

Unpolarized and linearly-polarized gluon distributions from JIMWLK evolution

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CM, E. Petreska and C. Roiesnel, JHEP 10 (2016) 065, arXiv:1608.02577

CM, C. Roiesnel and P. Taels, in preparation

Contents of the talk

- Unpolarized and linearly-polarized gluon TMDs
TMDs = transverse-momentum-dependent distributions
- The context for this talk: forward di-jets at the LHC
heavy-quark di-jets are sensitive linearly-polarized gluons
- Gluon TMDs in the small-x limit
their (non-linear) QCD evolution can be obtained from the so-called JIMWLK equation
- Numerical results
new insight regarding the low-momentum behavior (gluon saturation regime)

Unpolarized & linearly-polarized gluon TMDs

Generic definitions

I consider only hadronic/nuclear states that are *unpolarized*

$$2 \int \frac{d\xi^+ d^2 \xi_t}{(2\pi)^3 p_A^-} e^{ixp_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) F^{j-}(0)] | A \rangle$$
$$= \frac{\delta_{ij}}{2} \mathcal{F}(x, k_t) + \left(\frac{k_i k_j}{k_t^2} - \frac{\delta_{ij}}{2} \right) \mathcal{H}(x, k_t)$$

↓

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unpolarized gluon TMD linearly-polarized gluon TMD

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unpolarized gluon TMD linearly-polarized gluon TMD

- the goal is to compute them at small x , using the Color Glass Condensate effective description of the dense parton content of the wave function, in terms of the large gluon field A^- :

$$\frac{\langle A | \cdot | A \rangle}{\langle A | A \rangle} \rightarrow \langle \cdot \rangle_x = \int D A^- |\phi_x[A^-]|^2 .$$

Small-x QCD evolution

the evolution of the gluon TMDs with decreasing x can
be computed from the so-called JIMWLK equation

$$\frac{d}{d \ln(1/x)} \langle O \rangle_x = \langle H_{JIMWLK} O \rangle_x$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

a functional RG equation that resums the leading logarithms in

$$y = \ln(1/x)$$

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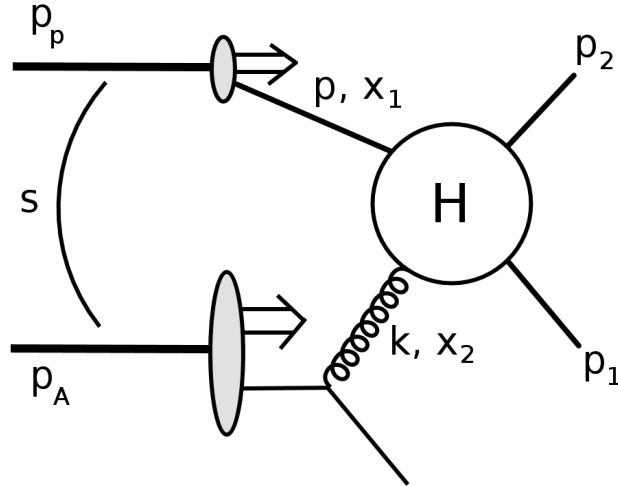
- the JIMWLK “Hamiltonian” reads:

$$H_{JIMWLK} = \int \frac{d^2\mathbf{x}}{2\pi} \frac{d^2\mathbf{y}}{2\pi} \frac{d^2\mathbf{z}}{2\pi} \frac{(\mathbf{x}-\mathbf{z}) \cdot (\mathbf{y}-\mathbf{z})}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \frac{\delta}{\delta A_c^-(\mathbf{x})} [1 + V_{\mathbf{x}}^\dagger V_{\mathbf{y}} - V_{\mathbf{x}}^\dagger V_{\mathbf{z}} - V_{\mathbf{z}}^\dagger V_{\mathbf{y}}]^{cd} \frac{\delta}{\delta A_d^-(\mathbf{y})}$$

with the adjoint Wilson line $V_{\mathbf{x}} = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) T^a \right]$

The context: forward di-jets

- large- x projectile (proton) on small- x target (proton or nucleus)



$$\langle k_{1t} \rangle \sim \Lambda_{QCD} \quad \langle k_{2t} \rangle \sim Q_s(x_2)$$

$$Q_s(x_2) \gg \Lambda_{QCD}$$

Incoming partons' energy fractions:

$$x_1 = \frac{1}{\sqrt{s}} (|p_{1t}| e^{y_1} + |p_{2t}| e^{y_2})$$

$$\xrightarrow{y_1, y_2 \gg 0}$$

$$x_1 \sim 1$$

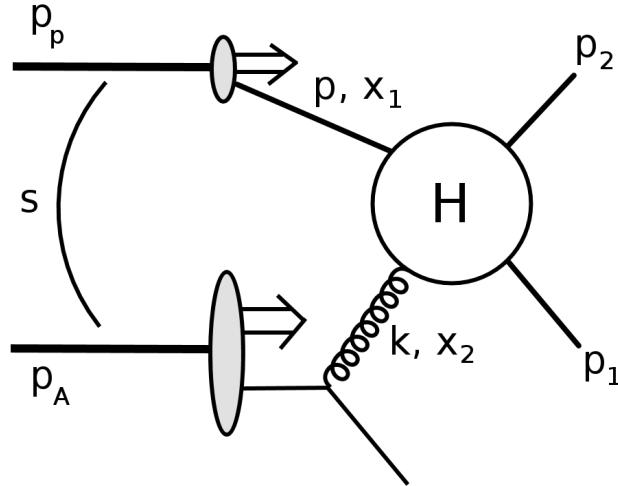
$$x_2 = \frac{1}{\sqrt{s}} (|p_{1t}| e^{-y_1} + |p_{2t}| e^{-y_2})$$

