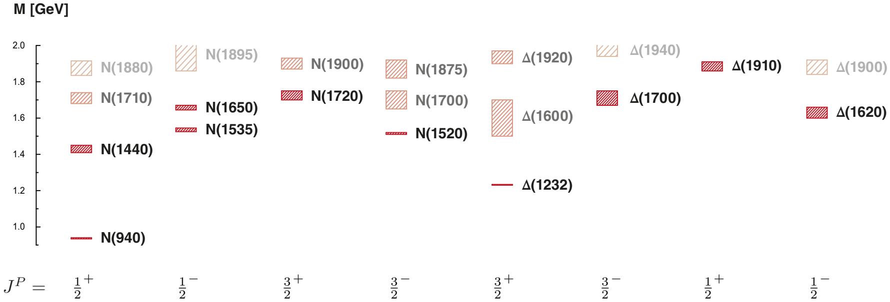
Microscopic approach to meson electroproduction

Gernot Eichmann
IST Lisboa, Portugal

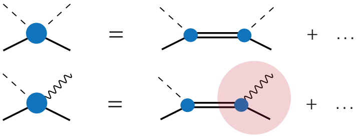
“Nucleon and Resonance Structure with Hard Exclusive Processes”

May 30, 2017
IPN Orsay, France

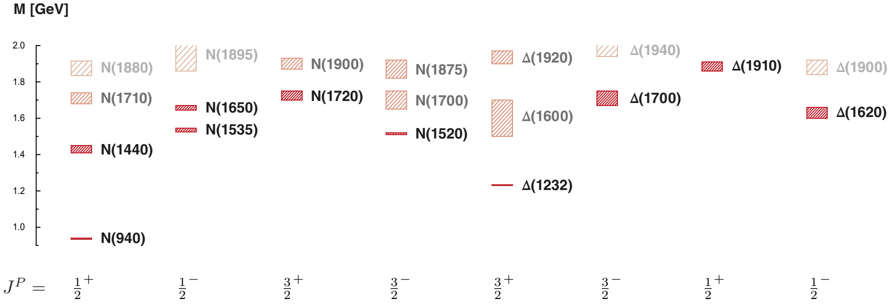
Light baryons



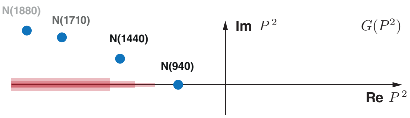
- Nature of **Roper** (level ordering)?
- Three-quark vs. **quark-diquark?**
- "Quark core" vs. meson-baryon **coupled channel effects?**
- **Hybrid baryons?**

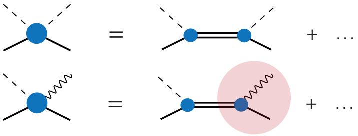


Light baryons



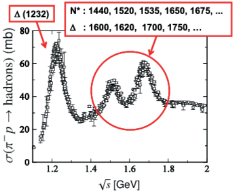
Resonances are **poles in complex plane:**





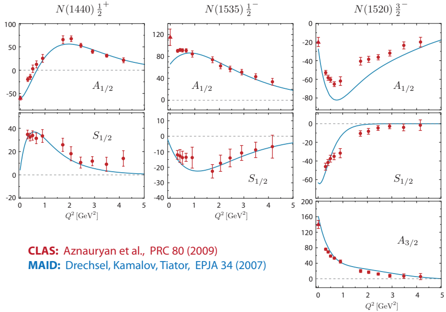
Light baryons

How to extract **transition FFs** from **cross sections?**



? →

H. Kamano,
PoS QNP2012



CLAS: Aznauryan et al., PRC 80 (2009)
MAID: Drechsel, Kamalov, Tiator, EPJA 34 (2007)

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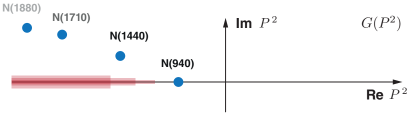
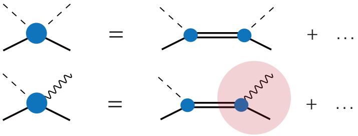
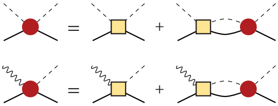




Reaction models

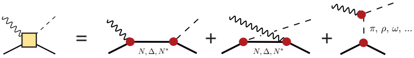
Hadronic coupled channel equations:

Sato-Lee/EBAC/ANL-Osaka, Dubna-Mainz-Taiwan, Valencia, Jülich-Bonn, GSI
JLab, MAID, SAID, KSU, Giessen, Bonn-Gatchina



$$\mathbf{T} = \mathbf{V} + \mathbf{V}\,\mathbf{G}\,\mathbf{T}\,, \qquad \mathbf{T} = \begin{pmatrix} T_{\pi\pi} & T_{\pi\gamma} \\ T_{\gamma\pi} & T_{\gamma\gamma} \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} V_{\pi\pi} & V_{\pi\gamma} \\ V_{\gamma\pi} & V_{\gamma\gamma} \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} G_{\pi} & 0 \\ 0 & G_{\gamma} \end{pmatrix}$$

Potentials from tree-level terms
of chiral effective Lagrangians:

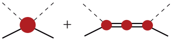


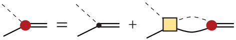
- **T matrix** vs. **K matrix:** simplify integral equations to algebraic relations

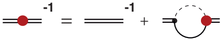
$$\mathbf{T} = \mathbf{V} + \mathbf{V}\,(\mathbf{G}_1 + \mathbf{G}_2)\,\mathbf{T}\,, \qquad \mathbf{K} = \mathbf{V} + \mathbf{V}\,\mathbf{G}_1\,\mathbf{K} \qquad \Rightarrow \qquad \mathbf{T} = \mathbf{K} + \mathbf{K}\,\mathbf{G}_2\,\mathbf{T}$$

- Split dressing effects: **"quark core"** vs. **meson-baryon effects**

V =


⇒ **T** =






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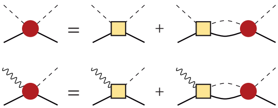

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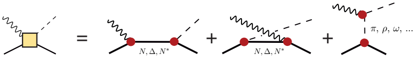
Potentials from tree-level terms of chiral effective Lagrangians:



- Poincaré invariance?
- Electromagnetic gauge invariance?
- Validity of effective hadronic Lagrangians?

  non-renormalizable ⇒ low-energy effective theory

  what is an "offshell hadron"?

  dynamical generation of resonances?

  microscopic effects?

