The Neutrinos

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Part 1

(c) Kamioka Observatory, ICRR(Institute for Cosmic Ray Research), The University of Tokyo

- Physics
- Experimental Findings
- Theoretical Ideas

The Neutrino Breakthrough
(1998 – …)

Neutrinos have nonzero masses!

The 2015 Nobel Prize in Physics went to **Takaaki Kajita** and **Art McDonald** for the experiments that proved this.

Super-
Kamiokande,
Japan

Sudbury
Neutrino
Observatory,
Canada

The 2016 Breakthrough Prize in Fundamental Physics went to these two experiments and four subsequent ones.

The Origin of Neutrino Mass

The fundamental constituents of matter
are the ***quarks***, the ***charged leptons***,
and the ***neutrinos***.

The discovery and study of the ***Higgs boson*** at CERN's Large Hadron Collider has provided strong evidence that the ***quarks*** and ***charged leptons*** derive their masses from an interaction with the ***Higgs field***.

Most theorists strongly suspect that the origin of the neutrino masses is different from the origin of the quark and charged lepton masses.

The Standard-Model ***Higgs field*** is probably still involved,
but there is probably something more —
something way outside the Standard Model —

Majorana masses.

More later ……….

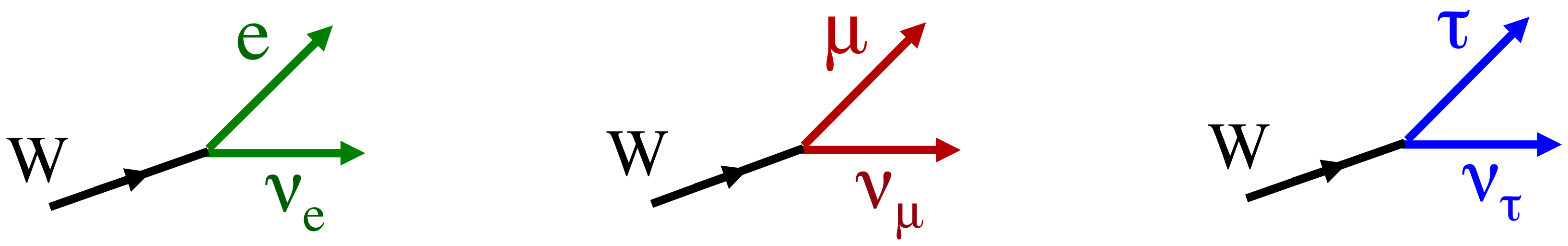
The discovery of neutrino mass comes from the observation of ***neutrino flavor change (neutrino oscillation)***.

The Physics of Neutrino Oscillation

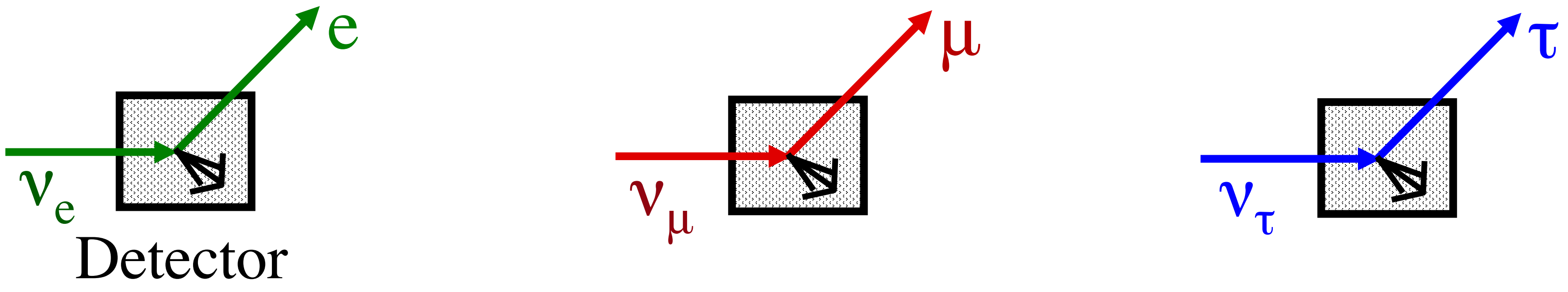
— Preliminaries

The Neutrino Flavors

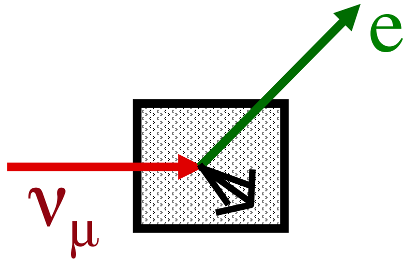
We ***define*** the known neutrinos of specific flavor, ν_e, ν_μ, ν_τ, by W boson decays:



As far as we know, when a neutrino of given flavor interacts and turns into a charged lepton, that charged lepton will always be of the same flavor as the neutrino.