$$x_2 \ll 1$$

so-called “dilute-dense” kinematics

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Gluon's transverse momentum (p_{1t} , p_{2t} imbalance):

$$|k_t|^2 = |p_{1t} + p_{2t}|^2 = |p_{1t}|^2 + |p_{2t}|^2 + 2|p_{1t}||p_{2t}| \cos \Delta\phi$$

relevant TMD regime here : $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

The gluon TMDs involved
in the di-jet process

TMD gluon distributions

- the naive operator definition is not gauge-invariant

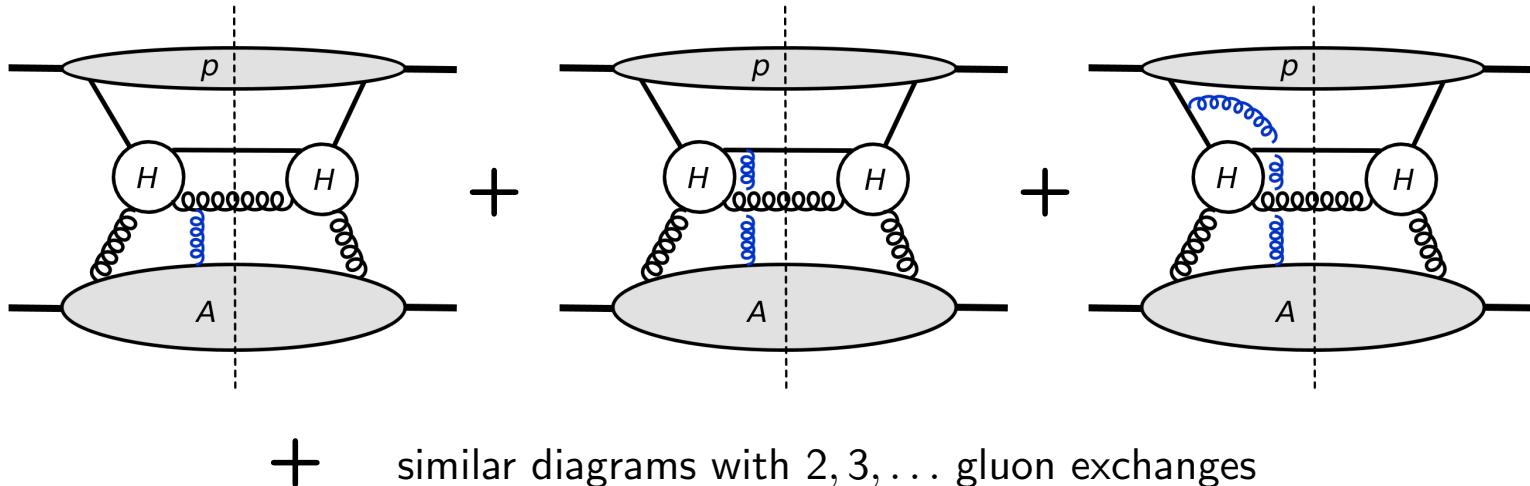
$$\mathcal{F}_{g/A}(x_2, k_t) \stackrel{\text{naive}}{=} 2 \int \frac{d\xi^+ d^2 \xi_t}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi_t} \langle A | \text{Tr} [F^{i-}(\xi^+, \xi_t) F^{i-}(0)] | A \rangle$$

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- a theoretically consistent definition requires to include more diagrams



They all contribute at leading power and need to be resummed.

this is done by including gauge links in the operator definition

Process-dependent TMDs

- the proper operator definition(s)

some gauge link $\mathcal{P} \exp \left[-ig \int_{\alpha}^{\beta} d\eta^{\mu} A^a(\eta) T^a \right]$

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- $U_{[\alpha, \beta]}$ renders gluon distribution gauge invariant

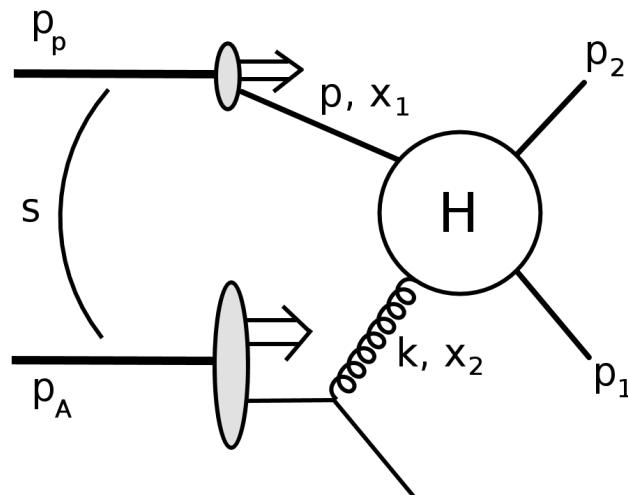
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however, the precise structure of the gauge link is process-dependent:

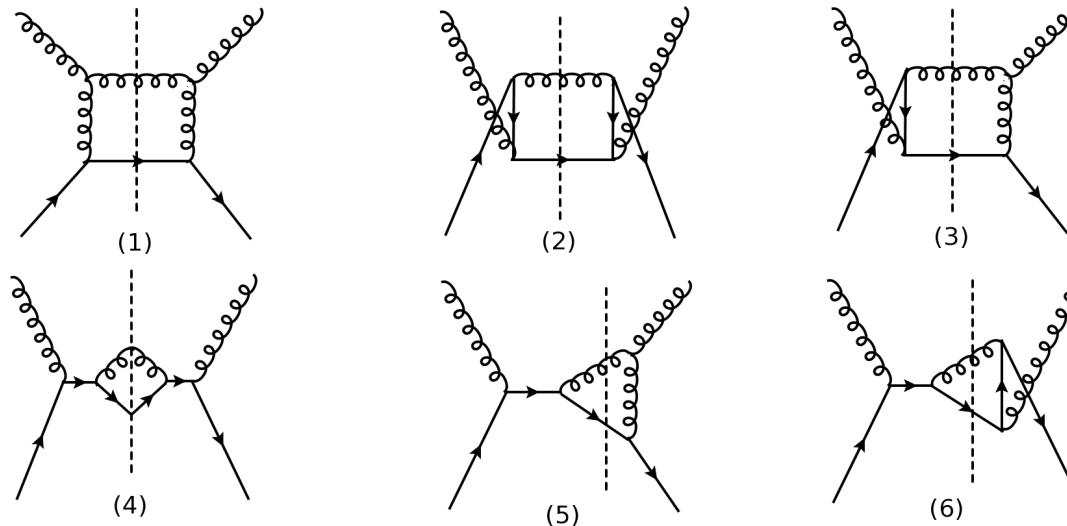
it is determined by the color structure of the hard process H

- in the large k_t limit, the process dependence of the gauge links disappears (like for the integrated gluon distribution), and a single gluon distribution is sufficient

TMDs for forward di-jets

- several gluon distributions are needed already for a single partonic sub-process

example for the $qg^* \rightarrow qg$ channel

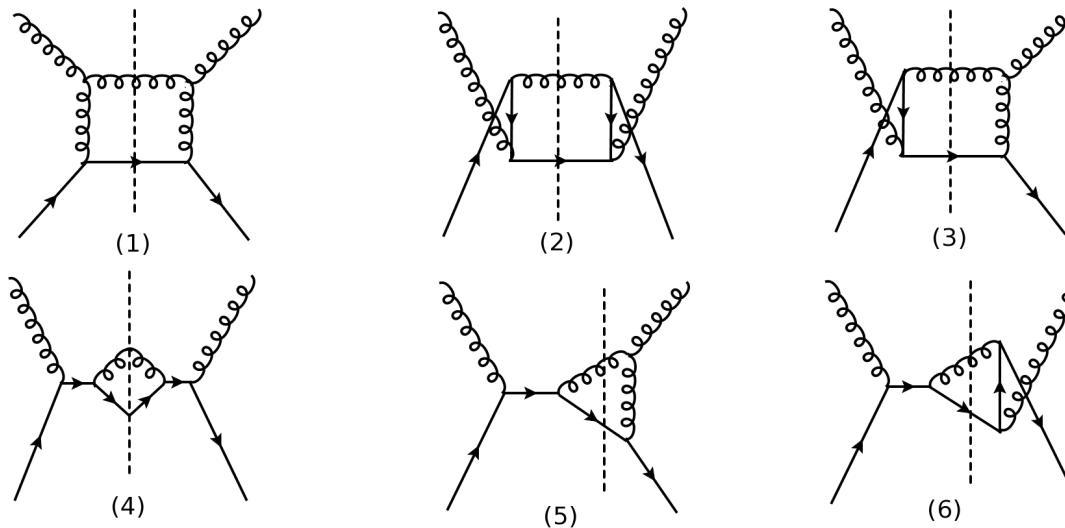


each diagram generates a different gluon distribution

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each diagram generates a different gluon distribution

2 unintegrated gluon distributions per channel, 6 in total: $\Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2)$

$$qg^* \rightarrow qg \quad gg^* \rightarrow q\bar{q} \quad gg^* \rightarrow gg \quad i = 1, 2$$

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

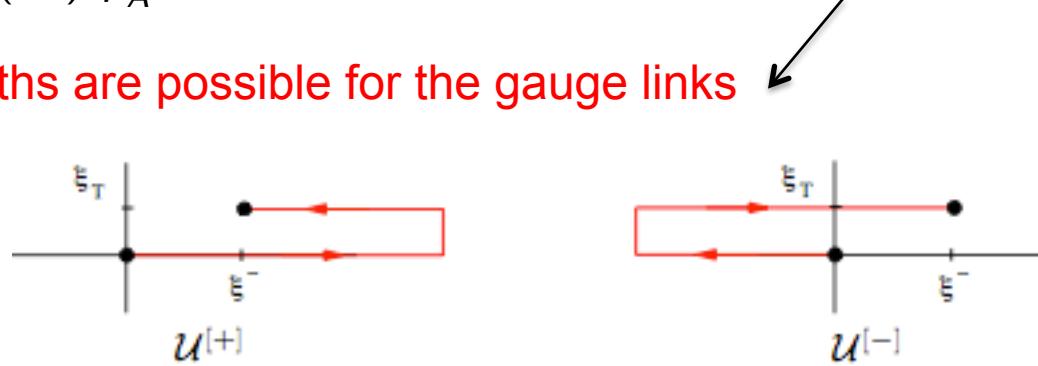
The six TMD gluon distributions

- correspond to a different gauge-link structure

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several paths are possible for the gauge links

examples :



