Probing saturation in 3-particle correlations in DIS

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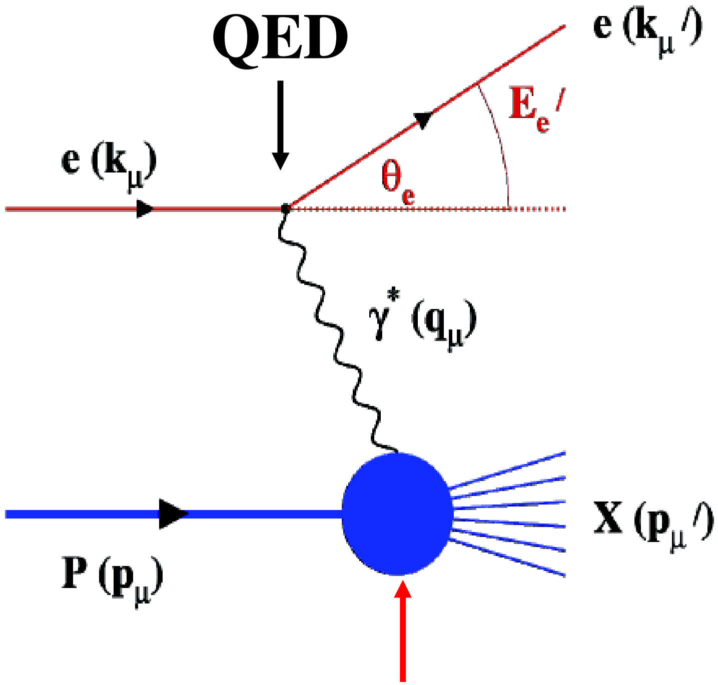
Deeply Inelastic Scattering (DIS)

probing hadron structure



(F₁, F₂ structure functions)

Kinematic Invariants

$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

Measure of resolution power

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\theta'_e}{2}\right)$$

Measure of inelasticity

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

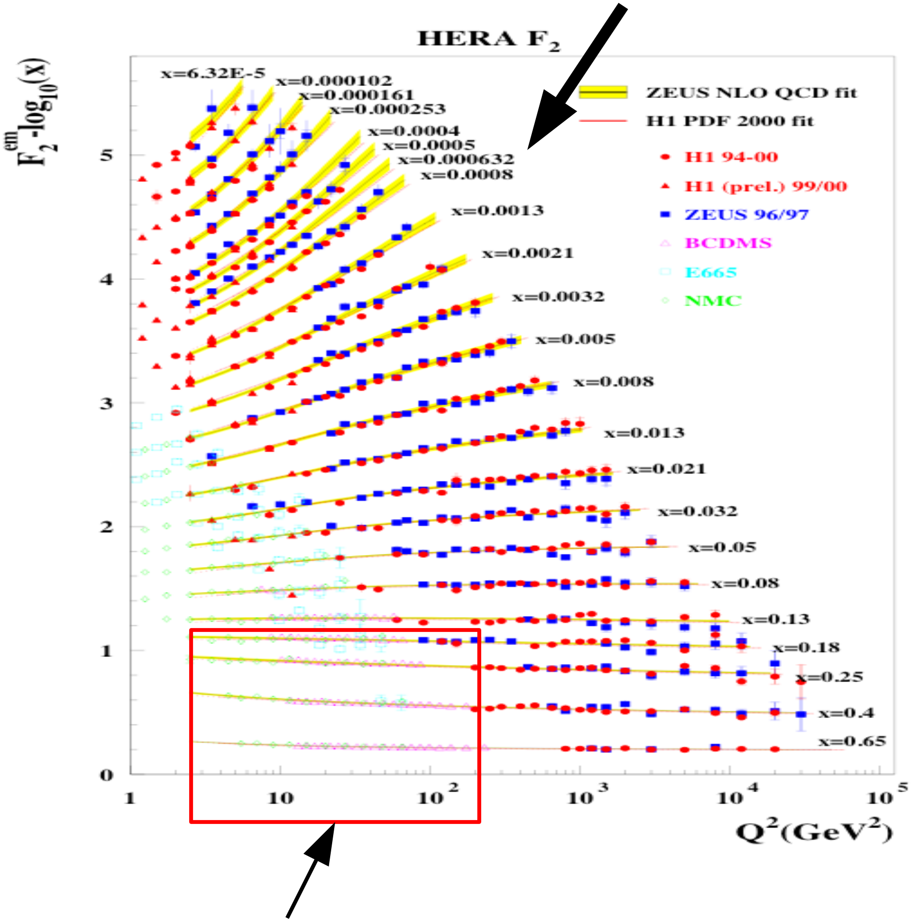
Measure of momentum fraction of struck quark

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

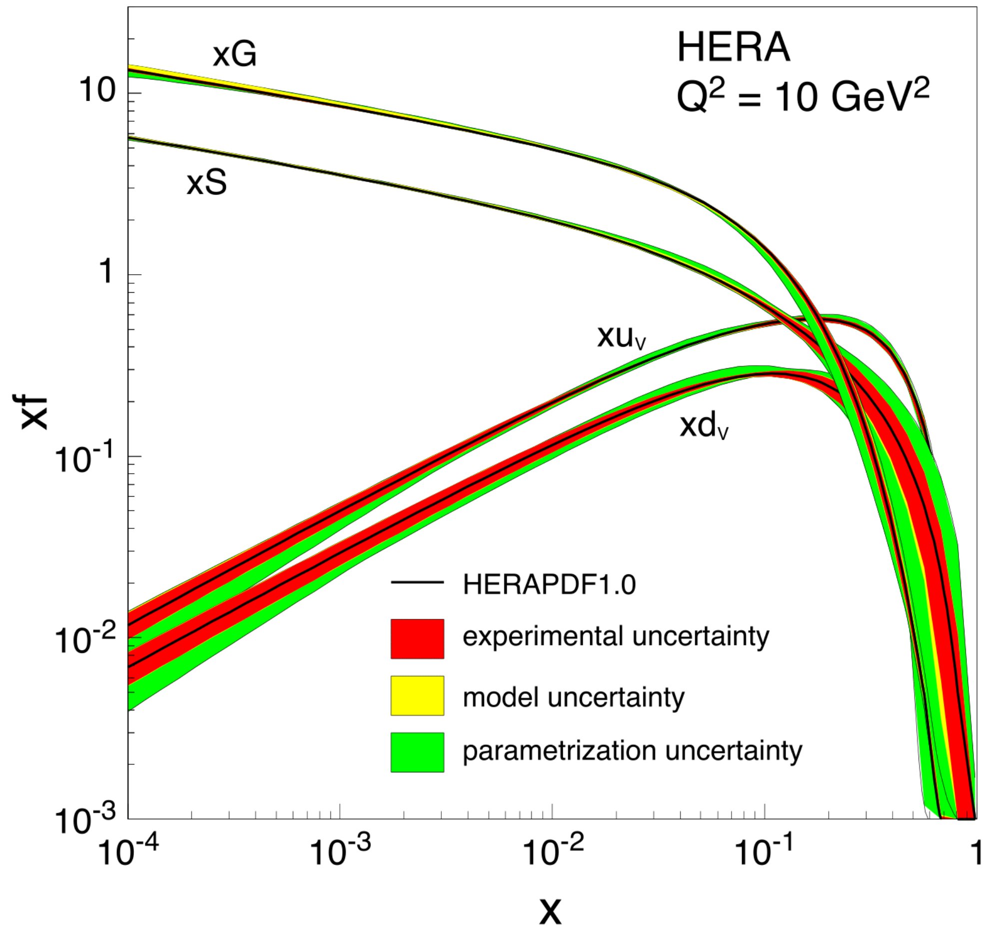
Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 \, x q(x, Q^2)$$

QCD: scaling violations



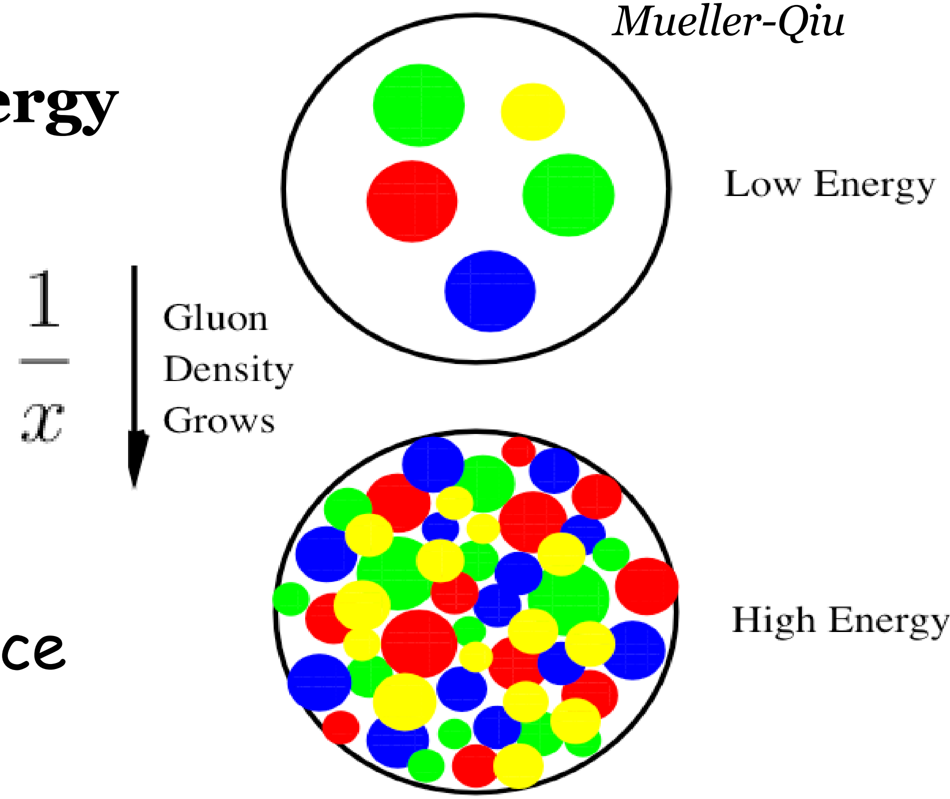
early experiments (SLAC,...):
scale invariance of hadron structure