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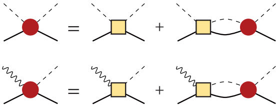

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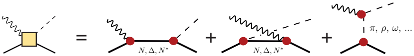

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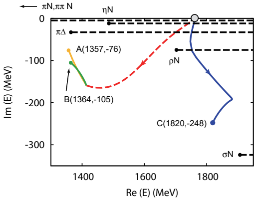

Suzuki et al., PRL 104 (2010)

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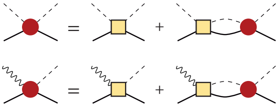

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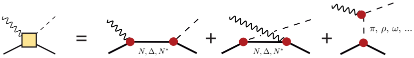

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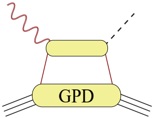
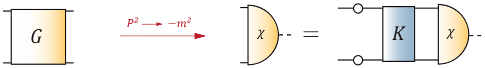
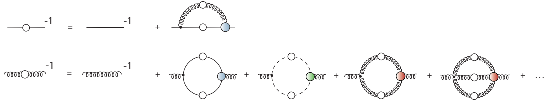


DSEs & BSEs

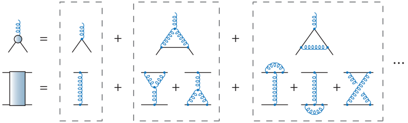
- Homogeneous **Bethe-Salpeter equation** for BS wave function:



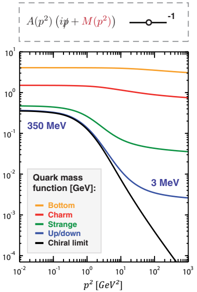
- Depends on QCD's n-point functions as input, satisfy **DSEs = quantum equations of motion**



- Kernel can be derived in accordance with **chiral symmetry:**



- **Quark propagator**

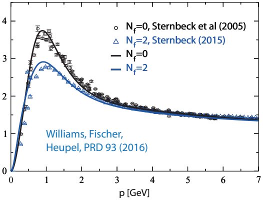


Dynamical chiral symmetry breaking generates 'constituent-quark masses'

DSEs & BSEs

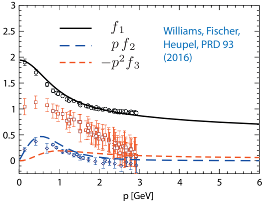
- **Gluon propagator**

$$\frac{D(p^2)}{p^2}\left(\delta^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)$$



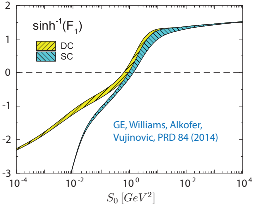
- **Quark-gluon vertex**

$$f_1\,\gamma^\mu + f_2\, ip^\mu + f_3\, p^\mu \not{p} + \dots$$



- **Three-gluon vertex**

$$F_1\left[\,\delta^{\mu\nu}(p_1-p_2)^\rho+\delta^{\nu\rho}(p_2-p_3)^\mu + \delta^{\rho\mu}(p_3-p_1)^\nu\right] + \dots$$



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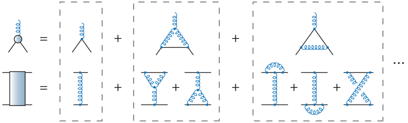
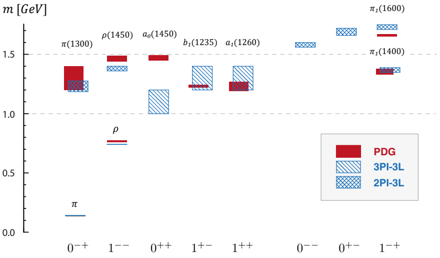


DSEs & BSEs



Light meson spectrum beyond rainbow-ladder:

Williams, Fischer, Heupel, PRD 93 (2016)

GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016)

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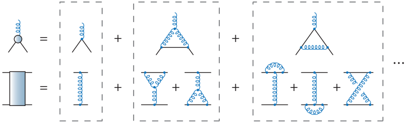
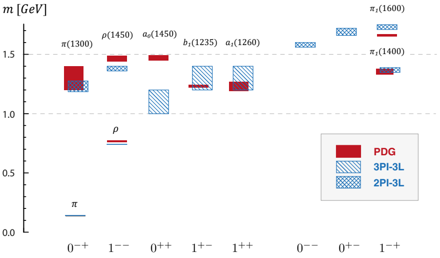


DSEs & BSEs



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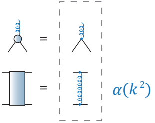
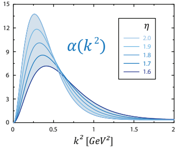




Rainbow-ladder:
effective gluon exchange

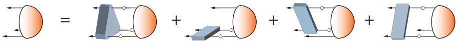
$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(\tfrac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999), Qin et al., PRC 84 (2011)

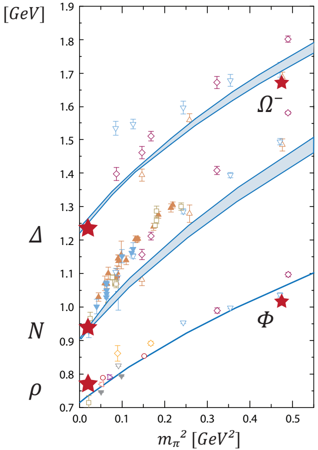
Baryons

Covariant Faddeev equation for baryons:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)

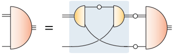
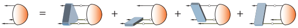


- 3-gluon diagram vanishes ⇒ **3-body effects small?**
- 2-body kernels same as for mesons, no further approximations: $M_N = 0.94\,\mathrm{GeV}$
- **Relativistic bound states** carry OAM: 64 (128) tensors for nucleon (Δ)
- Octet & decuplet baryons, pion cloud effects, first steps beyond rainbow-ladder
- **Baryon form factors:** nucleon and Δ FFs, $N \to \Delta\gamma$ transition, ...

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

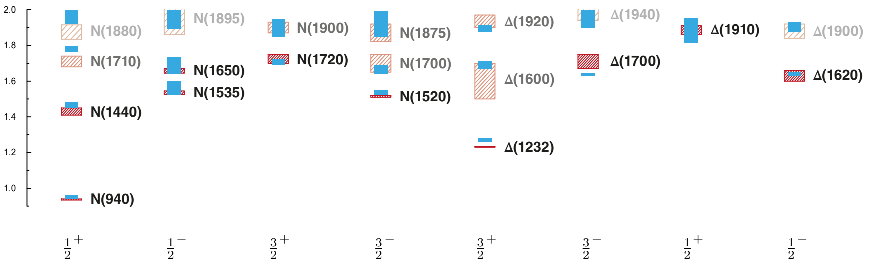


DSE / Faddeev landscape $N \to N^*\gamma$

	Quark-diquark 			Three-quark 		
	Contact interaction	**QCD-based model**	**DSE (RL)**	**RL**	**bRL**	**bRL + 3q**
N, Δ masses	√	√	√	√	√	$\dots$
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	$\dots$		
Roper	√	√	√	√	$\dots$	
$N \to N^*\gamma$	√	√	$\dots$	$\dots$		
$N^*(1535), \dots$	$\dots$	$\dots$	√	√	$\dots$	
$N \to N^*\gamma$	$\dots$	$\dots$	$\dots$	$\dots$		

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, Fischer, Sanchis-Alepuz, PRD 94 (2016)

