In-medium hadron properties in perspective of diquark structure

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``N and N* Structure with Hard Exclusive Process"
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Outline

I. **Motivations – phenomenology within iso-spin asymmetry**

II. QCD approaches – Diquark and QCD sum rules
- Symmetry energy
- Nucleon and hyperon self energy

III. Extreme limits and Future prospects

Iso-spin asymmetry – symmetry energy

- For finite nucleus

From equation of state

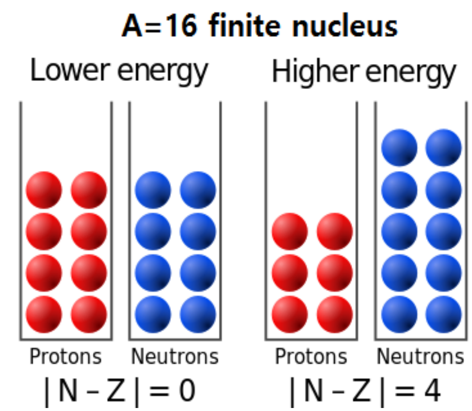
Bethe-Weizsäcker formula for liquid-drop model

$$M_{\rm nucl} = Nm_n + Zm_p - E_B/c^2,$$

$$E_B = a_V A - a_S A^{\frac{2}{3}} - a_C(Z(Z-1))A^{-\frac{1}{3}} - a_A I^2 A + \delta(A, Z)$$

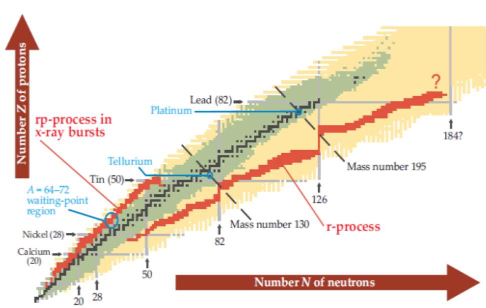
$$I = (N - Z)/A$$

Asymmetric term a_A (=32 MeV) accounts for shifted energy of nuclear matter per nucleon

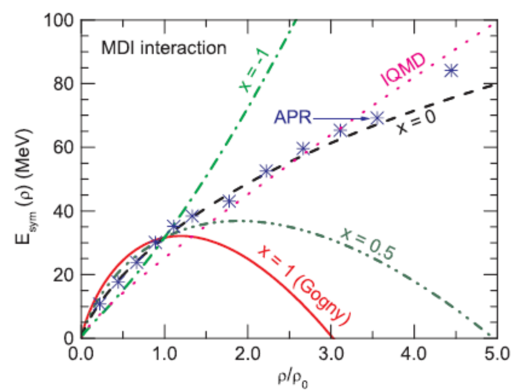


(Figure from Wikipedia)

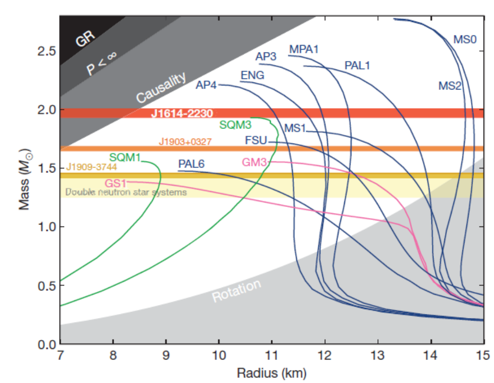
- From rare iso-tope to neutron star core



(Physics Today November 2008)



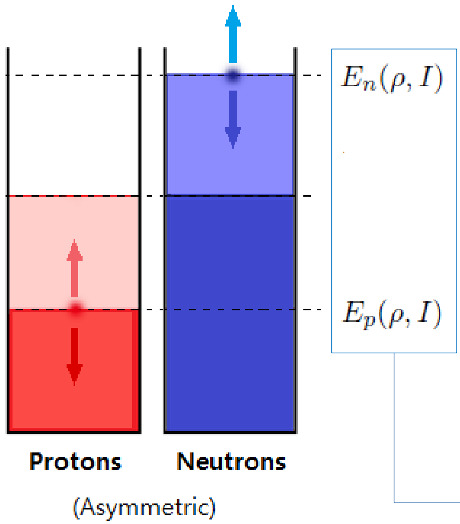
(PRL 102 (2009) 062502 Z. Xiao et al.)



(Nature 467 (2010) 1081 P. B. Demorest et al.)

Symmetry energy and quasi-nucleon

- For continuous (infinite) matter



Similarly, from equation of state

$$\frac{E(\rho_N, I)}{A} \equiv \bar{E}(\rho_N, I) = E_0(\rho_N) + E_{\rm sym}(\rho_N) I^2 + O(I^4) + \cdots$$

$$E_{sym}(\rho_N) = \frac{1}{2!}\frac{\partial^2}{\partial I^2}\bar{E}(\rho_N, I) \qquad I = (\rho_n - \rho_p)/\rho_N$$

If one assume linear density dependent potential, the symmetry energy can be easily read off from potential

$$E_{\rm sym} = \frac{1}{4I}\left(E_n(\rho, I) - E_p(\rho, I)\right) \longrightarrow$$ Linearly dependent on (ρ, I)

- Quasi-nucleon self-energies
  - In continuous matter, nuclear potential can be understood as self-energy of quasi-nucleon
  - Energy dispersion relation can be written in terms of self-energies (RMFT)

$$G(q) = -i\int d^4x e^{iqx}\langle\Psi_0|\mathrm{T}[\psi(x)\bar{\psi}(0)]|\Psi_0\rangle = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$ (near quasi-pole)

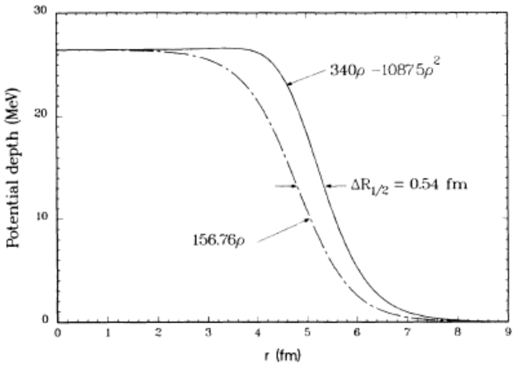
  - Self-energies can be calculated in QCD sum rules

Hyperons and neutron star

- Hyperons (S≠0) in medium (**In vacuum**, M_N~940 MeV, M_Λ~1115 MeV, M_Σ~1190 MeV)

  Λ is bounded (V ~ -30 MeV)

  $-U(r) = 56.67 f(r) - 30.21 f^2(r)$



(PRC38 (1988) 2700 D. J. Millener et. al.)

  Σ potential is repulsive (V ~ +100 MeV)

  $U(r) = (V_0 + iW_0) f(r) + V_{\rm spin}(r, \vec{l}\cdot\vec{\sigma}) + V_{\rm Coulomb}(r)$

	Σ-nucleus pot.	
	$U_\Sigma^{R\,a}$	$U_\Sigma^{S\,a}$
V_0 (MeV)	+150	-10
W_0 (MeV)	-15	-10
V_{SO} (MeV)	0	0
c (fm)	3.3[c]	3.3[c]
z (fm)	0.67	0.67

$f(r) = (1 + \exp[(r-c)/z])^{-1}$

$c = 1.1 \times (A-1)^{1/3}$

At normal density

(PRL89 (2002) 072301 H. Noumi et al.)

- If hyperon energy becomes lower than nucleon energy? ($\rho > \rho_0$, I=1)
  - New degree of freedom (hyperon) can appear in the nuclear matter
    → matter becomes softer → maximum neutron star mass will be bounded near 1.5$M_\odot$
  - 2$M_\odot$ neutron star has been observed (Nature 467 (2010) 1081 P. B. Demorest et al.)
    → should we exclude hyperons in neutron star? How such a stiff EOS could be constructed?
    → related with density behavior of hyperon self-energy and symmetry energy
  - Hyperon self-energies can be compared with nucleon self-energies in sum rules context

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- **Symmetry energy**
- **Nucleon and hyperon self energy**

III. Extreme limits and Future prospects

QCD Sum Rules: Overview

- Correlator for baryon current

$$\Pi(q) \equiv i \int d^4x \, e^{iqx} \langle \Psi_0 | \mathrm{T}[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$
$$= \Pi_s(q^2, q \cdot u) + \Pi_q(q^2, q \cdot u)\not{q} + \Pi_u(q^2, q \cdot u)\not{u}$$

Correlation of the quantum number contained in **η**
q stands for external momentum
u stands for medium velocity → (1,**0**) in rest frame

- Energy dispersion relation and OPE (in **QCD degree of freedom**)

$$\Pi_i(q_0, |\vec{q}|) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega, |\vec{q}|)}{\omega - q_0} + \text{polynomials},$$

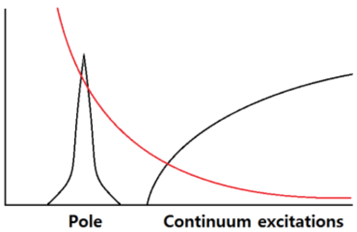
- Phenomenological Ansatz (in **hadronic degree of freedom**)

$$\Pi(q_0, |\vec{q}|) \sim \frac{1}{(q^\mu - \tilde{\Sigma}_v^\mu)\gamma_\mu - M_N^*}$$

Equating both sides, hadronic quantum number can be expressed in QCD degree of freedom

- Weighting - Borel transformation

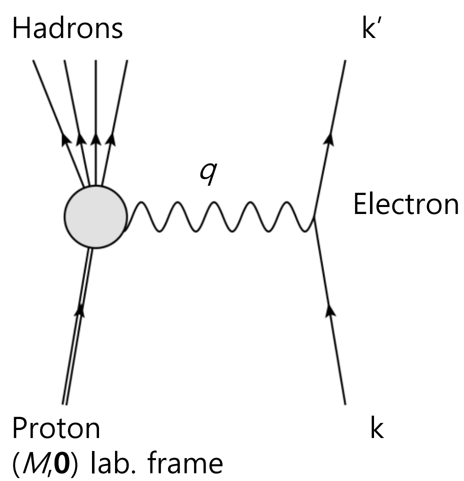
$$\mathcal{B}[\Pi_i(q_0, |\vec{q}|)] \equiv \lim_{\substack{-q_0^2, n \to \infty \\ -q_0^2/n = M^2}} \frac{(-q_0^2)^{n+1}}{n!} \left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0, |\vec{q}|)$$



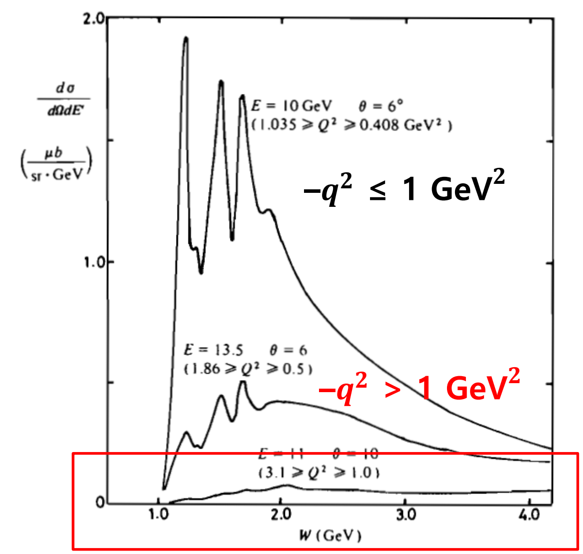
Interpolating fields – parton in hadron

- **Proton** is not a point-like particle

Inelastic scattering: **ep → e + hadrons**



Functions on invariant mass *W*

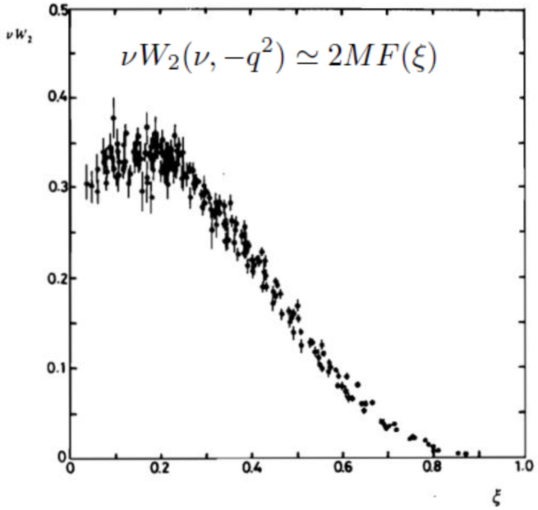


$$\frac{d\sigma}{d\Omega dE'} = \left(\frac{d\sigma}{d\Omega}\right)_M \left[2W_1(\nu,-q^2)\tan^2\frac{\theta}{2} + W_2(\nu,-q^2)\right] \Big/ 2M \qquad \nu = k'_0 - k_0$$

