# **DVCS and DVMP off Scalar Targets**

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### "Nucleon and Resonance Structure with Hard Exclusive Processes"

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# Outline

- JLab Kinematics (t <  $-|t_{min}| \neq 0$ )
- Original Formulation of DVCS with GPDs (Twist 2)
- Benchmark Barebone Calculation for JLab Kinematics (Exact Result vs. Reduced Result)
- Toward finding the Most General Hadronic Tensor Structure with CFFs (DNA method)
- Conclusion and Outlook

### JLab Kinematics t < $-|t_{min}| \neq 0$





$$t = \Delta^2 = -\frac{\varsigma^2 M^2 + \Delta_{\perp}^2}{1 - \varsigma} \quad ; \ \Delta^+ (\equiv \Delta^0 + \Delta^3) = -\varsigma \ p^+ \ ; \ \Delta_{\perp}^2 > \Delta_{\perp \min}^2 \neq 0$$

#### **Coincidence Experiment**



Figure 1.11: E00-110 schematic setup showing the three different detectors used to measured each of the particles in the final state. Carlos Muñoz CamachoThesis('05)

In[277]:=

Flatten[{{{"Q^2", "xBj", "k", "k'", "the", "thq", "thq'", "thp'", "q'", "t/Q^2"}}, Table[Block[{M = 0.938, Q = Sqrt[pars[[i, 1]]], xBj = pars[[i, 2]], Eb = pars[[i, 3]], the = pars[[i, 4]] Pi / 180, thetaC = 20 Pi / 180, thq = ArcCos[costhetaqT2]}, {Q^2, xBj, Eb, PeT1, the 180 / Pi, thq 180 / Pi, ArcCos[costhqf] 180 / Pi, ArcCos[ costhpf] 180 / Pi, qfT3mu[[1]], MandeltT2 / Q^2}], {i, 1, 12}}, 1] // MatrixForm

Out[277]//MatrixForm=

Q^2	хВј	k	k '	the	thq	thq'	thp'	q'	t/Q^2
1.9	0.36	5.75	2.93669	19.3	18.0503	11.9997	69.6549	2.65582	-0.1555
3.	0.36	6.6	2.15792	26.5	11.6529	6.69292	66.0322	4.23202	-0.131356
4.	0.36	8.8	2.87723	22.9	10.3184	5.96439	64.4845	5.66645	-0.120212
4.55	0.36	11.	4.26285	17.9	10.6859	6.58293	64.1816	6.45567	-0.116055
3.1	0.5	6.6	3.2951	22.5	19.5262	14.4829	60.4087	3.0229	-0.170658
4.8	0.5	8.8	3.68273	22.2	14.4748	10.3331	57.1229	4.76891	-0.13615
6.3	0.5	11.	4.28358	21.1	12.4174	8.76649	55.3539	6.31422	-0.119765
7.2	0.5	11.	3.32409	25.6	10.1755	6.74662	53.737	7.24243	-0.112945
5.1	0.6	8.8	4.26908	21.2	17.7604	13.7928	51.4698	4.06193	-0.172513
6.	0.6	8.8	3.46951	25.6	14.8072	11.1302	49.5692	4.82846	-0.156969
7.7	0.6	11.	4.1592	23.6	13.0416	9.77439	48.0193	6.28128	-0.136317
9.	0.6	11.	3.00426	30.2	10.1946	7.16227	46.0515	7.39496	0.125229

