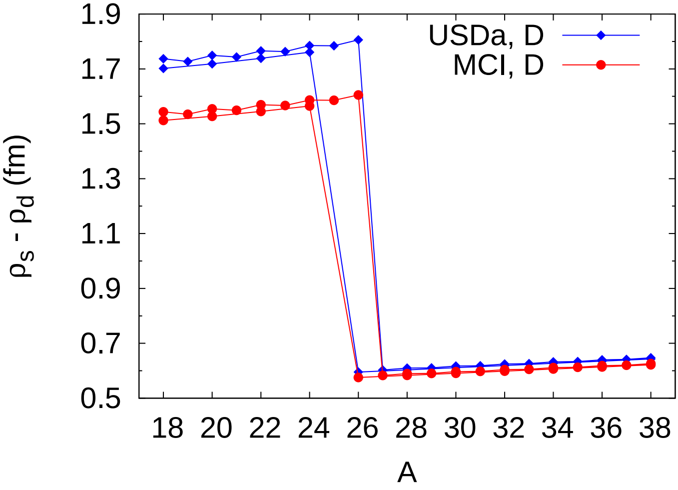
Radial behavior of the $s_{1/2}$ halo orbit

$$\delta = \begin{cases} \delta_< & \text{if} \quad N \text{ and } Z \quad < 14 \\ \delta_> & \text{if} \quad N \text{ or } Z \quad \geq 14. \end{cases} \tag{13}$$

Table : Results for the optimal δ's for D and AM sets of data, and for the USDa and MCI interactions for $t = 0, 1$ and 2. The rmsd are calculated with respect to all known radii in the *sd* shell (21 and 23 for D and AM respectively).

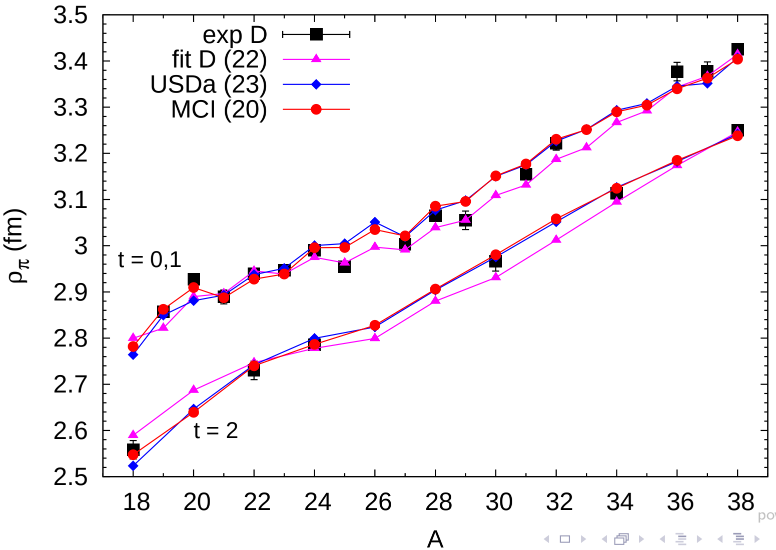
	Duflo		Angeli-Marinova	
	USDa	MCI	USDa	MCI
$\delta_<$	4.90	4.25	5.50	4.80
$\delta_>$	1.40	1.35	1.45	1.35
rmsd (fm)	0.023	0.020	0.023	0.018

Evolution of the $s_{1/2}$ halo orbit



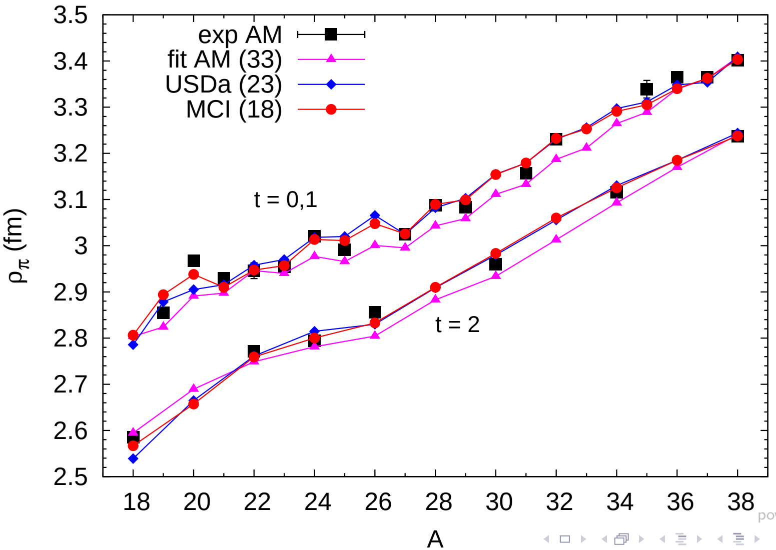
$\rho_s - \rho_d = \sqrt{\langle r^2_{s_{1/2}} \rangle} - \sqrt{\langle r^2_d \rangle}$ from Eq. (12)