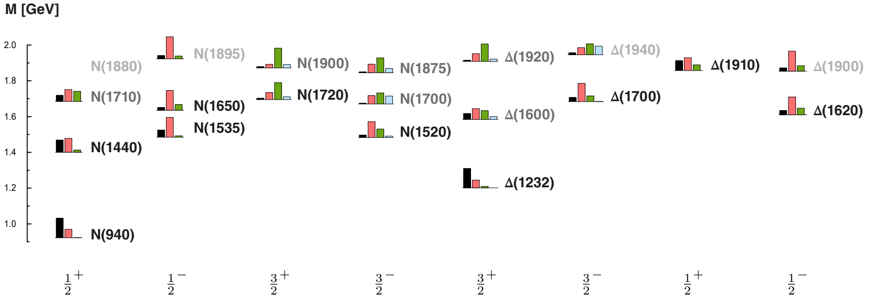


- **Quantitative agreement with experiment**
- N($\frac{1}{2}^+$) and Δ($\frac{3}{2}^+$) depend on sc + av diquarks; remaining ones “polluted” by ps + v diquarks
- Correct level ordering between **Roper and N(1535)**

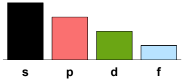
- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ–a_1 splitting
- η doesn’t change much

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: GE, FBS 58 (2017)



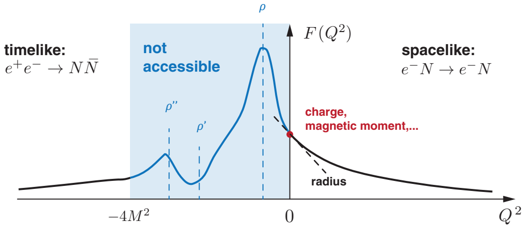
Partial-wave content:



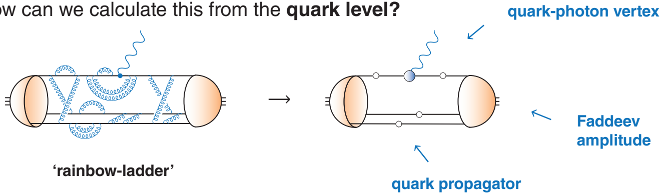
- N and Δ ground states dominated by **s waves,** negative-parity states typically by **p waves** (as expected)
- But 'quark-model forbidden' contributions are always present, e.g. **Roper: dominated by p waves** ⇒ **relativity is important!**

Matrix elements

Sketch of a generic electromagnetic form factor:

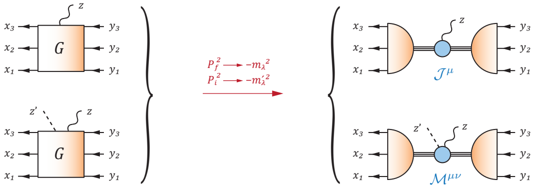


How can we calculate this from the **quark level?**



Matrix elements

Insert **spectral decomposition** in $\langle \cdots \psi(x_1) \cdots \bar{\psi}(y_1) \cdots j^\mu(z) \cdots \rangle$



"Gauging of equations":
Kvinikhidze, Blankleider, PRC 60 (1999)

GE, Fischer, PRD 85 (2012)
GE, Fischer, PRD 87 (2013)

Use properties of (functional) derivative, obtain
general expression for **current matrix elements** and **scattering amplitudes:**

$$\mathcal{J}^\mu = -\overline{\mathbf{\Psi}}_f \left(\mathbf{G}^{-1}\right)^\mu \mathbf{\Psi}_i \qquad \mathcal{M}^{\mu\nu} = \overline{\mathbf{\Psi}}_f \left[\left(\mathbf{G}^{-1}\right)^{\{\mu} \mathbf{G} \left(\mathbf{G}^{-1}\right)^{\nu\}} - \left(\mathbf{G}^{-1}\right)^{\mu\nu} \right] \mathbf{\Psi}_i$$

Relate **G** to elementary propagators, vertices and kernels:

$$\left(\mathbf{G}^{-1}\right)^\mu = \left(\mathbf{G_0}^{-1}\right)^\mu - \mathbf{K}^\mu = \left[\Gamma^\mu \otimes S^{-1} \otimes S^{-1} - \Gamma^\mu \otimes K_{(2)} - S^{-1} \otimes K^\mu_{(2)} + \text{perm.} \right] - K^\mu_{(3)}$$

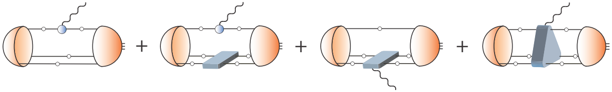
$$\left(\mathbf{G}^{-1}\right)^{\mu\nu} = \left(\mathbf{G_0}^{-1}\right)^{\mu\nu} - \mathbf{K}^{\mu\nu} = \left[\Gamma^{\mu\nu} \otimes S^{-1} \otimes S^{-1} - \Gamma^{\mu\nu} \otimes K_{(2)} + \Gamma^{\{\mu} \otimes \Gamma^{\nu\}} \otimes S^{-1} - \Gamma^{\{\mu} \otimes K^{\nu\}}_{(2)} - S^{-1} \otimes K^{\mu\nu}_{(2)} + \text{perm.} \right] - K^{\mu\nu}_{(3)}$$

Matrix elements

Current matrix element:

- impulse approximation + coupling to kernels
- **gauge invariance** is automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel

$\mathcal{J}^\mu$ =



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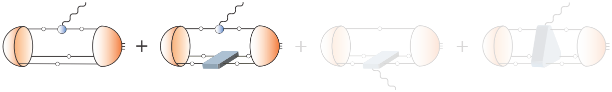
Matrix elements

Current matrix element:

- impulse approximation + coupling to kernels
- **gauge invariance** is automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel

$\mathcal{J}^\mu =$



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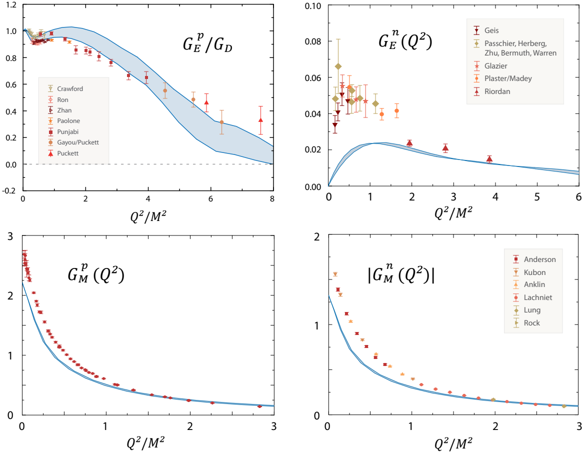
Nucleon em. form factors

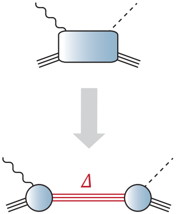


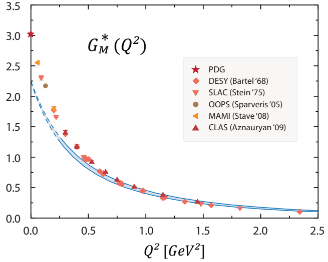
Three-body results:
all ingredients calculated,
model dependence shown
by bands GE, PRD 84 (2011)

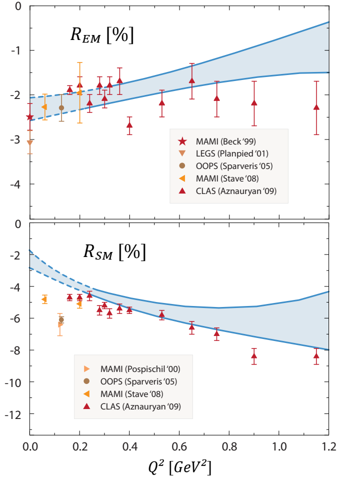
- **timelike vector-meson poles** automatically generated in quark-photon vertex
- **electric proton form factor:** consistent with data, possible zero crossing
- **magnetic form factors:** missing pion effects at low Q^2