but not

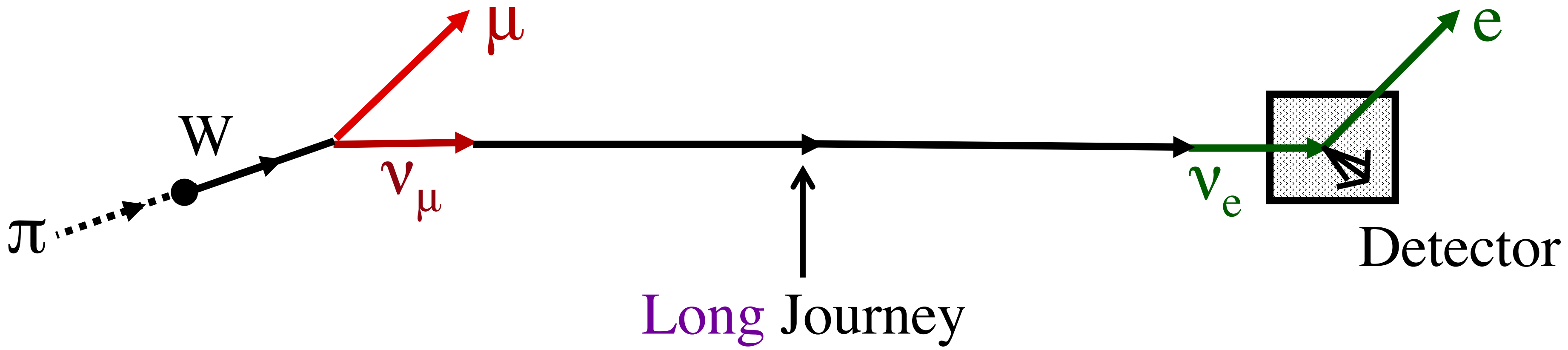


Lederman
Schartz
Steinberger

The weak interaction couples the neutrino of a given flavor only to the charged lepton of the same flavor.

Neutrino Flavor Change ("Oscillation")

If neutrinos have masses, and leptons mix,
we can have —



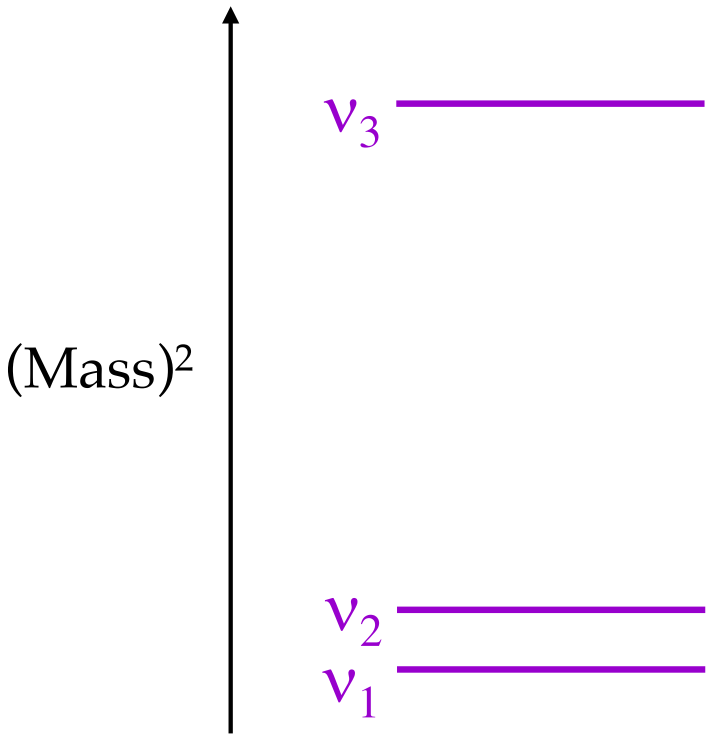
Give a ν time to change character, and you can have

for example: $\nu_\mu \longrightarrow \nu_e$

The last 19 years have brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum
of neutrino mass eigenstates ν_i:



$\text{Mass}\,(\nu_i) \equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e$ or $\mu\nu_\mu$ or $\tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i}\,|\nu_i> \quad .$$

Neutrino of flavor
α = e, μ, or τ

Neutrino of definite mass m_i

"PMNS" Leptonic Mixing Matrix

Notation: ℓ denotes a charged lepton. $\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$.

Since the only charged lepton ν_α couples to is ℓ_α ,
the 3 ν_α must be orthogonal.

To make up 3 orthogonal ν_α , we must have at least 3 ν_i .
Unless some ν_i masses are degenerate,
all ν_i will be orthogonal.

Then —

$$\delta_{\alpha\beta} = \langle \nu_\alpha | \nu_\beta \rangle = \left\langle \sum_i U^*_{\alpha i} \nu_i \middle| \sum_j U^*_{\beta j} \nu_j \right\rangle$$

$$= \sum_{i,j} U_{\alpha i} U^*_{\beta j} \langle \nu_i | \nu_j \rangle = \sum_i U_{\alpha i} U^*_{\beta i}$$

This says that
U is unitary,
but note the
unitary *U* may
not be 3 x 3.

