The Leptonic CP Phases

Shao-Feng Ge

(gesf02@gmail.com)

Max-Planck-Institut für Kernphysik, Heidelberg, Germany

2017-5-19

SFG, Duane A. Dicus, Wayne W. Repko, PLB **702**, 220 (2011) [arXiv:1104.0602]
SFG, Duane A. Dicus, Wayne W. Repko, PRL **108**, 041801 (2012) [arXiv:1108.0964]
Andrew D. Hanlon, **SFG**, Wayne W. Repko, PLB **729**, 185-191 (2014) [arXiv:1308.6522]
SFG [arXiv:1406.1985]
Jarah Evslin, **SFG**, Kaoru Hagiwara, JHEP **1602** (2016) 137 [arXiv:1506.05023]
SFG, Pedro Pasquini, M. Tortola, J. W. F. Valle, PRD **95** (2017) No.3, 033005 [arXiv:1605.01670]
SFG, Alexei Smirnov, JHEP **1610** (2016) 138 [arXiv:1607.08513]
SFG [arXiv:1704.08518]
SFG, Manfred Lindner, PRD **95** (2017) No.3, 033003 [arXiv:1608.01618]

Why neutrino mass & oscillation?

- Higgs boson for electroweak symmetry breaking & mass.
- Chiral symmetry breaking for mass.
- **The world seems not affected by the tiny neutrino mass!**
  - Neutrino mass $\Rightarrow$ Mixing
  - 3 Neutrino $\Rightarrow$ possible **CP violation**
  - CP violation $\Rightarrow$ Leptogenesis
  - Leptogenesis $\Rightarrow$ **Matter-Antimatter Asymmetry**
  - There is something left in the Universe.
  - Baryogenesis from quark mixing is not enough.

ν Oscillation Data

(for NH)	-1σ	Best Value	$+1\sigma$
$\mathbf{\Delta m_s^2} \equiv \Delta m_{12}^2\,(\mathbf{10^{-5}}\mathrm{eV}^{\mathbf{2}})$	7.33	**7.50**	7.69
$\lvert\mathbf{\Delta m_a^2} \equiv \Delta m_{13}^2\rvert\;(\mathbf{10^{-3}}\mathrm{eV}^{\mathbf{2}})$	2.484	**2.524**	2.563
$\sin^2\theta_s$ $(\theta_s \equiv \theta_{12})$	0.294 (32.81°)	0.306 (**33.56°**)	0.318 (34.33°)
$\sin^2\theta_a$ $(\theta_a \equiv \theta_{23})$	0.4200 (40.4°)	0.441 (**41.6°**)	0.468 (43.1°)
$\sin^2\theta_r$ $(\theta_r \equiv \theta_{13})$	0.02091 (8.41°)	0.02166 (**8.46°**)	0.02241 (8.61°)
δ_D, δ_{Mi}	?, **??**	?, **??**	?, **??**

Esteban, Gonzalez-Garcia, Maltoni, Martinez-Soler & Schwetz, arXiv:1611.01514

Evidence of μ–τ Symmetry

- **Two small deviations (1σ level):**

$$-3.5^\circ < \theta_a - 45^\circ < 5.8^\circ \qquad 8.4^\circ < \theta_r < 9.2^\circ$$

with **Best Fit Value**: $\boldsymbol{\theta_a - 45^\circ = -3.9^\circ}$ & $\boldsymbol{\theta_r = 8.8^\circ}$.

- **Zeroth Order Approximation:**

$$\boxed{\boldsymbol{\theta_a \approx 45^\circ, \qquad \theta_r \approx 0^\circ.}}$$

$\Rightarrow$ **CP** & **μ–τ Symmetric** Mass Matrix:

$$M_\nu^{(0)} = \begin{pmatrix} A & \mathbf{B} & \mathbf{B} \\ & \mathbf{C} & D \\ & & \mathbf{C} \end{pmatrix}$$

Mohapatra & Nussinov [hep-ph/9809415], Lam [hep-ph/0104116]

Horizontal Symmetry

[Lam, PRL**101**:121602(2008), PRD**78**:073015(2008)]

- **Mass Matrix** $\mathbf{M}_\nu$ invariant under **Transformation**:

$$\mathbf{G}_\nu^T \mathbf{M}_\nu \mathbf{G}_\nu = \mathbf{M}_\nu$$

- **Diagonalization**:

$$\mathbf{V}_\nu^T \mathbf{M}_\nu \mathbf{V}_\nu = \mathbf{D}_\nu$$

- **Rephasing**:

$$\mathbf{D}_\nu = \mathbf{d}_\nu^T \mathbf{D}_\nu \mathbf{d}_\nu$$

with $\mathbf{d}_\nu^2 = \mathbf{I}_3 \quad \Rightarrow \quad \mathbf{d}_\nu = \mathrm{diag}(\pm, \pm, \pm)$.