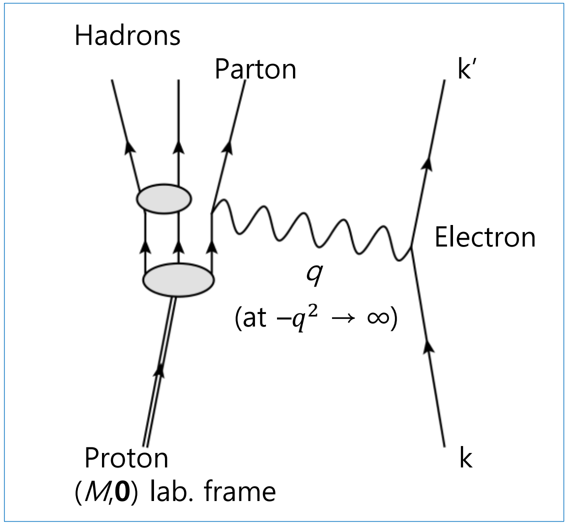
In Bjorken limit (large-momentum transferred region), there are no resonances
→ the scattering can be approximated by point-like free particles (partons)

Parton in hadron

- Bjorken scaling (at $-q^2 \to \infty$, $-q^2/(2M(k'_0 - k_0)) \to \xi$ (fixed) limit)



Behaves as well defined function of ξ

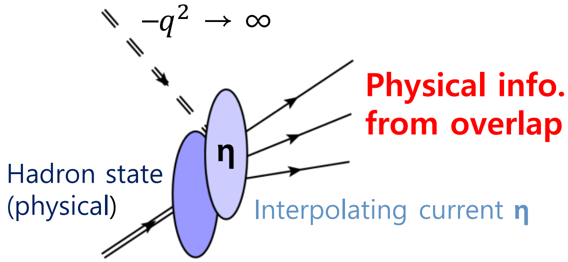


Point-like particle description leads $\nu W_2(\nu, -q^2)$ on function of fixed $\xi = -q^2/(2M(k'_0 - k_0))$
→ confirmed by experimental observation

OPE (at $-q^2 \to \infty$, $-q^2/(2M(k'_0 - k_0)) = \xi$) reproduces Bjorken scaling
→ quantum number of hadron can be interpolated with explicit QCD current

Interpolating current for baryons

- To obtain physical information



a. Quasi-particle state will be extracted from the overlap
b. We need to construct proper interpolating current which can be strongly overlapped with object hadron state
c. Our object: **N**, **P**, **Λ**, and **Σ family**

- Example: constructing **proton** current

Required quantum number: $\boldsymbol{I = 1/2,\ J^P = (1/2)^+}$

Simplest structure: **[$I = 0, J = 0$ di-quark structure] X [single quark with $I = 1/2,\ J = 1/2$]**

Di-quark structure: $\epsilon_{abc}(u_a^T C\gamma_5 d_b)$, $\epsilon_{abc}(u_a^T C d_b)$ ***u*** and ***d*** flavor in antisymmetric combination

Positive parity matching: $\eta_1 = \epsilon_{abc}(u_a^T C d_b)\gamma_5 u_c$, $\eta_2 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$

Ioffe's choice: $\eta = 2(\eta_1 - \eta_2) = \epsilon_{abc}(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu d_c$

→ chiral symmetry breaking term appears in leading order

- Ioffe's choice for hyperon (**Λ** and **Σ+**) current

$$\eta_\Lambda \sim \epsilon_{abc}\left[(u_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu d_c - (d_a^T C\gamma_\mu s_b)\gamma_5\gamma^\mu u_c\right]$$
$$\eta_{\Sigma+} \sim \epsilon_{abc}(u_a^T C\gamma_\mu u_b)\gamma_5\gamma^\mu s_c$$

Required quantum number
$I = 0,\ J^P = (1/2)^+$
$I = 1,\ J^P = (1/2)^+$

Ioffe's choice for Σ

- Σ_0 interpolating field in general combination

$$\eta_{\Sigma^0} = \epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c + [d_a^T C s_b]\gamma_5 u_c + t\left([u_a^T C\gamma_5 s_b]d_c + [d_a^T C\gamma_5 s_b]u_c\right)\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c.$$

Requirement: (1) spin-0 di-quark structure, (2) total *I=0* combination

- Using Fierz rearrangement

(a) $$\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{L,c} + [d_{R,a}^T C s_{R,b}]u_{L,c} - (L \leftrightarrow R)\right)$$

(b) $$\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c = 4\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{R,c} + [d_{R,a}^T C s_{R,b}]u_{R,c} - (L \leftrightarrow R)\right)$$

Quark propagation in perturbative regime (separation scale ~ 1 GeV)

$$\langle T[q_\beta^a(x)\bar{q}_\alpha^b(0)]\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i}{m_q - \not{p}} \simeq \underline{\frac{i}{2\pi^2}\delta_{ab}\frac{1}{(x^2)^2}[\not{x}]_{\alpha\beta}} - \underline{\frac{m_q}{4\pi^2}\frac{1}{x^2}\delta_{ab}\delta_{\alpha\beta}}$$

Light quark has chiral symmetry → propagation from each helicity state to itself
The symmetry is broken for strange quark ($m_s \neq 0$) → mixed propagation between helicity states

Correlator of each basis can be understood in diagrammatical way

Ioffe's choice for Σ

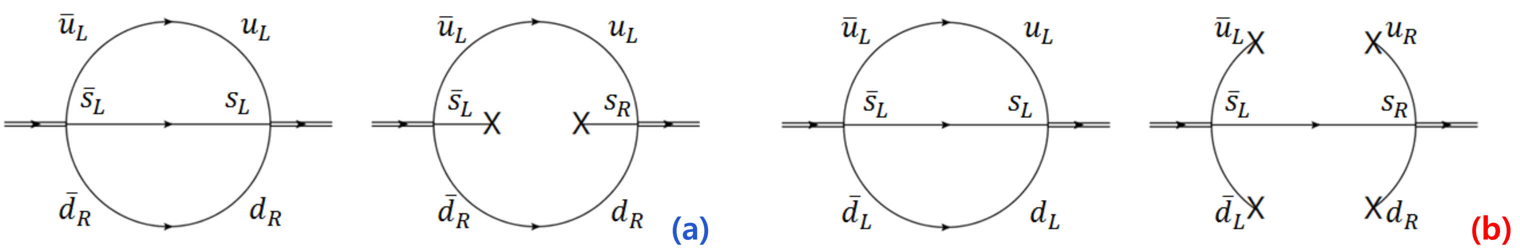
- **Σ₀** interpolating field (continued)

$$\eta_{\Sigma^0} = \epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c + [d_a^T C s_b]\gamma_5 u_c + t\left([u_a^T C\gamma_5 s_b]d_c + [d_a^T C\gamma_5 s_b]u_c\right)\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c.$$

(a) $\epsilon_{abc}[u_a^T C\gamma_\mu d_b]\gamma_5\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{L,c} + [d_{R,a}^T C s_{R,b}]u_{L,c} - (L \leftrightarrow R)\right)$

(b) $\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} d_b]\gamma_5\sigma^{\mu\nu} s_c = 4\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{R,c} + [d_{R,a}^T C s_{R,b}]u_{R,c} - (L \leftrightarrow R)\right)$

- **Lowest mass dimensional quark condensate**



Correlator of basis **(a)**: strange quark condensate (dim-3)
Correlator of basis **(b)**: four-quark condensate (dim-6)

Lack of clear information for four-quark condensate → choice of basis **(a)** can be better

Ioffe's choice for **proton**

- **Proton** case

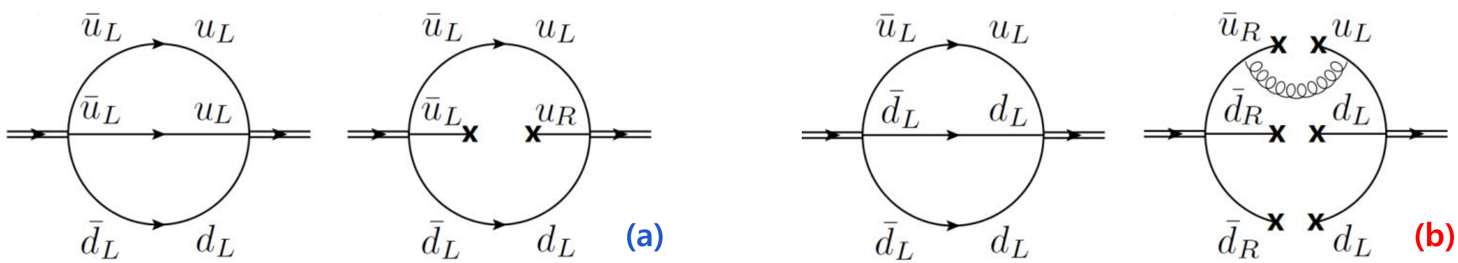
$$\eta_{p(t)} = 2\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 u_c + t[u_a^T C\gamma_5 d_b]u_c\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c$$

Requirement: (1) spin-0 di-quark structure, (2) total *I=0* combination

- After Fierz rearrangement

(a) $$\epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c = 4\epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]u_{L,c} - [u_{L,a}^T C d_{L,b}]u_{R,c}\right)$$

(b) $$\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c = 4\epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]u_{R,c} - [u_{L,a}^T C d_{L,b}]u_{L,c}\right)$$



Correlator of basis **(a)**: chiral condensate (dim-3)
Correlator of basis **(b)**: six-quark condensate (dim-9) with perturbative gluon attachment

Same reason for six-quark condensate → choice of basis **(a)** would be better (Ioffe's choice)

Generalized Interpolating field for Λ

- Special case: **Λ**

Possible I=0 combination with spin-0 di-quark structure

$$\{\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c,\ \epsilon_{abc}[u_a^T C\gamma_5 d_b]s_c,\ \underline{\epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c - [d_a^T C s_b]\gamma_5 u_c\right)},\ \underline{\epsilon_{abc}\left([u_a^T C\gamma_5 s_b]d_c - [d_a^T C\gamma_5 s_b]u_c\right)}\}$$

3rd and 4th basis can be expressed as

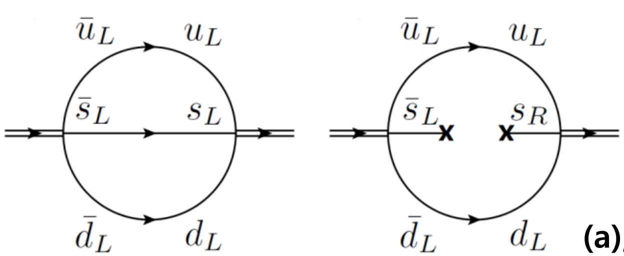
$$\epsilon_{abc}\left([u_a^T C s_b]\gamma_5 d_c - [d_a^T C s_b]\gamma_5 u_c\right) = \frac{1}{2}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + [u_a^T C\gamma_5 d_b]s_c - [u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

$$\epsilon_{abc}\left([u_a^T C\gamma_5 s_b]d_c - [d_a^T C\gamma_5 s_b]u_c\right) = \frac{1}{2}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + [u_a^T C\gamma_5 d_b]s_c + [u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

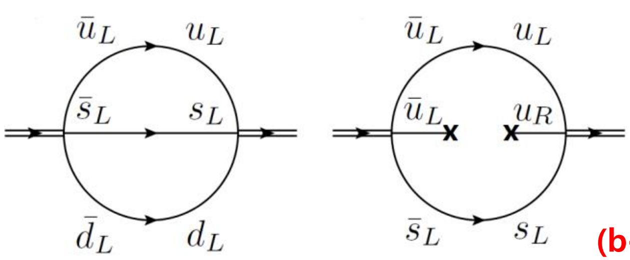
→ basis set can be reduced to 3-independent bases set