- when integrated, they all coincide

$$\int^{\mu^2} d^2 k_t \Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2) = x_2 f(x_2, \mu^2)$$

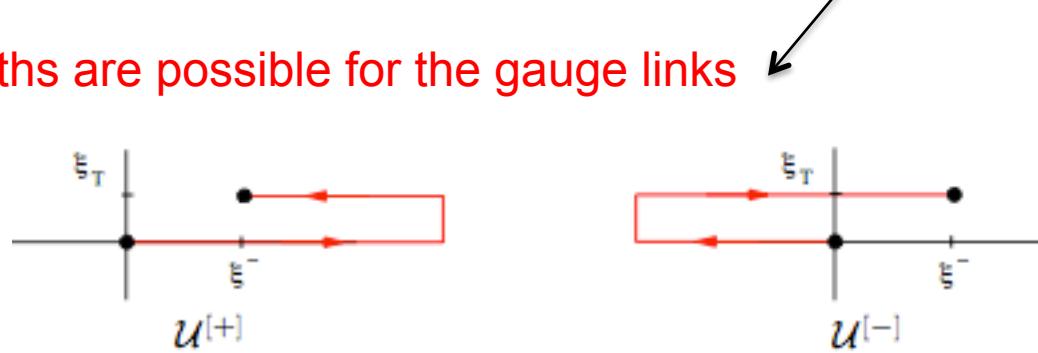
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- they are independent and in general they all should be extracted from data

only one of them has the probabilistic interpretation
of the number density of gluons at small x_2

The forward di-jet cross section

valid for $x_2 \ll x_1 \sim 1$ and $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$

$$\frac{d\sigma^{pA \rightarrow \text{dijets}+X}}{d^2P_t d^2k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/p}(x_1, \mu^2) \sum_{i=1}^2 K_{ag \rightarrow cd}^{(i)} \Phi_{ag \rightarrow cd}^{(i)} \frac{1}{1 + \delta_{cd}}$$

it involves six unpolarized gluon TMDs $\Phi_{ag \rightarrow cd}^{(i)}(x_2, k_t^2)$ (2 per channel)

their associated hard matrix elements $K_{ag \rightarrow cd}^{(i)}$ are on-shell (i.e. $k_t = 0$)

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it can be derived in two ways:

from the generic TMD factorization framework (valid up to power corrections):
by taking the small-x limit

Bomhof, Mulders and Pijlman (2006)

Kotko, Kutak, CM, Petreska, Sapeta and van Hameren (2015)

from the CGC framework (valid at small-x): by extracting the leading power

Dominguez, CM, Xiao and Yuan (2011)

CM, Petreska, Roiesnel (2016)

Heavy quark-antiquark di-jets

- for the $gg^* \rightarrow q\bar{q}$ channel, keeping a non-zero quark mass:
linearly-polarized gluon TMDs also appear, even with in unpolarized collisions !

CM, Roiesnel, Taels, in preparation

$$\begin{aligned} \frac{d\sigma(pA \rightarrow q(p_1)\bar{q}(p_2)X)}{dy_1 dy_2 d^2 p_{1t} d^2 p_{2t}} &= \frac{\alpha_s^2}{2C_F} \frac{z(1-z)}{(P_t^2 + m^2)^2} x_1 g(x_1, \mu^2) \left\{ \left(P_{qg}(z) + z(1-z) \frac{2m^2 P_t^2}{(P_t^2 + m^2)^2} \right) \right. \\ &\quad \times \left([(1-z)^2 + z^2] \mathcal{F}_{gg}^{(1)}(x_2, k_t) + 2z(1-z) \mathcal{F}_{gg}^{(2)}(x_2, k_t) - \frac{1}{N_c^2} \mathcal{F}_{gg}^{(3)}(x_2, k_t) \right) \\ &\quad + z(1-z) \frac{2m^2 P_t^2}{(P_t^2 + m^2)^2} \cos(2\phi) \\ &\quad \left. \times \left([(1-z)^2 + z^2] \mathcal{H}_{gg}^{(1)}(x_2, k_t) + 2z(1-z) \mathcal{H}_{gg}^{(2)}(x_2, k_t) - \frac{1}{N_c^2} \mathcal{H}_{gg}^{(3)}(x_2, k_t) \right) \right\} \end{aligned}$$

$$z = \frac{p_1^+}{p_1^+ + p_2^+} \quad k_t = p_{1t} + p_{2t} \quad \text{and} \quad P_t = (1-z)p_{1t} - zp_{2t}$$

linearly-polarized gluons come with a $\cos(2\phi)$ modulation
where ϕ is the angle between k_t and P_t