large number of gluons at small x

Gribov-Levin-Ryskin
Mueller-Qiu

Hadron/nucleus at high energy



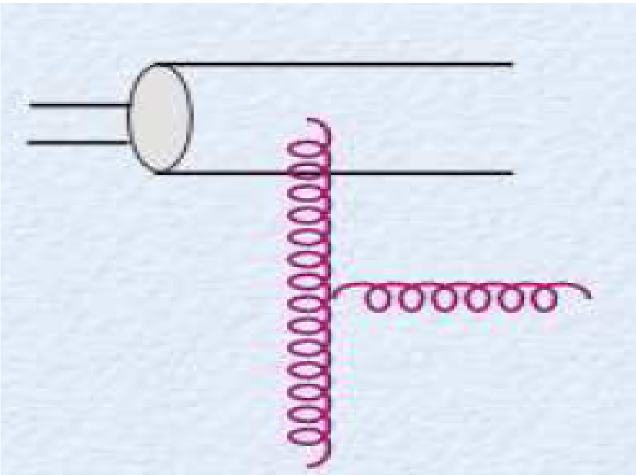
radiated gluons have the same size ($1/Q^2$) - the number of partons increase due to the increased longitudinal phase space

hadron/nucleus becomes a dense system of gluons:
gluon saturation

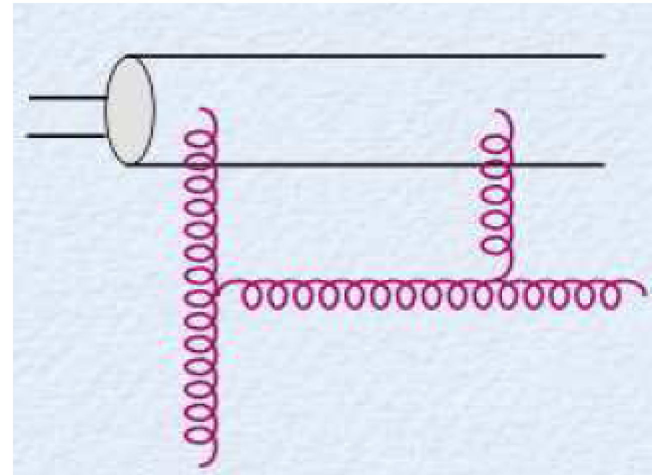
Physics of strong color fields in QCD, multi-particle production- possibly discover novel universal properties of theory in this limit

Perturbative QCD breaks down at small x

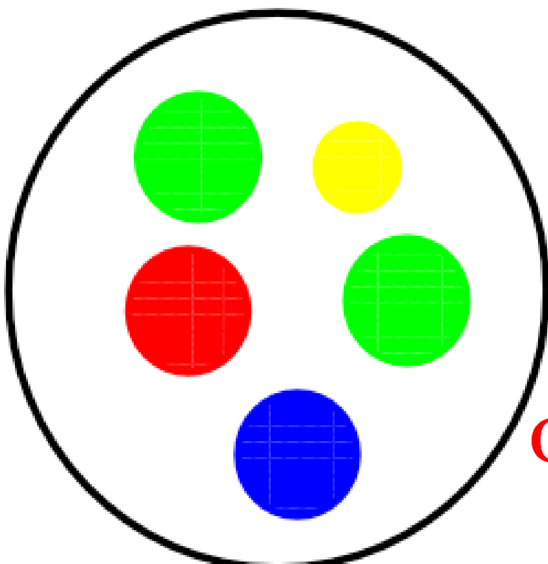
"attractive" bremsstrahlung *vs.* *"repulsive" recombination*



included in pQCD



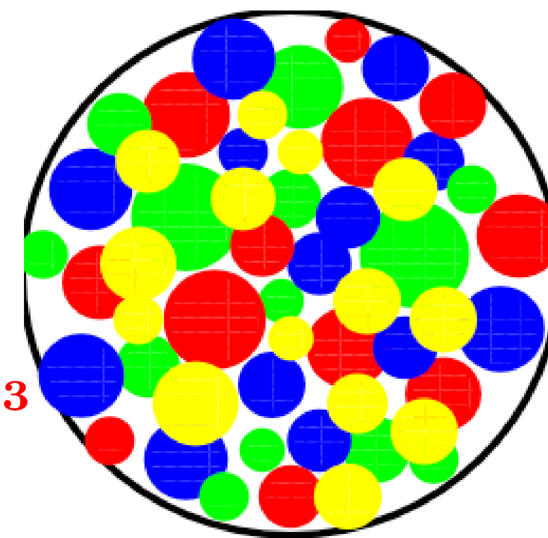
not included in pQCD
(collinear factorization)



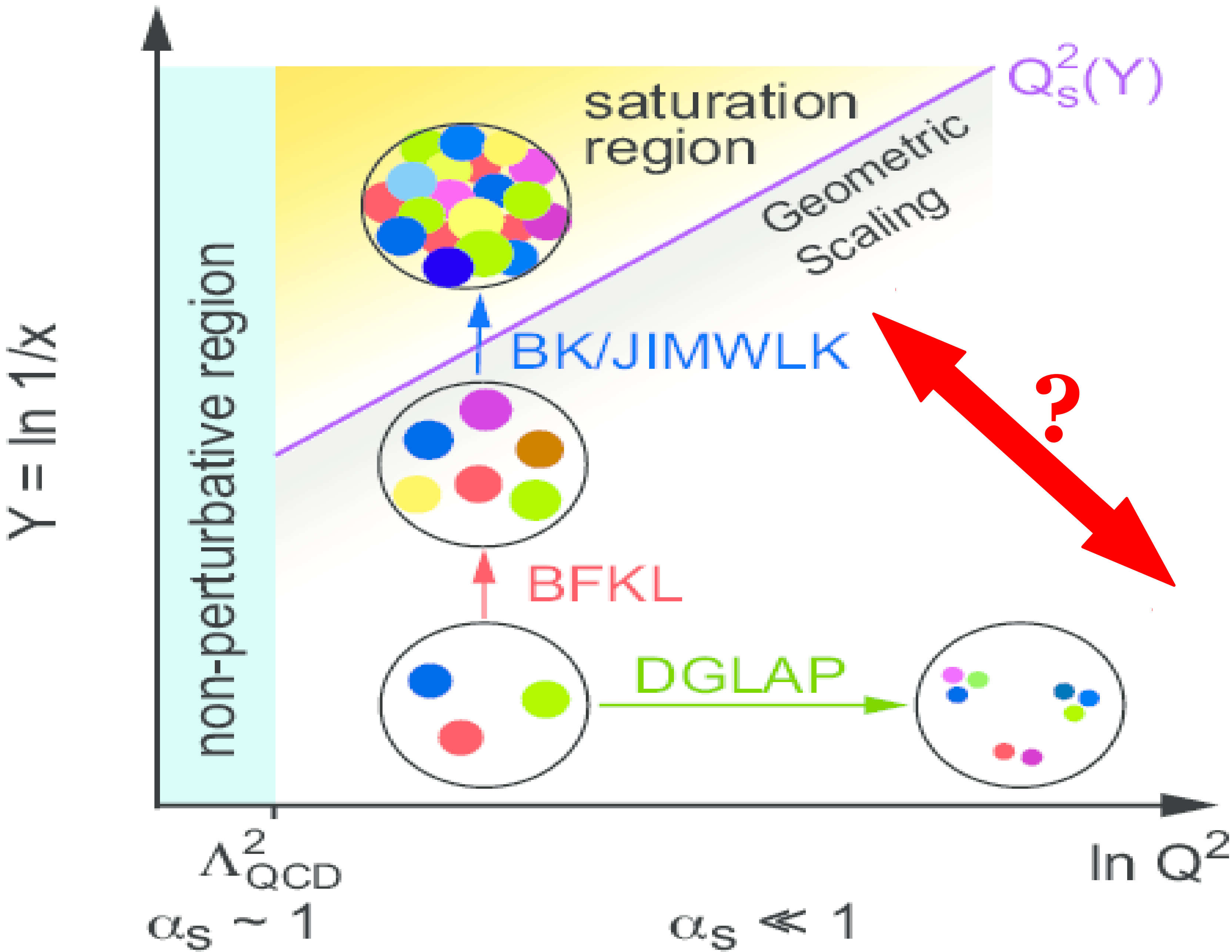
energy ~ 1/x

$$\frac{\alpha_s}{Q^2}\,\frac{xG(x,Q^2)}{\pi r^2} \sim 1$$

$$Q_s^2(x, b_t, A) \sim A^{1/3}\,(\frac{1}{x})^{0.3}$$



QCD at high energy: *saturation*



Probing saturation via correlations

polar angle (long-range rapidity correlations)