Table III in E12 - 06 - 114, Julie Roche et al. Jlab 12 GeV Exclusive Kinematics

## Nucleon GPDs in DVCS Amplitude

X.Ji,PRL78,610(1997): Eqs.(14) and (15)

$$\begin{split} \overline{p^{\mu} = \Lambda(\stackrel{c}{1}, \stackrel{x}{0}, \stackrel{y}{0}, \stackrel{z}{1})}, \\ n^{\mu} &= (\stackrel{c}{1}, \stackrel{x}{0}, \stackrel{y}{0}, \stackrel{z}{-1})/(2\Lambda) , \\ \overline{P^{\mu}} &= \frac{1}{2}(P+P')^{\mu} = p^{\mu} + \frac{M^{2} - \Delta^{2}/4}{2}n^{\mu}, \\ q^{\mu} &= -\xi p^{\mu} + \frac{Q^{2}}{2\xi}n^{\mu} , \\ \xi &= \frac{Q^{2}}{2\overline{P} \cdot q} , \\ \Delta^{\mu} &= -\xi \left[p^{\mu} - \frac{M^{2} - \Delta^{2}/4}{2}n^{\mu}\right] + \Delta^{\mu}_{\perp} . \end{split} \\ \begin{array}{l} T^{\mu\nu}(p,q,\Delta) &= -\frac{1}{2}(p^{\mu}n^{\nu} + p^{\nu}n^{\mu} - g^{\mu\nu}) \int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} + \frac{1}{x + \frac{\xi}{2} - i\varepsilon}\right) \\ &\times \left[H(x,\Delta^{2},\xi)\overline{U}(P')/\mu U(P) + E(x,\Delta^{2},\xi)\overline{U}(P') \frac{i\sigma^{\alpha\beta}n_{\alpha}\Delta_{\beta}}{2M}U(P)\right] \\ &- \frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}p_{\alpha}n_{\beta} \int_{-1}^{+1} dx \left(\frac{1}{x - \frac{\xi}{2} + i\varepsilon} - \frac{1}{x + \frac{\xi}{2} - i\varepsilon}\right) \\ &\times \left[\tilde{H}(x,\Delta^{2},\xi)\overline{U}(P')/\mu\gamma_{5}U(P) + \tilde{E}(x,\Delta^{2},\xi)\frac{\Delta \cdot n}{2M}\overline{U}(P')\gamma_{5}U(P)\right] \end{split}$$

Just above Eq.(14), ``To calculate the scattering amplitude, it is convenient to define <u>a special system of coordinates</u>."

Note here that 
$${q'}^2 = -\Delta_{\perp}^2 = 0$$
.

## Nucleon GPDs in DVCS Amplitude

A.V.Radyushkin, PRD56, 5524 (1997): Eq.(7.1)

At the beginning of Section 2E (Nonforward distributions), ``Writing the momentum of the virtual photon as  $q=q' - \zeta p$  is equivalent to using the Sudakov decomposition in the light-cone `plus' (p) and `minus' (q') components in a situation when there is no transverse momentum ."

Note that here 
$$(q-q')^2 = \Delta^2 = t = \zeta^2 M^2 > 0$$
  
while  $t < 0$  in DVCS.

### **Benchmark Calculation in JLab Kinematics**

 To see the effect of taking t<0, we mimic the kinematics at JLab and compute bare bone VCS amplitudes neglecting masses.

$$k^{\mu} = (xp^{+}, 0, 0, 0), \quad k'^{\mu} = \left( (x - \zeta_{eff}) p^{+}, \Delta_{\perp}, \frac{\Delta_{\perp}^{2}}{2(x - \zeta_{eff})p^{+}} \right)$$
$$q^{\mu} = \left( -\zeta p^{+}, 0, 0, \frac{Q^{2}}{2\zeta p^{+}} \right), \quad q'^{\mu} = \left( \alpha \frac{\Delta_{\perp}^{2}}{Q^{2}} p^{+}, -\Delta_{\perp}, \frac{Q^{2}}{2\alpha p^{+}} \right)$$
$$Here, \text{ for } Q \to \infty,$$
$$\zeta_{eff} = \zeta + \alpha \frac{\Delta_{\perp}^{2}}{Q^{2}} \to \zeta$$
$$\alpha = \frac{(x - \zeta)Q^{2}}{2\Delta_{\perp}^{2}} \left( 1 - \sqrt{1 - \frac{4\zeta}{x - \zeta}} \frac{\Delta_{\perp}^{2}}{Q^{2}} \right) \to \zeta$$
$$q'^{-} \to q^{-} = \frac{Q^{2}}{2\zeta p^{+}}$$