Evaluating the gluon TMDs at small- x

Gluon TMDs at small-x

- the gluon TMDs involved in the di-jet process are:

(showing here the $qg^* \rightarrow qg$ channel TMDs only)

$$\mathcal{F}_{qg}^{(1)} = 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - i \boldsymbol{k}_t \cdot \boldsymbol{\xi}} \left\langle \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

$$\mathcal{F}_{qg}^{(2)} = 2 \int \frac{d\xi^+ d^2\boldsymbol{\xi}}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - i \boldsymbol{k}_t \cdot \boldsymbol{\xi}} \left\langle \text{Tr} \left[F^{i-}(\xi) \frac{\text{Tr} [\mathcal{U}^{[\square]}]}{N_c} \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right\rangle$$

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- at small x they can be written as:

$$U_{\mathbf{x}} = \mathcal{P} \exp \left[ig \int_{-\infty}^{\infty} dx^+ A_a^-(x^+, \mathbf{x}) t^a \right]$$

$$\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \left\langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \right\rangle_{x_2}$$

$$\mathcal{F}_{qg}^{(2)}(x_2, |k_t|) = -\frac{4}{g^2} \int \frac{d^2x d^2y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \left\langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \text{Tr} [U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \right\rangle_{x_2}$$

these Wilson line correlators also emerge directly in CGC calculations
when $|p_{1t}|, |p_{2t}| \gg |k_t|, Q_s$ (the regime of validity of TMD factorization)

Outline of the derivation

- using $\langle p|p' \rangle = (2\pi)^3 2p^- \delta(p^- - p'^-) \delta^{(2)}(p_t - p'_t)$ and translational invariance

$$\int \frac{d\xi^+ d^2 \xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \langle A | O(0, \xi) | A \rangle = \frac{2}{\langle A | A \rangle} \int \frac{d^3 \xi d^3 \xi'}{(2\pi)^3} e^{ix_2 p_A^- (\xi^+ - \xi'^+) - ik_t \cdot (\xi - \xi')} \langle A | O(\xi', \xi) | A \rangle .$$

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- setting $\exp[ix_2 p_A^- (\xi^+ - \xi'^+)] = 1$ and denoting $\frac{\langle A|O(\xi', \xi)|A\rangle}{\langle A|A \rangle} = \langle O(\xi', \xi) \rangle_{x_2}$

we obtain e.g.

$$\mathcal{F}_{qg}^{(1)}(x_2, k_t) = 4 \int \frac{d^3 x d^3 y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} \left[F^{i-}(x) \mathcal{U}^{[-]\dagger} F^{i-}(y) \mathcal{U}^{[+]} \right] \right\rangle_{x_2}$$

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- then performing the x^+ and y^+ integrations using

$$\partial_i U_{\mathbf{y}} = ig \int_{-\infty}^{\infty} dy^+ U[-\infty, y^+; \mathbf{y}] F^{i-}(y) U[y^+, +\infty; \mathbf{y}]$$

we finally get $\mathcal{F}_{qg}^{(1)}(x_2, |k_t|) = \frac{4}{g^2} \int \frac{d^2 x d^2 y}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x} - \mathbf{y})} \left\langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \right\rangle_{x_2}$

The other (unpolarized) TMDs

- involved in the $gg^* \rightarrow q\bar{q}$ and $gg^* \rightarrow gg$ channels

$$\mathcal{F}_{gg}^{(1)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \frac{1}{N_c} \left\langle A \left| \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \text{Tr} \left[\mathcal{U}^{[\square]\dagger} \right] \right| A \right\rangle ,$$

$$\mathcal{F}_{gg}^{(2)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \frac{1}{N_c} \left\langle A \left| \text{Tr} \left[F^{i-}(\xi) \mathcal{U}_\xi^{[\square]\dagger} \right] \text{Tr} \left[F^{i-}(0) \mathcal{U}_0^{[\square]} \right] \right| A \right\rangle ,$$

$$\mathcal{F}_{gg}^{(3)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \left\langle A \left| \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \right| A \right\rangle ,$$

$$\mathcal{F}_{gg}^{(4)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \left\langle A \left| \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[-]\dagger} F^{i-}(0) \mathcal{U}^{[-]} \right] \right| A \right\rangle ,$$

$$\mathcal{F}_{gg}^{(5)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \left\langle A \left| \text{Tr} \left[F^{i-}(\xi) \mathcal{U}_\xi^{[\square]\dagger} \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}_0^{[\square]} \mathcal{U}^{[+]} \right] \right| A \right\rangle ,$$

$$\mathcal{F}_{gg}^{(6)}(x_2, k_t) = 2 \int \frac{d\xi^+ d^2\xi}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_t \cdot \xi} \frac{1}{N_c^2} \left\langle A \left| \text{Tr} \left[F^{i-}(\xi) \mathcal{U}^{[+]\dagger} F^{i-}(0) \mathcal{U}^{[+]} \right] \text{Tr} \left[\mathcal{U}^{[\square]} \right] \text{Tr} \left[\mathcal{U}^{[\square]\dagger} \right] \right| A \right\rangle .$$

Bomhof, Mulders and Pijlman (2006)