RESULTS D



exp D
fit D (22)
USDa (23)
MCI (20)
t = 0,1
t = 2
ρ_π (fm)
A
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
2.5
18
20
22
24
26
28
30
32
34
36
38

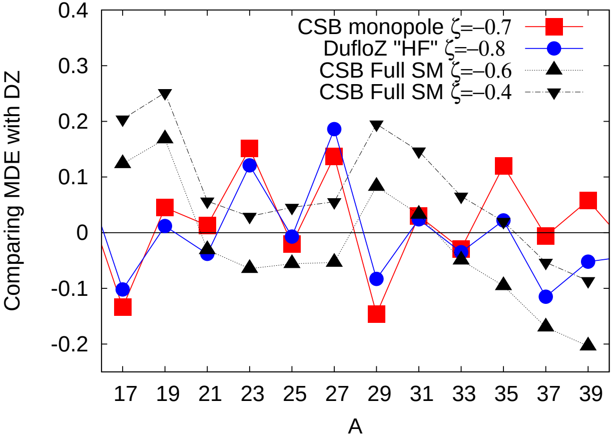
RESULTS AM



exp AM
fit AM (33)
USDa (23)
MCI (18)
t = 0,1
t = 2
ρπ (fm)
A
3.5
3.4
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
2.5
18
20
22
24
26
28
30
32
34
36
38

Neutron Radii

Recall DZ monopole (blue) and CSB monopole (red).



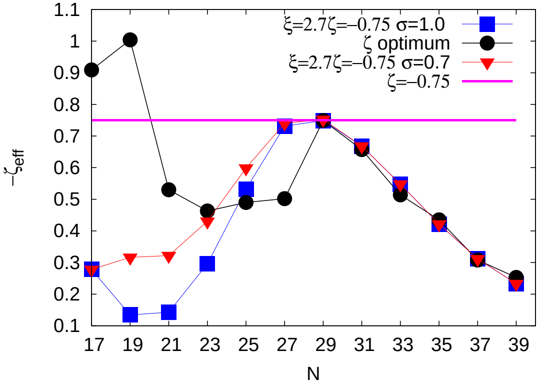
Detect problems with full SM. So naive
$\Delta r_{\nu\pi} = \rho_\nu - \rho_\pi = \frac{\zeta t}{A} e^{g/A}$. NO GO.
Find optimum ζ nucleus by nucleus: black dots next...

Full DZ*

Recall $\mathrm{DZ}* = \rho_*^2 = \rho_{0*}^2 + \mathcal{D}*$ and bring in the blue items in

$$\rho_{0*}^2 = A^{2/3}\left(\rho_0 + \frac{\tau}{2}\frac{t}{A^{4/3}} + \frac{\zeta}{2}\frac{t_z}{A^{4/3}}\right) e^{(g/A)} \tag{14}$$

$$\mathcal{D}* = \lambda(S_\pi + S_\nu)^2(1 + \xi\frac{e^{(g/A)}t_z}{2A^{4/3}}) + \mu(Q_\pi + Q_\nu)^2 \tag{15}$$



Revisit skins

Table : Comparing $\Delta r_{\nu\pi}$ for DZ and DZ*. with estimates (ests) [25,26] and measure (exp) [24] (fm).

	^{48}Ca	^{68}Ni	^{120}Sn	^{208}Pb	^{128}Pb
DZ$\Delta r_{\nu\pi}$	0.14	0.14	0.13	0.17	0.17
ests-exp	0.135(15)	0.17(2)	0.14(2)	0.16(3)	0.15(3)
ref.	[26]	[25]	[25]	[25]	[24]
DZ*$\Delta r_{\nu\pi}$	0.11	0.11	0.11	0.16	0.16

The DZ* estimates are definitely smaller than the DZ ones.
Note that for the only measured value in ^{128}Pb, the DZ* number remains safely inside error bars.