azimuthal angle (back to back)

signatures in production spectra

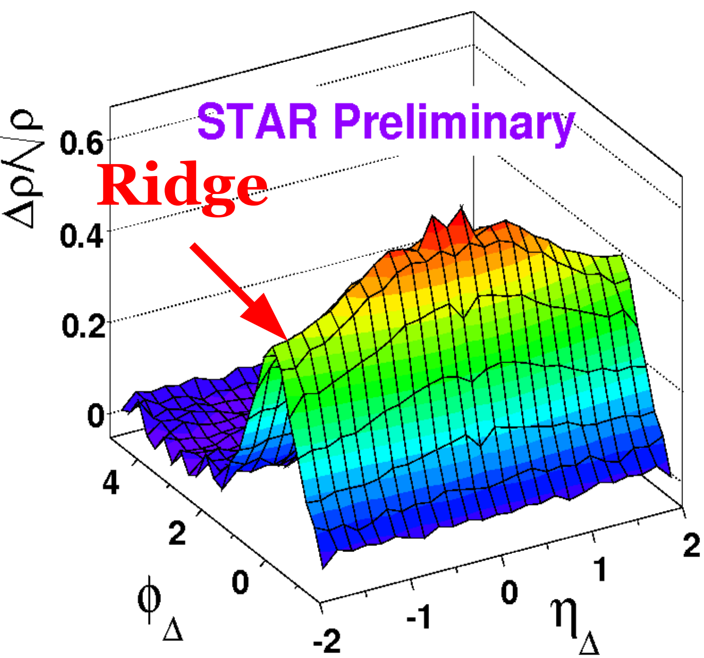
multiple scattering via Wilson lines:

p_t broadening

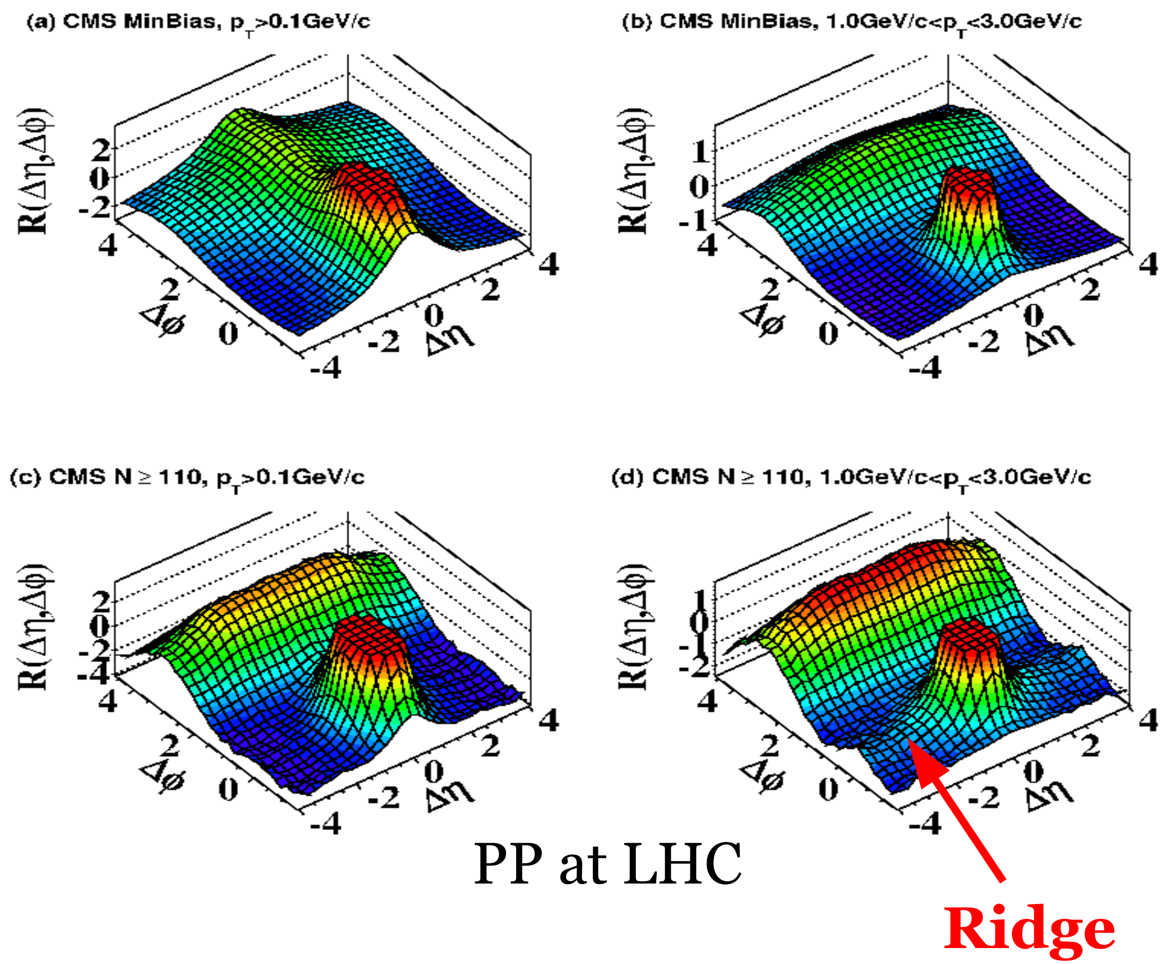
x-evolution via JIMWLK:

suppression of spectra/away side peaks

long-range rapidity correlations: the ridge



AA at RHIC

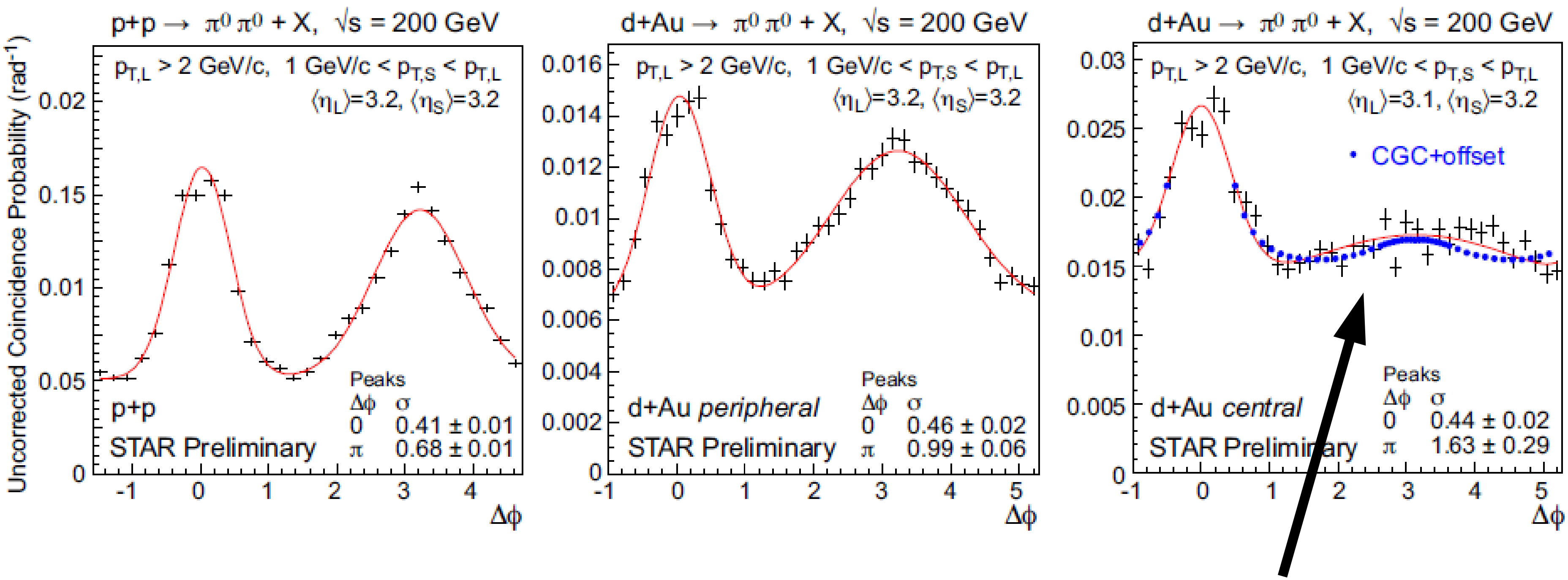


PP at LHC

Initial state vs final state ?
if final state, early or late times?

di-hadron correlations in pA

Recent STAR measurement (arXiv:1008.3989v1):