Representation in helicity bases

(a)	$\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c = \epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]s_{R,c} - [u_{R,a}^T C d_{R,b}]s_{L,c} - (L \leftrightarrow R)\right)$	P-scalar diquark
(b)	$\epsilon_{abc}[u_a^T C\gamma_5 d_b]s_c = \epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]s_{R,c} + [u_{R,a}^T C d_{R,b}]s_{L,c} - (L \leftrightarrow R)\right)$	Scalar diquark
(c)	$\epsilon_{abc}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{L,c} - [d_{R,a}^T C s_{R,b}]u_{L,c} - (L \leftrightarrow R)\right)$	Vector diquark



(a), (b), (c)



(b-c)

Generalized Interpolating field for Λ

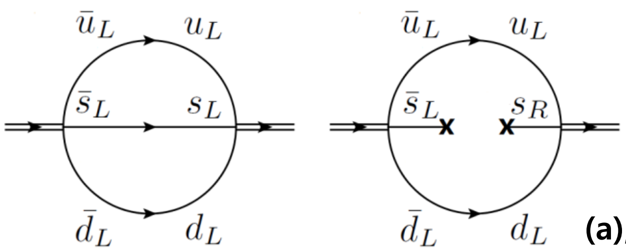
- Special case: Λ (continued)

Set of basis and **lowest mass dimensional quark condensate**

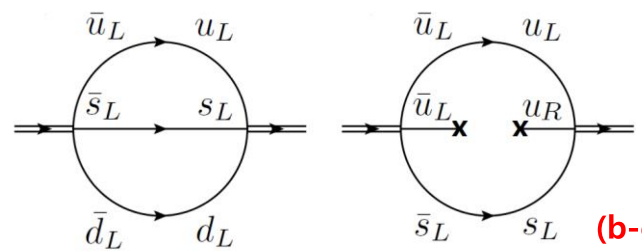
(a) $\epsilon_{abc}[u_a^T C d_b]\gamma_5 s_c = \epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]s_{R,c} - [u_{R,a}^T C d_{R,b}]s_{L,c} - (L \leftrightarrow R)\right)$

(b) $\epsilon_{abc}[u_a^T C\gamma_5 d_b]s_c = \epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]s_{R,c} + [u_{R,a}^T C d_{R,b}]s_{L,c} - (L \leftrightarrow R)\right)$

(c) $\epsilon_{abc}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c = 2\epsilon_{abc}\left([u_{R,a}^T C s_{R,b}]d_{L,c} - [d_{R,a}^T C s_{R,b}]u_{L,c} - (L \leftrightarrow R)\right)$



(a), (b), (c)



(b-c)

Correlator of basis **(a)**, **(b)**, **(c)**: strange quark condensate (dim-3)
Cross correlator of basis **(b-c)**: chiral condensate (dim-3)

General form of Λ interpolating field

$$\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + \tilde{a}[u_a^T C\gamma_5 d_b]s_c + \tilde{b}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

Where *A* determines overall normalization and coupling strength to physical Λ state

QCD Sum Rules: dispersion relation

- Simplest case: Nucleon in vacuum

$$\Pi(q) \equiv i \int d^4x e^{iqx} \langle 0|\mathrm{T}[\eta(x)\bar{\eta}(0)]|0\rangle = \Pi_s(q^2) + \Pi_q(q^2)\not{q}.$$

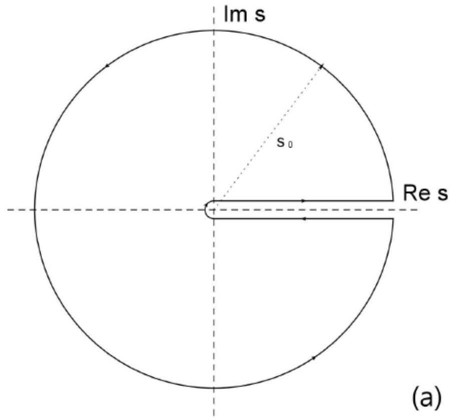
- Using Cauchy relation

$$\Pi_i(q^2) = \frac{1}{2\pi i}\int_0^\infty ds \frac{\Delta\Pi_i(s)}{s-q^2} + \text{polynomials}$$

$$\Delta\Pi_i(q^2) \equiv \lim_{\epsilon\to 0^+}[\Pi_i(q^2+i\epsilon) - \Pi_i(q^2-i\epsilon)]$$

$$= (2\pi)^4 i \sum_\alpha \left(\delta^4(q-p_\alpha)\langle 0|\eta(0)|\alpha\rangle\langle\alpha|\bar{\eta}(0)|0\rangle - \delta^4(q+p_\alpha)\langle 0|\bar{\eta}(0)|\alpha\rangle\langle\alpha|\eta(0)|0\rangle\right)$$

Imaginary part contains all possible hadronic resonance α



q^2 integration contour (Vacuum)

- Emphasizing ground state – Borel sum rules

$$\mathcal{B}[\Pi_i(q^2)] = \frac{1}{2\pi i}\int_0^\infty ds\, e^{-s/M^2}\Delta\Pi_i(s)$$

$$\equiv \lim_{\substack{-q^2, n\to\infty \\ -q^2/n = M^2}} \frac{(-q^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q^2}\right)^n \Pi_i(q^2) \equiv \hat{\Pi}_i(M^2)$$

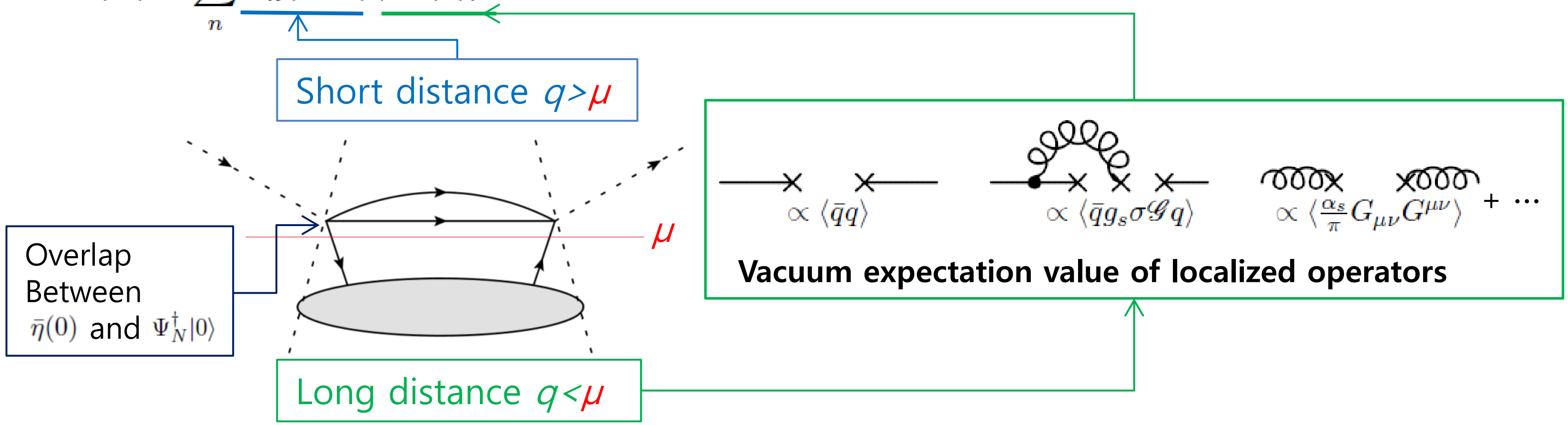
The continuum will be suppressed by setting M ~ hadronic mass scale

- The moment (Borel mass $-q^2/n = M^2$) is a fictitious value (non-physical) In principle, physical values such as mass should not depends on M
- As OPE is truncated, actually it depends → the value can be read off at plateau of Borel curve

QCD SR: operator product expansion

- Operator product expansion (Example: 2-quark condensate diagram)

$$\Pi_i(q^2) = \sum_n C_n^i(q^2, \mu)\langle \hat{O}_n(\mu) \rangle_0$$



- Separation scale is set to be hadronic scale (≤ 1 GeV)
  - **Wilson coefficient** contains perturbative contribution above separation scale – short-ranged partonic propagation in hadron
  - **Condensate** contains non-perturbative contribution below separation scale – long ranged correlation in low energy part of hadron
  - Quark confinement inside hadron is low energy QCD phenomenon
  - Genuine properties of hadron are reflected in **the condensates**

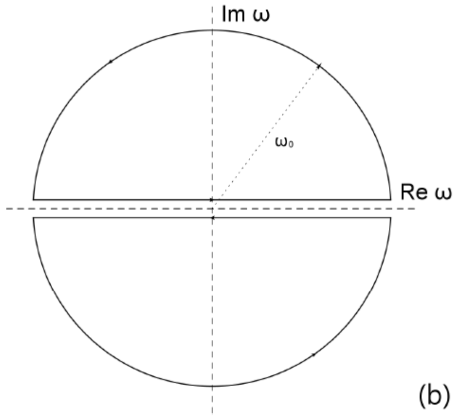
QCD Sum Rules: in-medium case

- Energy dispersion relation with fixed 3-momentum

$$\Pi_i(q_0,|\vec{q}|) = \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\omega \frac{\Delta\Pi_i(\omega,|\vec{q}|)}{\omega - q_0} + \text{polynomials},$$

$$\Delta\Pi_i(\omega,|\vec{q}|) \equiv \lim_{\epsilon\to 0^+}\left[\Pi_i(\omega+i\epsilon,|\vec{q}|) - \Pi_i(\omega - i\epsilon,|\vec{q}|)\right]$$

  - In-medium hadronic excitation is certainly not symmetric as in the vacuum case
  - Medium reference frame occurs – energy sum rules for quasi-particle state is proper choice



q_0 integration contour (In-medium)

- Energy Borel sum rules

$$\mathcal{B}[\Pi_i(q_0,|\vec{q}|)] = \frac{1}{2\pi i}\int_{-\omega_0}^{\omega_0} d\omega \ W(\omega)\Delta\Pi_i(\omega,|\vec{q}|)$$

$$\equiv \lim_{\substack{-q_0^2,n\to\infty \\ -q_0^2/n=M^2}} \frac{(-q_0^2)^{n+1}}{n!}\left(\frac{\partial}{\partial q_0^2}\right)^n \Pi_i(q_0,|\vec{q}|) \equiv \hat{\Pi}_i(M^2,|\vec{q}|),$$

$$W(\omega) = (\omega - \bar{E}_q)e^{-\omega^2/M^2}$$

Anti-state is suppressed, only quasi-particle part is emphasized

QCD Sum Rules: spectral ansatz

- According to relativistic mean field theory

$$G(q) = \frac{1}{\not{q} - M_n - \Sigma(q)} \rightarrow \lambda^2 \frac{\not{q} + M^* - \not{u}\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_0)}$$

Quasi-hadron propagator in **RMFT**

Each invariant can be assumed according to phenomenological Ansatz
Via Borel transformation, self-energies can be obtained in terms of invariants

$$\Pi_s(q_0, |\vec{q}|) = -\lambda_N^{*2} \frac{M_N^*}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots$$

$$\Pi_q(q_0, |\vec{q}|) = -\lambda_N^{*2} \frac{1}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots$$

$$\Pi_u(q_0, |\vec{q}|) = +\lambda_N^{*2} \frac{\Sigma_v}{(q_0 - E_q)(q_0 - \bar{E}_q)} + \cdots$$

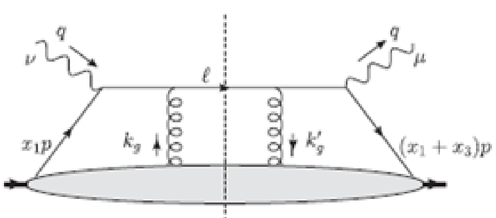
Borel transf. $\Rightarrow$

$$\bar{\mathcal{B}}[\Pi_s(q_0^2, |\vec{q}|)] = \lambda_N^{*2} M_h^* e^{-(E_q^2 - \vec{q}^2)/M^2}$$

$$\bar{\mathcal{B}}[\Pi_q(q_0^2, |\vec{q}|)] = \lambda_N^{*2} e^{-(E_q^2 - \vec{q}^2)/M^2}$$

$$\bar{\mathcal{B}}[\Pi_u(q_0^2, |\vec{q}|)] = \lambda_N^{*2} \Sigma_v^h e^{-(E_q^2 - \vec{q}^2)/M^2}$$

- Condensates and in-medium properties



DIS diagram

I. In-medium properties are included in low energy scale (long-ranged)

II. PCAC (Gellman-Oakes-Renner relation), Chiral perturbation theory, Lattice QCD, DIS experiment can be used to obtain in-medium condensates