Leptonic mixing is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

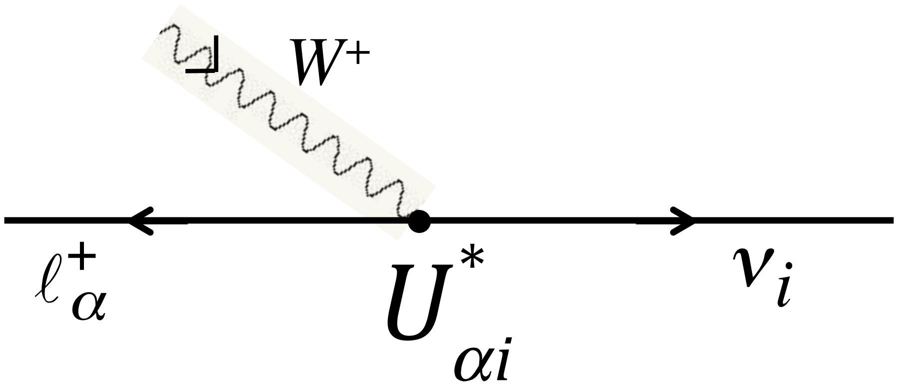
$$\mathcal{L}_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \bar{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

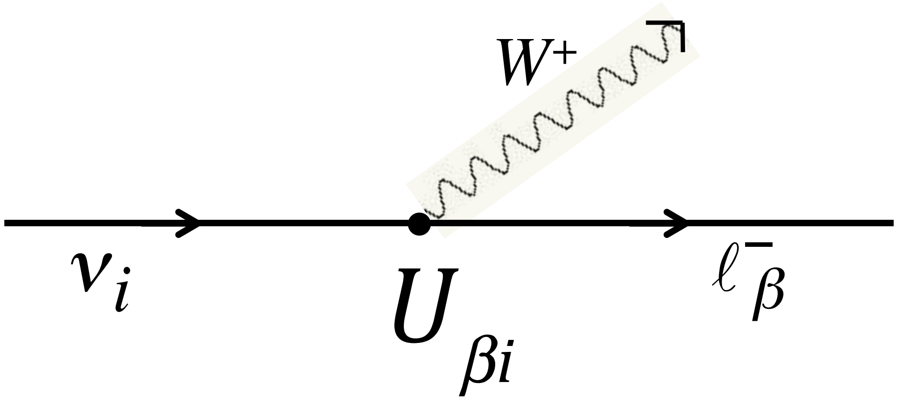
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

Taking mixing into account

The SM interaction conserves the Lepton Number L, defined by $L(\nu) = L(\ell^{-}) = -L(\bar{\nu}) = -L(\ell^{+}) = 1$.