Marquet, NPA (2007), Albacete + Marquet, PRL (2010)
Tuchin, NPA846 (2010)
A. Stasto + B-W. Xiao + F. Yuan, PLB716 (2012)
T. Lappi + H. Mantysaari, NPA908 (2013)

saturation effects de-correlate the hadrons

shadowing+energy loss: Z. Kang, I. Vitev, H. Xing, PRD85 (2012) 054024

Probing saturation in high energy collisions

"nucleus-nucleus" (dense-dense)
initial multiplicity

"proton-nucleus" (dilute-dense)
production spectra, correlations

<u>DIS</u>

structure functions (diffraction)
<u>NLO</u> di-hadron/jet correlations
<u>3-hadron/jet angular correlations</u>

need quite a bit of modeling

↓

much less modeling

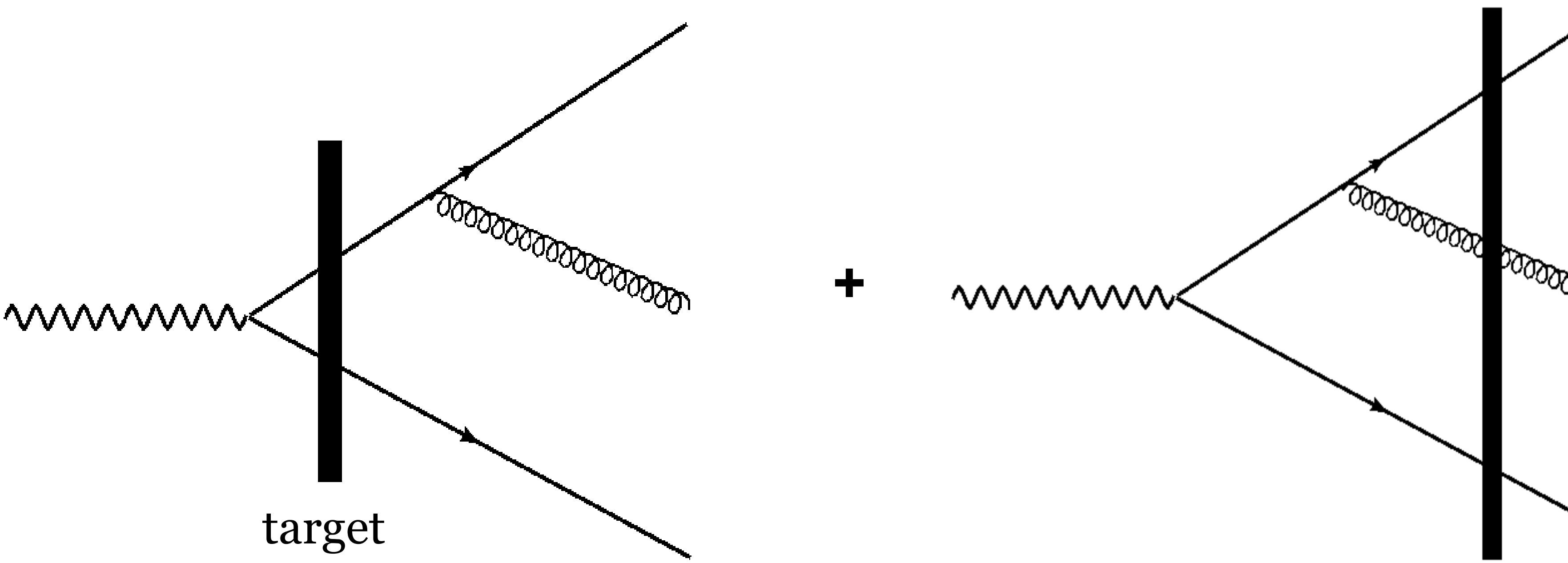
Azimuthal angular correlations

in

DIS

3-parton production in DIS

$$\gamma^\star \mathbf{T} \to \mathbf{q}\,\bar{\mathbf{q}}\,\mathbf{g}\,\mathbf{X}$$

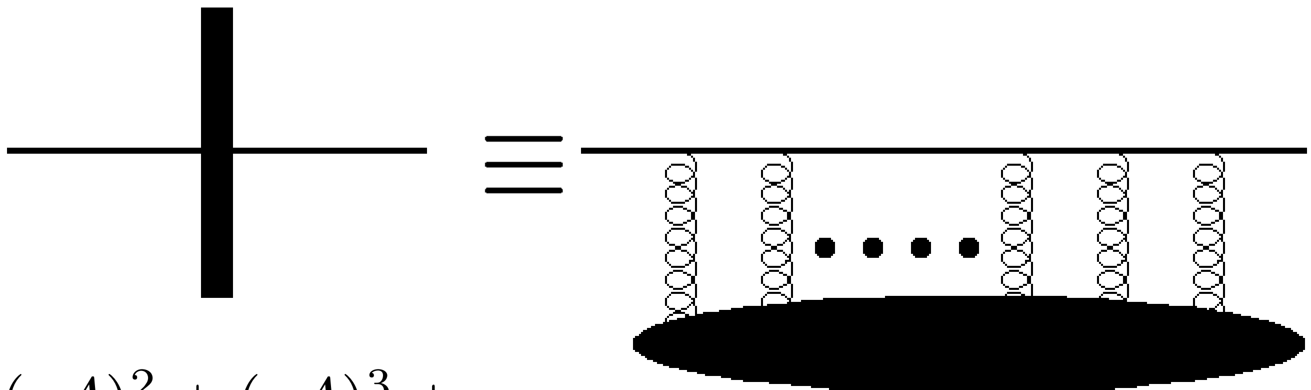


+ radiation from anti-quark

Ayala, Hentschinski , Jalilian-Marian, Tejeda-Yeomans; PLB761 (2016) 229

scattering of a quark from the target

target (proton, nucleus) as a classical color field
quark propagator in the background color field: Wilson line V



$$\sim gA + (gA)^2 + (gA)^3 + \cdots$$

$$\mathrm{S}_F(q,p) \equiv (2\pi)^4 \underbrace{\delta^4(p-q)\, S_F^0(p)}_{\text{no interaction}} + S_F^0(q)\, \underbrace{\tau_f(q,p)}_{\text{interaction}}\, S_F^0(p) \qquad \text{with} \qquad S_F^0(p) = \frac{i}{\not{p} + i\epsilon}$$

$$\tau_f(q,p) \equiv (2\pi)\delta(p^+ - q^+)\, \gamma^+ \int d^2x_t\, e^{i(q_t - p_t)\cdot x_t} \left\{\theta(p^+)[V(x_t) - 1] - \theta(-p^+)[V^\dagger(x_t) - 1]\right\}$$

$$\mathrm{V}(\mathrm{x}_t) = \hat{p}\, e^{ig \int dz^+ A^-(z^+, x_t)}$$

similar for gluon propagator

spinor helicity methods

Review:
L. Dixon, hep-ph/9601359

massless quarks: helicity eigenstates

$$u_\pm(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, u(k) \qquad \overline{u_\pm(k)} \equiv \overline{u(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

$$v_\mp(k) \equiv \frac{1}{2}(1 \pm \gamma_5)\, v(k) \qquad \overline{v_\mp(k)} \equiv \overline{v(k)}\, \frac{1}{2}(1 \mp \gamma_5)$$

helicity operator

$$\mathrm{h} \equiv \vec{\Sigma}\cdot\hat{p} = \begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}$$

$$\vec{\Sigma}\cdot\hat{p}\, u_\pm(p) = \pm u_\pm(p)$$
$$-\vec{\Sigma}\cdot\hat{p}\, v_\pm(p) = \pm v_\pm(p)$$

$$u_+(k) = v_-(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \\ \sqrt{k^+} \\ \sqrt{k^-} e^{i\phi_k} \end{bmatrix} \qquad u_-(k) = v_+(k) = \frac{1}{2^{1/4}} \begin{bmatrix} \sqrt{k^-} e^{-i\phi_k} \\ -\sqrt{k^+} \\ -\sqrt{k^-} e^{-i\phi_k} \\ \sqrt{k^+} \end{bmatrix}$$

with $e^{\pm i\phi_k} \equiv \dfrac{k_x \pm i k_y}{\sqrt{2k^+ k^-}} = \sqrt{2}\,\dfrac{k_t \cdot \epsilon_\pm}{k_t}$

$$n^\mu = (n^+ = 0, n^- = 1, n_\perp = 0)$$
$$\bar{n}^\mu = (\bar{n}^+ = 1, \bar{n}^- = 0, \bar{n}_\perp = 0)$$

and

$$k^\pm = \frac{E \pm k_z}{\sqrt{2}}$$
$$\epsilon_\pm = \frac{1}{\sqrt{2}}(1, \pm i)$$

spinor helicity methods

notation:

$$|i^\pm > \equiv |k_i^\pm > \equiv u_\pm(k_i) = v_\mp(k_i) \qquad < i^\pm| \equiv < k_i^\pm| \equiv \overline{u}_\pm(k_i) = \overline{v}_\mp(k_i)$$

basic spinor products:

$$< i\, j > \;\equiv\; < i^-|j^+ > = \overline{u}_-(k_i)\, u_+(k_j) \;=\; \sqrt{|s_{ij}|}e^{i\phi_{ij}}$$