- Together

$$\begin{aligned} \mathbf{M}_\nu &= \mathbf{G}_\nu^T \mathbf{M}_\nu \mathbf{G}_\nu = \underline{\mathbf{G}_\nu^T \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger \mathbf{G}_\nu} \\ &= \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger = \underline{\mathbf{V}_\nu^* \mathbf{d}_\nu^T \mathbf{D}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger} \end{aligned}$$

- **Consequence**: $\mathbf{V}_\nu^\dagger \mathbf{G}_\nu = \mathbf{d}_\nu \mathbf{V}_\nu^\dagger \Leftrightarrow \boxed{\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger}$

- **For Leptons:** $\underline{\mathbf{F}_\ell = \mathbf{V}_\ell \mathbf{d}_\ell \mathbf{V}_\ell^\dagger}$ with $\mathbf{d}_\ell = \mathrm{diag}(\mathbf{e}^{\mathbf{i}\beta_1}, \mathbf{e}^{\mathbf{i}\beta_2}, \mathbf{e}^{\mathbf{i}\beta_3})$.

Symmetry v.s. Mixing

[Lam, PRL**101**:121602(2008), PRD**78**:073015(2008)]

- **Two Nontrivial Independent** possibilities of $\mathbf{d}_\nu$:

$$\mathbf{d}_\nu^{(1)} = \mathrm{diag}(-1,1,1), \quad \mathbf{d}_\nu^{(2)} = \mathrm{diag}(1,-1,1), \qquad \mathbf{d}_\nu^{(3)} = -d_\nu^{(1)} d_\nu^{(2)} .$$

- $\boldsymbol{\theta}_\mathbf{s}$ parameterized in terms of $\mathbf{k}$: $\boxed{\tan\theta_s = \sqrt{2}/k}$

$$V_\nu(k) = \begin{pmatrix} \frac{k}{\sqrt{2+k^2}} & \frac{\sqrt{2}}{\sqrt{2+k^2}} & 0 \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mathbf{k}=\mathbf{2}$ $\quad \boldsymbol{\theta}_\mathbf{s} = \mathbf{35.3^\circ}$ **[TBM]**

$\mathbf{k} = \frac{\mathbf{3}}{\sqrt{\mathbf{2}}}$ $\quad \boldsymbol{\theta}_\mathbf{s} = \mathbf{33.7^\circ}$

$\mathbf{k} = \sqrt{\mathbf{6}}$ $\quad \boldsymbol{\theta}_\mathbf{s} = \mathbf{30.0^\circ}$

- **Two Independent** *Symmetry Transformations* $\mathbf{G_i} = \mathbf{V}_\nu \mathbf{d}_\nu^{(\mathbf{i})} \mathbf{V}_\nu^\dagger$

$$G_1 = \frac{1}{2+k^2}\begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}, \qquad G_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- $\mathbb{Z}_2^s(\times\overline{\mathbb{Z}}_2^s) \times \mathbb{Z}_2^{\mu\tau} \equiv \mathcal{G} = \{\mathbf{E}, \mathbf{G_1}, \mathbf{G_2}(\equiv G_1 G_3), \mathbf{G_3}\}$

Full v.s. Residual

[Lam, PRL**101**:121602(2008), PRD**78**:073015(2008)]

Full Symmetries:

$\mathcal{H} \equiv \mathcal{G} \times \mathcal{F}$	$\mathcal{G}$	$\mathcal{F}$
S_4	$\mathbb{Z}_2^s \times \mathbb{Z}_2^{\mu\tau}$	$\mathbb{Z}_3 \equiv \{I, F, F^2\}$
$\{G_1, G_3, F\}$	$G_1(G_2), G_3$	$F \equiv \text{diag}\,(1, \omega, \omega^2)$

Bottom-Up $\Uparrow$ $\Downarrow$ Top-Down

See also Smirnov et. al., 1204.0445, 1212.2149, 1510.00344

Residual Symmetries:

$\boldsymbol{\nu_i}$: $\mathcal{G} \equiv \mathbb{Z}_2^s(\overline{\mathbb{Z}}_2^s) \times \mathbb{Z}_2^{\mu\tau}$ for $d_\nu^i = \text{diag}\,(\pm 1, \pm 1, \pm 1)$

