# Flexible Spectator Model for Deeply Virtual Lepton Scattering Processes



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4) Wellesley College

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# OUTLINE

- GPDs, Model– Reggeized spectator "flexible parameterization"
- Valence quarks: Chiral Even & Odd  $\pi^0$  electroproduction
- Neutrinos extend "flexible"
- Gluon & Sea GPDs
- Preliminary results
- Some Observable quantities

# GPD definitions – 8 quark + 8 gluon (twist 2)

Momentum space nucleon matrix elements of quark field correlators

see, e.g. M. Diehl, Eur. Phys. J. C 19, 485 (2001).

$$\begin{split} \frac{1}{2} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^{+}\psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^{+}=0, \mathbf{z}_{T}=0} \\ &= \frac{1}{2P^{+}} \bar{u}(p', \lambda') \left[ H^{q} \gamma^{+} + E^{q} \frac{i\sigma^{+\alpha}\Delta_{\alpha}}{2m} \right] u(p, \lambda), \\ \frac{1}{2} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) \gamma^{+}\gamma_{5}\psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^{+}=0, \mathbf{z}_{T}=0} \\ &= \frac{1}{2P^{+}} \bar{u}(p', \lambda') \left[ \tilde{H}^{q} \gamma^{+}\gamma_{5} + \tilde{E}^{q} \frac{\gamma_{5}\Delta^{+}}{2m} \right] u(p, \lambda), \\ \langle J_{q}^{x} \rangle = \frac{1}{2} \int dx [H(x, 0, 0) + E(x, 0, 0)] x \\ \frac{1}{2} \int \frac{dz^{-}}{2\pi} e^{ixP^{+}z^{-}} \langle p', \lambda' | \bar{\psi}(-\frac{1}{2}z) i\sigma^{+i}\psi(\frac{1}{2}z) | p, \lambda \rangle \Big|_{z^{+}=0, \mathbf{z}_{T}=0} \\ &= \frac{1}{2P^{+}} \bar{u}(p', \lambda') \left[ H^{q}_{T} i\sigma^{+i} + \tilde{H}^{q}_{T} \frac{P^{+}\Delta^{i} - \Delta^{+}P^{i}}{m^{2}} \\ &+ E^{q}_{T} \frac{\gamma^{+}\Delta^{i} - \Delta^{+}\gamma^{i}}{2m} + \tilde{E}^{q}_{T} \frac{\gamma^{+}P^{i} - P^{+}\gamma^{i}}{m} \right] u(p, \lambda) \end{split}$$



## Normalizing & constraining quark GPDs – Chiral even

Form factor, Forward limit

 $\int H_q(x,\xi,t) dx = F_1^q(t), \quad H_q(x,0,0) = q(x) \quad \text{Integrates to charge}$ 

 $\int_{a}^{b} E_{q}(x,\xi,t) dx = F_{2}^{q}(t) \quad \Rightarrow \text{Anomalous magnetic moments}$ 

$$\int_{0} \tilde{H}_{q}(x,\xi,t) dx = g_{A}^{q}(t), \quad \tilde{H}_{q}(x,0,0) = \Delta q(x) = q_{\Rightarrow}^{\rightarrow}(x) - q_{\Rightarrow}^{\leftarrow}(x)$$

Integrates to axial charge

$$\int_{0}^{1} \tilde{E}_{q}(x,\xi,t) dx = g_{P}^{q}(t)$$

$$\begin{aligned} \zeta &= (\Delta q)/(Pq) \simeq Q^2/2(Pq) \equiv x_{Bj}, \ t = \Delta^2 \\ x &= \frac{k^+ + k'^+}{P^+ + P'^+} = \frac{X - \zeta/2}{1 - \zeta/2}, \ \xi &= \frac{2\Delta^+}{P^+ + P'^+} = \frac{\zeta}{2 - \zeta}. \end{aligned}$$

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#### The Model for valence quarks– Reggeized Diquarks

The Model – first for Chiral Even – Reggeized Diquark Spectator Diquark: Color anti-3, scalar & axial vector



"Flexible" covariant model

Gonzalez, GG, Liuti PRD84, 034007 (2011)

$$H = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \\ \times \int d^2 k_{\perp} \frac{\left[ (m + MX)(m + M\frac{X - \zeta}{1 - \zeta}) + \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp} \right]}{(k^2 - M_{\Lambda}^2)^2 (k'^2 - M_{\Lambda}^2)^2} \\ + \frac{\zeta^2}{4(1 - \zeta)} E,$$
(27)

$$E = \mathcal{N} \frac{1 - \zeta/2}{1 - X} \int d^2 k_{\perp} \times \frac{-2M(1 - \zeta)[(m + MX)\frac{\tilde{k}\cdot\Delta}{\Delta_{\perp}^2} - (m + M\frac{X - \zeta}{1 - \zeta})\frac{k_{\perp}\cdot\Delta}{\Delta_{\perp}^2}]}{(k^2 - M_{\Lambda}^2)^2(k'^2 - M_{\Lambda}^2)^2},$$
(28)

