

Kinematic Power Corrections in DVCS

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In this talk:

- Ambiguity of the leading-twist approximations
- Finite t and target mass corrections, t/Q^2 and m^2/Q^2

V. Braun, A. Manashov, D. Müller, B. Pirnay, Phys.Rev. D89 (2014) 074022

- Scalar targets

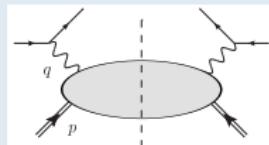
V. Braun, A. Manashov, B. Pirnay, Phys.Rev. D86 (2012) 014003



Planar vs. non-planar kinematics

- paradigm shift: finite t a “nuisance” \longrightarrow important tool

DIS



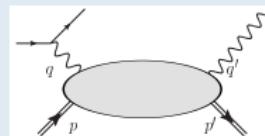
Define (p, q) as longitudinal plane:

$$p = (p_0, \vec{0}_\perp, p_z)$$

$$q = (q_0, \vec{0}_\perp, q_z)$$

\Rightarrow parton fraction = Bjorken x

DVCS



Many choices possible:

$$p = (p_0, \vec{0}_\perp, p_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

or

$$p + p' = (P_0, \vec{0}_\perp, P_z), \quad q = (q_0, \vec{0}_\perp, q_z)$$

etc.

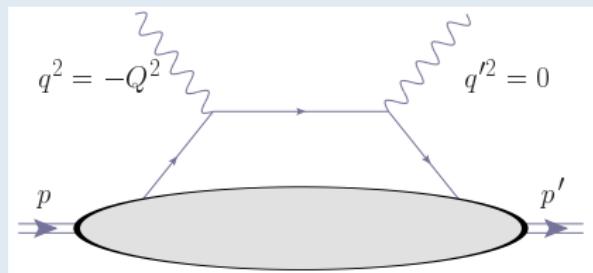
\Rightarrow parton fraction $2\xi = x_B [1 + \mathcal{O}\left(\frac{t}{Q^2}\right)]$,
redefinition of helicity amplitudes

- Ambiguity is resolved by adding “kinematic” power corrections $t/Q^2, m^2/Q^2$



“Photon” reference frame

Braun, Manashov, Pirnay: PRD **86** (2012) 014003



longitudinal plane (q, q')

$$n = q', \quad \tilde{n} = -q + \frac{Q^2}{Q^2 + t} q'$$

with this choice $\Delta = q - q'$ is longitudinal and

$$|P_\perp|^2 = -m^2 - \frac{t}{4} \frac{1 - \xi^2}{\xi^2} \sim t_{\min} - t$$

where

$$P = \frac{1}{2}(p + p'), \quad \xi_{\text{BMP}} = -\frac{(\Delta \cdot q')}{2(P \cdot q')} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

photon polarization vectors

$$\varepsilon_\mu^0 = -\left(q_\mu - q'_\mu q^2/(qq')\right)/\sqrt{-q^2},$$

$$\varepsilon_\mu^\pm = (P_\mu^\perp \pm i\bar{P}_\mu^\perp)/(\sqrt{2}|P_\perp|), \quad \bar{P}_\mu^\perp = \epsilon_{\mu\nu}^\perp P^\nu$$



Relating CFFs in the laboratory and photon reference frame

$$\begin{aligned}\mathcal{F}_{++}^{\text{lab}} &= \mathcal{F}_{++}^{\text{phot}} + \frac{\kappa}{2} \left[\mathcal{F}_{++}^{\text{phot}} + \mathcal{F}_{-+}^{\text{phot}} \right] - \kappa_0 \mathcal{F}_{0+}^{\text{phot}}, \\ \mathcal{F}_{0+}^{\text{lab}} &= -(1 + \kappa) \mathcal{F}_{0+}^{\text{phot}} + \kappa_0 \left[\mathcal{F}_{++}^{\text{phot}} + \mathcal{F}_{-+}^{\text{phot}} \right]\end{aligned}$$

$$\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

where

$$\kappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}, \quad \kappa \sim (t_{\min} - t)/Q^2$$

and different skewedness parameter

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$



Defining the Leading Twist approximation

Kumerički-Müller convention (KM)

$$\text{LT}_{\text{KM}} : \begin{cases} \mathcal{F}_{++}^{\text{lab}} = T_0 \otimes F, & \mathcal{F}_{0+}^{\text{lab}} = 0, \\ \mathcal{F}_{-+}^{\text{lab}} = 0, & \xi_{\text{KM}} = \xi^{\text{lab}} \end{cases}$$