$\boldsymbol{\ell_i}$: $\mathcal{F} \in U(1) \times U(1)$ for $d_\ell^i = \text{diag}\,(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$

Residual Symmetry as Effective Theory

- **Full symmetry HAS TO be Broken!**
  - Fermion needs to acquire mass.
  - Non-trivial mixing $\boxed{V_{\rm PMNS} = V_\ell^\dagger V_\nu}$
- If mixing is **TRUELY determined by symmetry**, it has to be **residual symmetry**
  - VEVs
  - Yukawa couplings
- **Residual Symmetry** as **Custodial Symmetry**
  - **Gauge symmetry has to be broken**. Otherwise, no mixing.
  - **Weak mixing angle** is a function of gauge couplings, which **cannot be dictated by gauge symmetry** (and VEV).
  - **Weak mixing angle is related to** the physical observables, the **gauge boson masses**, by **custodial symmetry**.

Example Zee-A_4 Model

hep-ph/0508278

Lepton's Representation:

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \sim \mathbf{3}\,, \quad \begin{matrix} e_R \sim \mathbf{1} \\ \mu_R \sim \mathbf{1}'\,, \\ \tau_R \sim \mathbf{1}'' \end{matrix} \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \mathbf{3}\,.$$

A_4 invariant Lagrangian:

$$\begin{aligned} \mathcal{L}_\ell &= \mathbf{y_1} \overline{e}_R (\mathbf{1} \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \mathbf{1} \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y_2} \overline{\mu}_R (\omega \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega^2 \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y_3} \overline{\tau}_R (\omega^2 \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega \varphi_3^\dagger \tau_L)\,. \end{aligned}$$

Mass term with $\langle \varphi_i \rangle = v_i$:

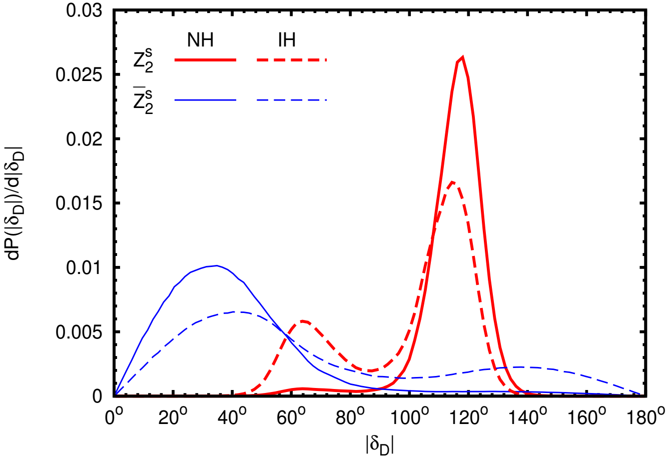
$$\mathcal{L}_\ell = \begin{pmatrix} \overline{e}_R & \overline{\mu}_R & \overline{\tau}_R \end{pmatrix} \begin{pmatrix} \mathbf{y_1} & & \\ & \mathbf{y_2} & \\ & & \mathbf{y_3} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \omega & \mathbf{1} & \omega^2 \\ \omega^2 & \mathbf{1} & \omega \end{pmatrix} \begin{pmatrix} \mathbf{v_1} & & \\ & \mathbf{v_2} & \\ & & \mathbf{v_3} \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}\,.$$

$$\mathbf{v_1} = \mathbf{v_2} = \mathbf{v_3} = \mathbf{v} \Rightarrow U_{\ell,R} = I,\ \mathbf{U_{\ell,L}}(\omega),\ m_{\ell,i} = \mathbf{y_i v}.$$

$$\mathbf{y_1} = \mathbf{y_2} = \mathbf{y_3} = \mathbf{y} \Rightarrow U_{\ell,L} = I,\ \mathbf{U_{\ell,R}}(\omega),\ m_{\ell,i} = \mathbf{y v_i}.$$

Prediction of Large δ_D by $\mathbb{Z}_2^s$ and $\overline{\mathbb{Z}}_2^s$

$$\cos\delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r} \qquad \cos\delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r}$$

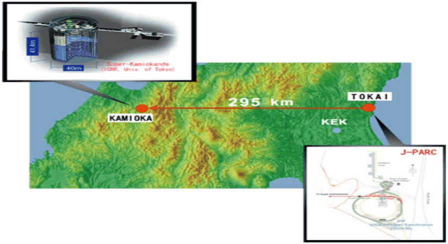


1σ Indication for $\delta_D = -74^\circ(-110^\circ)$ [Schwetz et.al. 1108.1376]