Mixing Matrix Elements
At the Vertices

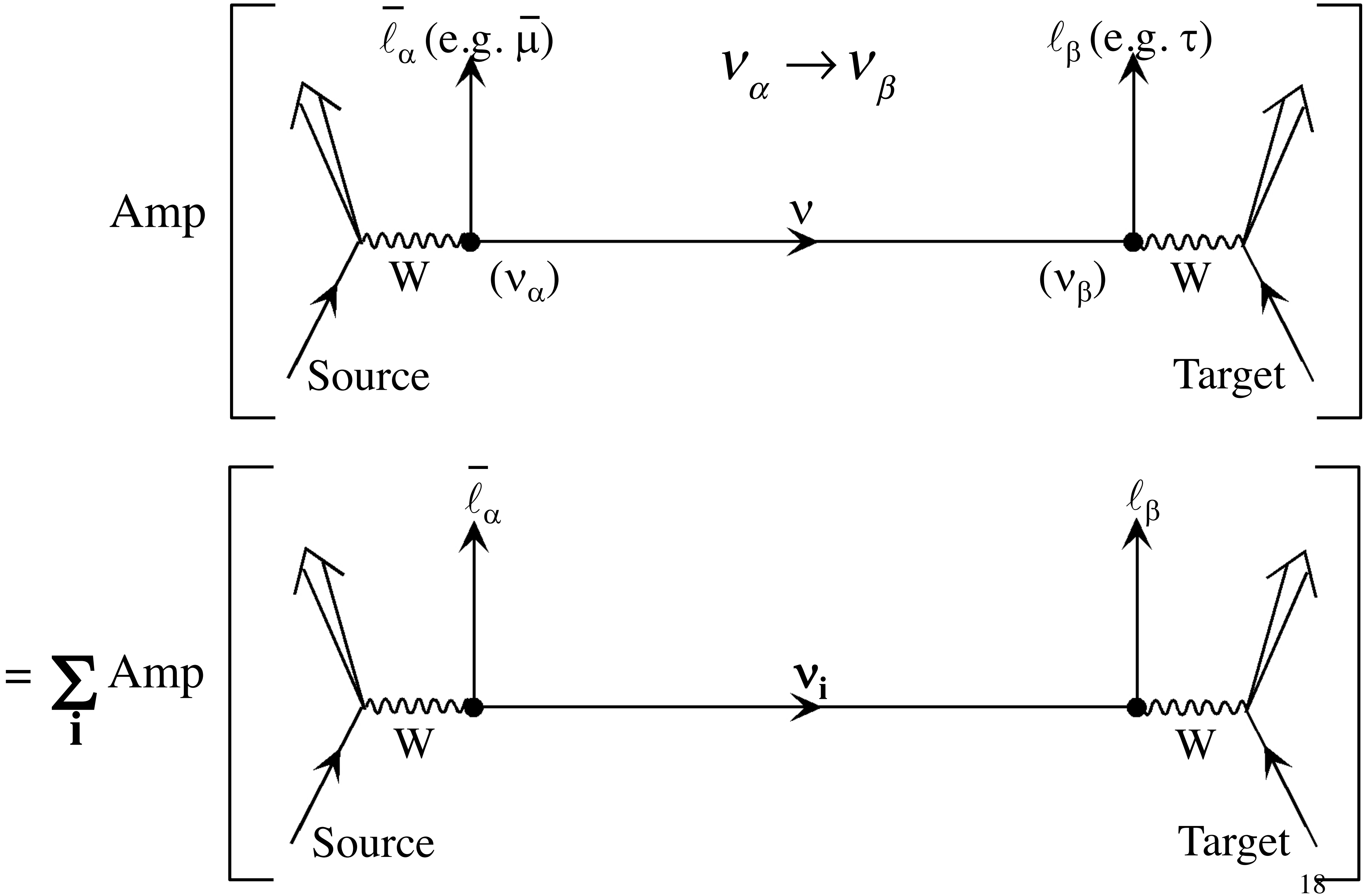




How Neutrino Oscillation Works

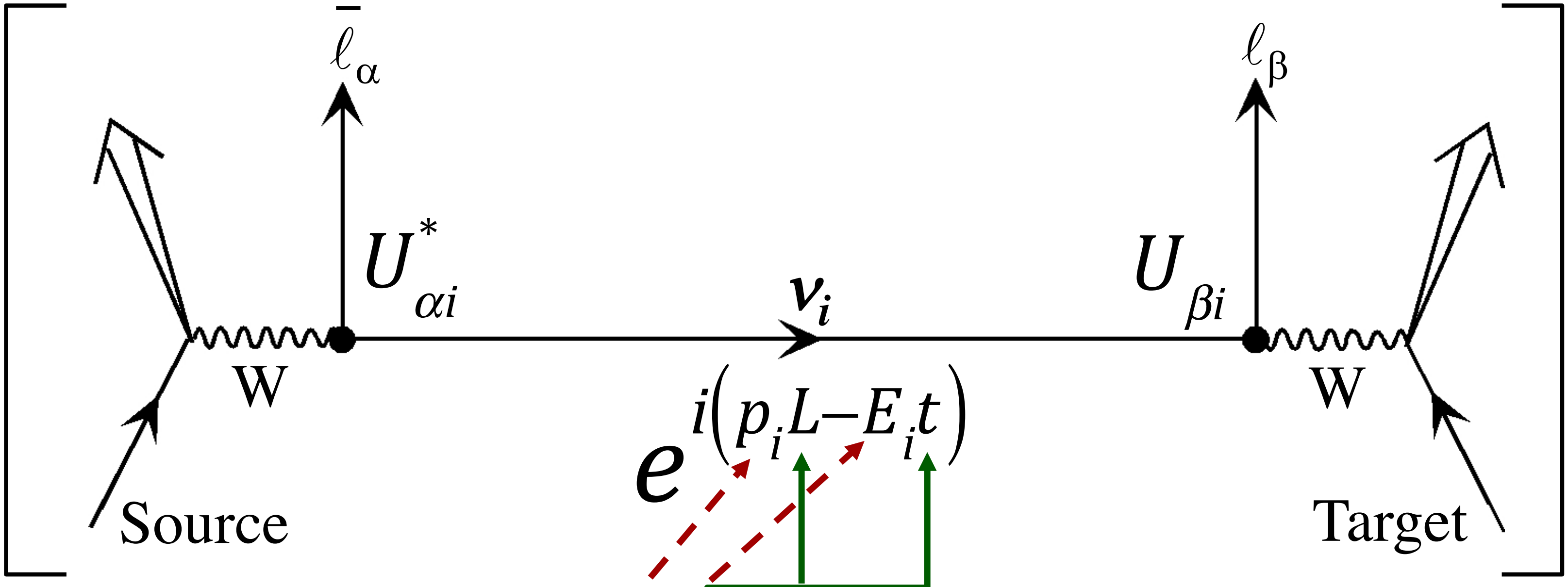
Neutrino Flavor Change (“Oscillation”)

(Approach of B.K. and Stodolsky)



$$\text{Amp}\left(\nu_\alpha \to \nu_\beta\right)$$

$$= \sum_{\mathbf{i}} \text{Amp}$$



$$e^{i\left(p_i L - E_i t\right)}$$

Momentum and energy of ν_i

Coordinates of detection point, taking source point as (0, 0)

Neutrino sources are ~ constant in time.

Averaged over time, the

$$e^{-iE_1t} - e^{-iE_2t} \quad \text{interference}$$

is —

$$\left\langle e^{-i(E_1-E_2)t} \right\rangle_t = 0$$ *unless* $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ν_i,