#### "Bare Bone" VCS Operators & Amplitudes



$$egin{aligned} &\mathcal{H}(\{s',s\}\{h',h\}) &= &ar{u}(k';s')(\mathcal{O}_s+\mathcal{O}_u)u(k;s)\ &\mathcal{L}(\{\lambda',\lambda\}h) &= &ar{u}(\ell';\lambda'){
otives}^*(q;h)u(\ell;\lambda)\ &\mathcal{M}=\sum_h \mathcal{L}(\{\lambda',\lambda\}h)rac{1}{q^2}\mathcal{H}(\{s',s\}\{h',h\}) \end{aligned}$$

Usin  
and  

$$\underbrace{k-\Delta}_{k-\Delta}_{v} = k'_{\gamma^{\alpha} + i\epsilon^{\mu\alpha\nu\beta}\gamma_{\beta}\gamma_{5}}$$
we compare the exact amplitude  

$$\underbrace{H(h_{q}^{\mu\nu}, h_{q'}^{=}, s_{k} = \sum_{k} [(\{(k^{+}_{*} + q^{+})n^{\mu}(+), q^{+}, q^{\oplus}(p^{\mu}_{q}), (n^{\nu}_{s}) + T_{U}^{q})]n^{\nu}(+) + \{(k^{+} + eq^{+})n^{\nu}(+) + q^{-}n^{\nu}(-) + q_{\perp}^{\nu}\}n^{\mu}(+) - g^{\mu\nu}q^{-}) \\ \times \bar{u}(k'; s') \not(-)u(k; s) - i\epsilon^{\mu\nu\alpha\beta}\{(k^{+} + q^{+})n_{\alpha}(+) + q^{-}n_{\alpha}(-) + q_{\perp}_{\alpha}\}n_{\beta}(+) \\ \times \bar{u}(k'; s') \not(-)\gamma_{5}u(k; s)]$$

and the reduced amplitude that agrees in the DVCS limit

$$T_{s}^{\mu\nu}\gamma^{\mu}\underline{\gamma}^{\alpha}\gamma^{\nu}\underline{q}^{=}[\overset{a}{s}[\overset{a}{n}\overset{\mu}{n}\overset{\gamma}{(-)}\overset{\mu}{n}\overset{g}{(+)}\overset{a}{+}\overset{\mu}{n}\overset{g}{(-)}\overset{\mu}{n}\overset{g}{(+)}\overset{\mu}{+}\overset{g}{(+)}\overset{\mu}{+}\overset{g}{(+)}\overset{\mu}{+}\overset{g}{(+)}\overset{g}{+}\overset{\mu}{+}\overset{g}{)}\overset{g}{}_{5}$$
$$\times \overline{u}(k';s')\not(-)u(k;s)$$
$$-i\epsilon^{\mu\nu\alpha\beta}n_{\alpha}(-)n_{\beta}(+)\times \overline{u}(k';s')\not(-)\gamma_{5}u(k;s)]$$

€

€

The tensor structure of the reduced amplitude is identical to the ones given by X. Ji and A.V. Radyushkin.

# Sanity Checks of Amplitudes

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



# **Checking Amplitudes**

- Gauge invariance of each and every polarized amplitude including the longitudinal polarization for the virtual photon.
- Klein-Nishina Formula in RCS.
- Angular Momentum Conservation.



## Comparison

Complete DVCS amplitudes,  $\sum_{h} \mathcal{L}(\{\lambda', \lambda\}, h) \frac{1}{q^2} \mathcal{H}(\{h', h\} \{s', s\})$  in three approaches, ours, A.V. Radyushkin, and X. Ji. Because the hadrons and leptons are massless,  $\lambda' = \lambda$  and s' = s.