$$[i\, j] \;\equiv\; < i^+|j^- > = \overline{u}_+(k_i)\, u_-(k_j) \;=\; -\sqrt{|s_{ij}|}e^{-i\phi_{ij}}$$

$$\cos\phi_{ij} = \frac{k_i^x k_j^+ - k_j^x k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}}$$

$$\sin\phi_{ij} = \frac{k_i^y k_j^+ - k_j^y k_i^+}{\sqrt{|s_{ij}| k_i^+ k_j^+}}$$

with

$$s_{ij} = (k_i + k_j)^2 = 2k_i \cdot k_j = - < ij > [ij]$$

and

$$< ii > = [ii] = 0$$
$$< ij] = [ij >= 0$$

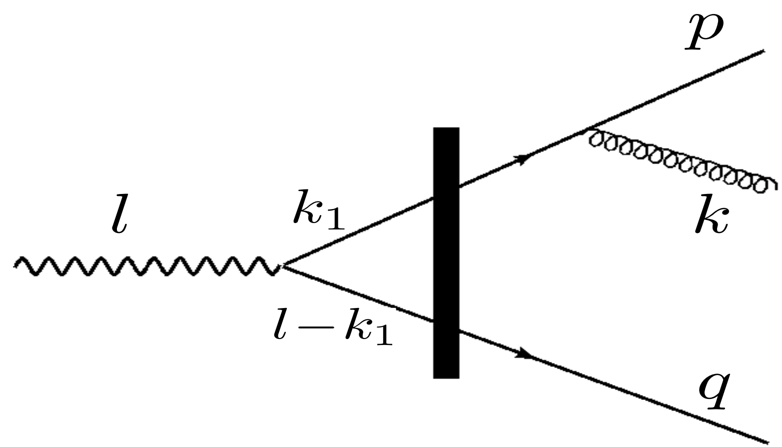
charge conjugation $< i^+|\gamma^\mu|j^+ > = < j^-|\gamma^\mu|i^- >$

Fierz identity $< i^+|\gamma^\mu|j^+ >< k^+|\gamma^\mu|l^+ > = 2[ik] \; < lj >$

any off-shell momentum $k^\mu \equiv \bar{k}^\mu + \frac{k^2}{2k^+} n^\mu$ where $\bar{k}^\mu$ is on-shell $\bar{k}^2 = 0$

any on-shell momentum $\not{p} = |p^+ >< p^+| + |p^- >< p^-|$

Diagram A1



Numerator: Dirac Algebra

$$a_1 \equiv \overline{u}(p)\,(k)\,(\not p + \not k)\;\not k_1\;(l)\,(\not k_1 - \not l)\;v(q)$$

longitudinal photons

quark anti-quark gluon helicity: + - +

$$\not l = l^+ \not{\overline{n}} - \frac{Q^2}{2l^+}\not n$$

$$a_1^{L;+-+} \;=\; -\frac{\sqrt{2}}{[n\,k]}\frac{Q}{l^+}\,[n\,p] < k\,p > [n\,p] < n\,\overline{k}_1 > [n\,\overline{k}_1] < n\,q >$$

$$(< n\,\overline{k}_1 > [n\,\overline{k}_1] - l^+ < n\,\overline{n} > [n\,\overline{n}])$$

with

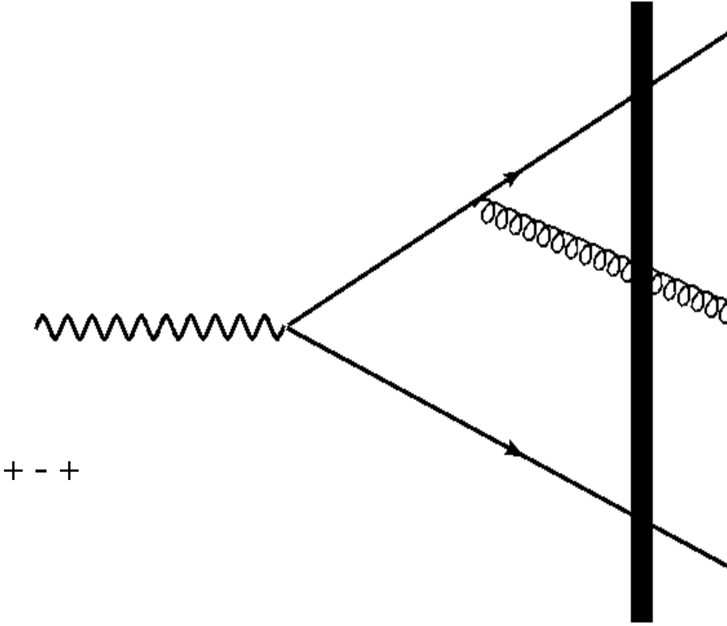
$$< np >= -[np] = \sqrt{2p^+}$$

transverse photons: +

$$a_1^{\perp=+;+-+} = -\frac{\sqrt{2}}{[nk]}[pn] < kp > [pn] < nk_1 > [k_1 n] < \bar{n}k_1 > [k_1 n] < nq >$$

Diagram A3

Numerator: Dirac Algebra



longitudinal photons

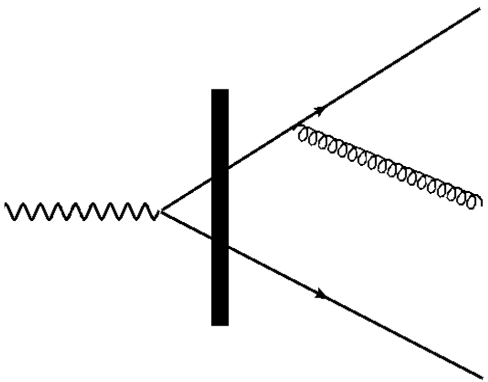
quark anti-quark gluon helicity: + - +

$$
\begin{aligned}
a_3^{L;+-+} &= \frac{\sqrt{2}Q}{l^+\,[n\bar{k}_2]}[pn]\left(< n\bar{k}_1 > [\bar{k}_1 n] - < n\bar{k}_2 > [\bar{k}_2 n]\right) < \bar{k}_2\bar{k}_1 > [\bar{k}_1 n] \\
&\quad \left(< n\bar{k}_1 > [\bar{k}_1 n] - l^+ < n\bar{n} > [\bar{n}n]\right) < nq > \\
&= -2^4 Q(l^+)^2\,\frac{(z_1 z_2)^{3/2}}{z_3}\,\left[z_3\, k_{1\,t}\cdot\epsilon - (z_1+z_3)k_{2\,t}\cdot\epsilon\right]
\end{aligned}
$$

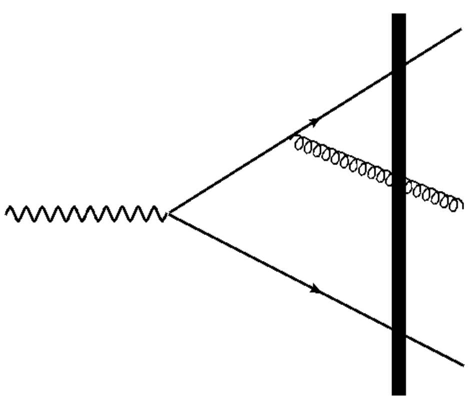
the rest is some standard integrals, we know how to compute the numerators efficiently

add up the amplitudes, add, square.. : **get (trace of) products of Wilson lines**

structure of Wilson lines: amplitude



$$\Big(\quad \Big)_{ij} = \left[V^\dagger(y_t)\, V(x_t)\, t^a\right]_{ij}$$



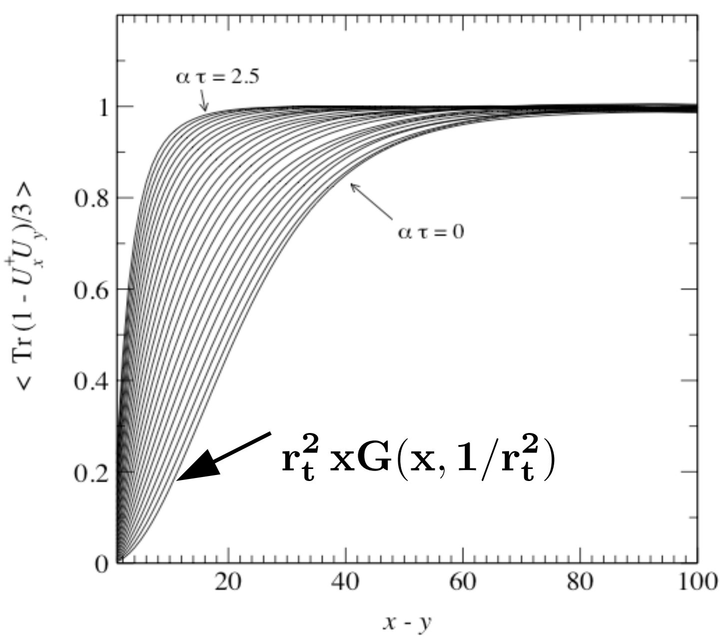
$$\Big(\quad \Big)_{ij} = \left[V^\dagger(y_t)\, t^b\, V(x_t)\right]_{ij}\; U^{ba}(z_t)$$