The other TMDs at small-x

- involved in the $gg^* \rightarrow q\bar{q}$ and $gg^* \rightarrow gg$ channels

$$\begin{aligned}\mathcal{F}_{gg}^{(1)}(x_2, k_t) &= \frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \langle \text{Tr} [(\partial_i U_{\mathbf{y}})(\partial_i U_{\mathbf{x}}^\dagger)] \text{Tr} [U_{\mathbf{x}} U_{\mathbf{y}}^\dagger] \rangle_{x_2}, \\ \mathcal{F}_{gg}^{(2)}(x_2, k_t) &= -\frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger] \text{Tr} [(\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \rangle_{x_2}, \\ \mathcal{F}_{gg}^{(4)}(x_2, k_t) &= -\frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{x}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{y}}^\dagger] \rangle_{x_2}, \\ \mathcal{F}_{gg}^{(5)}(x_2, k_t) &= -\frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger U_{\mathbf{x}} U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \rangle_{x_2}, \\ \mathcal{F}_{gg}^{(6)}(x_2, k_t) &= -\frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \frac{1}{N_c^2} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \text{Tr} [U_{\mathbf{x}} U_{\mathbf{y}}^\dagger] \text{Tr} [U_{\mathbf{y}} U_{\mathbf{x}}^\dagger] \rangle_{x_2}.\end{aligned}$$

with a special one singled out: the Weizsäcker-Williams TMD

$$\mathcal{F}_{gg}^{(3)}(x_2, k_t) = -\frac{4}{g^2} \int \frac{d^2 \mathbf{x} d^2 \mathbf{y}}{(2\pi)^3} e^{-ik_t \cdot (\mathbf{x}-\mathbf{y})} \langle \text{Tr} [(\partial_i U_{\mathbf{x}}) U_{\mathbf{y}}^\dagger (\partial_i U_{\mathbf{y}}) U_{\mathbf{x}}^\dagger] \rangle_{x_2}$$

Evolution of the "dipole" TMD

(in a mean-field type approximation)

- the Balitsky-Kovchegov (BK) evolution

Balitsky (1996), Kovchegov (1998)

$$\frac{d}{dY} f_Y(k) = \bar{\alpha} \int \frac{dk'^2}{k'^2} \left[\underbrace{\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}}}_{\text{BFKL}} \right] - \bar{\alpha} f_Y^2(k)$$
$$Y = \ln \left(\frac{1}{x} \right) \quad \bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

non-linearity important
when the gluon density
becomes large

Evolution of the "dipole" TMD

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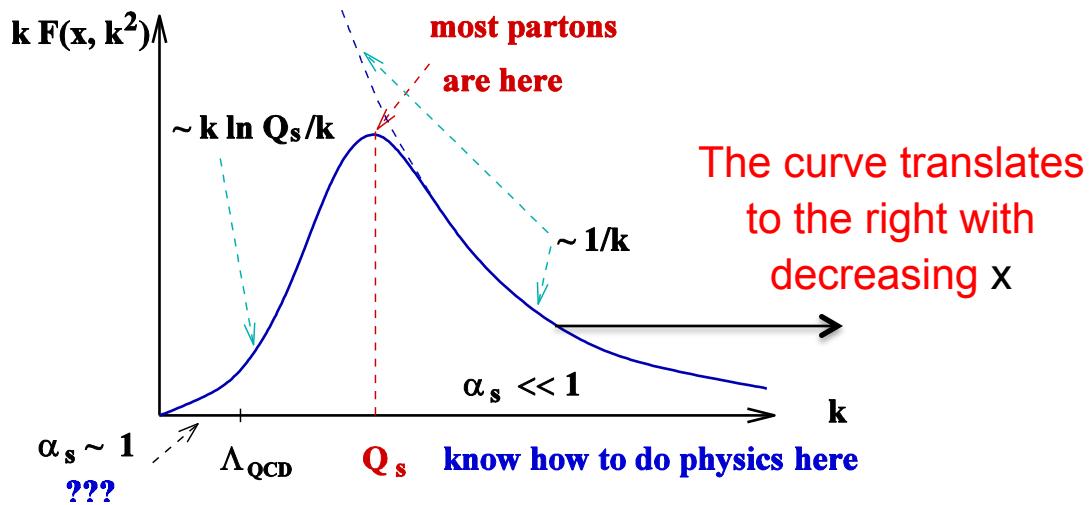
Balitsky (1996), Kovchegov (1998)

$$\frac{d}{dY} f_Y(k) = \bar{\alpha} \int \frac{dk'^2}{k'^2} \left[\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}} \right] - \bar{\alpha} f_Y^2(k)$$

Y = $\ln\left(\frac{1}{x}\right)$ $\bar{\alpha} = \frac{\alpha_s N_c}{\pi}$ BFKL

non-linearity important
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- solutions: qualitative behavior



Evolution of the "dipole" TMD

(in a mean-field type approximation)

