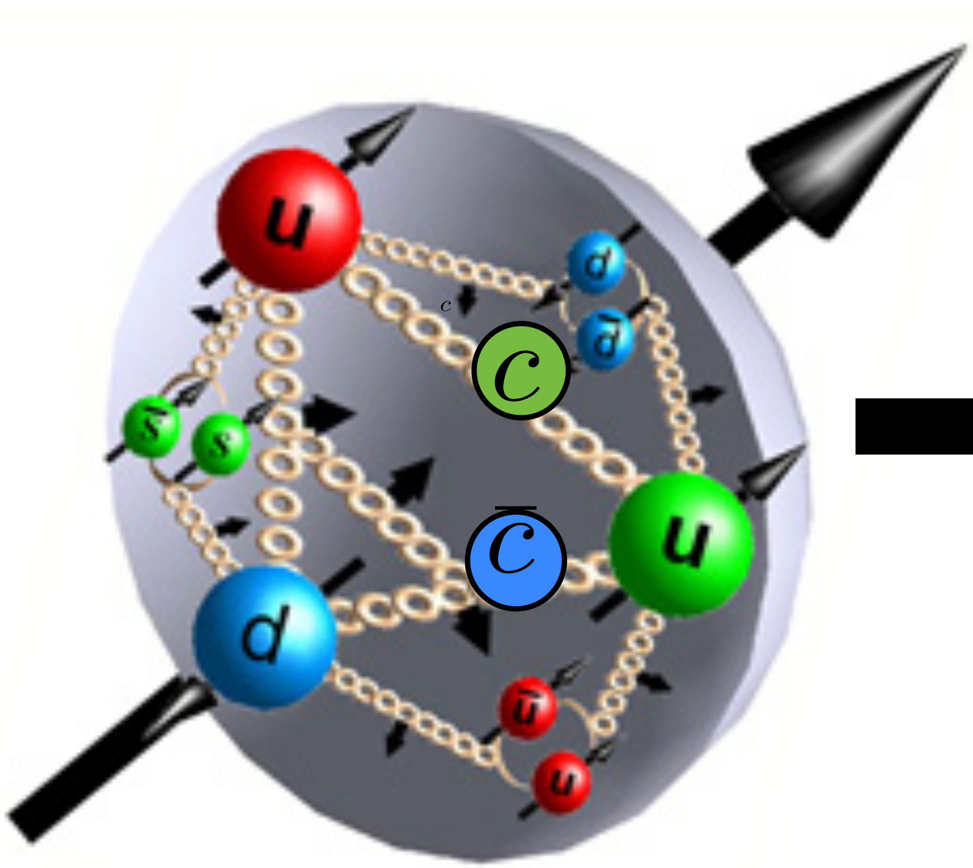
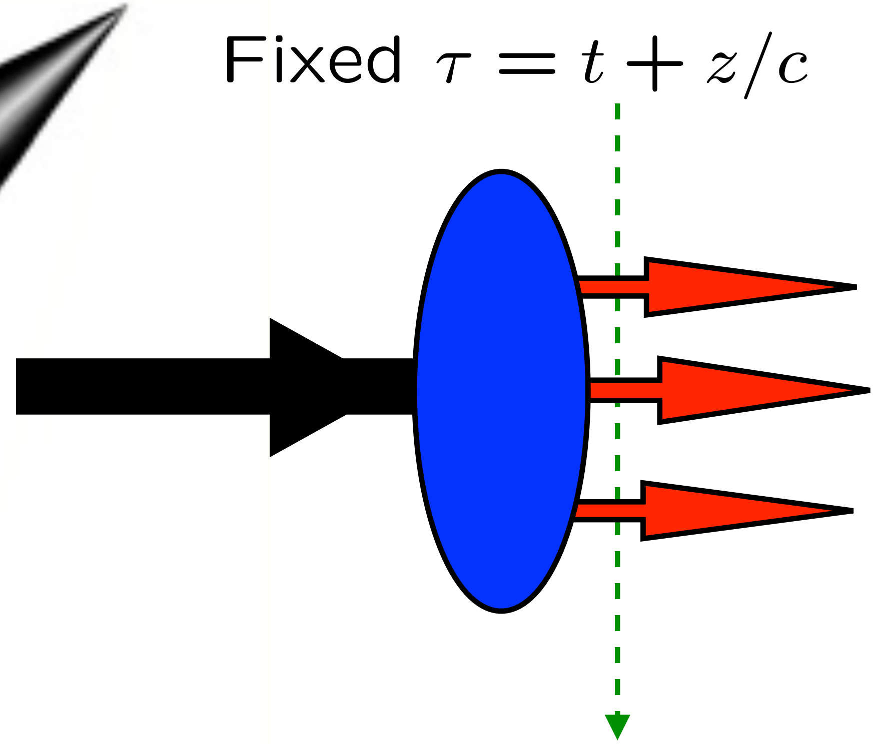
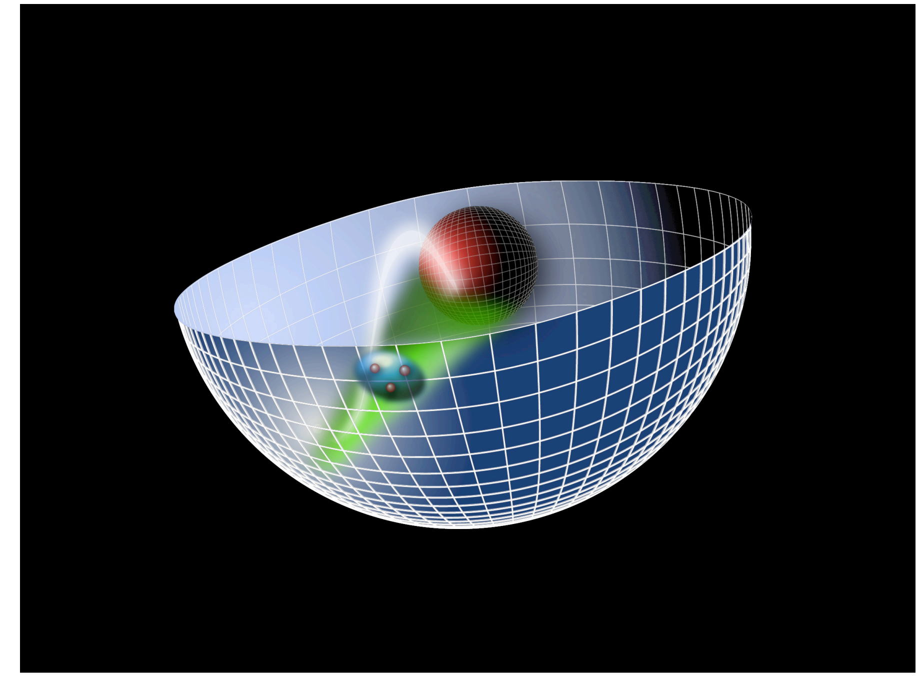
Supersymmetric Properties of Hadron Physics and Predictions for Exclusive Processes from Light-Front Holography and Superconformal Algebra







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY · DIE LUFT DER FREIHEIT WEHT · 1891

with Guy de Tèramond, Hans Günter Dosch, C. Lorce, K. Chiu, R. S. Sufian, A. Deur

May 29-31, 2017

Nucleon and Resonance Structure with Hard Exclusive Processes

IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY

Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

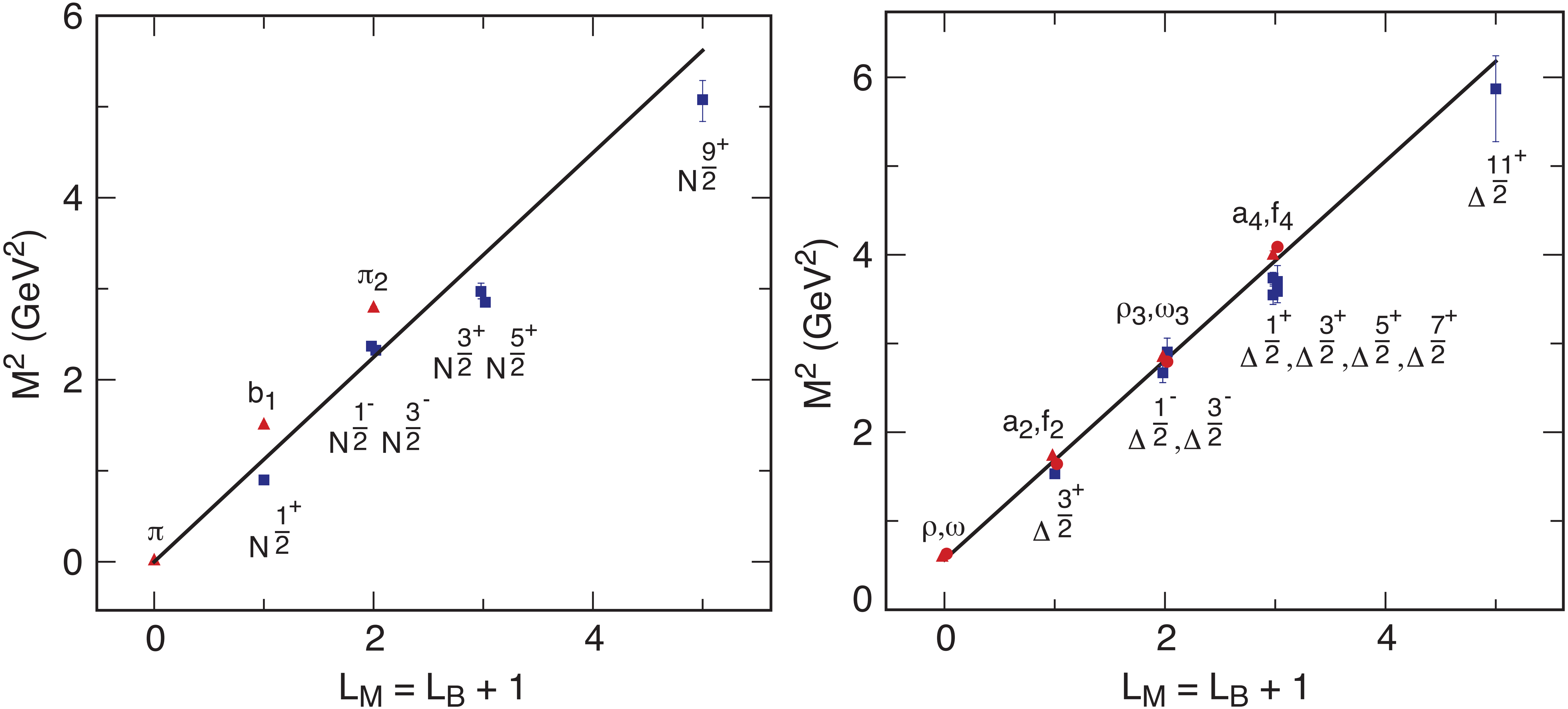
Relativistic, Frame-Independent, Color-Confining

Origin of hadronic mass scale

AdS/QCD
Light-Front Holography
Superconformal Algebra

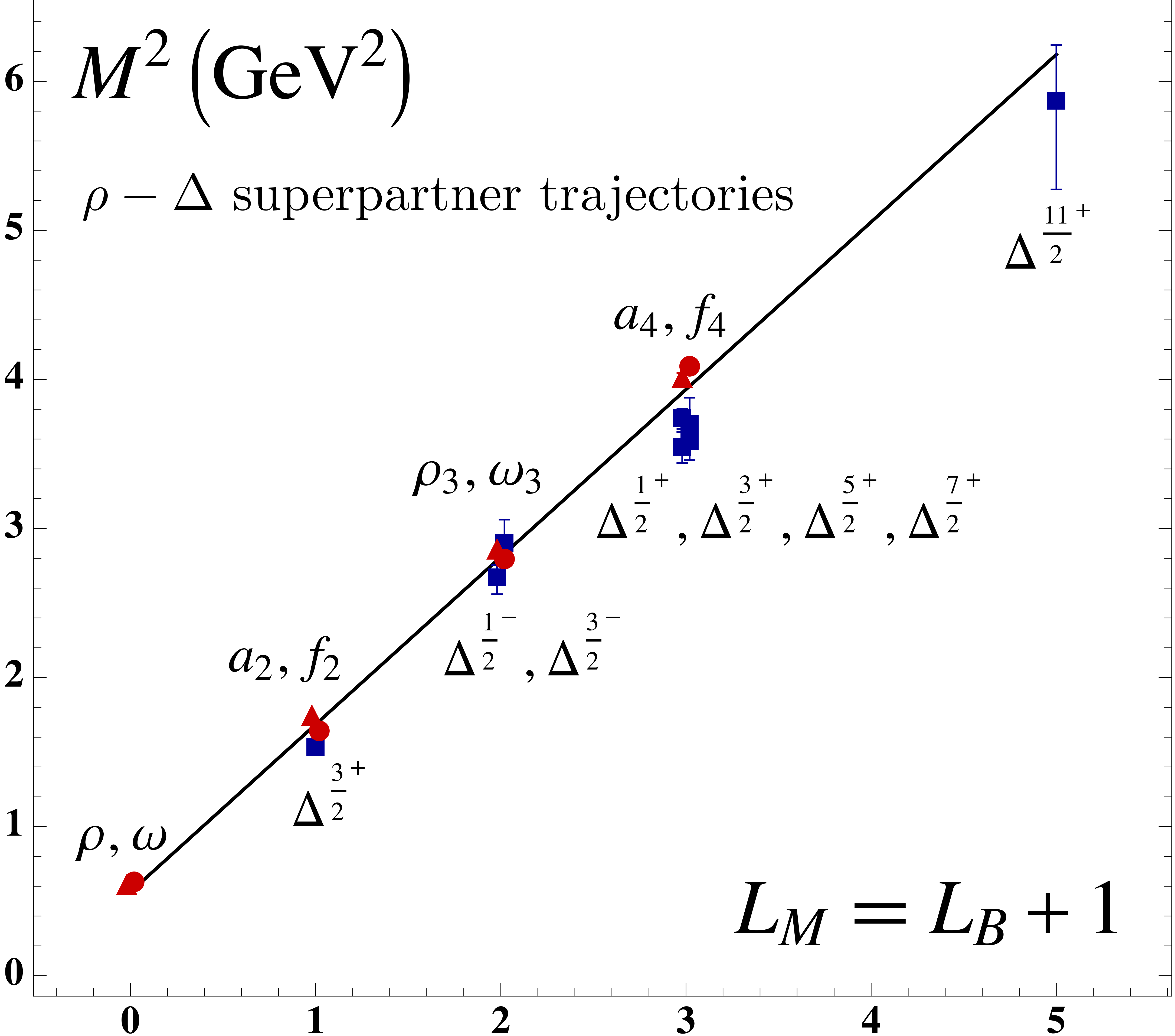
Spectroscopy and Dynamics

Solid line: κ = 0.53 GeV



Superconformal meson-nucleon partners

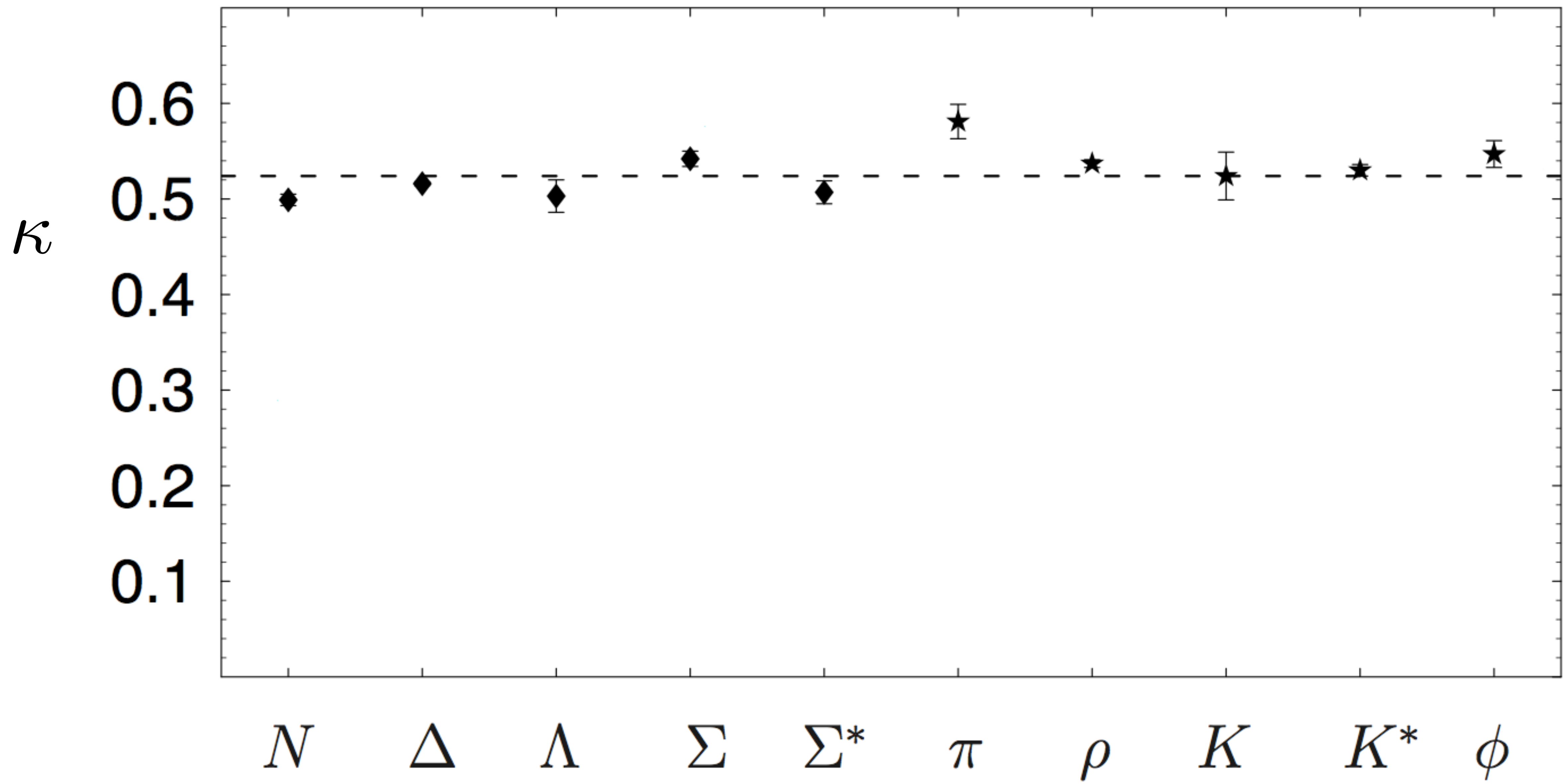
de Tèramond, Dosch, Lorce, sjb



Dosch, de Teramond, sjb

Dosch, de Teramond, Lorce, sjb

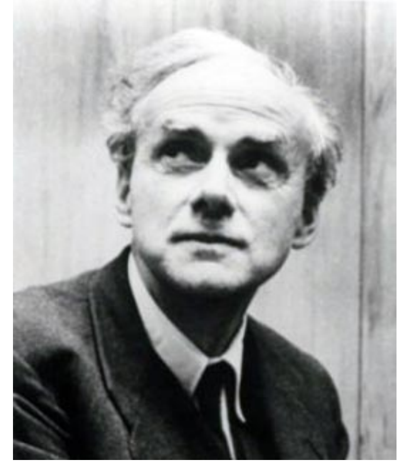
$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$



Fit to the slope of Regge trajectories, including radial excitations

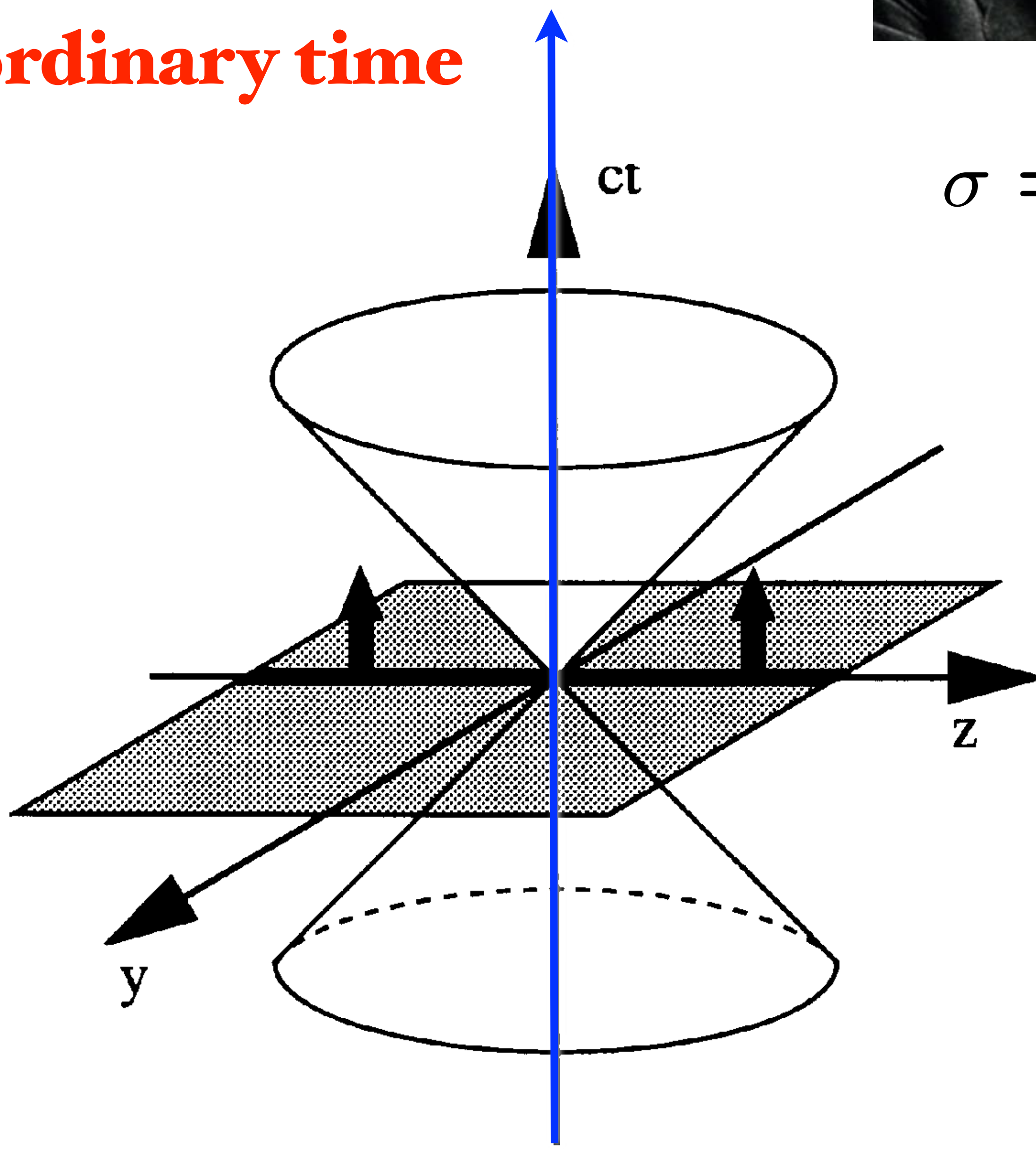
Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Dirac's Amazing Idea: The "Front Form"



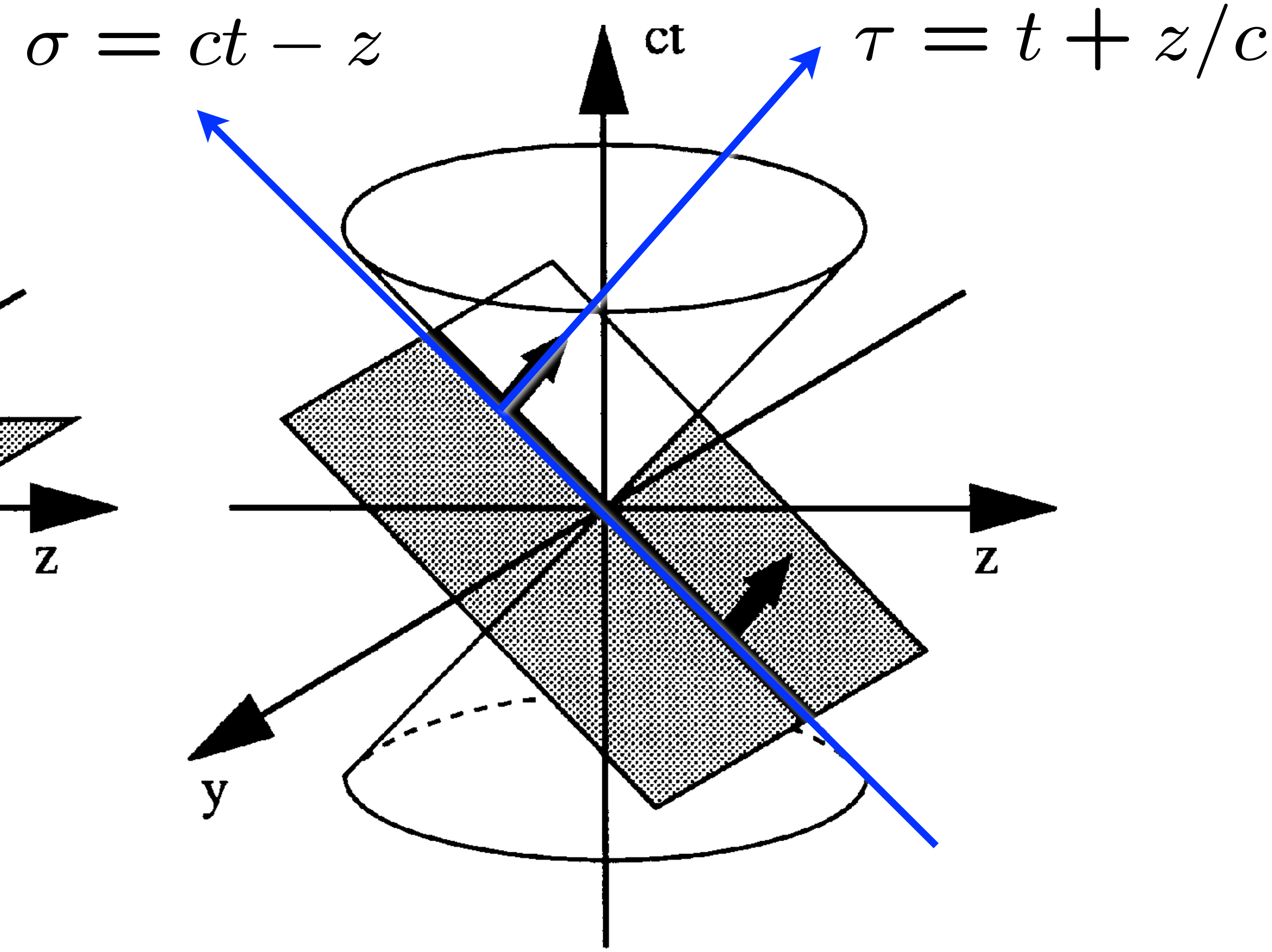
P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time



Instant Form

Evolve in light-front time!



Front Form

Causal, Boost Invariant!

- *Satisfies Poincarè Invariance*

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

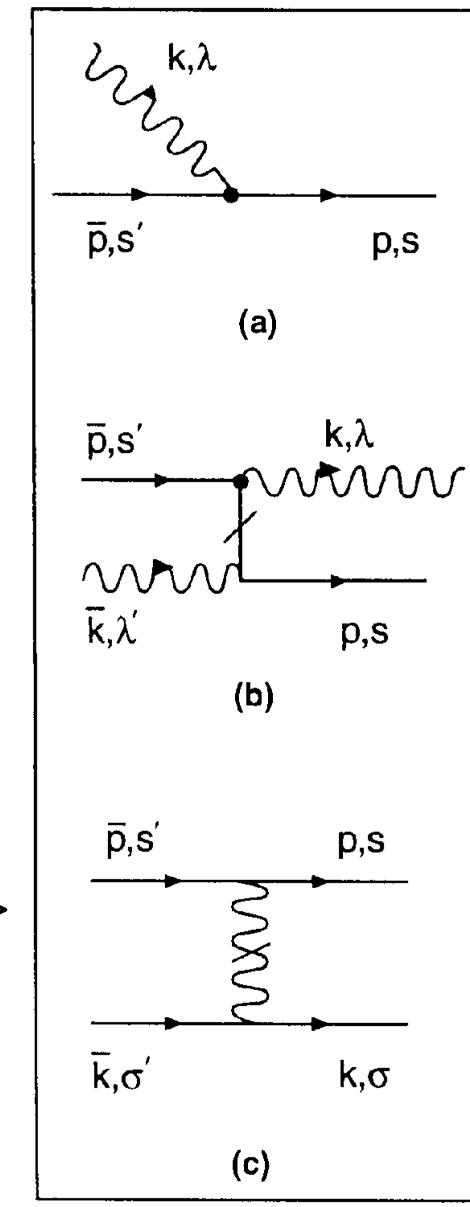
H^{int}_{LF}: Matrix in Fock Space

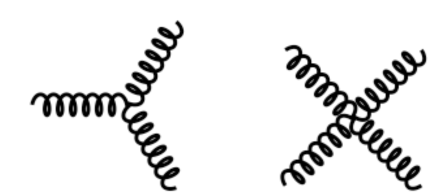
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





H^{int}_{LF}

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

$$H_{LF} = P^+P^- - \vec{P}_\perp^2$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

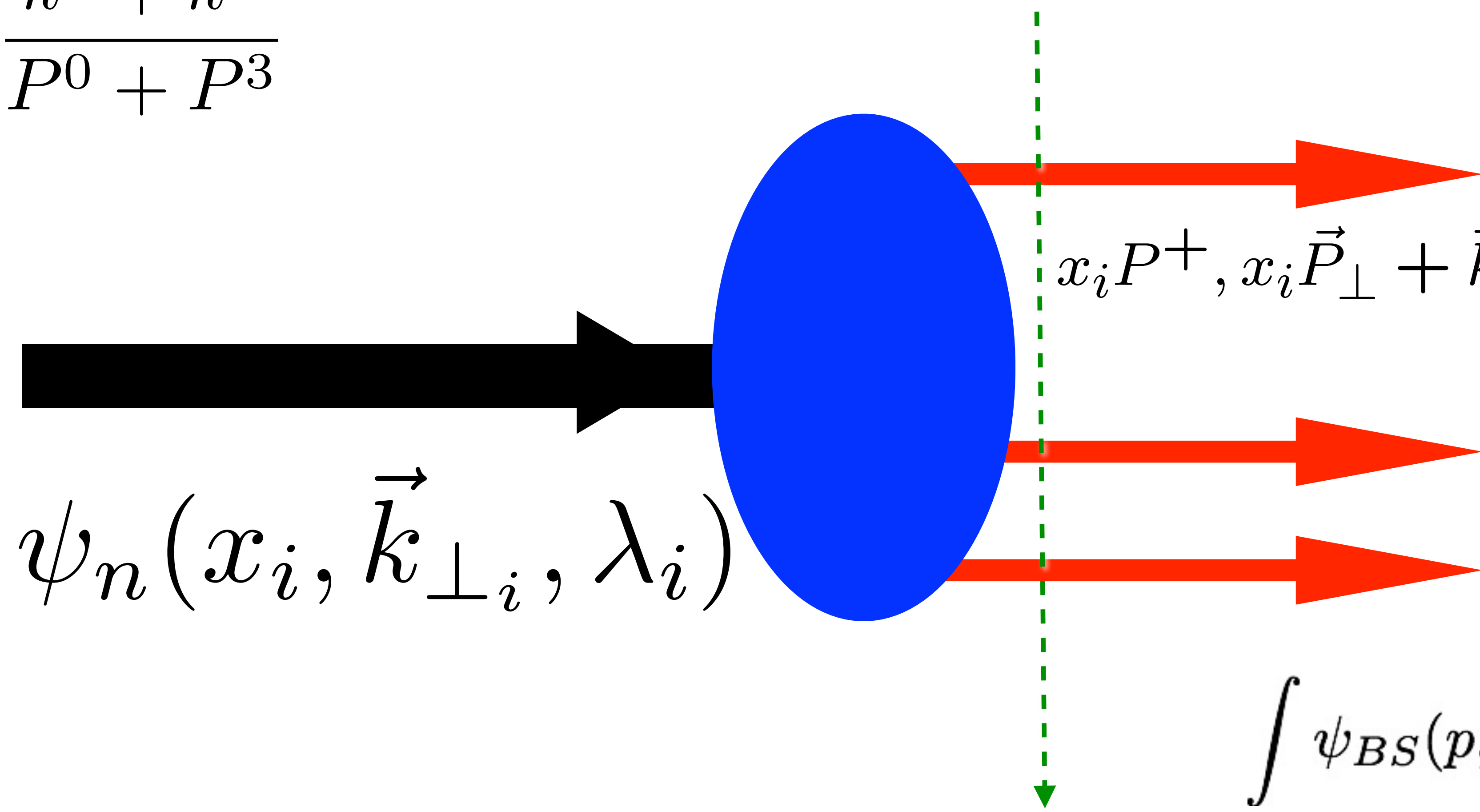
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian $H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$\int \psi_{BS}(p,k)dk^- \to \psi_{LF}$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

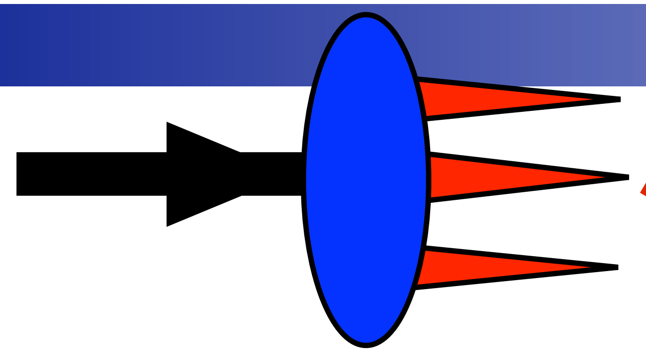
Invariant under boosts! Independent of P^μ

Off-shell in P- and invariant mass

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

- *Poincarè Invariance*

• Light Front Wavefunctions:



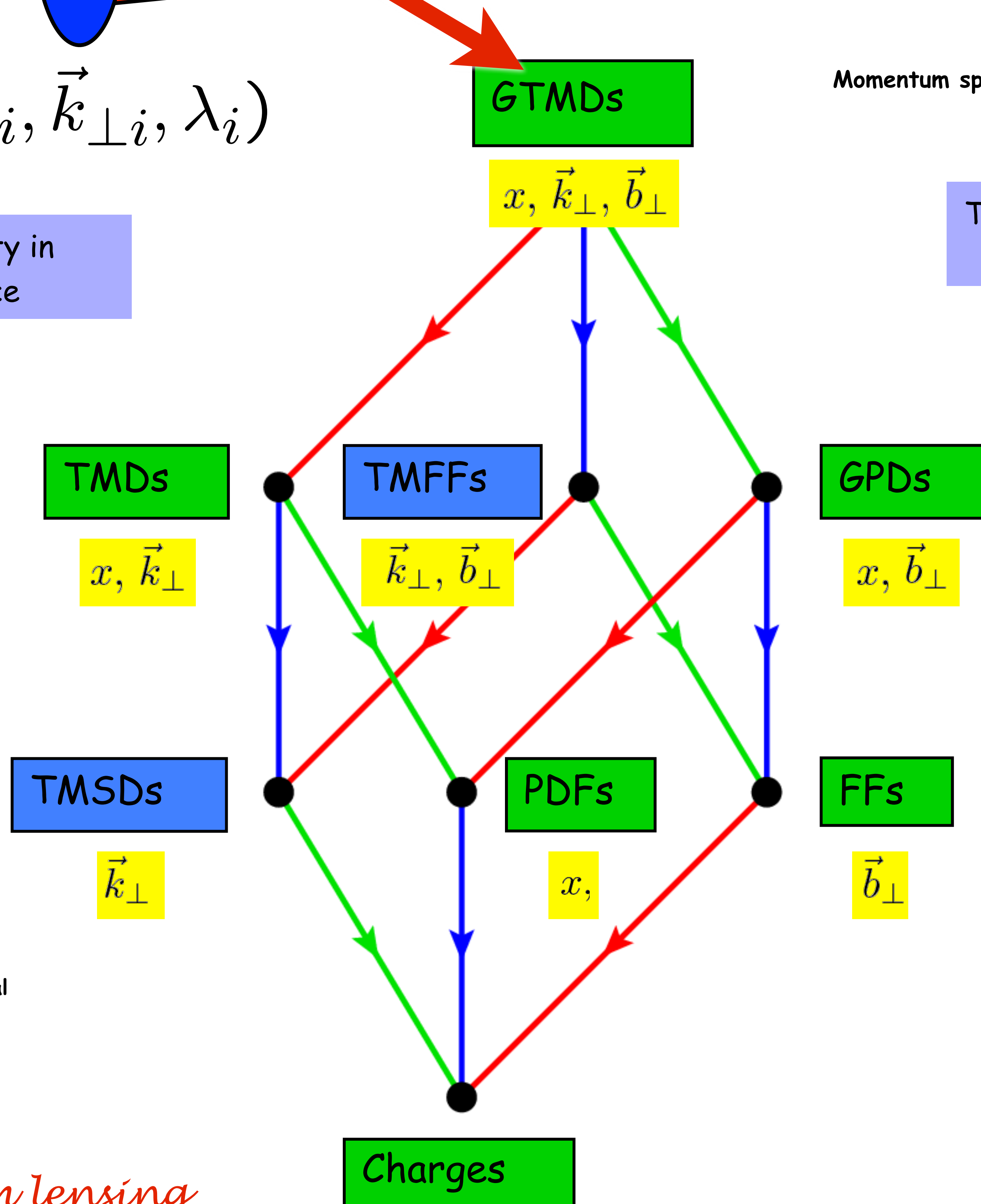
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