Dipoles at large N_c : BK eq

$$T \equiv 1 - S$$

$$\frac{d}{dy} T(x_t - y_t) = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2 (y_t - z_t)^2} \times$$

$$[T(x_t - z_t) + T(z_t - y_t) - T(x_t - y_t) - T(x_t - z_t) T(z_t - y_t)]$$



$$\tilde{T}(p_t) \to \log\left[\frac{Q_s^2}{p_t^2}\right]$$ **saturation region**

$$\tilde{T}(p_t) \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]^\gamma$$

$$R_{pA} \sim \frac{1}{A^{\frac{\gamma}{3}}}$$

$$\gamma < 1$$

$$\tilde{T}(p_t) \to \frac{1}{p_t^2}\left[\frac{Q_s^2}{p_t^2}\right]$$ **pQCD region**

Rummukainen-Weigert, NPA739 (2004) 183
NLO: Balitsky-Kovchegov-Weigert-Gardi-Chirilli (2007-2008)

The quadrupole

$$Q(r,\bar{r},\bar{s},s) \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$$

line config.: $r = \bar{s},\ \bar{r} = s,\ z \equiv r - \bar{r}$

square config.: $r - \bar{s} = \bar{r} - s = r - \bar{r} = \cdots \equiv z$

"naive" Gaussian: $Q = S^2$ $\quad S(r,\bar{r}) \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r}) >$

Gaussian $\quad Q_|(z) \approx \frac{N_c+1}{2}[S(z)]^{2\frac{N_c+2}{N_c+1}} - \frac{N_c-1}{2}[S(z)]^{2\frac{N_c-2}{N_c-1}}$

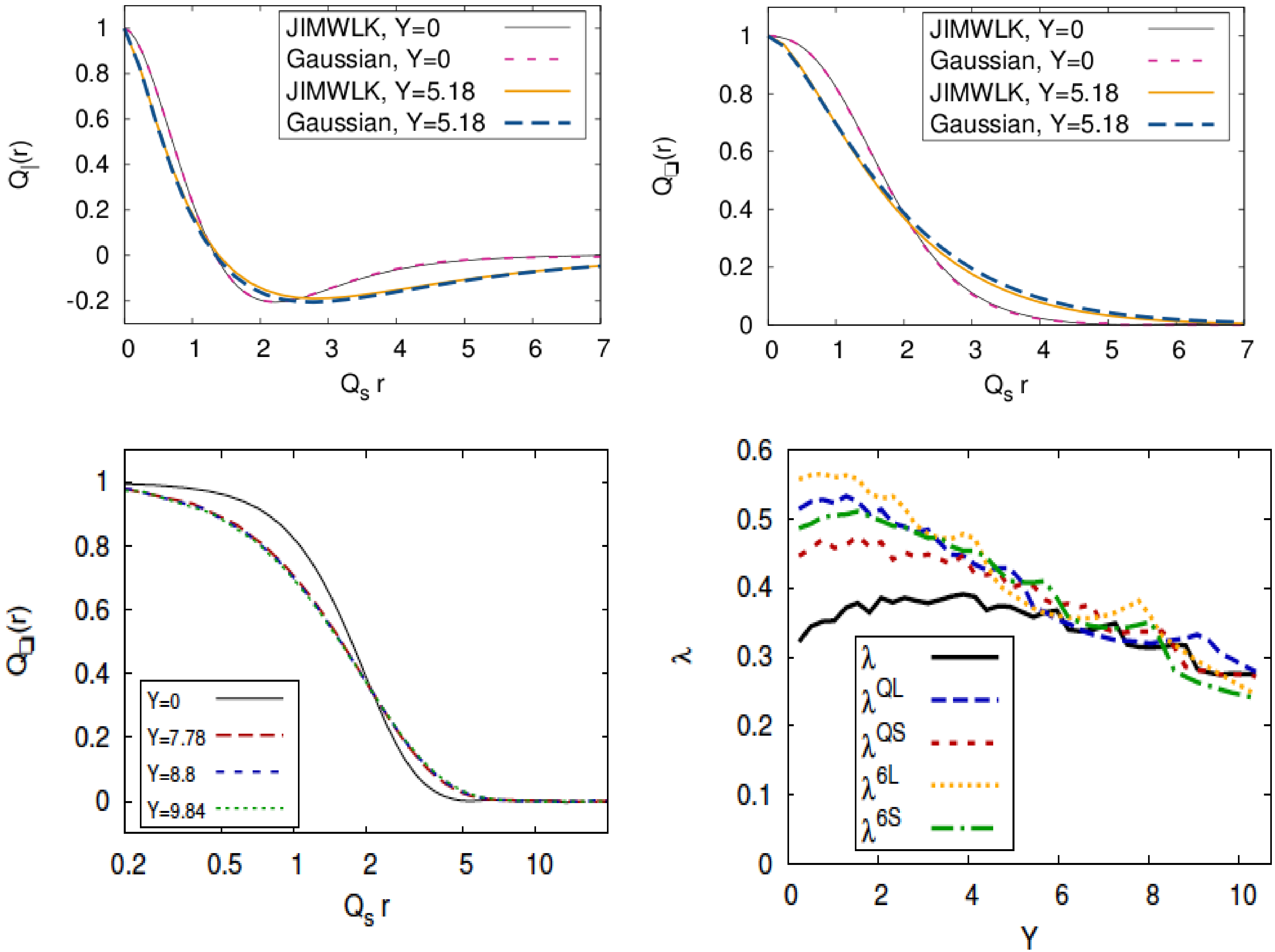
$$Q_{sq}(z) = [S(z)]^2 \left[\frac{N_c+1}{2} \left(\frac{S(z)}{S(\sqrt{2}z)} \right)^{\frac{2}{N_c+1}} - \frac{N_c-1}{2} \left(\frac{S(\sqrt{2}z)}{S(z)} \right)^{\frac{2}{N_c-1}} \right]$$

Gaussian + large Nc $\quad Q_|(z) \to S^2(z)[1 + 2\,log[S(z)]]$

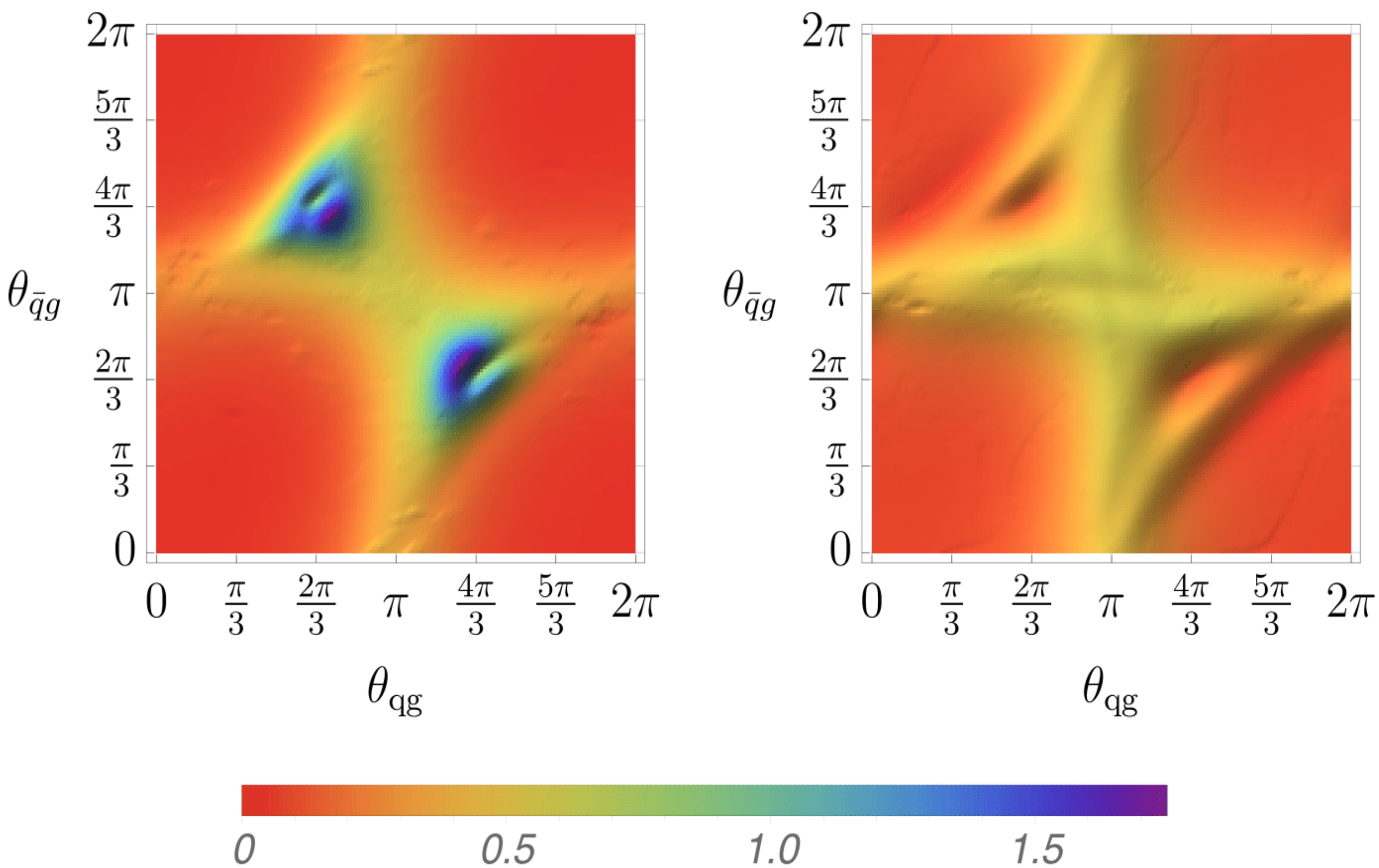
$$Q_{sq}(z) = S^2(z) \left[1 + 2 \ln \left(\frac{S(z)}{S(\sqrt{2}z)} \right) \right]$$