$$E = \mathcal{N} \frac{1-\chi}{1-\chi} \int d^{2}k_{\perp}$$

$$K = k - \Delta$$

$$P' = P - \Delta$$

$$K' = k - \Delta$$

$$P' = P - \Delta$$

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$$P' = P - \Delta$$

$$E = \mathcal{N} \frac{1-\zeta/2}{1-\chi}$$

$$K' = k - \Delta$$

$$E = \mathcal{N} \frac{1-\zeta/2}{1-\chi} \int d^{2}k_{\perp}$$

$$\frac{\zeta^{2}}{4(1-\zeta)} \tilde{E},$$

$$E = \mathcal{N} \frac{1-\zeta/2}{1-\chi} \int d^{2}k_{\perp}$$

$$\frac{(m + MX)(m + M\frac{X-\zeta}{1-\zeta}) - \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}]}{(k^{2} - M_{\Lambda}^{2})^{2}(k^{\prime 2} - M_{\Lambda}^{2})^{2}}$$

$$E = \mathcal{N} \frac{1-\zeta/2}{1-\chi} \int d^{2}k_{\perp}$$

$$\frac{(m + MX)(m + M\frac{X-\zeta}{1-\zeta}) - \mathbf{k}_{\perp} \cdot \tilde{\mathbf{k}}_{\perp}]}{(k^{2} - M_{\Lambda}^{2})^{2}(k^{\prime 2} - M_{\Lambda}^{2})^{2}},$$

$$E = \mathcal{N} \frac{1-\zeta/2}{1-\chi} \int d^{2}k_{\perp}$$

$$\times \frac{-\frac{4M(1-\zeta)}{\zeta}[(m + MX)\frac{\tilde{k}\cdot\Delta}{1-\zeta}](m + MX)\frac{\tilde{k}\cdot\Delta}{1-\zeta}]}{(k^{2} - M_{\Lambda}^{2})^{2}(k^{\prime 2} - M_{\Lambda}^{2})^{2}},$$

$$(30)$$



# Reggeizing diquark model

Diquark+quark & fixed masses (e.g. at  $\xi=0$ )

$$\begin{split} H_{M_{X}^{q},m_{q}}^{M_{\Lambda}^{q}} &= \mathcal{N}_{q} \int \frac{d^{2}k_{\perp}}{1-x} \frac{\left[(m_{q}+Mx)(m_{q}+Mx)+\mathbf{k}_{\perp}\cdot\tilde{\mathbf{k}}_{\perp}\right]}{\left[\mathcal{M}_{q}^{2}(x)-k_{\perp}^{2}/(1-x)\right]^{2} \left[\mathcal{M}_{q}^{2}(x)-\tilde{k}_{\perp}^{2}/(1-x)\right]^{2}},\\ E_{M_{X}^{q},m_{q}}^{M_{\Lambda}^{q}} &= \mathcal{N}_{q} \int \frac{d^{2}k_{\perp}}{1-x} \frac{-2M/\Delta_{\perp}^{2}[(m_{q}+Mx)\tilde{\mathbf{k}}_{\perp}\cdot\mathbf{\Delta}_{\perp}-(m_{q}+Mx)\mathbf{k}_{\perp}\cdot\mathbf{\Delta}_{\perp}]}{\left[\mathcal{M}_{q}^{2}(x)-k_{\perp}^{2}/(1-x)\right]^{2} \left[\mathcal{M}_{q}^{2}(x)-\tilde{k}_{\perp}^{2}/(1-x)\right]^{2}},\end{split}$$

Diquark mass "spectrum" as in Brodsky, Close & Gunion Phys. Rev. D8, 3678 (1973)

$$F_T^q(X,\zeta,t) = \mathcal{N}_q \int_0^\infty dM_X^2 \rho(M_X^2) F_T^{(m_q,M_\Lambda^q)}(X,\zeta,t;M_X).$$

$$\rho(M_X^2) \approx \begin{cases} (M_X^2)^\alpha & M_X^2 \to \infty \\ \\ \delta(M_X^2 - \overline{M}_X^2) & M_X^2 \text{ few GeV}^2 \end{cases}$$

 $F_T^q(X,\zeta,t) \approx \mathcal{N}_q X^{-\alpha_q + \alpha'_q(X)t} F_T^{(m_q,M^q_\Lambda)}(X,\zeta,t;\overline{M}_X) = R_{p_q}^{\alpha_q,\alpha'_q}(X,\zeta,t) G_{M_X,m}^{M_\Lambda}(X,\zeta,t)$ 

#### **R≭**Dq



## EM Form Factors (t dependence)

precision mesurements  $\rightarrow$  tighter parameters



O.Gonzalez, GRG, S.Liuti, K.Kathuria PRC88, 065206(2013) data: G.D. Cates, et al. PRL106,252003 (2011).





#### Chiral even GPDs



From GPDs with evolution to Compton **Form Factors** CFFs to helicity amps helicity amps to observables <-> parameters