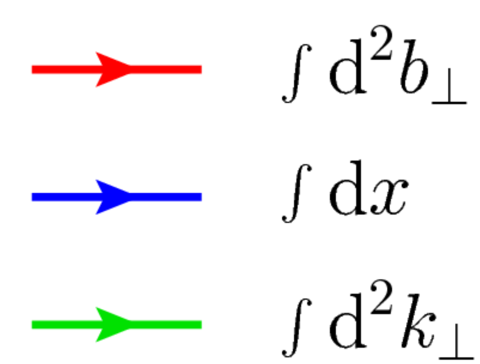
Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

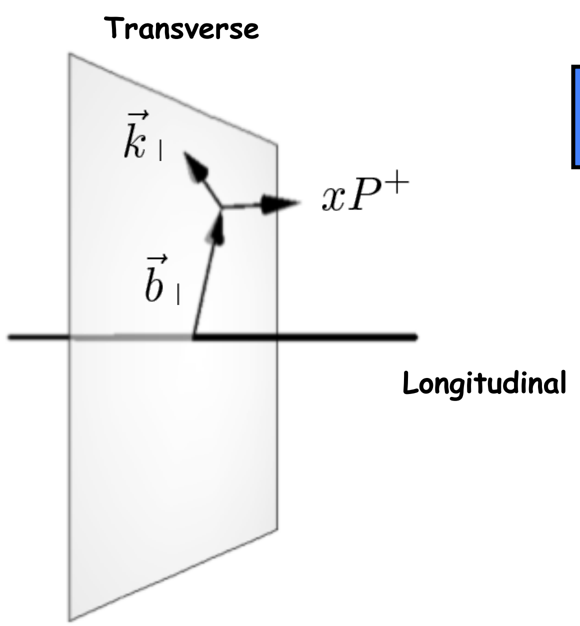
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



Lorce, Pasquini





Sivers, T-odd from lensing

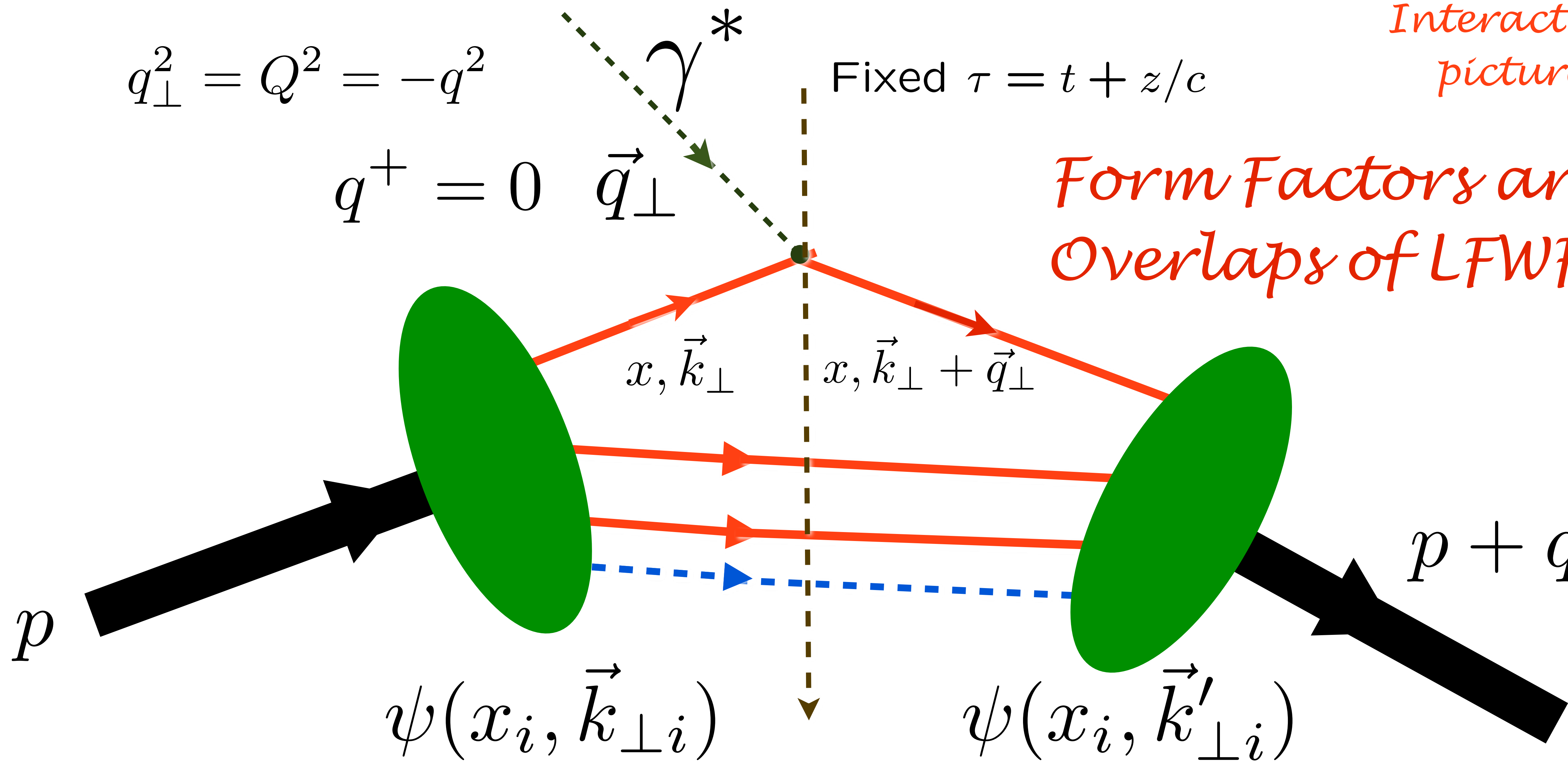
Properties of Hard Exclusive Amplitudes

- ***Form Factors (Elastic and Transition) are overlaps of Light-Front Wavefunctions***
- ***Key Input Hard Exclusive Processes: Distribution amplitudes***
- ***Factorization Theorems*** $\phi_M(x,Q) = \int^Q d^2k_\perp \psi_{q\bar{q}}(x,\vec{k}_\perp)$
- ***Hard Scattering Exclusive Hadron Amplitudes => Distribution amplitudes convoluted with hard subprocesses***
- ***ERBL Evolution of Distribution Amplitudes***
- ***Counting rules reflect leading twist LFWFS***
- ***Hadron-Helicity Conservation (Chiral Theory)***
- ***Quark Interchange Dominance***
- ***Color Transparency***
- ***Hidden Color***

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Exact LF Formula for Pauli Form Factor

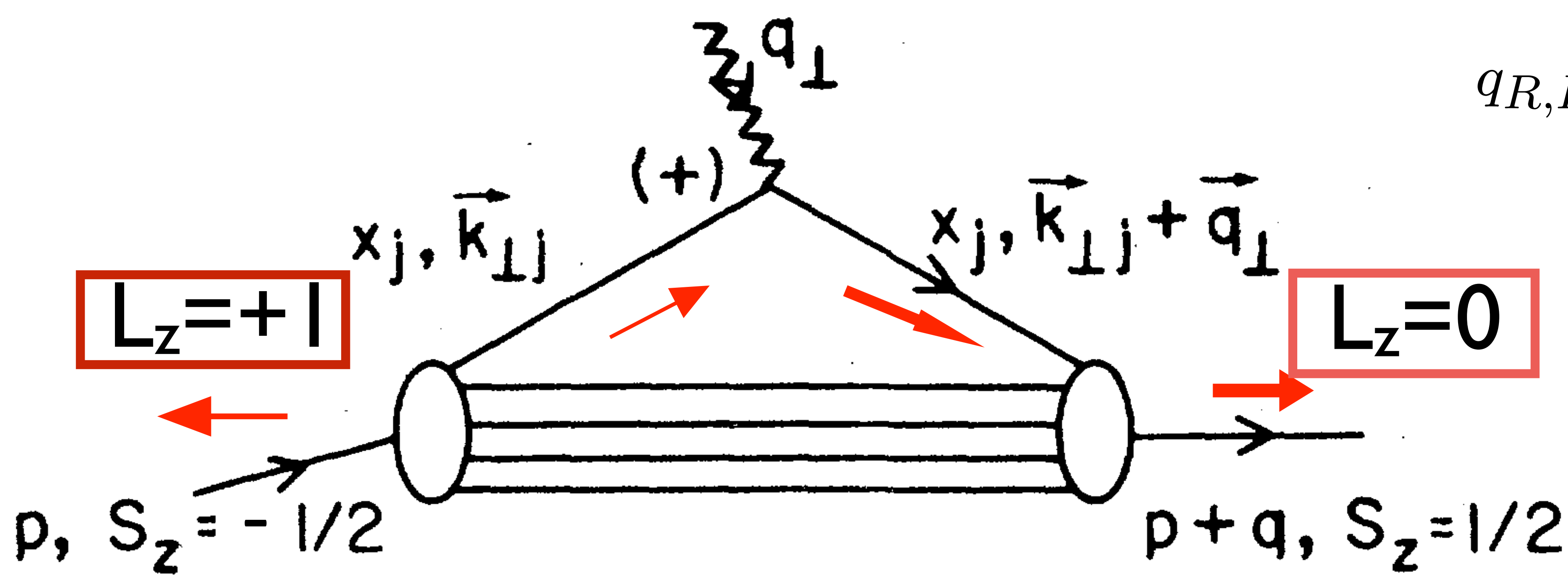
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

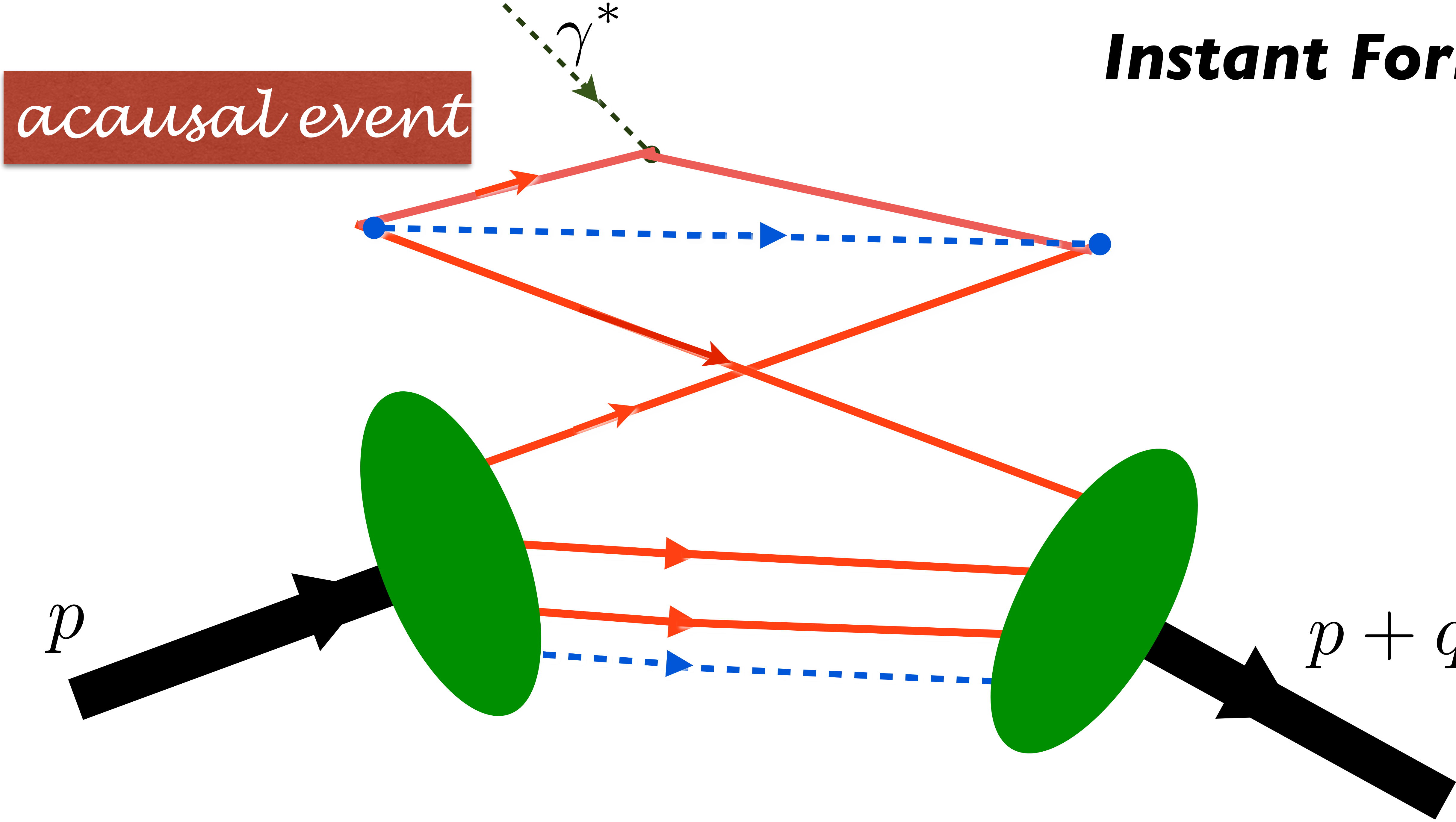
$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Instant Form



Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form

Boosts are dynamical in instant form

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

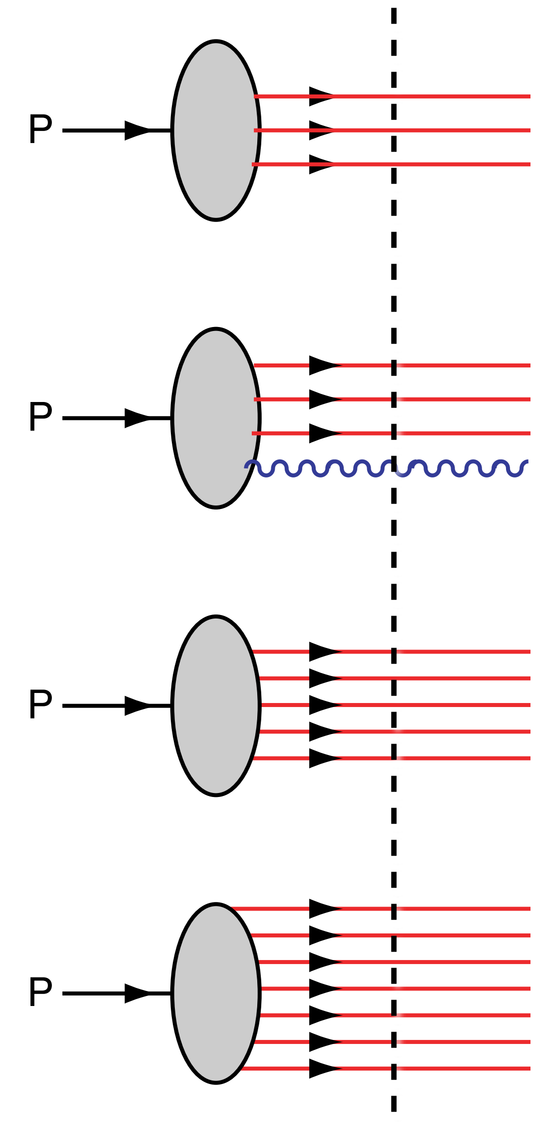
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

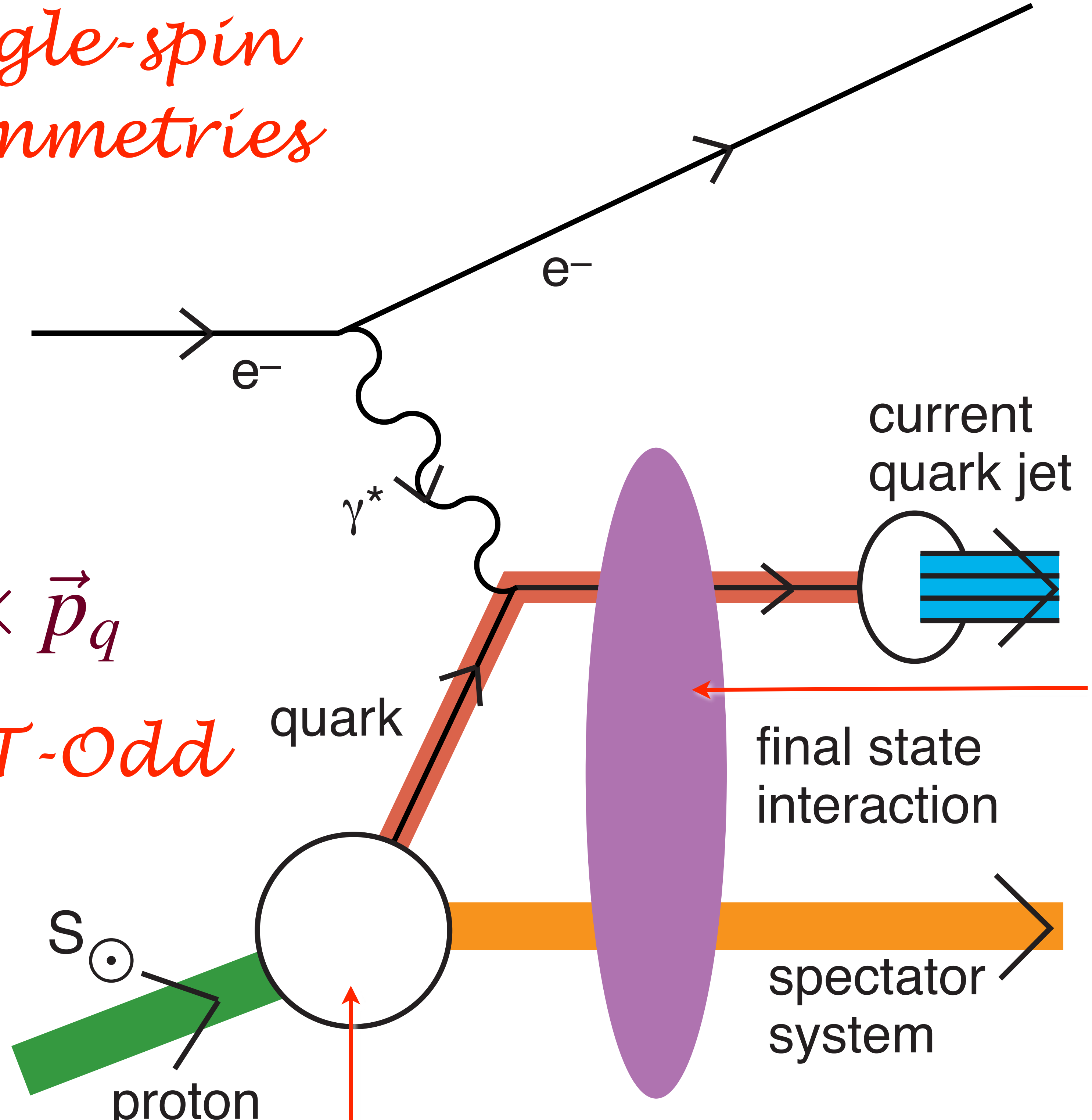
$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Hidden Color

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb



Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i \, \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

QCD S- and P- Coulomb Phases --Wilson Line

Pseudo- T-Odd

"Lensing Effect"

"Lensing" involves soft scales

Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

Advantages of the Dirac's Front Form for Hadron Physics
Poincare' Invariant

Physics Independent of Observer's Motion

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no vacuum condensates!
- Profound implications for Cosmological Constant

Terrell, Penrose

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

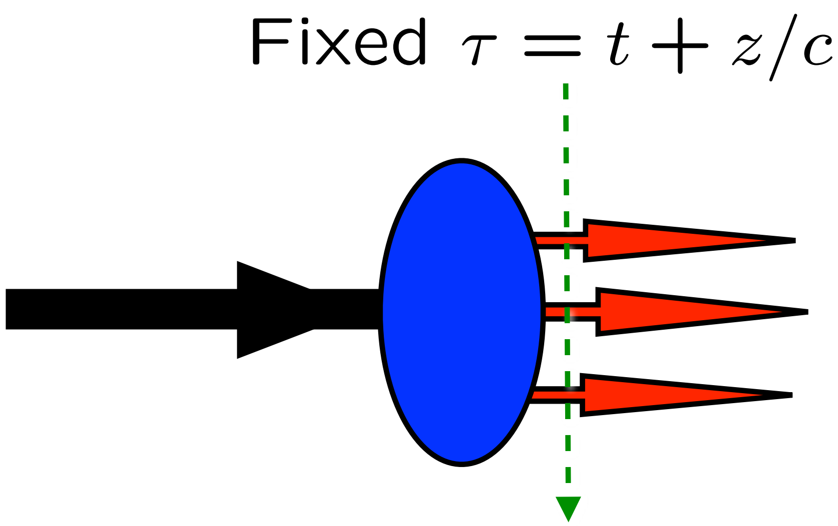
- *"History": Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!*
- *Wick Theorem applies, but few amplitudes since all $k^+ > 0$.*
- *J_z Conservation at every vertex* $$\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n$$ *at order g^n*

  K. Chiu, sjb
- *Unitarity is explicit*
- *Loop Integrals are 3-dimensional* $\int_0^1 dx \int d^2 k_\perp$
- *hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

QED atoms: positronium and muonium

$$H_{QED}$$

$$(H_0 + H_{int}) \; |\Psi> = E \; |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})] \; \psi(\vec{r}) = E \; \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)] \; \psi(r) = E \; \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

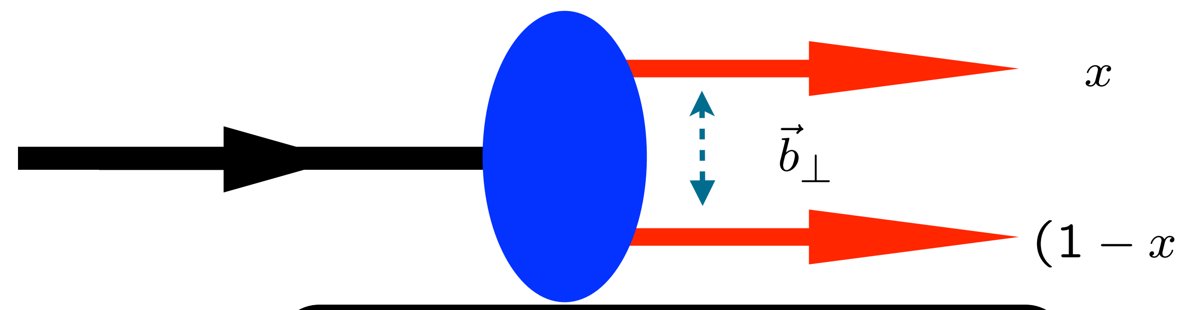
Bohr Spectrum

Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b_\perp^2$$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$\left[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}\right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \; \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Azimuthal Basis ζ, ϕ

$m_q = 0$

Single variable ζ

AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential!

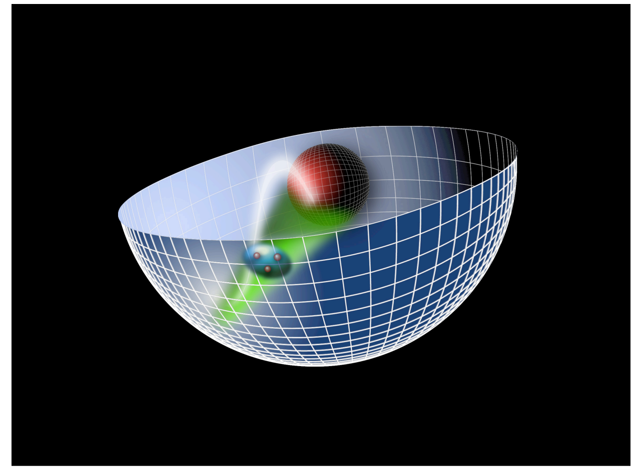
Semiclassical first approximation to QCD

Sums an infinite # diagrams

de Tèramond, Dosch, Lorcè, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)]\psi = M^2\psi$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Single variable ζ

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

de Tèramond, Dosch, Lorcè, sjb

AdS/QCD
Soft-Wall Model

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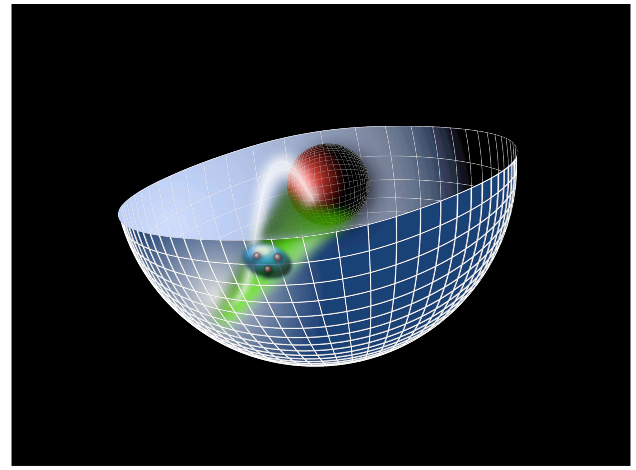

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)]\psi = M^2\psi$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

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- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial\tau}|\psi(\tau) >$$

New term

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4} x^2 \right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)]\psi = M^2\psi$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

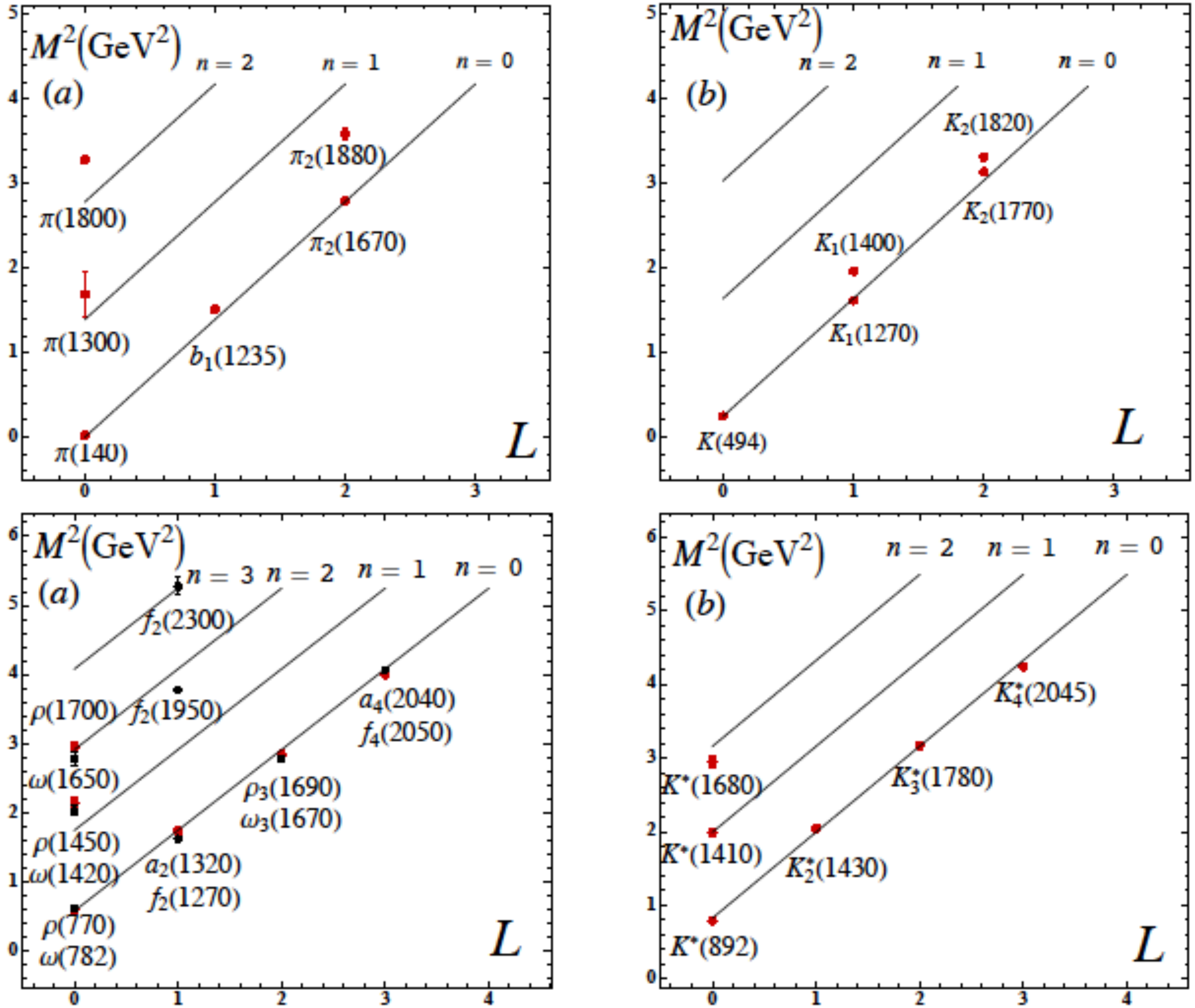
$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

G. de Teramond, H. G. Dosch, sjb

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

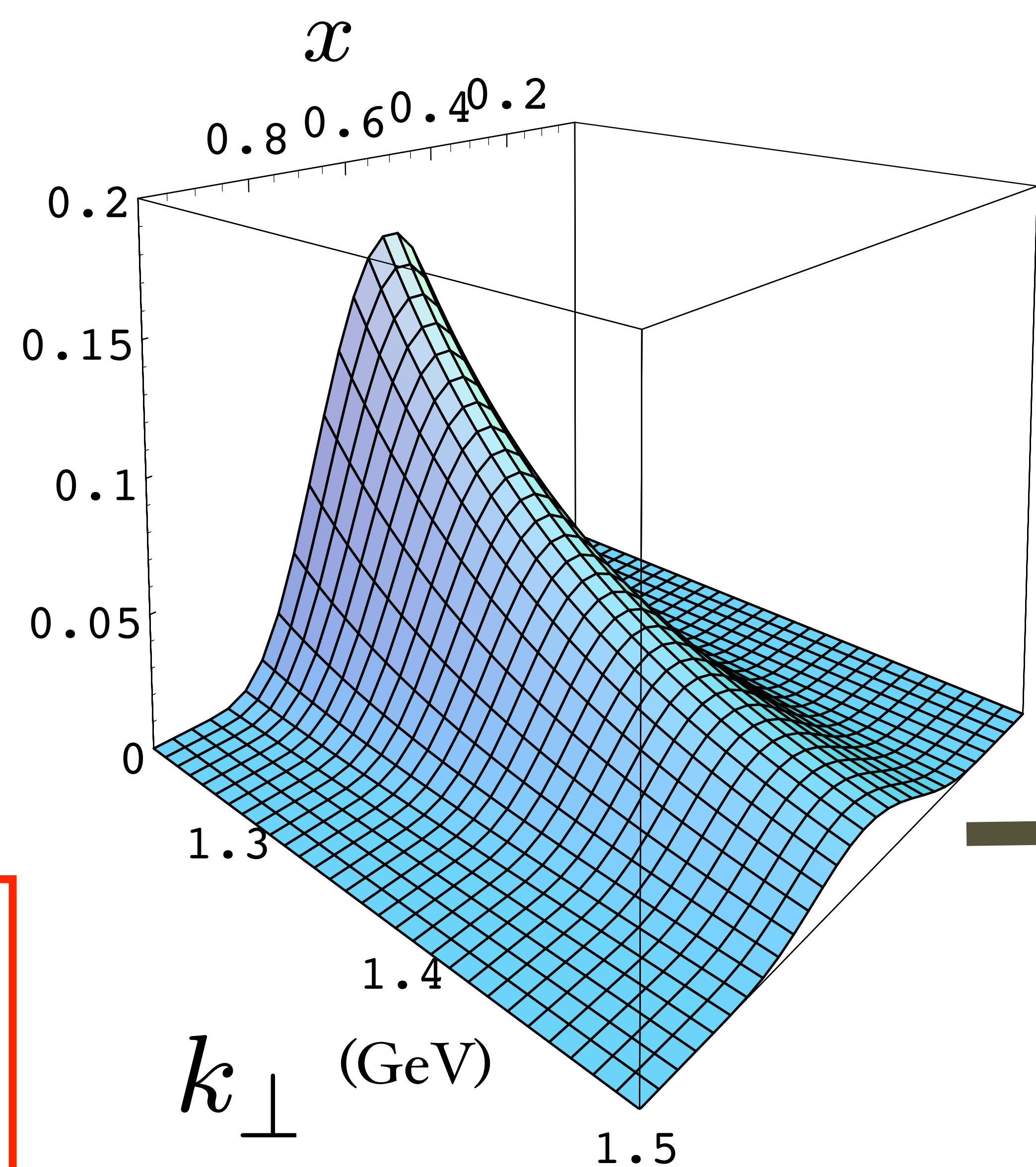
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Prediction from AdS/QCD: Meson LFWF

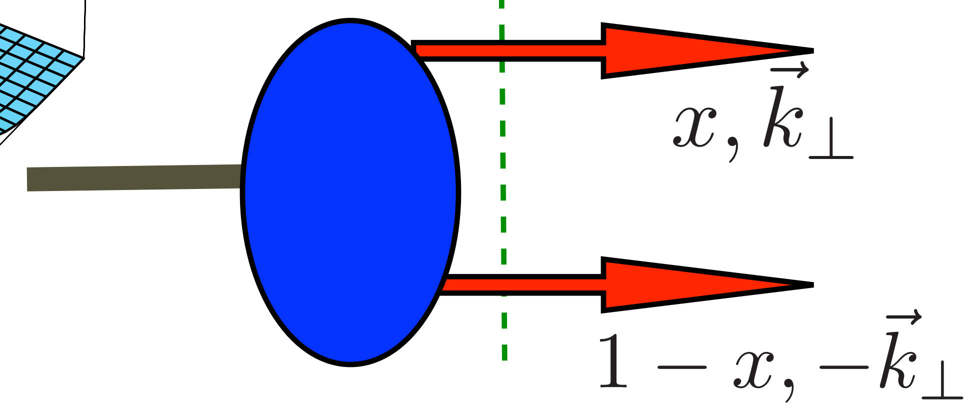
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