In-medium condensates

- Simplest guess: linear Fermi gas approximation

$$\begin{aligned}\langle \hat{O} \rangle_{\rho,I} &= \langle \hat{O} \rangle_{\text{vac}} + \langle n|\hat{O}|n\rangle \rho_n + \langle p|\hat{O}|p\rangle \rho_p \\ &= \langle \hat{O} \rangle_{\text{vac}} + \tfrac{1}{2}(\langle n|\hat{O}|n\rangle + \langle p|\hat{O}|p\rangle)\rho \\ &\quad + \tfrac{1}{2}(\langle n|\hat{O}|n\rangle - \langle p|\hat{O}|p\rangle) I \rho.\end{aligned}$$

[Vacuum condensate] +
[nucleon expectation value] **x** [density]

Iso-spin symmetric and asymmetric part

- Example: chiral condensates

Iso-spin symmetric part

$$\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_{\text{vac}} + \frac{\sigma_N}{2m_q}\rho$$

Nucleon-pion sigma term

$$\sigma_N = \frac{1}{3}\sum_{a=1}^{3}\left(\langle \tilde{N}|[\mathcal{Q}_A^a,[\mathcal{Q}_A^a,H_{QCD}]]|\tilde{N}\rangle - \langle 0|[\mathcal{Q}_A^a,[\mathcal{Q}_A^a,H_{QCD}]]|0\rangle\right)$$

$$= 2m_q \int d^3x \left(\langle \tilde{N}|\bar{q}q|\tilde{N}\rangle - \langle 0|\bar{q}q|0\rangle\right) \equiv 2m_q \langle N|\bar{q}q|N\rangle$$

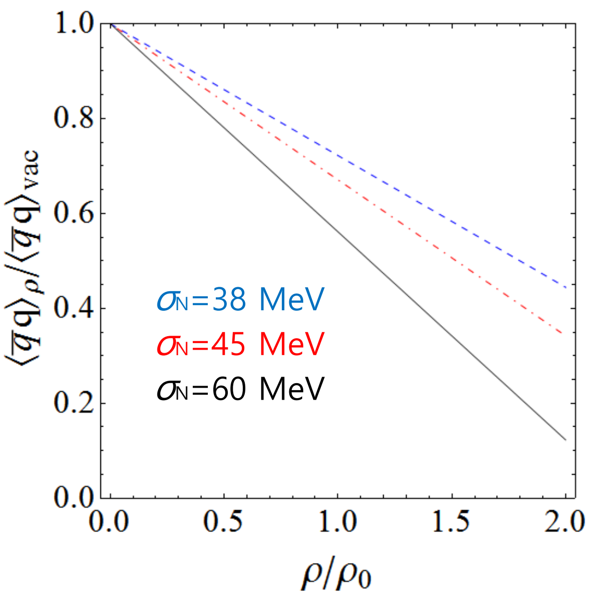
where, $H_{QCD} = \int d^3x (2m_q \bar{q}q + m_s \bar{s}s + \cdots)$

With Hellman-Feynman theorem

$$2m_q \langle \psi|\bar{q}q|\psi\rangle = m_q \frac{d}{dm_q}\langle \psi|H_{QCD}|\psi\rangle$$

and linear density approximation $\mathcal{E} \sim M_N \rho$

Sigma term determines dropping rate

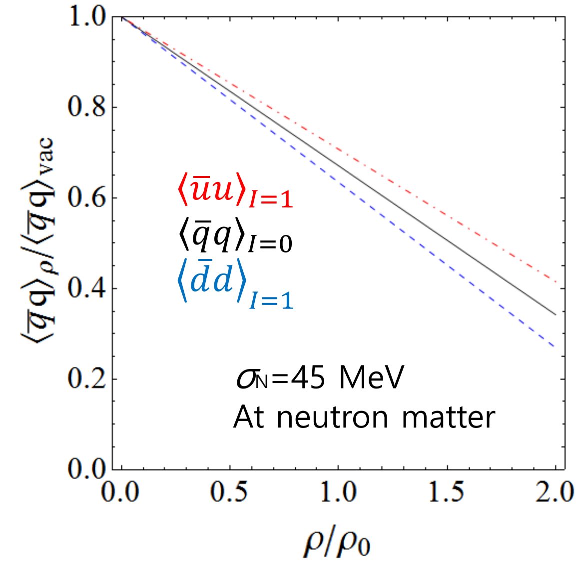


In-medium condensates

- ## Asymmetric part

From trace anomaly and heavy quark expansion

$$T^{\mu}_{\ \mu} = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \sum_{h=c,t,b} m_h \bar{h}h + \cdots$$
$$= \left(-\frac{9\alpha_s}{8\pi}\right) G^2 + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + O(\mu^2/4m_h^2)$$



Low-lying baryon octet mass relation

$$m_p = A + m_u B_u + m_d B_d + m_s B_s$$
$$m_n = A + m_u B_d + m_d B_u + m_s B_s$$
$$m_{\Sigma^+} = A + m_u B_u + m_d B_s + m_s B_d$$
$$m_{\Sigma^-} = A + m_u B_s + m_d B_u + m_s B_d$$
$$m_{\Xi^0} = A + m_u B_d + m_d B_s + m_s B_u$$
$$m_{\Xi^-} = A + m_u B_s + m_d B_d + m_s B_u$$

$$\Rightarrow \quad \frac{1}{2}\left(\langle p|\bar{u}u|p\rangle - \langle p|\bar{d}d|p\rangle\right) = \frac{1}{2}\left(\frac{(m_{\Xi^0} + m_{\Xi^-}) - (m_{\Sigma^+} + m_{\Sigma^-})}{2m_s - (m_u + m_d)}\right)$$

where $A \equiv \langle (\bar{\beta}/4\alpha_s) G^2 \rangle_p, \ B_u \equiv \langle \bar{u}u \rangle_p, \ B_d \equiv \langle \bar{d}d \rangle_p$

- ## Strange contents

$$\langle \bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_{\rm vac} + \langle \bar{s}s \rangle_N \rho$$
$$= (0.8)\langle \bar{q}q \rangle_{\rm vac} + y \frac{\sigma_N}{2m_q} \rho$$
$$y = \langle \bar{s}s \rangle_N / \langle \bar{q}q \rangle_N$$

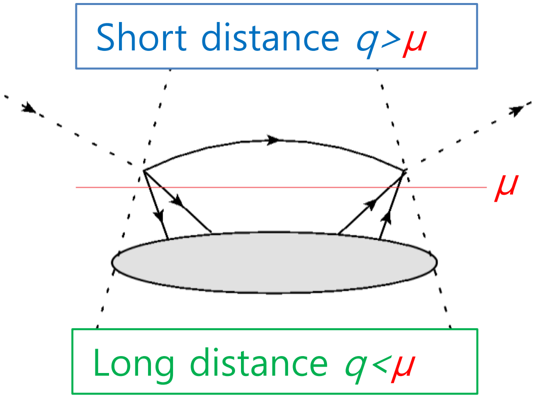
Ratio **0.8** is determined from vacuum sum rule for hyperon

y can be determined from direct lattice QCD
→ recent lattice results says ***y*** should be small
***y*~0.05** (PRD87 074503, PRD91 051502, PRD94 054503)

We confined ***y* → 0.1**

4-quark condensates – baryon sum rules

- In 3-quark constituted baryon sum rules



- There is no loop in leading order diagram
  → no suppression factor comes from loop diagram
- Can be numerically important in sum rules
- Indeed, 4-quark condensates give non-negligible contribution to baryon sum rules
- For twist-4 condensates, DIS data can be used (within linear density approximation)

- Twist-4 ops. in baryon OPE

Operator type	$\gamma-\gamma$	$\gamma_5\gamma-\gamma_5\gamma$	$\sigma-\sigma$
t^A-t^A	$\langle\bar{q}_1\gamma_5\gamma t^A q_1\bar{q}_2\gamma_5\gamma t^A q_2\rangle_{p,\mathrm{s.t.}}\equiv T^1_{q_1q_2}$	$\langle\bar{q}_1\gamma t^A q_1\bar{q}_2\gamma t^A q_2\rangle_{p,\mathrm{s.t.}}\equiv T^2_{q_1q_2}$	$\langle\bar{q}_1\sigma t^A q_1\bar{q}_2\sigma t^A q_2\rangle_{p,\mathrm{s.t.}}\equiv T^5_{q_1q_2}$
$I-I$	$\langle\bar{q}_1\gamma_5\gamma q_1\bar{q}_2\gamma_5\gamma q_2\rangle_{p,\mathrm{s.t.}}\equiv T^3_{q_1q_2}$	$\langle\bar{q}_1\gamma q_1\bar{q}_2\gamma q_2\rangle_{p,\mathrm{s.t.}}\equiv T^4_{q_1q_2}$	$\langle\bar{q}_1\sigma q_1\bar{q}_2\sigma q_2\rangle_{p,\mathrm{s.t.}}\equiv T^6_{q_1q_2}$

$$\langle p|\bar{q}_1\Gamma_i^\alpha q_1\bar{q}_2\Gamma_i^\beta q_2|p\rangle_{\mathrm{s.t.}}=\left(u^\alpha u^\beta-\frac{1}{4}g^{\alpha\beta}\right)\frac{1}{4\pi\alpha_s}\frac{M_n}{2}T^i_{q_1q_2}$$

q_1 and q_2 stand for light quark flavor

Matrix elements can be obtained from DIS experiment data

Borel transformed OPE

- Nucleon OPE (**neutron**)

$$
\begin{aligned}
\overline{\mathcal{W}}_M[\Pi_{n,s}(q_0^2,|\vec{q}|)] &= \lambda_n^{*2} M_n^* e^{-(E_{n,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{n,s}^e(q_0^2,|\vec{q}|)] - \bar{E}_{n,q}\mathcal{B}[\Pi_{n,s}^o(q_0^2,|\vec{q}|)] \\
&= -\frac{1}{4\pi^2}(M^2)^2 E_1 \langle \bar{u}u\rangle_{\rho,I}, \\
\overline{\mathcal{W}}_M[\Pi_{n,q}(q_0^2,|\vec{q}|)] &= \lambda_n^{*2} e^{-(E_{n,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{n,q}^e(q_0^2,|\vec{q}|)] - \bar{E}_{n,q}\mathcal{B}[\Pi_{n,q}^o(q_0^2,|\vec{q}|)] \\
&= \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} + \frac{1}{32\pi^2} M^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \\
&\quad - \left[\frac{1}{9\pi^2} M^2 E_0 - \frac{4}{9\pi^2}\vec{q}^2\right] \langle u^\dagger i D_0 u\rangle_{\rho,I} L^{-\frac{4}{9}} - \left[\frac{4}{9\pi^2} M^2 E_0 - \frac{4}{9\pi^2}\vec{q}^2\right] \langle d^\dagger i D_0 d\rangle_{\rho,I} L^{-\frac{4}{9}} \\
&\quad - \frac{1}{2}\langle \bar{d}\gamma d \bar{d}\gamma d\rangle_{\text{tr.}} - \frac{1}{2}\langle \bar{d}\gamma_5\gamma d \bar{d}\gamma_5\gamma d\rangle_{\text{tr.}} + \frac{3}{2}\langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\text{tr.}} + \frac{5}{2}\langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\text{tr.}} \\
&\quad - \frac{1}{2}\langle \bar{d}\gamma d \bar{d}\gamma d\rangle_{\text{s.t.}} + \frac{1}{2}\langle \bar{d}\gamma_5\gamma d \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}} - \frac{1}{2}\langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\text{s.t.}} + \frac{1}{2}\langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}} \\
&\quad + \bar{E}_{p,q}\left[\frac{1}{6\pi^2} M^2 \left[\langle u^\dagger u\rangle_{\rho,I} + \langle d^\dagger d\rangle_{\rho,I}\right] E_0 L^{-\frac{4}{9}}\right], \\
\overline{\mathcal{W}}_M[\Pi_{n,u}(q_0^2,|\vec{q}|)] &= \lambda_n^{*2} \Sigma_v^n e^{-(E_{n,q}^2-\vec{q}^2)/M^2} \mathcal{B}[\Pi_{n.u}^e(q_0^2,|\vec{q}|)] - \bar{E}_{n,q}\mathcal{B}[\Pi_{n.u}^o(q_0^2,|\vec{q}|)] \\
&= \frac{1}{12\pi^2}(M^2)^2 \left[7\langle d^\dagger d\rangle_{\rho,I} + \langle u^\dagger u\rangle_{\rho,I}\right] E_1 L^{-\frac{4}{9}} \\
&\quad + \bar{E}_{p,q}\left[\frac{4}{9\pi^2} M^2 \langle u^\dagger i D_0 u\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} + \frac{16}{9\pi^2} M^2 \langle d^\dagger i D_0 d\rangle_{\rho,I} E_0 L^{-\frac{4}{9}} \right. \\
&\qquad \left. + 2\left[\langle \bar{d}\gamma d \bar{d}\gamma d\rangle_{\text{s.t.}} - \langle \bar{d}\gamma_5\gamma d \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}} + \langle \bar{u}\gamma u \bar{d}\gamma d\rangle_{\text{s.t.}} - \langle \bar{u}\gamma_5\gamma u \bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}}\right]\right].
\end{aligned}
$$