Dirac CP Phase Measurement

CP Measurement @ Accelerator Exps

- T2K



- NOνA

- DUNE,T2KII/T2HK/T2KK/T2KO, MOMENT/ADS-CI, Super-PINGU

The Dirac CP Phase δ_D @ Accelerator Exp

- To leading order in $\alpha = \frac{\delta M^2_{21}}{|\delta M^2_{31}|} \sim 3\%$, the oscillation probability relevant to measuring δ_D @ **T2(H)K**,

$$P_{\substack{\nu_\mu \to \nu_e \\ \overline{\nu}_\mu \to \overline{\nu}_e}} \approx 4 s^2_a c^2_r s^2_r \sin^2 \phi_{31} - 8 c_a s_a c^2_r s_r c_s s_s \sin \phi_{21} \sin \phi_{31} \left[\cos \delta_D \cos \phi_{31} \pm \sin \delta_D \sin \phi_{31}\right]$$

for ν & $\overline{\nu}$, respectively. $[\phi_{ij} \equiv \frac{\delta m^2_{ij} L}{4 E_\nu}]$

- $\nu_\mu \to \nu_\mu$ Exps measure $\sin^2(2\theta_a)$ precisely, but not $\sin^2 \theta_a$.

- Run both ν & $\overline{\nu}$ modes @ first peak $[\phi_{31} = \frac{\pi}{2}, \phi_{21} = \alpha \frac{\pi}{2}]$,

$$P_{\overline{\nu}_\mu \to \overline{\nu}_e} + P_{\nu_\mu \to \nu_e} = 2 s^2_a c^2_r s^2_r \,,$$

$$P_{\overline{\nu}_\mu \to \overline{\nu}_e} - P_{\nu_\mu \to \nu_e} = \alpha \pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos \theta_r \sin \delta_D \,.$$

The Dirac CP Phase δ_D @ Accelerator Exp

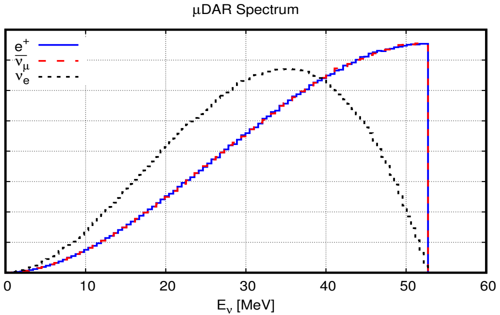
Accelerator experiment, such as **T2(H)K**, uses off-axis beam to compare ν_e & $\overline{\nu}_e$ appearance @ the oscillation maximum.

- **Disadvantages**:
  - **Efficiency**:
    - Proton accelerators produce ν more efficiently than $\overline{\nu}$ ($\sigma_\nu > \sigma_{\overline{\nu}}$).
    - The $\overline{\nu}$ mode needs more beam time [$\mathbf{T}_{\overline{\nu}} : \mathbf{T}_\nu = \mathbf{2} : \mathbf{1}$].
    - Undercut statistics $\Rightarrow$ Difficult to reduce the uncertainty.
  - **Degeneracy**:
    - Only $\sin \delta_D$ appears in $P_{\nu_\mu \to \nu_e}$ & $P_{\overline{\nu}_\mu \to \overline{\nu}_e}$.
    - Cannot distinguish δ_D from $\pi - \delta_D$.
  - **CP Uncertainty** $\frac{\partial P_{\mu e}}{\partial \delta_D} \propto \cos \delta_D \Rightarrow \Delta(\delta_D) \propto 1 / \cos \delta_D$.
- **Solution**:

  Measure $\overline{\nu}$ mode with μ^+ decay @ rest (μDAR)

μDAR $\bar{\nu}$ Oscillation Experiments

- A cyclotron produces 800 MeV proton beam @ fixed target.
- Produce $\pi^\pm$ which stops &
  - π^- is absorbed,
  - π^+ decays @ rest: $\pi^+ \to \mu^+ + \nu_\mu$.
- μ^+ stops & decays @ rest: $\boldsymbol{\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e}$.