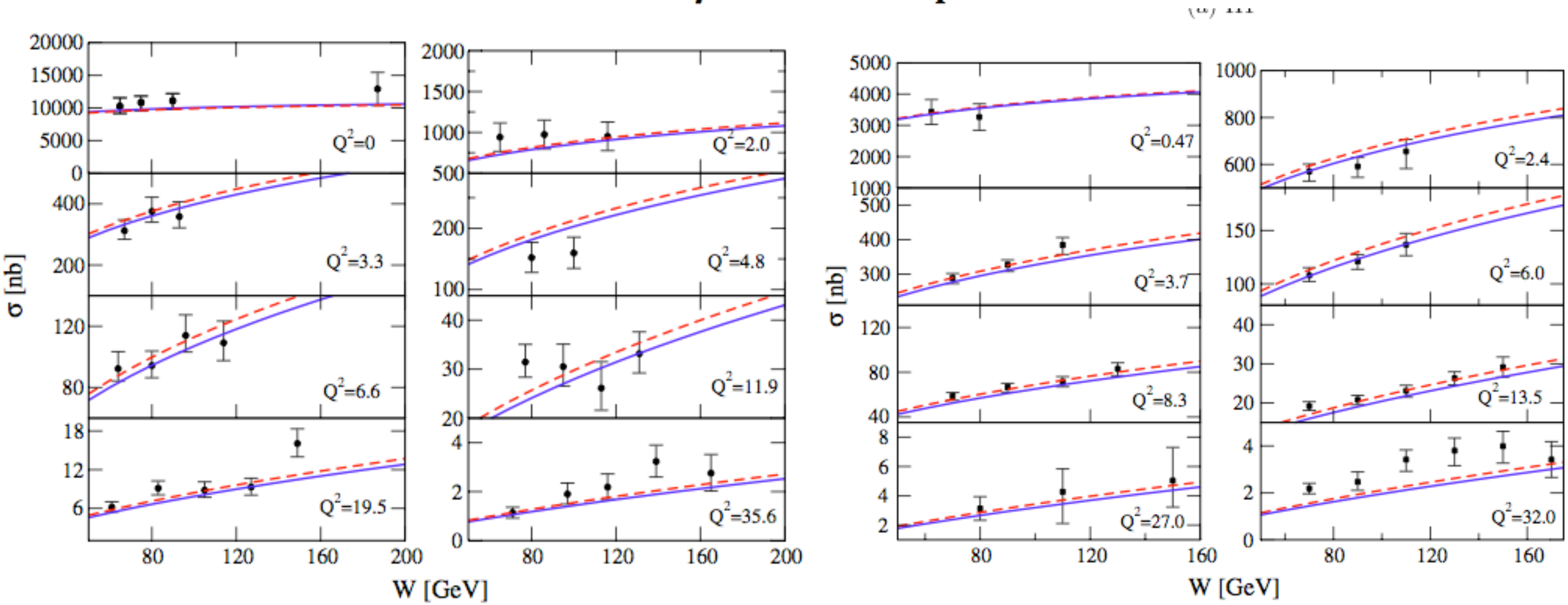
$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

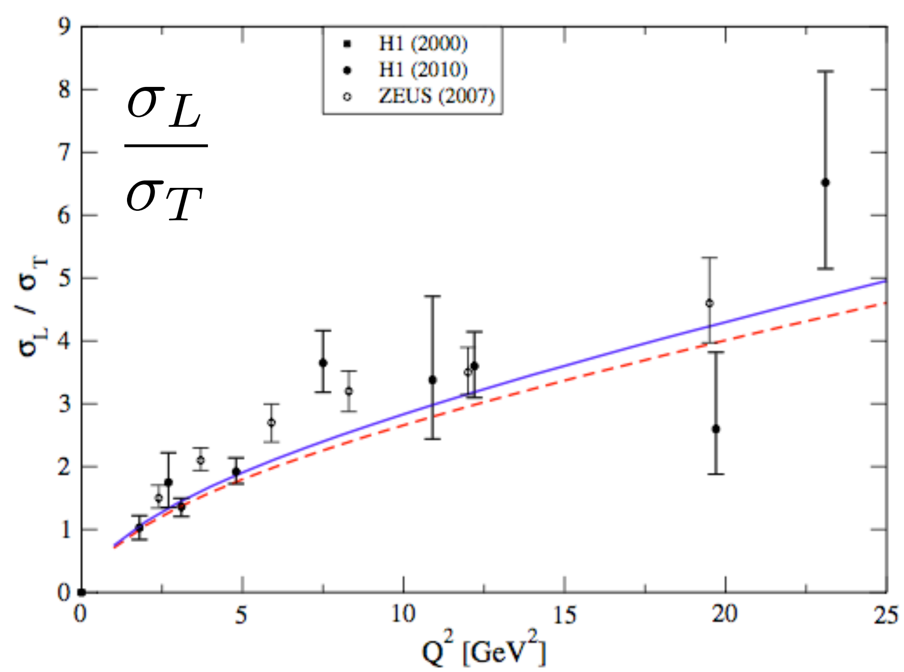
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H1 (b) ZEUS

J. R. Forshaw, R. Sandapen

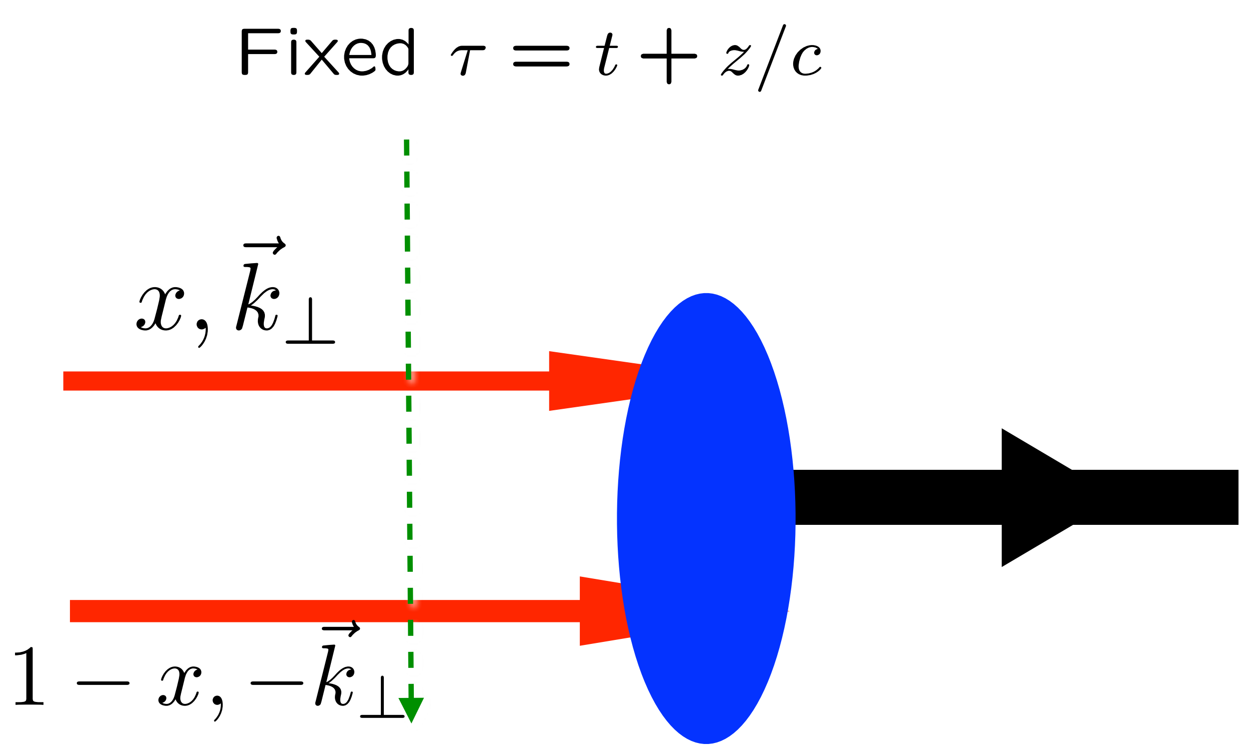
$$\gamma^* p \rightarrow \rho^0 p'$$



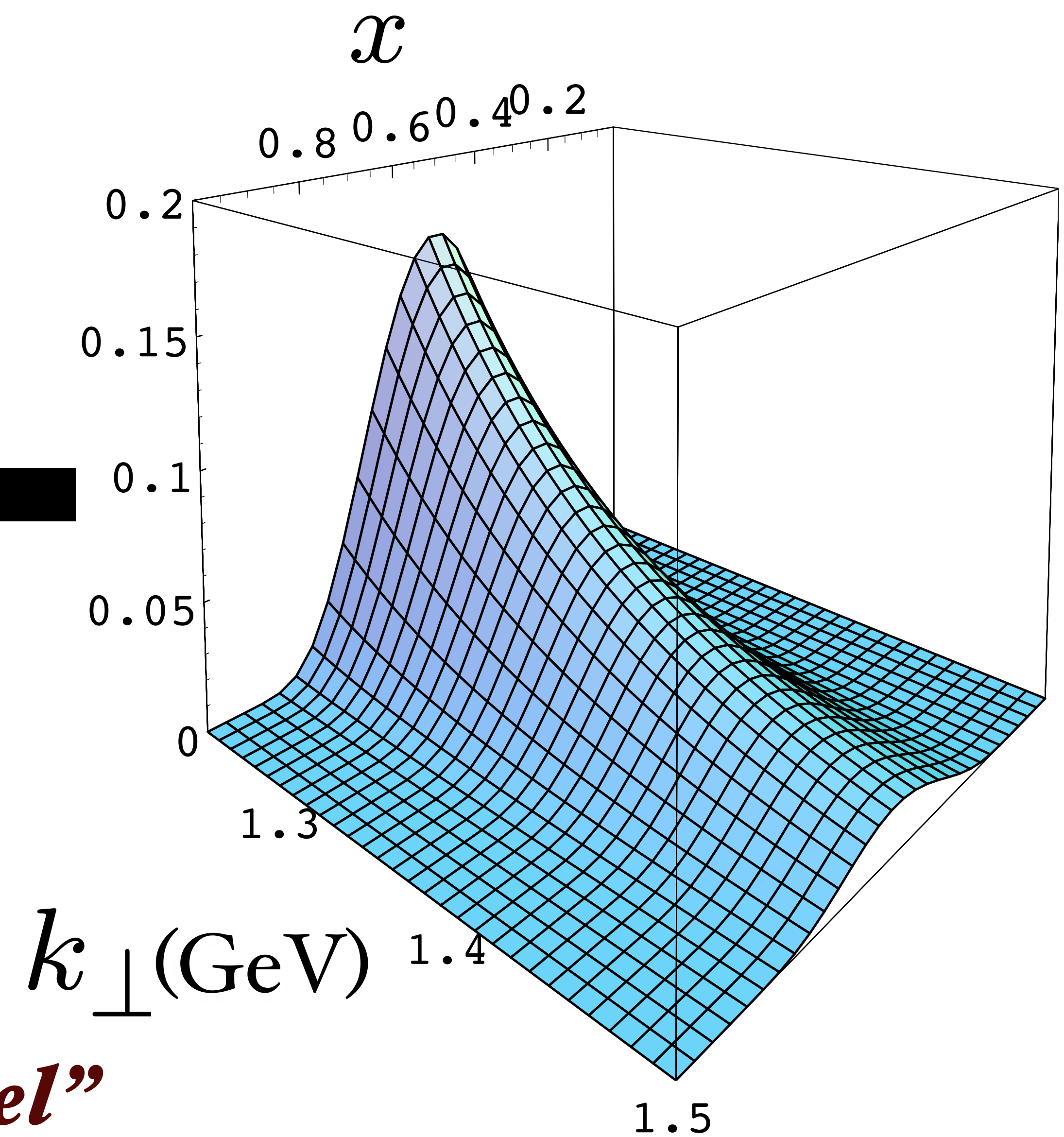
$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$

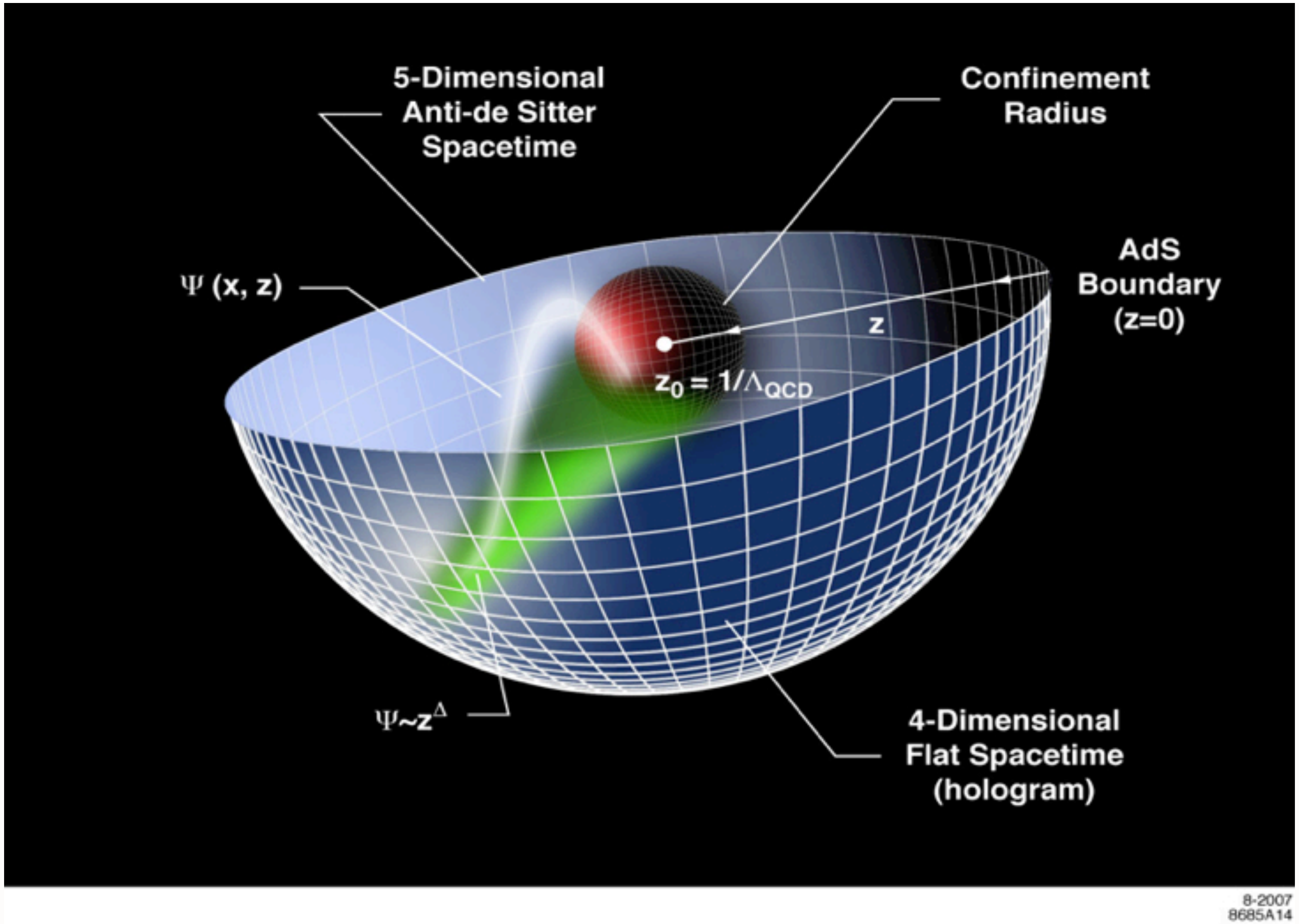


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



"Hadronization at the Amplitude Level"

Boost-invariant LFWF connects confined quarks and gluons to hadrons

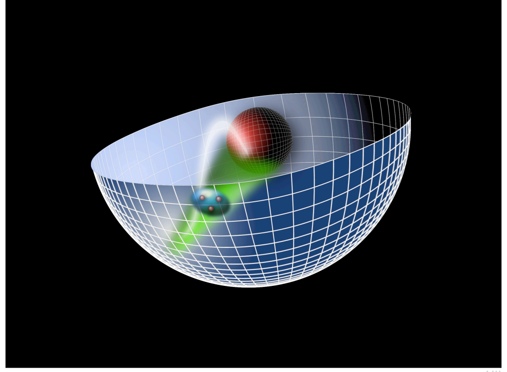


Changes in physical length scale mapped to evolution in the 5th dimension z

AdS_5

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).
- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS$_5$



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

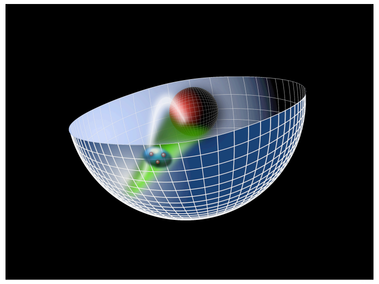
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

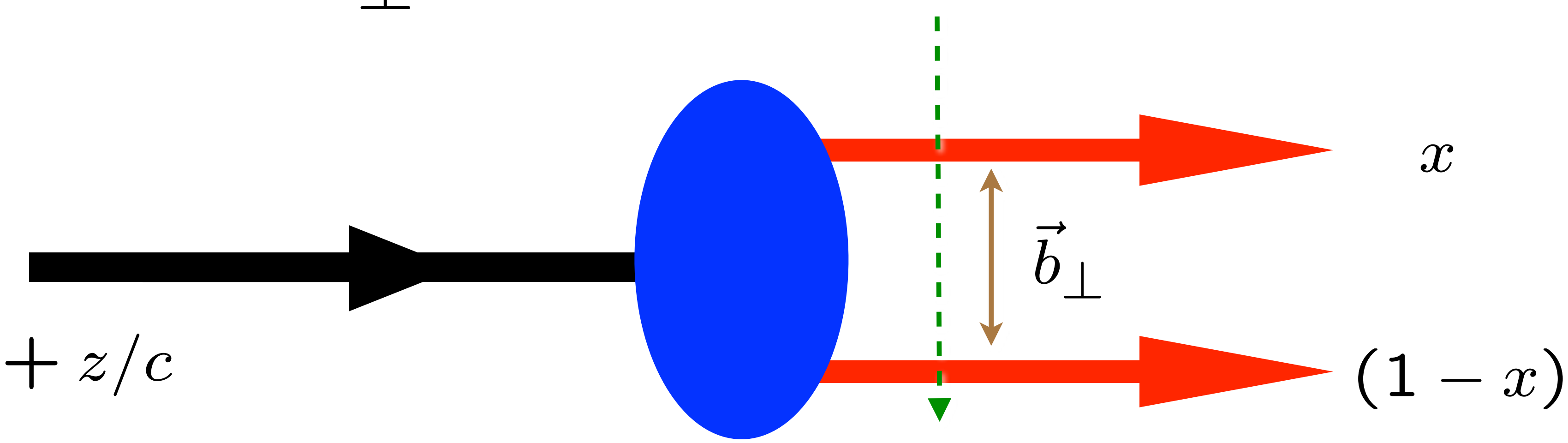
de Teramond, sjb

LF(3+1) $\longleftrightarrow$ AdS_5

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



Fixed $\tau = t + z/c$

$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

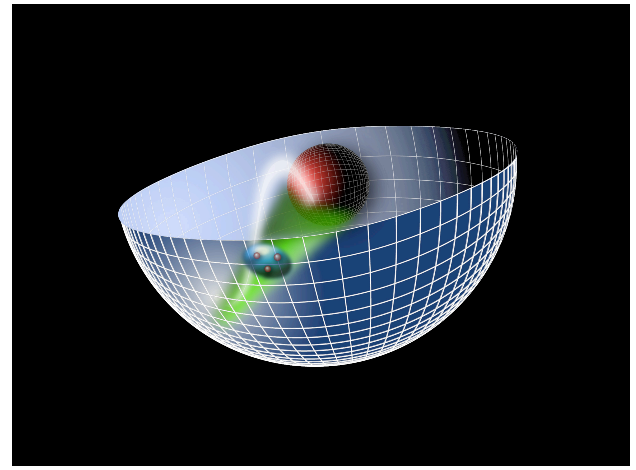
$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, sjb

AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp .$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Single variable ζ

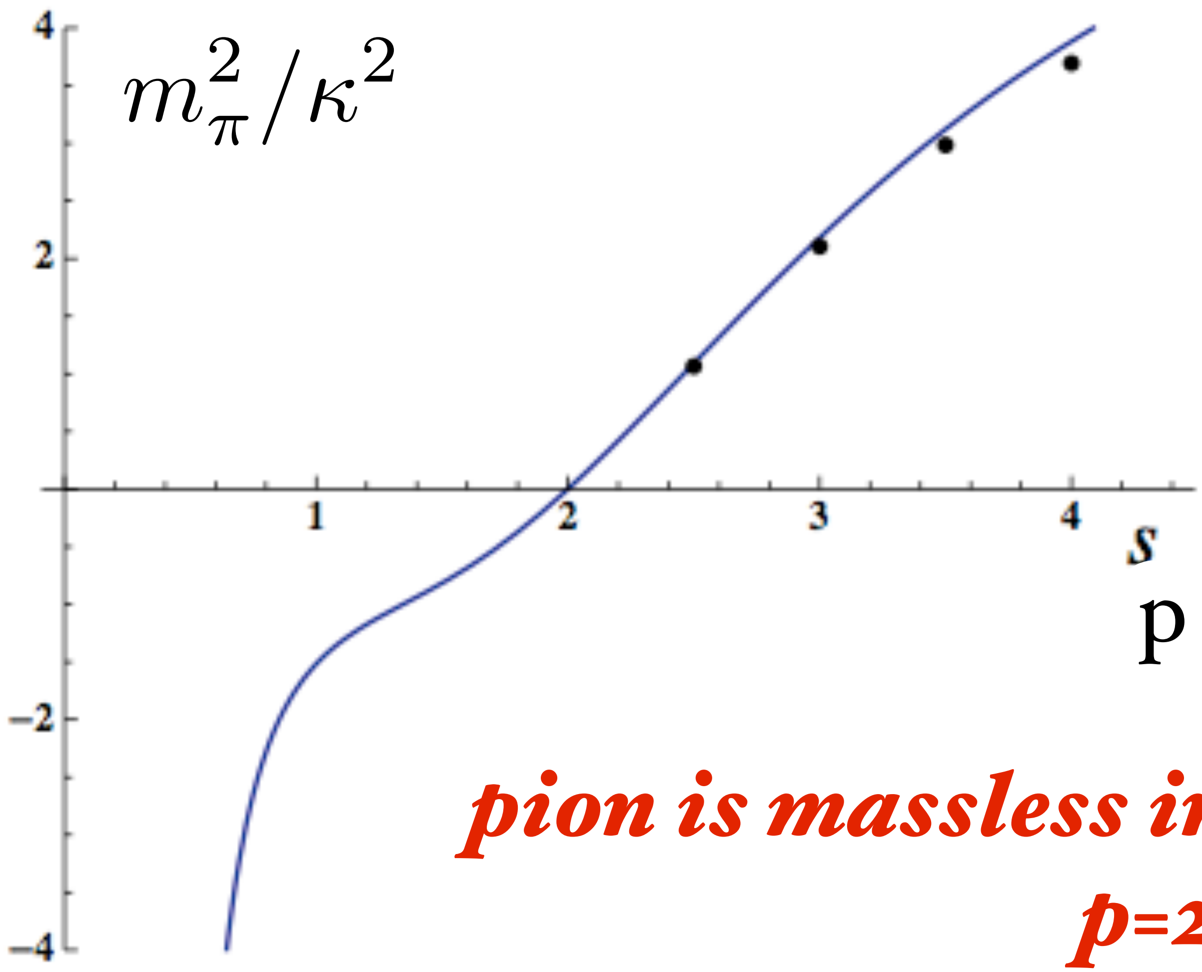
Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

- **Dosch, de Tèramond, sjb**

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

$$[H,D] = i\,H, \quad [H, K] = 2\,i\,D, \quad [K, D] = -\,i\,K$$

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Fubini and Rabinovici

Baryon Equation

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Consider $R_w = Q + wS;$

w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \big(-\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \big)$$

$$G_{22} = \big(-\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \big)$$

Identify $f - \frac{1}{2} = L_B \ , \quad w = \kappa^2$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\Big)\psi_J^+ = M^2\psi_J^+$$

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\Big)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\Big(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\Big)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ!

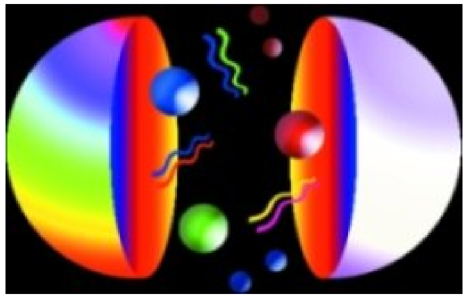
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

Quark Chiral Symmetry of Eigenstate!

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- "Chiral partners"

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Nucleon: Equal Probability for L=0,1

AdS/QCD + Light Front Holography: Proton is bound state of a quark + scalar diquark

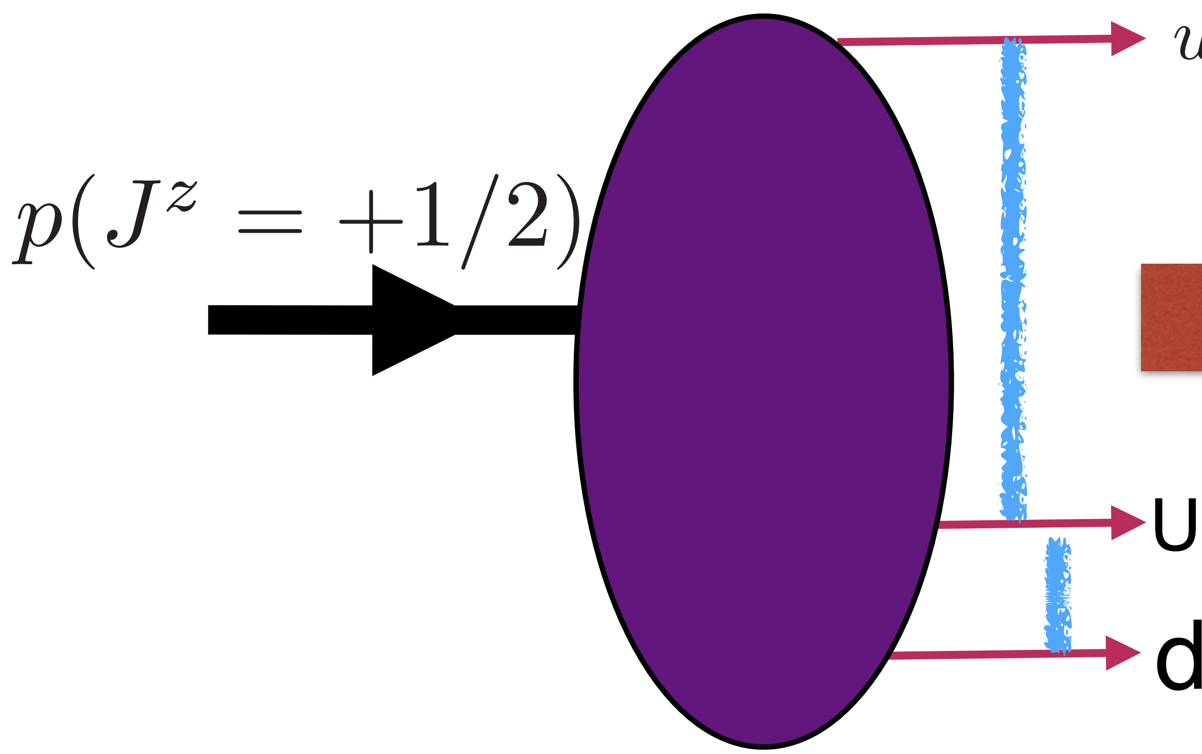
de Teramond, Dosch, Lorce, sjb

Skyrme model: Ellis, Karliner, sjb

LF J^z conservation: K. Chiu, sjb

$$3_C \times 3_C = \bar{3}_C + \cancel{6}_C$$

$$|p> = |u_{3C}[ud]_{\bar{3}C}>$$



$u(S^z = +1/2)$ $u(S^z = -1/2)$

$L^z = 0$ $L^z = -1$

Equal probability L=0, L=1

J=0

Quark chiral symmetry
Anomalous moment nonzero
Leading Twist Sivers Effect

Gluonic distribution reflects quark+diquark color structure of the proton

Color confinement potential —> high density gluon field: flux tube

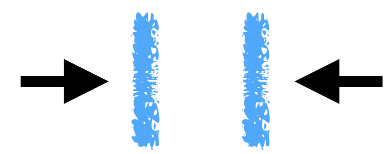
Collisions of flux tubes of protons

Color confinement potential —> high density gluon field: flux tube

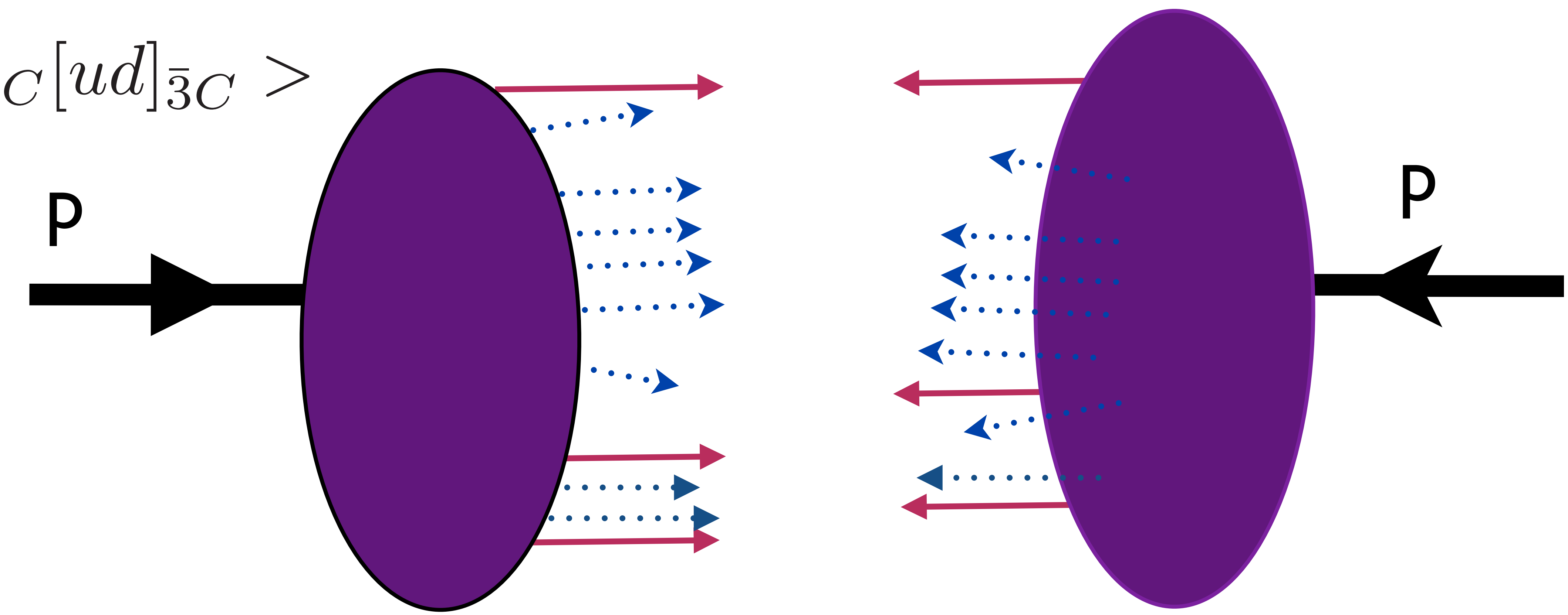
Highest hadron multiplicity produced when the two flux tubes are aligned and overlap completely along their length.