$\lambda$	h' s	this work	AVR	LX
$\frac{1}{2}$	$1 \frac{1}{2}$	$rac{4}{Q}\sqrt{rac{x}{x-\zeta}}\left(1+rac{\zeta}{2(x-\zeta)}rac{\mathbf{\Delta}_{\perp}^{2}}{Q^{2}} ight)$	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_{\perp}^{2}}{Q^{2}}\right)$	0
$\frac{1}{2}$	$1 - \frac{1}{2}$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_{\perp}^{2}}{Q^{2}}\right)$	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_{\perp}^{2}}{Q^{2}}\right)$	0
<u>1</u> 2	$-1 \frac{1}{2}$	$-\frac{4}{Q^3}\frac{\zeta^2}{\sqrt{x(x-\zeta)}(x-\zeta)}\frac{\Delta_{\perp}^2}{Q^2}$	0	$\frac{4}{Q}\sqrt{\frac{x}{x-\zeta}}\left(1-\frac{\zeta}{2(x-\zeta)}\frac{\Delta_{\perp}^{2}}{Q^{2}}\right)$
<u>1</u> 2	$-1 - \frac{1}{2}$	0	0	$\frac{4}{Q}\sqrt{\frac{x-\zeta}{x}}\left(1+\frac{\zeta}{2(x-\zeta)}\frac{\Delta_{\perp}^{2}}{Q^{2}}\right)$

AVR=XJ, taking into account the real photon helicity swap for the exact collinear kinematics vs. the nonlinear kinematicsin LFD:

$$q'^{\mu} = \left( lpha \frac{\Delta_{\perp}^{2}}{Q^{2}} p^{+}, -\Delta_{\perp}, \frac{Q^{2}}{2\alpha p^{+}} 
ight) \leftrightarrow \left( 0, 0_{\perp}, \frac{Q^{2}}{2\zeta p^{+}} 
ight) + h' \leftrightarrow - h'$$

C.Carlson and C.Ji, Phys.Rev.D67,116002 (2003); B.Bakker and C.Ji, Phys.Rev.D83,091502(R) (2011).

### For any orders in Q

#### Exact

#### Reduced



$$D = \frac{4\zeta\Delta^2}{(x-\zeta)Q^2}, \quad D_{\pm} = 1 \pm \sqrt{1-D}$$

Here,  $\Delta = |\Delta_{\perp}|$ 



CFFs should be generalized for  $t = \Delta^2 \neq 0$  beyond the leading twist  $p^+$  $p^{\mu} = \Lambda(1, 0, 0)$  $q^{\mu} = -\xi p^{\mu} + \frac{Q^{2}}{2\xi} n^{\mu} , \quad \xi = \frac{Q^{2}}{2\overline{P} \cdot q} ,$  $\Delta^{\mu} = -\xi \left[ p^{\mu} - \frac{M^2 - \Delta^2 / 4}{2} n^{\mu} \right] + \Delta^{\mu}_{\perp} .$ 

#### Number of Independent Amplitudes in VCS



12 independent tensor structures

M.Perrottet, Lett. Nuovo Cim. 7, 915 (1973); R.Tarrach, Nuovo Cim. 28A, 409 (1975); D.Drechsel et al.,PRC57,941(1998); A.V.Belitsky, D.Mueller and A.Kirchner, NPB629, 323(2002); A.V.Belitsky and D.Mueller, PRD82, 074010(2010)

