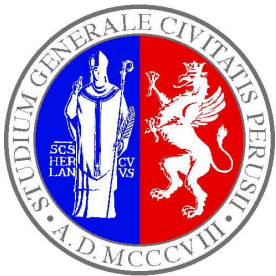


3D nuclear parton structure: from DVCS to double parton scattering at the LHC

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with

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Marco Traini (Trento), Vicente Vento (Valencia)**



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Outline

Two topics towards the investigation of the 3D structure of hadronic matter:



1 - Nuclear DVCS (GPDs):

nuclei as a Lab to study the parton structure of the *bound* nucleon

From ^3He

(S.S., PRC 70 (2004); PRC 79 (2009); M. Rinaldi and S.S. PRC 85 (2012); PRC 87 (2013)):

results but neither data nor proposals @ JLab... EIC?

to ^4He

data and proposals at JLab; we have no results of calculations, yet, just starting



2 - Double parton scattering at the LHC:

*

parton correlations in p-p scattering

some recent results of ours

*

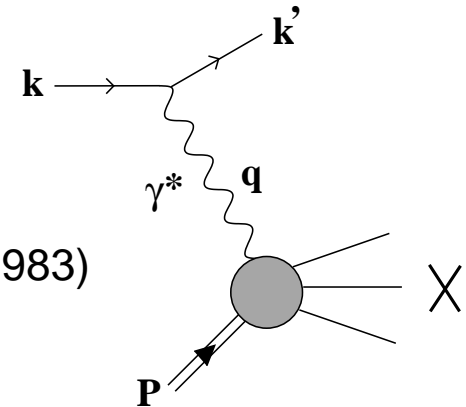
prospects to study parton correlations in p-A scattering:

nuclei as a Lab to study the *free* nucleon (not yet my results)

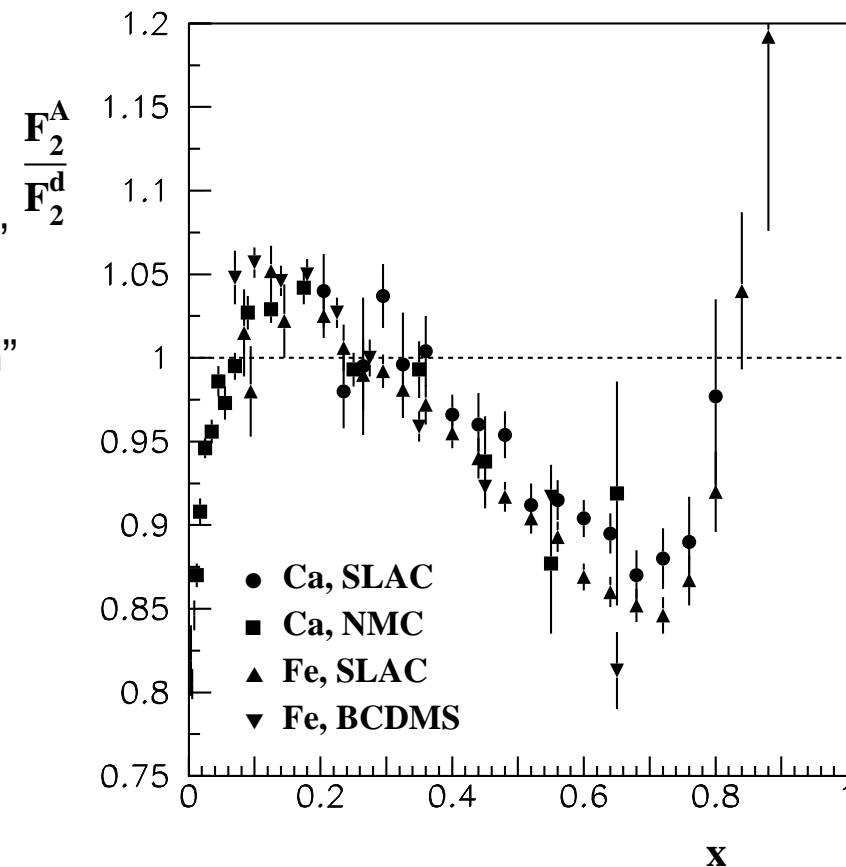
EMC effect in A-DIS

Measured in $A(e, e')X$, ratio of A to d SFs F_2 (EMC Coll., 1983)

One has $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$



- $x \leq 0.1$ "Shadowing region"
- $0.1 \leq x \leq 0.2$ "Enhancement region"
- $0.2 \leq x \leq 0.8$ "EMC (binding) region"
- $0.8 \leq x \leq 1$ "Fermi motion region"
- $x \geq 1$ "TERRA INCOGNITA"



EMC effect: explanations?

In general, with a few parameters any model explains the data:

EMC effect = “Everyone’s Model is Cool” (G. Miller)

Situation: basically not understood. Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

- the knowledge of nuclear parton distributions is crucial for the data analysis of heavy ions collisions;
- the partonic structure of the neutron is measured with nuclear targets and several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

Inclusive measurements cannot distinguish between models

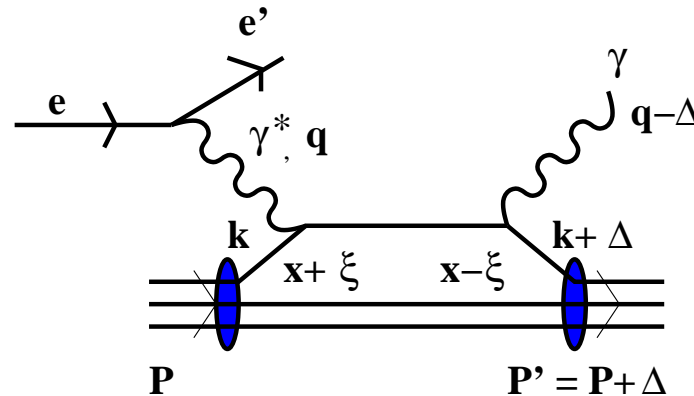
One has to go beyond:

R. Dupré and S.S., EPJA 52 (2016) 159

- SIDIS: not here
- **Hard Exclusive Processes (GPDs)**

GPDs: Definition (X. Ji PRL 78 (97) 610)

For a $J = \frac{1}{2}$ target,
in a hard-exclusive process,
($Q^2, \nu \rightarrow \infty$)
such as (**coherent**) DVCS:



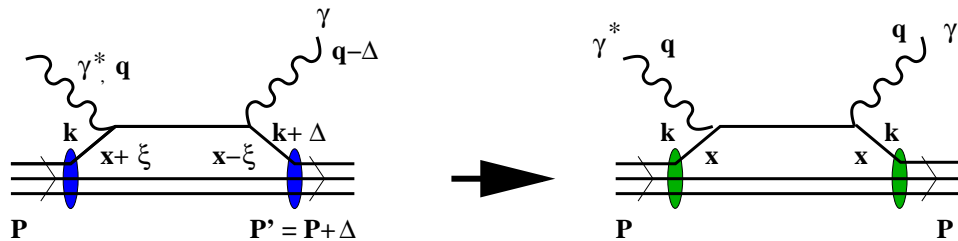
the GPDs $H_q(x, \xi, \Delta^2)$ and $E_q(x, \xi, \Delta^2)$ are introduced:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle = H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) \\ + E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots$$

- $\Delta = P' - P$, $q^\mu = (q_0, \vec{q})$, and $\bar{P} = (P + P')^\mu / 2$
- $x = k^+ / P^+$; $\xi = \text{"skewness"} = -\Delta^+ / (2\bar{P}^+)$
- $x \leq -\xi \longrightarrow$ GPDs describe *antiquarks*;
 $-\xi \leq x \leq \xi \longrightarrow$ GPDs describe $q\bar{q}$ *pairs*; $x \geq \xi \longrightarrow$ GPDs describe *quarks*

GPDs: limits

- when $P' = P$, i.e., $\Delta^2 = \xi = 0$, one recovers the usual PDFs:



$$H_q(x, \xi, \Delta^2) \implies H_q(x, 0, 0) = q(x); \quad E_q(x, 0, 0) \text{ unknown}$$

- the x -integration yields the q-contribution to the Form Factors (ffs)

$$\int dx \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle =$$

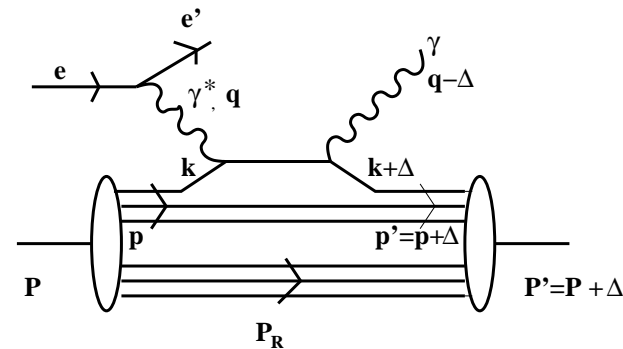
$$\int dx H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) + \int dx E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots$$

$$\implies \int dx H_q(x, \xi, \Delta^2) = F_1^q(\Delta^2) \quad \int dx E_q(x, \xi, \Delta^2) = F_2^q(\Delta^2)$$

$$\implies \text{Defining } \boxed{\tilde{G}_M^q = H_q + E_q} \quad \text{one has } \int dx \tilde{G}_M^q(x, \xi, \Delta^2) = G_M^q(\Delta^2)$$

Nuclei: why?

ONE of the reasons is understood by studying coherent DVCS in I.A.:



In a symmetric frame ($\bar{p} = (p + p')/2$) :

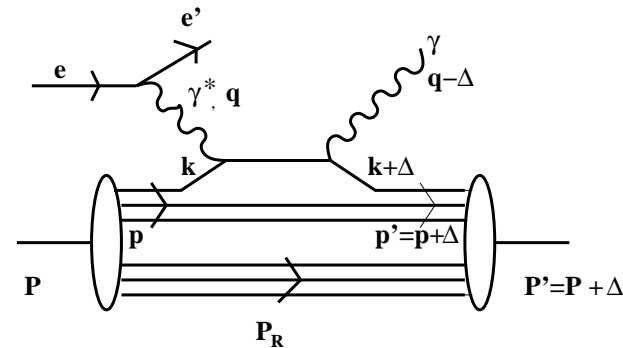
$$\begin{aligned} k^+ &= (x + \xi)\bar{P}^+ = (x' + \xi')\bar{p}^+ , \\ (k + \Delta)^+ &= (x - \xi)\bar{P}^+ = (x' - \xi')\bar{p}^+ , \end{aligned}$$

one has, for a given GPD, H_q or \tilde{G}_M^q ,

$$GPD_q(x, \xi, \Delta^2) \simeq \int \frac{dz^-}{4\pi} e^{ix\bar{P}^+ z^-} {}_A \langle P' S' | \hat{O}_q^\mu | PS \rangle_A \Big|_{z^+=0, z_\perp=0} .$$

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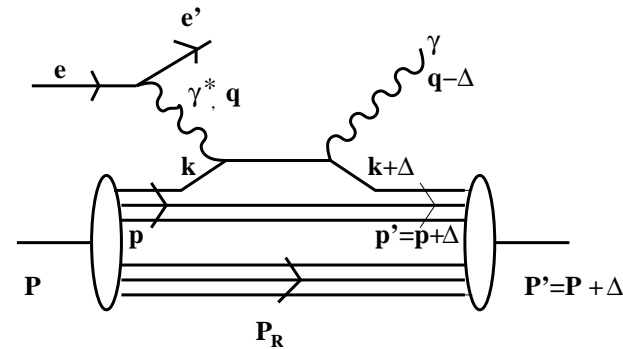
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By properly inserting complete sets of states for the interacting nucleon and the recoiling system :

Nuclei: why?

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$$\begin{aligned} GPD_q(x, \xi, \Delta^2) &= \int \frac{dz^-}{4\pi} e^{ix'\bar{p}^+z^-} \langle P' S' | \sum_{\vec{P}'_R, S'_R, \vec{p}', s'} \{ |P'_R S'_R\rangle | p' s' \rangle \} \langle P'_R S'_R | \\ &\quad \langle p' s' | \hat{O}_q^\mu \sum_{\vec{P}_R, S_R, \vec{p}, s} \{ |P_R S_R\rangle | p s \rangle \} \{ \langle P_R S_R | \langle p s | \} | PS \rangle , \end{aligned}$$

and, since $\{ \langle P_R S_R | \langle p s | \} | PS \rangle = \langle P_R S_R, p s | PS \rangle (2\pi)^3 \delta^3(\vec{P} - \vec{P}_R - \vec{p}) \delta_{S, S_R} \delta_{s, s}$,

Why nuclei?

a convolution formula can be obtained (S.S. PRC 70, 015205 (2004)):

$$H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int_{x-\xi}^{1-\xi} \frac{dz}{z} h_N^A(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right)$$

in terms of $H_q^N(x', \xi', \Delta^2)$, the GPD of the free nucleon N , and of the light-cone off-diagonal momentum distribution:

$$h_N^A(z, \xi, \Delta^2) = \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(z + \xi - \frac{p^+}{\bar{P}^+}\right)$$

where $P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E)$, is the one-body off-diagonal spectral function for the nucleon N in the nucleus,

$$\begin{aligned} P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) &= \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{R,s} \langle \vec{P}' M | (\vec{P} - \vec{p}) S_R, (\vec{p} + \vec{\Delta}) s \rangle \\ &\times \langle (\vec{P} - \vec{p}) S_R, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_R^*) . \end{aligned}$$

Why nuclei?