Bjorken, Goldhaber, sjb

Hadrons produced from the collisions of flux tubes



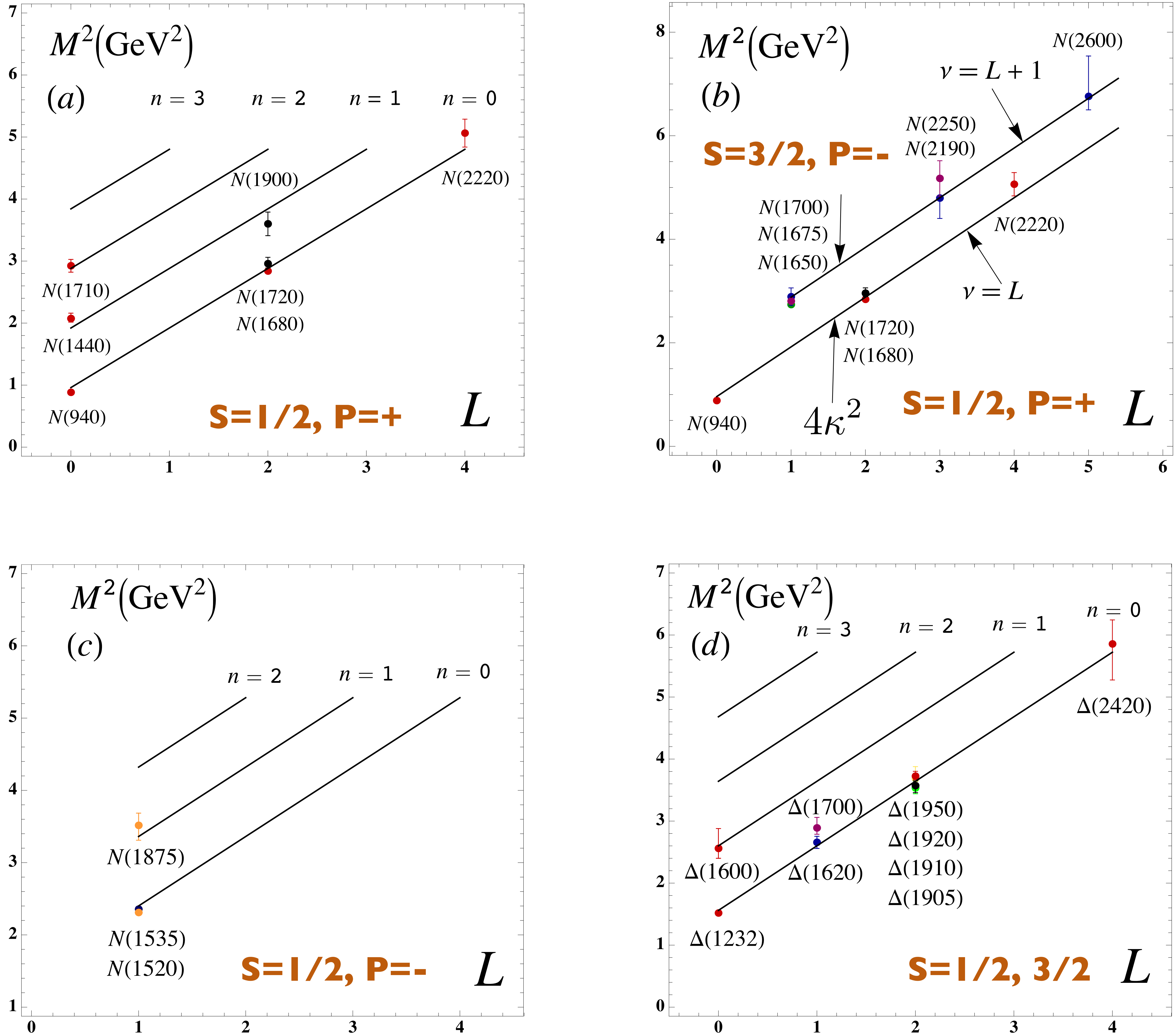
$$|p>=|u_{3C}[ud]_{\bar{3}C}>$$



Gluonic distribution reflects quark+diquark color structure of the protons

v_2 (dominant) + v_3 (from `Y' quark + diquark configurations)

- ***Strangeness and charm enhancements***



$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
ν = L + 1
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
ν = L
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 3
n = 2
n = 1
n = 0
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

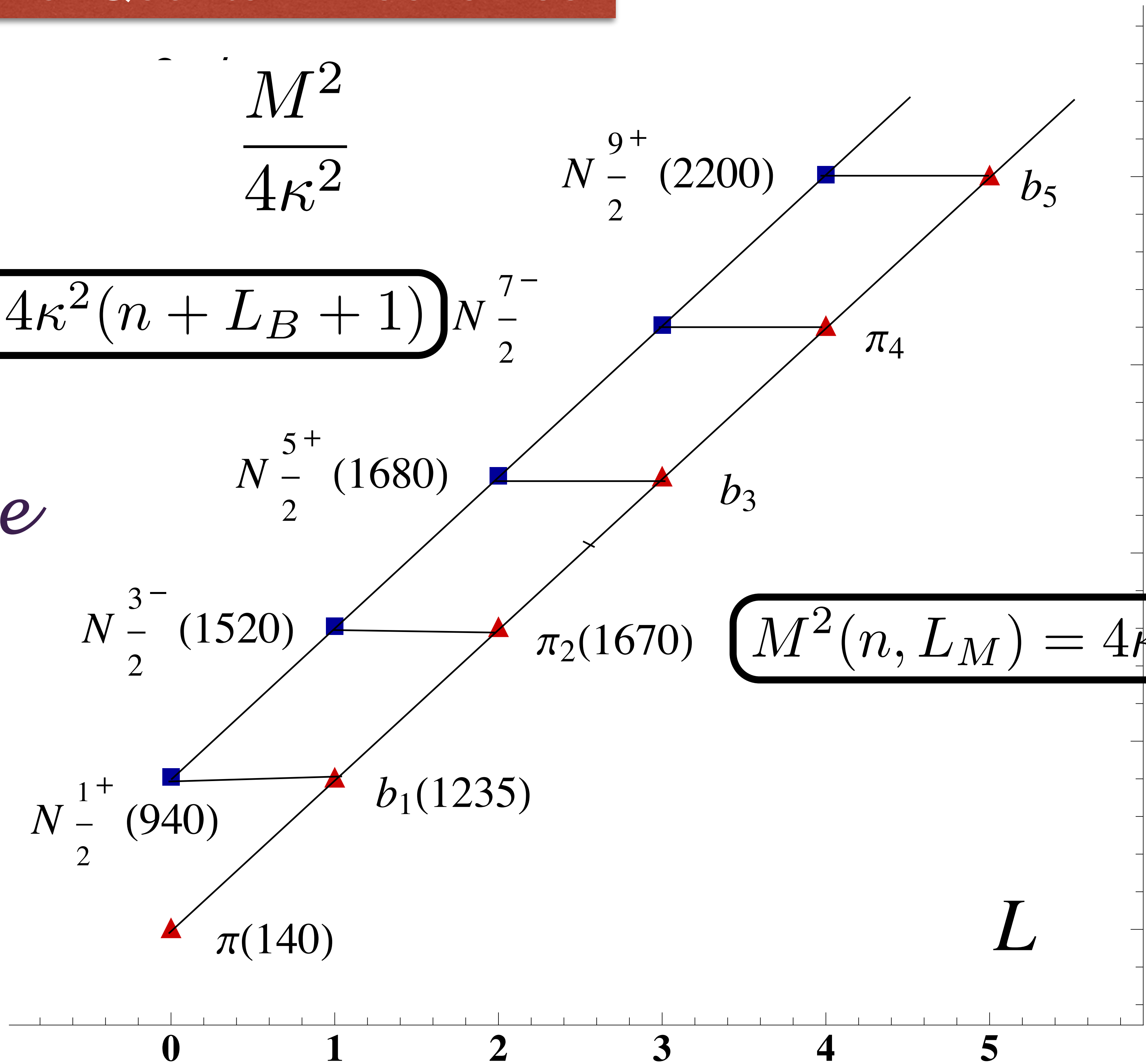
Superconformal Quantum Mechanics

de Tèramond, Dosch, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope

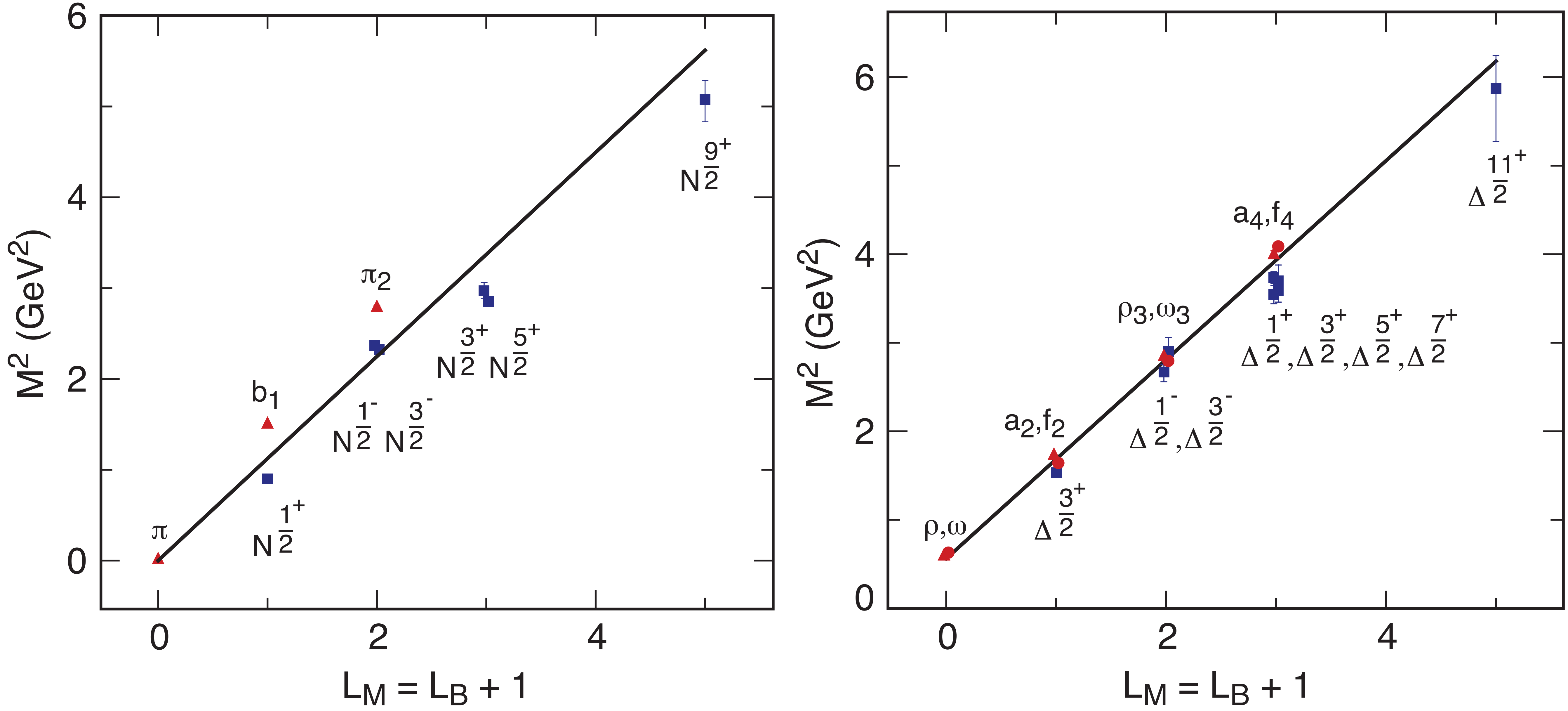
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$



$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

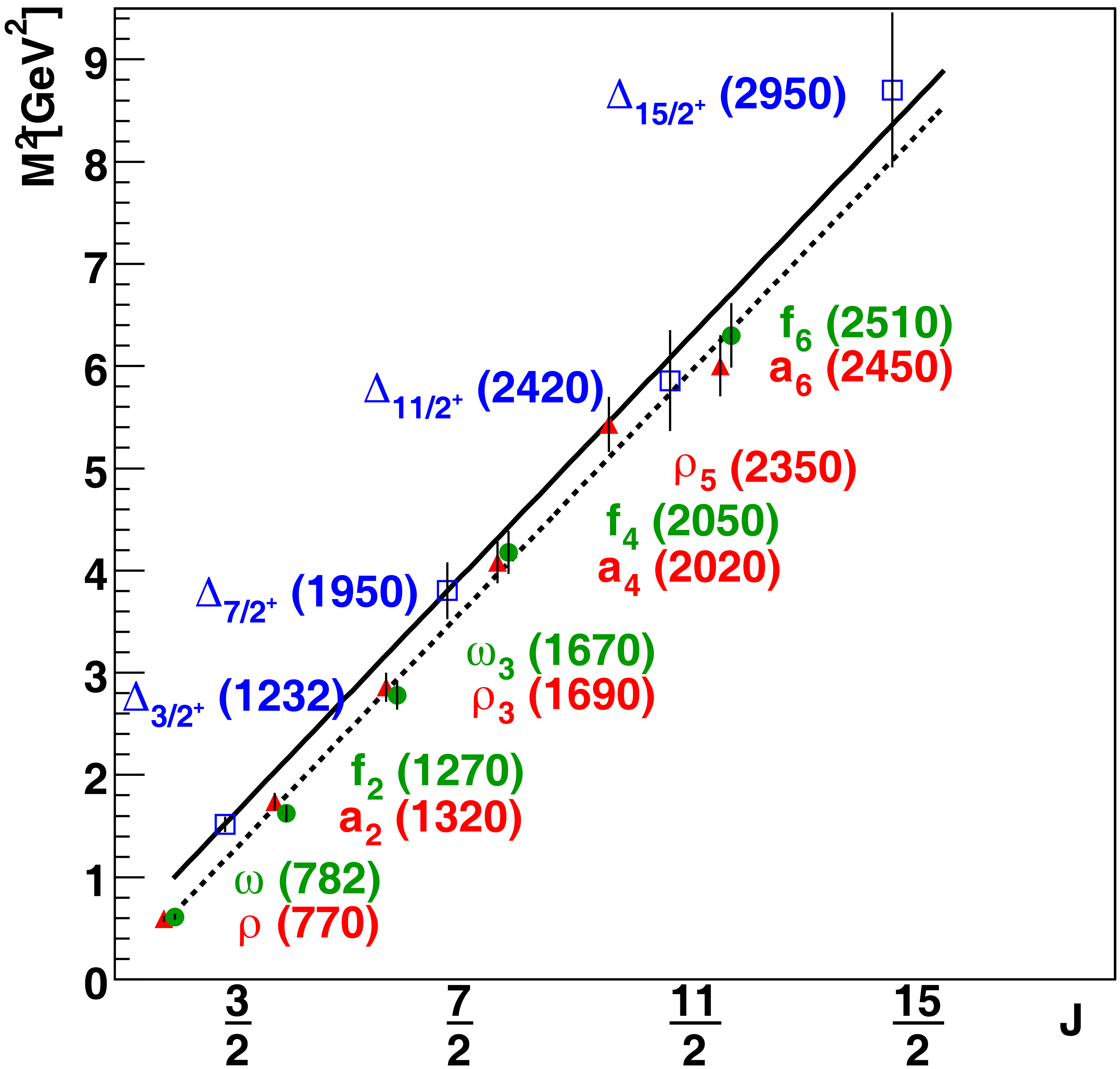
Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$

Solid line: $\varkappa$ = 0.53 GeV



Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb

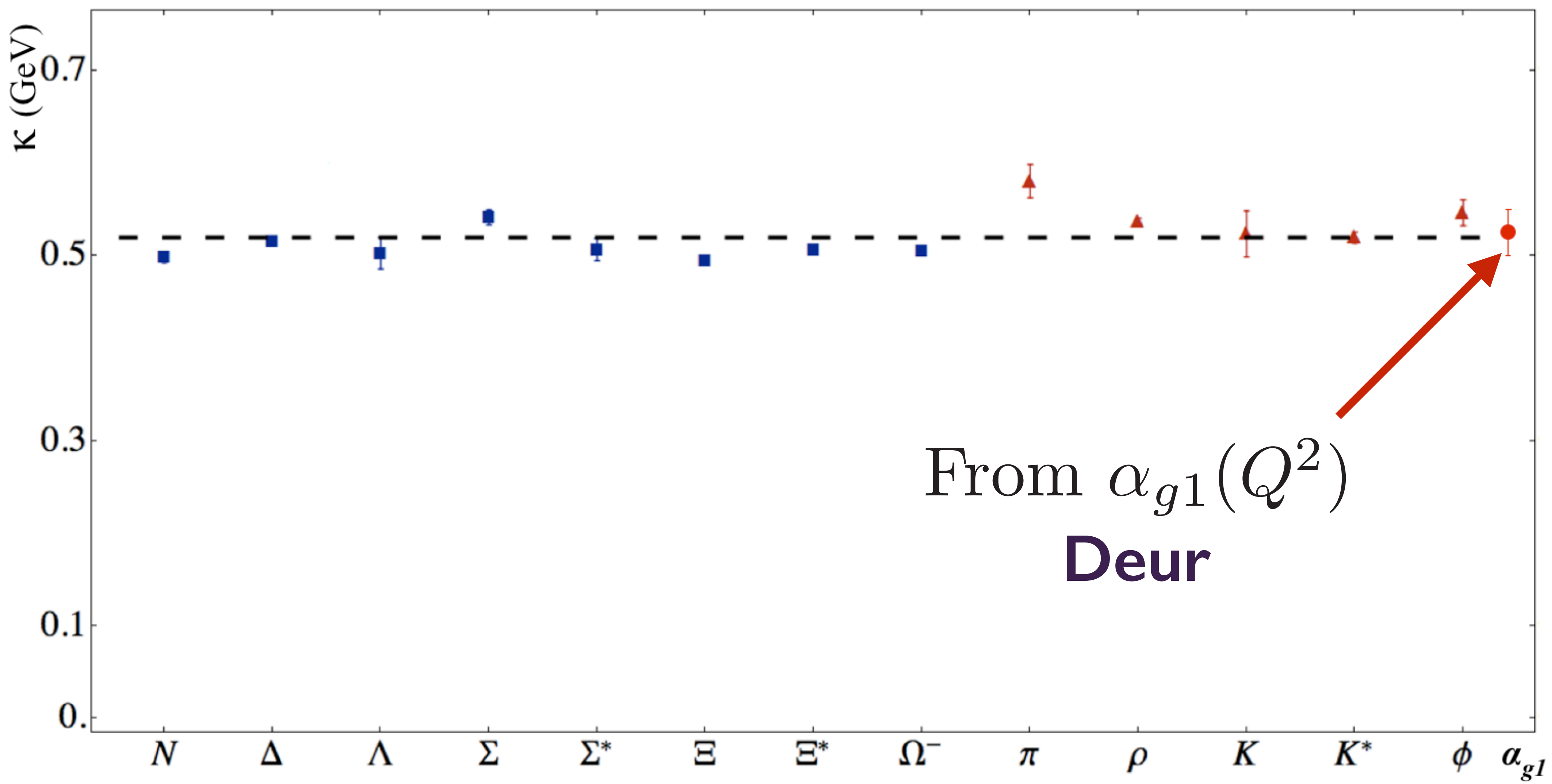


The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with J = L+S.

Dosch, de Teramond, Lorce, sjb

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$

κ



Fit to the slope of Regge trajectories, including radial excitations

Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates (even the pion) have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$J^z = +1/2 :< L^z >= 1/2, < S^z_q >= 0$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

No mass -degenerate parity partners!

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} \, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad ({F_1}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \, \Psi_+^2(z) = 1$$

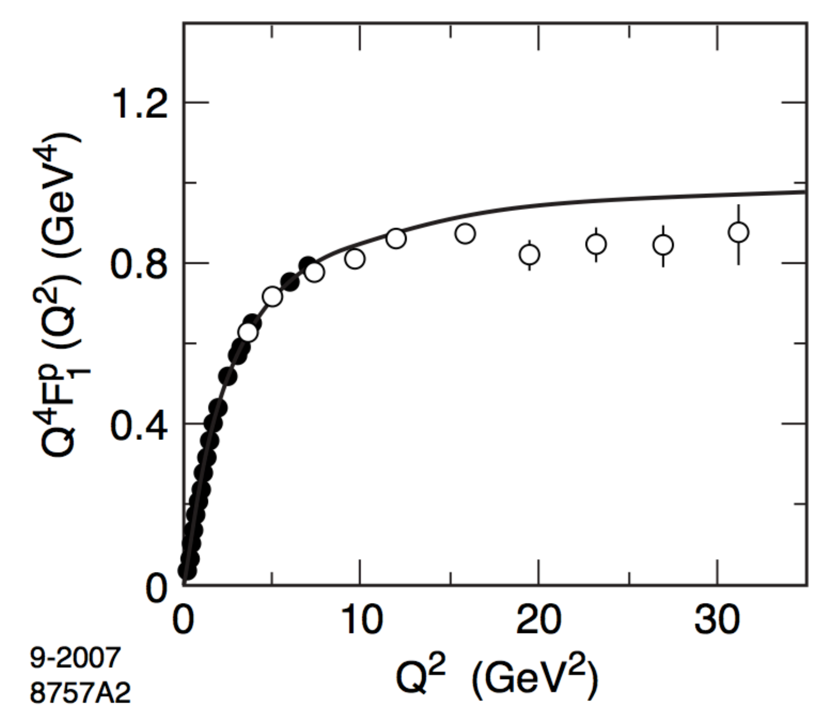
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} \, e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with ${\mathcal{M}_{\rho_n}}^2 \to 4\kappa^2(n+1/2)$



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$

- Transition form factor

$$F_1{}^p_{N\to N^*}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_1{}^p_{N\to N^*}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_1{}^p_{N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

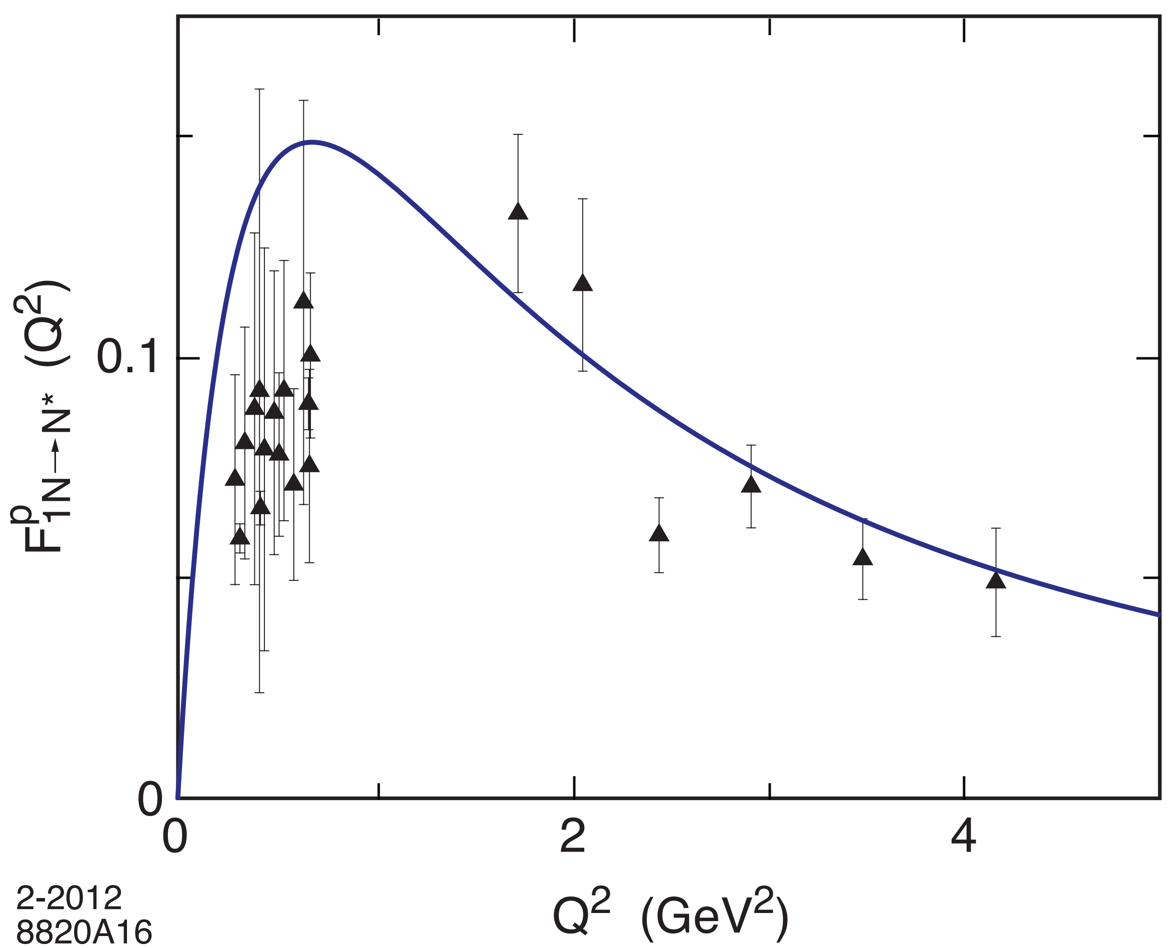
with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

Nucleon Transition Form Factors

$$F^{p}_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\,\frac{\frac{Q^2}{\mathcal{M}_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}.$$

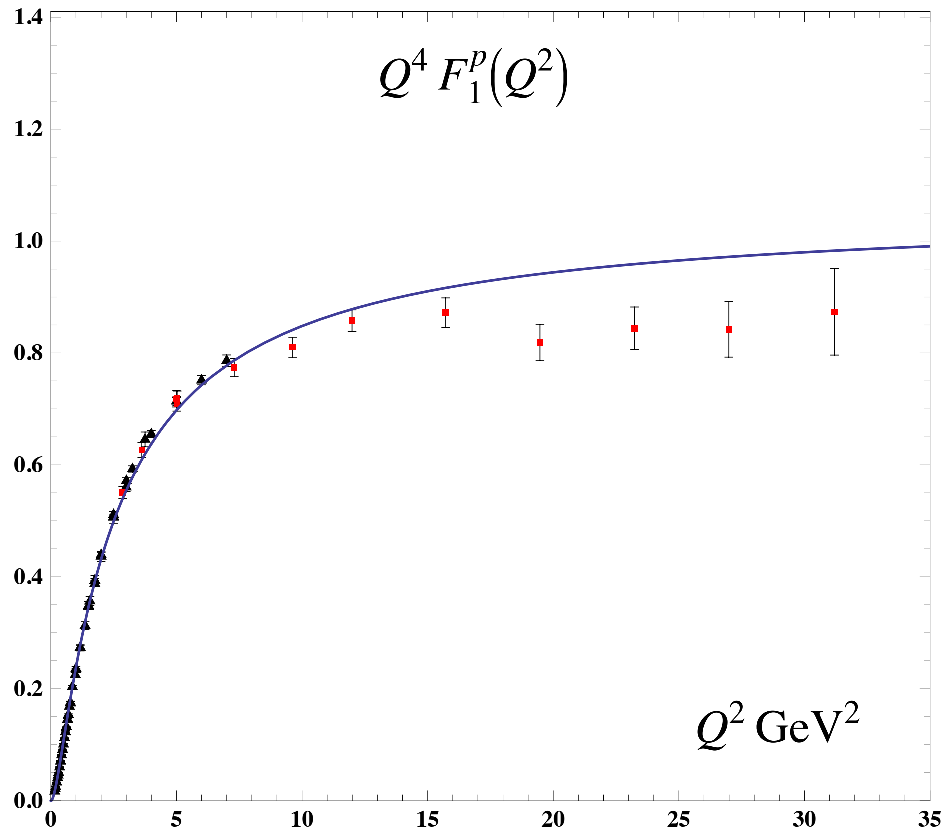


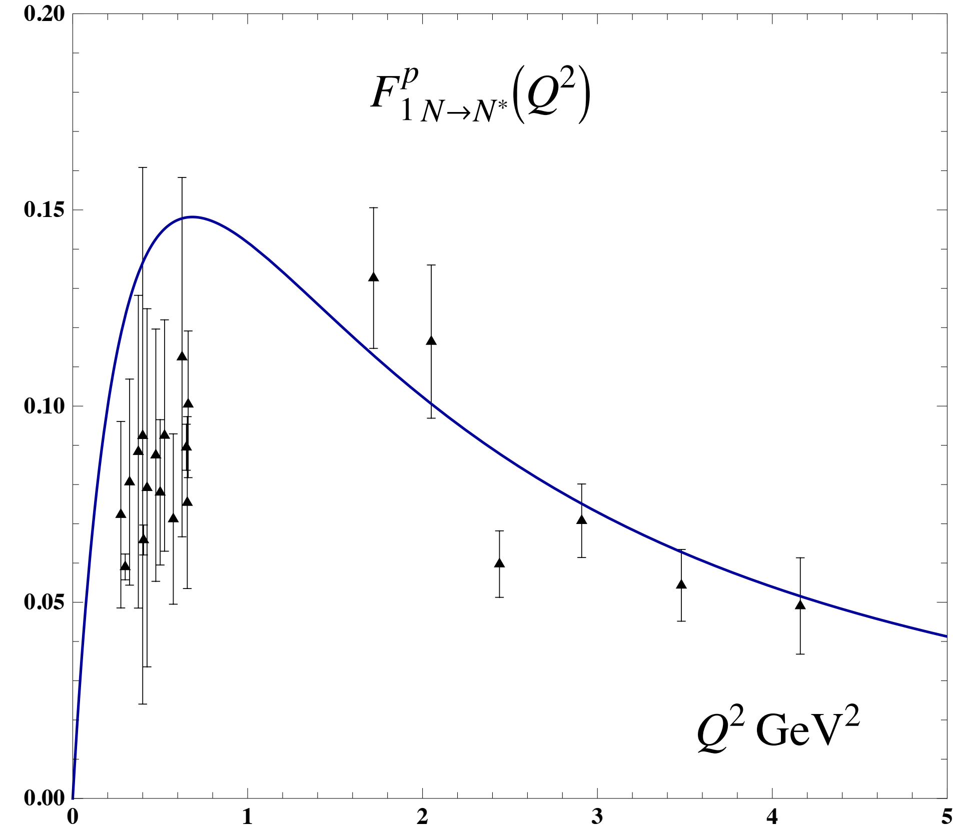
Proton transition form factor to the first radial excited state. Data from JLab

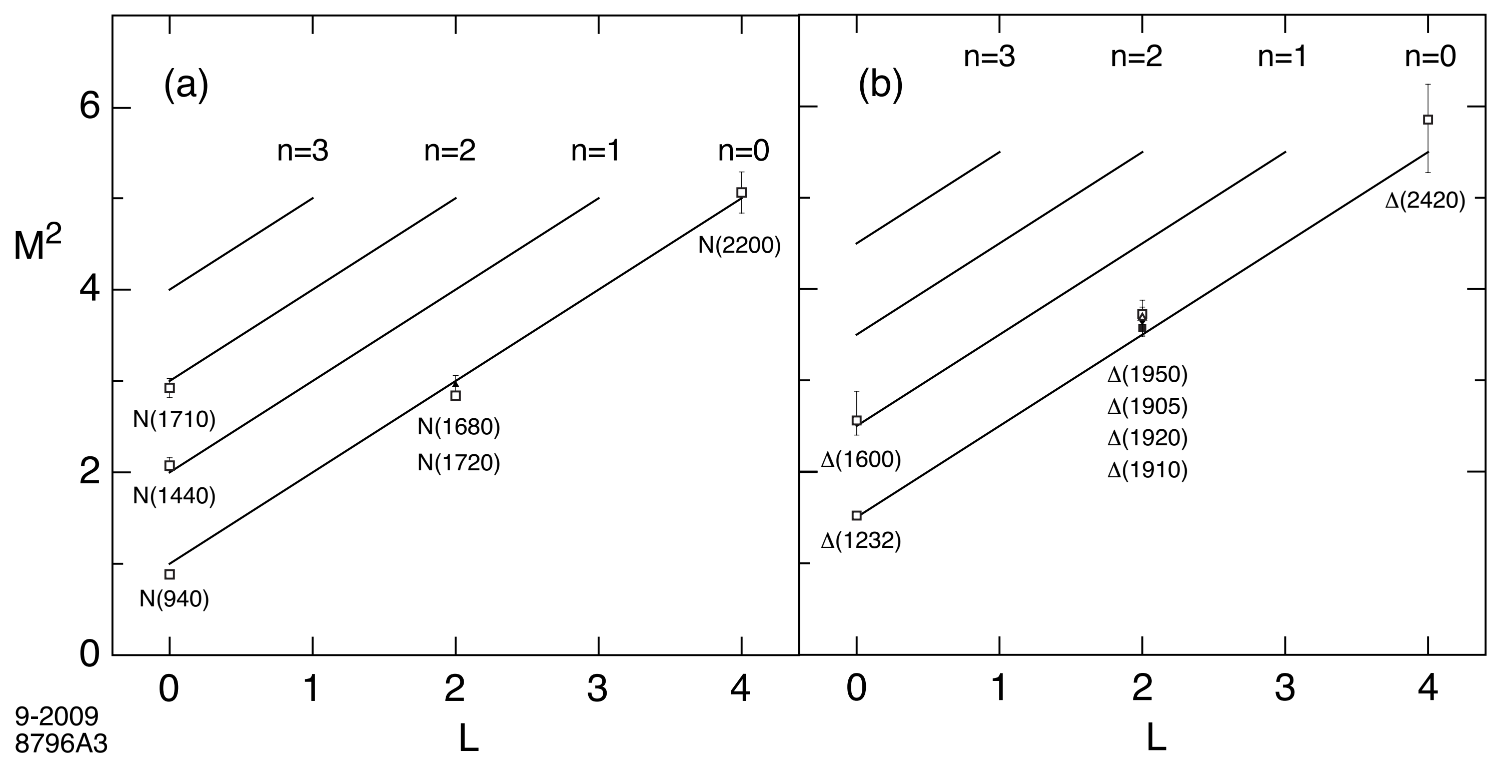
Predict hadron spectroscopy and dynamics

Excited Baryons in Holographic QCD

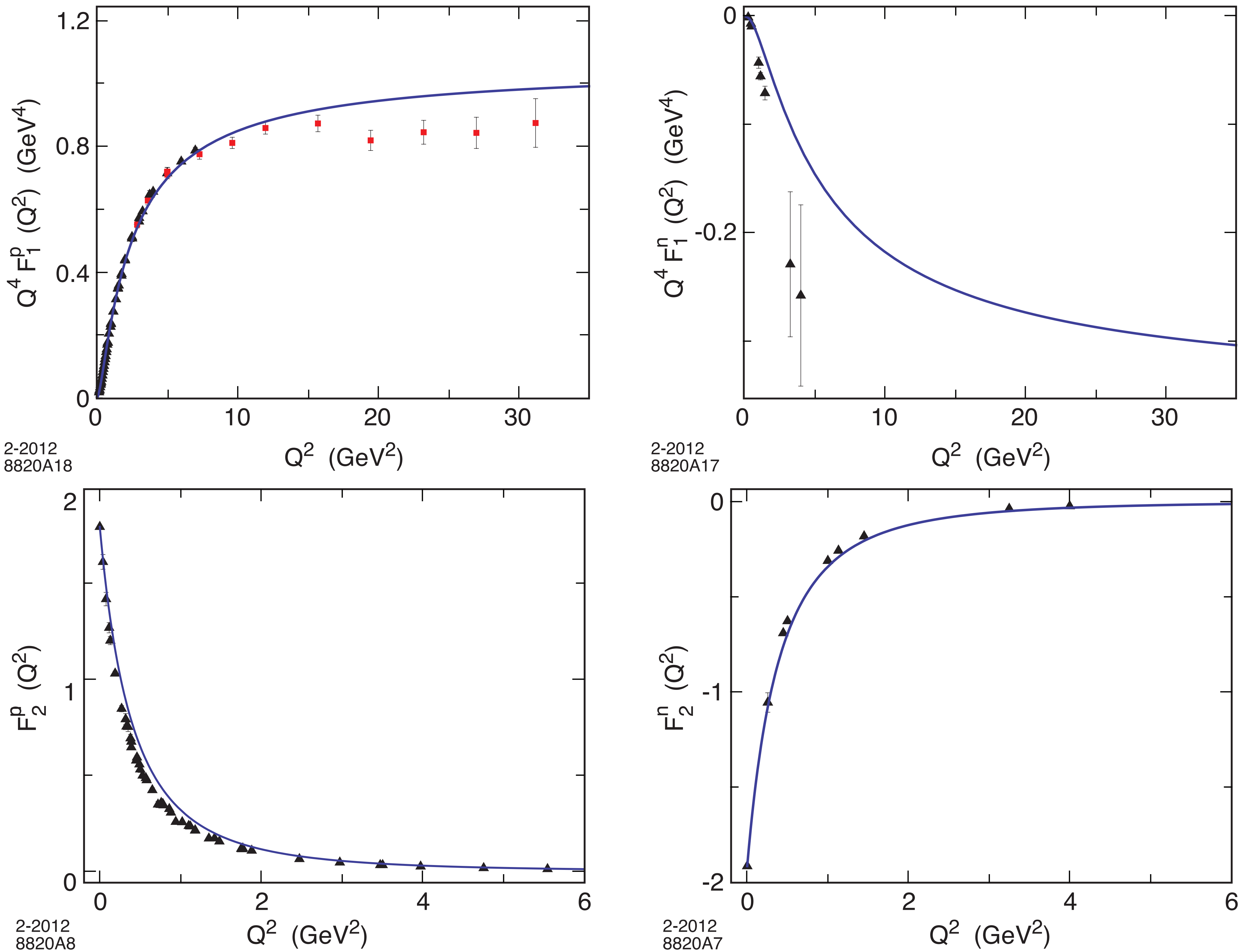
G. de Teramond & sjb





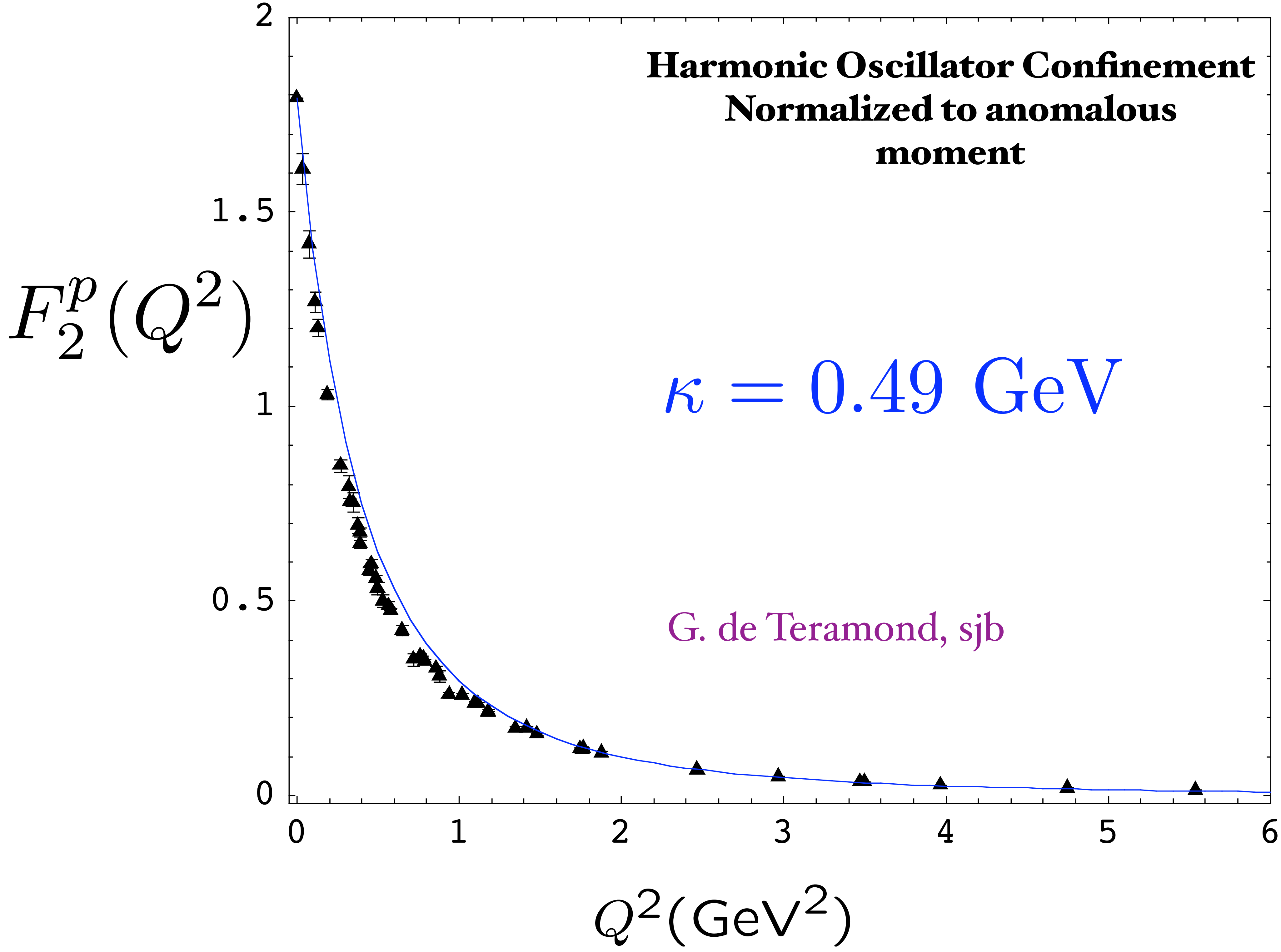


Using $SU(6)$ flavor symmetry and normalization to static quantities



Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



Nucleon structure in a light-front quark model consistent with quark counting rules and data

Gutsche, Lyubovitskij, Schmidt Vega

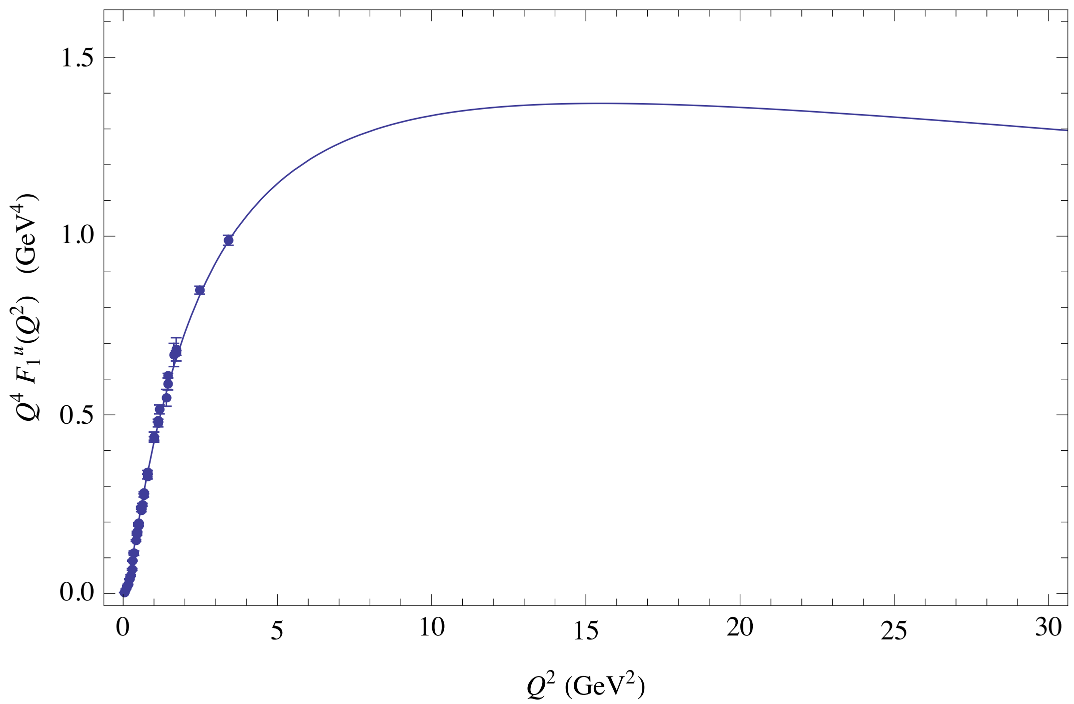


FIG. 9: Dirac u quark form factor multiplied by Q^4.