Braun-Manashov-Pirnay convention (BMP)

$$\text{LT}_{\text{BMP}} : \begin{cases} \mathcal{F}_{++}^{\text{phot}} = T_0 \otimes F, & \mathcal{F}_{0+}^{\text{phot}} = 0, \\ \mathcal{F}_{-+}^{\text{phot}} = 0, & \xi_{\text{BMP}} = \xi^{\text{phot}} \end{cases}$$



$$\text{LT}_{\text{BMP}} : \begin{cases} \mathcal{F}_{++}^{\text{lab}} = \left(1 + \frac{\kappa}{2}\right) T_0 \otimes F, & \mathcal{F}_{0+} = \kappa_0 T_0 \otimes F \\ \mathcal{F}_{-+}^{\text{lab}} = \frac{\kappa}{2} T_0 \otimes F, & \xi = \xi_{\text{BMP}}, \end{cases}$$

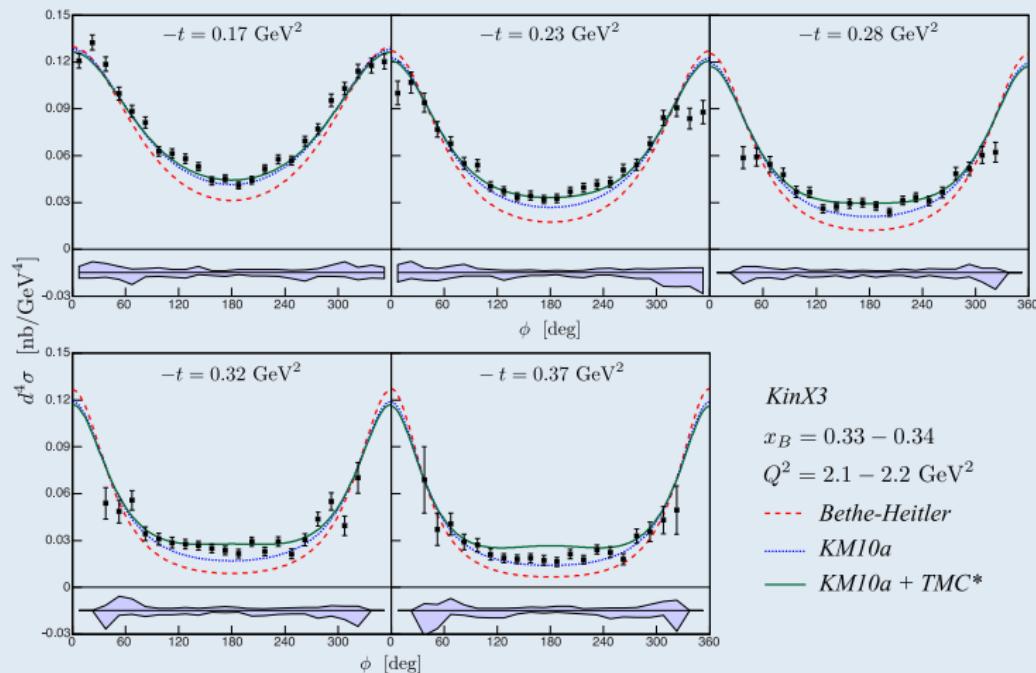
- **Changing frame of reference results in**
 - Different skewedness parameter for a given x_B
 - Numerically significant excitation of helicity-flip CFFs