#### **RESULT:** Chiral odd GPDs



# Chiral odd GPD Compton form factors







#### Cross sections for $\pi^{0}$

$$\frac{d^{4}\sigma}{dx_{Bj}dyd\phi dt} = \Gamma \left\{ \left[ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} + \sqrt{\epsilon(\epsilon+1)} \cos \phi F_{UU}^{\cos \phi} + h \sqrt{\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] + S_{\parallel} \left[ \sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} + h \left( \sqrt{1-\epsilon^{2}} F_{LL} + \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right) \right] - S_{\perp} \left[ \sin(\phi - \phi_{S}) \left( F_{UT,T}^{\sin(\phi - \phi_{S})} + \epsilon F_{UT,L}^{\sin(\phi - \phi_{S})} \right) + \frac{\epsilon}{2} \left( \sin(\phi + \phi_{S}) F_{UT}^{\sin(\phi + \phi_{S})} + \sin(3\phi - \phi_{S}) F_{UT}^{\sin(3\phi - \phi_{S})} \right) + \sqrt{\epsilon(1+\epsilon)} \left( \sin \phi_{S} F_{UT}^{\sin \phi_{S}} + \sin(2\phi - \phi_{S}) F_{UT}^{\sin(2\phi - \phi_{S})} \right) \right] + S_{\perp} h \left[ \sqrt{1-\epsilon^{2}} \cos(\phi - \phi_{S}) F_{LT}^{\cos(\phi - \phi_{S})} + \sqrt{\epsilon(1-\epsilon)} \left( \cos \phi_{S} F_{LT}^{\cos \phi_{S}} + \cos(2\phi - \phi_{S}) F_{LT}^{\cos(2\phi - \phi_{S})} \right) \right] \right\}$$

$$(70)$$

$$\begin{split} F_{UU,T} &= \frac{d\sigma_T}{dt}, \quad F_{UU,L} = \frac{d\sigma_L}{dt}, \quad F_{UU}^{\cos\phi} = \frac{d\sigma_{LT}}{dt}, \\ F_{UU}^{\cos 2\phi} &= \frac{d\sigma_{TT}}{dt}, \quad F_{LU}^{\sin\phi} = \frac{d\sigma_{LT'}}{dt} \end{split}$$
See: GG, Gonzalez Hernandez, Liuti 1311.0483.pdf;  
J.Phys.G39,115001(2012); PRD84, 034007(2011).  
Notation: Bacchetta, et al., JHEP 0702, 093 (2007);  
angular form: Diehl & Sapeta, Eur. Phys. J. C 41, 515 (2005).



#### **Helicity amps** (q'+N->q+N') are linear combinations of GPDs

$$\begin{split} A_{++,--} &= \sqrt{1-\xi^2} \underbrace{\left[H_T\right]} \cdot \frac{t_0 - t}{4M^2} \widetilde{H}_T - \frac{\xi^2}{1-\xi^2} E_T + \frac{\xi}{1-\xi^2} \widetilde{E}_T\right] & H_{\mathsf{T}}(\mathsf{x},\xi,\mathsf{t}) \\ &= \frac{\sqrt{1-\zeta}}{1-\zeta/2} \left[H_T + \frac{t_0 - t}{4M^2} \widetilde{H}_T - \frac{\zeta^2/4}{1-\zeta} E_T + \frac{\zeta/2}{1-\zeta} \widetilde{E}_T\right] & H_{\mathsf{T}}(\mathsf{x},0,0) = h_1(\mathsf{x}) \\ A_{+-,-+} &= -e^{i2\varphi} \sqrt{1-\xi^2} \frac{t_0 - t}{4M^2} \widetilde{H}_T = -e^{i2\varphi} \frac{\sqrt{1-\zeta}}{1-\zeta/2} \frac{t_0 - t}{4M^2} \widetilde{H}_T \\ A_{++,+-} &= e^{i\varphi} \frac{\sqrt{t_0 - t}}{4M} \left[ 2\widetilde{H}_T + (1-\xi) \left( E_T + \widetilde{E}_T \right) \right] \\ &= e^{i\varphi} \frac{\sqrt{t_0 - t}}{2M} \left[ \widetilde{H}_T + \frac{1-\zeta}{1-\zeta/2} \left( E_T + \widetilde{E}_T \right) \right] , \\ A_{-+,--} &= e^{i\varphi} \frac{\sqrt{t_0 - t}}{4M} \left[ 2\widetilde{H}_T + (1+\xi) \left( E_T - \widetilde{E}_T \right) \right] \\ &= e^{i\varphi} \frac{\sqrt{t_0 - t}}{4M} \left[ \widetilde{H}_T + \frac{1}{1-\zeta/2} \left( E_T - \widetilde{E}_T \right) \right] . \end{split}$$

Chiral odd helicity amps

See Diehl, Phys.Rept. 388, 41 (2003)

#### In diquark spectator models $A_{++;++}$ , etc. are calculated directly. Inverted -> GPDs



<u>How to single out chiral odd GPDs?</u> Exclusive Lepto-production of  $\pi^{0}$  Or  $\eta, \eta'$ to measure chiral odd GPDs & Transversity



t-channel J <sup>PC</sup> quantum numbers enhance chiral odd see GG, Gonzalez Hernandez, Liuti, J.Phys.G39,115001(2012)



Reggeization



$$A = \mathcal{N} \int \frac{dk_X^2 dk^2}{(k^2 - m^2 - i\epsilon)(k'^2 - m^2 - i\epsilon)} \frac{\rho(k_X^2, k^2) \times (spin \ structure)}{(k_X^2 - M_X^2 - i\epsilon)}$$

Landshoff, Polkinghorn, Short '71 Brodsky, Close, Gunion '71 Regge behavior required for Compton Ahmad, Honkanen,Liuti,Taneja '07, '09 Gorshteyn & Szczepaniak (PRD, 2010) Brodsky, Llanes-Estrada '07 Brodsky, Llanes, Szczepaniak '08

Gonzalez, GG, Liuti, arXiv:1201.6088 [hep-ph] J. Phys. G: Nucl. Part. Phys. 39 115001 (2012)

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# How well do the parameters fixed with DVCS data reproduce $\pi^{\circ}$ electroproduction data?