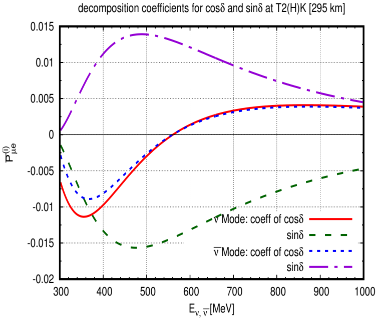


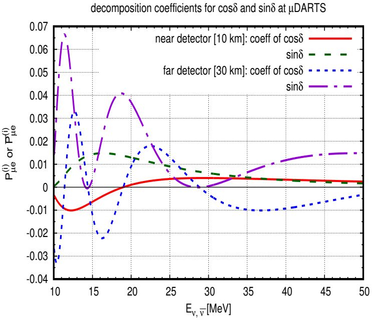
- $\overline{\nu}_\mu$ travel in all directions, oscillating as they go.
- A detector measures the $\overline{\nu}_e$ from $\boldsymbol{\overline{\nu}_\mu \to \overline{\nu}_e}$ **oscillation**.

Accelerator + μDAR Experiments

Combining $\nu_\mu \to \nu_e$ **@ accelerator** [narrow **peak @ 550 MeV**] & $\overline{\nu}_\mu \to \overline{\nu}_e$ **@ μDAR** [wide **peak $\sim$ 45 MeV**] solves the 2 problems:

- **Efficiency**:
  - $\overline{\nu}$ @ high intensity, μDAR is plentiful enough.
  - Accelerator Exps can devote all run time to the ν mode. With same run time, the statistical uncertainty drops by $\sqrt{3}$.
- **Degeneracy**: (**decomposition in propagation basis** [1309.3176])





New Proposals

1 μDAR source + **2** detectors

Advantages:

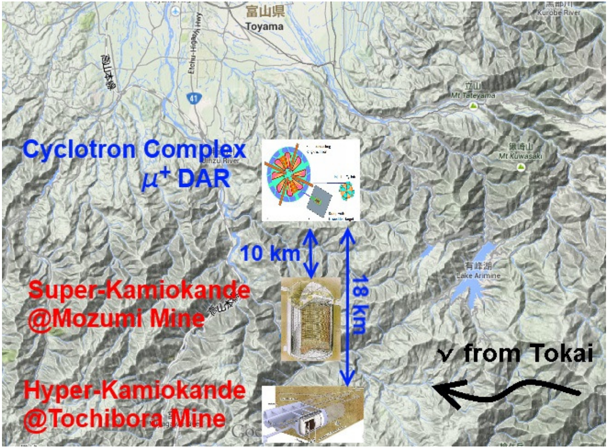
- Full (**100%**) duty factor!
- **Lower** intensity: $\sim$ **9mA** [$\sim$ **4**$\times$ lower than DAEδALUS]
- Not far beyond the current state-of-art technology of cyclotron [**2.2mA** @ Paul Scherrer Institute]
- MUCH **cheaper** & technically **easier**.
  - Only one cyclotron.
  - Lower intensity.

Disadvantage?

- A second detector!
  - **μDAR** with **T**wo **S**cintillators (**μDARTS**) [1401.3977]
  - **T**okai '**N** **T**oyama to(**2**) **K**amioka (**TNT2K**) [1506.05023]

TNT2K

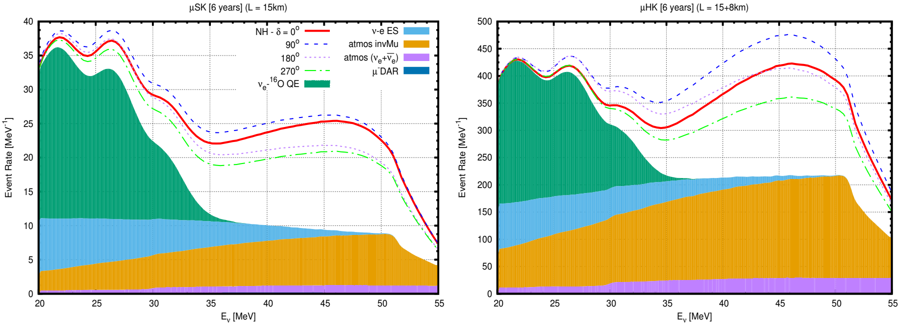
- T2(H)K + μSK + μHK



- μDAR is also useful for **material**, **medicine** industries in Toyama

Event Shape @ TNT2K

Evslin, Ge & Hagiwara [1506.05023]