$\Rightarrow$ **"quark core without pion-cloud effects"**

Nucleon-Δ-γ transition







- **Magnetic dipole transition (G_M^*)** dominant: quark spin flip (s wave). "Core + 25% pion cloud"
- **Electric & Coulomb quadrupole ratios** small & negative, encode deformation. Reproduced without pion cloud: **OAM** from **p waves!**

  GE, Nicmorus, PRD 85 (2012)

Matrix elements

Scattering amplitude:

$\mathcal{M}^{\mu\nu}$ =

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general expression for **current matrix elements** and **scattering amplitudes:**

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Matrix elements

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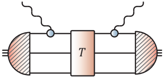
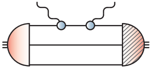
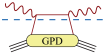
$\mathcal{M}^{\mu\nu}$ =  + 

Nucleon resonances

quark **Born:** reproduces perturbative handbag



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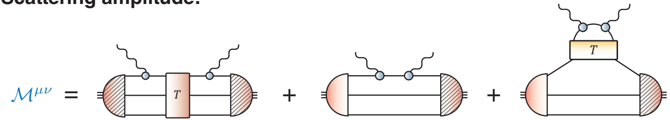
Matrix elements

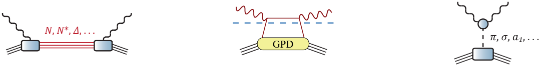
Scattering amplitude:



Nucleon resonances

quark **Born:** reproduces perturbative handbag

quark **1PI:** reproduces t-channel meson poles



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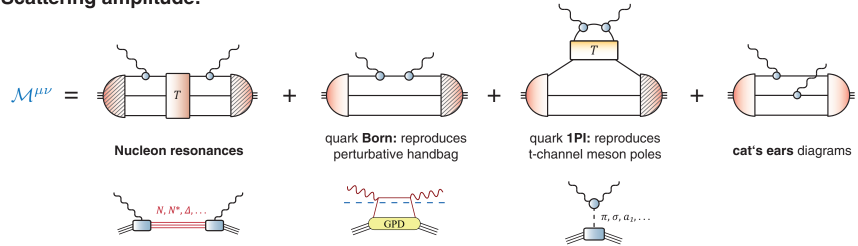
Matrix elements

Scattering amplitude:



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Matrix elements

Scattering amplitude:

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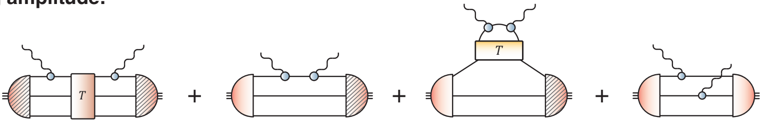
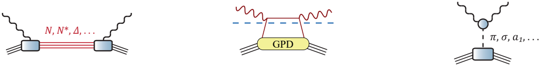


Nucleon resonances | quark **Born:** reproduces perturbative handbag | quark **1PI:** reproduces t-channel meson poles | **cat's ears** diagrams



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Matrix elements

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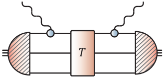
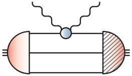
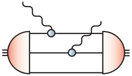
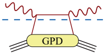
$\mathcal{M}^{\mu\nu}$ = 

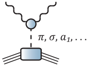
+  + 

Nucleon resonances

quark **Compton vertex:** reproduces
perturbative handbag & t-channel meson poles

cat's ears diagrams





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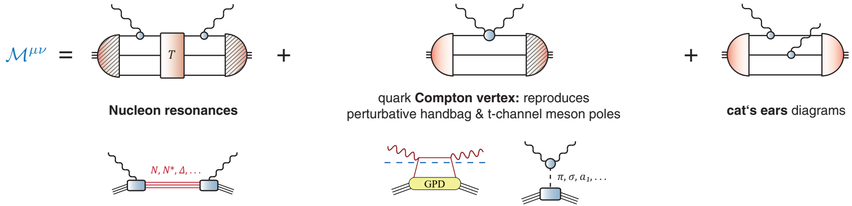
Matrix elements

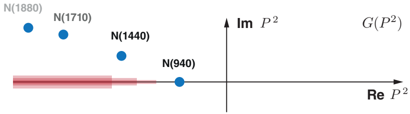
Scattering amplitude:



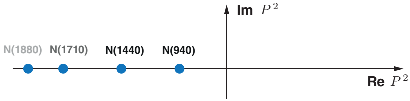
- **Poincaré covariance** and **crossing symmetry** are automatic
- **gauge invariance** and **chiral symmetry** are automatic,
  as long as all ingredients calculated from same symmetry-preserving kernel
- **perturbative processes** are included
- **s, t, u channel poles** are generated dynamically, no need for “offshell hadrons”
- hadronic rescattering is implicit $\Rightarrow$ **unitarity?**

Resonances?

Branch cuts & widths generated by **meson-baryon interactions:** Roper → $N\pi$, etc.



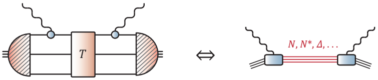
Without them: **bound states without widths**



Spectrum generated by quark-gluon interactions, meson-baryon effects would mainly shift poles into complex plane?

Resonances must be generated dynamically at the **quark level:** complicated topologies beyond rainbow-ladder

In scattering amplitude, they would mainly appear here:

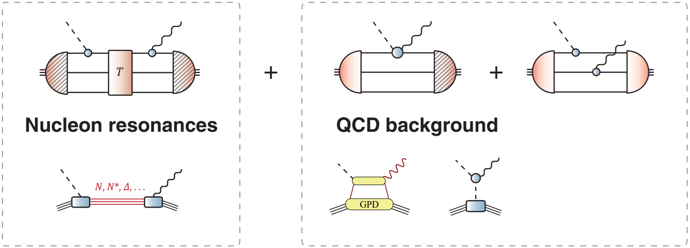


but pion cloud effects will also contribute to “QCD background”

Meson electroproduction

Scattering amplitude:

$\mathcal{M}^{\mu\nu} =$



Need:

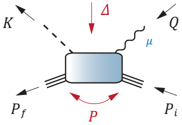
- quark propagator
- nucleon Faddeev amplitude
- quark-photon vertex
- quark-meson vertex
- **quark-meson-photon vertex**

+ a lot of CPU power!!