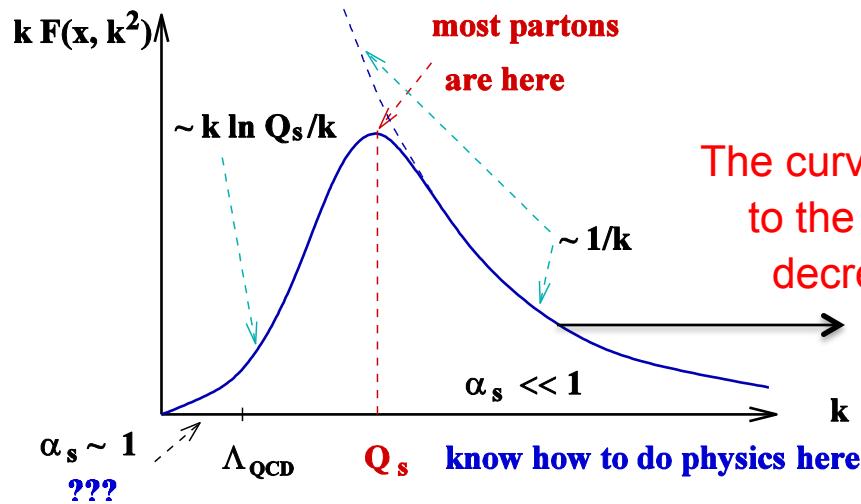
- the Balitsky-Kovchegov (BK) evolution

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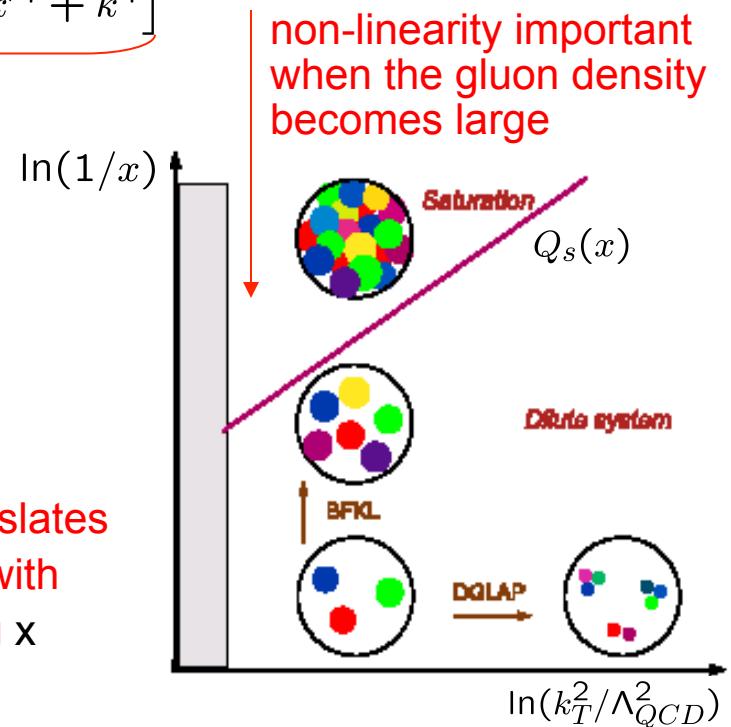
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BFKL

- solutions: qualitative behavior



Balitsky (1996), Kovchegov (1998)



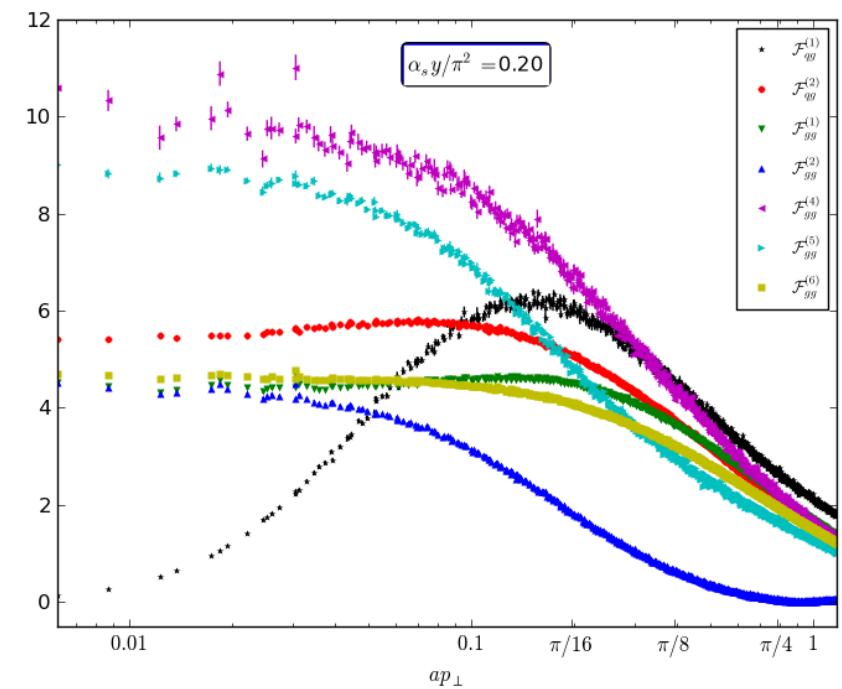
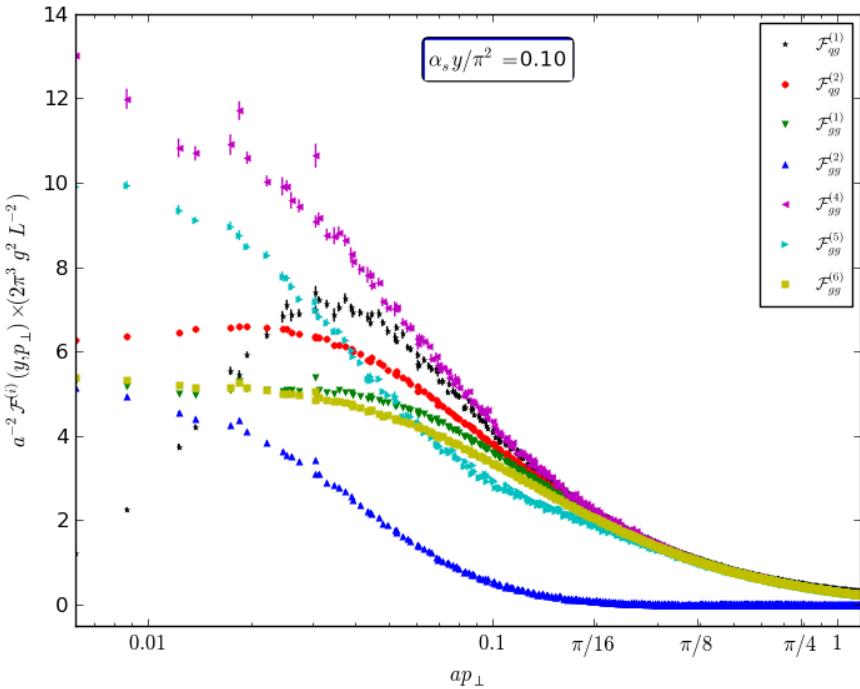
the distribution of partons as a function of x and k_T