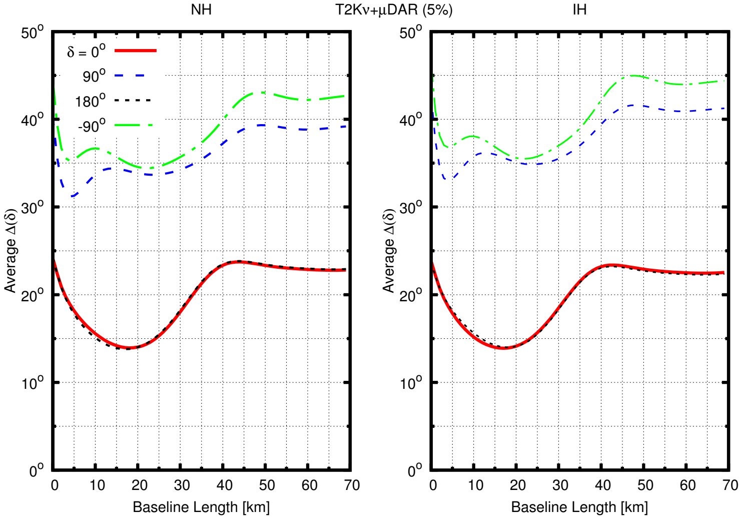


Expected μDAR IBD signal from 6 yrs of running @ SK (15km) & HK (23km) with NH.

Simulated by **NuPro**, http://nupro.hepforge.org/

δ_D Precision @ TNT2K

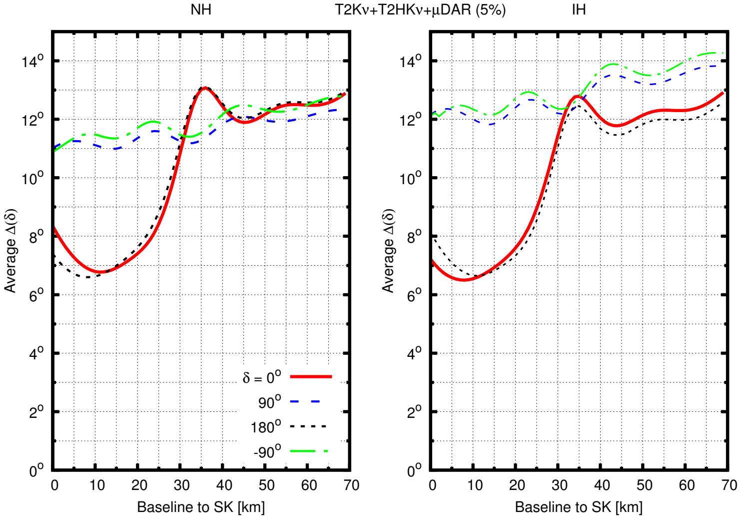
Evslin, Ge & Hagiwara [1506.05023]



Simulated by **NuPro**, http://nupro.hepforge.org/

δ_D Precision @ TNT2K

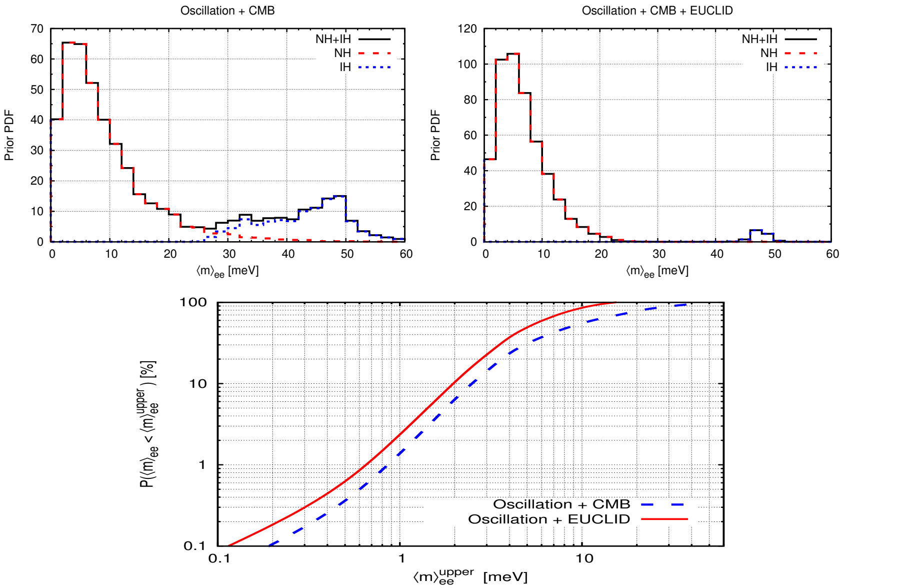
Evslin, Ge & Hagiwara [1506.05023]



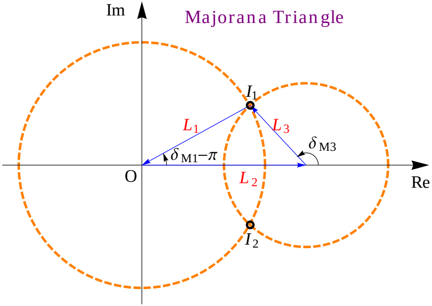
Simulated by **NuPro**, http://nupro.hepforge.org/

Majorana CP Phase Measurement

Preference of NH $\Rightarrow$ Non-Observation of $0\nu2\beta$?