- **Different results for experimental observables**



Large effects for the total cross section

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



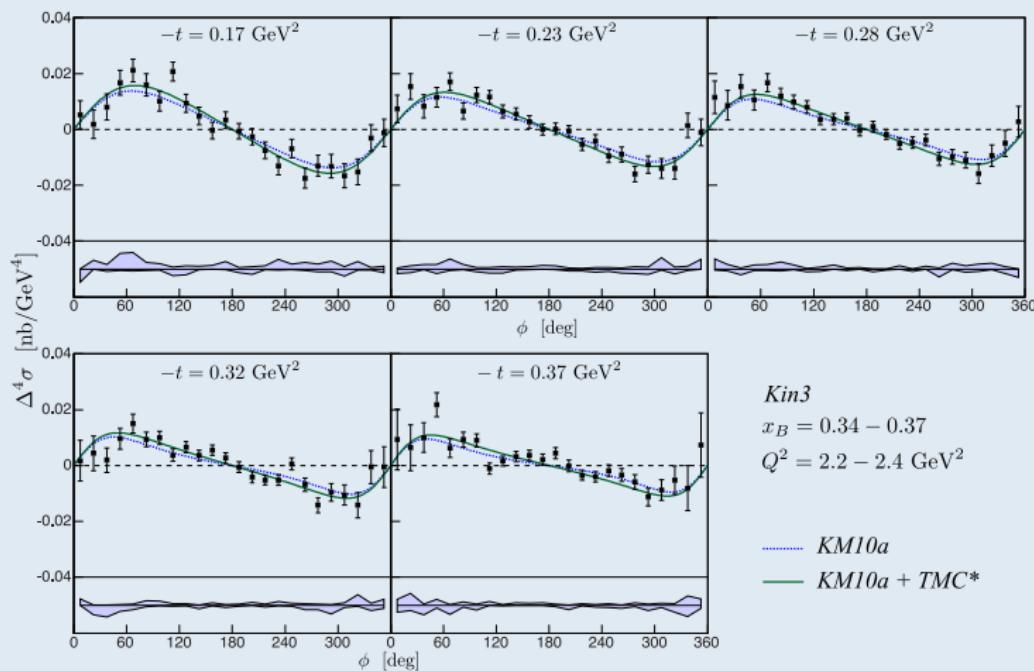
- TMC* curves very close to BMP LT

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B 841 (2010) 1)



Small/moderate effects for asymmetries

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- TMC* curves very close to BMP LT

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B 841 (2010) 1)



Summary of this part:

- **noncomplanarity makes separation of collinear directions ambiguous**
 - hence “leading twist approximation” ambiguous
 - related to violation of translation invariance and EM Ward identities
- **have to be repaired by adding power corrections of special type, “kinematic” PC**



BMP helicity amplitudes

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

$$\begin{aligned}\mathcal{A}_{\mu\nu}(q, q', p) &= i \int d^4x e^{-i(z_1 q - z_2 q')x} \langle p', s' | T\{J_\mu(z_1 x) J_\nu(z_2 x)\} | p, s \rangle \\ &= \varepsilon_\mu^+ \varepsilon_\nu^- \mathcal{A}^{++} + \varepsilon_\mu^- \varepsilon_\nu^+ \mathcal{A}^{--} + \varepsilon_\mu^0 \varepsilon_\nu^- \mathcal{A}^{0+} \\ &\quad + \varepsilon_\mu^0 \varepsilon_\nu^+ \mathcal{A}^{0-} + \varepsilon_\mu^+ \varepsilon_\nu^+ \mathcal{A}^{+-} + \varepsilon_\mu^- \varepsilon_\nu^- \mathcal{A}^{-+} + q'_\nu \mathcal{A}_\mu^{(3)}\end{aligned}$$

for the calculation to the twist-4 accuracy one needs

- $\mathcal{A}^{++}, \mathcal{A}^{--}$: $1 + \frac{1}{Q^2}$
- $\mathcal{A}^{0+}, \mathcal{A}^{0-}$: $\frac{1}{Q}$ ← agree with existing results
- $\mathcal{A}^{-+}, \mathcal{A}^{+-}$: $\frac{1}{Q^2}$ ← straightforward



BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

$$A_q^{a\pm} = \mathbb{H}_{a\pm}^q h + \mathbb{E}_{a\pm}^q e \mp \tilde{\mathbb{H}}_{a\pm}^q \tilde{h} \mp \tilde{\mathbb{E}}_{a\pm}^q \tilde{e}$$

with, e.g.

Belitsky, Müller, Ji: NPB 878 (2014) 214

$$h = \frac{\bar{u}(p') (\not{q} + \not{q}') u(p)}{P \cdot (\not{q} + \not{q}')} \quad \dots$$

- The results read

Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\begin{aligned} \mathbb{H}_{++} &= T_0 \circledast H + \frac{t}{Q^2} \left[-\frac{1}{2} T_0 + T_1 + 2\xi \mathbf{D}_\xi T_2 \right] \circledast H + \frac{2t}{Q^2} \xi^2 \partial_\xi \xi T_2 \circledast (H+E) \\ \mathbb{H}_{0+} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q} \left[\xi \partial_\xi T_1 \circledast H + \frac{t}{Q^2} \partial_\xi \xi T_1 \circledast (H+E) \right] - \frac{t}{\sqrt{2}Q|\xi P_\perp|} \xi T_1 \circledast \left[\xi (H+E) - \tilde{H} \right] \\ \mathbb{H}_{-+} &= \frac{4|\xi P_\perp|^2}{Q^2} \left[\xi \partial_\xi^2 \xi T_1^{(+)} \circledast H + \frac{t}{Q^2} \partial_\xi^2 \xi^2 T_1^{(+)} \circledast (H+E) \right] \\ &\quad + \frac{2t}{Q^2} \xi \left[\xi \partial_\xi \xi T_1^{(+)} \circledast (H+E) + \partial_\xi \xi T_1 \circledast \tilde{H} \right] \end{aligned}$$