Quadrupole: $Q(r, \bar{r}, \bar{s}, s) \equiv \frac{1}{N_c} < Tr\, V(r)\, V^\dagger(\bar{r})\, V(\bar{s})\, V^\dagger(s) >$

Dumitru-Jalilian-Marian-Lappi-Schenke-Venugopalan:PLB706 (2011) 219



3-parton azimuthal angular correlations



multiple scattering:

broadening ***of the peak***

x-evolution:

reduction ***of magnitude***

some thoughts/ideas/dreams/......

cold matter energy loss

how important is cold matter Eloss in single inclusive production in the forward rapidity region?

cold matter energy loss?
Kopeliovich, Frankfurt and Strikman
Neufeld,Vitev,Zhang, PLB704 (2011) 590

Munier, Peigne, Petreska, arXiv:1603.01028

$$z\frac{dI}{dz} \equiv \frac{\frac{d\sigma a+A\to a+g+X}{dydy'd^2p_t}}{\frac{d\sigma a+A\to a+X}{dyd^2p_t}}$$

the difference between a nuclear target and a proton target is the medium induced energy loss

used to estimate the energy loss in single inclusive processes in the forward kinematics at RHIC and the LHC

can also do this for di-jets in DIS (3-parton production/2-parton production)

Possible extensions to other processes?

real photons: $Q^2 \to 0$

ultra-peripheral nucleus-nucleus collisions

inclusive 3-jet production

NLO inclusive di-jet production

crossing symmetry:

$$\gamma^{(\star)}\, T \longrightarrow q\, \bar{q}\, g\, X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\, T \longrightarrow q\, g\, \gamma^{(\star)}\, X \\ \bar{q}\, T \longrightarrow \bar{q}\, g\, \gamma^{(\star)}\, X \\ g\, T \longrightarrow q\, \bar{q}\, \gamma^{(\star)}\, X \end{array} \right\}$$

proton-nucleus collisions (collinear factorization in proton?)

di-jet + photon production in pA

$$pA \longrightarrow h_1\, h_2\, \gamma^{(\star)}\, X$$

Possible extensions to other processes?

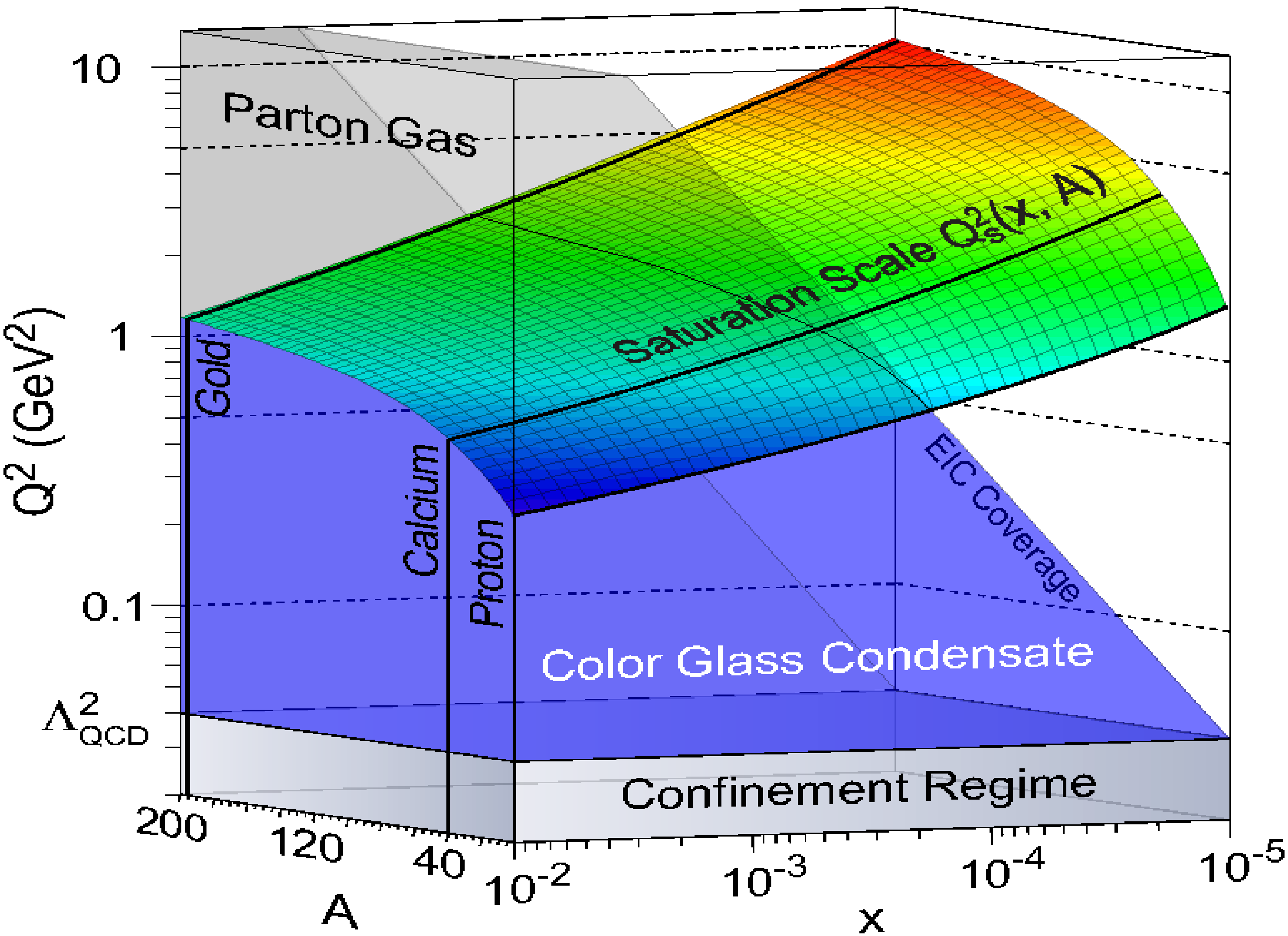
MPI (double/triple parton scattering)

$$\gamma^{(\star)}\, T \longrightarrow q\,\bar{q}\,g\,X \quad \Longleftrightarrow \quad \left\{ \begin{array}{l} q\,\bar{q}\,T \longrightarrow g\,\gamma^{(\star)}\,X \\ g\,\bar{q}\,T \longrightarrow \bar{q}\,\gamma^{(\star)}\,X \\ g\,q\,T \longrightarrow q\,\gamma^{(\star)}\,X \end{array} \right\}$$

$$pA \longrightarrow h\,\gamma^{(\star)}\,X$$

if one *assumes* target is accurately described by CGC at small x
this will tell us about DPS (proton "GPD" at large x)

The Saturation Scale Q_s



x 9/4 for gluons

SUMMARY

CGC is a systematic approach to high energy collisions

high gluon density: re-sum multiple soft scatterings

high energy: re-sum large logs of energy (rapidity or log 1/x)

Leading Log CGC works (too) well

it has been used to fit a wealth of data; ep, eA, pp, pA, AA

Precision (NLO) studies are needed

available for DIS, single inclusive forward production in pp, pA

Azimuthal angular correlations offer a unique probe of CGC

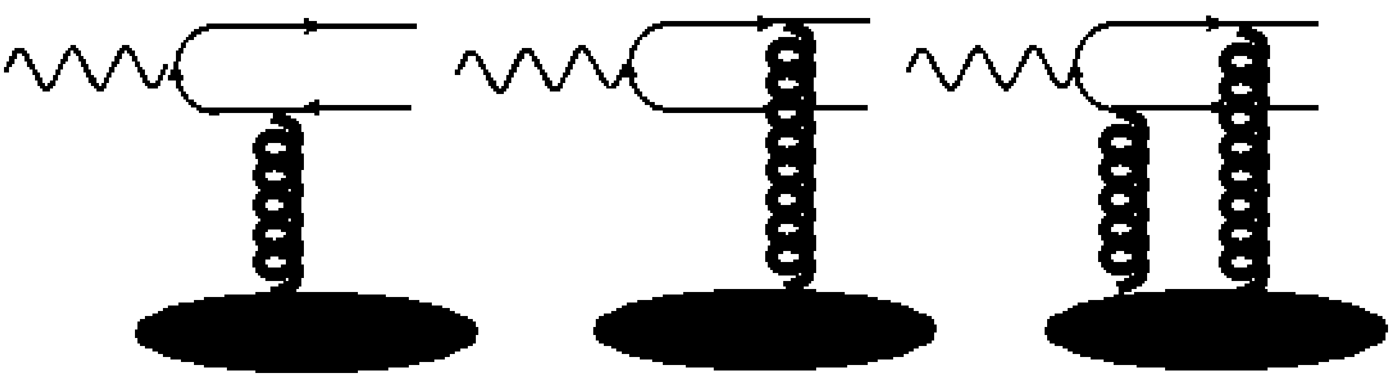
3-hadron/jet correlations should be even more discriminatory