Hall B data, Kubarovsky& Stoler, PoS ICHEP 2010 & PRL 109, 112001 (2012)



#### CLAS π<sup>0</sup> arXiv:1206.6355v1 [hep-ex] PRL 109, 112001 (2012) GG, Gonzalez-Hernandez, Liuti compared to Goloskokov & Kroll



-t, GeV<sup>2</sup> -t, GeV<sup>2</sup> FIG. 2: The extracted structure functions vs. t for the bins with the best kinematic coverage and for which there are theoretical calculations. The data and curves are as follows: black- $\sigma_U (= \sigma_T + \epsilon \sigma_L)$ , blue- $\sigma_{TT}$ , and red- $\sigma_{LT}$ . The shaded bands reflect the experimental systematic uncertainties. The curves are theoretical predictions produced with the models of Refs. [14] (solid) and [15] (dashed).

- The t-> 0 feature for us is that f<sub>10</sub><sup>+-</sup> nonflip <u>dominates</u> & it is driven by H<sub>T.</sub>
- But  $f_{10}^{++} \& f_{10}^{--}$  also contribute as  $\sim V(t_0^{-}$ t), however weaker. For  $\xi \rightarrow 0$  they  $\supset$ CFFs  $2\tilde{H}_T + E_T \equiv \overline{E}_T$
- f<sub>10</sub><sup>++</sup> & f<sub>10</sub><sup>--</sup> are <u>not equal</u> in magnitude, particularly vs. ζ or ξ ⊃ mixes "natural & unnatural parity"
- In  $A_{LL} \sim |f_{10}^{++}|^2 + |f_{10}^{+-}|^2 |f_{10}^{-+}|^2 |f_{10}^{--}|^2$ sensitive to differences of  $f_{10}^{++} \& f_{10}^{--}$
- c.f. Goloskokov & Kroll different
   dominant amps.



#### **Some results** GG, Gonzalez Hernandez, Liuti, PRD84, 034007 (2011) . . . (2016);

Beam charge asymmetry HERMES A<sub>UT</sub> coefficients





Beam spin asymmetry (CLAS data -DeMasi, et al. )





### Extending Phenomenology of Flexible Model

## Neutrino Production - What Processes? How to Measure?



## From exclusive electroproduction to neutrino-production



$$f_{\Lambda_{w}\Lambda;0,\Lambda'} = \sum_{\lambda,\lambda'} g_{\Lambda_{w}\lambda;0,\lambda'}(X,\zeta,t,Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(X,\zeta,t),$$

 $g_{\Lambda W,\lambda;0,\lambda'} = same \ helicity \ structure \ as \ W+p \rightarrow \pi+p'$ with  $W^{\mu} \sim c_V \gamma^{\mu} - c_A \gamma^{\mu} \gamma^5 \& \pi \sim \varphi_A q'_V \gamma^{\nu} \gamma^5 + \varphi_P \gamma^5$  $A_{\Lambda',\lambda';\Lambda,\lambda} \rightarrow 2 \ge 6 \ helicity \ amps \ \supseteq \ GPDs$ 



# Hard subprocesses

Consider  $\pi$  production V<sup>\*</sup> + q  $\rightarrow \pi$  + q' with DA for  $\pi \supset q'_{\mu}\gamma^{\mu}\gamma^{5}$  twist2 &  $\gamma^{5}$  twist 3 Helicity amps: Longitudinal with non-flip quark – leading ~ Q/M:  $q'_{\mu}\gamma^{\mu}\gamma^{5}$ Transverse with flip quark ~ constant (or  $\Delta^{2}/Q^{2}$  cross channel) :  $\gamma^{5}$ Transverse non-flip ~  $\Delta/M$  : :  $q'_{\mu}\gamma^{\mu}\gamma^{5}$  ( $\Delta$  by angular momentum conservation) Longitudinal with flip ~  $\Delta/Q$  ( $\Delta$  by angular momentum conservation)

Question: Does Twist 3 dominate v production of π as in electroproduction?
 i.e. will measurements be sensitive to this twist 3 DA π mechanism?
 Will Transversity be important?
 See Kopeliovich, Schmidt, Siddikov: Twist 2 & longitudinal dominate.
 See Pire & Szymanowski: Twist 3 dominates for heavy quarks.



## Neutral Current interaction

• NC & parity violating electroproduction





Contribution to  $\pi^0$  exclusive  $\nu$  production cross section,

 $d\sigma_{L'T}/dt$  from model GPDs (c.f. Regge description).

Few GeV region accessible in current & future neutrino experiments.



GRG, Gonzalez-Hernandez, Liuti, McAskill, Proc. NuFAct09, 0911.0495; in preparation 2017.