The obtained expressions have the correct **limits**:

- the **x-integral** gives the f.f. $F_q^A(\Delta^2)$ in **I.A.**:

$$\int dx H_q^A(x, \xi, \Delta^2) = F_q^N(\Delta^2) \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) = F_q^A(\Delta^2)$$

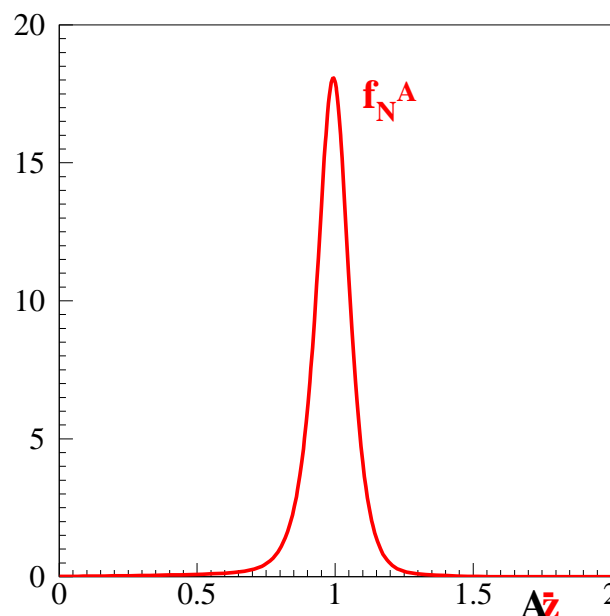
- forward limit** (standard DIS):

$$q^A(x) \simeq \sum_N \int_x^1 \frac{d\tilde{z}}{\tilde{z}} f_N^A(\tilde{z}) q^N\left(\frac{x}{\tilde{z}}\right)$$

with the **light-cone momentum distribution**:

$$f_N^A(\tilde{z}) = \int dE d\vec{p} P_N^A(\vec{p}, E) \delta\left(\tilde{z} - \frac{p^+}{P^+}\right),$$

which is strongly peaked around $A\tilde{z} = 1$:



Why nuclei?

In DIS, the contribution of nucleons with large longitudinal momentum \tilde{z} can be observed only at high x , where the cross sections are vanishingly small, since q^N is also vanishing; the same does not happen for GPDs, since one has:

$$H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int_{x-\xi}^{1-\xi} \frac{dz}{z} h_N^A(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right) \quad \text{with:}$$

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so that, by tuning the independent variable ξ ,
one can select regions where nucleons with large
 \tilde{z} dominate, even if x is low ($x - \xi \leq z = \tilde{z} - \xi$)
and H_q^N is not vanishing.

(e.g., sensitivity to non-nucleonic
degrees of freedom, firstly observed in:
Berger, Cano, Diehl and Pire,
PRL 87 (2001) 142302)

Why nuclei?

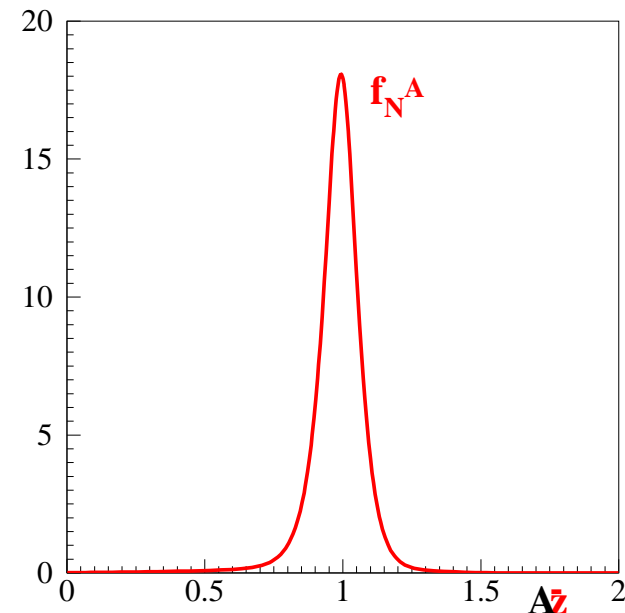
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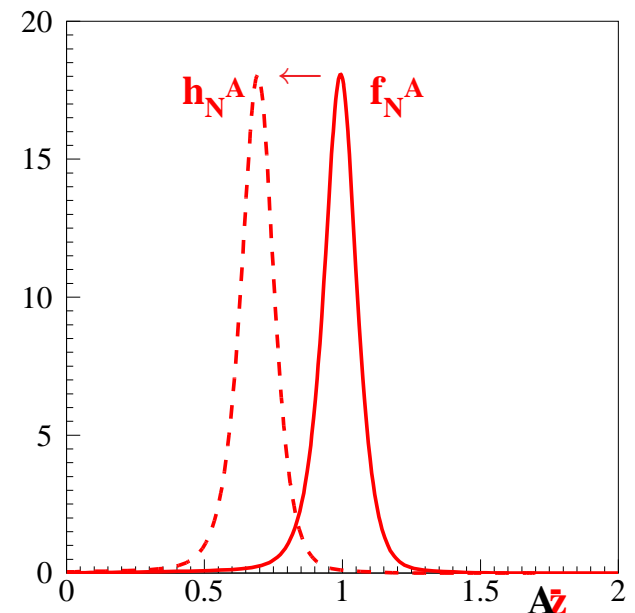
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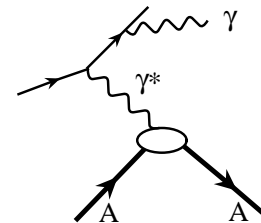
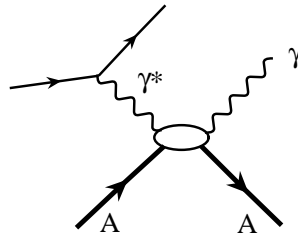
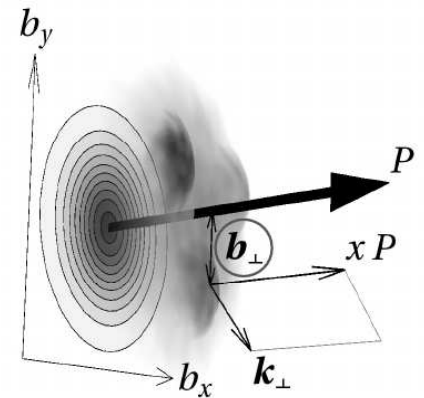
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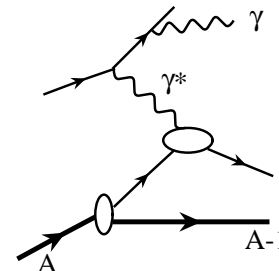
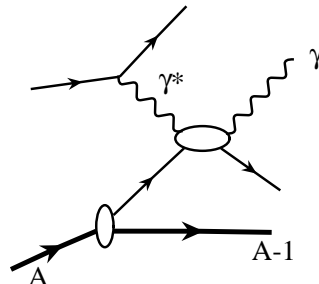
Nuclei and DVCS tomography

In impact parameter space, GPDs are *densities*:

$$\rho_q(x, \vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H^q(x, 0, \Delta^2) + ..$$



Coherent DVCS: nuclear tomography



Incoherent DVCS: tomography of bound nucleons: realization of the EMC effect

Why nuclei? - (many reasons...)

I work with **light nuclei**, in **IA**, so far for the **coherent channel**...

But there is much good theoretical work:



beyond the coherent channel:

Liuti & Taneja PRC 72 032201 (R) 2005: spin-0 nuclei, applied to ^4He ;

Guzey, arXiv:0801.3235 (nucl-th): constraining the neutron information from incoherent DVCS off nuclei at large t (spin-0 nuclei);



beyond IA (Shadowing: low x_{Bj} , large distances):

Freund & Strikman, PRC 69 (2004) 015203;

Goeke et al., arXiv:0802.0669 (hep-ph);



for finite-heavy nuclei:

Guzey & Strikman, PRC 68 (2003) 015204

Kirchner & Müller, EPJC 32, 347 (2003)



discussing **other issues:**

Color Transparency phenomena, Liuti & Taneja, PRD 70, 074019 (2004);

Energy-momentum tensor in nuclei:

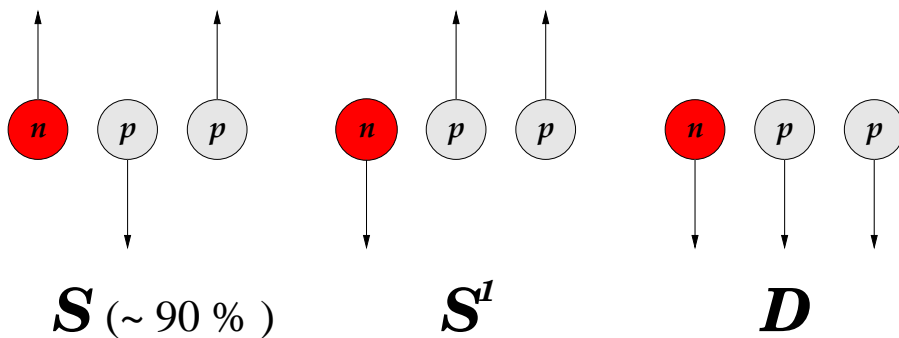
Polyakov PLB 555 (2003) 57

Guzey & Siddikov J. Phys. G 32, 251 (2006)

...

GPDs for ^3He : why?

- ^3He is theoretically well known. Even a relativistic treatment may be implemented.
- ^3He has been used extensively as an effective neutron target, especially to unveil the spin content of the free neutron, due to its peculiar spin structure:



In S -wave
 $^3\vec{H}e = \vec{n}!$

^3He always promising when the neutron angular momentum properties have to be studied. To what extent for OAM and \tilde{G}_M^q ? The answer here.

- To this aim, ^3He is a unique target:

- * in isoscalar systems, such as ^2H and ^4He , the contribution of the neutron E_q is basically cancelled by that of the proton one ($\kappa_p \simeq -\kappa_n$); very difficult to extract the neutron E_q , crucial to access OAM, in coherent experiments;
- * heavier targets do not allow refined theoretical treatments.

GPDs of ^3He in IA

H_q^A can be obtained in terms of H_q^N (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$H_q^A(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} \sum_S \sum_s P_{SS,ss}^N(\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} H_q^N(x', \Delta^2, \xi'),$$

and $\tilde{G}_M^{3,q}$ in terms of $\tilde{G}_M^{N,q}$ (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$\tilde{G}_M^{3,q}(x, \Delta^2, \xi) = \sum_N \int dE \int d\vec{p} \left[P_{+-,+ -}^N - P_{+-,- +}^N \right] (\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} \tilde{G}_M^{N,q}(x', \Delta^2, \xi'),$$

where $P_{SS,ss}^N(\vec{p}, \vec{p}', E)$ is the one-body, spin-dependent, off-diagonal spectral function for the nucleon N in the nucleus,

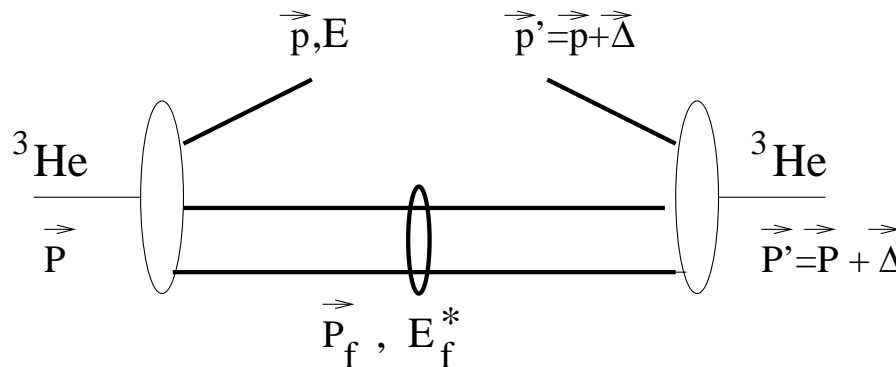
$$P_{SS',ss'}^N(\vec{p}, \vec{p}', E) = \frac{1}{(2\pi)^6} \frac{M\sqrt{ME}}{2} \int d\Omega_t \sum_{s_t} \langle \vec{P}' S' | \vec{p}' s', \vec{t}_{s_t} \rangle_N \langle \vec{p} s, \vec{t}_{s_t} | \vec{P} S \rangle_N,$$

evaluated by means of a **realistic** treatment based on **Av18 wave functions** (w.f. from A. Kievsky *et al* NPA 577, 511 (1994), overlaps from A. Kievsky *et. al*, PRC 56, 64 (1997)).