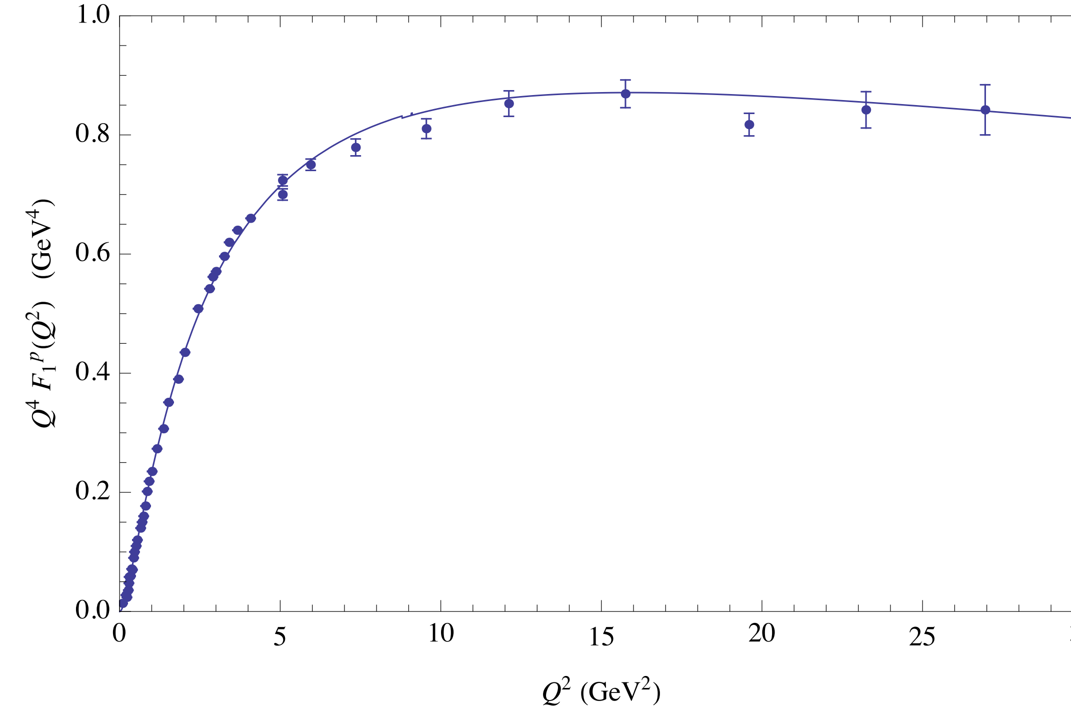


FIG. 13: Dirac proton form factor multiplied by Q^4.

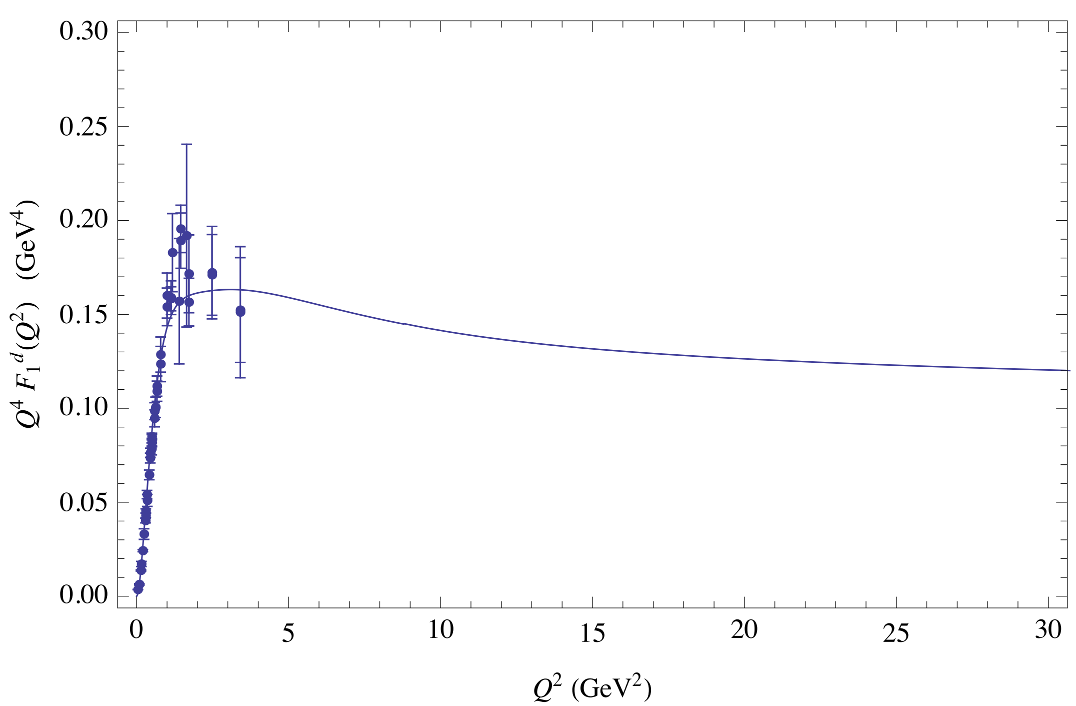


FIG. 10: Dirac d quark form factor multiplied by Q^4.

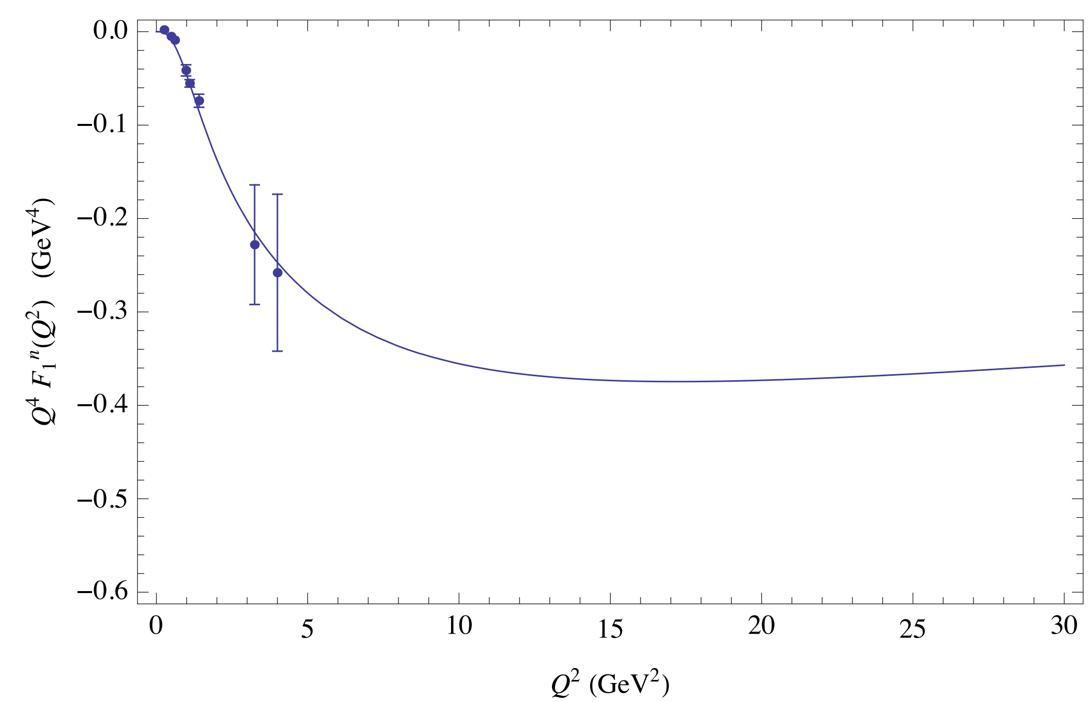
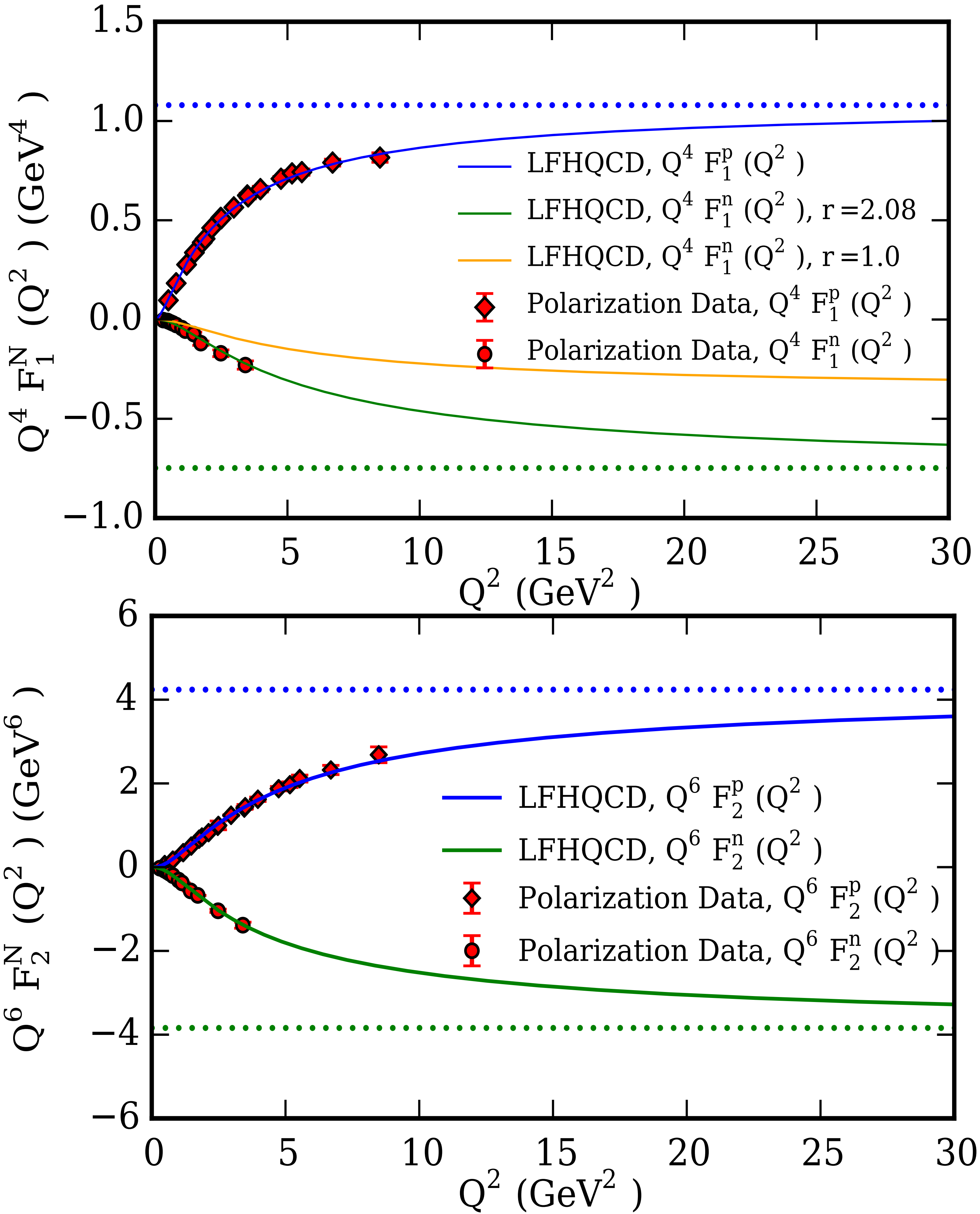


FIG. 14: Dirac neutron form factor multiplied by Q^4.

Sufian, de Teramond, Deur, Dosch, sjb



$Q^4 F_1^p(Q^2)$

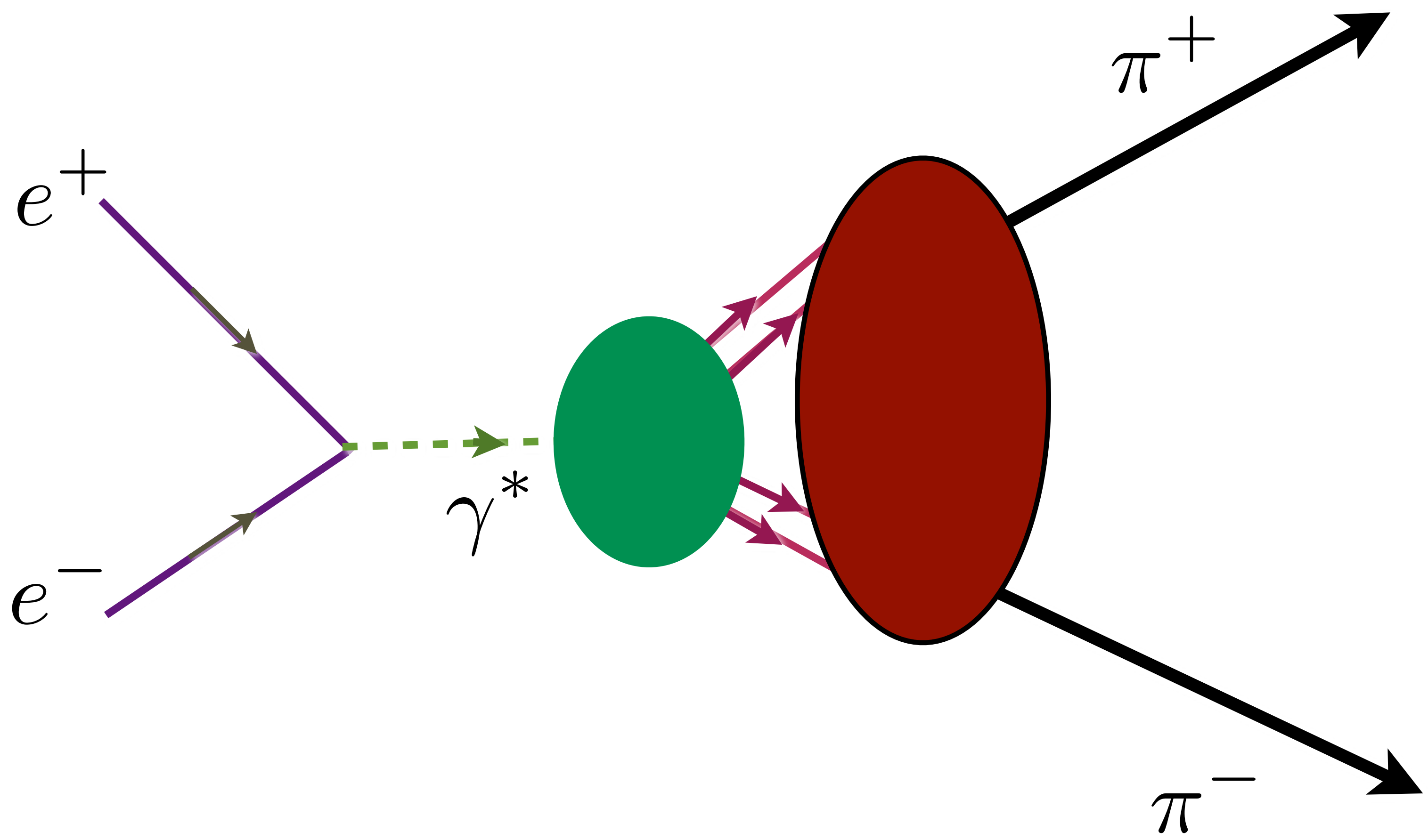
$Q^4 F_1^n(Q^2)$

Includes 5-quark Fock states

$Q^6 F_2^p(Q^2)$

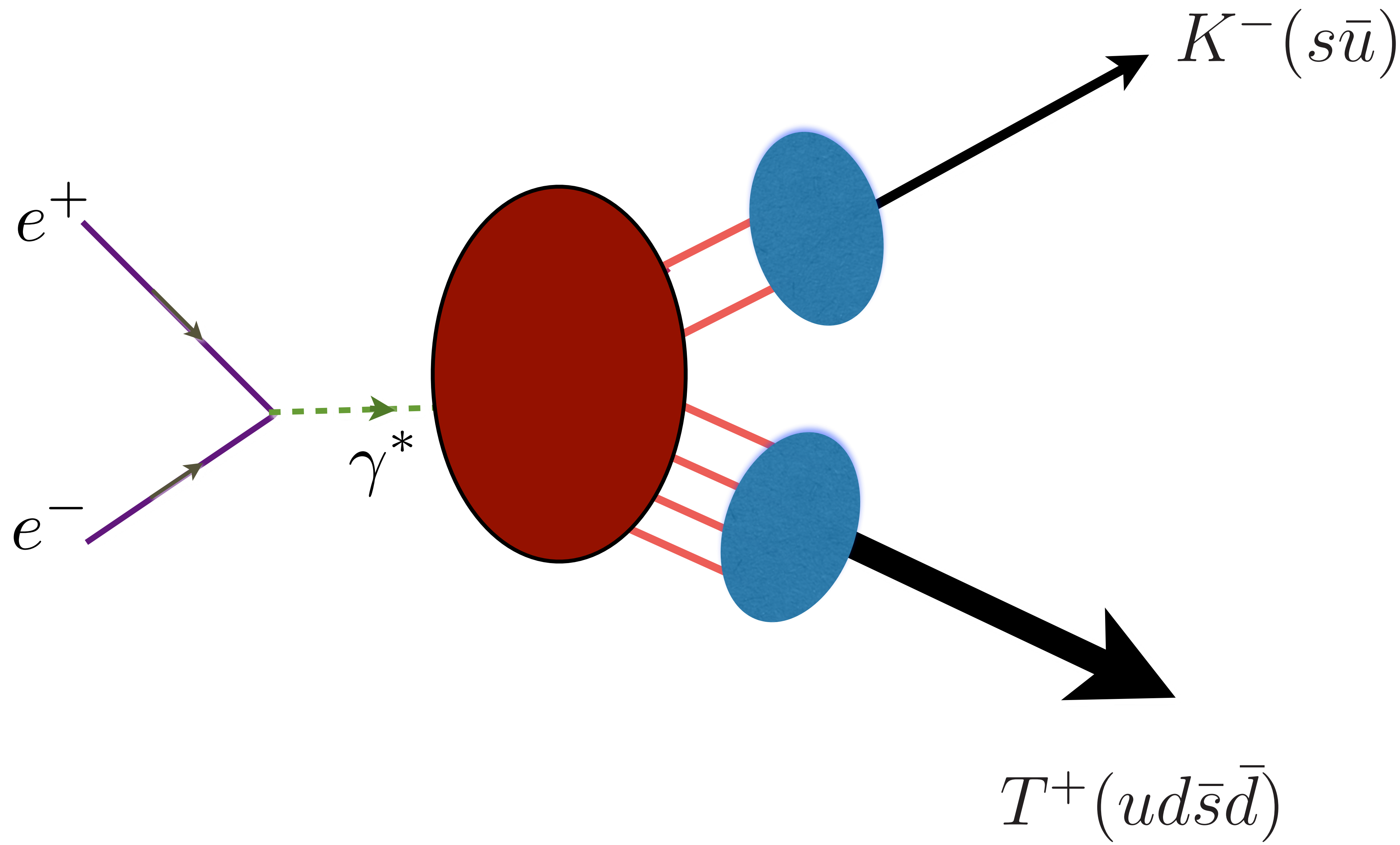
$Q^6 F_2^n(Q^2)$

Dressed soft-wall current brings in higher Fock states and more vector meson poles



Use Counting Rules to Verify Composition of Tetraquark

$$\mathcal{A}(e^+e^- \to \bar{M}(q\bar{q}) + T([qq][\bar{q}\bar{q}]) \sim \frac{1}{\sqrt{s}^{(2+4-1-1)}} = \frac{1}{s^2}$$



Same fall-off as $\mathcal{A}(e^+e^- \to \bar{B}(q[qq]) + B(q[\bar{q}\bar{q}]) \sim \frac{1}{\sqrt{s}^{(3+3-1-1)}} = \frac{1}{s^2}$

Current Matrix Elements in AdS Space (SW)

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

Dressed Current in Soft-Wall Model

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z) \Phi(z).$$

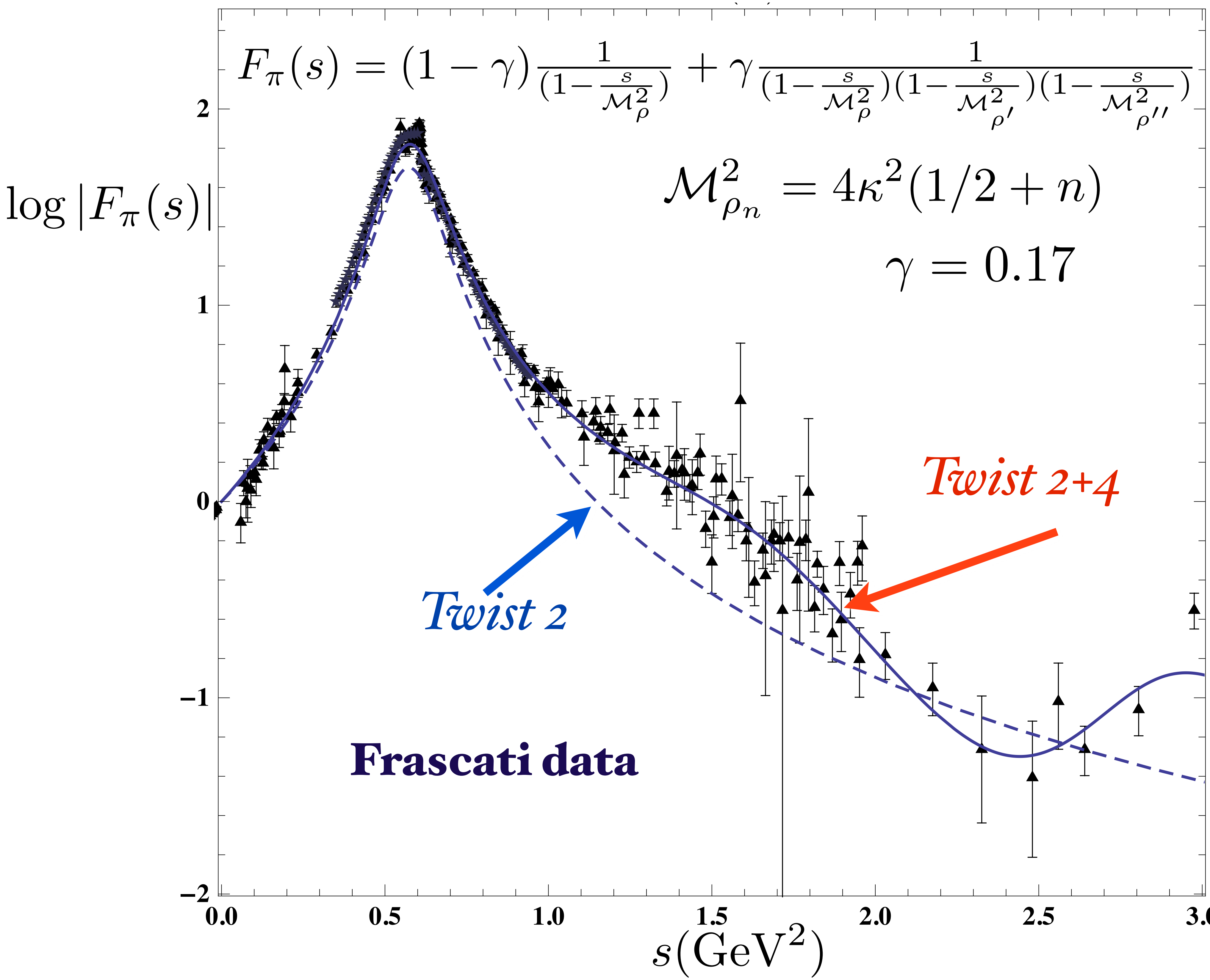
- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

de Tèramond & sjb
Grigoryan and Radyushkin

Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



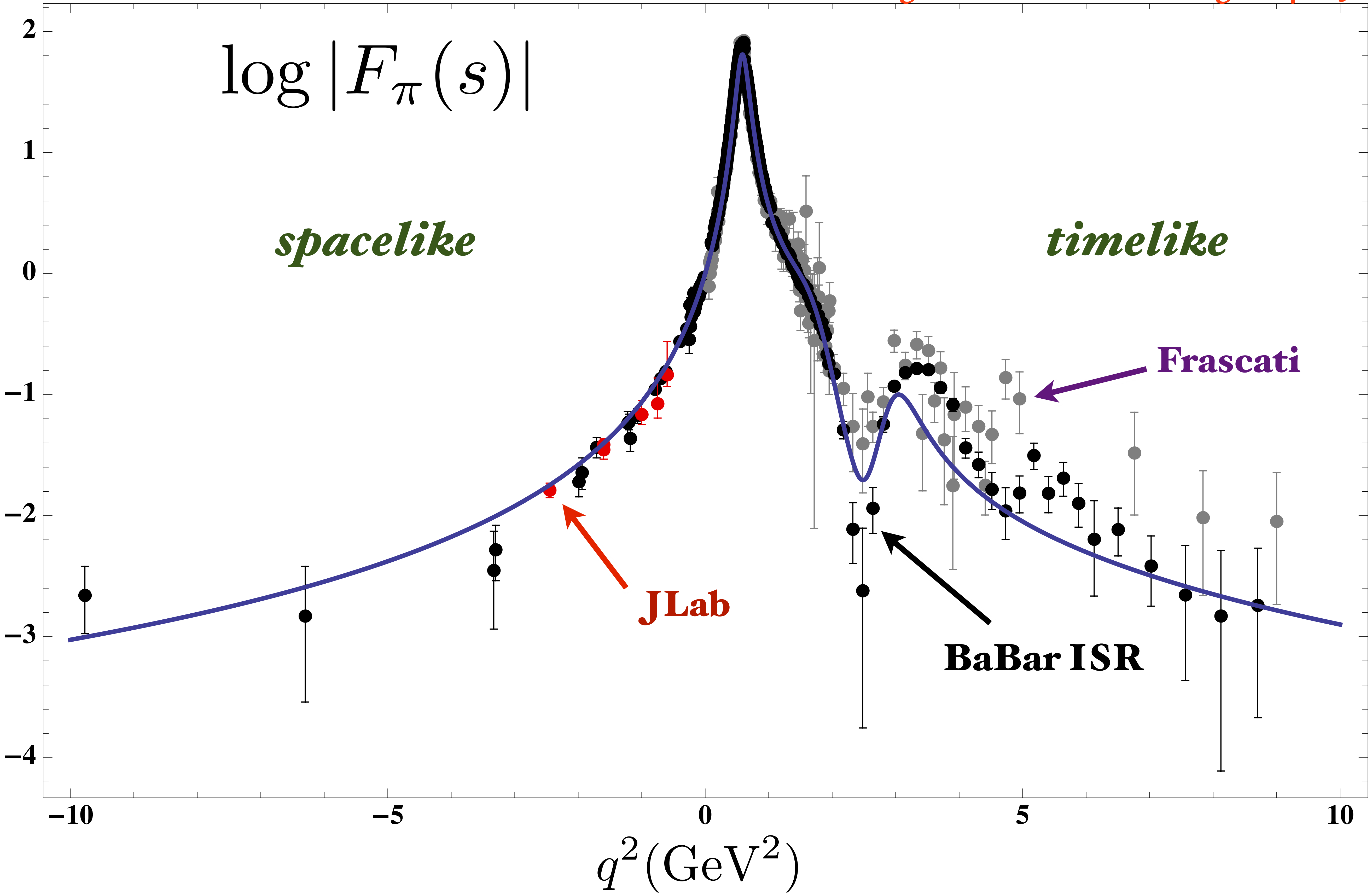
Prescription for Timelike poles :

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

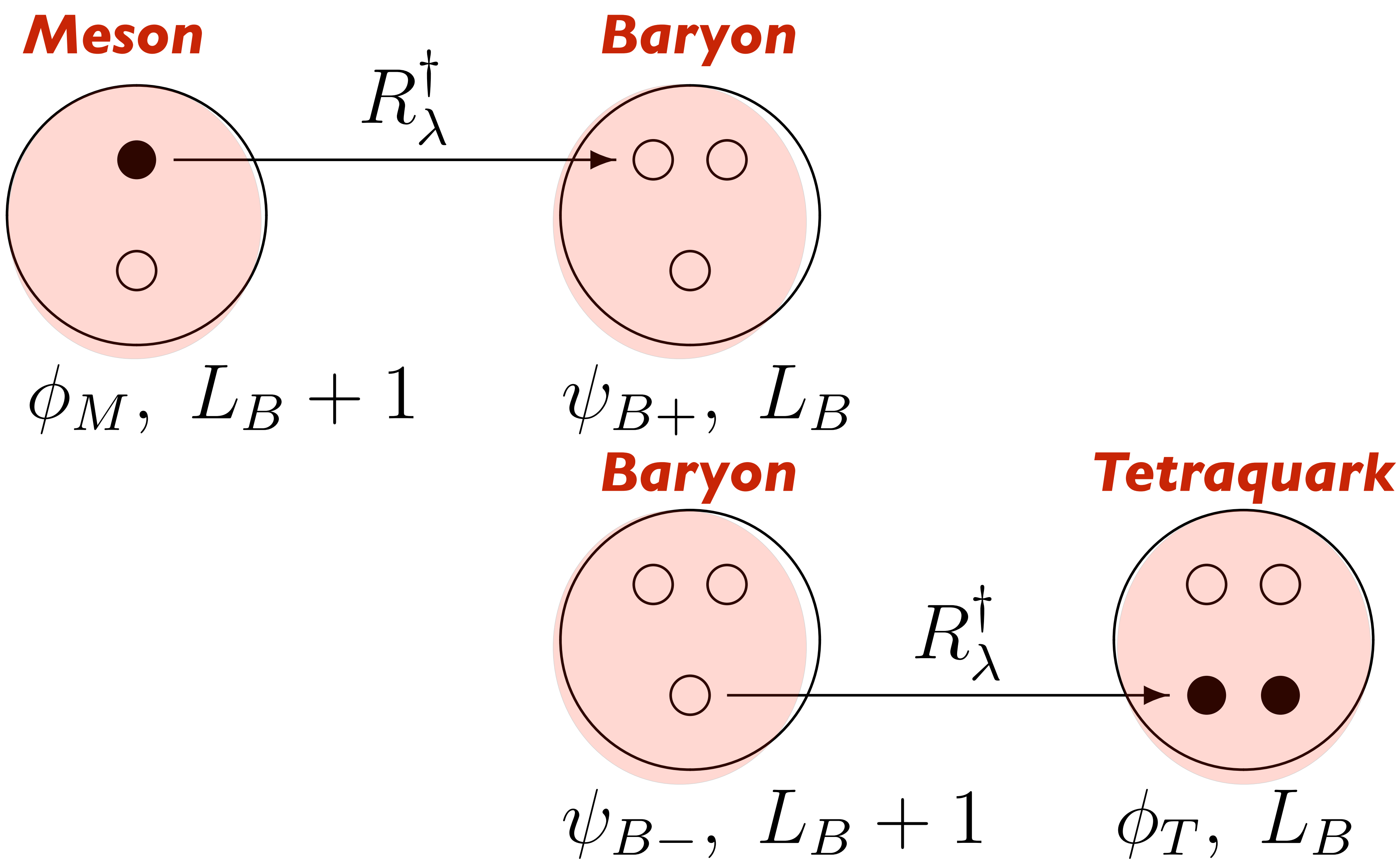
Pion Form Factor from AdS/QCD and Light-Front Holography



Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: quark + scalar diquark $|q(qq)>$
(Equal weight: $L = 0, L = 1$)

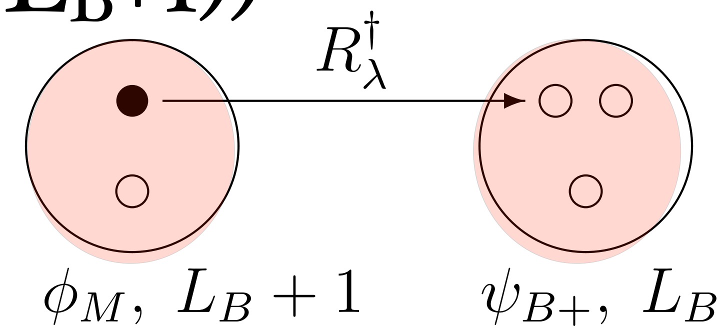
de Tèramond, Dosch, Lorce, sjb

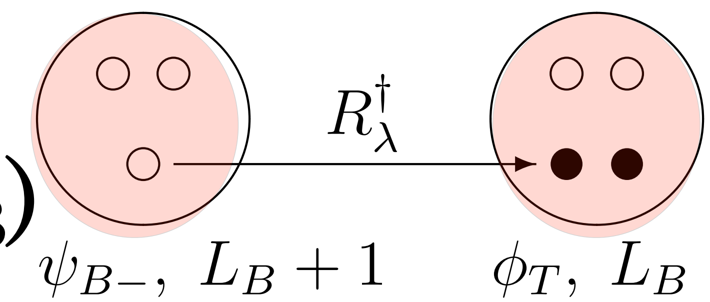
Superconformal Algebra

2X2 Hadronic Multiplets

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

- quark-antiquark meson ($L_M = L_B+1$))
- quark-diquark baryon (L_B)
- quark-diquark baryon (L_B+1)
- diquark-antidiquark tetraquark ($L_T = L_B$)
- Universal Regge slopes $\lambda = \kappa^2$





$$M_H^2/\lambda = \overbrace{\underbrace{(2n + L_H + 1)}_{kinetic} + \underbrace{(2n + L_H + 1)}_{potential}}^{contribution\ from\ 2\text{-}dim\ light\text{-}front\ harmonic\ oscillator} + \overbrace{2(L_H + s) + 2\chi}^{contribution\ from\ AdS\ and\ superconformal\ algebra} + < \sum_i \frac{m_i^2}{x_i} >$$

$\chi(mesons) = -1$ $\chi(baryons, tetraquarks) = +1$

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 with same mass eigenvalue
- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\quad S^z = \pm 1/2$
- Proton spin carried by quark L^z

$$< J^z >= \frac{1}{2}(S_q^z = \frac{1}{2}, L^z = 0) + \frac{1}{2}(S_q^z = -\frac{1}{2}, L^z = 1) =< L^z >= \frac{1}{2}$$

- Mass-degenerate meson “superpartner” with $L_M = L_B + 1$. *“Shifted meson-baryon Duality”*

Mesons and baryons have same κ !