Any chance of obtaining some information?



$$\langle m \rangle_{ee} \equiv \overrightarrow{L_1} + \overrightarrow{L_2} + \overrightarrow{L_3} \, ,$$

with

$$\overrightarrow{L_1} \equiv m_1 U_{e1}^2 = m_1 c_r^2 c_s^2 e^{i\delta_{\rm M1}} \, ,$$

$$\overrightarrow{L_2} \equiv m_2 U_{e2}^2 = \sqrt{m_1^2 + \Delta m_s^2} c_r^2 s_s^2 \, ,$$

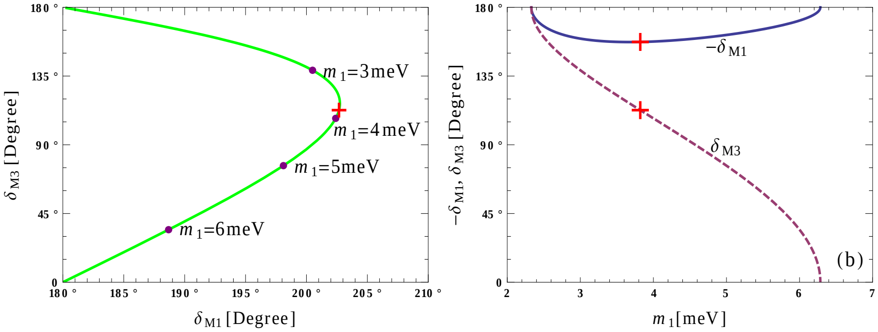
$$\overrightarrow{L_3} \equiv m_3 U_{e3}^2 = \sqrt{m_1^2 + \Delta m_a^2} s_r^2 e^{i\delta_{\rm M3}} \, .$$

Determine 2 Majorana Phases Simultaneously

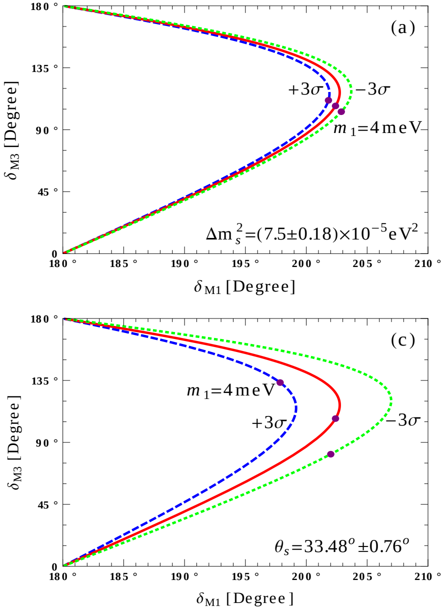
$$|L_1 - L_3| \leq L_2 \leq L_1 + L_3 \,.$$

$$\cos\delta_{\rm M1} = -\frac{L_1^2 + L_2^2 - L_3^2}{2L_1L_2} = -\frac{m_1^2 c_r^4 c_s^4 + m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_1 m_2 c_r^4 c_s^2 s_s^2}\,,$$

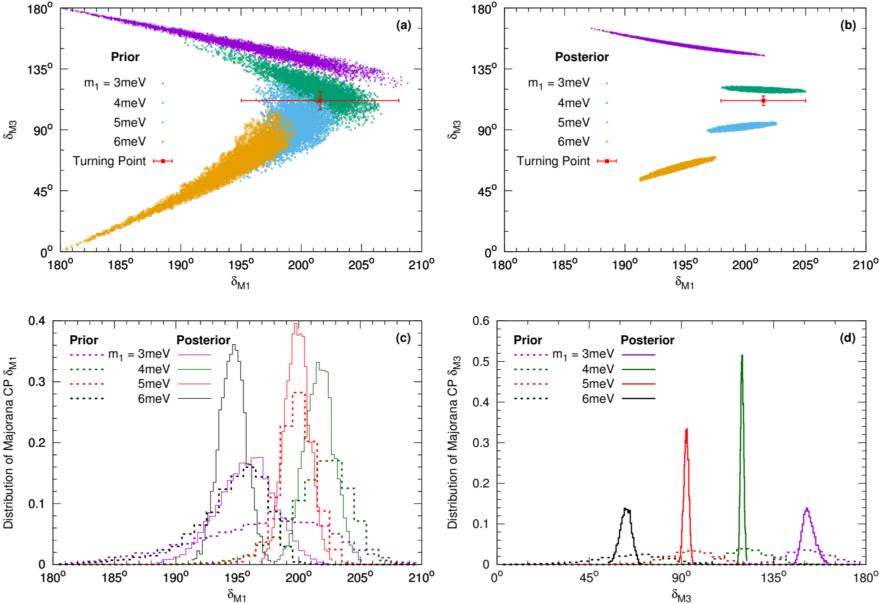
$$\cos\delta_{\rm M3} = +\frac{L_1^2 - L_2^2 - L_3^2}{2L_2L_3} = +\frac{m_1^2 c_r^4 c_s^4 - m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2m_2 m_3 c_r^2 s_r^2 s_s^2}\,.$$