Nucleon GPDs: initially, given by a model

A few words about $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$:

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{f,s} \langle \vec{P}' M | (\vec{P} - \vec{p}) S_f, (\vec{p} + \vec{\Delta}) s \rangle \\ \times \langle (\vec{P} - \vec{p}) S_f, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_f^*).$$

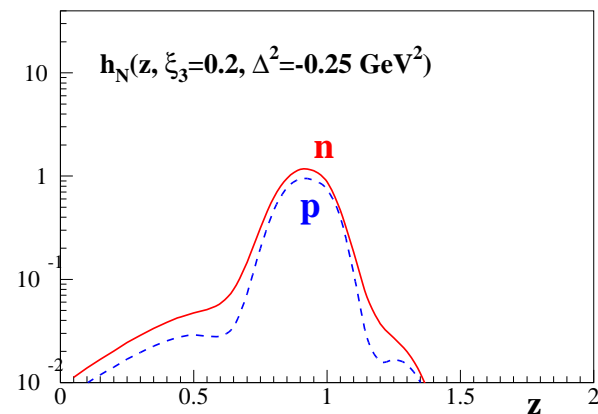
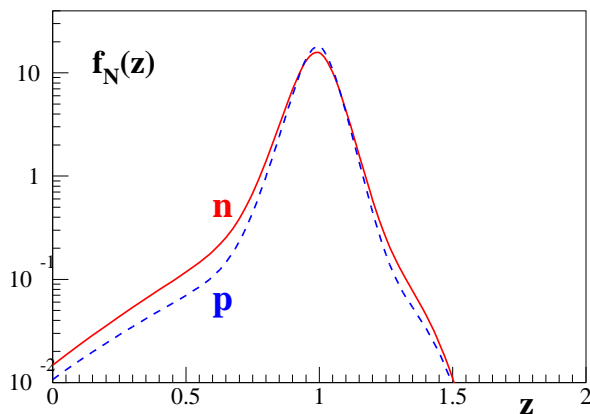
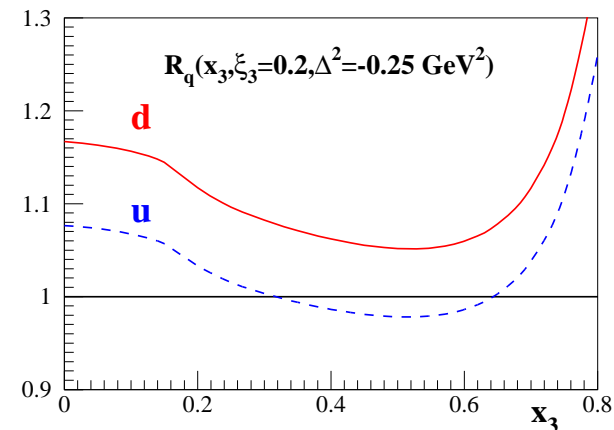
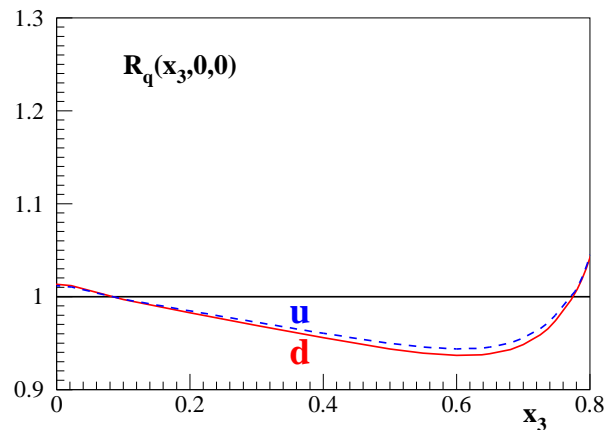


- the two-body recoiling system can be either the deuteron or a scattering state;
- when a deeply bound nucleon, with high removal energy $E = E_{min} + E_f^*$, leaves the nucleus, the recoiling system is left with high excitation energy E_f^* ;
- the three-body bound state and the two-body bound or scattering state are evaluated within the same (Av18) interaction: the extension of the treatment to heavier nuclei would be extremely difficult

Example of nuclear effects - flavor dependence



The **d** and **u** distributions follow the pattern of the **neutron** and **proton** light-cone momentum distributions, respectively (R_q = ratio of bound to free GPDs):



Nuclear GPDs: from ^3He to ^4He



Our results, for ^3He :

- * An instant form, I.A. calculation of H^3 , \tilde{G}_M^3 , \tilde{H}^3 , within AV18;
- * an extraction procedure of the neutron information, able to take into account all the nuclear effects encoded in an IA analysis;
- * Interesting features: sensitivity of the results to nuclear dynamics: binding effects bigger than in DIS; different realistic potentials give different results
- * control on conventional nuclear effects, possible evidence of exotic ones; One can imagine a relativistic (LF) extension...
- * No data; no proposals at JLAB... EIC?

BUT



Actually, data for ^4He are becoming available from the CLAS collaboration at JLab (M. Hattawy, Ph.D. Thesis, Orsay, September 2015);

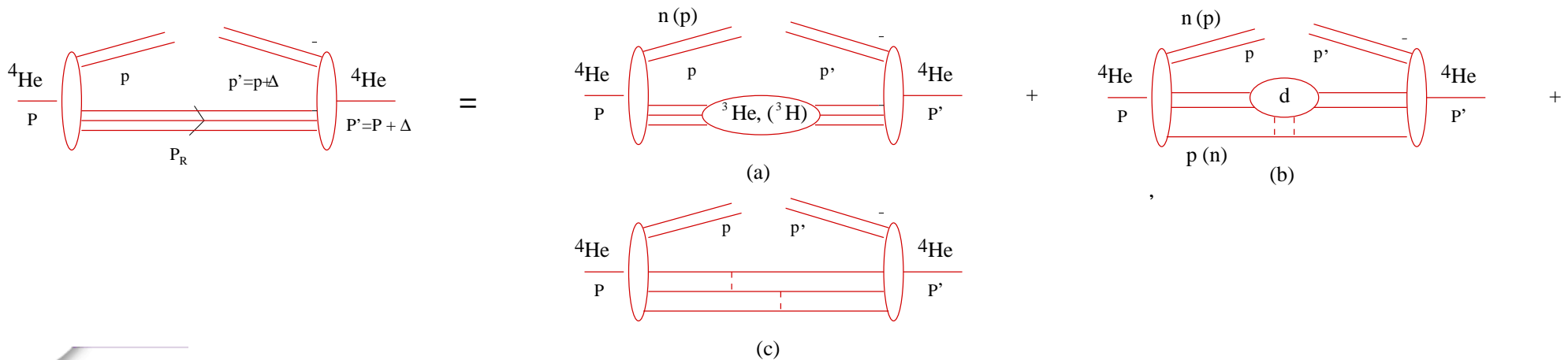
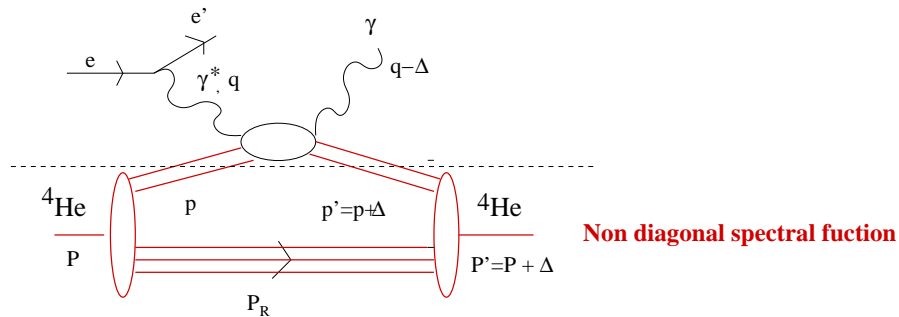
New impressive proposals presented at the JLab PAC, this month

Experimentally, for DVCS, ^4He is the simplest target one can imagine (only one GPD, H). A true nucleus, deeply bound!

DVCS off ^4He

- Mohammad thesis demonstrates that measurements are possible, separating the coherent and incoherent channels;
- Realistic microscopic calculations are necessary. A collaboration has started with Sara Fucini (Perugia, graduating student), Michele Viviani (INFN Pisa).

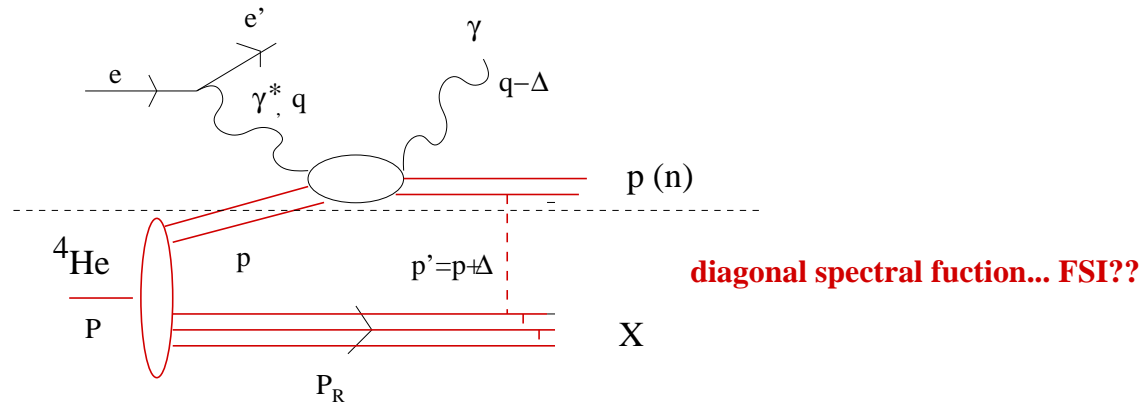
Coherent channel in IA:



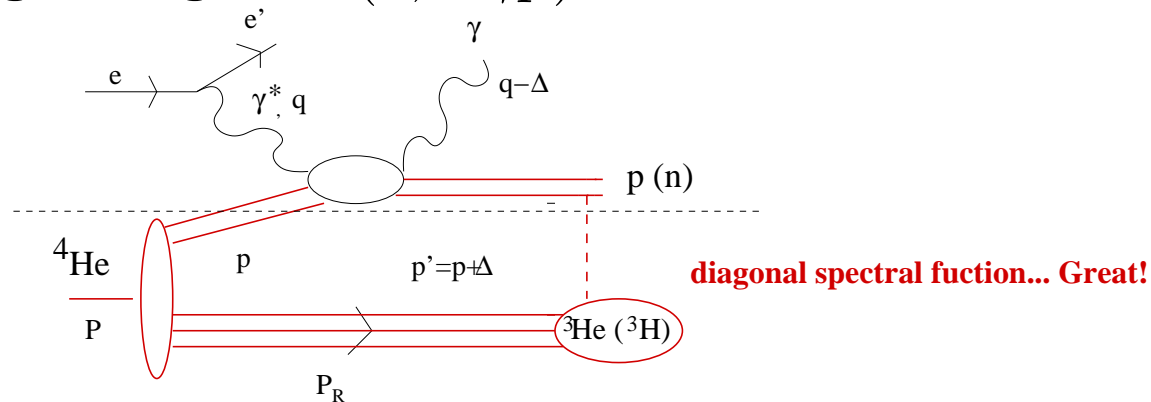
we are working on a); b) is feasible; c) is really challenging

Incoherent DVCS off ^4He in IA

$^4\text{He}(e, e' \gamma p(n))X$



Tagged! e.g., $^4\text{He}(e, e' \gamma p)^3\text{H}$



If Sara gets a Ph.D grant I think we can complete the analysis and evaluate all the relevant X-sections

2 - Moving to collider Physics

- **Hard multiple parton interactions (MPI) and double parton scattering (DPS) @LHC; double Parton Distribution Functions (dPDFs) and 3D proton structure: 2-parton correlations**

- **Our contribution:**
model calculation of dPDFs and DPS cross sections.

M. Rinaldi, S.S. and V. Vento, PRD 87, 114021 (2013);

M. Rinaldi, S.S., M. Traini and V. Vento, JHEP 12 (2014); PLB 752 (2016) 40; JHEP 10 (2016) 063;

M. Traini, M. Rinaldi, S.S. and V. Vento, PLB 752 (2016) 40

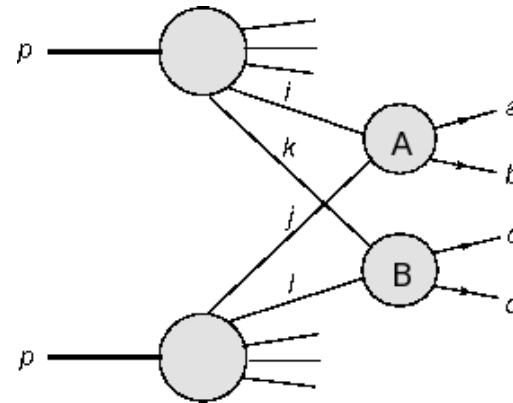
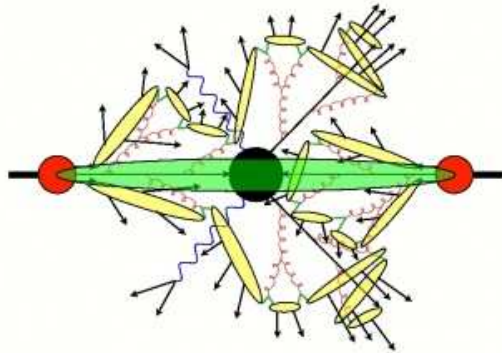
F.A. Ceccopieri, M. Rinaldi, S.S, arXiv:1702.05363 [hep-ph]

- **Prospects in $p - A$ collisions**

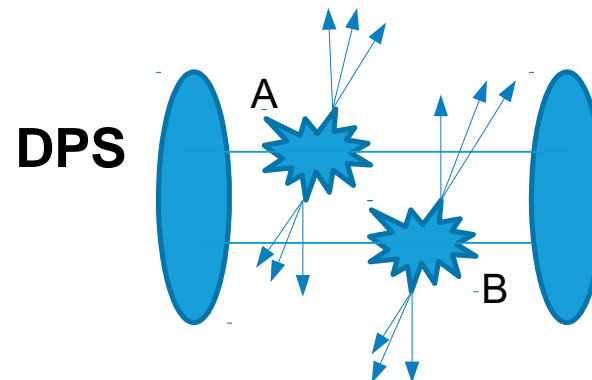
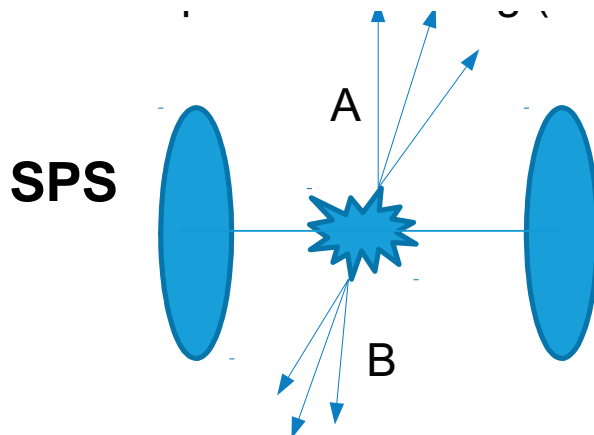
I have no results yet

Hard MPI and Double Parton Scattering

- In an LHC collision, MPI can occur:



- MPI are a wide subject; I will discuss *hard* double parton scattering (DPS). In the reaction $p + p \rightarrow A + B + X$, two (sets of) hard objects A and B , with associated scales Q_A and Q_B , can be produced through DPS, in addition to single parton scattering (SPS):



Hard MPI and Double Parton Scattering

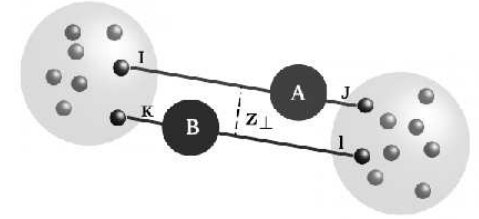
In terms of the total cross section, the DPS mechanism is power suppressed with respect to SPS (see, e.g., M. Diehl, D. Ostermeier, A. Schäfer JHEP 03 (2012) 089):

$$\sigma_{DPS}/\sigma_{SPS} = \Lambda^2/Q^2$$

However:

- DPS can compete with SPS if SPS process is suppressed by small or multiple coupling constants (e.g., same sign WW J. Gaunt et al EPJC 69 (2010) 53).
- DPS populates the final state phase space in a different way from SPS. In particular, it tends to populate the region of small $q_{A,T}$, $q_{B,T}$ and it is competitive with SPS in this region (used in experiments).
- DPS becomes more important relative to SPS as the collider energy grows. Smaller x values are probed where there is a larger density of partons (LHC!).
- DPS is important: a background for the search of new Physics
- **DPS is important for us:** it reveals new information about the **structure of the nucleon**. In particular, **correlations** between partons in the proton.