$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E}\ .$$

Then the plane-wave factor $e^{i(p_i L - E_i t)}$ is —

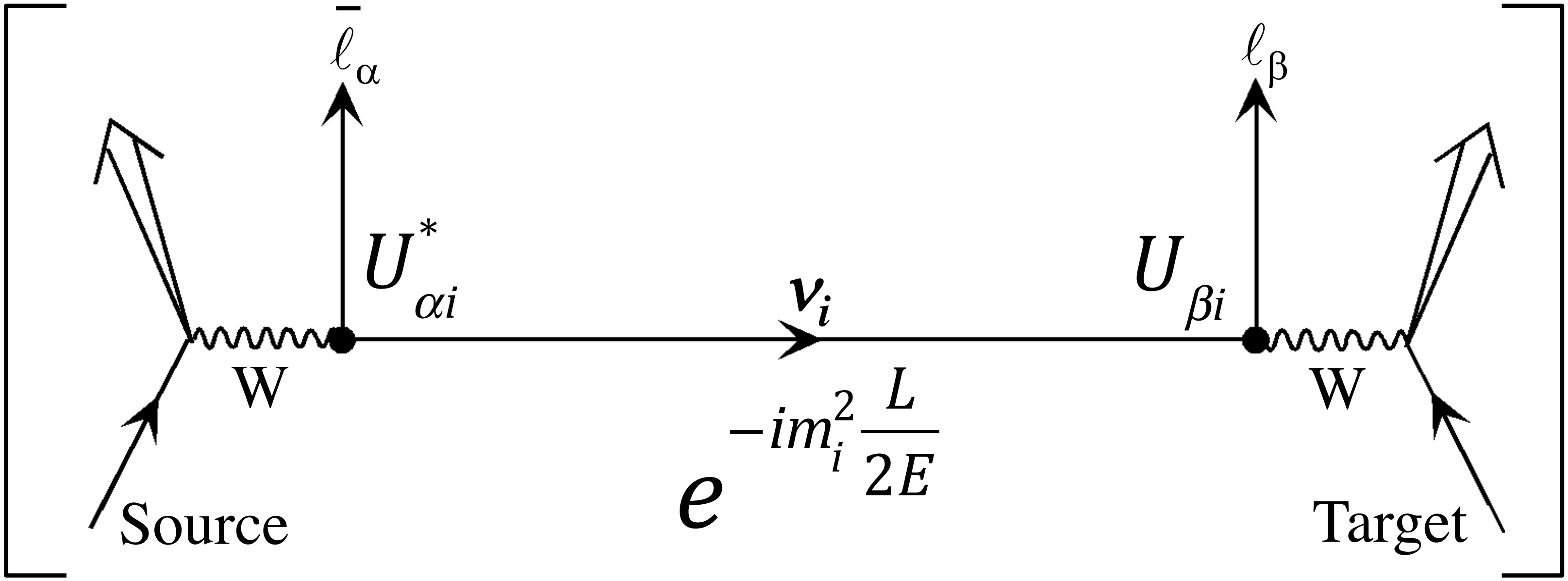
$$e^{i(p_i L - E_i t)} \cong e^{i\left\{\left(E - \frac{m_i^2}{2E}\right)L - Et\right\}} = e^{iE(L-t)}\, e^{-im_i^2 \frac{L}{2E}}$$

Irrelevant overall phase factor

Then —

$$\mathrm{Amp}\left(\nu_\alpha \to \nu_\beta\right)$$

$$= \sum_{\mathbf{i}} \mathrm{Amp}$$



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability of Neutrino Oscillation in Vacuum

$$P\left(\nu_\alpha \to \nu_\beta\right) = \left|\mathrm{Amp}\left(\nu_\alpha \to \nu_\beta\right)\right|^2 =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin^2\left(\Delta m^2_{ij} \frac{L}{4E}\right)$$

$$+ 2\sum_{i>j} \mathrm{Im}\left(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}\right) \sin\left(\Delta m^2_{ij} \frac{L}{2E}\right)$$

where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$.

Neutrino flavor change implies neutrino mass!

Neutrinos vs. Antineutrinos

$$\left[\bar{\nu}_\alpha(\mathrm{RH}) \rightarrow \bar{\nu}_\beta(\mathrm{RH})\right] = \mathrm{CP}\left[\nu_\alpha(\mathrm{LH}) \rightarrow \nu_\beta(\mathrm{LH})\right]$$

A difference between the probabilities of these two oscillations in vacuum would be a leptonic violation of CP invariance.

Assuming CPT invariance —

$$P\left[\bar{\nu}_\alpha(\mathrm{RH}) \rightarrow \bar{\nu}_\beta(\mathrm{RH})\right] = P\left[\nu_\beta(\mathrm{LH}) \rightarrow \nu_\alpha(\mathrm{LH})\right]$$