- Spin-dependent interaction from embedding in AdS space

  [S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé, PLB **759**, 171 (2016)]

$$\boxed{G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda\, S} \qquad S = 0, 1$$

  where S is the spin of the meson or the spectator cluster in the baryon

- Light hadron spectrum (Add quadratic mass correction Δm_q^2 from light quark masses)

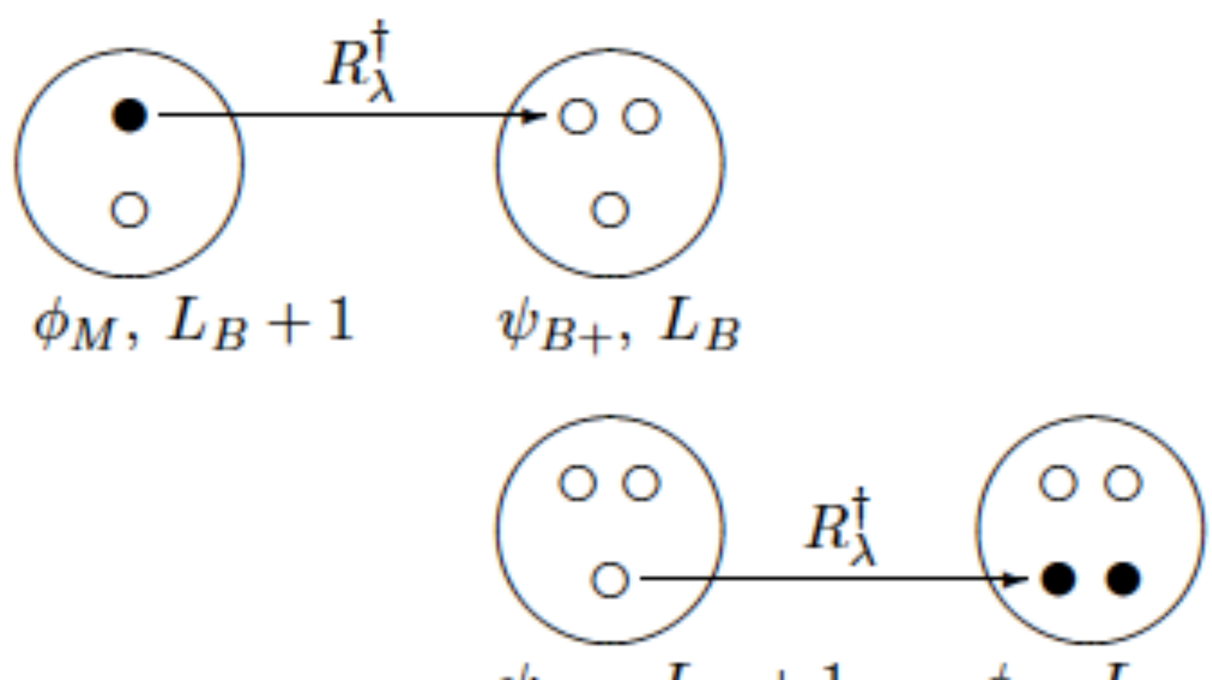
$$\text{Mesons} : M^2 = 4\lambda\,(n + L_M) + 2\lambda$$

$$\text{Baryons} : M^2 = 4\lambda\,(n + L_B + 1) + 2\lambda S$$

$$\text{Tetraquarks} : M^2 = 4\lambda\,(n + L_T + 1) + 2\lambda S$$

- Supersymmetric quadruplet

  $\{\phi_M, \psi_{B+}, \psi_{B-}, \phi_T\}$



de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

- Diquark: Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states $\bar{3}_C \times 3_C = 1_C$

$$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

$$2\left[\sigma([\{qq\}N) + \sigma(qN)\right] - [\sigma(qN) + \sigma(\bar{q}N)] = [\sigma(\{qq\}N) + \sigma(\{qq\}N)]$$

Candidates $f_0(980) I = 0, J^P = 0^+$, partner of proton

$a_1(1260) I = 0, J^P = 1^+$, partner of $\Delta(1233)$

Universal Hadronic Features

- ***Universal quark light-front kinetic energy***

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1 + 2n + L)$$

Equal: Virial Theorem!

- ***Universal quark light-front potential energy***

$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1 + 2n + L)$$

- ***Universal Constant Term***

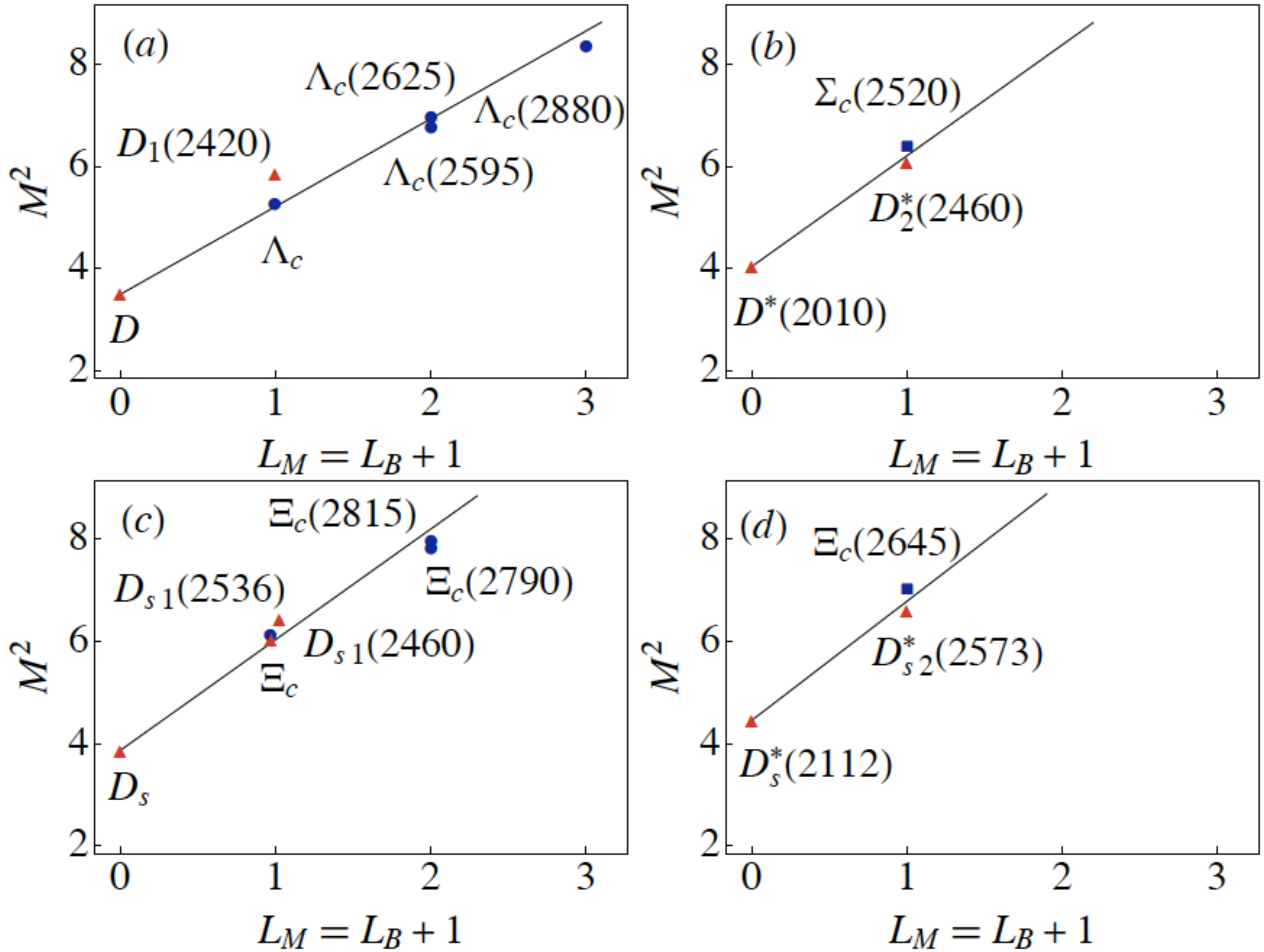
$$\mathcal{M}^2_{spin} = 2\kappa^2(S + L - 1 + 2n_{diquark})$$

$$M^2 = \Delta\mathcal{M}^2_{LFKE} + \Delta\mathcal{M}^2_{LFPE} + \Delta\mathcal{M}^2_{spin}$$

$$+ < \sum_i \frac{m_i^2}{x_i} >$$

Dosch, de Teramond, sjb

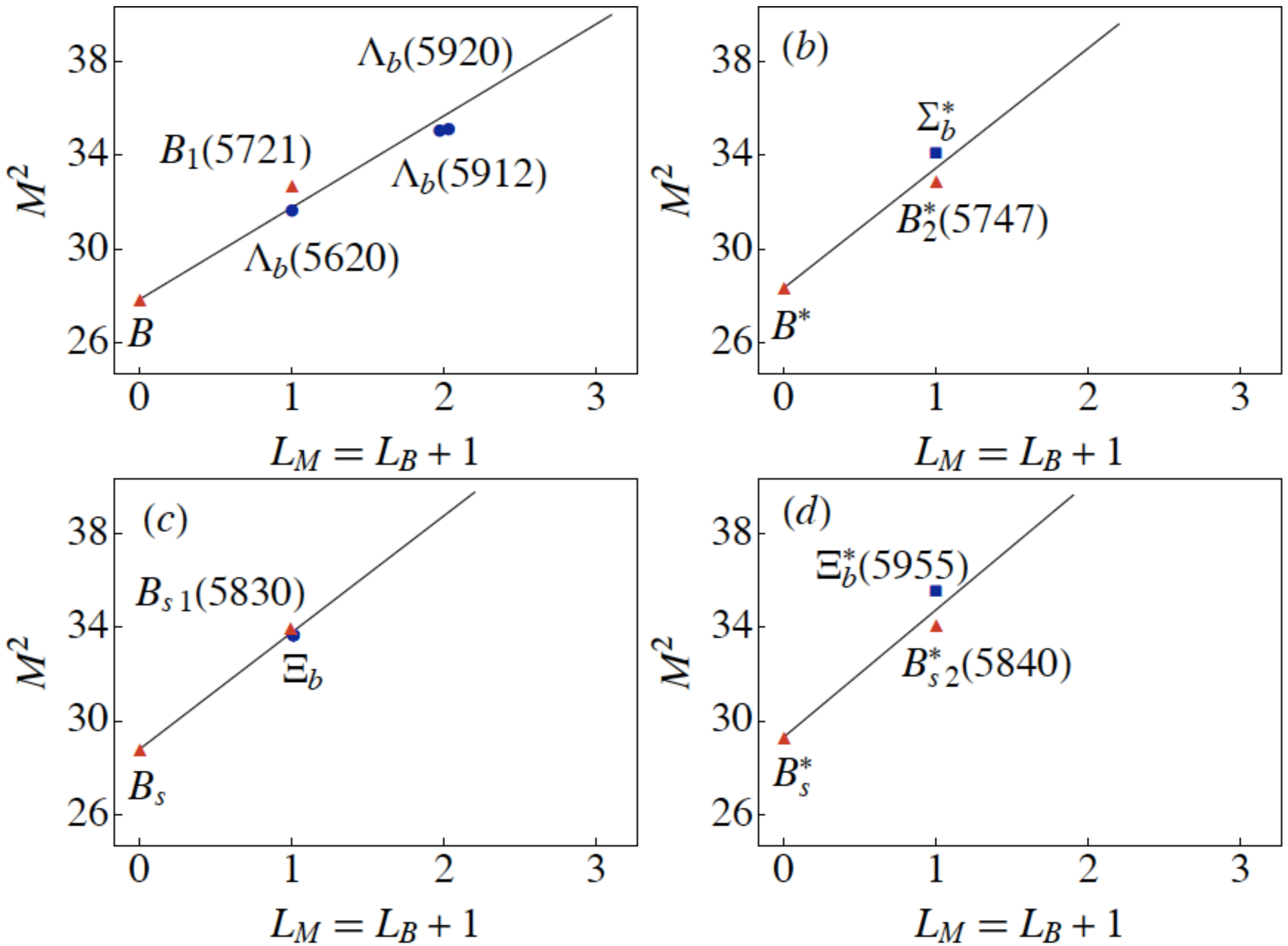
Supersymmetry across the light and heavy-light spectrum



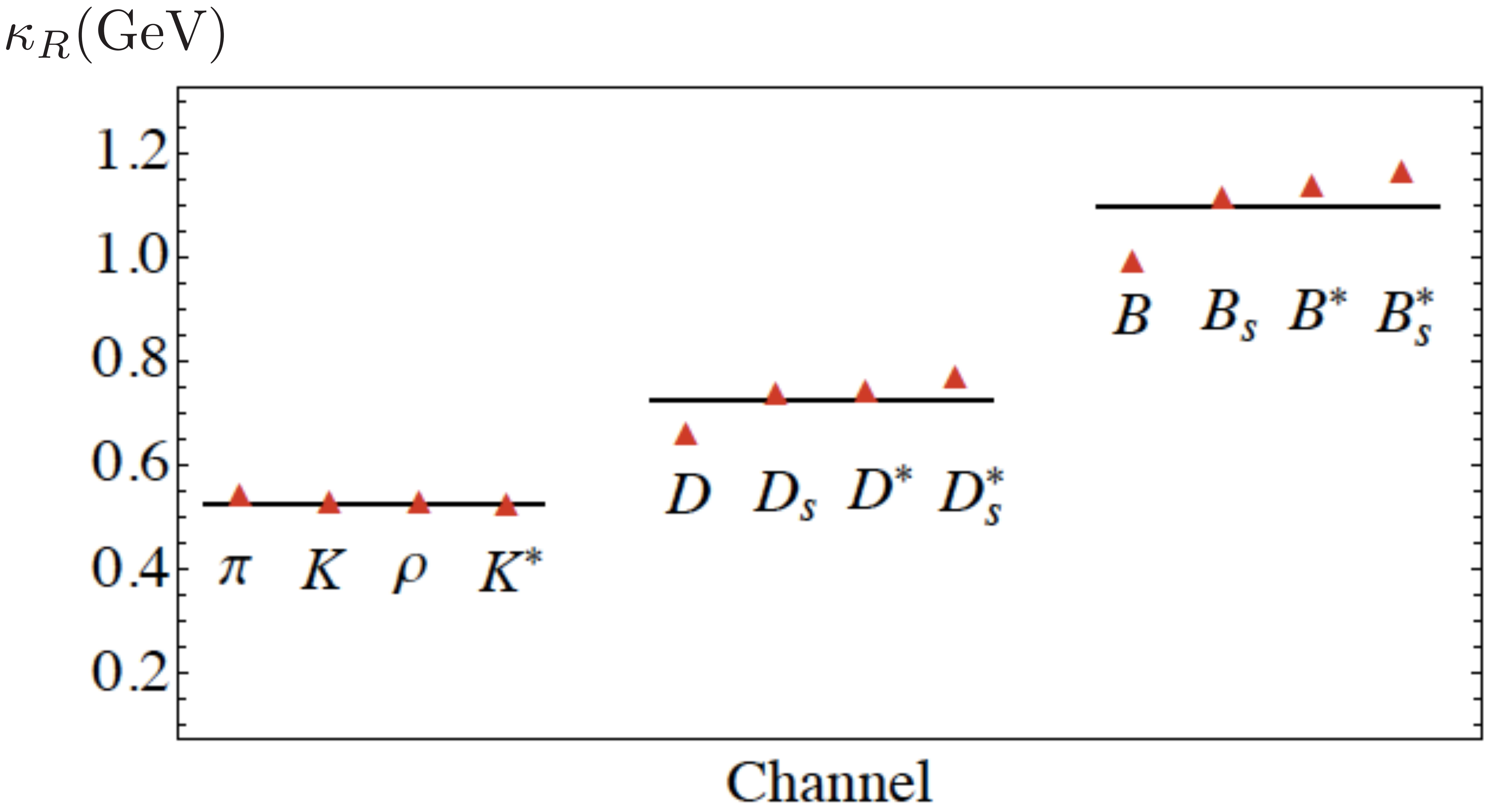
Heavy charm quark mass does not break supersymmetry

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy bottom quark mass does not break supersymmetry



Regge slope for heavy-light mesons, baryons: increases with heavy quark mass

Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

$\Updownarrow$

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

Foundations of Light-Front Holography

- *The QCD Lagrangian for m_q =0 has no mass scale.*
- *What determines the hadron mass scale?*
- *DAFF principle: add terms linear in D and K to Conformal Hamiltonian: Mass scale κ appears, but action remains scale invariant —> unique harmonic oscillator potential*
- *Apply DAFF to the Poincare' invariant LF Hamiltonian: Unique color-confining potential*
- *Fixes AdS_5 dilaton: predicts Spin and Spin-Orbit Interactions*
- *Apply DAFF to Superconformal representation of the Lorentz group*
- *Predicts Meson, Baryon, Tetraquark spectroscopy, dynamics*
- *Supersymmetric Features of Spectrum*

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}} \quad \text{from dilaton } e^{\kappa^2 z^2}$$

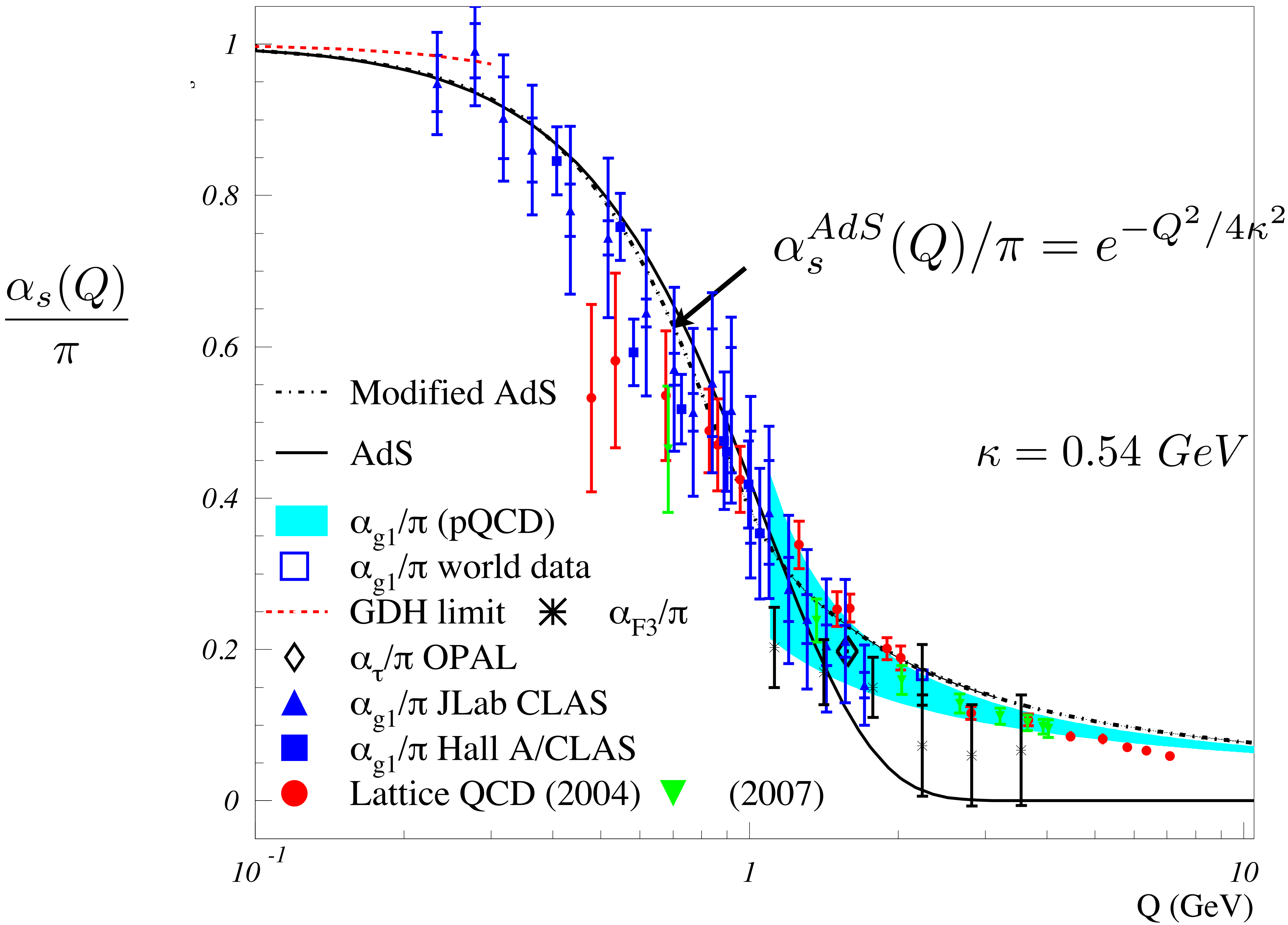
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal*** β_0, β_1

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

Deur, de Tèramond, sjb

All-Scale QCD Coupling

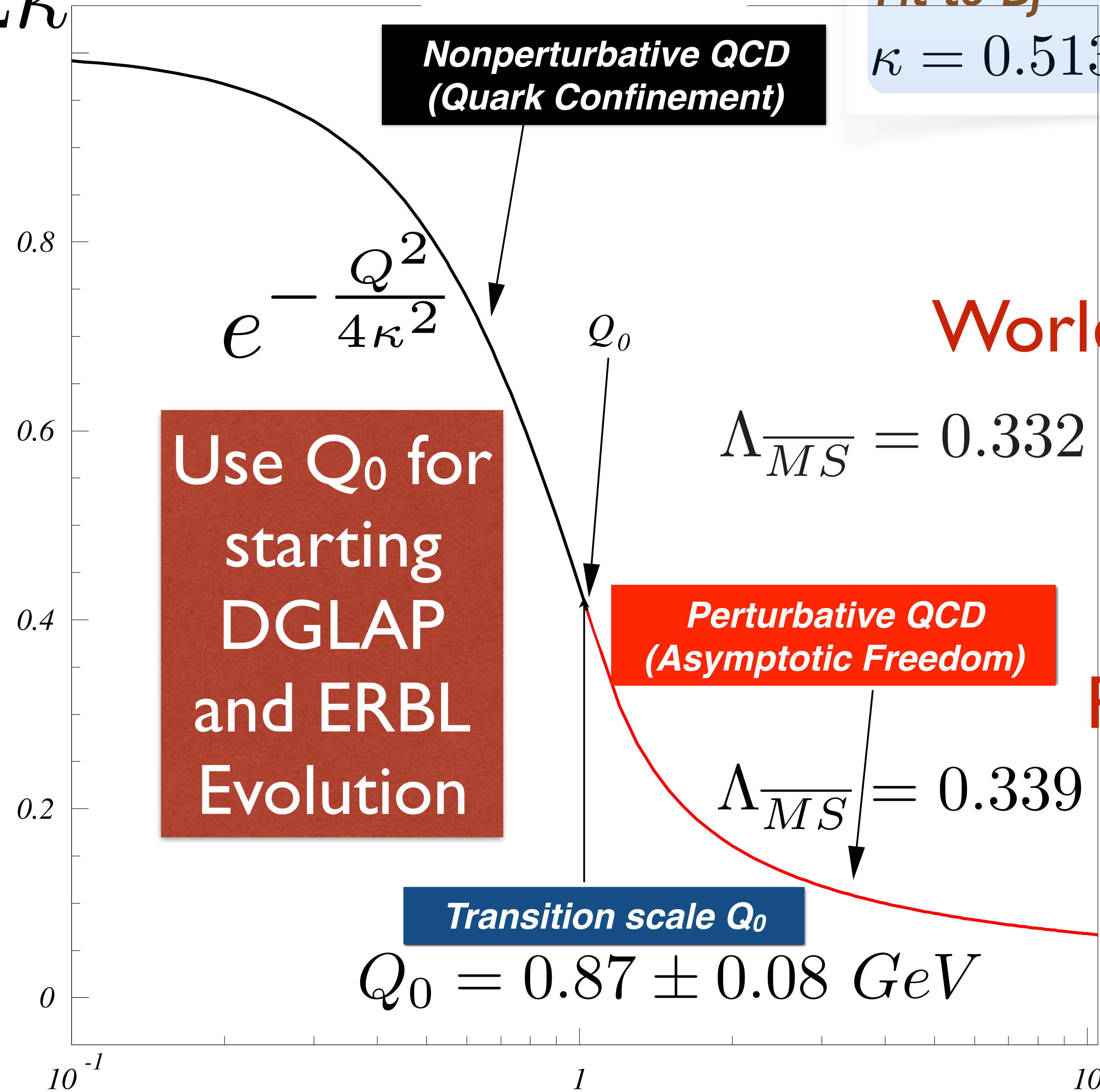
$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



World Data:

$$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ \mathrm{GeV}$$

Prediction

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$$

$$Q_0 = 0.87 \pm 0.08\ GeV$$

$\overline{MS}$ scheme

$$\lambda \equiv \kappa^2$$

Features of LF Holographic QCD

- ***Regge spectroscopy—same slope in n,L for mesons, baryons***
- ***Chiral features for m_q=0: m_π =0, chiral-invariant proton!***
- ***Hadronic LFWFs***
- ***Counting Rules***
- ***Connection between hadron masses and*** $\Lambda_{\overline{MS}}$

Superconformal AdS Light-Front Holographic QCD (LFHQCD)

Meson-Baryon Mass Degeneracy for $L_M=L_B+1$

Fundamental Hadronic Features of Hadrons

Virial Theorem

- Partition of the Proton's Mass: Potential vs. Kinetic Contributions

$$\Delta\mathcal{M}^2_{LFKE} = \kappa^2(1+2n+L)$$
$$\Delta\mathcal{M}^2_{LFPE} = \kappa^2(1+2n+L)$$

- Color Confinement $U(\zeta^2) = \kappa^4\zeta^2$
- Role of Quark Orbital Angular Momentum in the Proton

Equal L=0,1

- Quark-Diquark Structure
- Quark Mass Contribution $\Delta M^2 =< \frac{m_q^2}{x} >$

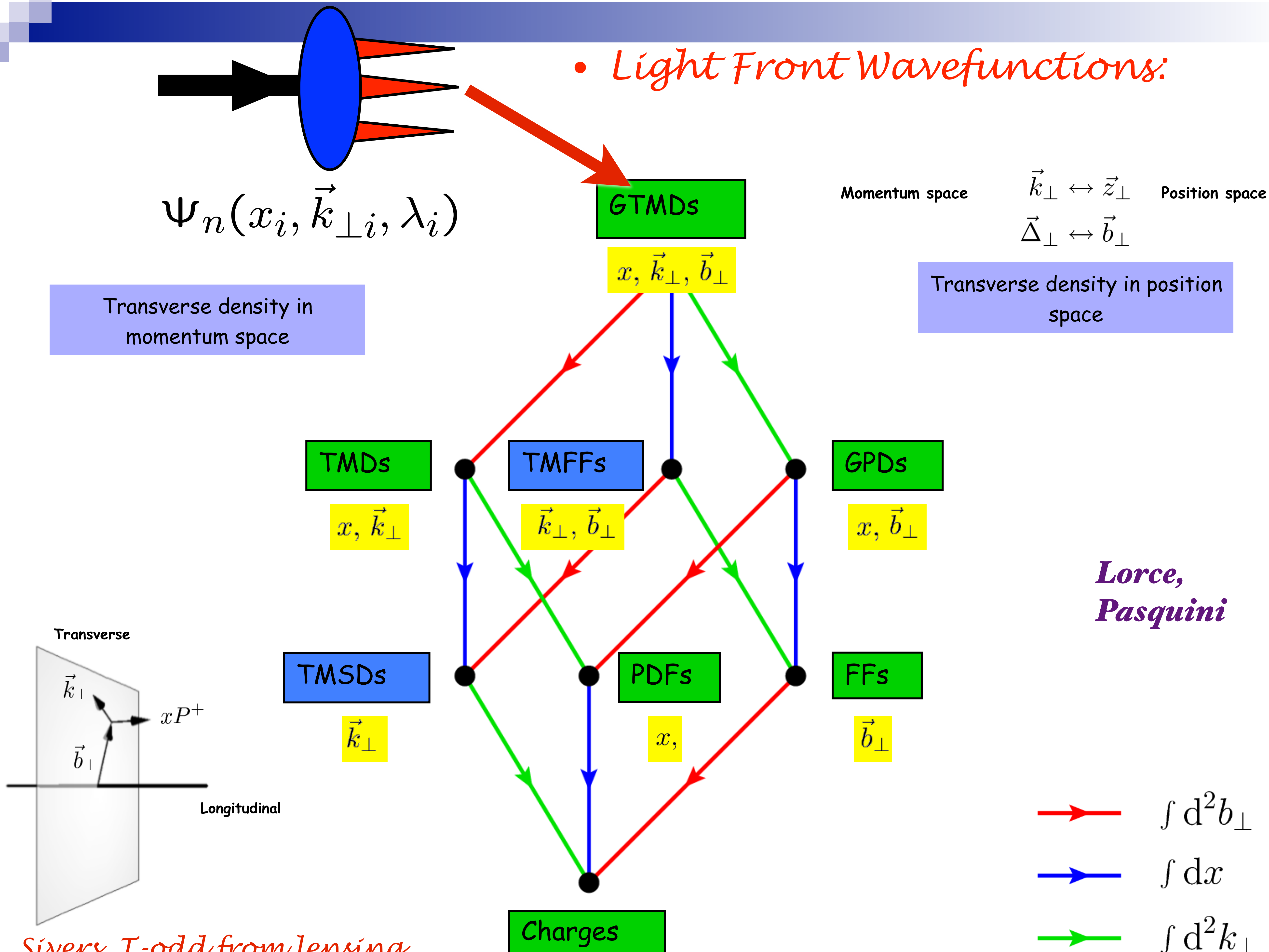
from the Yukawa coupling to the Higgs zero mode

- Baryonic Regge Trajectory $M^2_p(n, L_B) = 4\kappa^2(n + L_B + 1)$
- Mesonic Supersymmetric Partners $L_M = L_B + 1$
- Proton Light-Front Wavefunctions and Dynamical Observables

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

- Form Factors, Distribution Amplitudes, Structure Functions
- Non-Perturbative - Perturbative QCD Transition $Q_0 = 0.87 \pm 0.08\ GeV\ \overline{MS}\ scheme$
- Dimensional Transmutation: $m_p \simeq 3.21\ \Lambda_{\overline{MS}}$ $m_\rho \simeq 2.2\ \Lambda_{\overline{MS}}$

- *Light Front Wavefunctions:*



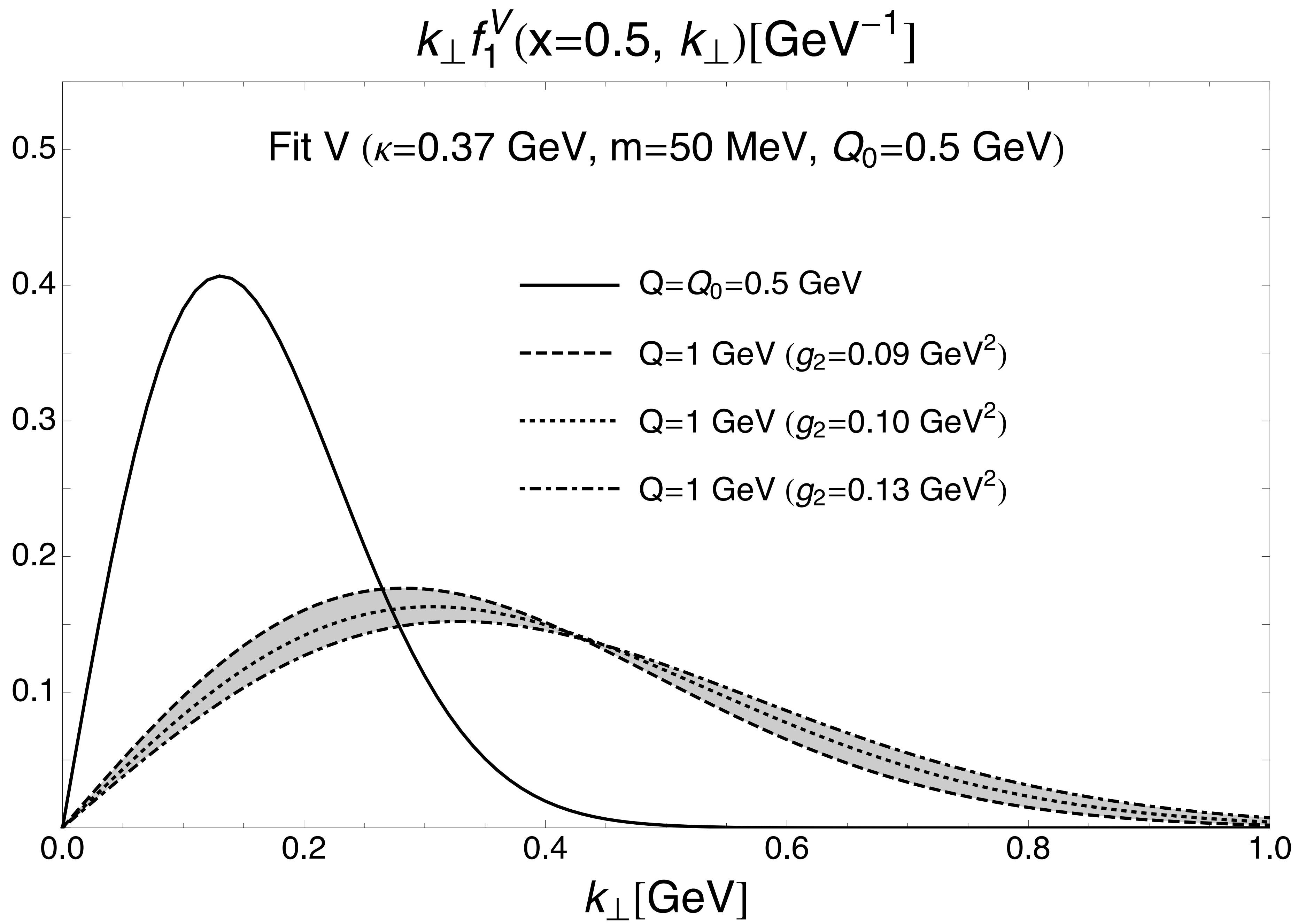
Lorce, Pasquini

Sivers, T-odd from lensing

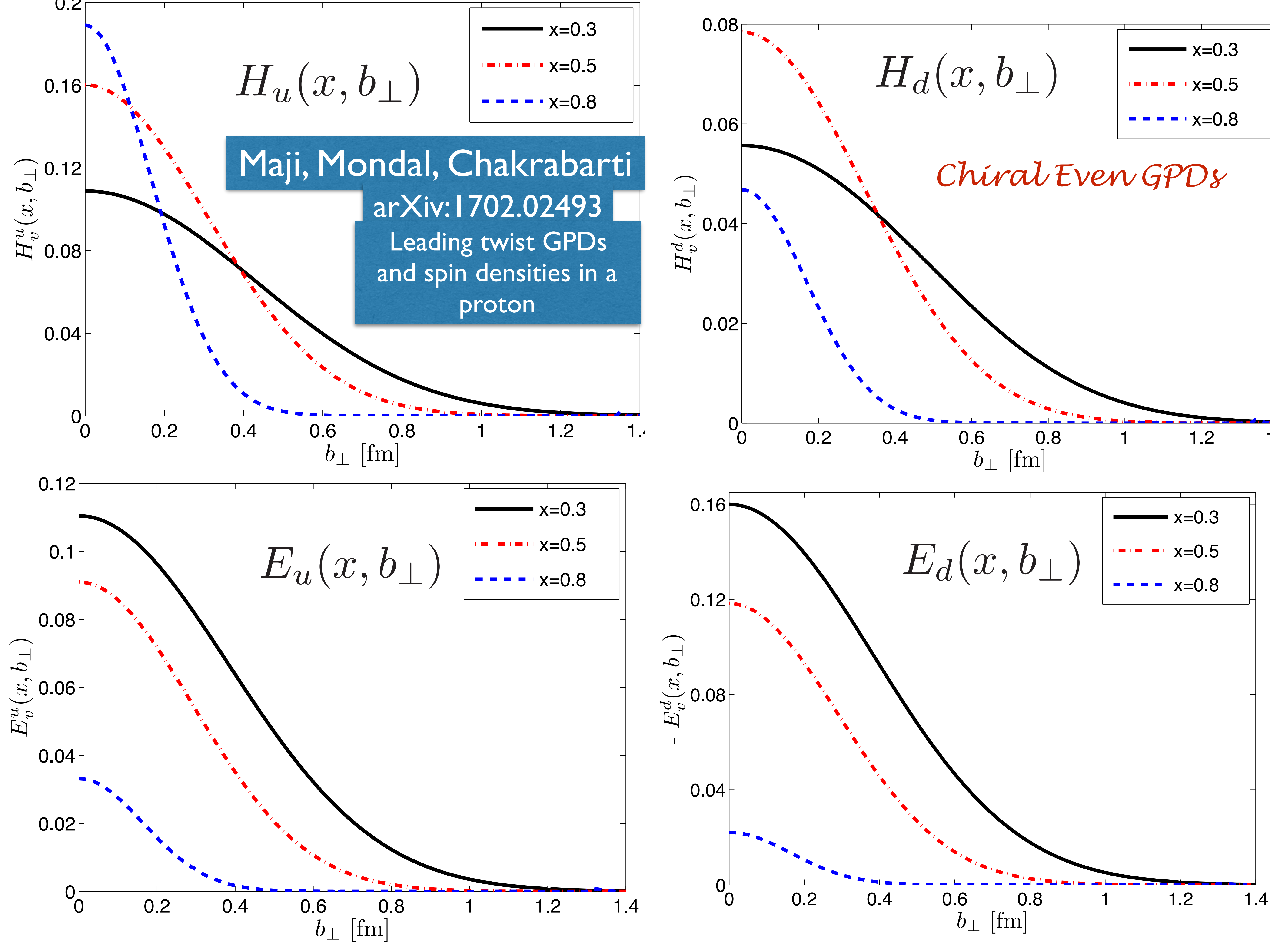
The transverse structure of the pion in momentum space inspired by the AdS/QCD correspondence