DPS and Double Parton Distributions (dPDFs)



DPS cross section (N. Paver, D. Treleani, Nuovo Cimento 70A, 215 (1982); M. Mekhfi PRD 32, 2371 (1985)) - proof of fact. still missing (but see M. Diehl, J. Gaunt... JHEP 1 (2016) 76)

$$d\sigma = \frac{1}{S} \sum_{i,j,k,l} \hat{\sigma}_{ij}(x_1, x_3, \mu_A) \hat{\sigma}_{kl}(x_2, x_4, \mu_B) \times \int d^2 \vec{b}_\perp F_{ik}(x_1, x_2, \mathbf{b}_\perp, \mu_A, \mu_B) F_{jl}(x_3, x_4, \mathbf{b}_\perp, \mu_A, \mu_B)$$

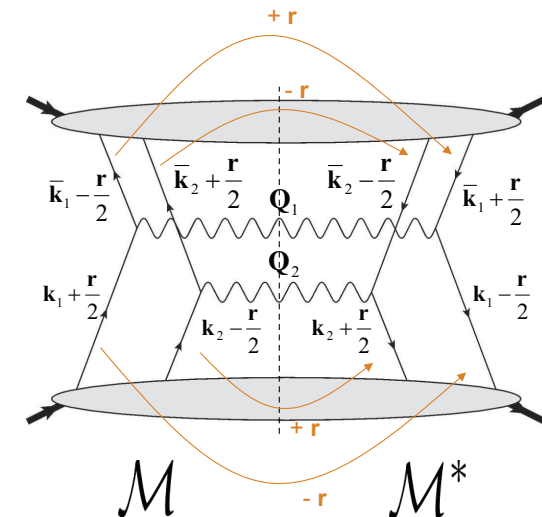
x_i = momentum fraction carried by the parton inside the hadron;

$\mu_{A,B}$ = momentum scale; \mathbf{b}_\perp = **transverse distance** between the two partons the dPDF $F_{ik}(x_1, x_2, \mathbf{b}_\perp, \mu_A, \mu_B)$ in one of the protons is very interesting!

In momentum space:

transverse momentum of partons, in general, is different in amplitude and conjugate

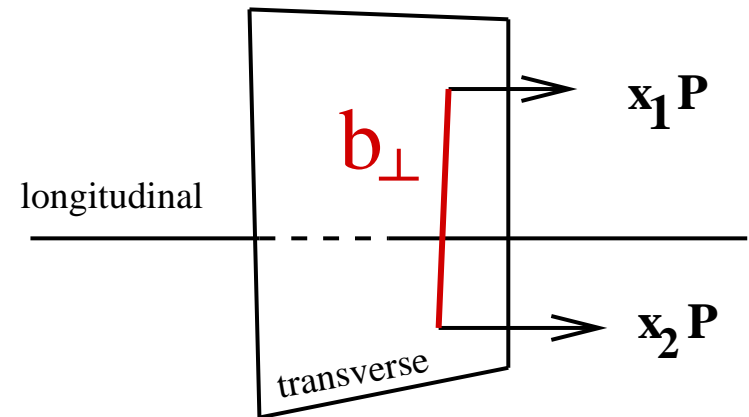
\mathbf{k}_\perp = **momentum imbalance** of a parton line between amplitude and conjugate



Nucleon 3D structure from dPDFs?

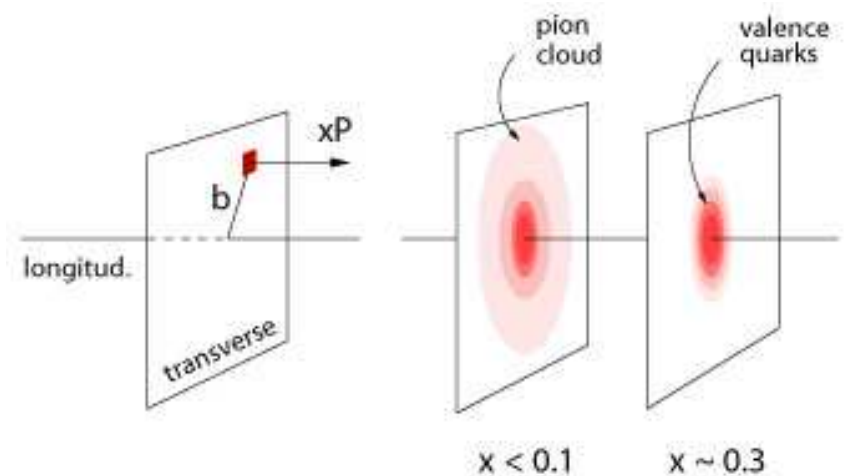
$F_{ij}(x_1, \mu_A, x_2, \mu_B, \mathbf{b}_\perp)$ is a **2-body** density

- It is dimensioned;
- Its F.T. wrt \mathbf{b}_\perp , dimensionless, is NOT a density (sometimes it is called ${}_2GPD$)



“Nucleon tomography” is done through **GPDs** (in Impact parameter space)

- $H(x, \xi = 0, \mathbf{b}, \mu)$ is a **1-body** density
- Its F.T. wrt \mathbf{b} , dimensionless, is NOT a density (standard GPD)



Nucleon tomography

F_{ij} is very interesting: **2-body** quantities are always theoretically intriguing (their measurement, challenging). The difference between a 2-body quantity and the product of two 1-body quantities is a measurement of **CORRELATIONS**

dPDFs and correlations

F_{ij} is usually factorized as follows ($(x_1, x_2) - k_\perp$ factorization):

$$F_{ij}(x_1, x_2, \vec{k}_\perp, \mu_A, \mu_B) = F_{ij}(x_1, x_2, \mu) T(\vec{k}_\perp, \mu)$$

AND (x_1, x_2) factorization):

$$F_{ij}(x_1, x_2, \mu) = \overbrace{q_i(x_1, \mu) q_j(x_2, \mu)}^{\text{PDF}} \theta(1 - x_1 - x_2) (1 - x_1 - x_2)^n$$

NO CORRELATION ANSATZ

This means that correlations between the quarks in the proton are neglected.

- Is this a safe approximation? Important for LHC fundamental studies (actually it is broken even by QCD evolution)
- Taking into account correlations in the analysis, can one understand better the proton structure through MPI observation?

Double Parton Correlations (DPCs)

- In principle, correlations are there. At low x , due to the large population of partons, they may be less relevant **BUT** theoretical estimates are necessary.

We are not alone in addressing this issue

(Calucci and Treleani (1999), Korotkikh et al. (2004), Gaunt and Stirling (2010), Diehl and Schäfer (2011), Snigirev (2011), Blok et al. (2012), Strikman et al. (2013)...)

- Difficult to study **DPCs**, non perturbative quantities, from first principles

- Our contribution: a quark model analysis

Constituent Quark Models **CQM** provide a correlated framework.

Widely used to guide measurements of NP quantities (e.g., TMDs, GPDs): they reproduce the gross features of experimental PDFs in the valence region...

BUT present LHC data are at low x ...

One has to perform a perturbative QCD evolution to the experimental scale, Q^2 from a low momentum scale, μ_o^2



$$f(x, \mu_o^2) \xrightarrow{\text{R.G.E., p. QCD}} f(x, Q^2), \text{DIS}$$

Twist-2

$$L.O. = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]}$$

+ N.L.O. (2 loops)

dPDFs in a Light-Front approach

M. Rinaldi, S.S., M. Traini and V.Vento, JHEP, 1412 (2014) 028

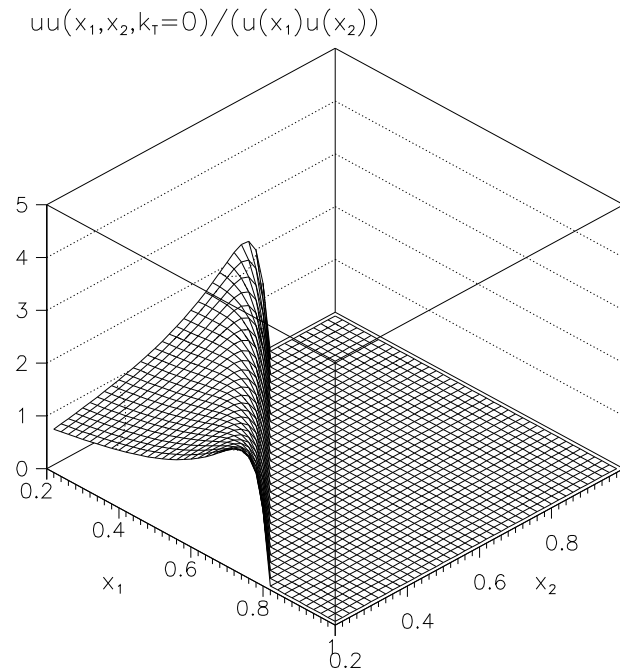
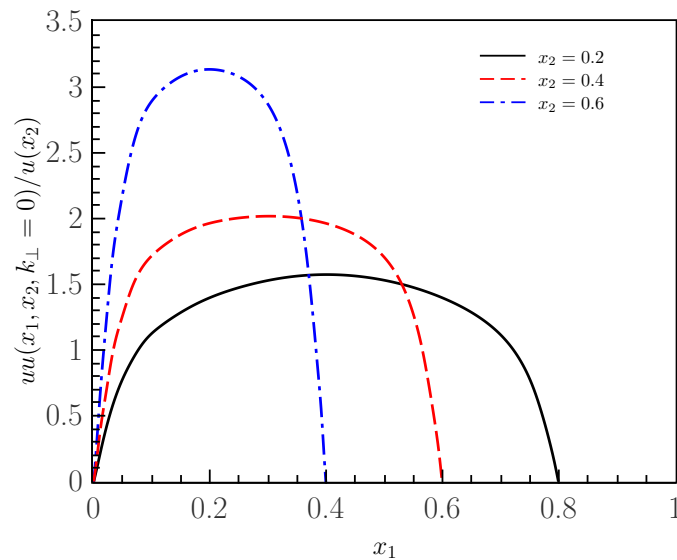
To improve the approach of our first calculation (M. Rinaldi, S.S. and V.Vento, PRD 87, 114021 (2013)), we implemented Relativity using a **Light-Front (LF)** approach. In the Relativistic Hamiltonian Dynamics (**RHD**) of an interacting system, (Dirac, 1949), one has:

- full Poincaré covariance
- fixed number of on-mass-shell constituents

Among the 3 possible forms of **RHD**, the **LF** (initial hypersurface: $x^+ = x_0 + x_3 = 0$; in standard, “Instant Form” QM: $x_0 = 0$). has several advantages. The most relevant here:

- 7 Kinematical generators (maximum number): i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii) P^+ , \mathbf{P}_\perp , iii) Rotation around z .
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion from the global one (as in the non relativistic (NR) case).
- in a specific construction of the Poincaré generators (Bakamjian-Thomas) it is possible to obtain a Mass equation, Schrödinger-like. A clear connection to NR.
- The IMF description of DIS is easily included. Systematically applied to calculate FFs, PDFs, GPDs, TMDs...

Results: $x_1 - x_2$ -factorization



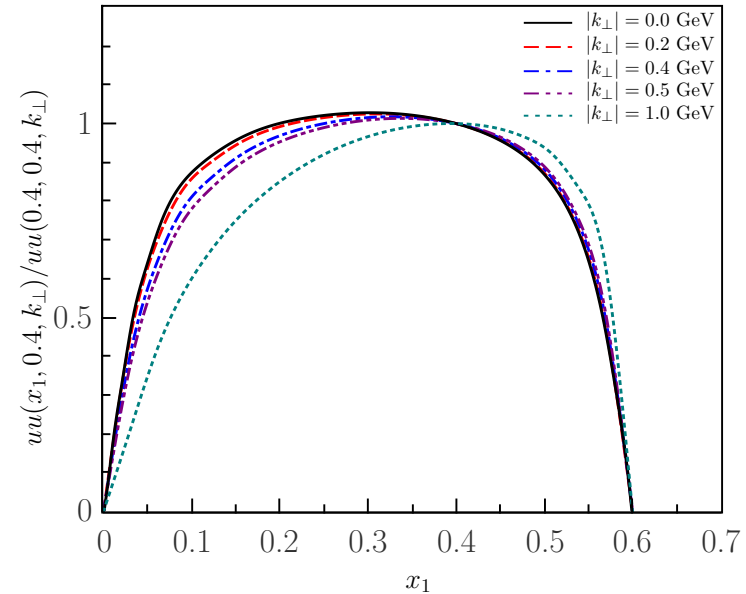
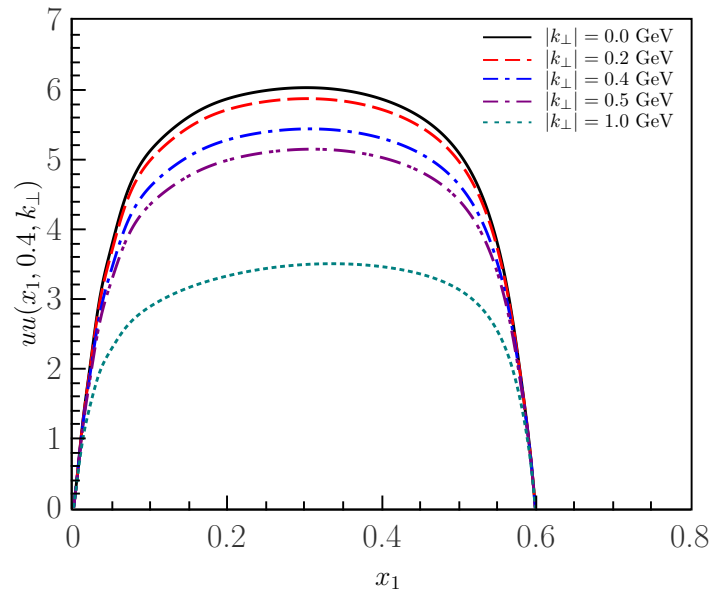
LEFT: The ratio $uu(x_1, x_2, k_\perp = 0)/u(x_2)$, for three different values of x_2 .
There should be no x_2 dependence in the ratio if the $x_1 - x_2$ -factorization were realized;

RIGHT: The ratio $uu(x_1, x_2, k_\perp = 0)/(u(x_2)u(x_1))$.