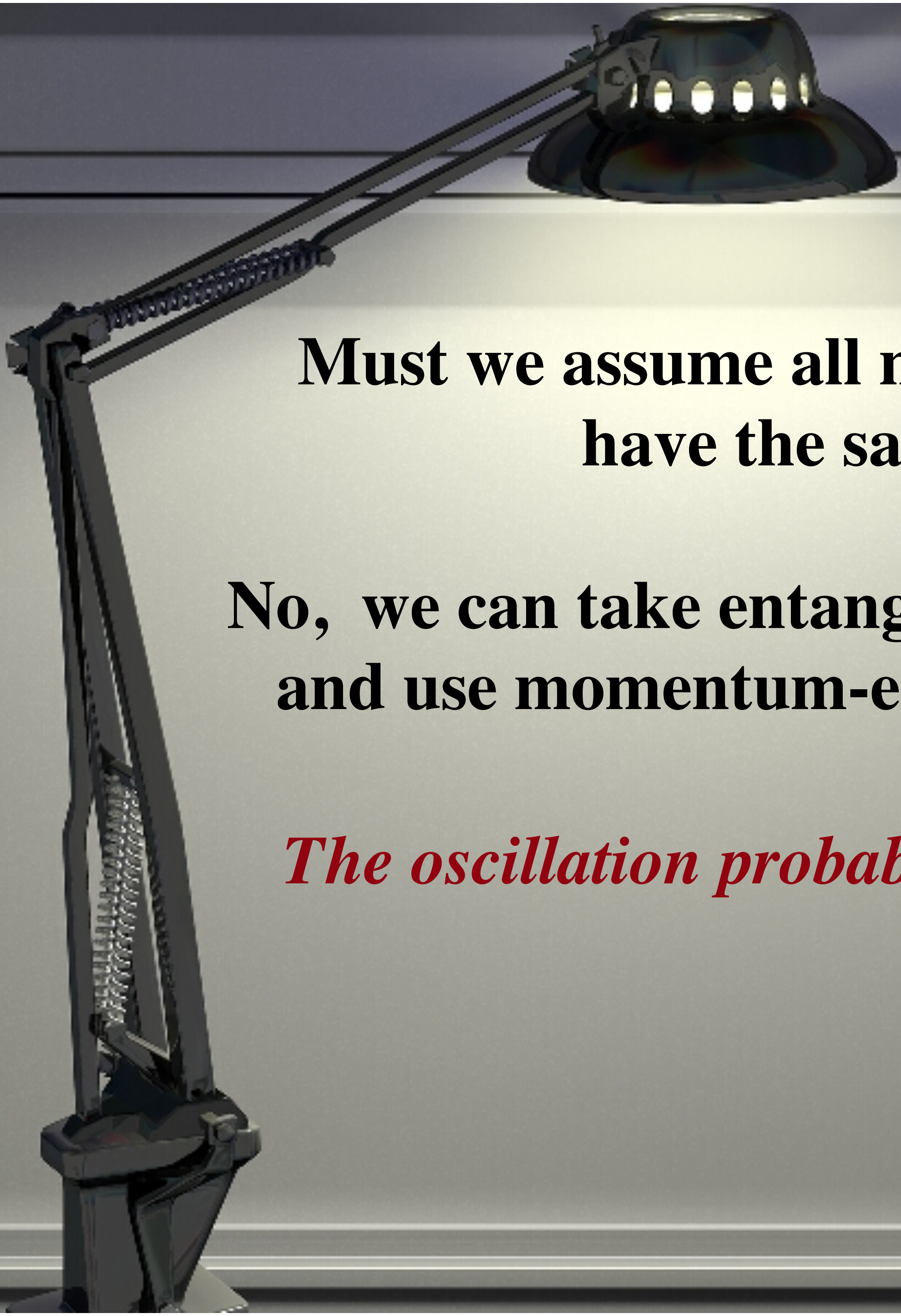
$$P\left(\overset{(-)}{\nu}_{\alpha} \to \overset{(-)}{\nu}_{\beta}\right) =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^{*}_{\alpha i} U_{\beta i} U_{\alpha j} U^{*}_{\beta j}\right) \sin^{2}\left(\Delta m^{2}_{ij} \frac{L}{4E}\right)$$

$$\overset{+}{\underset{(-)}{}} 2\sum_{i>j} \mathrm{Im}\left(U^{*}_{\alpha i} U_{\beta i} U_{\alpha j} U^{*}_{\beta j}\right) \sin\left(\Delta m^{2}_{ij} \frac{L}{2E}\right)$$

In neutrino oscillation, CP non-invariance comes from phases in the leptonic mixing matrix U.

Note: Including $\hbar$ and c, $\Delta m^{2}_{ij} \frac{L}{4E} = 1.27 \Delta m^{2}_{ij}\left(\mathrm{eV}^{2}\right) \frac{L\left(\mathrm{km}\right)}{E\left(\mathrm{GeV}\right)}$



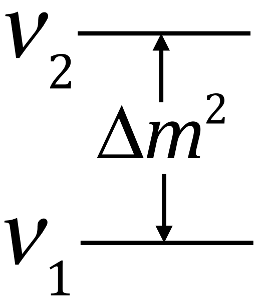
Must we assume all mass eigenstates have the same *E*?

No, we can take entanglement into account, and use momentum-energy conservation.

The oscillation probabilities are still the same.

(B.K.)

When Only Two Flavors and Two Mass Eigenstates Matter



Does not affect oscillation

$$U = \begin{pmatrix} U_{\alpha 1} & U_{\alpha 2} \\ U_{\beta 1} & U_{\beta 2} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{i\xi} & 0 \\ 0 & 1 \end{pmatrix}$$

Mixing angle

For $\beta \neq \alpha$, $P\left(\overset{(-)}{\nu}_{\alpha} \leftrightarrow \overset{(-)}{\nu}_{\beta}\right) = \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right)$

For no flavor change,

$$P\left(\overset{(-)}{\nu}_{\alpha} \rightarrow \overset{(-)}{\nu}_{\alpha}\right) = 1 - \sin^2 2\theta \sin^2\left(\Delta m^2 \frac{L}{4E}\right)$$

For Some Applications, the Plane Wave Treatment of Neutrino Oscillation Is Wrong

The probability of neutrino oscillation depends on the distance L between the neutrino source and the point of detection.