Borel transformed OPE

- Σ^+ hyperon OPEs

$$\overline{\mathcal{W}}_M[\Pi_{\Sigma^+,s}(q_0^2,|\vec{q}|)] = \lambda_{\Sigma^+}^{*2} M_{\Sigma^+}^* e^{-(E_{\Sigma^+,q}^2-\vec{q}^2)/M^2} = \bar{\mathcal{B}}[\Pi_{\Sigma^+,s}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Sigma^+,q}\bar{\mathcal{B}}[\Pi_{\Sigma^+,s}^o(q_0^2,|\vec{q}|)]$$
$$= \frac{m_s}{16\pi^4}(M^2)^3 E_2 L^{-\frac{8}{9}} - \frac{1}{4\pi^2}(M^2)^2 E_1 \langle \bar{s}s\rangle_{\rho,I} + m_s\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\text{tr.}} - m_s\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\text{tr.}}$$
$$+ \bar{E}_{\Sigma^+,q}\left[\frac{m_s}{2\pi^2}M^2\langle u^\dagger u\rangle_{\rho,I}E_0 L^{-\frac{8}{9}} - \frac{4}{3}\langle \bar{s}s\rangle_{\text{vac}}\langle u^\dagger u\rangle_{\rho,I}\right],$$

$$\overline{\mathcal{W}}_M[\Pi_{\Sigma^+,q}(q_0^2,|\vec{q}|)] = \lambda_{\Sigma^+}^{*2} e^{-(E_{\Sigma^+,q}^2-\vec{q}^2)/M^2} = \bar{\mathcal{B}}[\Pi_{\Sigma^+,q}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Sigma^+,q}\bar{\mathcal{B}}[\Pi_{\Sigma^+,q}^o(q_0^2,|\vec{q}|)]$$
$$= \frac{1}{32\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} - +\frac{1}{32\pi^2}M^2\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}}$$
$$- \left[\frac{1}{9\pi^2}M^2E_0 - \frac{4}{9\pi^2}\vec{q}^2\right]\langle s^\dagger iD_0 s\rangle_{\rho,I}L^{-\frac{4}{9}} - \left[\frac{4}{9\pi^2}M^2E_0 - \frac{4}{9\pi^2}\vec{q}^2\right]\langle u^\dagger iD_0 u\rangle_{\rho,I}L^{-\frac{4}{9}}$$
$$- \frac{1}{2}\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\text{tr.}} - \frac{1}{2}\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\text{tr.}} + \frac{3}{2}\langle \bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\text{tr.}} + \frac{5}{2}\langle \bar{u}\gamma u\bar{s}\gamma s\rangle_{\text{tr.}}$$
$$- \frac{1}{2}\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\text{s.t.}} + \frac{1}{2}\langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\text{s.t.}} - \frac{1}{2}\langle \bar{u}\gamma u\bar{s}\gamma s\rangle_{\text{s.t.}} + \frac{1}{2}\langle \bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\text{s.t.}}$$
$$+ \bar{E}_{\Sigma^+,q}\left[\frac{1}{6\pi^2}M^2\langle u^\dagger u\rangle_{\rho,I}E_0 L^{-\frac{4}{9}}\right],$$

$$\overline{\mathcal{W}}_M[\Pi_{\Sigma^+,u}(q_0^2,|\vec{q}|)] = \lambda_{\Sigma^+}^{*2}\Sigma_v^{\Sigma^+} e^{-(E_{\Sigma^+,q}^2-\vec{q}^2)/M^2}\bar{\mathcal{B}}[\Pi_{\Sigma^+,u}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Sigma^+,q}\bar{\mathcal{B}}[\Pi_{\Sigma^+,u}^o(q_0^2,|\vec{q}|)]$$
$$= \frac{7}{12\pi^2}(M^2)^2\langle u^\dagger u\rangle_{\rho,I}E_1 L^{-\frac{4}{9}}$$
$$+ \bar{E}_{\Sigma^+,q}\left[\frac{4}{9\pi^2}M^2\langle s^\dagger iD_0 s\rangle_{\rho,I}E_0L^{-\frac{4}{9}} + \frac{16}{9\pi^2}M^2\langle u^\dagger iD_0 u\rangle_{\rho,I}E_0 L^{-\frac{4}{9}}\right.$$
$$\left. + 2\left[\langle \bar{u}\gamma u\bar{u}\gamma u\rangle_{\text{s.t.}} - \langle \bar{u}\gamma_5\gamma u\bar{u}\gamma_5\gamma u\rangle_{\text{s.t.}} + \langle \bar{u}\gamma u\bar{s}\gamma s\rangle_{\text{s.t.}} - \langle \bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\text{s.t.}}\right]\right].$$

Borel transformed OPE

- **Λ** hyperon OPE with **Generalized interpolating field**

$$\overline{\mathcal{W}}_M[\Pi_{\Lambda,s}(q_0^2,|\vec{q}|)] = \lambda_\Lambda^{*2} M_\Lambda^* e^{-(E_{\Lambda,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi^e_{\Lambda,s}(q_0^2,|\vec{q}|)] - \bar{E}_{\Lambda,q}\mathcal{B}[\Pi^o_{\Lambda,s}(q_0^2,|\vec{q}|)]$$

$$= -\frac{(1-\tilde{a}^2+2\tilde{b}^2)}{64\pi^4} m_s (M^2)^3 E_2 L^{-\frac{8}{9}}$$

$$+\frac{(1-\tilde{a}^2+2\tilde{b}^2)}{16\pi^2}(M^2)^2\langle\bar{s}s\rangle_{\rho,I}E_1 - \frac{\tilde{a}\tilde{b}}{4\pi^2}(M^2)^2\langle\bar{q}q\rangle_{\rho,I}E_1$$

$$-\frac{(1-\tilde{a}^2-2\tilde{b}^2)}{128\pi^2} m_s M^2 \left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_{\rho,I} E_0 L^{-\frac{8}{9}}$$

$$+\frac{(1+\tilde{a}^2+4\tilde{b}^2)}{4} m_s\langle\bar{u}u\bar{d}d\rangle_{\rm tr.} + \frac{(1+\tilde{a}^2-4\tilde{b}^2)}{4} m_s\langle\bar{u}\gamma_5 u\bar{d}\gamma_5 d\rangle_{\rm tr.}$$

$$-\frac{(1-\tilde{a}^2-2\tilde{b}^2)}{4} m_s\langle\bar{u}\gamma u\bar{d}\gamma d\rangle_{\rm tr.} - \frac{(1-\tilde{a}^2+2\tilde{b}^2)}{4} m_s\langle\bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\rm tr.} - \frac{(1+\tilde{a}^2)}{8} m_s\langle\bar{u}\sigma u\bar{d}\sigma d\rangle_{\rm tr.}$$

$$-\bar{E}_{\Lambda,q}\left[\frac{(1-\tilde{a}^2+2\tilde{b}^2)}{8\pi^2} m_s M^2\langle q^\dagger q\rangle_{\rho,I}E_0 L^{-\frac{8}{9}} + \frac{2\tilde{a}\tilde{b}}{3}\langle q^\dagger q\rangle_{\rho,I}\langle\bar{q}q\rangle_{\rm vac} - \frac{(1-\tilde{a}^2+4\tilde{b}^2)}{3}\langle q^\dagger q\rangle_{\rho,I}\langle\bar{s}s\rangle_{\rm vac}\right]$$

$$\overline{\mathcal{W}}_M[\Pi_{\Lambda,u}(q_0^2,|\vec{q}|)] = \lambda_\Lambda^{*2}\Sigma_v^\Lambda e^{-(E_{\Lambda,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi^e_{\Lambda,u}(q_0^2,|\vec{q}|)] - \bar{E}_{\Lambda,q}\mathcal{B}[\Pi^o_{\Lambda,u}(q_0^2,|\vec{q}|)]$$

$$= \frac{(1+\tilde{a}^2+14\tilde{b}^2)}{48\pi^2}(M^2)^2\langle q^\dagger q\rangle_{\rho,I}E_1 L^{-\frac{4}{9}} - \frac{2\tilde{a}\tilde{b}}{3} m_s\langle q^\dagger q\rangle_{\rho,I}\langle\bar{s}s\rangle_{\rm vac}$$

$$+\bar{E}_{\Lambda,q}\Bigg[\tilde{b}^2\langle\bar{u}\gamma u\bar{d}\gamma d\rangle_{\rm s.t.} - \tilde{b}^2\langle\bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\rm s.t.} + \tilde{b}^2\langle\bar{u}\sigma u\bar{d}\sigma d\rangle_{\rm s.t.}$$

$$+\frac{(1+\tilde{a}^2-2\tilde{b}^2)}{2}\langle\bar{u}\gamma u\bar{s}\gamma s\rangle_{\rm s.t.} + (\tilde{a}+\tilde{b}^2)\langle\bar{u}\gamma_5\gamma u\bar{s}\gamma_5\gamma s\rangle_{\rm s.t.} - \tilde{a}\tilde{b}\langle\bar{u}\sigma u\bar{s}\sigma s\rangle_{\rm s.t.}\Bigg].$$

Borel transformed OPE

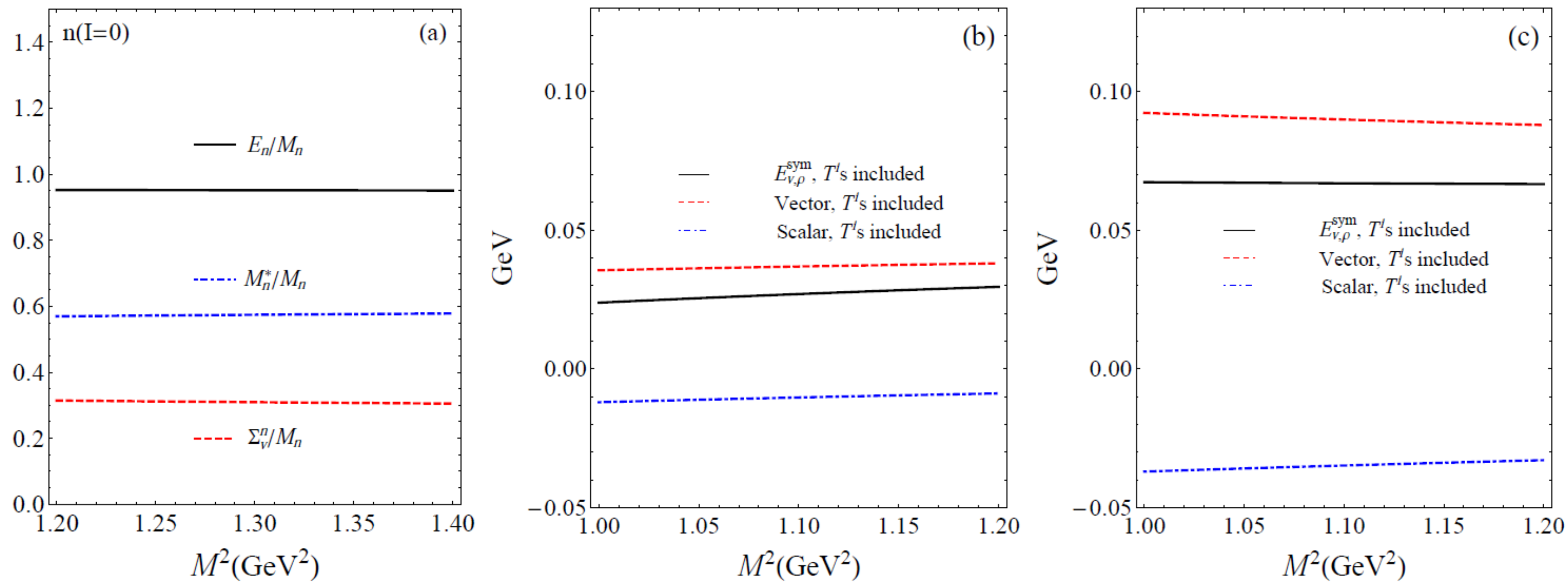
- **Λ** hyperon OPE with **Generalized interpolating field** (continued)