It should be 1 everywhere if the $x_1 - x_2$ -factorization were realized;

- The $x_1 - x_2$ -factorization is badly violated in the valence region
- Already found in bag model and NR calculations

Results: $(x_1, x_2) - k_\perp$ factorization



LEFT: General trend increasing k_\perp

RIGHT: The ratio $uu(x_1, x_2 = 0.4, k_\perp) / uu(x_1 = 0.4, x_2 = 0.4, k_\perp)$,
for different k_\perp values: **there should be no k_\perp dependence if factorization worked**



mildly violated as in the **NR** model (and in the Bag);

Model independent lesson

We have a fully correlated model with correct symmetries and dynamical
(non-factorized) k_\perp dependence. So far, at the model scale μ_0^2 ...

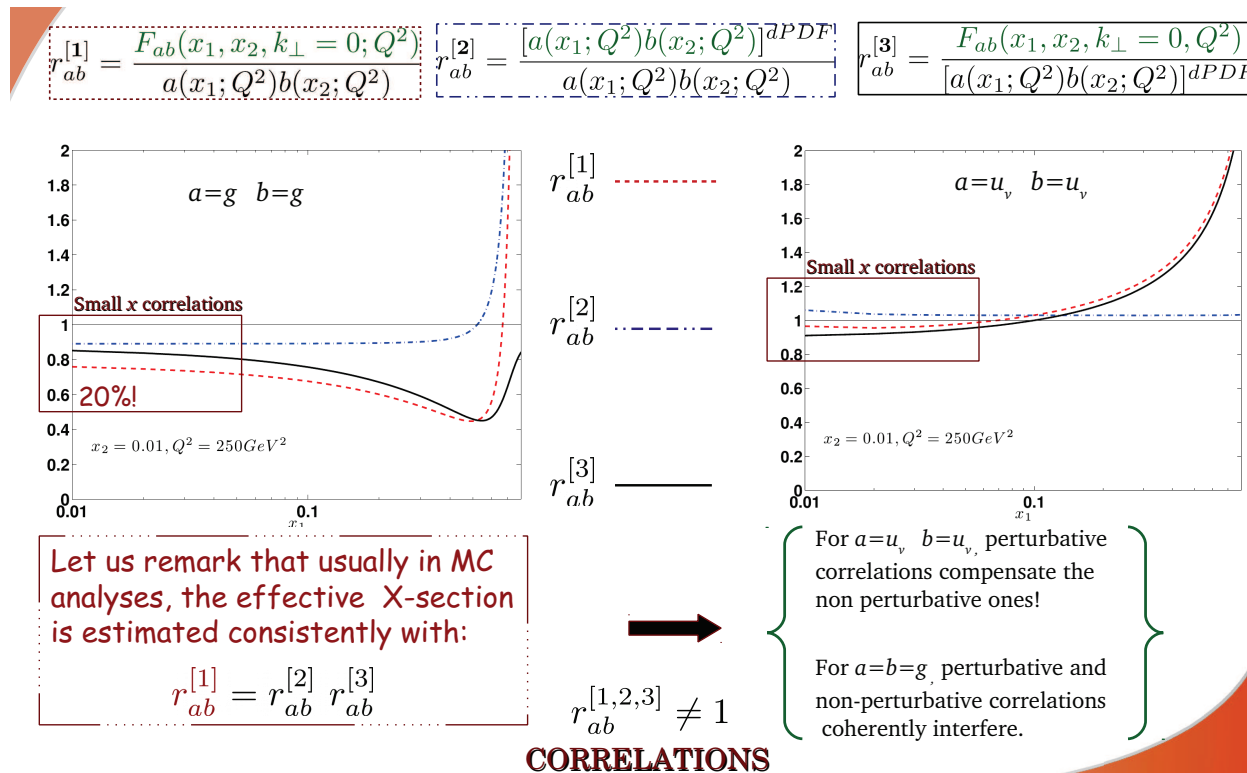
pQCD evolution of the LF dPDFs

M. Rinaldi, S.S., M. Traini and V.Vento, JHEP 10 (2016) 063

dPDFs evolution known (Kirshner et al., 1982...).

We solved the LO evolution equations by inversion of double Mellin transforms.

Do correlations survive after evolution?



Strong correlations in the valence region also at Q^2 . Weaker, but sizeable, at low x .

Found also for polarized dPDFs (e.g., M. Diehl, T. Kasemets and S. Keane, JHEP 05 (2014) 118)

Comparison with existing data?

The effective Cross Section σ_{eff}

Defined through:

$$\sigma_{double}^{pp} = \frac{m}{2} \frac{\sigma_A^{pp'} \sigma_B^{pp'}}{\sigma_{eff}}$$

σ_{eff} is the only measured quantity

$$\sigma_A^{pp'}(x_1, x'_1) = \text{single scattering} = \sum_{i,k} q_i^p(x_1) q_k^{p'}(x'_1) \hat{\sigma}_{ik}^A(x_1, x'_1),$$

$$\begin{aligned} \sigma_{double}^{pp}(x_1, x'_1, x_2, x'_2) &= \frac{m}{2} \sum_{i,j,k,l} \int D_{ij}(x_1, x_2; \mathbf{k}_\perp) \hat{\sigma}_{ik}^A(x_1, x'_1) \\ &\times \hat{\sigma}_{jl}^B(x_2, x'_2) D_{kl}(x'_1, x'_2; -\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}. \end{aligned}$$

If (reasonable!): $\hat{\sigma}_{ij}(x, x') = C_{ij} \bar{\sigma}(x, x')$ with $C_{gg} : C_{qg} : C_{qq} = 1 : (4/9) : (4/9)^2$ then

$$\sigma_{eff}(x_1, x'_1, x_2, x'_2) = \frac{\sum_{i,k,j,l} q_i(x_1) q_k(x'_1) q_j(x_2) q_l(x'_2) C_{ik} C_{jl}}{\sum_{i,j,k,l} C_{ik} C_{jl} \int D_{ij}(x_1, x_2; \mathbf{k}_\perp) D_{kl}(x'_1, x'_2; -\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}}$$

BUT \longrightarrow if $D_{ij} = q q f(\mathbf{k}_\perp) \longrightarrow \sigma_{eff} = \frac{1}{\int f^2(\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}} = \frac{1}{\int \tilde{f}^2(\mathbf{b}_\perp) d\mathbf{b}_\perp}$

No x —dependence, no scale dependence... No correlations

σ_{eff} : experimental situation

- model dependent extractions;
 σ_{pp}^{double} not measured...
- Older data at lower \sqrt{S}
- “constant” (large errorbars)
- Different ranges in x_i accessed in different experiments.

Kinematics:

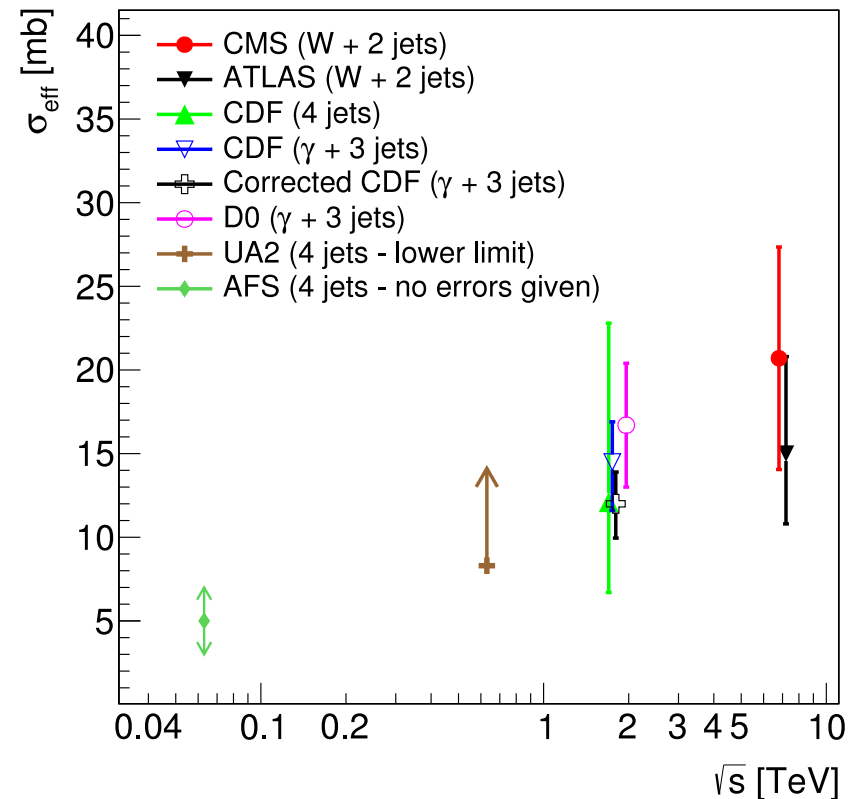
$$x_{1,2} = \sqrt{\tau} e^{\pm y} \quad \tau = x_1 x_2 = \frac{s}{S} \quad y = \frac{1}{2} \ln \frac{x_1}{x_2} \simeq \eta = -\ln \left(\tan \frac{\theta}{2} \right)$$

High x for hard jets (heavy particles detected, large partonic s)

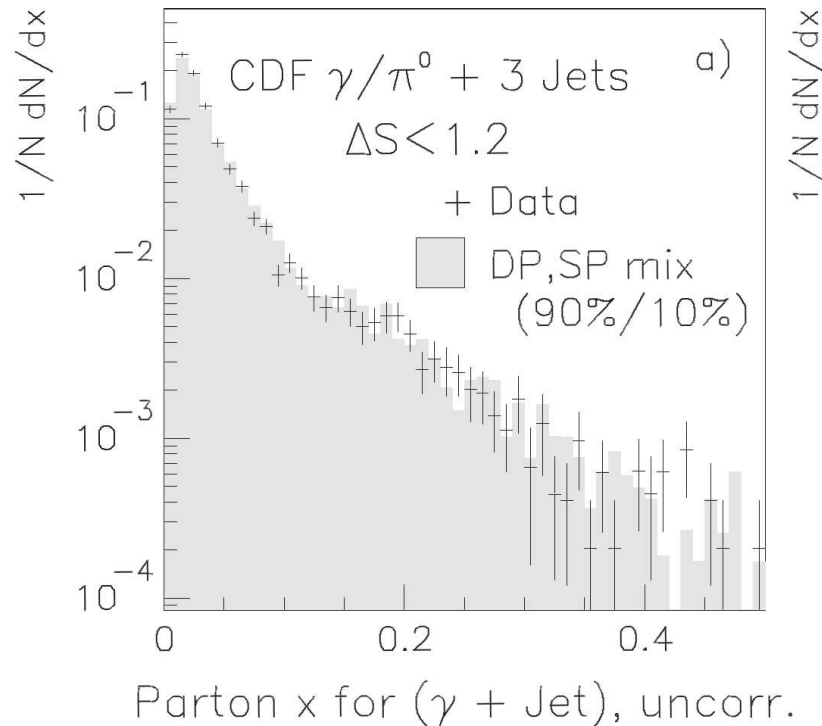
For example: AFS, $y \simeq 0$, $x_1 = x_2$ in $[0.2, 0.3]$

CDF: x_1, x_2, x'_1, x'_2 in $[0.02, 0.4]$

Valence region included...



σ_{eff} : x dependence (?)



CDF, F. Abe et al.
PRD 56, 3811 (1997)

Shaded area: Montecarlo *without* correlations in x

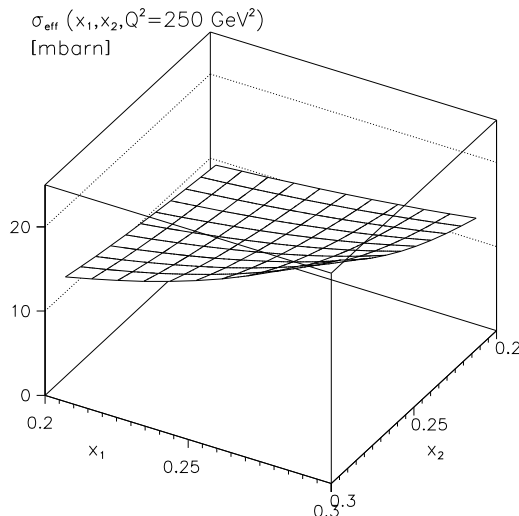
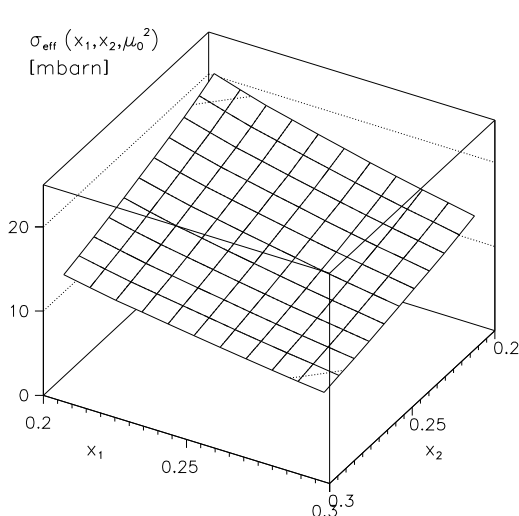
Data well described (?) for x_1, x_2, x'_1, x'_2 in $[0.02, 0.4]$ (also in the valence region...)