BMP Compton form factors (CFFs)

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$$\mathcal{A}_q^{a\pm} = \mathbb{H}_{a\pm}^q h + \mathbb{E}_{a\pm}^q e \mp \tilde{\mathbb{H}}_{a\pm}^q \tilde{h} \mp \tilde{\mathbb{E}}_{a\pm}^q \tilde{e}$$

with, e.g.

Belitsky, Müller, Ji: NPB **878** (2014) 214

$$h = \frac{\bar{u}(p') (\not{q} + \not{q}') u(p)}{P \cdot (\not{q} + \not{q}')} \quad \dots$$

- The results read

Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\begin{aligned}\mathbb{E}_{++} &= T_0 \circledast E + \frac{t}{Q^2} \left[-\frac{1}{2} T_0 + T_1 + 2\xi \mathbf{D}_\xi T_2 \right] \circledast E - \frac{8m^2}{Q^2} \xi^2 \partial_\xi \xi T_2 \circledast (H + E) \\ \mathbb{E}_{0+} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q} \left[\xi \partial_\xi T_1 \circledast E \right] + \frac{4m^2}{\sqrt{2}Q|\xi P_\perp|} \xi T_1 \circledast \left[\xi (H + E) - \tilde{H} \right] \\ \mathbb{E}_{-+} &= \frac{4|\xi P_\perp|^2}{Q^2} \left[\xi \partial_\xi^2 \xi T_1^{(+)} \circledast E \right] - \frac{8m^2}{Q^2} \xi \left[\xi \partial_\xi \xi T_1^{(+)} \circledast (H + E) + \partial_\xi \xi T_1 \circledast \tilde{H} \right]\end{aligned}$$

etc.



where $F = H, E, \tilde{H}, \tilde{E}$ are C-even GPDs

$$T \circledast F = \sum_q e_q^2 \int_{-1}^1 \frac{dx}{2\xi} T\left(\frac{\xi + x - i\epsilon}{2(\xi - i\epsilon)}\right) F(x, \xi, t)$$

the coefficient functions T_k^\pm are given by the following expressions:

$$\begin{aligned} T_0(u) &= \frac{1}{1-u} \\ T_1(u) \equiv T_1^{(-)}(u) &= -\frac{\ln(1-u)}{u} \\ T_1^{(+)}(u) &= \frac{(1-2u)\ln(1-u)}{u} \\ T_2(u) &= \frac{\text{Li}_2(1) - \text{Li}_2(u)}{1-u} + \frac{\ln(1-u)}{2u} \end{aligned}$$

and

$$\mathbf{D}_\xi = \partial_\xi + 2 \frac{|\xi P_\perp|^2}{t} \partial_\xi^2 \xi = \partial_\xi - \frac{t - t_{\min}}{2t} (1 - \xi^2) \partial_\xi^2 \xi$$



Main features:

- Complete results available to $t/Q^2, m^2/Q^2$ accuracy
 - translation and gauge invariance restored
 - factorization valid
 - correct threshold behavior $t \rightarrow t_{\min}, \xi \rightarrow 1$
 - for many observables, complete results close to LT in “photon frame”
- Two expansion parameters

$$\frac{t}{Q^2}; \quad \frac{t - t_{\min}}{Q^2} \sim \frac{|\xi P_\perp|^2}{Q^2}$$

- Most of mass corrections absorbed in $t_{\min} = -4m^2\xi^2/(1-\xi^2)$;
always overcompensated by finite- t corrections in the physical region
- Some extra m^2/Q^2 corrections for nucleon due to spinor algebra;
disappear in certain CFF combinations and for scalar targets