# Gluon GPDs



## **Gluon GPDs**

$$\begin{split} \frac{1}{\bar{P}^{+}} \int \frac{dz^{-}}{2\pi} e^{ix\bar{P}^{+}z^{-}} \langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)G^{+i}(\frac{1}{2}z)|P,\Lambda\rangle \Big|_{z^{+}=0,\vec{z}_{T}=0} = \\ \frac{1}{2\bar{P}^{+}} \bar{U}(P',\Lambda')[H^{g}(x,\xi,t)\gamma^{+} + E^{g}(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_{\alpha})}{2M}]U(P,\Lambda) \end{split}$$

Even t-channel parity & Gluon helicity conserving

$$\begin{split} \frac{-i}{\bar{P}^{+}} \int \frac{dz^{-}}{2\pi} e^{ix\bar{P}^{+}z^{-}} \langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)\tilde{G}^{+i}(\frac{1}{2}z)|P,\Lambda\rangle \Big|_{z^{+}=0,\vec{z}_{T}=0} = \\ \frac{1}{2\bar{P}^{+}} \bar{U}(P',\Lambda')[\tilde{H}^{g}(x,\xi,t)\gamma^{+}\gamma_{5} + E_{x}^{g}(x,\xi,t)\frac{\gamma_{5}(-\Delta^{+})}{2M}]U(P,\Lambda) \end{split}$$

Odd t-channel parity & Gluon helicity conserving



# Gluon & Sea quark distributions Spectator Model

generalize Regge-diquark spectator model

- N→g + "color octet N" spectator (8⊗8⊃1) (could be spin ½ or 3/2)
- (N→ *anti-u* + color 3 "tetraquark"uuud)
- How to normalize?

 $H_g(x,\xi,t)_Q^2 \rightarrow H_g(x,0,0)_Q^2 = xG(x)_Q^2$ Evolution & small x phenomenology

- Sea quark distributions H<sub>anti-u</sub>(x,0,0) . . .
- Use pdf's to fix x dependence
- Small x ~ Pomeron



$$\begin{array}{c} \lambda, \mathbf{k}^{+} = \mathbf{X} \mathbf{P}^{+} \\ \mathbf{P}^{+$$

GG, Gonzalez Hernandez, Liuti, Poage, in progress



#### After pdf's vs. $Q^2 \rightarrow fix x$ dependence Regge behavior determines *t* dependence Spectator determines $\zeta$ dependence



from J. Poage

## Extension to Gluon "Transversity"



$$\begin{split} -\frac{1}{P^+} \int \frac{dz^-}{2\pi} \, e^{ixP^+z^-} \langle p', \lambda' | \, \mathbf{S}F^{+i}(-\frac{1}{2}z) \, F^{+j}(\frac{1}{2}z) \, |p,\lambda\rangle \Big|_{z^+=0,\,\mathbf{z}_T=0} \\ &= \left. \mathbf{S} \, \frac{1}{2P^+} \, \frac{P^+\Delta^j - \Delta^+ P^j}{2mP^+} \right. \\ &\times \left. \bar{u}(p',\lambda') \left[ H^g_T \, i\sigma^{+i} + \tilde{H}^g_T \, \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} \right. \\ &+ E^g_T \, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}^g_T \, \frac{\gamma^+P^i - P^+\gamma^i}{m} \right] u(p,\lambda) \end{split}$$

4 GPDs: M.Diehl, EPJC19, 485 (2001)



## Construct helicity flip amps Spectator Model, then GPDs

$$\begin{split} A_{++,+-} &= \sqrt{1-\xi^2} \, \frac{t_0 - t}{4M^2} \Big( \tilde{H}_T^g + (1-\xi) \frac{E_T^g + \tilde{E}_T^g}{2} \Big) \\ A_{-+,--} &= \sqrt{1-\xi^2} \, \frac{t_0 - t}{4M^2} \Big( \tilde{H}_T^g + (1+\xi) \frac{E_T^g - \tilde{E}_T^g}{2} \Big) \\ A_{++,--} &= +e^{-i\phi} (1-\xi^2) \frac{\sqrt{t_0 - t}}{2M} \Big( H_T^g + \frac{t_0 - t}{M^2} \tilde{H}_T^g - \frac{\xi^2}{1-\xi^2} E_T^g + \frac{\xi}{1-\xi^2} \tilde{E}_T^g \Big) \\ A_{-+,+-} &= -e^{i\phi} (1-\xi^2) \frac{\sqrt{t_0 - t}^3}{8M^3} \tilde{H}_T^g, \end{split}$$

Compare to spectator model results

$$\tilde{H}_T^g = 0$$

$$(1-X)A^0_{-+,--} = (1-X')A^0_{++,+-}$$

 $\tilde{E}_T^g = 0.$ 

As in Hoodbhoy & Ji, PRD58, 054006 (1998)



#### Using the Reggeized Spectators Model

#### How to Measure? What Processes? Long standing question.

M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B411, 193 (1997).
X. Ji and J. Osborne, UMD PP#98-074, hep-ph/9801260.
P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. A598, 435 (1996).
P. Hoodbhoy & X. Ji, PRD58, 054006 (1998)

# **A**<sub> $\Lambda',-1;\Lambda,+1$ </sub> **Gluon Transversity** contributes to DVCS ~ $\alpha_{S}$

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi} \sum_{q} e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda'g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi-x-i\varepsilon)(\xi+x-i\varepsilon)} C'(x,\xi,Q^2)$$
  
DVCS cross sections: do/dt  $\infty \sum |M_{\Lambda',\dots}|^2$ 

\*\*\*Interference with Bethe-Heitler contains  $cos3\phi$  modulation to distinguish from (leading twist) quark contribution \*\*\*



$$\mathcal{T}^{\mu\nu\alpha\beta} = \frac{1}{x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'S' | F^{(\mu\alpha}(-\frac{\lambda}{2}n)F^{\nu\beta}(\frac{\lambda}{2}n) | PS \rangle \ .$$

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# Measuring Gluons in Nucleons



$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T|^2$$
$$T(k, p, k', q', p') = T_{DVCS}(k, p, k', q', p') + T_{BH}(k, p, k', q', p'),$$