May be not enough accuracy for high x ? No x dependence?

Actually, our understanding is that, in the valence region, x dependence has to be seen. Let the model guide us...

σ_{eff} : x dependence from the LF model

(M. Rinaldi, S.S., M. Traini, V. Vento PLB 752 (2016) 40)



Shown at $y \simeq 0$, $x_i = x'_i$ in $[0.2, 0.3]$, at μ_0^2 (left) and at $Q^2=250 \text{ GeV}^2$ (AFS kin.)
Not constant at all. A factor of 2 easily found. **Expected!** Remember:

$$\sigma_{eff}(x_1, x_2) = \frac{\sum_{i,k,j,l} q_i(x_1) q_k(x_1) q_j(x_2) q_l(x_2) C_{ik} C_{jl}}{\sum_{i,j,k,l} C_{ik} C_{jl} \int D_{ij}(x_1, x_2; \mathbf{k}_\perp) D_{kl}(x_1, x_2; -\mathbf{k}_\perp) \frac{d\mathbf{k}_\perp}{(2\pi)^2}}$$

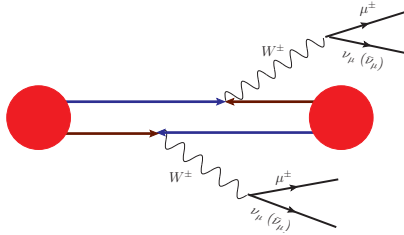
Numerator and denominator decrease with x with different velocity (one must have $x_1 + x_2 > 1 \rightarrow D_{12}(x_1, x_2, k_\perp) = 0$). **Model independent result!**

It ranges between 10 and 20 mbarn... But taking the global average in $x_{1,2}$, we get **10.9 mbarn...** (Encouraging agreement! but this is a model dependent result...)

x dependence of σ_{eff} and 3D proton structure

- The x dependence of σ_{eff} , recently found also in AdS/QCD (M. Traini et al. PLB 752 (2016) 40), could give information on the 3D nucleon structure
- Our model calculation shows that either such a dependence is found or something is not well posed in the definition of σ_{eff}
- Other Authors, with different arguments, reach similar conclusions (M. Diehl, talk at DIS 2013; Calucci Treleani 1999; Frankfurt, Strikman, Weiss 2003, 2004)
- This information is a very interesting, DYNAMICAL, one: we could learn if fast (slow) partons like to stay close to (far from) each other (in transverse plane)
- **Accessible in specific processes?**

σ_{eff} : x dependence from W^+W^+ , W^-W^- production
(F.A. Ceccopieri, M. Rinaldi, S.S., arXiv:1702.05363[hep-ph])



$$\sigma_{eff} = \frac{m}{2} \frac{\sigma_{SPS}^{pp} \sigma_{SPS}^{pp}}{\sigma_{DPS}^{pp}}$$

$$\sigma_{DPS}^{pp} = \frac{d^4 \sigma^{pp \rightarrow \mu^\pm \mu^\pm X}}{d\eta_1 dp_{T,1} d\eta_2 dp_{T,2}} = \sum_{i,k,j,l} \frac{1}{2} \int d^2 \vec{b}_\perp D_{ij}(x_1, x_2, \vec{b}_\perp, M_W) D_{kl}(x_3, x_4, \vec{b}_\perp, M_W)$$

$$\frac{d^2 \sigma_{ik}^{pp \rightarrow \mu^\pm X}}{d\eta_1 dp_{T,1}} \frac{d^2 \sigma_{jl}^{pp \rightarrow \mu^\pm X}}{d\eta_2 dp_{T,2}} \mathcal{J}(\eta_i, p_{T,i}) .$$

$\mathcal{J}(\eta_i, p_{T,i})$ implements experimental cuts; D_{kl} from LF model.

$$\sigma_{SPS}^{pp} = \frac{d^2 \sigma^{pp \rightarrow W^- (\rightarrow l^- \bar{\nu}) X}}{d\eta dE_T} = \frac{G_F^2}{6s\Gamma_W} \frac{1}{\sqrt{A^2 - 1}} V_{D\bar{U}}^2 \left[f_D(x_a) f_{\bar{U}}(x_b) u^2 + f_{\bar{U}}(x_a) f_D(x_b) t^2 \right]$$

$A = M_W / (2E_T)$; $D = d, s, b$; $U = u, c, t$, f from LF model

σ differential in lepton pseudo-rapidity, η , and E_T (ν unobserved)

σ_{eff} : x dependence in W^+W^+ , W^-W^- production @ LHC
(F.A. Ceccopieri, M. Rinaldi, S.S., arXiv:1702.05363[hep-ph])

$$pp, \sqrt{s} = 13 \text{ TeV}$$

$$p_{T,\mu}^{leading} > 20 \text{ GeV}, \quad p_{T,\mu}^{subleading} > 10 \text{ GeV}$$

$$|p_{T,\mu}^{leading}| + |p_{T,\mu}^{subleading}| > 45 \text{ GeV}$$

$$|\eta_\mu| < 2.4$$

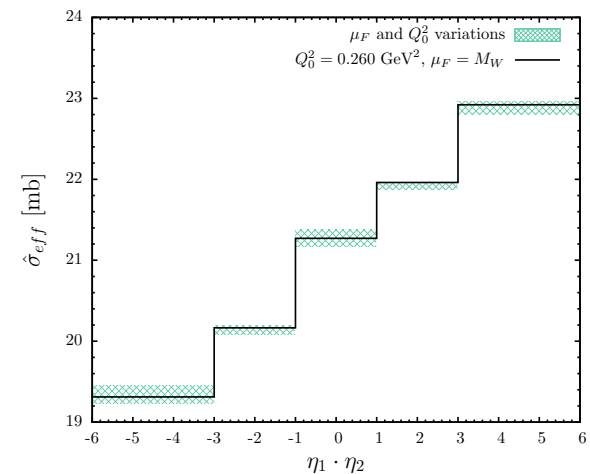
$$20 \text{ GeV} < M_{inv} < 75 \text{ GeV} \text{ or } M_{inv} > 105 \text{ GeV}$$

- Cuts mutuated from : CMS Collaboration, CMS-PAS-FSQ-13-001



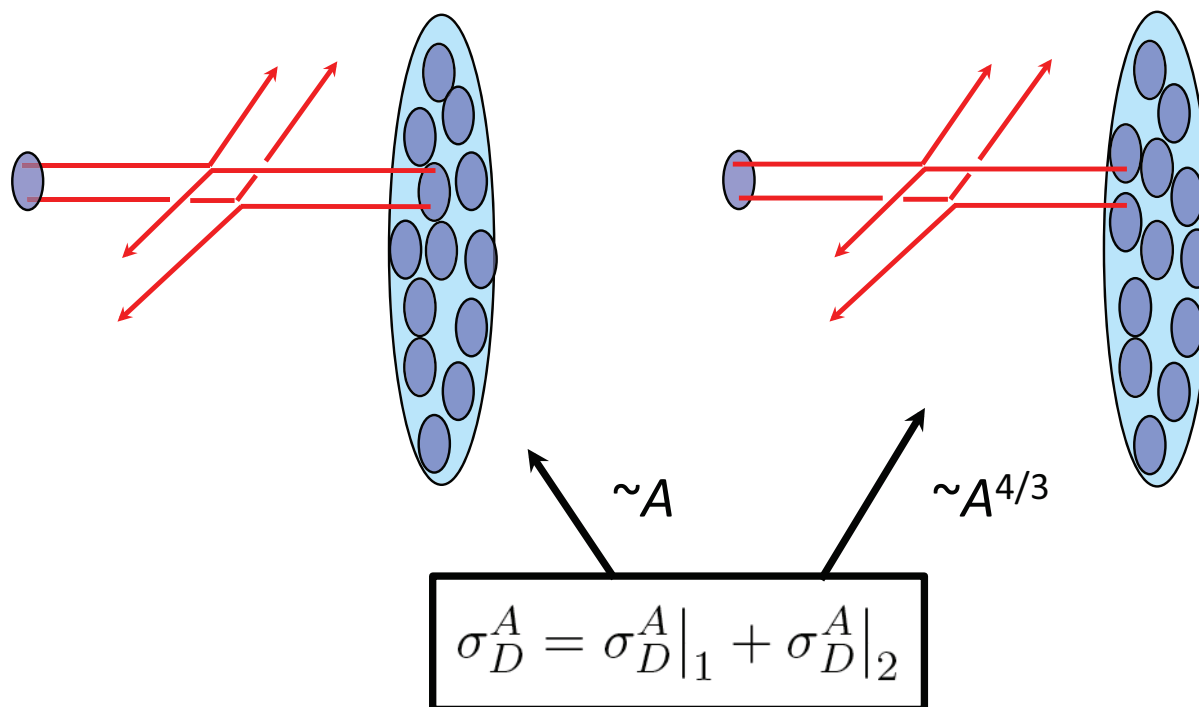
Within this particular model,
non-constant $\hat{\sigma}_{eff}$
could be appreciated IF:
 $\mathcal{L} > 1000 \text{ fb}^{-1}$ at 68% C.L.

$$\langle \hat{\sigma}_{eff} \rangle = 21.04 \pm_{-0.07}^{+0.07} (\delta Q_0) \pm_{-0.07}^{+0.06} (\delta \mu_F) \text{ mb}$$



DPS in $p - A$ Collisions

Significant enhancement of DPS in $p - A$ collisions, due to the fact that, in the nucleus, the two interacting partons can originate from two different nucleons:

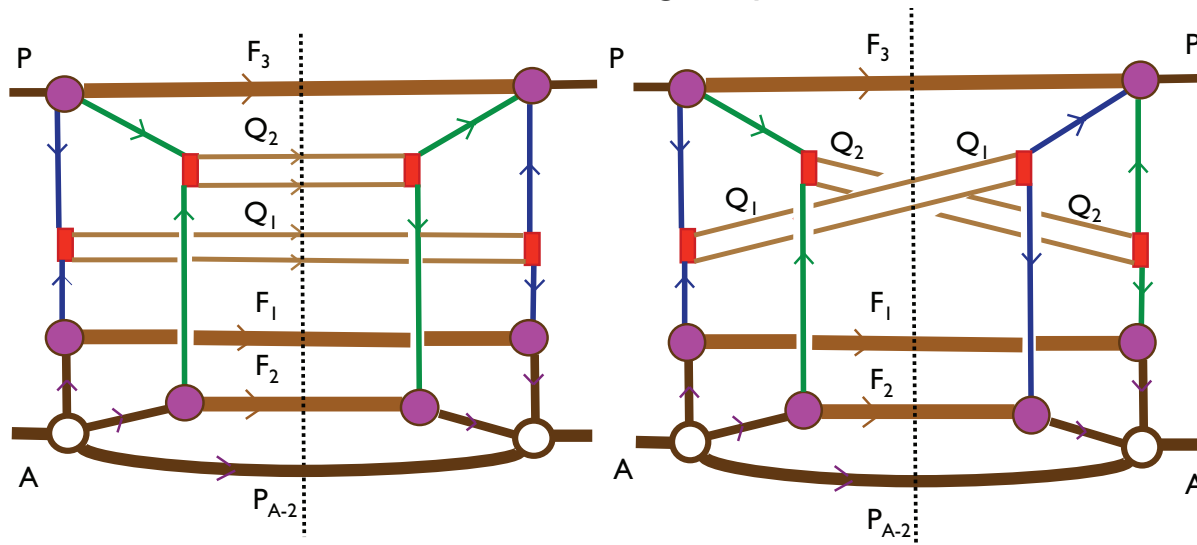


Easier access to DPS with respect to $p - p$

see M. Strikman and D. Treleani, PRL 88 (2002) 031801

GTMDs from DPS in $p - A$ Collisions (?)

Two terms in the forward scattering amplitude, in case of two active target nucleons



When the two target partons are identical, in addition to the usually considered diagonal term one needs in fact also an interference term (S. Salvini, D. Treleani et al. PRD 89 (2014) 016020), which contains naturally the function

$$\hat{H}(x, \xi, \tilde{\mathbf{b}}, \Delta^2) = \int d\mathbf{k}_T e^{i\mathbf{k}_T \cdot \tilde{\mathbf{b}}} H(x, \xi, \mathbf{k}_T, \Delta^2) ,$$

i.e., it is sensitive to a (F.T.) GTMD (for $\Delta_{\perp} = 0$), not easily accessed otherwise.

Probably very difficult to be extracted but the information is there...