### **DNA Method**

$$\begin{split} d^{\mu\nu\alpha\beta} &= g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha} \\ G^{\mu\nu}(q'q) &= q'_{\alpha}d^{\mu\nu\alpha\beta}q_{\beta} = q' \cdot q \ g^{\mu\nu} - q^{\mu}q'^{\nu}, \\ G^{\mu\nu}(qq) &= q_{\alpha}d^{\mu\nu\alpha\beta}q_{\beta} = q^{2} \ g^{\mu\nu} - q^{\mu}q^{\nu}, \\ G^{\mu\nu}(q'q') &= q'_{\alpha}d^{\mu\nu\alpha\beta}q'_{\beta} = q'^{2} \ g^{\mu\nu} - q'^{\mu}q'^{\nu}, \\ G^{\mu\nu}(\overline{P}q) &= \overline{P}_{\alpha}d^{\mu\nu\alpha\beta}q_{\beta} = \overline{P} \cdot q \ g^{\mu\nu} - q^{\mu}\overline{P}^{\nu}, \\ G^{\mu\nu}(q'\overline{P}) &= q'_{\alpha}d^{\mu\nu\alpha\beta}\overline{P}_{\beta} = \overline{P} \cdot q' \ g^{\mu\nu} - \overline{P}^{\mu}q'^{\nu}. \end{split}$$

$$\tilde{T}^{\mu\nu}_{\text{DNA}} := \sum_{i=1}^{5} S_{i} \ \tilde{T}^{(i)\ \mu\nu}_{\text{DNA}} = S_{1} \ G^{\mu\nu}(q'q) \\ + S_{2} \ G^{\mu\lambda}(q'\overline{P}) \ G_{\lambda}^{\nu}(\overline{P}q) \\ + S_{3} \ G^{\mu\lambda}(q'\overline{P}) \ G_{\lambda}^{\nu}(\overline{P}q) \\ + S_{4} \ [G^{\mu\lambda}(q'\overline{P}) \ G_{\lambda}^{\nu}(\overline{P}q). \end{split}$$

Compton Form Factors (CFFs) :  $S_i$ , i = 1, 2, ..., 5

B.Bakker and C.Ji, Few-Body Syst. 58,1 (2017)

Most General Hadronic Tensor for Scalar Target

$$T^{\mu\nu} = G_{qq'}^{\mu\nu} S_1 + G_q^{\mu\lambda} G_{q'\lambda}^{\nu} S_2 + G_{q\overline{P}}^{\mu\lambda} G_{\overline{P}q'\lambda}^{\nu} S_3$$
$$+ (G_{q\overline{P}}^{\mu\lambda} G_{q'\lambda}^{\nu} + G_q^{\mu\lambda} G_{\overline{P}q'\lambda}^{\nu}) S_4 + G_q^{\mu\lambda} \overline{P}_{\lambda} \overline{P}_{\lambda'} G_{q'}^{\lambda'\nu} S_5$$
$$G_{qq'}^{\mu\nu} = g^{\mu\nu} q \cdot q' - q'^{\mu} q^{\nu}$$
$$G_a^{\mu\nu} = g^{\mu\nu} q^2 - q^{\mu} q^{\nu}$$

$$G_{q'}^{\mu\nu} = g^{\mu\nu}q'^2 - q'^{\mu}q'^{\nu}$$

$$G_{q\overline{P}}^{\mu\nu} = g^{\mu\nu}q \cdot \overline{P} - \overline{P}^{\mu}q^{\nu}$$

$$G_{\overline{P}q'}^{\mu\nu} = g^{\mu\nu}q' \cdot \overline{P} - q'^{\mu}\overline{P}^{\nu}$$

For  $q'^2 = 0$ , only  $S_{1,} S_2$  and  $S_4$  contribute.