Bacchetta, Cotogno, Pasquini

1703.07669



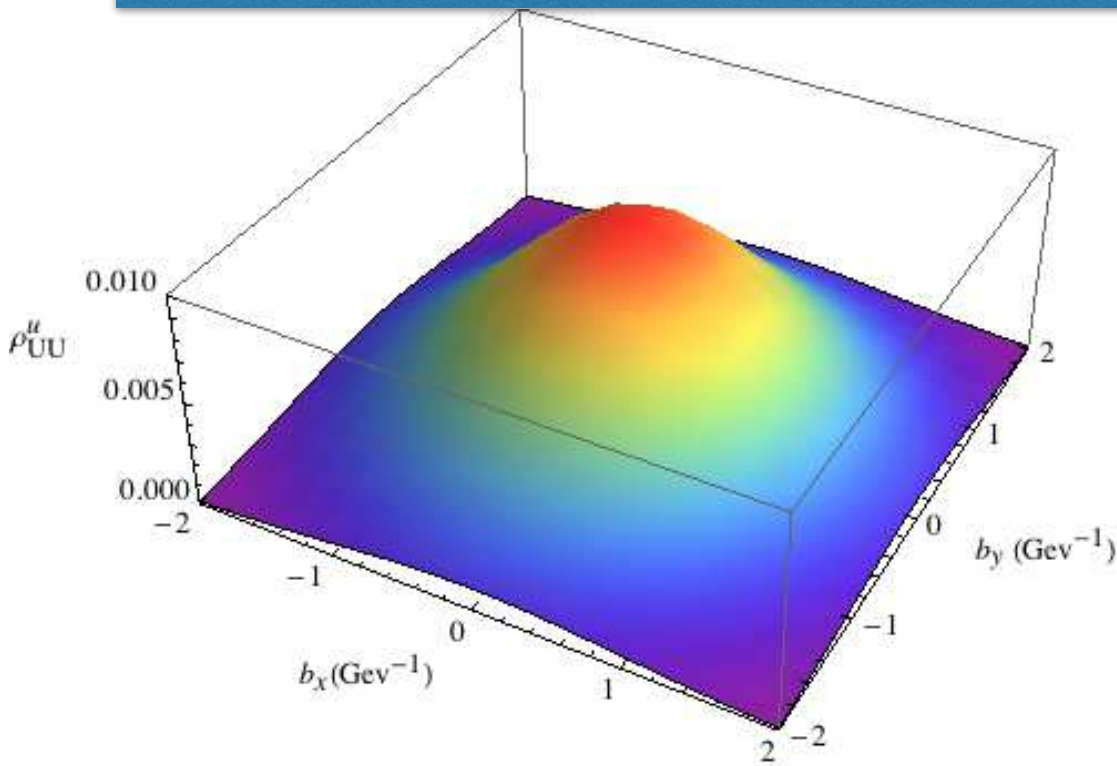
Results for the quark TMD of the pion, multiplied by $k_\perp$, from the pure-valence LFWF for the m = 50 MeV scenario, as function of $k_\perp$ and at fixed x = 0.5. The solid curve shows the result at the scale of the model, Q_0 = 0.5 GeV, corresponding with the initial scale for the TMD evolution. The shaded band gives the spread of the results after evolution of the TMD to 1 GeV



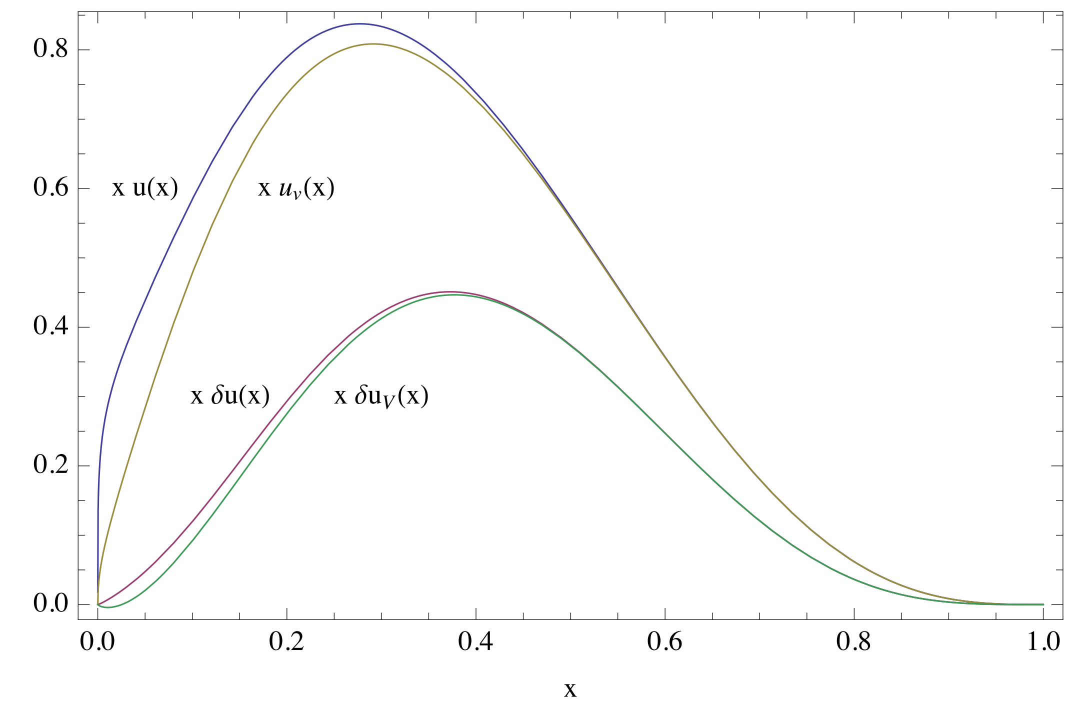
Maji, Mondal, Chakrabarti
arXiv:1702.02493
Leading twist GPDs and spin densities in a proton
Chiral Even GPDs
$H_u(x, b_\perp)$
$H_d(x, b_\perp)$
$E_u(x, b_\perp)$
$E_d(x, b_\perp)$
$H_v^u(x, b_\perp)$
$H_v^d(x, b_\perp)$
$E_v^u(x, b_\perp)$
- $E_v^d(x, b_\perp)$
$b_\perp$ [fm]
x=0.3
x=0.5
x=0.8

Nucleon parton distributions in a light-front quark model
Gutsche, Lyubovitskij, Schmidt

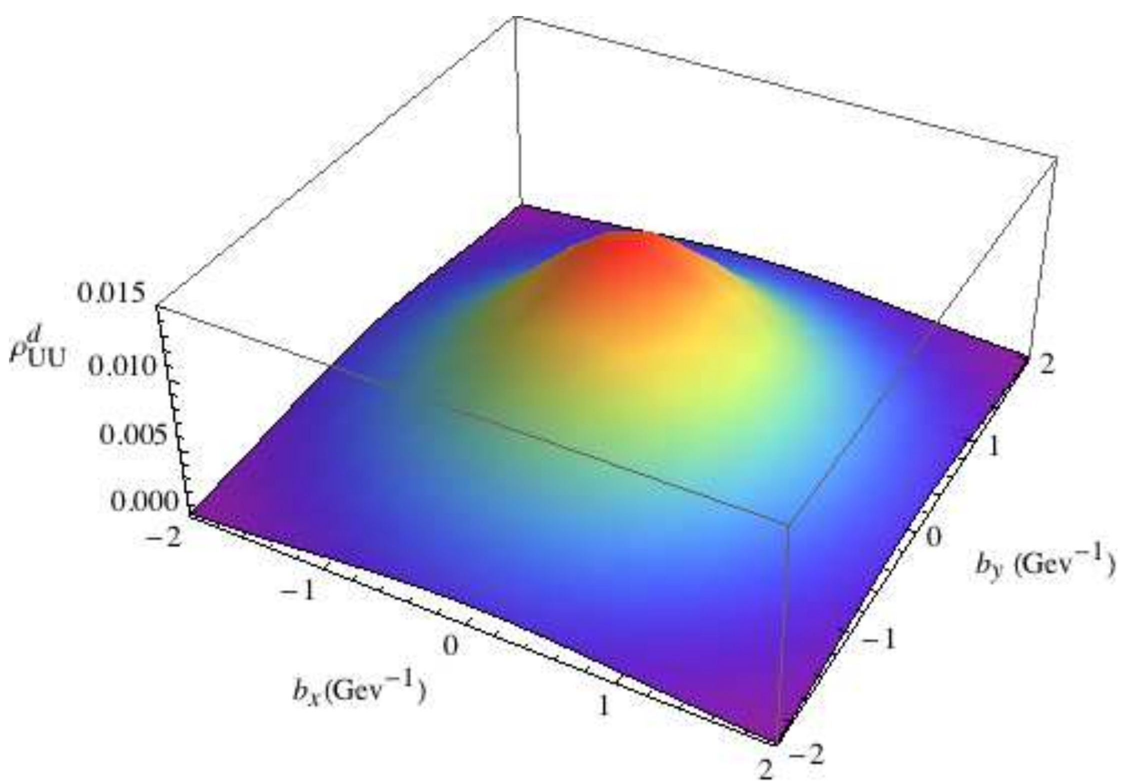
arXiv:1605.03526



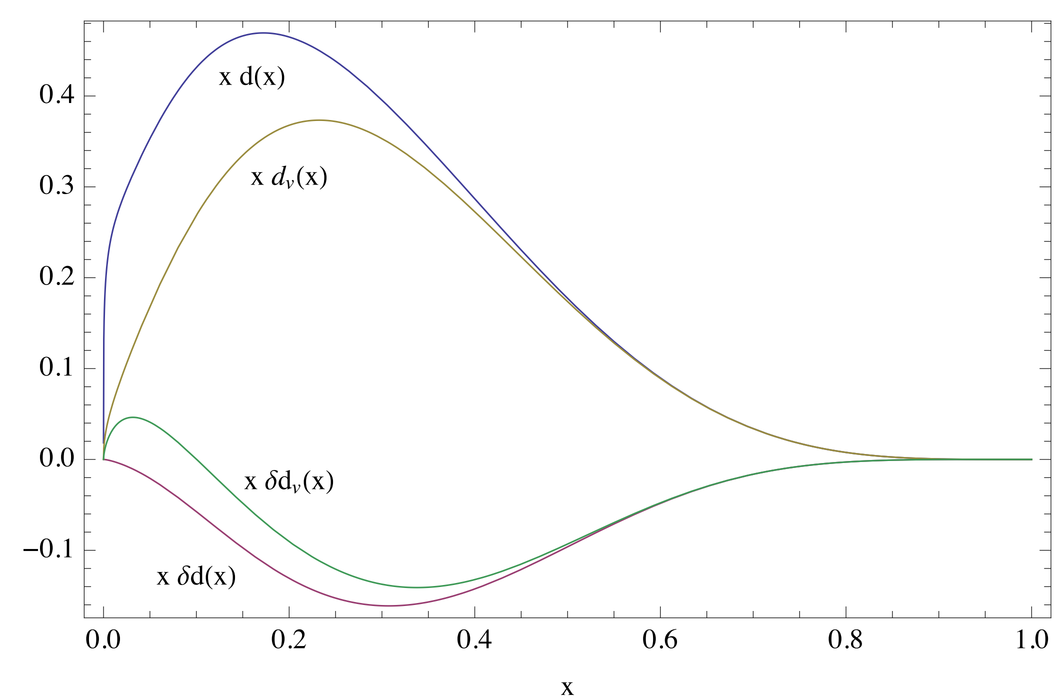
Wigner distribution $\rho^u_{UU}(x, \mathbf{b}_\perp, \mathbf{k}_\perp)$ at $x = 0.5$, $k_x = k_y = 0.5$ GeV.



u quark PDFs multiplied with x.



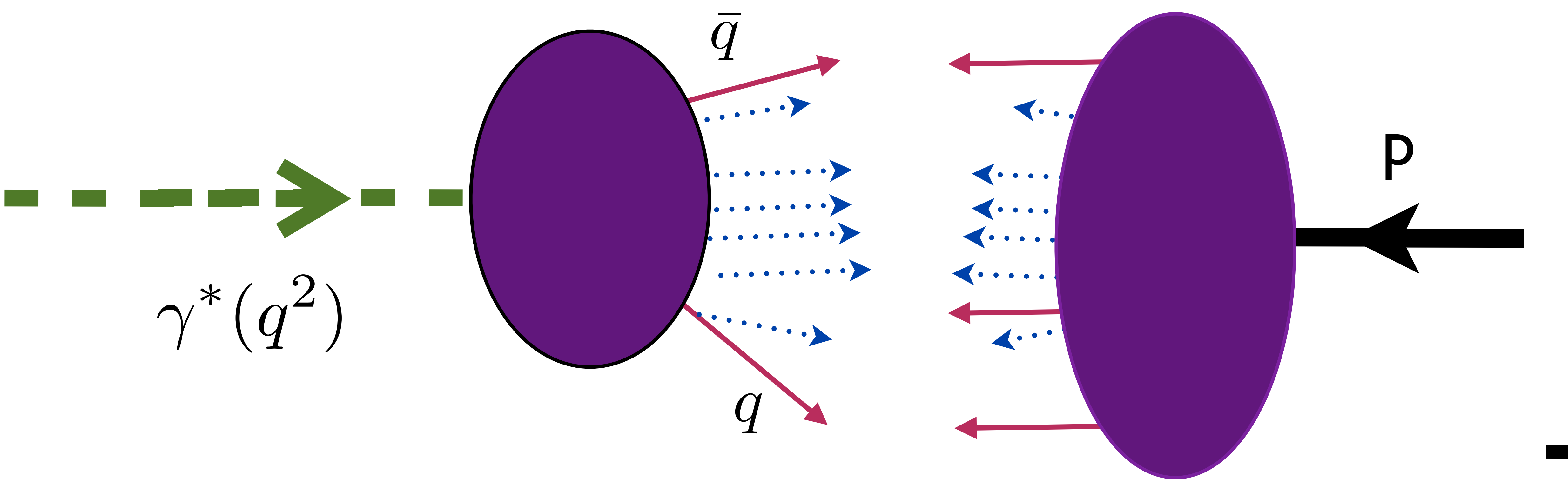
Wigner distribution $\rho^d_{UU}(x, \mathbf{b}_\perp, \mathbf{k}_\perp)$ at $x = 0.5$, $k_x = k_y = 0.5$ GeV.



d quark PDFs multiplied with x.

LHeC: Virtual Photon-Proton Collider

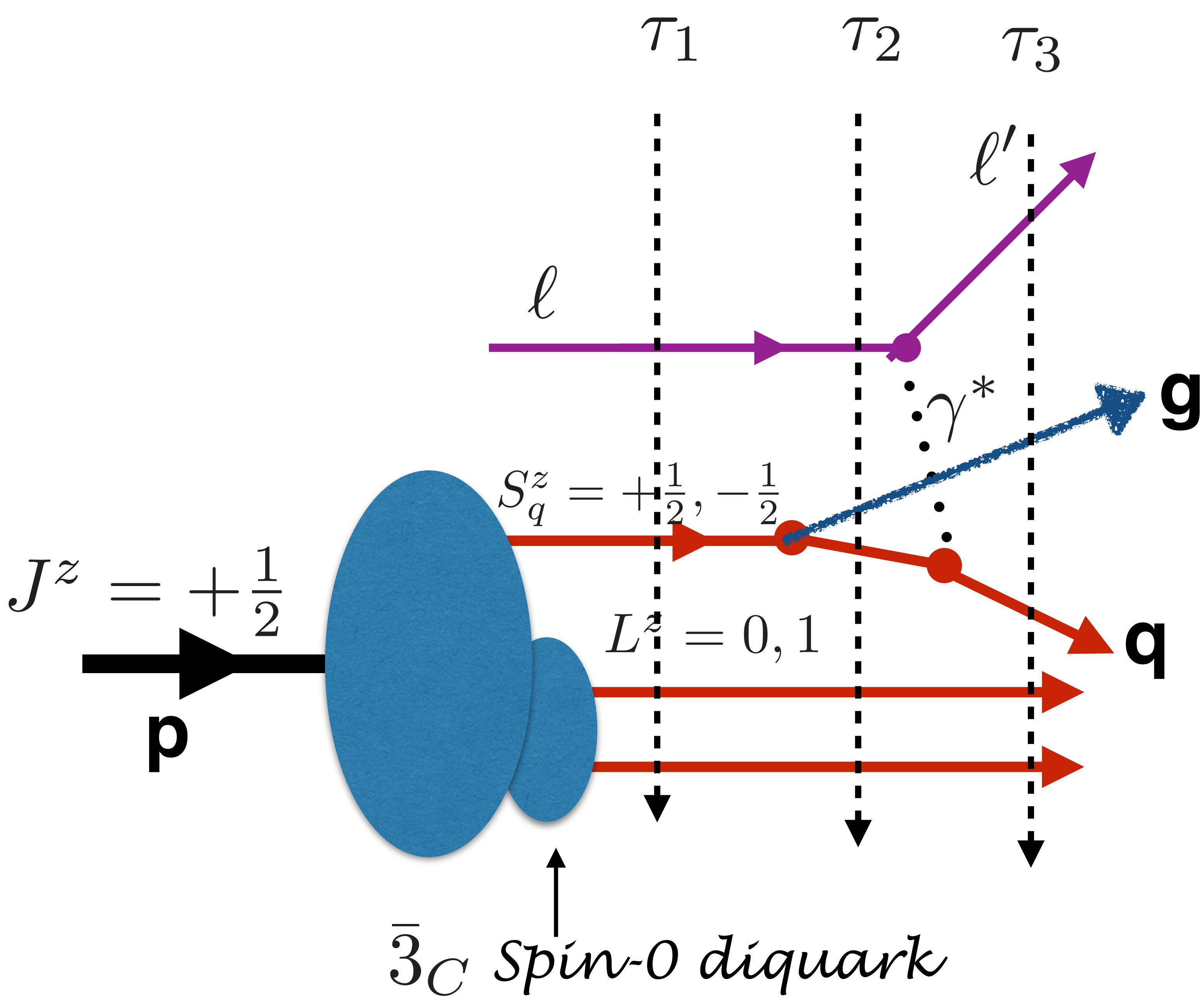
Perspective from the photon-proton collider frame



proton or ions

variable spacelike photon virtuality
various primary flavors
Study Ridge Phenomena with
Controlled source

Deep Inelastic Scattering on |uudg> LF Fock state



Gluon carries away momentum and orbital angular momentum of quark

$$|p> = |u[ud]_0> \to |u[ud]_0 g>$$

In each case the gluon is emitted with $S_g^z = +1$

Tony Zee

de Tèramond, Dosch, sjb

"Quantum Field Theory in a Nutshell"

Dreams of Exact Solvability

"In other words, if you manage to calculate m_P it better come out proportional to Λ_{QCD} since Λ_{QCD} is the only quantity with dimension of mass around.

Light-Front Holography:

Similarly for m_ρ.

$$m_p \simeq 3.21\ \Lambda_{\overline{MS}}$$

$$m_\rho \simeq 2.2\ \Lambda_{\overline{MS}}$$

Put in precise terms, if you publish a paper with a formula giving m_ρ/m_P in terms of pure numbers such as 2 and π, the field theory community will hail you as a conquering hero who has solved QCD exactly."

$$(m_q = 0)$$

$$m_\pi = 0$$

$$\frac{m_\rho}{m_P} = \frac{1}{\sqrt{2}}$$

$$\frac{\Lambda_{\overline{MS}}}{m_\rho} = 0.455 \pm 0.031$$

Hadronization at the Amplitude Level

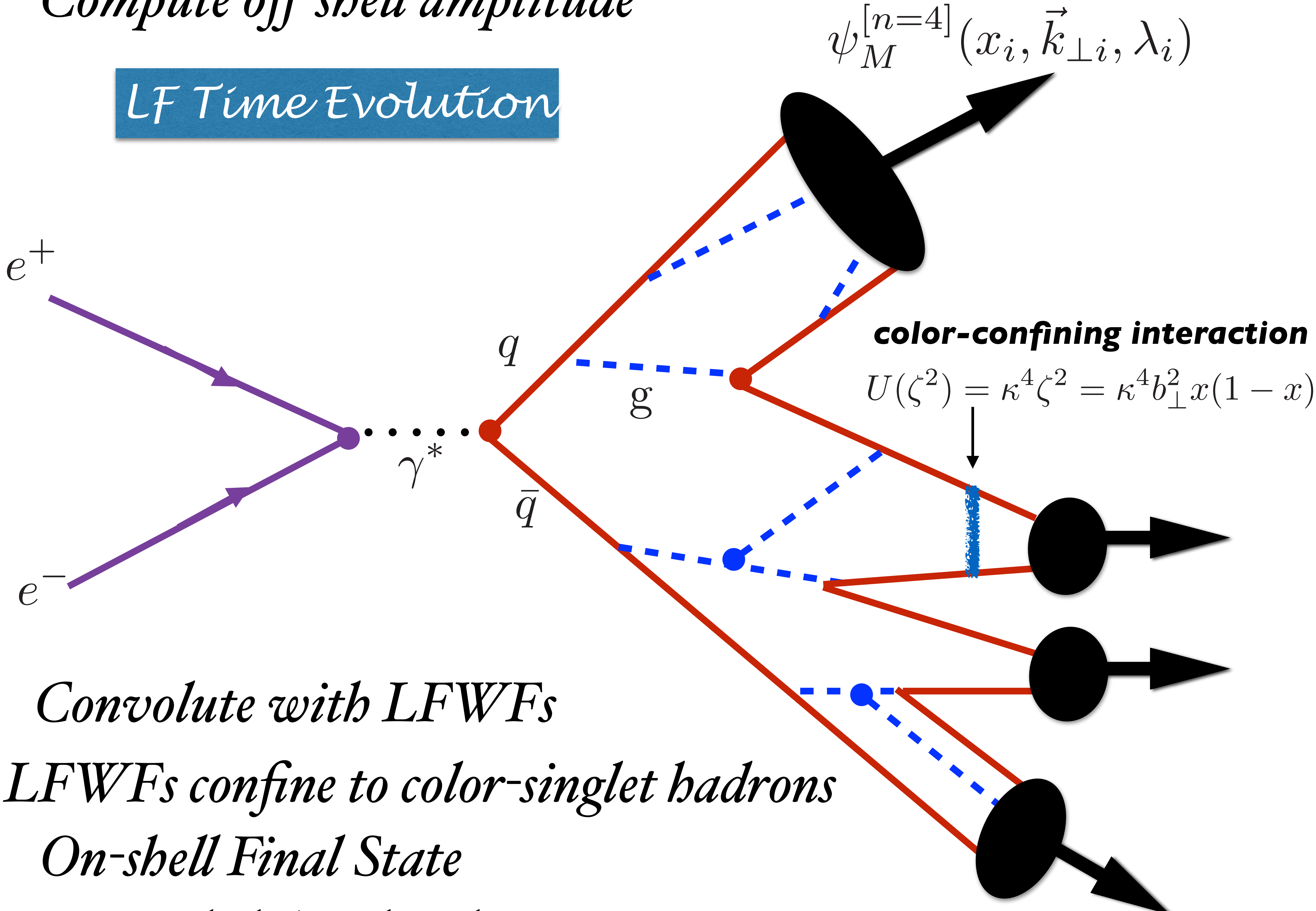
- ***Quarks and Gluons are confined; do not appear as asymptotic states***
- ***Hadron LFWFs: Arbitrarily Off-Shell in parton invariant mass; Fock state expansion***
- ***Hadron LFWFs: Amplitudes that convert quarks and gluons to hadrons, Fock state by Fock state***
- ***J^z conservation each and every state: entanglement***
- ***Harmonic oscillator confinement — potential energy between colored partons grows as***

$$U(\zeta^2) = \kappa^4 \zeta^2 = \kappa^4 b_\perp^2 x(1-x)$$

- ***Must compute processes at amplitude level***

Compute off-shell amplitude

LF Time Evolution



Convolute with LFWFs

LFWFs confine to color-singlet hadrons

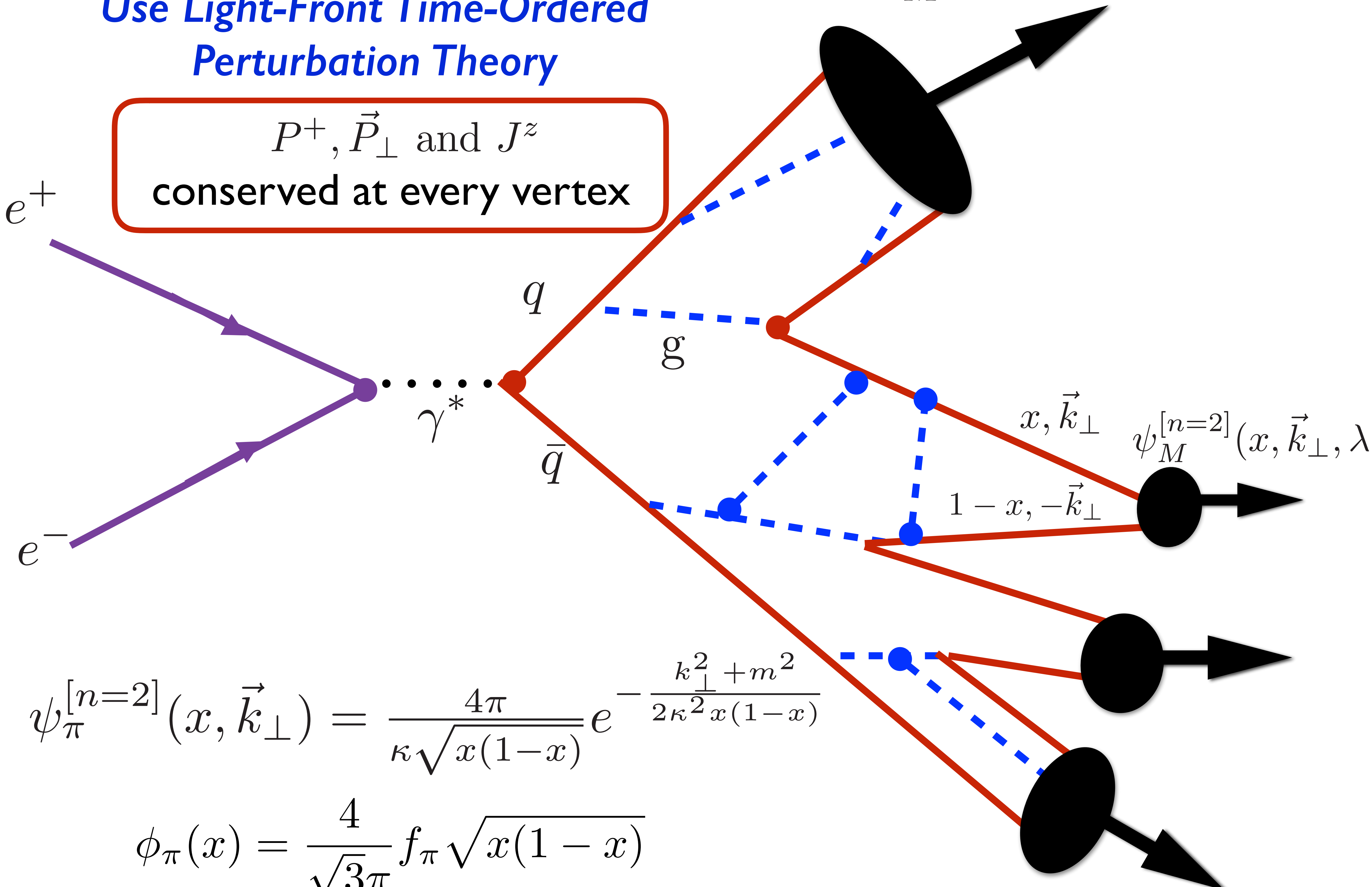
On-shell Final State

Entangled Amplitude

LF Time Evolution

Use Light-Front Time-Ordered Perturbation Theory

$P^+, \vec{P}_\perp$ and J^z
conserved at every vertex



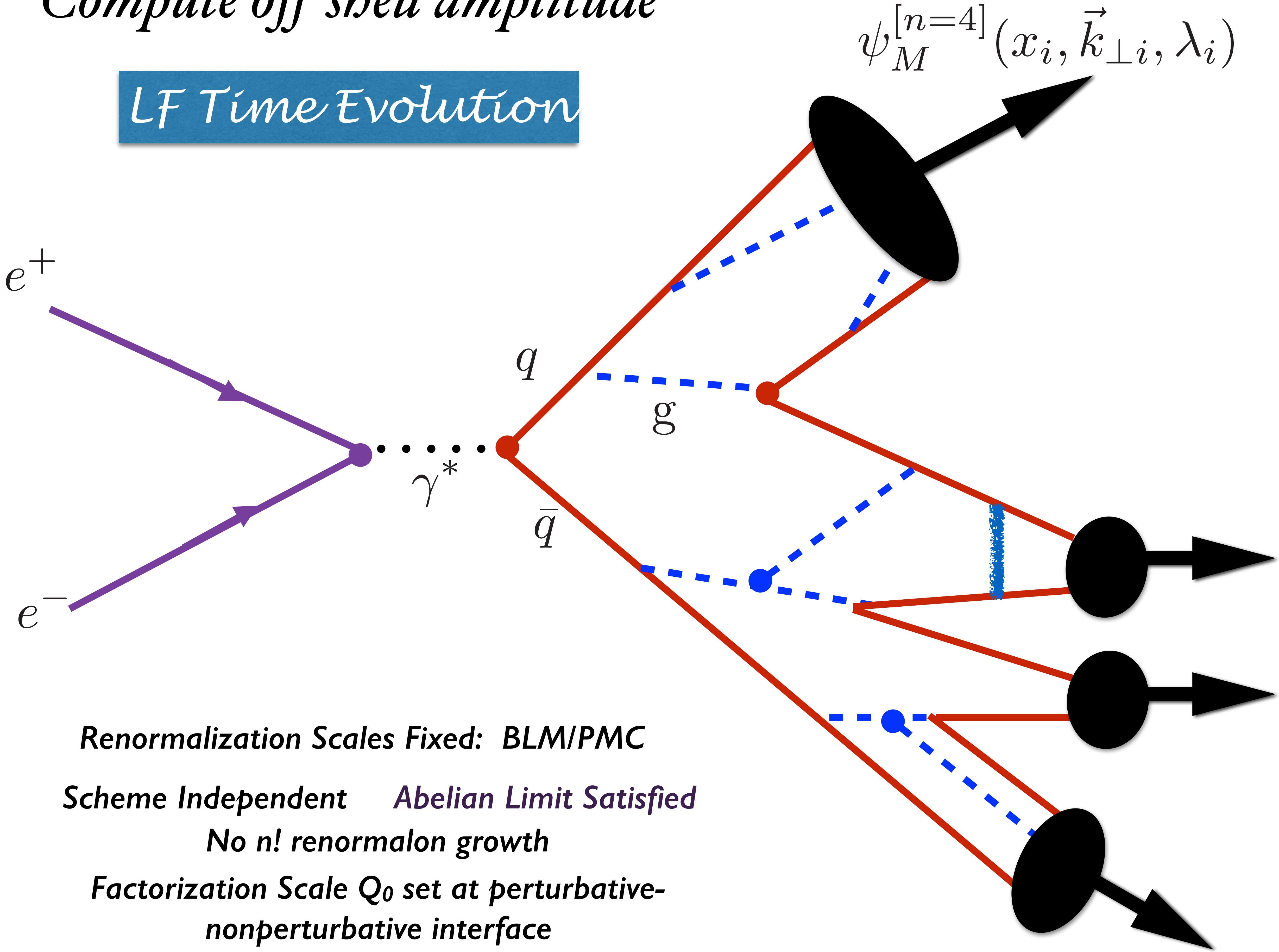
$$\psi_\pi^{[n=2]}(x, \vec{k}_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2 + m^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