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}$$
.

$$\mathcal{I} = T_{BH}^* T_{DVCS} + T_{DVCS}^* T_{BH}.$$

For unpolarized  $e+p \rightarrow e'+\gamma+p'$  cross section depends on azimuthal angle  $\phi$ . cos3 $\phi$  term in interference d $\sigma$  measures gluon transversity GPDs (CFF's)

$$\frac{\sqrt{t_0 - t}^3}{8M^3} \left[ H_T^g F_2 - E_T^g F_1 - 2 \tilde{H}_T^g \left( F_1 + \frac{t}{4M^2} F_2 \right) \right] \cos 3\phi$$

 $\mathscr{H}_{T}^{g} \sim \int dx H^{g}_{T} / (x-\xi)(x+\xi) CFF's$ 

See Diehl, *et al*. PLB411, 193 (1997); Diehl, EPJC25, 223 (2002); Belitsky, Mueller, PLB486, 369 (2000).



# Summary

- Flexible parameterization for chiral even valence quarks from form factors, pdfs & DVCS R\*Dq
- Extended R\*Dq to R\*Spectator
- Extend phenomenology to v production
- New Extension to gluons & the sea
- Considered Gluon sector
  - Helicity conserving & Helicity →gluon *Transversity*
- Measurements?
- More phenomenology to come

## Backup & extra slides

#### Connecting to exclusive processes (DVCS, DVMP...)





Convolution of "hard part" with quark-proton Helicity amplitudes

$$f_{\Lambda_{\gamma},\Lambda;\Lambda'_{\gamma},\Lambda'} = \sum_{\lambda,\lambda'} g_{\lambda,\lambda'}^{\Lambda_{\gamma},\Lambda'_{\gamma(M)}}(x,k_T,\zeta,t;Q^2) \otimes A_{\Lambda',\lambda';\Lambda,\lambda}(x,k_T,\zeta,t),$$

$$\lambda = +(-) \lambda' \text{ chiral even (odd)}$$
See Ahmad, et al. PRD75, 0904003 (2007);  
ibid, EPJC63, 407 (2009).
$$see Ahmad, GG, Liuti, PRD79, 054014, (2009)$$
for first chiral odd GPD parameterization  
for prodez, GG, Liuti, PRD84, 034007 (2011) chiral even GPD

Gonzalez, GG, Liuti PRD84, 034007 (2011) chiral even GPD



#### Chiral odd quark GPDs One question is: how do we normalize chiral-odd GPDs?

The only Physical constraints on the various chiral-odd GPDs are

Forward limit

$$H_{_T}(x,0,0) = q_{\Uparrow}^{\uparrow}(x) - q_{\Uparrow}^{\downarrow}(x) = h_{_1}(x)$$
 Transversity

Form Factors

Integrates to tensor charge  $\delta_a$ 

$$\begin{split} \int &H_T^q(x,\xi,t)\,dx = \delta q(t) \\ &\int \bar{E}_T^q(x,\xi,t)\,dx = \int \Bigl(2\tilde{H}_T^q + E_T^q\Bigr)dx = \kappa_T^q(t) \\ &\int \tilde{E}_T(x,\xi,t)\,dx = 0 \\ & \text{No direct interpretation of } \mathsf{E}_{\mathsf{T}}\,. \end{split}$$

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## What about Gluon "transversity"? Double helicity flip *does not mix* with quark distributions

Transversity for on-shell gluons or photons : no |0> helicity

$$|+1\rangle_{trans} = \{|+1\rangle + |-1\rangle\}/2 = |-1\rangle_{trans}$$

$$|0\rangle_{trans} = \{|+1\rangle - |-1\rangle\}/\sqrt{2}$$
helicity  $|\pm 1\rangle = \{-/+x - iy\}/\sqrt{2}$ 

$$x = (-10)_{trans} = P_{parallel}$$
Linear polarization in the plane
$$y = i\sqrt{2} |+1\rangle_{trans} = P_{normal}$$
Linear polarization normal to the plane

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).



## Gluon GPDs: Helicity flipping Gluon "Transversity"

$$\begin{split} &-\frac{1}{\bar{P}^+} \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P',\Lambda' | \mathbf{S}G^{+j}(-\frac{1}{2}z)G^{+k}(\frac{1}{2}z) | P,\Lambda \rangle \Big|_{z^+=0,\vec{z}_T=0} \\ &= \mathbf{S}\frac{1}{2\bar{P}^+} \frac{\bar{P}^+\Delta^j - \Delta^+\bar{P}^j}{2M\bar{P}^+} \\ &\quad \times \bar{U}(P',\Lambda') \left[ H^g_T(x,\xi,t)i\sigma^{+k} + \tilde{H}^g_T \frac{\bar{P}^+\Delta^k - \Delta^+\bar{P}^k}{M^2} \right. \\ &\qquad \qquad + E^g_T(x,\xi,t) \frac{\gamma^+\Delta^k - \Delta^+\gamma^k}{2M} + \tilde{E}^g_T \frac{\gamma^+\bar{P}^k - \bar{P}^+\gamma^k}{M} \right] U(P,\Lambda) \end{split}$$





FIG. 16 (color online). Hall A data [49] for the "sum" (upper panel) and "difference" (lower panel) of the two electron beam polarizations. Shown are curves, including the contribution of the  $\zeta$ -dependent factor from Eq. (34) (solid lines) and neglecting it (dashed lines). All terms (DVCS, Interference, and Total) are shown for the sum graph. The wide yellow bands in both panels represent the error of the data fit. The green band in the asymmetry graph is the theoretical error from our parametrization.