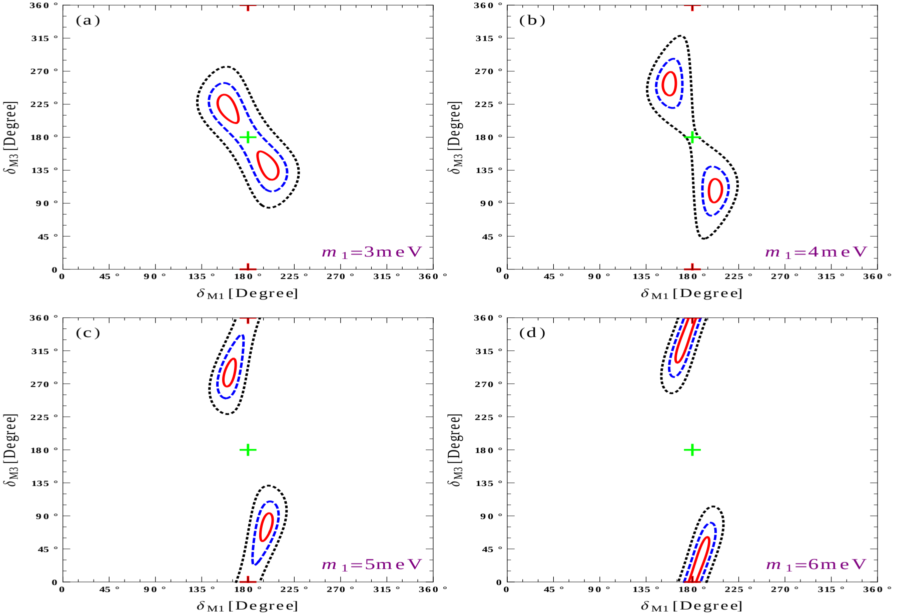
Uncertainties from Oscillation Parameters



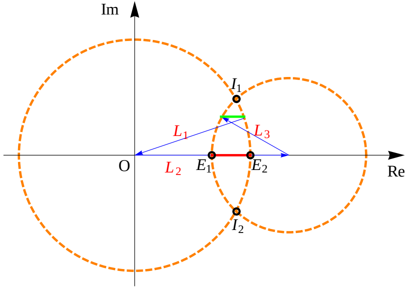
Uncertainties from Oscillation Parameters

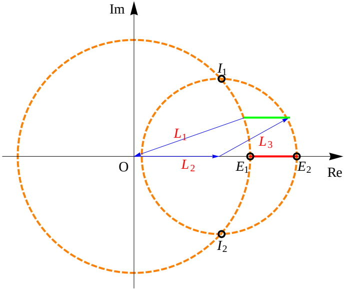


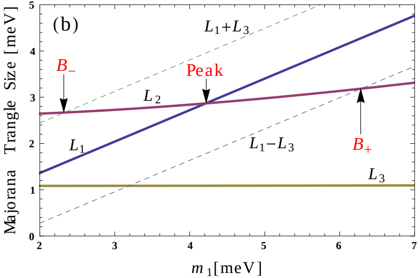
Uncertainties from Oscillation Parameters

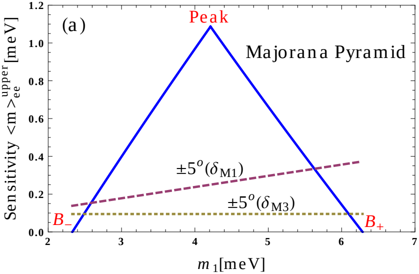


Majorana Pyramid









Prey of Leptonic CP Phases

Thank You!