DVCS on a scalar target

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

- Helicity amplitudes

$$\begin{aligned} \mathcal{A}_{\mu\nu} = & -g_{\mu\nu}^\perp \mathcal{A}^{(0)} + \frac{1}{\sqrt{-q^2}} \left(q_\mu - q'_\mu \frac{q^2}{(qq')} \right) g_{\nu\rho}^\perp P^\rho \mathcal{A}^{(1)} \\ & + \frac{1}{2} \left(g_{\mu\rho}^\perp g_{\nu\sigma}^\perp - \epsilon_{\mu\rho}^\perp \epsilon_{\nu\sigma}^\perp \right) P^\rho P^\sigma \mathcal{A}^{(2)} + \cancel{g_{\nu\rho}^\perp \mathcal{A}_\mu^{(3)}} \end{aligned}$$

where

$$\begin{aligned} \mathcal{A}^{(0)} = & -2 \left\{ \left(1 - \frac{t}{2Q^2} \right) \int dx \frac{H(x, \xi, t)}{x + \xi - i\epsilon} + \frac{t}{Q^2} \int dx \frac{H(x, \xi, t)}{x - \xi} \ln \left(\frac{x + \xi}{2\xi} - i\epsilon \right) \right. \\ & \left. - \frac{2}{Q^2} \left(\frac{t}{\xi} + 2|\xi P_\perp|^2 \partial_\xi \right) \xi^2 \partial_\xi \int dx \frac{H(x, \xi, t)}{x - \xi} \left[\frac{1}{2} \ln \left(\frac{x + \xi}{2\xi} - i\epsilon \right) + \text{Li}_2 \left(\frac{x + \xi}{2\xi} + i\epsilon \right) - \text{Li}_2(1) \right] \right\} \end{aligned}$$

$$\mathcal{A}^{(1)} = \frac{8}{Q} \xi^2 \partial_\xi \int dx \frac{H(x, \xi, t)}{x - \xi} \ln \left(\frac{x + \xi}{2\xi} - i\epsilon \right),$$

$$\mathcal{A}^{(2)} = \frac{8}{Q^2} \xi^3 \partial_\xi^2 \int dx \frac{x H(x, \xi, t)}{x - \xi} \ln \left(\frac{x + \xi}{2\xi} - i\epsilon \right)$$



DVCS on a scalar target — *continued*

Braun, Manashov, Pirnay: PRD **86** (2012) 014003

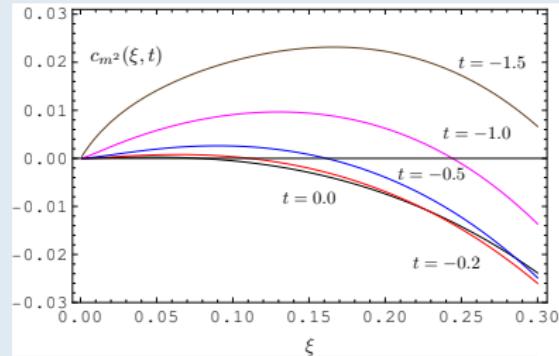
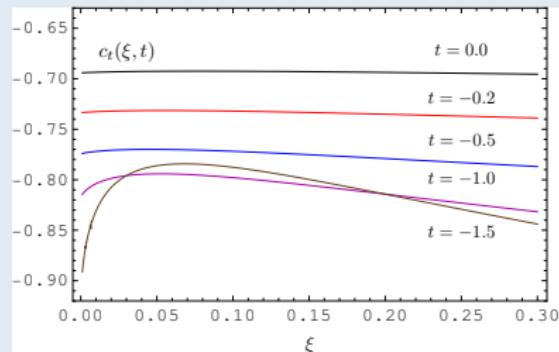
define

$$\frac{\text{Im}\mathcal{A}^{(0)} - \text{Im}\mathcal{A}_{LO}^{(0)}}{\text{Im}\mathcal{A}_{LO}^{(0)}} = \frac{t}{Q^2} c_t(\xi, t) + \frac{m^2}{Q^2} c_{m^2}(\xi, t)$$

$$\text{Im}\mathcal{A}_{LO}^{(0)} = 2 H(\xi, \xi, t)$$

$$\left. \frac{\text{Im}\mathcal{A}^{(0)} - \text{Im}\mathcal{A}_{LO}^{(0)}}{\text{Im}\mathcal{A}_{LO}^{(0)}} \right|_{t=t_{\min}} \simeq -(0.62 - 0.65) \frac{t_{\min}}{Q^2}$$

$$\sim \frac{\xi^2 m^2}{Q^2}$$



What can/should be done?

short/medium term

- Bulk of the twist-four corrections captured in “photon” frame for generic H, E ?
- Direct calculation of DVCS observables starting from “photon” frame
- “Standard” code combining twist-4 + NLO

long(er) term

- resummation of $(t/Q^2)^k$ and $(m^2/Q^2)^k$ corrections to all powers
- NLO corrections to $(t/Q^2)^k$ and $(m^2/Q^2)^k$, gluon contributions
- Matching $(t/Q^2)^k$ and $(m^2/Q^2)^k$ with BFKL resummation in the small- x limit