FIG. 18 (color online). Calculations at Hermes kinematics [52,53,56]. Shown is  $A_{LU}(90^\circ)$  vs -t,  $Q^2$ , and  $x_{BJ}$ , respectively, calculated at each kinematical bin provided by Hermes [56] (curve denoted as "Hermes kinematics") and at the nominal average values presented in each panel. It is interesting to notice that, due to the correlation between  $x_{Bj}$  and  $Q^2$  in the data, different features arise when using the average bin values. In the lower panels, we also show the effect of disregarding the DVCS term in the denominator (dashed curves).



# Hadrons to quarks



 $A_{\Lambda',\lambda';\Lambda,\lambda} \supset GPDs$ 

One of 4 terms: 
$$\frac{-i}{2}\bar{u}(k')i\gamma_5 \left[\frac{i\left((k+q)^{\mu}\gamma_{\mu}+m_3\right)}{(k+q)^2-m_3^2}\varepsilon^{\nu}\gamma_{\nu}(c_V-c_A\gamma_5)\right]u(k)$$

+ u-channel



# Hard subprocess & Q

v or anti-v,  $\mu$  or  $\mu^+$ v or anti-v V or anti-v V or  $\mu^+$  or  $W^-$  Q V or  $\rho, \omega, \pi, \eta, \eta'$  QN(P)

Consider  $\pi$  production V<sup>\*</sup> + q  $\rightarrow \pi$  + q<sup>\*</sup> with DA for  $\pi \supset q'_{\mu}\gamma^{\mu}\gamma^{5}$  twist2 &  $\gamma^{5}$  twist3 Helicity amps:

**Longitudinal** with non-flip quark – leading ~ Q/M:  $q'_{\mu}\gamma^{\mu}\gamma^{5}$  **Transverse** with flip quark ~ constant (or  $\Delta^2/Q^2$  cross channel) :  $\gamma^5$  **Transverse** non-flip ~  $\Delta/M$  : :  $q'_{\mu}\gamma^{\mu}\gamma^5$  ( $\Delta$  by angular momentum conservation) **Longitudinal** with flip ~  $\Delta/Q$  ( $\Delta$  by angular momentum conservation)



# Differential cross sections

Neutral Current:

$$\frac{d^4\sigma}{dx_{Bj}dQ^2d|t|d\phi} = \frac{G_F^2 x_{Bj}^2}{32\pi^3\cos^4(\theta_W)Q^2(1+Q^2/M_Z^2)^2(1+\gamma^2)^{3/2}} \left|T\right|^2$$

Charge Current: replace  $M_z$  with  $M_w$  and set  $\theta_w$ =0.

Separating the Vector boson + nucleon scattering:

$$\begin{split} \frac{d\sigma_Z}{dt} &=\; \frac{\sqrt{2}G_F M_Z^2 x_{Bj}^2 \mid T \mid^2}{16\pi Q^4 (1+\gamma^2)} \\ \frac{d\sigma_W}{dt} &=\; \frac{G_F M_Z^2 x_{Bj}^2 \mid T \mid^2}{16\pi \sqrt{2}Q^4 (1+\gamma^2)} . \end{split}$$



# Azimuthal correlations for neutrinos to separate *P* violating parts

$$\frac{d^{4}\sigma}{d\Omega d\varepsilon_{2}d\phi dt} = \Gamma \left[ \frac{d\sigma_{T}}{dt} + \varepsilon_{L} \frac{d\sigma_{L}}{dt} + \varepsilon \cos 2\phi \frac{d\sigma_{TT}}{dt} + \sqrt{2\varepsilon_{L}(\varepsilon+1)}\cos\phi \frac{d\sigma_{LT}}{dt} + \varepsilon \sin 2\phi \frac{d\sigma_{T'T}}{dt} + \varepsilon \sin 2\phi \frac{d\sigma_{T'T}}{dt} + \sqrt{2\varepsilon_{L}(\varepsilon+1)}\sin\phi \frac{d\sigma_{L'T}}{dt} \right]$$



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# Construct Gluon helicity amps from Spectator Model, then GPDs

$$\begin{aligned} A^{0}_{++++} &= \mathcal{A}\Big(\frac{\vec{k}_{\perp} \cdot \vec{k}_{\perp}}{XX'} + ((1-X)M - M_{X})((1-X')M - M_{X})\Big) \\ A^{0}_{-+++} &= \mathcal{A}\frac{\vec{k}_{\perp} \cdot \vec{k}_{\perp}}{XX'}((1-X)(1-X')) \\ A^{0}_{-+++} &= \mathcal{A}((1-X')((1-X)M - M_{X})\frac{\vec{k}_{X} + i\vec{k}_{y}}{X'} \\ A^{0}_{-+++} &= \mathcal{A}((1-X)((1-X')M - M_{X})\frac{(k_{x} - ik_{y})}{X} \\ \text{with } \mathcal{A} &= \mathcal{N}\frac{1}{\sqrt{1-X}\sqrt{1-X'}(1-X)}\frac{1}{(k^{2} - M_{A}^{2})^{2}(k'^{2} - M_{A}^{2})^{2}} = \frac{\sqrt{1-\zeta}}{(1-X)^{2}}\frac{1}{(k^{2} - M_{A}^{2})^{2}(k'^{2} - M_{A}^{2})^{2}} \\ \text{Invert Helicity amps to obtain GPDs} \end{aligned} \qquad A_{++,++} &= \sqrt{1-\xi^{2}}\Big(\frac{H^{g} + \tilde{H}^{g}}{2} - \frac{\xi^{2}}{1-\xi^{2}}\frac{E^{g} + \tilde{E}^{g}}{2}\Big) \\ A_{-+,-+} &= -e^{-i\phi}\frac{\sqrt{t_{0}-t}}{2M}\Big(\frac{E^{g} - \xi\tilde{E}^{g}}{2}\Big) \\ A_{-+,++} &= e^{i\phi}\frac{\sqrt{t_{0}-t}}{2M}\Big(\frac{E^{g} + \xi\tilde{E}^{g}}{2}\Big), \end{aligned}$$