$$\overline{\mathcal{W}}_M[\Pi_{\Lambda,q}(q_0^2,|\vec{q}|)] = \lambda_\Lambda^{*2} e^{-(E_{\Lambda,q}^2-\vec{q}^2)/M^2} = \mathcal{B}[\Pi_{\Lambda,q}^e(q_0^2,|\vec{q}|)] - \bar{E}_{\Lambda,q}\mathcal{B}[\Pi_{\Lambda,q}^o(q_0^2,|\vec{q}|)]$$
$$= \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{256\pi^4}(M^2)^3 E_2 L^{-\frac{4}{9}} + \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{256\pi^2} M^2 \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{\rho,I} E_0 L^{-\frac{4}{9}}$$
$$- \frac{\tilde{a}\tilde{b}}{4\pi^2} m_s M^2 \langle \bar{q}q \rangle_{\rho,I} E_0 L^{-\frac{4}{9}} + \frac{(1+\tilde{a}^2+4\tilde{b}^2)}{32\pi^2} m_s M^2 \langle \bar{s}s \rangle_{\rho,I} E_0 L^{-\frac{4}{9}}$$
$$- \frac{(1-\tilde{a}^2+2\tilde{b}^2)}{4}\langle \bar{u}u\bar{d}d\rangle_{\text{tr.}} - \frac{(1-\tilde{a}^2-2\tilde{b}^2)}{4}\langle \bar{u}\gamma_5 u\bar{d}\gamma_5 d\rangle_{\text{tr.}} + \frac{(1+\tilde{a}^2-\tilde{b}^2)}{4}\langle \bar{u}\gamma u\bar{d}\gamma d\rangle_{\text{tr.}}$$
$$+ \frac{(1+\tilde{a}^2+\tilde{b}^2)}{4}\langle \bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\text{tr.}} + \frac{(1-\tilde{a}^2)}{8}\langle \bar{u}\sigma u\bar{d}\sigma d\rangle_{\text{tr.}} + \tilde{a}\tilde{b}\langle \bar{q}q\bar{s}s\rangle_{\text{tr.}} + \tilde{b}\langle \bar{q}\gamma_5 q\bar{s}\gamma_5 s\rangle_{\text{tr.}}$$
$$+ \frac{(1+\tilde{a}^2-10\tilde{b}^2)}{8}\langle \bar{q}\gamma q\bar{s}\gamma s\rangle_{\text{tr.}} + \frac{(\tilde{a}-3\tilde{b}^2)}{4}\langle \bar{q}\gamma_5\gamma q\bar{s}\gamma_5\gamma s\rangle_{\text{tr.}} - \frac{\tilde{a}\tilde{b}}{4}\langle \bar{q}\sigma q\bar{s}\sigma s\rangle_{\text{tr.}}$$
$$- \frac{\tilde{b}^2}{4}\langle \bar{u}\gamma u\bar{d}\gamma d\rangle_{\text{s.t.}} + \frac{\tilde{b}^2}{4}\langle \bar{u}\gamma_5\gamma u\bar{d}\gamma_5\gamma d\rangle_{\text{s.t.}} - \frac{\tilde{b}^2}{4}\langle \bar{u}\sigma u\bar{d}\sigma d\rangle_{\text{s.t.}}$$
$$- \frac{(1+\tilde{a}^2-2\tilde{b}^2)}{8}\langle \bar{q}\gamma q\bar{s}\gamma s\rangle_{\text{s.t.}} + \frac{(\tilde{a}+\tilde{b}^2)}{4}\frac{1}{q^2}\langle \bar{q}\gamma_5\gamma q\bar{s}\gamma_5\gamma s\rangle_{\text{s.t.}} + \frac{\tilde{a}\tilde{b}}{4}\langle \bar{q}\sigma q\bar{s}\sigma\gamma s\rangle_{\text{s.t.}}$$
$$+ \bar{E}_{\Lambda,q}\left[\frac{(1+\tilde{a}^2+2\tilde{b}^2)}{24\pi^2} M^2 \langle q^\dagger q\rangle_{\rho,I} E_0\right],$$

Sum rule result I - nucleons

- **Neutron** sum rules and symmetry energy (at $\rho=\rho_0$)

Neutron sum rules (***I=0***) | Sym. energy without twist-4 ops. | Sym. energy with twist-4 ops.

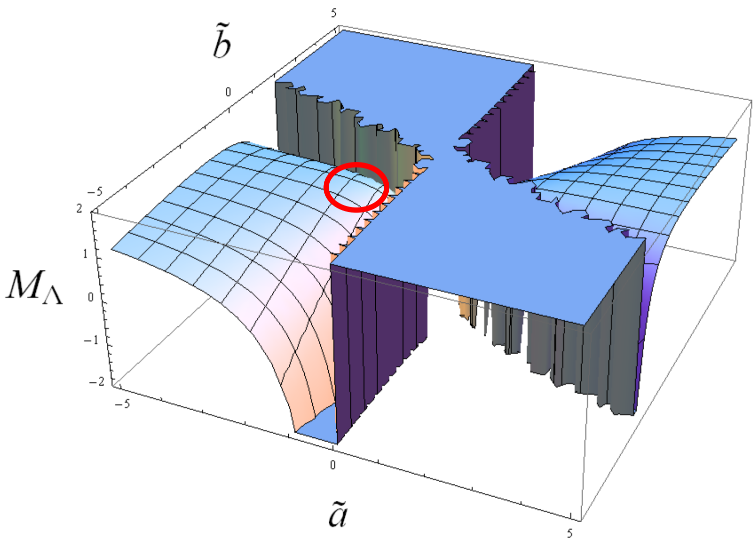


1. The quasi-**neutron** energy is slightly reduced → represents bounding at $\rho=\rho_0$
2. Large cancelation mechanism in both of the quasi-neutron and the symmetry energy
3. Twist-4 matrix elements enhance the strength of cancelation mechanism
   → simple linear gas approximation would not be good choice

Determination of $\{\tilde{a},\tilde{b}\}$ in Λ sum rules

- 3D plot with $\tilde{a}$ and $\tilde{b}$

Vacuum sum rules with $\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + \tilde{a}[u_a^T C\gamma_5 d_b]s_c + \tilde{b}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$



1. Self-energies will be obtained by taking ratio
   $M_\Lambda = \bar{\mathcal{B}}[\Pi_{\Lambda,s}(q_0^2,|\vec{q}|)]/\bar{\mathcal{B}}[\Pi_{\Lambda,q}(q_0^2,|\vec{q}|)]$
   $\Sigma_v^\Lambda = \bar{\mathcal{B}}[\Pi_{\Lambda,v}(q_0^2,|\vec{q}|)]/\bar{\mathcal{B}}[\Pi_{\Lambda,q}(q_0^2,|\vec{q}|)]$
2. Overall normalization A becomes meaningless in practical calculation → free parameter reduces to $\tilde{a}$ and $\tilde{b}$
3. Plane $\{\tilde{a},\tilde{b}\}$ has stable/unstable region

- **Ioffe's choice** corresponds to $\{\tilde{a},\tilde{b}\} = \{-1,-1/2\}$ and $A_{(-1,-1/2)} = \sqrt{8/3}$

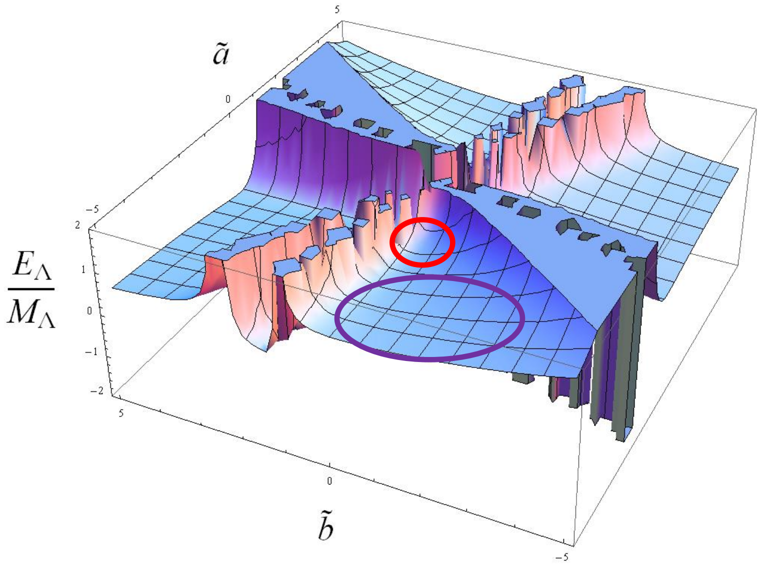
$$\eta_{\Lambda(-1,-1/2)} \Rightarrow \sqrt{\frac{2}{3}}\epsilon_{abc}\left([u_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu d_c - [d_a^T C\gamma_\mu s_b]\gamma_5\gamma^\mu u_c\right)$$

- This linear combination is located on boundary of stable region
  → mass can be drastically changed via even small variation of $\{\tilde{a},\tilde{b}\}$

Determination of $\{\tilde{a},\tilde{b}\}$ in Λ sum rules

- 3D plot with $\tilde{a}$ and $\tilde{b}$

In-medium sum rules with $\eta_{\Lambda(\tilde{a},\tilde{b})} = A_{(\tilde{a},\tilde{b})}\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + \tilde{a}[u_a^T C\gamma_5 d_b]s_c + \tilde{b}[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$



1. The OPE **does not contain** the derivative expansion and $\bar{s}\gamma s$ dependent Ops.
2. **Ioffe's choice** is located on unstable point and the quasi-Λ energy/M_Λ ~ 1.5
3. To control the repulsive tendency of the quasi-Λ energy, one can try derivative expansion

$$\bar{s}\gamma_\mu s\bar{q}q = (\bar{s}\gamma_\mu s\bar{q}q) + x^\nu\,(\bar{s}\gamma_\mu D_\nu s\bar{q}q)$$

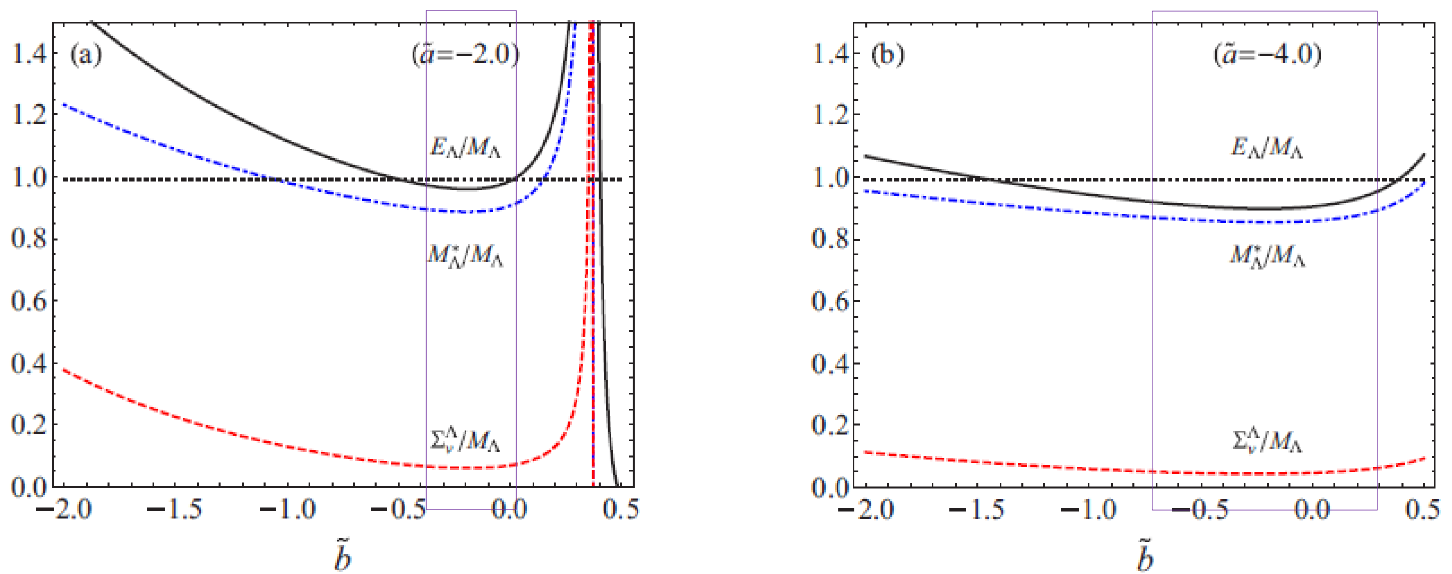
$$\langle\bar{s}\gamma_\mu D_\nu s\bar{q}q\rangle = \underline{\frac{1}{4}g_{\mu\nu}m_s\langle\bar{s}s\bar{q}q\rangle} + \frac{4}{3}\left(\langle\bar{s}\gamma_0 D_0 s\bar{q}q\rangle - \frac{1}{4}m_s\langle\bar{s}s\bar{q}q\rangle\right)\left(u_\mu u_\nu - \frac{1}{4}g_{\mu\nu}\right)$$