Meson electroproduction

3 independent variables (↔ s, t, u):

$$\tau = \frac{Q^2}{4m^2}\,, \qquad \eta = \frac{K\cdot Q}{m^2}\,, \qquad \lambda = -\frac{P\cdot Q}{m^2} = -\frac{P\cdot K}{m^2}$$



Amplitude depends on **6 Lorentz-invariant functions:**

$$\mathcal{M}^\mu(P,K,Q) = \bar{u}(P_f)\left(\sum_{i=1}^{6} A_i(\tau,\eta,\lambda)\, M_i^\mu(P,K,Q)\right) u(P_i)$$

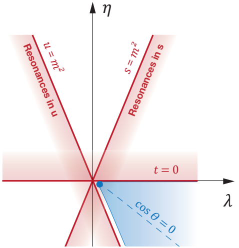
with appropriate tensor basis: no kinematic singularities!
GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 (2016), 1606.09602

(in contrast to Dennery amplitudes)
Fubini, Nambu, Wataghin 1958, Dennery 1961

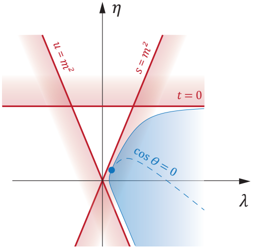
$$M^\mu_{1\dots 6}(P,K,Q) = i\gamma_5\left\{ [\gamma^\mu, \not{Q}]\,, \quad t^{\mu\nu}_{QK}\,P^\nu, \quad t^{\mu\nu}_{QQ}\,P^\nu, \quad t^{\mu\nu}_{QP}\,i\gamma^\nu, \quad \lambda\, t^{\mu\nu}_{QK}\,i\gamma^\nu, \quad \lambda\, t^{\mu\nu}_{QQ}\,i\gamma^\nu \right\}$$

$$t^{\mu\nu}_{AB} = A\cdot B\,\delta^{\mu\nu} - B^\mu A^\nu$$

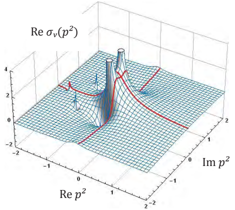
Photo-production
($\tau = 0$):



Electro-production
($\tau > 0$):



Meson electroproduction

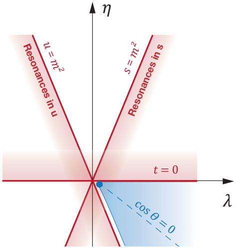


Singularity structure of quark propagator prevents **direct kinematic access** to all relevant regions

Strategies:

- if amplitudes free of kinematic singularities, **only physical poles and cuts** $\Rightarrow$ extrapolate from unphysical regions (or offshell kinematics)
- clean solution (expensive): use **contour deformations**

Photo-production ($\tau = 0$):



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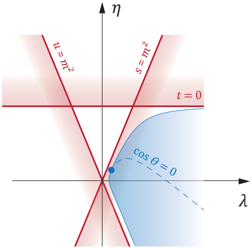
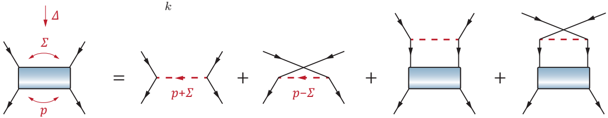


A toy model

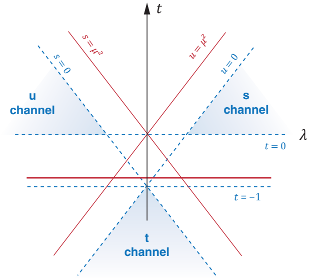
Scattering amplitude for two **massive scalar particles** (mass m) with **massive exchange particle** (mass μ):

$$T(p,\Sigma,\Delta) = K(p,\Sigma) + \int_k T(p,k,\Delta)\, D(k_+)\, D(k_-)\, K(k,\Sigma)$$



Onshell amplitude: Mandelstam plane

- $t = \dfrac{\Delta^2}{4m^2}, \quad \lambda = -\dfrac{p\cdot\Sigma}{m^2}$
- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel



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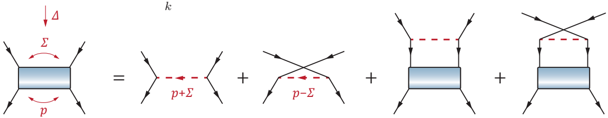
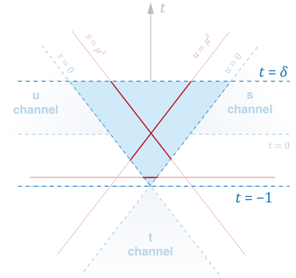


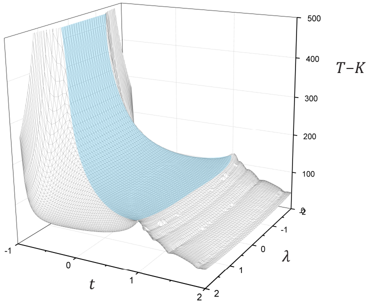
Onshell amplitude: Mandelstam plane

- $t = \frac{\Delta^2}{4m^2}, \quad \lambda = -\frac{p\cdot\Sigma}{m^2}$
- Born terms for exchange particle produce s- and u-channel poles
- Bound state pole in t channel
- **Poles** in propagators and exchange particle pose **restrictions:**

$$-1 < t < \delta, \quad |\lambda| < 1 + t, \quad \delta = \frac{\mu^2}{m^2} - 1$$



A toy model



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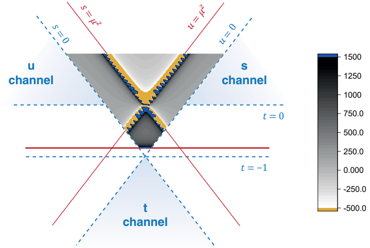
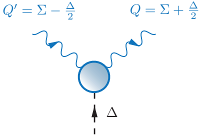
Subtract Born terms to get rid of s- and u-channel poles (↔ 1PI part):

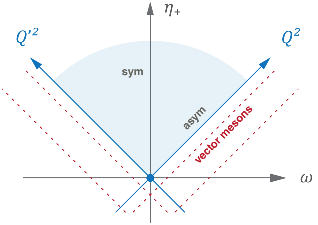
- rise is due to t-channel bound state
- outside blue region: naive integration over poles (wrong)
- scattering amplitude almost independent of λ!



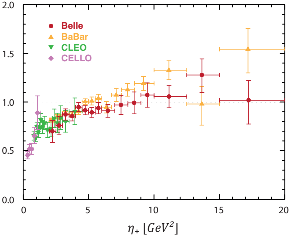
Pion transition form factor



$$= e^2 \, \frac{F(Q^2, Q'^2)}{4\pi^2 f_\pi} \, \varepsilon^{\mu\nu\alpha\beta} \, Q'^\alpha \, Q^\beta$$



- $F(0,0) = 1$ in chiral limit (Abelian anomaly)
- $\frac{\eta_+ F(Q^2, Q'^2)}{4\pi^2 f_\pi^2} \xrightarrow{\eta_+ \to \infty} \frac{2}{3} \dots 1(?)$ Lepage, Brodsky, PRD 22 (1980)

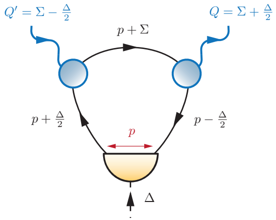


$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$

$$\omega = \frac{Q^2 - Q'^2}{2}$$

$$\eta_- = Q \cdot Q'$$

Pion transition form factor



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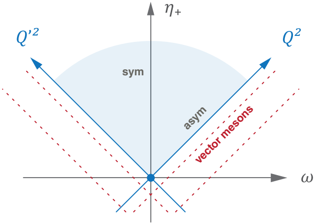
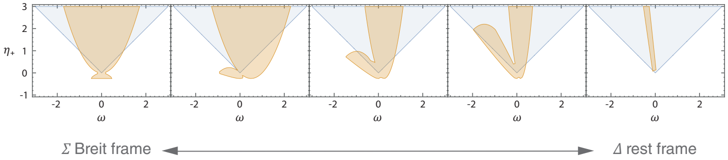