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### Construct Gluon helicity flip amps Spectator Model, then GPDs

$$\begin{aligned} A_{++,+-}^{g} &= \int d^{2}k_{\perp} \mathcal{A}^{g} \left[ (-1)(1-X) \frac{\left(k_{1}+ik_{2}\right)\left(\tilde{k}_{1}+i\tilde{k}_{2}\right)}{XX'} \right] \\ A_{-+,--}^{g} &= \int d^{2}k_{\perp} \mathcal{A}^{g} \left[ (-1)(1-X') \frac{\left(k_{1}+ik_{2}\right)\left(\tilde{k}_{1}+i\tilde{k}_{2}\right)}{XX'} \right] \\ A_{++,--}^{g} &= \int d^{2}k_{\perp} \mathcal{A}^{g} \left[ \left( (1-X)M - M_{X} \right) \frac{\left(\tilde{k}_{1}+i\tilde{k}_{2}\right)}{X'} - \left( (1-X')M - M_{X} \right) \frac{\left(k_{1}+ik_{2}\right)}{X} \right] \\ A_{-+,+-}^{g} &= 0 \end{aligned}$$
(5.15)

Invert Helicity amps to obtain GPDs

$$\begin{vmatrix} A_{++,+-} &= \sqrt{1-\xi^2} \, \frac{t_0 - t}{4M^2} \Big( \tilde{H}_T^g + (1-\xi) \frac{E_T^g + \tilde{E}_T^g}{2} \Big) \\ A_{-+,--} &= \sqrt{1-\xi^2} \, \frac{t_0 - t}{4M^2} \Big( \tilde{H}_T^g + (1+\xi) \frac{E_T^g - \tilde{E}_T^g}{2} \Big) \\ A_{++,--} &= +e^{-i\phi} (1-\xi^2) \frac{\sqrt{t_0 - t}}{2M} \Big( H_T^g + \frac{t_0 - t}{M^2} \tilde{H}_T^g - \frac{\xi^2}{1-\xi^2} E_T^g + \frac{\xi}{1-\xi^2} \tilde{E}_T^g \Big) \\ A_{-+,+-} &= -e^{i\phi} (1-\xi^2) \frac{\sqrt{t_0 - t}^3}{8M^3} \tilde{H}_T^g, \end{aligned}$$



#### pdf's fix x dependence

Gluon



Figure 6: The plot above shows the distribution  $H_g(X,0,0,25 \text{ GeV}^2)$  for the two fits. Alekhin's distribution Xg(X) used in the fit procedure is included with an error band of one half of the error for the set a02m\_lo. The X points used in the fit procedure are indicated by black dots.

Figure 3: The plot above shows the distribution  $XH_{\bar{u}}(X,0,0,25 \text{ GeV}^2)$  for the two fits. Alekhin's distribution  $X\bar{u}(X)$  used in the fit procedure is included with an error band of one half of the error for the set a02m\_lo. The X points used in the fit procedure are indicated by black dots.

anti-u

#### Single Q<sup>2</sup> value shown --- fit known pdf's all Q<sup>2</sup> from J. Poage

GG, Gonzalez Hernandez, Liuti, Poage, in progress



## Gluon & sea distributions J. Poage



GG, Gonzalez Hernandez, Liuti, Poage, in progress



### Fitting gluon pdf's

c.f. Alekhin, .. etc.



GG, **T**onzalez Hernandez, Liuti, Poage, in progress



#### Preliminary: x and t dependence of $H_q(x, 0, t)$ for input scale



'igure 9: The plot above displays the distribution  $H_g(X,0,t)$  for a range of t values.

#### GG, Gonzalez Hernandez, Liuti, Poage, in progress



H<sub>g</sub>(x,0,t) What constrains t-dependence? Spectator t-dependence w/o Regge small x behavior: & hybrid Regge-Spectator model combines



**Compare Gluon form factor via QCD sum rules** Braun, Gornicki, Mankiewicz, Schaefer, Phys.Lett.B302, 291 (1993)



# Abstract

Chiral Even and Odd Generalized Parton Distributions for • valence quarks were obtained in a "flexible" spectator model based on the covariant scattering matrix approach. The parametrization of the GPDs was constrained by nucleon form factors, PDFs and some deeply virtual Compton scattering data. The model is extended to sea quarks and gluons. A broad range of measured and measurable electron, muon and neutrino processes, including cross sections and polarization asymmetries, are compared with existing data. Predictions are made for processes sensitive to the newly parameterized GPDs.