To determine L, we must know where the neutrino started, and where it was detected.

A plane wave has a definite, precise momentum p.

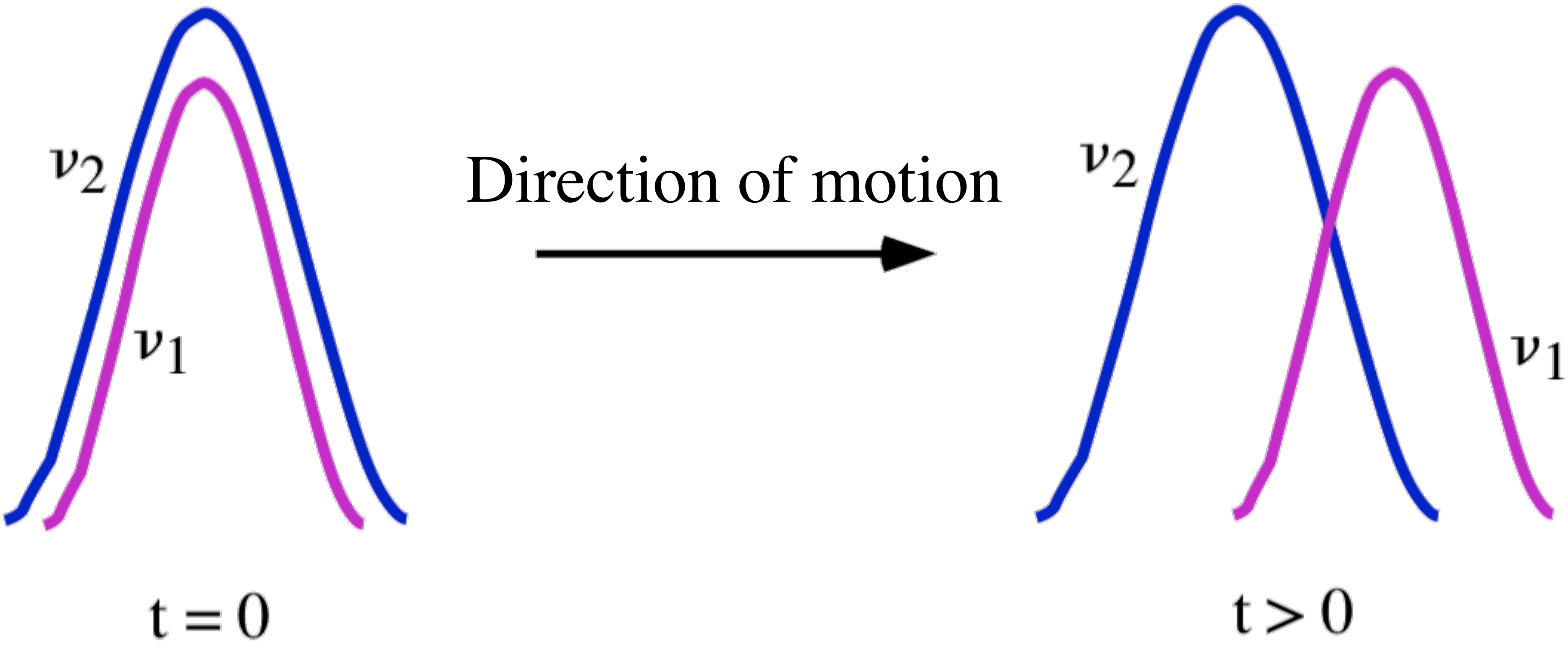
Heisenberg: $\Delta x \Delta p \geq \hbar/2$.

If we know precisely the momentum with which a neutrino was born, we know nothing about where it was born.

The Wave Packet Picture

Each mass eigenstate is described by a wave packet.

Suppose ν_2 is heavier than ν_1.



Direction of motion

Eventually the wave packets will separate.

No more oscillation.

How soon do the wave packets separate??