COMPUTATIONAL OUTLOOK

Tempting but dangerous is

$$\rho_\pi^2 = \frac{41.47}{\hbar\omega_\pi} \sum_i z_i (p_i + 3/2 + \delta_i)/Z \tag{16}$$

$$\rho_\nu^2 = \frac{41.47}{\hbar\omega_\nu} \sum_i n_i (p_i + 3/2 + \delta_i)/N \tag{17}$$

So try (using $r_i^2 = (p_i + 3/2 + \delta_i)$), and write

$$\rho^2 = \frac{41.47}{\hbar\omega} \sum_i [(n_i r_i^2 + z_i r_i^2) + \alpha|n_i r_i^2 - z_i r_i^2| + \beta(n_i r_i^2 - z_i r_i^2)]/A \tag{18}$$

The Game Changer

So far paradigm is:
We know interaction(s). We know how to calculate.
Therefore: if something wrong Three Body
The game changer says
We know interaction(s). We do not know how to calculate well enough.
Therefore: Look for hints:
RHF [3], nonlocal [(19),20, 21, 22];
and/or calculate better [23]

IT SOUNDS IMPROBABLE, BUT AS SHERLOCK HOLMES PUT IT

Once you eliminate the impossible, whatever remains, no matter how improbable, must be the truth.

Bibliography 1

[1 AM] I. Angeli and K. Marinova, Atomic Data and Nuclear Data Tables 99, 69 (2013).
[2 DZ] J. Duflo and A. P. Zuker, Phys. Rev. C 66, 051304 (2002).
[3] G. A. Lalazissis, T. Niksic, D. Vretenar, and P. Ring, Phys. Rev. C 71, 024312 (2005).
[4] I. Angeli, Y. P. Gangrsky, K. P. Marinova, I. N. Boboshin, S. Y. Komarov, B. S. Ishkhanov, and V. V. Varlamov, Journal of Physics G: Nuclear and Particle Physics 36, 085102 (2009).
[5] http://www.nndc.bnl.gov/amdc/web/dz.html. Tables used in DZ
[6] E. Nadjakov, K. Marinova, and Y. Gangrsky, Atomic Data and Nuclear Data Tables 56, 133 (1994).
[7] J. Bonnard, S. M. Lenzi, and A. P. Zuker, Phys. Rev. Lett. 116, 212501 (2016), arXiv:1606.02168 [nuclth].
[8] J. Bonnard and A. P. Zuker, arXiv:1606.03345 nucl-th.
[9] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C 51, 38 (1995).
[10] A. Bohr and B. R. Mottelson, Nuclear Structure, vol. I (Benjamin, New York, 1969).
[11] J. Ehrman, Phys. Rev. 81, 412 (1951).
[12] R. G. Thomas, Phys. Rev. 88, 1109 (1952).

Bibliography 2

[13] E. Caurier, G. MartíPinedo, F. Nowacki, A. Poves, and A. P. Zuker, Rev. Mod. Phys. 77, 427 (2005).
[14] K. Kreim et al., Phys. Lett. B731, 97 (2014).
[15] R. F. García Ruiz et al., Nature Physics 12(2016)594.
[16] J. Mendoza-Temis, J. G. Hirsch, and A. P. Zuker, Nuclear Physics A 843, 14 (2010).
[17] J. A. Nolen and J. P. Schifer, Phys. Lett. B29, 396 (1968), J. A. Nolen and J. P. Schifer, Ann., Rev. Nucl. Sci. 19, 471 (1969).
[18] A. P. Zuker Czech. Journal of Physics B 25, 311 (1975).
[19]R Utama and Wei-Chia Chen and J Piekarewicz, J. Phys. G 43(2016)114002
[20] F. Perey and B. Buck, Nuclear Physics 32, 353 (1962).
[21] G. H. Rawitscher, Nuclear Physics A 886, 1 (2012).
[22] G. H. Rawitscher,10.1103/PhysRevC.31.1173 (1985)
[23] Jansen, G. R. and Schuster, M. D. and Signoracci, A. and Hagen, G. and Navrátil, P. http://link.aps.org/doi/10.1103/PhysRevC.94.011301
[24 Pbskin] C. M. Tarbert et al., Phys. Rev. Lett. 112, 242502 (2014).
[25 rocam] X. RocaMaza et al., Phys. Rev. C 92, 064304 (2015). [26 hagen] G. Hagen et al., Nat. Phys. 12(2016)186 DOI: 10.1038/NPHYS3529