di-hadron (azimuthal) angular correlations in DIS

DIS total cross section (F_L, F_2): ***dipoles*** $< \mathrm{Tr}\, V\, V^\dagger >$

di-jet production in DIS: ***quadrupoles*** $< \mathrm{Tr}\, V\, V^\dagger\, V\, V^\dagger >$

$$\text{LO:} \qquad \gamma^\star T \to q\,\bar{q}\,X$$



di-hadron production in DIS

$$\gamma^\star(\mathbf{k})\,\mathbf{p} \to \mathbf{q}(\mathbf{p})\,\bar{\mathbf{q}}(\mathbf{q})\,\mathbf{X}$$

$$\mathcal{A}^\mu(k,q,p) = \frac{i}{2}\int \frac{d^2 l_\perp}{(2\pi)^2} d^2x_\perp d^2y_\perp\, e^{i(p_\perp + q_\perp - k_\perp - l_\perp)\cdot y_\perp}$$
$$e^{i l_\perp \cdot x_\perp}\,\overline{u}(q)\,\Gamma^\mu(k^\pm, k_\perp, q^-, p^-, q_\perp - l_\perp)\,v(p)$$
$$\left[V(x_\perp)V^\dagger(y_\perp) - 1\right]$$

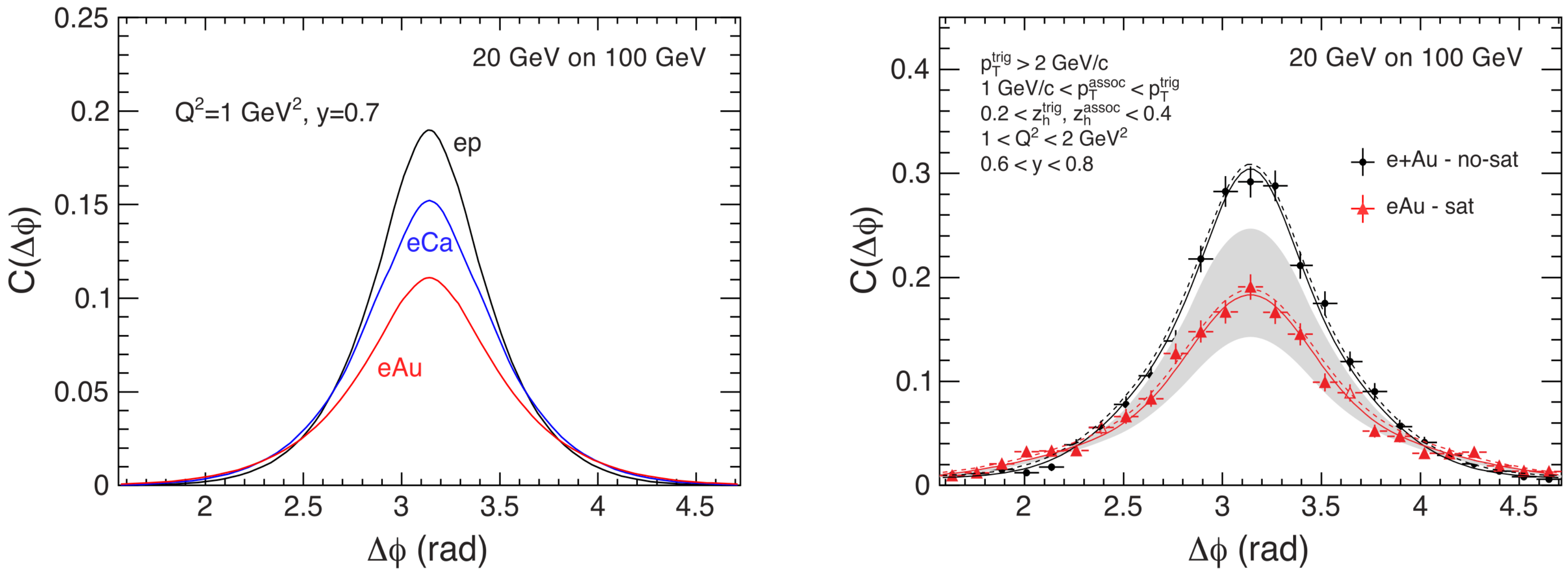
with

$$\Gamma^\mu \equiv \frac{\gamma^-(\not{q} - \not{l} + m)\gamma^\mu(\not{q} - \not{k} - \not{l} + m)\gamma^-}{p^-[(q_\perp - l_\perp)^2 + m^2 - 2q^- k^+] + q^-[(q_\perp - k_\perp - l_\perp)^2 + m^2]}$$

F. Gelis and J. Jalilian-Marian, PRD67 (2003) 074019

Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037

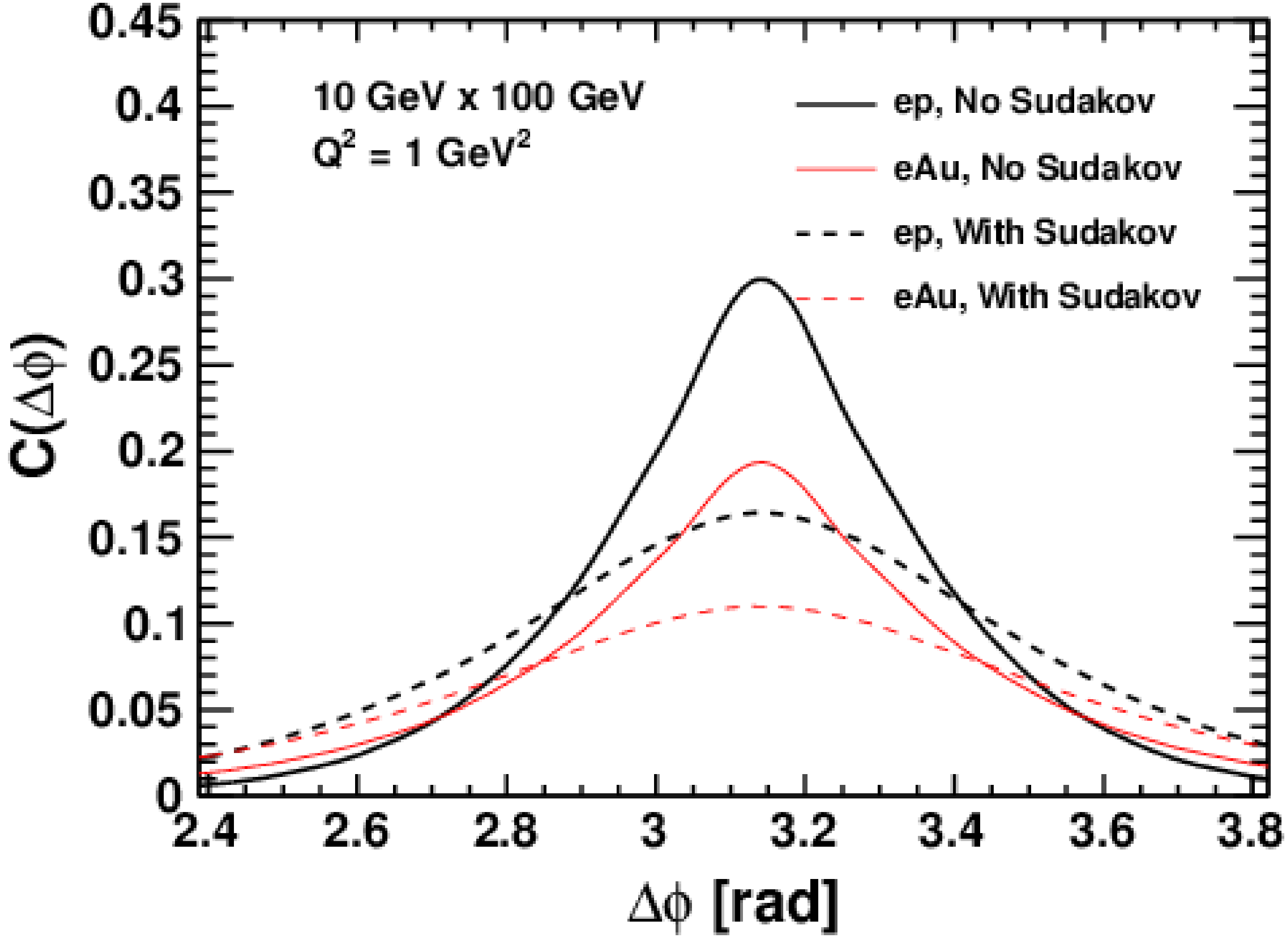
Di-hadron azimuthal correlations in DIS



Electron Ion Collider...., A. Accardi et al., arXiv:1212.1701

Zheng-Aschenauer-Lee-Xiao, PRD89 (2014)7, 074037

Azimuthal correlations in DIS



Zheng + Aschenauer + Lee + Xiao, PRD89 (2014)7, 074037