JIMWLK numerical results

using a code written by Claude Roiesnel

initial condition at $y=0$: MV model
evolution: JIMWLK at leading log

CM, Petreska, Roiesnel (2016)

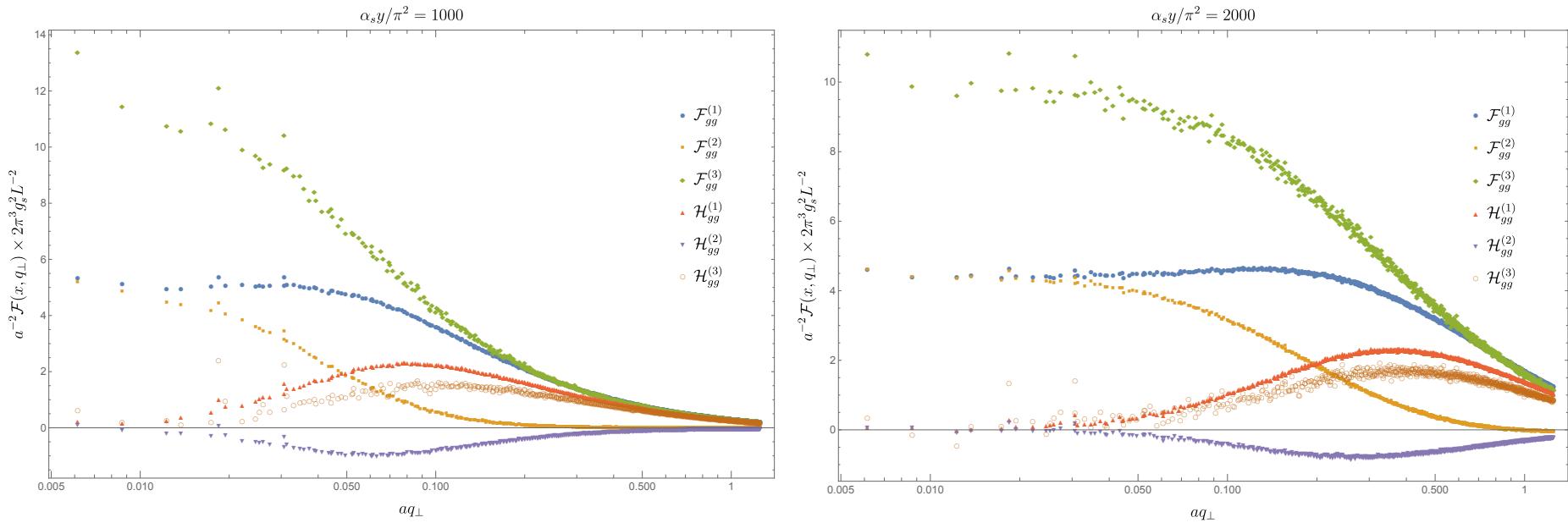


saturation effects impact the various gluon TMDs in very different ways

Linearly-polarized gluon TMDs

CM, Roiesnel, Taels, in preparation

they are equal to their unpolarized partners at large momentum
however they vanish at low momentum



gluons with non-zero polarization mostly have $k_t \sim Q_S(x_2)$

Conclusions

- different processes involve different gluon TMDs, with different operator definitions
- each operator definition provides an unpolarized gluon TMD and a linearly-polarized one
- given an initial condition, they can all be obtained at smaller values of x , from the JIMWLK equation
- as expected, the various gluon TMDs coincide at large transverse momentum, in the linear regime
- however, they differ significantly from one another at low transverse momentum, in the non-linear saturation regime
- we have quantified these differences and they are not negligible