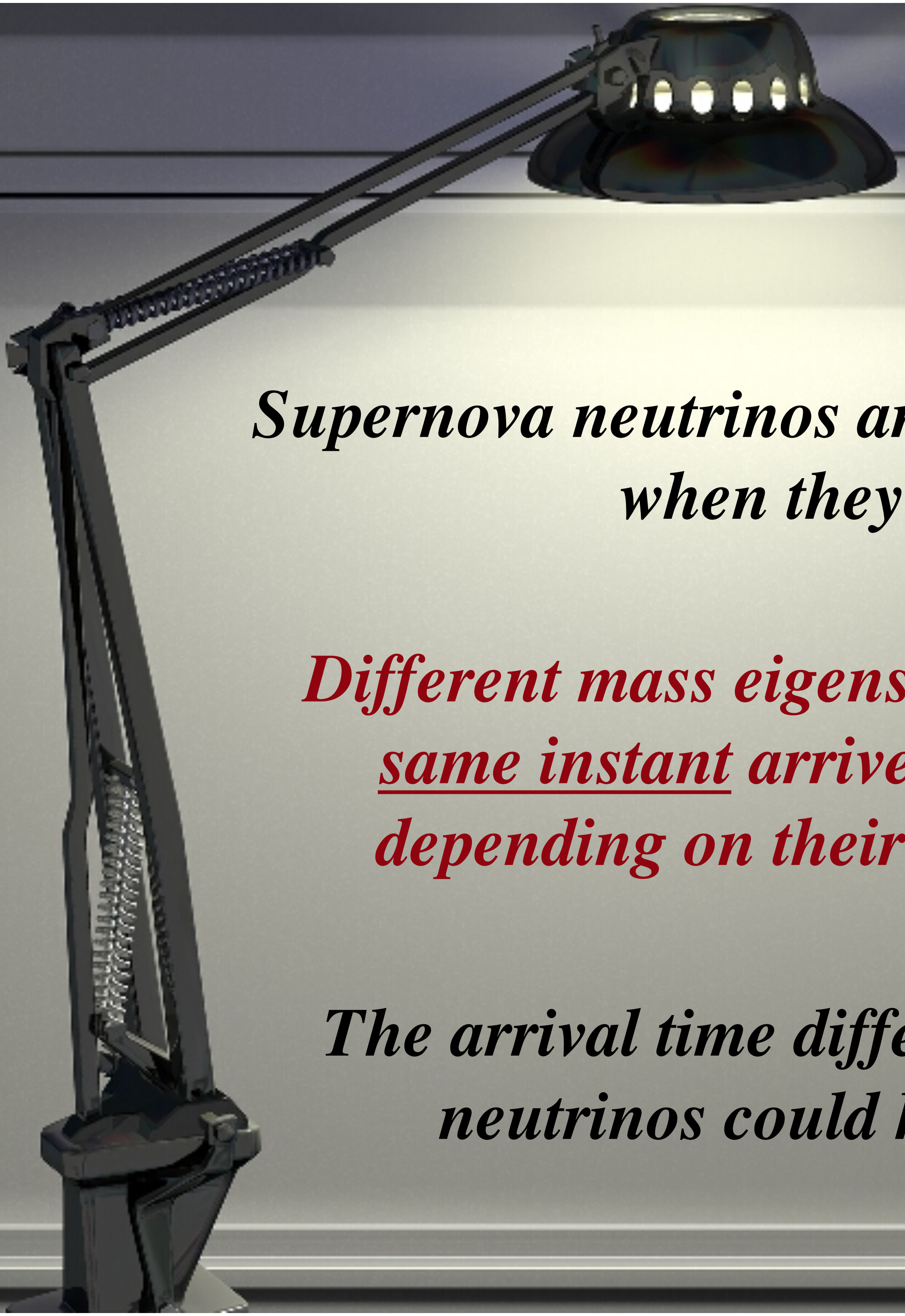
For accelerator neutrinos with energy $E = 1$ GeV, and a wave packet width equal to the length of the pion decay region where the neutrinos are born, the bigger $\Delta m^2 = 2.4 \times 10^{-3}$ eV^2 leads to wave packet separation in

10^{20} km.

This separation may be safely ignored!

However, for supernova neutrinos from SN 1987A, with energy $E \sim 10$ MeV, and a wave packet width equal to an *estimated* inter-nucleon distance within the star, separation occurs in

10^{3} km.

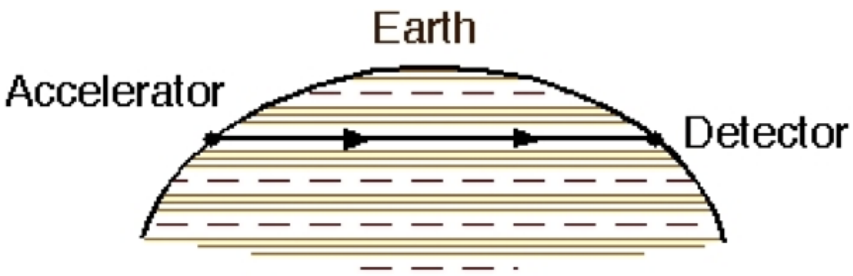


Supernova neutrinos are no longer oscillating when they reach us.

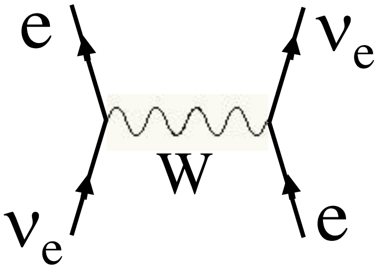
Different mass eigenstates produced at the same instant arrive at separate times, depending on their individual speeds.

The arrival time difference for the SN 1987 A neutrinos could have been ~10^{-4} sec.

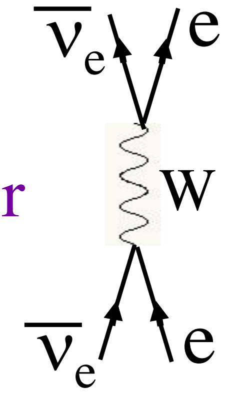
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this
W-exchange interaction leads to
an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant

Electron density

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\overbrace{[\sqrt{2}G_F N_e]}^{\text{Interaction energy}} \;/\; \overbrace{[\Delta m^2/2E]}^{\text{Vacuum energy}} \equiv x \; .$$

The matter effect —

- — Grows with neutrino energy E
- — Is sensitive to Sign(Δm^2)
- — Reverses when ν is replaced by $\bar{\nu}$

This last is a “fake CP violation” that has to be taken into account in searches for genuine CP violation.