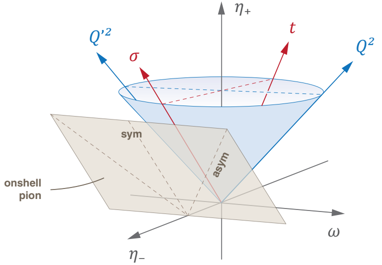
Quark singularities complicate matters: symmetric limit ok, but asymmetric limit only up to ~4 GeV^2 Maris, Tandy, PRC 65 (2002)



exploit Lorentz invariance to change frame

Weil, GE, Fischer, Williams, 1704.06046 [hep-ph]

Pion transition form factor



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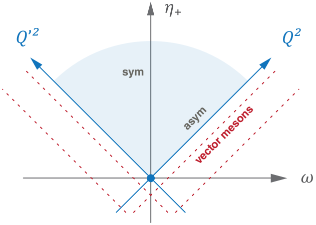
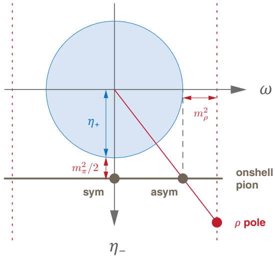
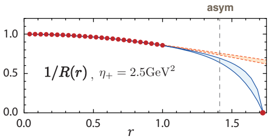




Idea:

- calculate FF inside cone
- interpolate to physical plane using VM pole as constraint
- can be done for arbitrary Q^2



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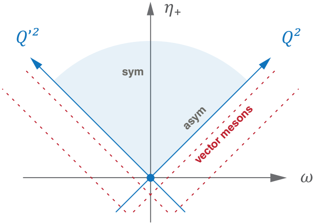
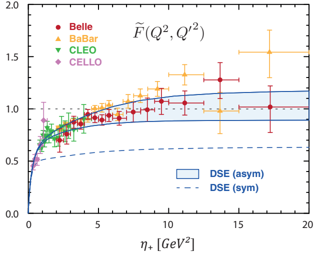
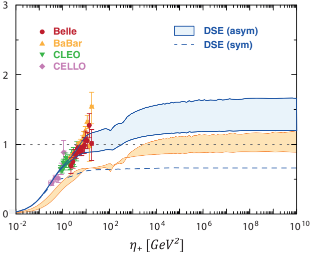
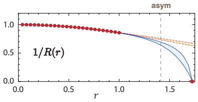




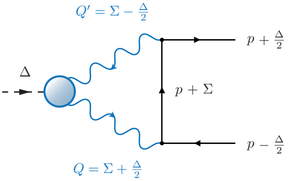


VM poles modify asymptotic scaling!

GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]



Rare pion decay $\pi^0 \to e^+e^-$



- After reanalysis of radiative corrections still 2σ discrepancy in branching ratio between exp and theory:

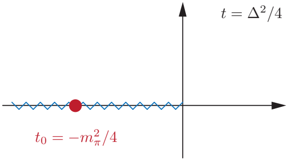
$6.87(36) \times 10^{-8}$ **KTeV Collab.:** Abouzaid et al., PRD 75 (2007); Husek, Kampf, Novotny, EPJ C74 (2014)

$6.23(09) \times 10^{-8}$ Dorokhov, JETP Lett. 91 (2010), Masjuan, Sanchez-Puertas, 1504.07001

- Depends on **pion transition FF** as input: GE, Fischer, Weil, Williams, 1704.05774

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2} \,.$$

- cannot be calculated directly in Euclidean kinematics because of **photon and lepton poles**



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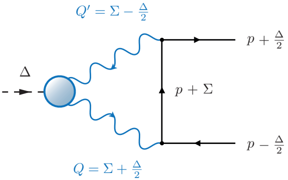

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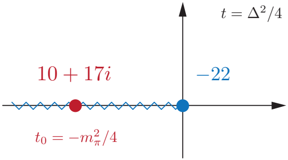
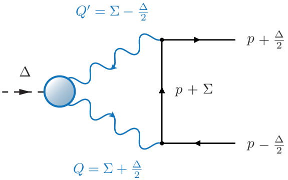


- cannot be calculated directly in Euclidean kinematics because of **photon and lepton poles**

- workaround with dispersion relations:

$$\text{Im}\ \mathcal{A}^{\text{LO}}(t) = \frac{\pi \ln\gamma(t)}{2\beta(t)} F(0,0) \quad \Rightarrow \quad \text{Re}\ \mathcal{A}(t) = \mathcal{A}(0) + \frac{\ln^2\gamma(t) + \frac{1}{3}\pi^2 + 4\,\text{Li}_2(-\gamma(t))}{4\beta(t)}$$

Rare pion decay $\pi^0 \to e^+e^-$



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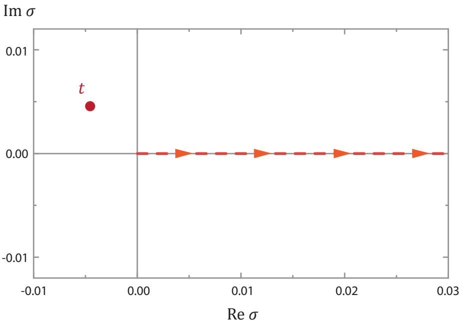
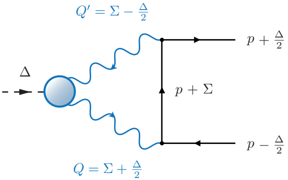
Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$



Rare pion decay $\pi^0 \to e^+e^-$



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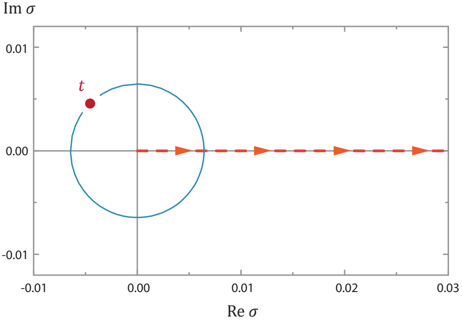

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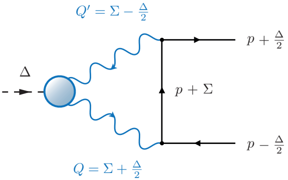

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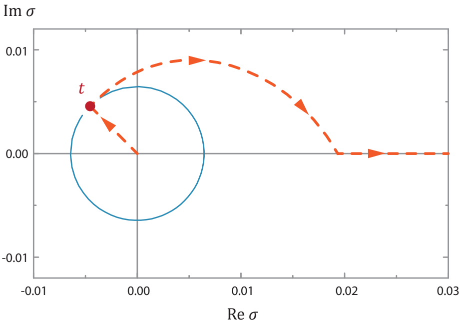

Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$
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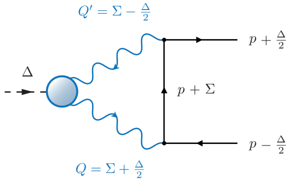
- deform integration contour:
  cut opens at t

Rare pion decay $\pi^0 \to e^+e^-$



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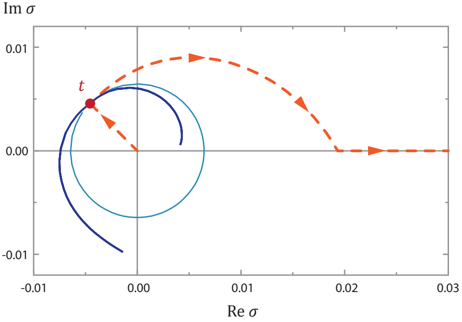

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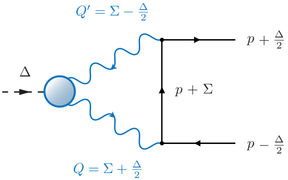
- but lepton cut does **not** open at t!