→ **Trace part can reduce** the quasi-Λ energy but contains large uncertainty
→ It is worthwhile to **try new linear combination** with $\tilde{a}$ and $\tilde{b}$

Stable $\{\tilde{a},\tilde{b}\}$ for **Λ** sum rules

- Confining stable points on $\{\tilde{a}, \tilde{b}\}$ plain

Cross section with fixed $\tilde{a} = -2.0$ and $\tilde{a} = -4$



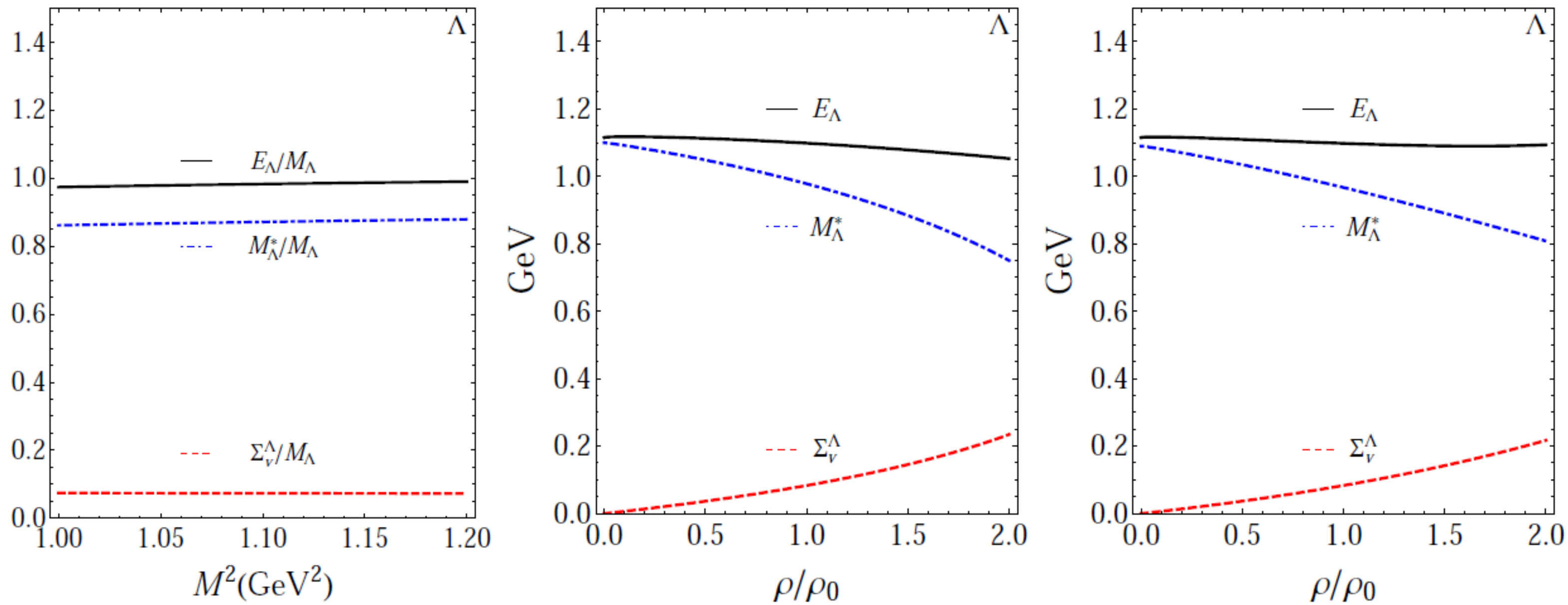
Sum rules have been obtained by averaging results on following 9 points:
$\{\tilde{a},\tilde{b}\}$ = {(-1.80,-0.10), (-1.80,-0.15), (-1.80,-0.30), (-2.00,-0.10), (-2.00,-0.20), (-2.00,-0.30), (-2.20,-0.10), (-2.20,-0.30), (-2.20,-0.50)}

As $|\tilde{a}|$ becomes large, sum rules become **stable** and weakly dependent on $\tilde{b}$
→ enhanced **scalar diquark basis** gives stable and reasonable sum rules

Sum rule result II – Λ hyperon

- Λ sum rules with new interpolating field

Λ sum rules | Constant negative pole | Density dependent negative pole

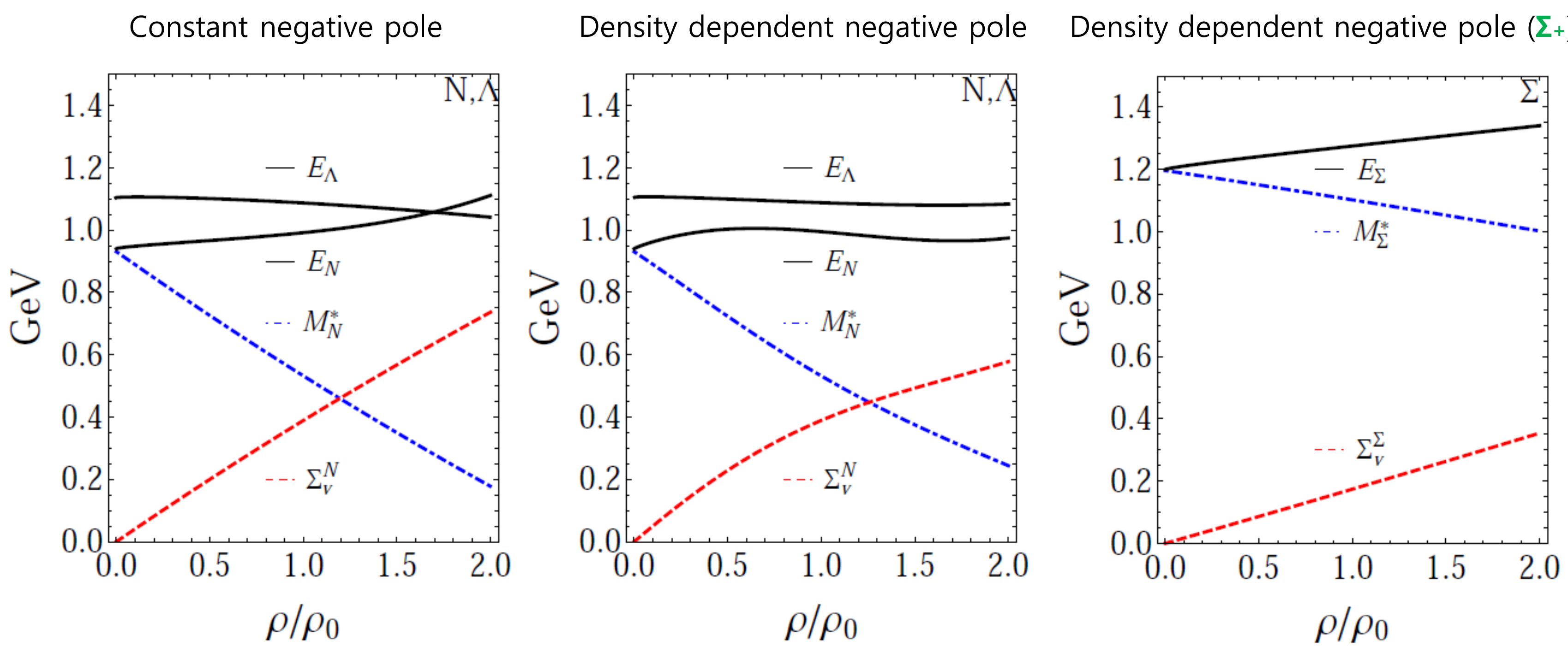


1. The quasi-Λ energy is slightly reduced → represents bounding at normal nuclear density
2. Weak attraction and weak repulsion → **scalar**: $V_{s\Lambda}$ / V_{sN} ~ 0.31 **vector**: $V_{v\Lambda}$ / V_{vN} ~ 0.26
   → naïve quark # counting for determination of N-H force strength may not be good
3. Constant negative anti-Λ pole case (2nd graph) and density dependent case (3rd graph)

$$\bar{E}_q = \Sigma_v(\bar{E}_q) - \sqrt{\vec{q}^2 + M^*(\bar{E}_q)^2}$$ (anti-Λ pole)

Sum rule result III – density behavior

- Comparison of density behavior (neutron matter)



1. Constant negative pole case: the quasi energy of **Λ** and **neutron** crosses at $\rho/\rho_0 = 1.8$
2. Density dependent case: never crosses
3. In **Σ+** sum rules, there is only small difference between constant- and density dependent-case
4. **Within new interpolating field for Λ, the early onset of the hyperon in the dense nuclear matter is unlikely**

Outline

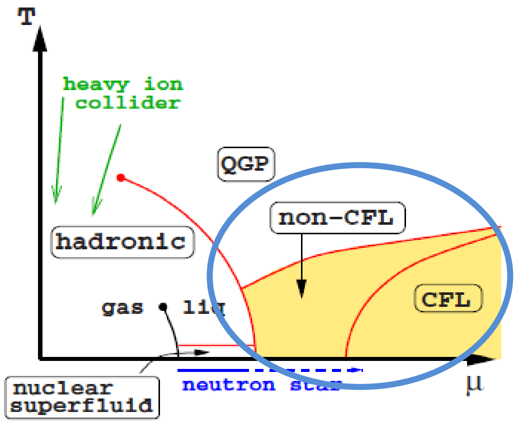
I. Motivations – phenomenology within iso-spin asymmetry

II. QCD approaches – Diquark and QCD sum rules
- Symmetry energy
- Nucleon and hyperons

III. Extreme limits and Future prospects

At extremely high density?