...This is just an aspect of a wide subject

- MPI: a fast-moving field; annual conference:



- Crucial theoretical aspects still under debate:
 - * Proof of factorization (encouraging results by M. Diehl et al. JHEP 1 2016 76)
 - * Contribution of “inhomogeneous” evolution to DPS (A. Snigirev, B. Blok, M. Strikman, M. Diehl, J. Gaunt...)
 - * Polarization effects (e.g., M. Echevarria et al. JHEP 4 (2015) 34), color correlations (X. Artru et al, PRD 37 (1988) 1628)...
- A white paper to be published in 2017, edited by P. Bartalini and J. Gaunt (some 25 invited authors)

Conclusions

Aspects of two directions in 3D nuclear structure studies:

Nuclear Tomography from DVCS

- * Interesting ^3He ... At the future EIC?
- * Good prospects for ^4He @JLab

Nuclear “Imaging” (?) from DPS: a fascinating subject rediscovered thanks to the LHC

- * parton correlation studies in p - p scattering
- * prospects for parton correlation studies in p - A scattering

Nuclear effects - the binding

General IA formula: $H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int_x^1 \frac{dz}{z} h_N^A(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right)$

where

$$h_N^A(z, \xi, \Delta^2) = \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(z + \xi - \frac{p^+}{P^+}\right)$$

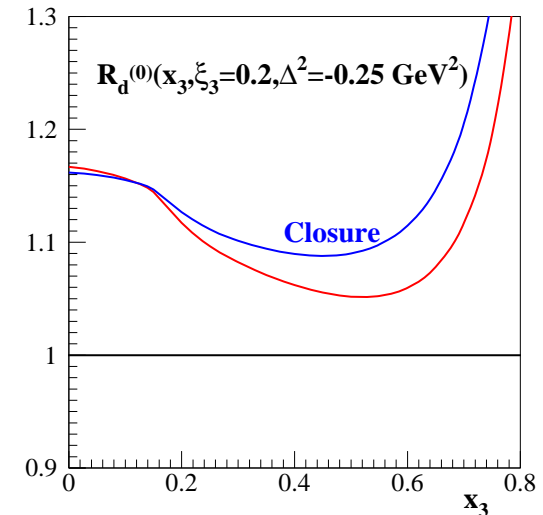
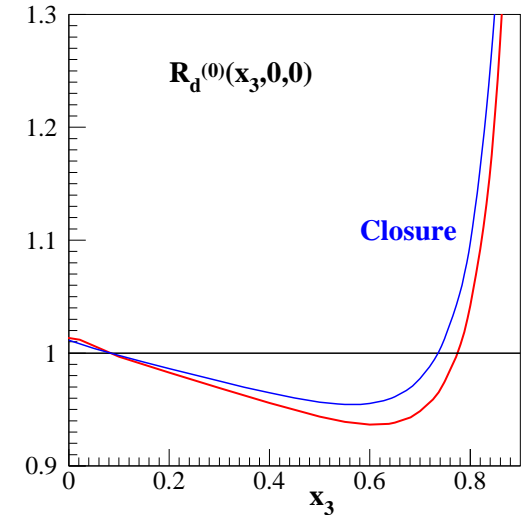
$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \bar{\sum}_M \sum_{s,f} \langle \vec{P}' M | \vec{P}_f, (\vec{p} + \vec{\Delta}) s \rangle \\ \times \langle \vec{P}_f, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_f^*)$$

using the Closure Approximation, $E_f^* = \bar{E}$:

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) \simeq \bar{\sum}_M \sum_s \langle \vec{P}' M | a_{\vec{p}+\vec{\Delta},s} a_{\vec{p},s}^\dagger | \vec{P} M \rangle$$

$$\delta(E - E_{min} - \bar{E}) = \\ = n(\vec{p}, \vec{p} + \vec{\Delta}) \delta(E - E_{min} - \bar{E}) ,$$

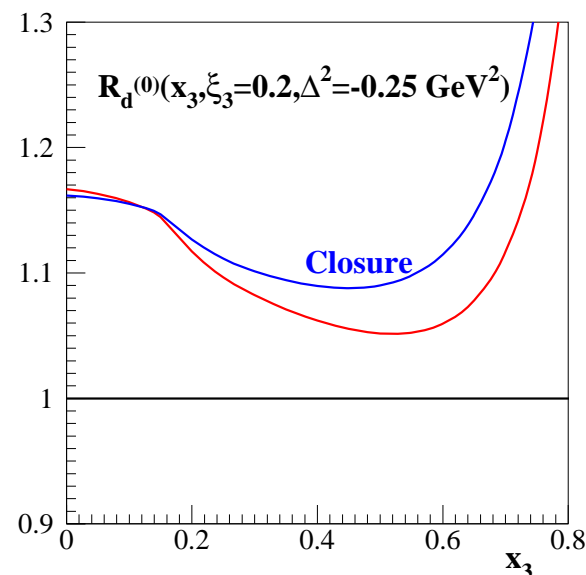
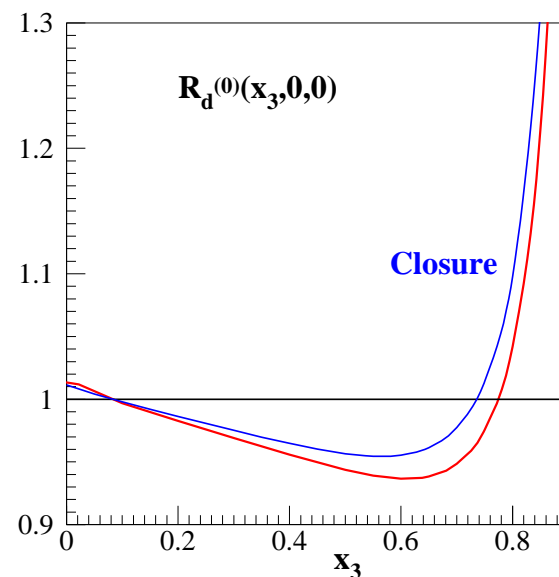
Spectral function substituted by a Momentum distribution



Nuclear effects - the binding

Nuclear effects are bigger than in the forward case: dependence on the binding

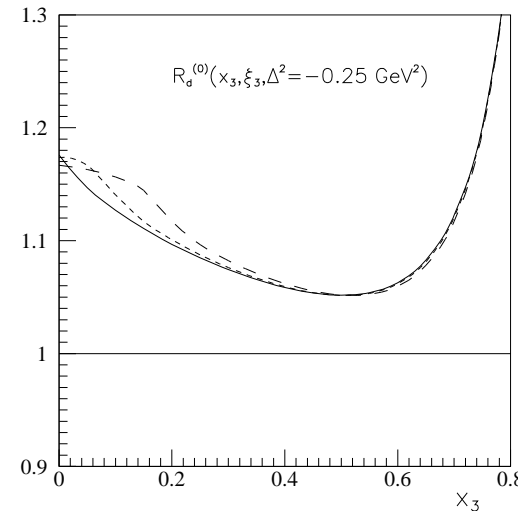
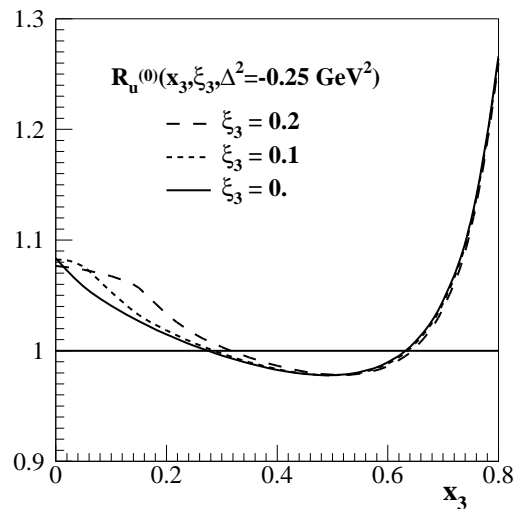
- In calculations using $n(\vec{p}, \vec{p} + \vec{\Delta})$ instead of $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$, in addition to the IA, also the Closure approximation has been assumed;
- 5 % to 10 % binding effect between $x = 0.4$ and 0.7 - much bigger than in the forward case;
- for $A > 3$, the evaluation of $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$ is difficult - such an effect is not under control: Conventional nuclear effects can be mistaken for exotic ones;
- for ${}^3\text{He}$ it is possible : this makes it a unique target, even among the Few-Body systems.



Example of nuclear effects - flavor dependence



Nuclear effects are bigger for the d flavor rather than for the u flavor:



$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2)}$$

$$H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2)$$

$R_q^{(0)}(x, \xi, \Delta^2)$ would be one if there were no nuclear effects;



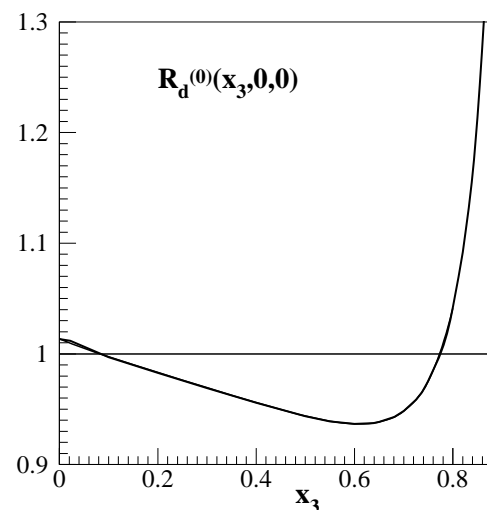
This is a typical **conventional, IA** effect (spectral functions are different for p and n in ^3He , not isoscalar!); if (not) found, clear indication on the reaction mechanism of **DIS off nuclei**.

Dependence on the NN interaction

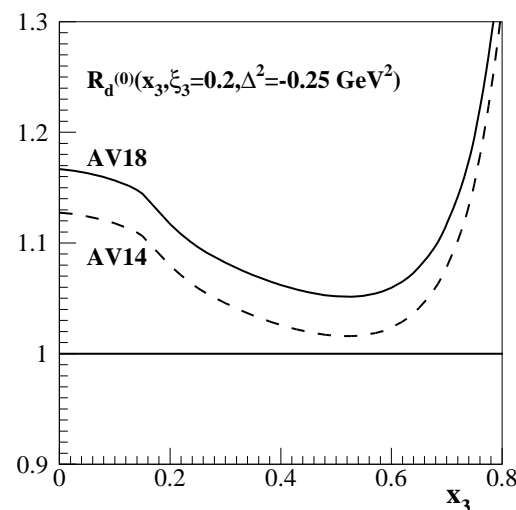
Nuclear effects are bigger than in the forward case: dependence on the potential



Forward case: Calculations using the **AV14** or **AV18** interactions are **indistinguishable**



Non-forward case: Calculations using the **AV14** and **AV18** interactions **do differ**:

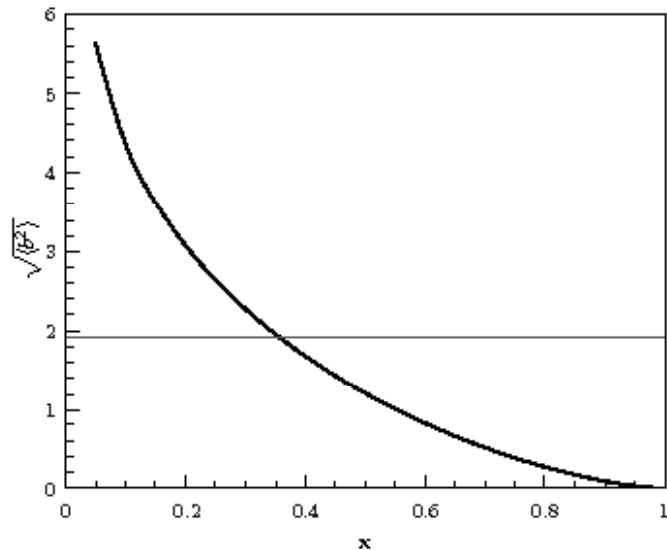


³He tomography (preliminary)

Analysis of the “ x -dependent charge radius” of ³He.

From GPDs in impact parameter space, this quantity, $\sqrt{\langle b^2 \rangle(x)}$, can be obtained from:

$$\langle b^2 \rangle(x) = \int d\vec{b}_\perp b_\perp^2 (\rho_u(x, |\vec{b}_\perp|) + \rho_d(x, |\vec{b}_\perp|))$$



reference line: $\sqrt{\langle b^2 \rangle}$ from the calculations of the ³He charge f.f. in IA. (see [L.E. Marcucci et al. PRC 58 \(1998\)](#)).

$\sqrt{\langle r^2 \rangle}$ corresponding to the Av18 calculation is similar to $\sqrt{\langle b^2(x) \rangle}$ in the valence region.

Backup: dPDF, formally

$$\begin{aligned}
 F_{q_1 q_2}(x_1, x_2, \mathbf{z}_\perp) &= -8\pi M^2 \int \frac{dz_1^+}{4\pi} \frac{dz_2^+}{4\pi} \frac{dz_3^+}{4\pi} e^{-ix_1 M z_1^+/2} e^{-ix_2 M z_2^+/2} e^{ix_1 M z_3^+/2} \\
 &\times \langle P, \mathbf{p} = \mathbf{0} | \left[\bar{q}_1 \left(z_1^+ \frac{\bar{n}}{2} + z_\perp \right) \frac{\vec{\eta}}{2} \right]_c \\
 &\times \left[\bar{q}_2 \left(z_2^+ \frac{\bar{n}}{2} \right) \frac{\vec{\eta}}{2} \right]_d q_{1,c} \left(z_3^+ \frac{\bar{n}}{2} + z_\perp \right) q_{2,d}(0) | P, \mathbf{p} = \mathbf{0} \rangle . \quad (-36)
 \end{aligned}$$

Link between dPDFs and GPDs

The **dPDF** is formally defined through the Light-cone correlator:

$$F_{12}(x_1, x_2, \vec{z}_\perp) \propto \sum_X \int dz^- \left[\prod_{i=1}^2 dl_i^- e^{ix_i l_i^- p^+} \right] \langle p | O(z, l_1) | X \rangle \langle X | O(0, l_2) | p \rangle \Big|_{\substack{\vec{l}_{1\perp} = \vec{l}_{2\perp} = 0 \\ l_1^+ = l_2^+ = z^+ = 0}}$$

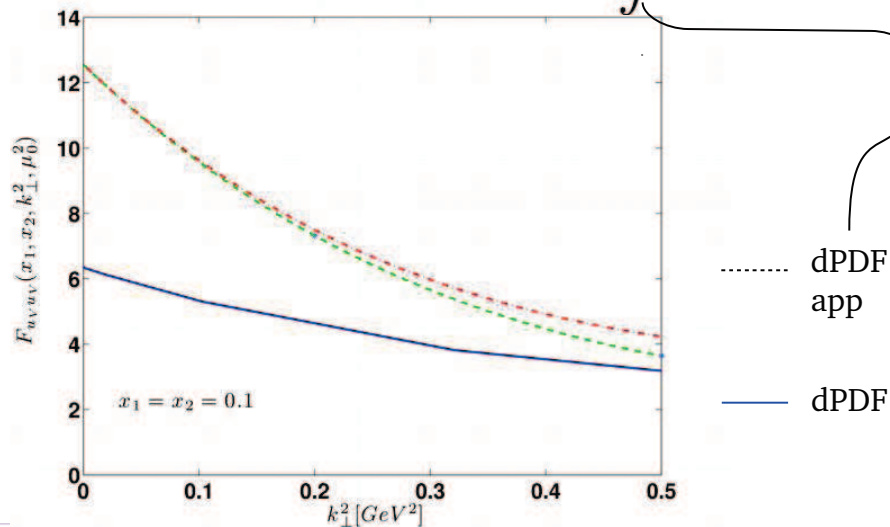
M. Diehl, D. Ostermeier, A. Schafer, JHEP 03 (2012) 089