#### Metz's approach $S_1 = -B_1, S_2 = B_3, S_3 = -B_2, S_4 = B_4, S_5 = B_{19}$

The method using the projectors introduces a kinematical singularity at  $q' \cdot q = 0$ . In Tarrach's paper a method is described to remove these kinematic poles. Here we give the final result of that algorithm as obtained in the thesis of Metz<sup>3</sup>. His CFFs are denoted as  $B_1$ ,  $B_2$ ,  $B_3$ ,  $B_4$ , and  $B_{19}$ . They are implicitly given in terms of the elementary tensor by the following equations:

$$\begin{split} M^{\mu\nu} &= B_1 M_1^{\mu\nu} + B_2 M_2^{\mu\nu} + B_3 M_3^{\mu\nu} + B_4 M_4^{\mu\nu} + B_{19} M_{19}^{\mu\nu}, \\ M_1^{\mu\nu} &= -q' \cdot q \, g^{\mu\nu} + q^{\mu} q'^{\nu}, \\ M_2^{\mu\nu} &= -(\bar{P} \cdot q)^2 \, g^{\mu\nu} - q' \cdot q \, \bar{P}^{\mu} \bar{P}^{\nu} + \bar{P} \cdot q \, (\bar{P}^{\mu} q'^{\nu} + q^{\mu} \bar{P}^{\nu}), \\ M_3^{\mu\nu} &= q'^2 q^2 \, g^{\mu\nu} + q' \cdot q \, q'^{\mu} q^{\nu} - q^2 \, q'^{\mu} q'^{\nu} - q'^2 \, q^{\mu} q^{\nu}, \\ M_4^{\mu\nu} &= \bar{P} \cdot q \, (q'^2 + q^2) \, g^{\mu\nu} - \bar{P} \cdot q \, (q'^{\mu} q'^{\nu} + q^{\mu} q^{\nu}) \\ &- q^2 \, \bar{P}^{\mu} q'^{\nu} - q'^2 \, q^{\mu} \bar{P}^{\nu} + q' \cdot q \, (\bar{P}^{\mu} q^{\nu} + q'^{\mu} \bar{P}^{\nu}), \\ M_{19}^{\mu\nu} &= (\bar{P} \cdot q)^2 \, q'^{\mu} q^{\nu} + q'^2 q^2 \, \bar{P}^{\mu} \bar{P}^{\nu} - \bar{P} \cdot q \, q^2 \, q'^{\mu} \bar{P}^{\nu} - \bar{P} \cdot q \, q'^2 \, \bar{P}^{\mu} q^{\nu}. \end{split}$$

<sup>3</sup>A. Metz, *Virtuelle Comptonstreuung un die Polarisierbarkeiten de Nukleons* (in German), PhD thesis, Universität mainz, 1997.

#### Gauge invariance requires more than handbag amplitudes







$$J_{S}^{\mu} = F_{S1} \left( (\Delta \cdot q) q^{\mu} - q^{2} \Delta^{\mu} \right)$$

$$\stackrel{e'}{+} F_{S2} \left( (\Delta \cdot q) (P_{2-x_{B}}^{\chi_{B}} q^{\mu}) - (q^{2} + P \cdot q) \Delta^{\mu} \right)$$

$$\stackrel{\gamma^{*}}{+} \frac{\gamma, \pi, \eta, \rho, \omega, K}{(1+\xi)P} \left( (1-\xi)P \right)$$

$$\stackrel{\gamma, \pi, \eta, \rho, \omega, K}{+} \frac{\varphi^{\mu\nu\alpha\beta}}{(1+\xi)P} P_{\nu} \Delta_{\alpha} q_{\beta}$$

$$H, E- \text{ unpolarized, } \tilde{H}, \tilde{E} - \text{ polarized GPD}$$
The ODDe Define Muchaer Structure

The GPDs Define Nucleon Structure

## **Conclusion and Outlook**

- Although the existing formulation meant already good progress, the realistic experimental setup requires the extension of the formalism to cover the broader kinematic regions of the DVCS experiments.
- The determination of most general hadron tensor structure is important not only for CFFs also for the discussion of GPDs.
- Maintaining EM gauge invariance is an important constraint.
- The DNA of the most general hadronic tensor structure for scalar target is found and applicable to DVCS and DVMP off <sup>4</sup>H<sub>e</sub>.