- QCD phase transition



$E \sim \mu$ — Hard scale (separation point between slow-fast modes)

$E \sim g\mu$ — Soft scale for screening and damping

$E \sim \mu \exp(-1/g)$ — Gap scale for color superconductivity

$E \sim \mu \exp(-1/g^2)$ — Non-Fermi liquid effect

Fermi surface $p_F = \mu > \Lambda_{QCD}$

In $q \geq \mu > \Lambda_{QCD}$ region, **QCD** can be directly applicable

Static quantities can be obtained from partition function for dense QCD

$$\mathcal{Z}_\Omega = \mathrm{Tr} \exp\left[-\beta(\hat{H} - \vec{\mu}\cdot\hat{\vec{N}})\right] = \int \mathcal{D}(\bar{\psi}, \psi, A, \eta) \exp\left[-\int_0^\beta d\tau \int d^3x \mathcal{L}_E(\bar{\psi}, \psi, A, \eta)\right]$$

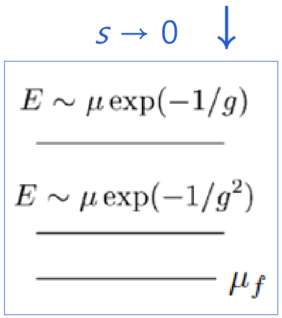
Dense QCD Lagrangian (Euclidean)

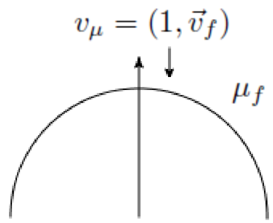
$$\mathcal{L}_E = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 + \bar{\eta}^a(\partial^2\delta_{ab} + g f_{abc}\partial_\mu A^c_\mu)\eta^b + \sum_f^{n_f}\left[\psi^\dagger_f \partial_\tau \psi_f + \bar{\psi}_f(-i\gamma^i\partial_i + m_f)\psi_f - \mu_f \psi^\dagger_f \psi_f - g\bar{\psi}_f A\!\!\!/ \psi_f\right]$$

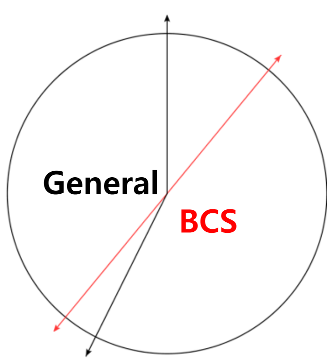
Full QCD and effective approach within scale hierarchy

Extremely low temperature

- **At T~0 limit,** quark is mainly confined near Fermi sea







If one scales longitudinal momentum to near Fermi surface

$$\int d^4p \to \mu_f^2 \int d\Omega \int dl^2 s^2 \quad \text{where} \quad l = (l_0, (\vec{l} \cdot \vec{v}_f)\vec{v}_f)$$

Free fermion part should be invariant under scaling

$$\int d^2 l s^2 \psi^\dagger_{\vec{v}_f} s(l_0 - l_\parallel)\psi_{\vec{v}_f} \quad \to \quad \psi \sim s^{-\frac{3}{2}}$$

Four-quark interaction

General scattering

$$\int \Pi_i^4 \left(dk_\perp^2 dl^2\right)_i \left[\psi^\dagger(k_3)\psi(k_1)V(k)\psi^\dagger(k_4)\psi(k_2)\right] \delta(k_1 + k_2 - (k_3 + k_4))$$

scales as s^2 : irrelevant in $s \to 0$ scaling

Interaction **between opposite velocity (BCS type)**

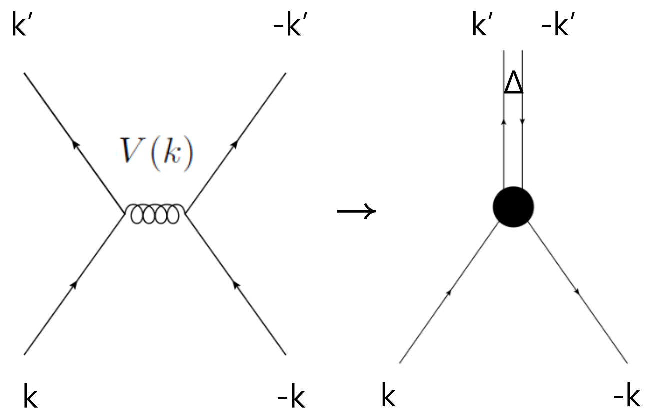
$$\int \Pi_i^4 \left(dk_\perp^2 dl^2\right)_i \left[\psi^\dagger(k_3)\psi(k_1)V(k)\psi^\dagger(-k_3)\psi(-k_1)\right] \delta(l_1 + l_2 - (l_3 + l_4))$$

scales as s^0: **marginal** in $s \to 0$ scaling

In QCD, there is no relevant interaction which scales as s^{-n}
→ **BCS** type interaction becomes most important at scaling

Color BCS paired states

- **4 quark interaction in QCD** (N_c=3)



Anti-triplet channel is attractive (V<0)

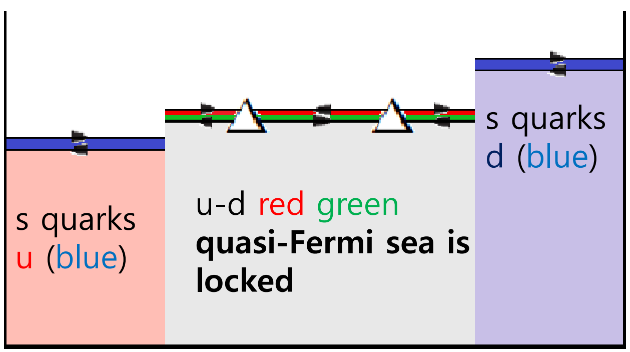
$$\tau^a_{ij}\tau^a_{kl} = \frac{1}{6}(\delta_{ij}\delta_{kl}+\delta_{il}\delta_{kj}) - \frac{1}{3}(\delta_{ij}\delta_{kl}-\delta_{il}\delta_{kj})$$

→ BCS condensation in low energy limit

To take entire Fermi surface, spin-0 condensate is favored → in same helicity (asymmetric in spin)

For asymmetric wave function as for fermion, flavor should be in asymmetric configuration

- Modification of Fermi-sea



In non negligible M_s^2/μ , **2SC** state is favored

$$\langle \psi^\alpha_a C\gamma_5 \psi^\beta_b\rangle \sim \Delta_1\epsilon^{\alpha\beta1}\epsilon_{ab1}+\Delta_2\epsilon^{\alpha\beta2}\epsilon_{ab2}+\Delta_3\epsilon^{\alpha\beta3}\epsilon_{ab3}$$

$$\langle\psi^T_{L,\alpha i}C\psi_{L,\beta j}\rangle = -\langle\psi^T_{R,\alpha i}C\psi_{R,\beta j}\rangle = \frac{\Delta}{2}\epsilon_{\alpha\beta3}\epsilon_{ij3}$$

$$\mathcal{L}_\Delta = -\frac{\Delta}{2}\psi^T_L C\epsilon\psi_L\epsilon - (L\to R) + \text{h.c.}$$

In 2SC phase, u-d red-green states are trapped in gap and only s quarks and u-d blue quarks can be liberated

Naïve future plan – diquark

- Diquark paring pattern

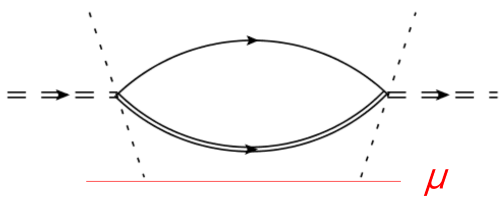
I. In **Λ** structure, scalar ($I=0$) light diquark structure should be emphasized

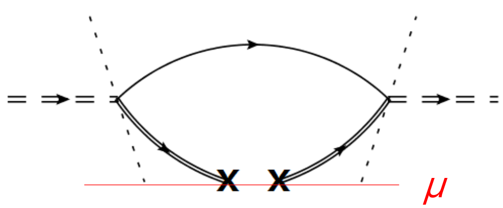
$$\eta_\Lambda = \epsilon_{abc}\left([u_a^T C d_b]\gamma_5 s_c + (\tilde{a} \leq -2)[u_a^T C\gamma_5 d_b]s_c + (\tilde{b} \sim -1/8)[u_a^T C\gamma_5\gamma_\mu d_b]\gamma^\mu s_c\right)$$

II. 2SC BCS paring at cold dense matter

$$\langle \psi^T_{L,\alpha i} C\psi_{L,\beta j}\rangle = -\langle \psi^T_{R,\alpha i} C\psi_{R,\beta j}\rangle = \frac{\Delta}{2}\epsilon_{\alpha\beta 3}\epsilon_{ij3}$$

- The diquark structure corresponds to the light 4q ops.

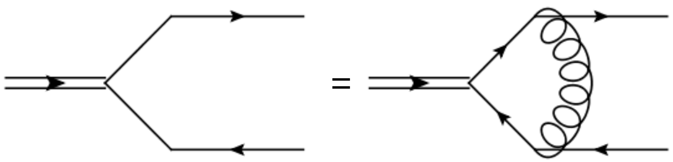




- Just above QCD scale ($q > \mu \sim \Lambda_{QCD}$), diquark may be weakly bounded state in perturbative interaction
- But below the scale ($q < \mu \sim \Lambda_{QCD}$), diquark contribution may mainly overlapped with four-quark condensates
- $\langle \bar{q}_{1a'}\Gamma q_{1a}\bar{q}_{2b'}\Gamma q_{2b}\rangle \simeq \langle \bar{q}_{1a'}\tilde{\Gamma}\bar{q}_{2b'}q_{1a}\tilde{\Gamma}q_{2b}\rangle$
- Scalar/Twist-4 condensates correspond to \`good/bad' diquark (s=0/s=1) correlation
- GPD analysis for internal structure of baryons can contribute to dense matter physics

Naïve future plan – similarities

- Dense matter and heavy quark system



Singlet state can be obtained by solving Bethe-Salpeter equation

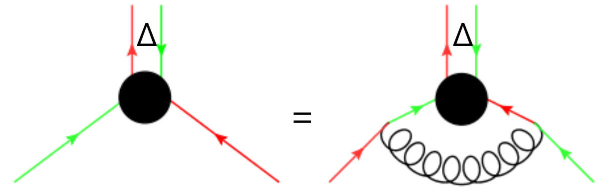
$$\Gamma_\mu(p_1,-p_2) = iC_{\text{color}} \int \frac{d^4k}{(2\pi)^4} g^2 V(k)\gamma^\nu \Delta(p_1+k) \times \Gamma_\mu(p_1+k,-p_2+k)\Delta(-p_2+k)\gamma_\nu$$

In non-relativistic and heavy mass limit

$$\Gamma_\mu(q/2+p,-q/2+p) = -\left(\varepsilon - \frac{\mathbf{p}^2}{m}\right)\sqrt{\frac{M_\Phi}{N_c}}\psi(\mathbf{p})\frac{1+\gamma_0}{2}\gamma_i\delta_{\mu i}\frac{1-\gamma_0}{2}$$

Coulombic bound state

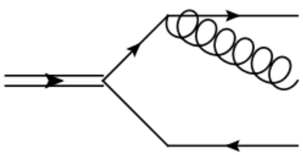
$$\left(\varepsilon - \frac{\mathbf{p}^2}{m}\right)\psi(\mathbf{p}) = -g^2 C_{\text{color}} \int \frac{d^3k}{(2\pi)^3} V(\mathbf{k})\psi(\mathbf{p}+\mathbf{k})$$



Large gap size can be obtained by solving gap equation with one gluon exchange

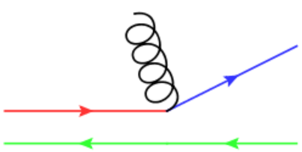
$$\Delta(k) = ig^2 \int \frac{d^4q}{(2\pi)^4}\left(\gamma_\mu \frac{\lambda^a}{2}\right)^T S_{21}(q)\left(\gamma_\nu \frac{\lambda^a}{2}\right) D_{\mu\nu}(q-k)$$

$$\Delta/\mu = (b/g^5)\exp\left(-3\pi^2/\sqrt{2}g\right) \qquad b = 256\pi^4$$



Singlet → Octet

adj. 4,5,6,7



Gap → Ungapped

External gluon attachment can dissolve the bound state

For color BCS state, Meissner mass screens dissociation of gapped state → requires large momentum transf.

Needs more improvement

- Diquark in nucleon
  - Cross between two bases does not have perturbative contrbution

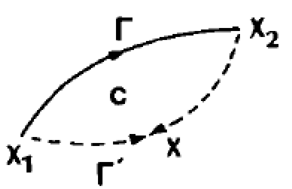
$$\eta_{p(t)} = 2\epsilon_{abc}\left([u_a^T C d_b]\gamma_5 u_c + t[u_a^T C\gamma_5 d_b]u_c\right)$$
$$= \left(\frac{1-t}{2}\right)\epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c + \left(\frac{1+t}{4}\right)\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c$$

(a) $\epsilon_{abc}[u_a^T C\gamma_\mu u_b]\gamma_5\gamma^\mu d_c = 4\epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]u_{L,c} - [u_{L,a}^T C d_{L,b}]u_{R,c}\right)$

(b) $\epsilon_{abc}[u_a^T C\sigma_{\mu\nu} u_b]\gamma_5\sigma^{\mu\nu} d_c = 4\epsilon_{abc}\left([u_{R,a}^T C d_{R,b}]u_{R,c} - [u_{L,a}^T C d_{L,b}]u_{L,c}\right)$

  - However chiral symmetry breaking effect appears in 2quark and 4quark condensates
    → Ioffe's choices may not the only option

- Non local nature is missing
  - The interpolating fields are defined as local operator
  - Even in the point like limit, the gauge connection effect should be concerned



$$O^{(a)}_{\mu_1\cdots\mu_n}(x, C) = \bar{q}(x)\gamma_{\mu_1}\vec{D}_{\mu_2}\cdots\vec{D}_{\mu_n}U(x, x; C)q(x)$$

All the local divergence occurs at two endpoints, does not depend on the path
→ x1-x2=0 limit gives Wilson loop phase factor

  - GPD studies may provide experimental constraints