Approximated by the proton state!

$$\int \frac{dp'^+ d\vec{p}'_\perp}{p'^+} |p'\rangle \langle p'|$$

$$F_{12}(x_1, x_2, \vec{z}_\perp) \sim \int d\vec{b} f(x_1, 0, \vec{b} + \vec{z}_\perp) f(x_2, 0, \vec{b})$$

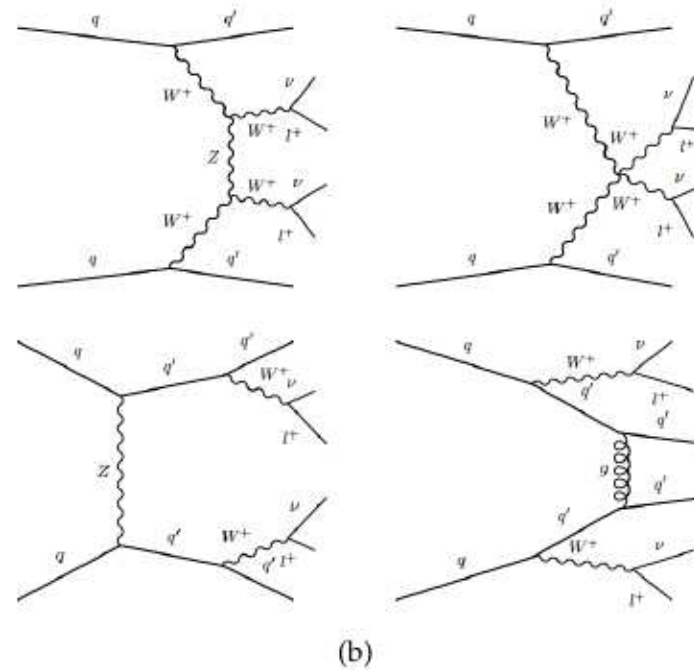
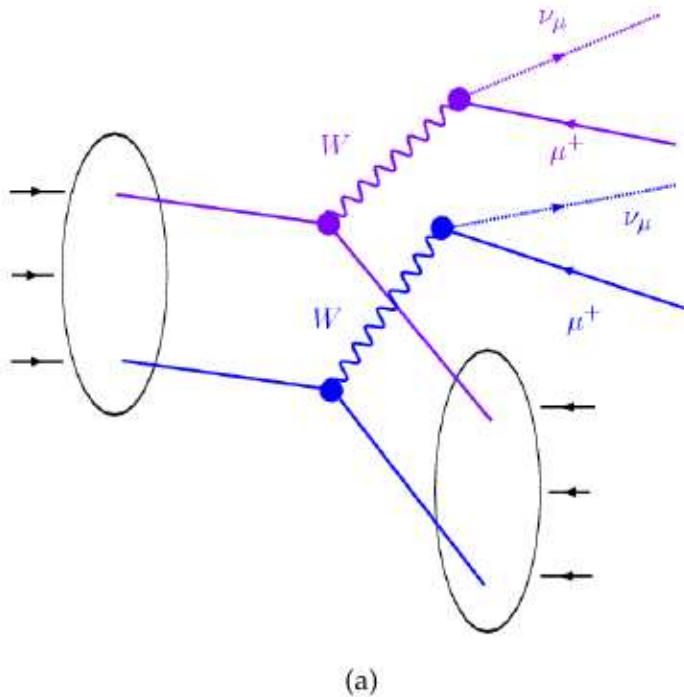
GPDs depending on the impact parameter



In GPDs, the variables \vec{b} and x are correlated!

Correlations between \vec{z}_\perp and x_1, x_2 could be present in **dPDFs** !

Backup: $W^\pm W^\pm$ from DPS (a) and SPS (b)



The preliminar cross section for charged inclusive same sign W boson production in pp collisions at 13 TeV, $E_T^\mu > 25$ GeV, $|\eta^\mu| < 2.4$ is of the order of the femtobarn, calculated within the LF model of dPDFs and evolved according to homogeneous evolution equations at fixed b .

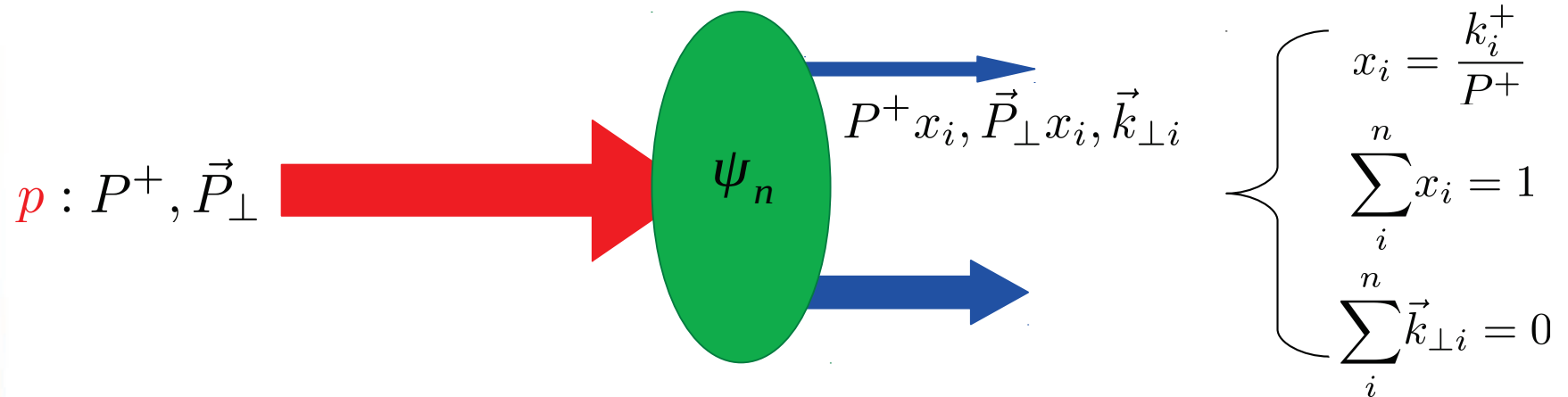
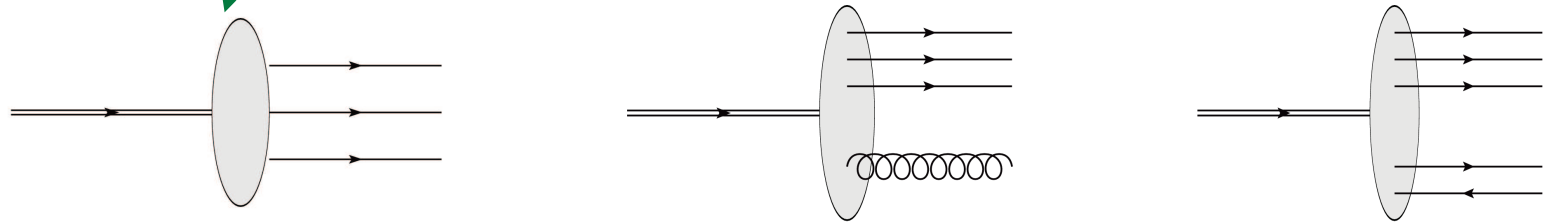
The effective cross sections calculated within this model is around 20 mb for this particular process in the given kinematics.

A Light-Front wave function representation

The proton wave function can be represented in the following way:

see e.g.: S. J. Brodsky, H. -C. Pauli, S. S. Pinsky, Phys.Rept. 301, 299 (1998)

$$|p, P^+ \vec{P}_\perp\rangle = \psi_{qqq}|qqq\rangle + \psi_{qqqg}|qqqg\rangle + \psi_{qqq\bar{q}q}|qqq\bar{q}q\rangle + \dots$$



$$\psi_n^{[l]}(x_i, \vec{k}_\perp i, \lambda_i) \longleftrightarrow \text{Invariant under LF boosts!}$$

A Light-Front wave function representation

In our approach, it is possible to connect **LF** states to **IF** ones, through the Melosh

Rotations $D_{\lambda\lambda'}^{1/2}(R_{il}(\vec{k}))$

(see, e.g., B.D. Keister, W.N. Polyzou, Adv. Nucl. Phys. 20, 225 (1991))

$$|\vec{k}_\perp, \lambda, \tau\rangle_{[l]} \propto \sqrt{2k_0} \sum_{\lambda'} D_{\lambda\lambda'}^{1/2}(R_{il}(\vec{k})) |\vec{k}_\perp, \lambda', \tau\rangle_{[i]},$$

so that a relation between the **LF** $\psi_\lambda^{[l]}$ and the **IF** $\psi_\lambda^{[i]}$ is obtained

$$\begin{aligned} \psi_\lambda^{[l]}(\beta_1, \beta_2, \beta_3) &\propto \left[\frac{\omega_1 \omega_2 \omega_3}{M_0 x_1 x_2 x_3} \right] \sum_{\mu_1 \mu_2 \mu_3} D_{\mu_1 \lambda_1}^{1/2*}(R_{il}(\vec{k}_1)) D_{\mu_2 \lambda_2}^{1/2*}(R_{il}(\vec{k}_2)) D_{\mu_3 \lambda_3}^{1/2*}(R_{il}(\vec{k}_3)) \\ &\times \psi_\lambda^{[i]}(\alpha_1, \alpha_2, \alpha_3) \end{aligned}$$

with $\beta_i = \{x_i, \vec{k}_{i\perp}, \lambda_i, \tau_i\}$, $\alpha_i = \{\vec{k}_i, \mu_i, \tau_i\}$, $\omega_i = k_{i0}$, $M_0 = \sum_i \omega_i = \sum_i \sqrt{m^2 + \vec{k}_i^2}$

Now this formalism can be used in the definition of the dPDF...

dPDFs in the Light-Front approach

quark-quark **dPDFs** are defined through Light-Cone quantized states and fields (see, e.g., M. Diehl, D. Ostermeier, A. Schäfer **JHEP 03 (2012) 089**). From that, extending a procedure used, e.g., in Pasquini, Boffi and Traini **NPB 649 (2003) 243** for GPDs, the LF dPDF is obtained, in mom. space, in the intrinsic frame $\vec{k}_1 + \vec{k}_2 + \vec{k}_3 = 0$:

$$F_{12}(x_1, x_2, \vec{k}_\perp) = 3(\sqrt{3})^3 \int \prod_{i=1}^3 d\vec{k}_i \delta\left(\sum_{i=1}^3 \vec{k}_i\right) \Phi^*(\{\vec{k}_i\}, \vec{k}_\perp) \Phi(\{\vec{k}_i\}, -\vec{k}_\perp) \\ \times \delta\left(x_1 - \frac{k_1^+}{M_0}\right) \delta\left(x_2 - \frac{k_2^+}{M_0}\right)$$

with

$$\Phi(\{\vec{k}_i\}, \vec{k}_\perp) = \Phi\left(\vec{k}_1 \pm \frac{\vec{k}_\perp}{2}, \vec{k}_2 \mp \frac{\vec{k}_\perp}{2}, \vec{k}_3\right)$$

(**NB**: here \vec{k}_\perp is the momentum conjugated to \vec{b}_\perp ; it is a *relative* momentum, it is **NOT** the argument of TMDs. Sorry for a possibly confusing notation)

$$\Phi(\vec{k}_1, \vec{k}_2, \vec{k}_3) = D^{1/2*}(R_{il}(\vec{k}_1)) D^{1/2*}(R_{il}(\vec{k}_2)) D^{1/2*}(R_{il}(\vec{k}_3)) \psi^{[i]}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$$

Now one always gets the correct support: $x_1 + x_2 > 1 \rightarrow F_{12}(x_1, x_2, k_\perp) = 0$

A model is needed for the usual IF wave function $\psi^{[i]}$

Example: dPDFs in a LF Hyper-central CQM

Hyper-central CQMs have a long tradition (see, i.e., **Giannini and Santopinto, arXiv:1501.03722**). We use here the relativistic version developed in **Faccioli, Traini, Vento NPA656,400 (1999)**. The proton w.f.

$$\psi^{[i]} = \frac{1}{\pi\sqrt{\pi}} \Psi(k_\xi) \times SU(6)_{spin-isospin}$$

is solution of the mass equation

$$(M_0 + V)\Psi(k_\xi) \equiv \left(\sum_{i=1}^3 \sqrt{m^2 + \vec{k}_i^2} - \frac{\tau}{\xi} + \kappa_l \xi \right) \Psi(k_\xi) = M\Psi(k_\xi)$$

with $k_\xi = \sqrt{2(\vec{k}_1^2 + \vec{k}_2^2 + \vec{k}_1 \cdot \vec{k}_2)}$, $\tau = 3.30$ $\kappa_l = 1.80 \text{ fm}^{-2}$ and

$$\Psi(k_\xi) = \sum_{\nu=0}^{16} c_\nu \frac{(-1)^\nu}{\alpha^3} \left[\frac{2\nu!}{(\nu+2)!} \right]^{1/2} e^{-k_\xi^2/(2\alpha^2)} \sum_{m=0}^{\nu} \frac{(-1)^m}{m!} \frac{(\nu+2)}{(\nu-m)!(m+2)!} \left(\frac{k_\xi^2}{\alpha^2} \right)^m$$

The parameters have been chosen to reproduce the light baryon spectrum. Successful despite its simplicity. Used in several calculations of PDFs and GPDs (see **Pasquini, Boffi and Traini NPB 649 (2003) 243... Traini PRD (2014) 3 034021**)