Rare pion decay $\pi^0 \to e^+e^-$



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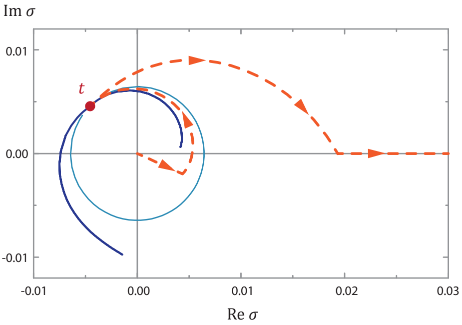
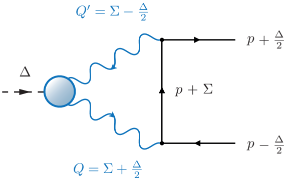


Photon and lepton poles produce **branch cuts** in complex $\Sigma^2 = \sigma$ plane:

- 'Euclidean integration': $0 < \sigma < \infty$
- not possible: circular photon cut
- deform integration contour: cut opens at t
- but lepton cut does **not** open at t!
- **deform contour** such that it never crosses any cut!

Rare pion decay $\pi^0 \to e^+e^-$



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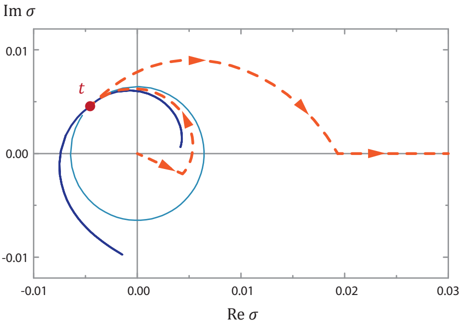
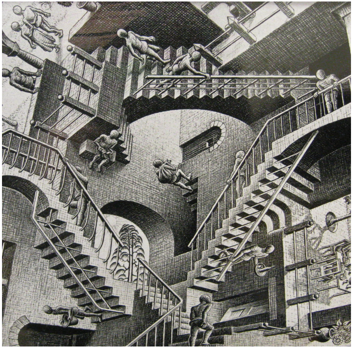
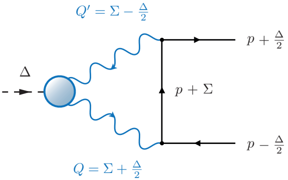




Rare pion decay $\pi^0 \to e^+e^-$



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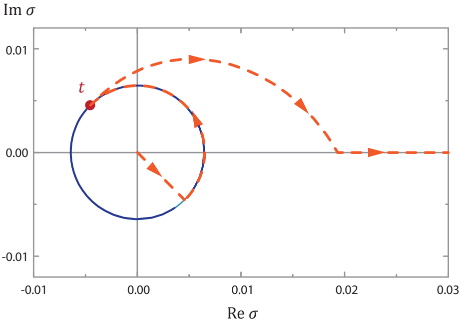
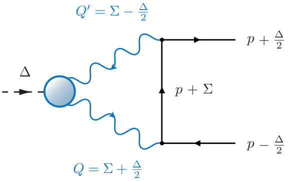


Rare pion decay $\pi^0 \to e^+e^-$



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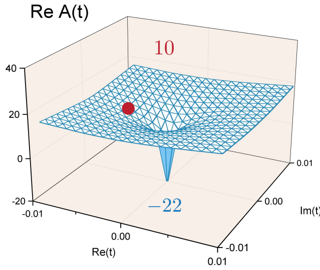
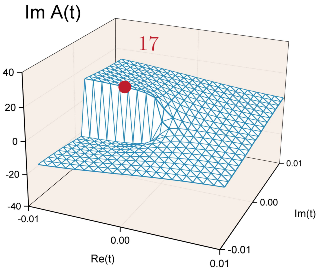


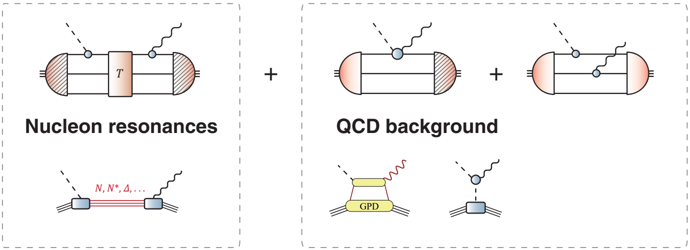


- Algorithm is stable & efficient
- Can be applied to any integral as long as **singularity locations** known
- solves problems with **Wick rotation!**

Weil, GE, Fischer, Williams, 1704.06046 [hep-ph]

Summary

$\mathcal{M}^{\mu\nu}$ =



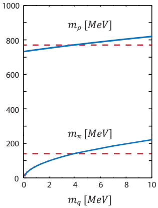
- in principle, full expression for **meson electroproduction amplitude**
- complete calculation not (yet?) feasible
  ⇒ treat as fixed **background potential** in coupled-channel equations
- **kinematic access** to physical region difficult (singularities in integrands)
  ⇒ contour deformation methods in development
  ⇒ smart workarounds: extrapolations from offshell kinematics, etc.
- also relevant for GlueX! **Exotic mesons** can be (relativistic) $q\bar{q}$ states . . .

Thank you!

Backup slides

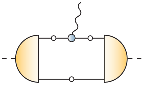
Mesons

- Pion is **Goldstone boson:** ${m_\pi}^2 \sim m_q$



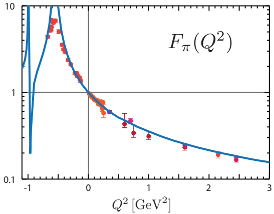
- **Pion em. form factor:**

Maris & Tandy, PRC 61 (2000),
Chang et al., PRL 111 (2013)



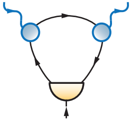
Timelike VM poles automatically generated in quark-photon vertex!

A. Krassnigg, Schladming 2010

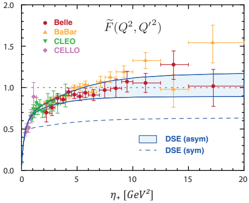


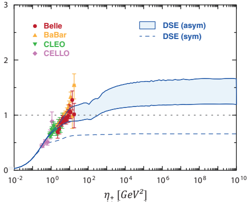
- **Pion transition form factor**

GE, Fischer, Weil, Williams, 1704.05774 [hep-ph]



VM poles modify asymptotic scaling!

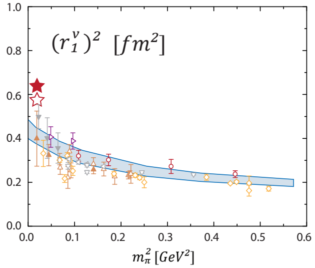




Nucleon em. form factors

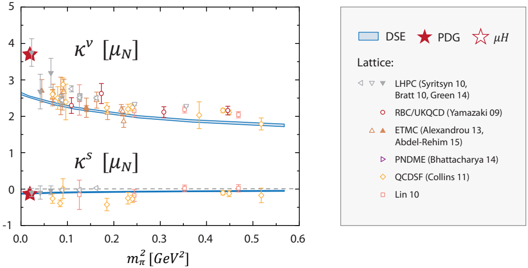
Nucleon charge radii:

isovector (p-n) Dirac (F1) radius

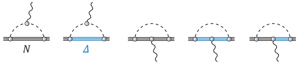


Nucleon magnetic moments:

isovector (p-n), isoscalar (p+n)



- **Pion-cloud effects** missing (⇒ divergence!), agreement with lattice at larger quark masses.



- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$

Calc: $\kappa^s = -0.12(1)$

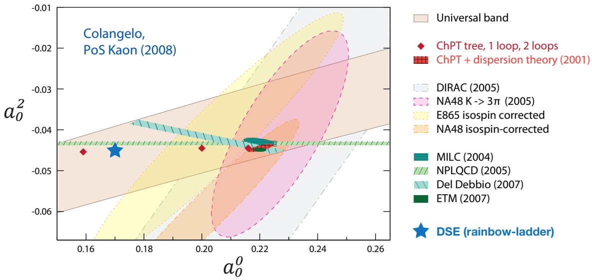
!!

GE, PRD 84 (2011)

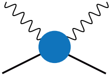
. . . and more

Scattering amplitudes from quark level:

- $\pi\pi$ **scattering**
  Bicudo et al., PRD 65 (2002),
  Cotanch, Maris, PRD 66 (2002)

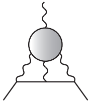


- **Nucleon Compton scattering**



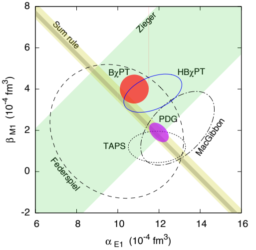
  GE, Fischer, PRD 85 (2012) & PRD 87 (2013), GE, FBS 57 (2016)

- **Hadronic light-by-light scattering**
  Goecke, Fischer, Williams, PLB 704 (2011),
  GE, Fischer, Heupel, PRD 92 (2015)



Compton scattering

Nucleon polarizabilities:
ChPT & dispersion relations
Hagelstein, Miskimen, Pascalutsa, PPNP 88 (2016)



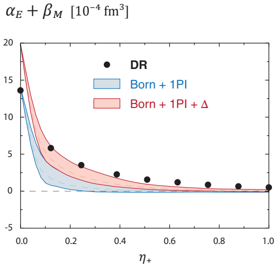
First DSE results:
GE, FBS 57 (2016)

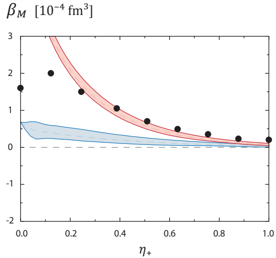
- Quark Compton vertex (Born + 1PI) calculated, added Δ exchange
- compared to DRs
  Pasquini et al., EPJ A11 (2001),
  Downie & Fonvieille, EPJ ST 198 (2011)
- α_E dominated by handbag, β_M by Δ contribution

⇒ **large "QCD background"!**

In total: polarizabilities ≈

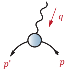
Quark-level effects ↔ Baldin sum rule
+ nucleon resonances (mostly Δ)
+ pion cloud (at low η_+)?





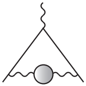
Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ

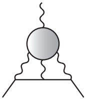


$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

- Theory uncertainty dominated by **QCD:**
  Is QCD contribution under control?



Hadronic vacuum polarization

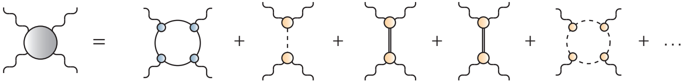


Hadronic light-by-light scattering

$a_\mu\ [10^{-10}]$ — Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

Exp:	11 659 208.9	(6.3)	
QED:	11 658 471.9	(0.0)	
EW:	15.3	(0.2)	
Hadronic:			
• VP (LO+HO)	685.1	(4.3)	
• **LBL**	**10.5**	**(2.6)**	**?**
SM:	11 659 182.8	(4.9)	
Diff:	26.1	(8.0)	

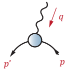
- **LbL amplitude:** ENJL & MD model results
  Bijnens 1995, Hakayawa 1995, Knecht 2002, Melnikov 2004, Prades 2009, Jegerlehner 2009, Pauk 2014



Quark loop	pseudoscalar exchange	scalar exchange	axialvector exchange	π, K loop	
2	8 ... 11	-1	2	-2	$(\times 10^{-10})$

Muon g-2

- **Muon anomalous magnetic moment:**
  total SM prediction deviates from exp. by ~3σ



$$= ie\,\bar{u}(p')\left[F_1(q^2)\,\gamma^\mu - F_2(q^2)\,\frac{\sigma^{\mu\nu}q_\nu}{2m}\right]u(p)$$

a_μ [10^{-10}]

Jegerlehner, Nyffeler, Phys. Rept. 477 (2009)

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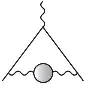
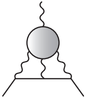
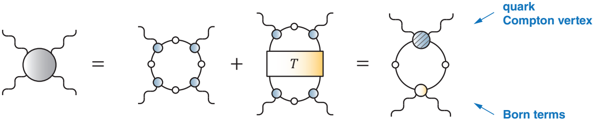


Hadronic vacuum polarization



Hadronic light-by-light scattering

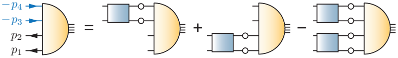
- **LbL amplitude** at quark level, derived from **gauge invariance:**
  GE, Fischer, PRD 85 (2012), Goecke, Fischer, Williams, PRD 87 (2013)



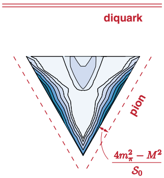
- **no double-counting, gauge invariant!**
- need to understand **structure of amplitude**
  GE, Fischer, Heupel, PRD 92 (2015)

Tetraquarks *are* resonances

- **Light scalar mesons** σ, κ, a_0, f_0 as **tetraquarks:** solution of four-body equation reproduces mass pattern
  GE, Fischer, Heupel, PLB 753 (2016)



BSE dynamically generates **meson poles** in wave function, drive σ mass from 1.5 GeV to ~350 MeV



Four quarks rearrange to **"meson molecule"**

Tetraquarks are "dynamically generated **resonances**" (but from the quark level!)