ERBL Evolution of Distribution Amplitudes for $Q^2 > Q_0^2$

Compute off-shell amplitude

LF Time Evolution



Renormalization Scales Fixed: BLM/PMC

Scheme Independent *Abelian Limit Satisfied*

No n! renormalon growth

Factorization Scale Q_0 set at perturbative-nonperturbative interface

Counting Rules: Hard Exclusive Processes, x ~ 1

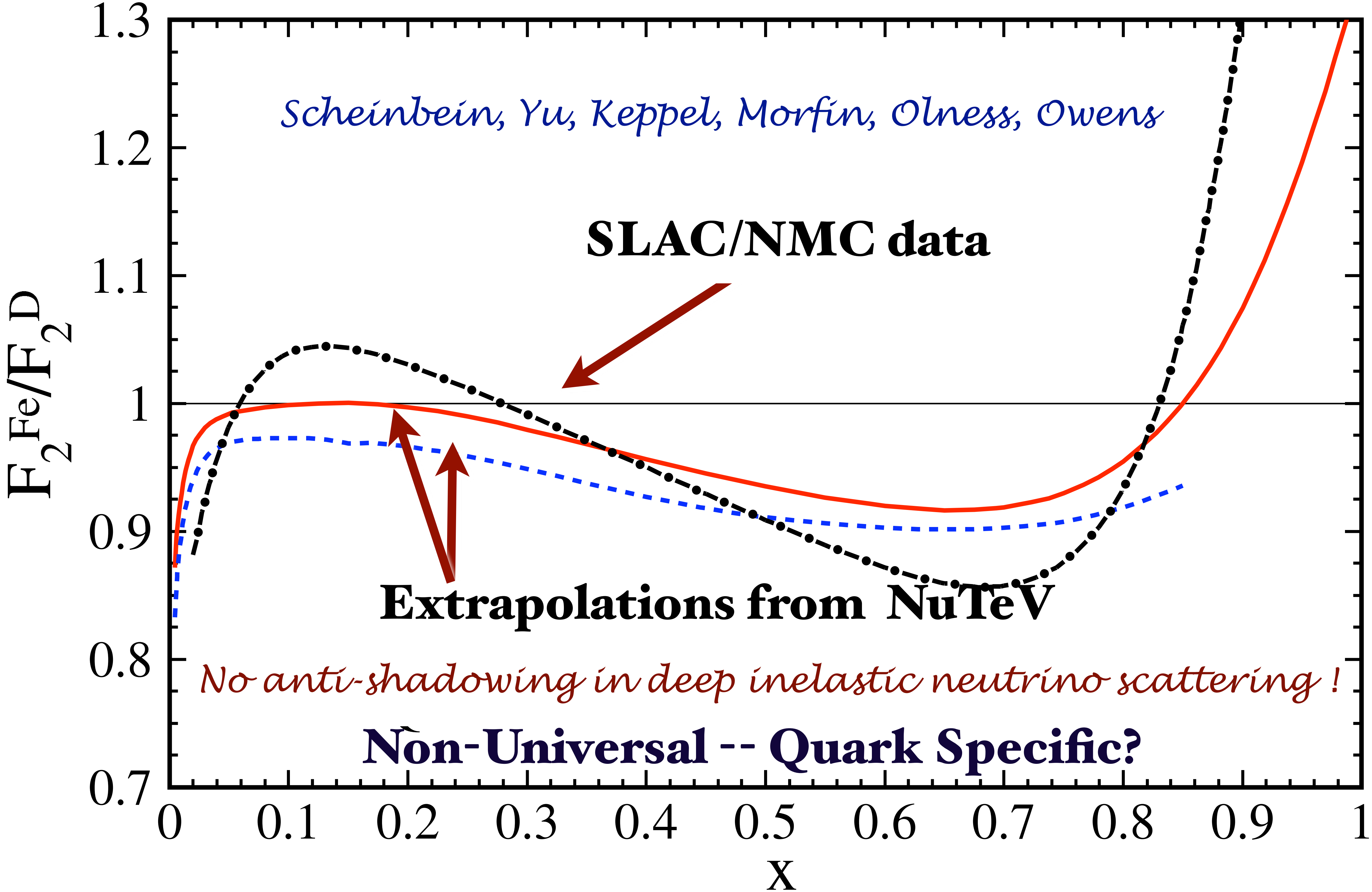
Invariance Principles of Quantum Field Theory

- *Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- *Causality: Information within causal horizon: Front Form*
- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

Novel QCD

- Flavor-Dependent Anti-Shadowing
- LF Vacuum and Cosmological Constant: No QCD condensates
- Principle of Maximum Conformality (PMC): Eliminate renormalization anomaly; scheme independent
- Match Perturbative and Non-Perturbative Domains
- Hadronization at Amplitude Level
- Intrinsic Heavy Quarks from AdS/QCD: Higgs at high x_F
- Ridge from flux tube collisions
- Baryon-to-meson anomaly at high p_T

$Q^2 = 5 \text{ GeV}^2$



"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology

Elements of the solution:
(A) Light-Front Quantization: causal, frame-independent vacuum
(B) New understanding of QCD "Condensates"
(C) Higgs Light-Front Zero Mode

Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- Independent of observer frame
- Causal
- Lowest invariant mass state M= 0.
- Trivial up to k^+=0 zero modes-- already normal-ordering
- Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)
- QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.
- QED vacuum; no loops
- Zero cosmological constant from QED, QCD, EW

Goals

- Test QCD to maximum precision at the LHC
- Maximize sensitivity to new physics
- High precision determination of fundamental parameters
- Determine renormalizations scales without ambiguity
- Eliminate scheme dependence

Predictions for physical observables cannot depend on theoretical conventions such as the renormalization scheme

QCD Principles

- **Extended Conformal Invariance: AdS/QCD** $\kappa \to C\kappa$
- **Chiral QCD only predicts mass ratios**
- **Supersymmetric Features of QCD: Superconformal algebra**
- **Unique Confinement Potential, Nonperturbative Running Coupling**
- **Physics Independent of Observer Frame: LF!**
- **Physics Independent of Conventions such as MSbar: PMC**
- **Zero Cosmological Constant for Causal Frame-Independent LF Vacuum**
- **Leading Twist Factorization-Breaking Corrections from ISI, FSI**
- **Nuclear Shadowing and Antishadowing not in nuclear LFWF**
- **Nuclear PDFS do not obey sum rules**

PRL **110,** 192001 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Systematic All-Orders Method to Eliminate Renormalization-Scale and Scheme Ambiguities in Perturbative QCD

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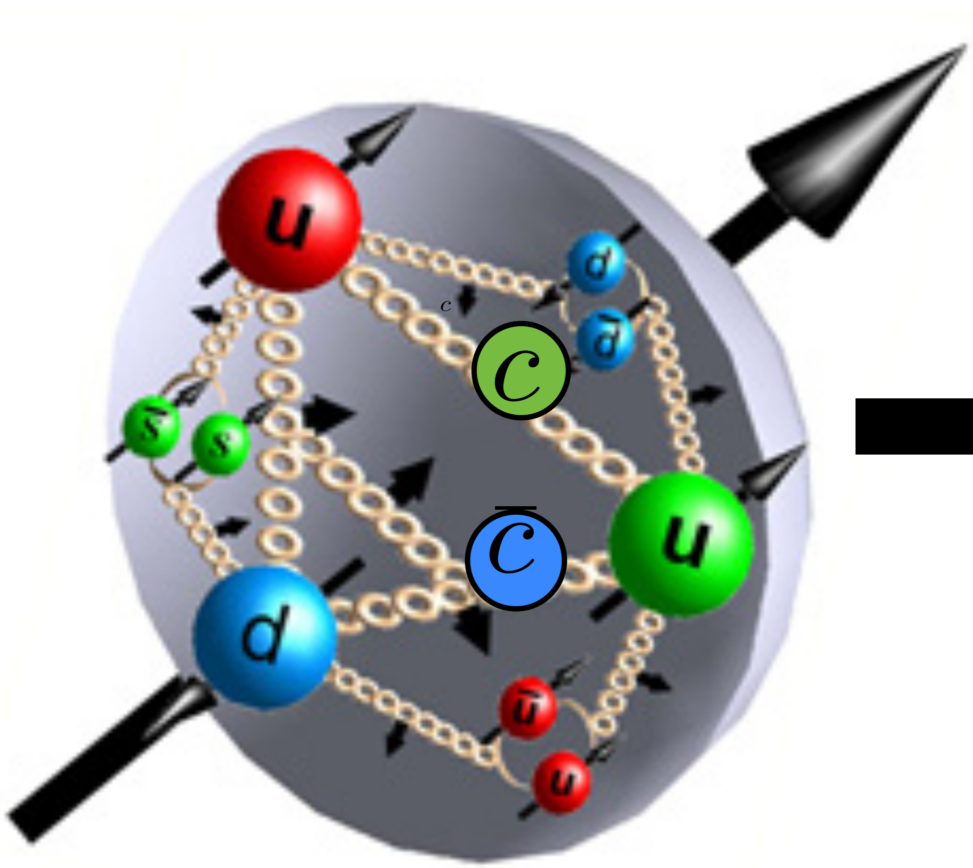
We introduce a generalization of the conventional renormalization schemes used in dimensional regularization, which illuminates the renormalization scheme and scale ambiguities of perturbative QCD predictions, exposes the general pattern of nonconformal $\{\beta_i\}$ terms, and reveals a special degeneracy of the terms in the perturbative coefficients. It allows us to systematically determine the argument of the running coupling order by order in perturbative QCD in a form which can be readily automatized. The new method satisfies all of the principles of the renormalization group and eliminates an unnecessary source of systematic error.

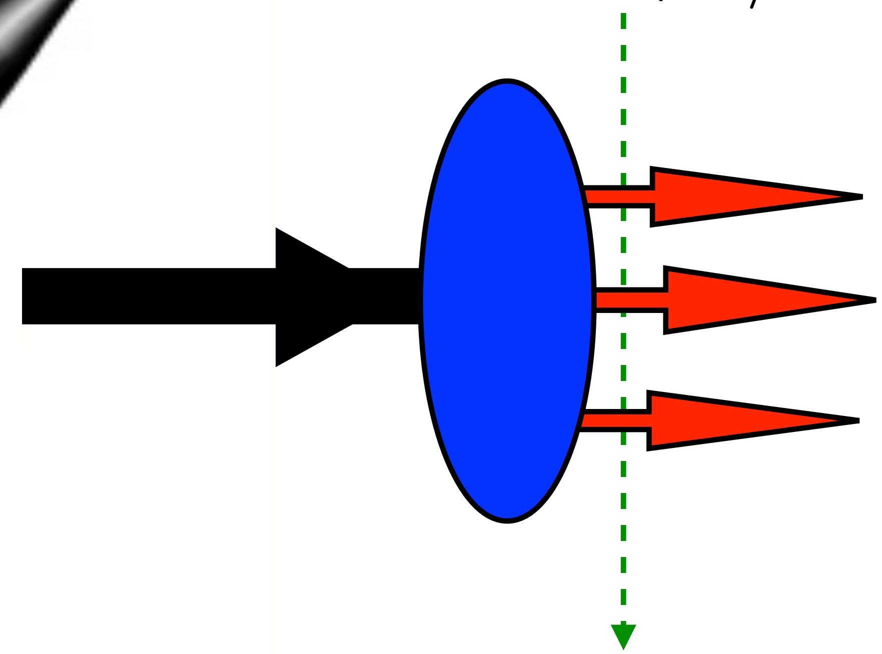
Principle of Maximum Conformality (PMC)

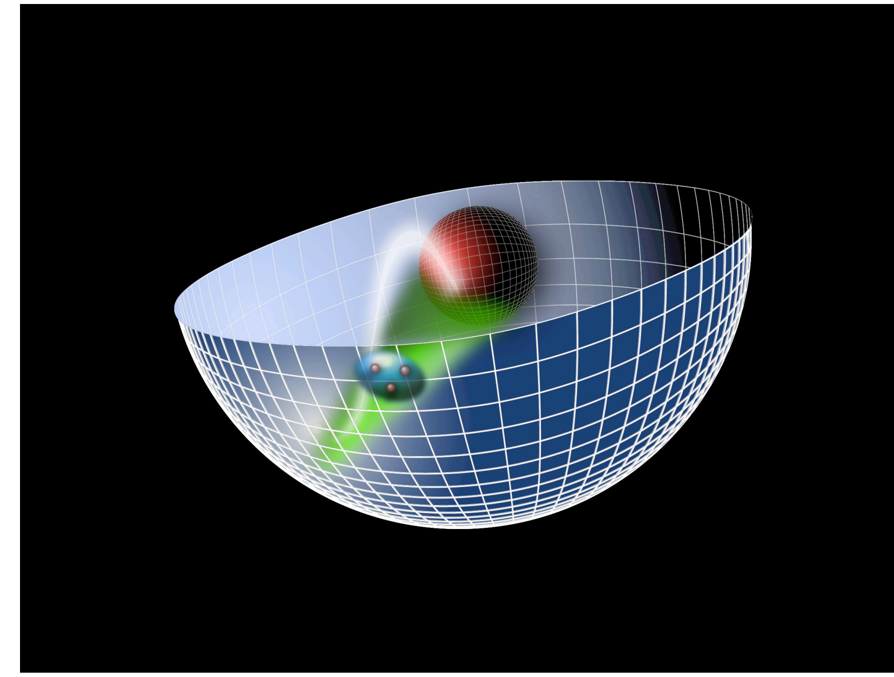
Features of BLM/PMC

- ***Predictions are scheme-independent***
- ***Matches conformal series***
- ***Commensurate Scale Relations between observables: Generalized Crewther Relation (Kataev, Lu, Rathsman, sjb)***
- ***No n! Renormalon growth***
- ***New scale at each order; n_F determined at each order***
- ***Multiple Physical Scales Incorporated***
- ***Rigorous: Satisfies all Renormalization Group Principles***
- ***Realistic Estimate of Higher-Order Terms***
- ***Eliminates unnecessary theory error***

Supersymmetric Properties of Hadron Physics and Predictions for Exclusive Processes from Light-Front Holography and Superconformal Algebra







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LELAND STANFORD JUNIOR UNIVERSITY

with Guy de Tèramond, Hans Günter Dosch, C. Lorce, K. Chiu, R. S. Sufian, A.Deur

May 29-31, 2017

Nucleon and Resonance Structure with Hard Exclusive Processes

INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY

Electron-Electron Scattering in QED

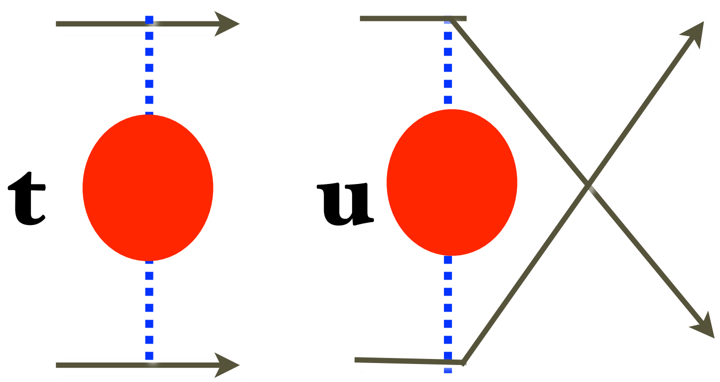
$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$

$$\alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)}$$

Gell-Mann--Low Effective Charge

Electron-Electron Scattering in QED

$$\mathcal{M}_{ee\to ee}(++;++) = \frac{8\pi s}{t}\,\alpha(t) + \frac{8\pi s}{u}\,\alpha(u)$$



- **Two separate physical scales: t, u = photon virtuality**
- **Gauge Invariant. Dressed photon propagator**
- **Sums all vacuum polarization, non-zero beta terms into running coupling. This is the purpose of the running coupling!**
- **If one chooses a different initial scale, one must sum an infinite number of graphs -- but always recover same result!**
- **Number of active leptons correctly set**
- **Analytic: reproduces correct behavior at lepton mass thresholds**
- **No renormalization scale ambiguity!**

δ-$\mathcal{R}$enormalization Scheme ($\mathcal{R}_\delta$ scheme)

In dim. reg. $1/\epsilon$ poles come in powers of [Bollini & Gambiagi, 't Hooft & Veltman, '72]

$$\ln \frac{\mu^2}{\Lambda^2} + \frac{1}{\epsilon} + c$$

In the modified minimal subtraction scheme (MS-bar) one subtracts together with the pole a constant [Bardeen, Buras, Duke, Muta (1978) on DIS results]:

$$\ln(4\pi) - \gamma_E$$

This corresponds to a shift in the scale:

$$\mu^2_{\overline{\rm MS}} = \mu^2 \exp(\ln 4\pi - \gamma_E)$$

A finite subtraction from infinity is arbitrary. *Let's make use of this!*

Subtract an arbitrary constant and keep it in your calculation: $\mathcal{R}_\delta$-scheme

M. Mojaza, Xing-Gang Wu, sjb

$$\ln(4\pi) - \gamma_E - \delta,$$

$$\mu_\delta^2 = \mu^2_{\overline{\rm MS}} \exp(-\delta) = \mu^2 \exp(\ln 4\pi - \gamma_E - \delta)$$

M. Mojaza, Xing-Gang Wu, sjb

Exposing the Renormalization Scheme Dependence

Observable in the $\mathcal{R}_\delta$-scheme:

$$\rho_\delta(Q^2) = r_0 + r_1 a(\mu) + [r_2 + \beta_0 r_1 \delta] a(\mu)^2 + [r_3 + \beta_1 r_1 \delta + 2\beta_0 r_2 \delta + \beta_0^2 r_1 \delta^2] a(\mu)^3 + \cdots$$

$$\mathcal{R}_0 = \overline{\text{MS}} \ , \quad \mathcal{R}_{\ln 4\pi - \gamma_E} = \text{MS} \qquad \mu^2 = \mu_{\overline{\text{MS}}}^2 \ \exp(\ln 4\pi - \gamma_E) \ , \quad \mu_{\delta_2}^2 = \mu_{\delta_1}^2 \ \exp(\delta_2 - \delta_1)$$

Note the divergent ‘renormalon series’ $n! \beta^n \alpha_s^n$

Renormalization Scheme Equation

$$\frac{d\rho}{d\delta} = -\beta(a) \frac{d\rho}{da} \stackrel{!}{=} 0 \quad \longrightarrow \text{PMC}$$

$$\rho_\delta(Q^2) = r_0 + r_1 a_1(\mu_1) + (r_2 + \beta_0 r_1 \delta_1) a_2(\mu_2)^2 + [r_3 + \beta_1 r_1 \delta_1 + 2\beta_0 r_2 \delta_2 + \beta_0^2 r_1 \delta_1^2] a_3(\mu_3)^3$$

The $\delta_k^p a^n$-term indicates the term associated to a diagram with $1/\epsilon^{n-k}$ divergence for any p. Grouping the different δ_k-terms, one recovers in the $N_c \to 0$ Abelian limit the dressed skeleton expansion.

M. Mojaza, Xing-Gang Wu, sjb

General result for an observable in any $\mathcal{R}_\delta$ renormalization scheme:

$$\rho(Q^2) = r_{0,0} + r_{1,0}a(Q) + [r_{2,0} + \beta_0 r_{2,1}]a(Q)^2 + [r_{3,0} + \beta_1 r_{2,1} + 2\beta_0 r_{3,1} + \beta_0^2 r_{3,2}]a(Q)^3 + [r_{4,0} + \beta_2 r_{2,1} + 2\beta_1 r_{3,1} + \frac{5}{2}\beta_1\beta_0 r_{3,2} + 3\beta_0 r_{4,1} + 3\beta_0^2 r_{4,2} + \beta_0^3 r_{4,3}]a(Q)^4 + \mathcal{O}(a^5)$$

PMC scales thus satisfy

$$r_{1,0}a(Q_1) = r_{1,0}a(Q) - \beta(a)r_{2,1}$$

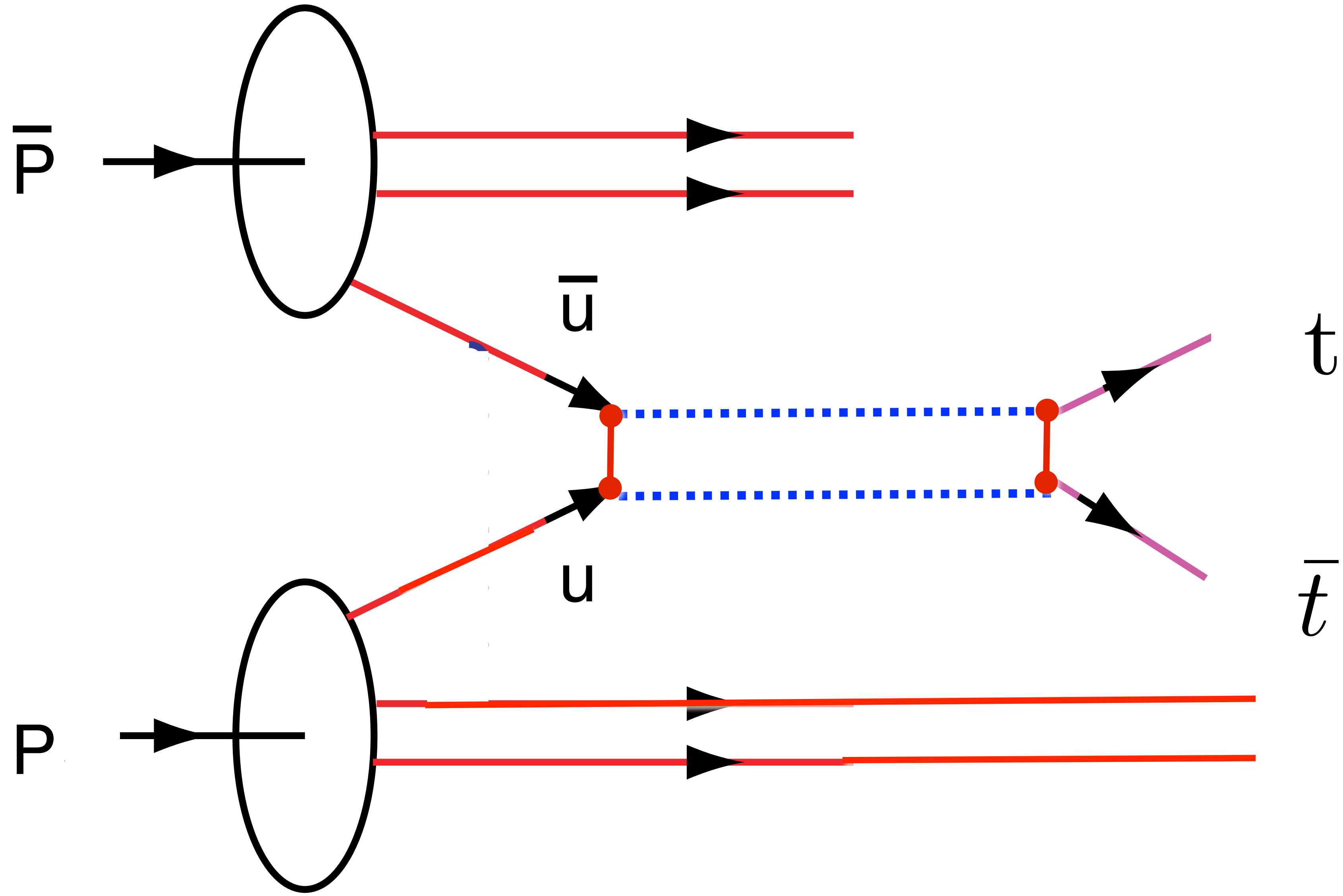
$$r_{2,0}a(Q_2)^2 = r_{2,0}a(Q)^2 - 2a(Q)\beta(a)r_{3,1}$$

$$r_{3,0}a(Q_{\underset{3}{2}})^3 = r_{3,0}a(Q)^3 - 3a(Q)^2\beta(a)r_{4,1}$$

$$\vdots$$

$$r_{k,0}a(Q_k)^k = r_{k,0}a(Q)^2 - k\ a(Q)^{k-1}\beta(a)r_{k+1,1}$$

Implications for the $\bar{p}p \to t\bar{t}X$ asymmetry at the Tevatron



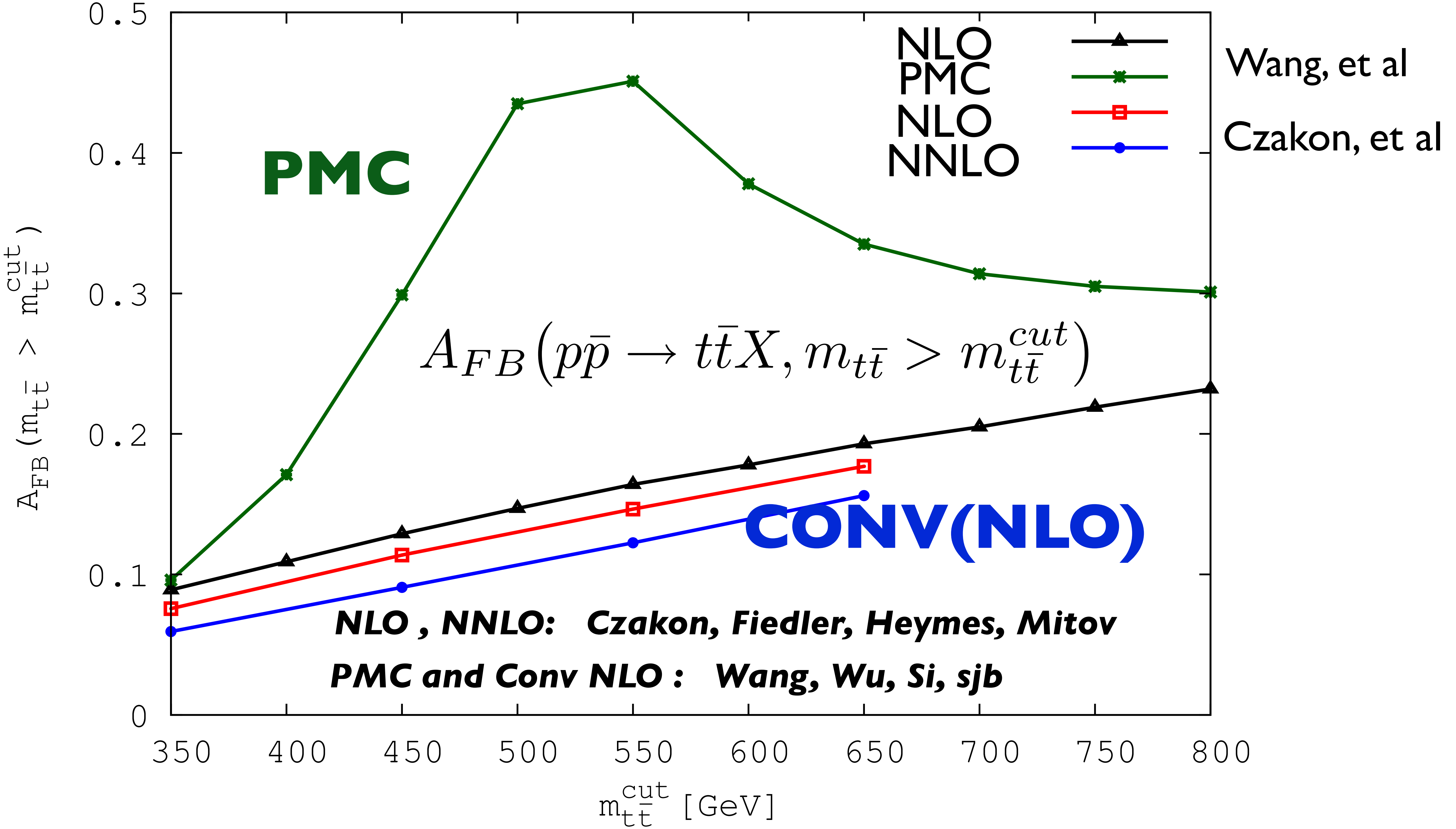
Interferes with Born term.

Smaller value of renormalization scale increases asymmetry, just as in QED

Xing-Gang Wu, sjb

NNLO QCD predictions for fully-differential top-quark pair production at the Tevatron arXiv:1601.05375

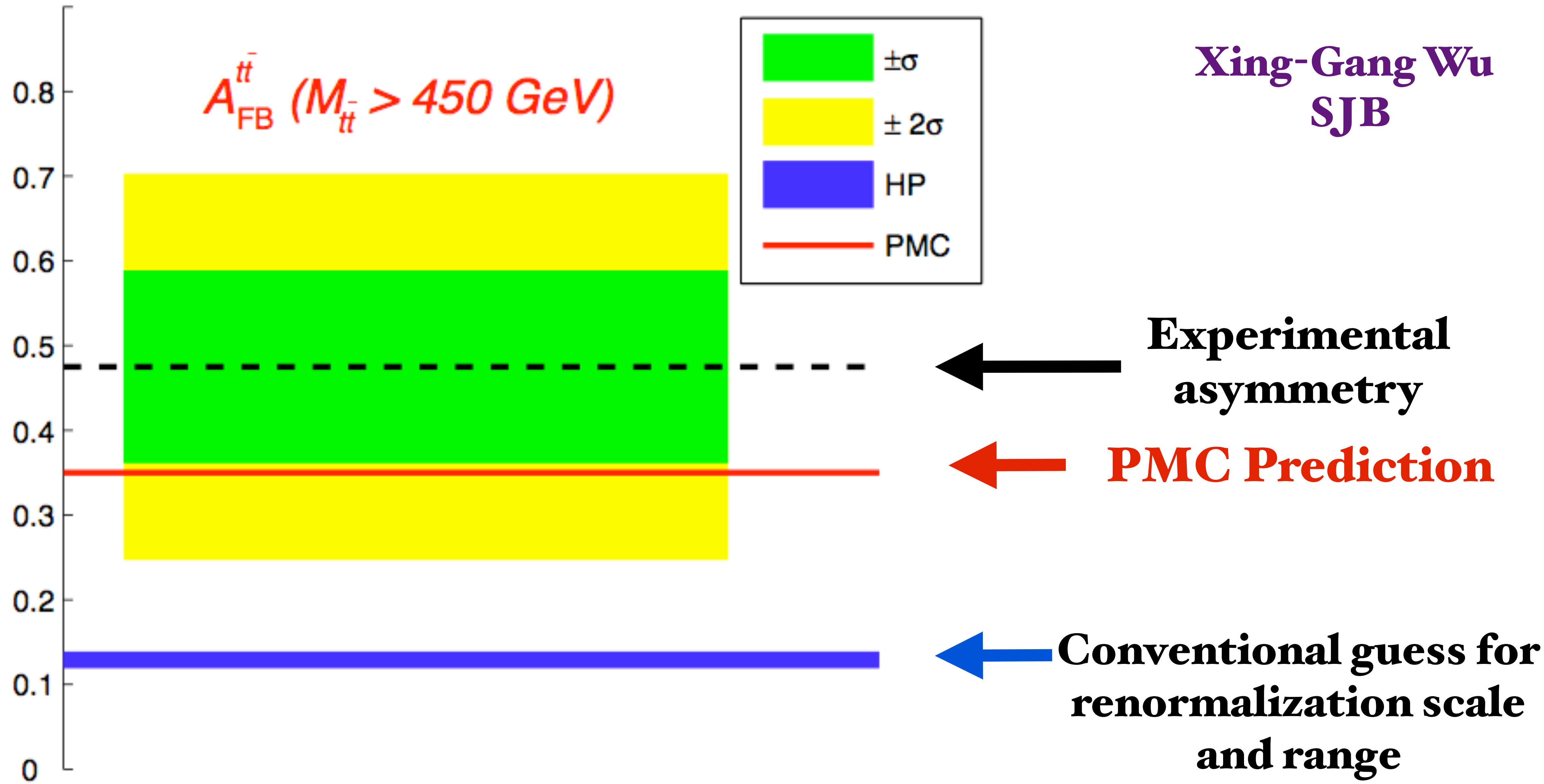
Michał Czakon,[a] Paul Fiedler,[a] David Heymes[b] and Alexander Mitov[b]



Predictions for the cumulative front-back asymmetry.

The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)

Xing-Gang Wu
SJB



Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting

Set multiple renormalization scales -- Lensing, DGLAP, ERBL Evolution ...

Choose renormalization scheme; e.g. $\alpha_s^R(\mu_R^{\rm init})$

↓

Choose μ_R^{init}*; arbitrary initial renormalization scale*

↓

Identify β_i via δ-dependence

↓

Shift scale of α_s *to* $\mu_R^{\rm PMC}$ *to eliminate* $\{\beta_i^R\}$ *− terms*

↓

Conformal Series

Result is independent of $\mu_R^{\rm init}$ *and scheme at fixed order*

PMC/BLM

No renormalization scale ambiguity!

Result is independent of Renormalization scheme and initial scale!

QED Scale Setting at N_C=0

Eliminates unnecessary systematic uncertainty

Scale fixed at each order

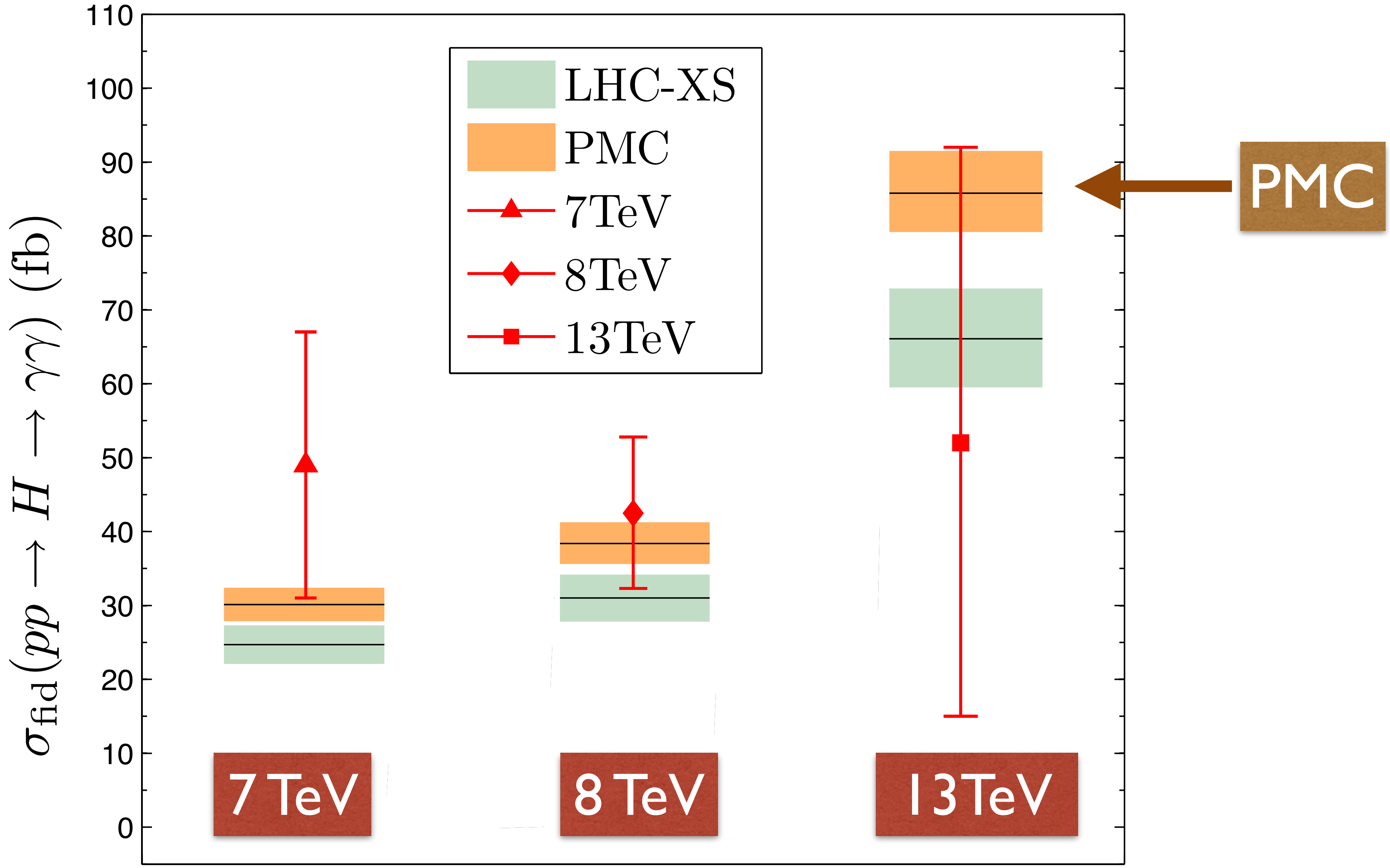
δ-Scheme automatically identifies β-terms!

Principle of Maximum Conformality

Xing-Gang Wu, Matin Mojaza
Leonardo di Giustino, SJB

S-Q Wang, X-G Wu, sjb

$\sigma(pp \to HX \to \gamma\gamma X)$



Comparison of the PMC predictions for the fiducial cross section $\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$ with the ATLAS measurements at various collision energies. The LHC-XS predictions are presented as a comparison.

$\sigma_{\rm fid}(pp \to H \to \gamma\gamma)$	7 TeV	8 TeV	13 TeV
ATLAS data [48]	49 ± 18	$42.5^{+10.3}_{-10.2}$	52^{+40}_{-37}
LHC-XS [3]	24.7 ± 2.6	31.0 ± 3.2	$66.1^{+6.8}_{-6.6}$
PMC prediction	$30.1^{+2.3}_{-2.2}$	$38.4^{+2.9}_{-2.8}$	$85.8^{+5.7}_{-5.3}$

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

Huet, sjb

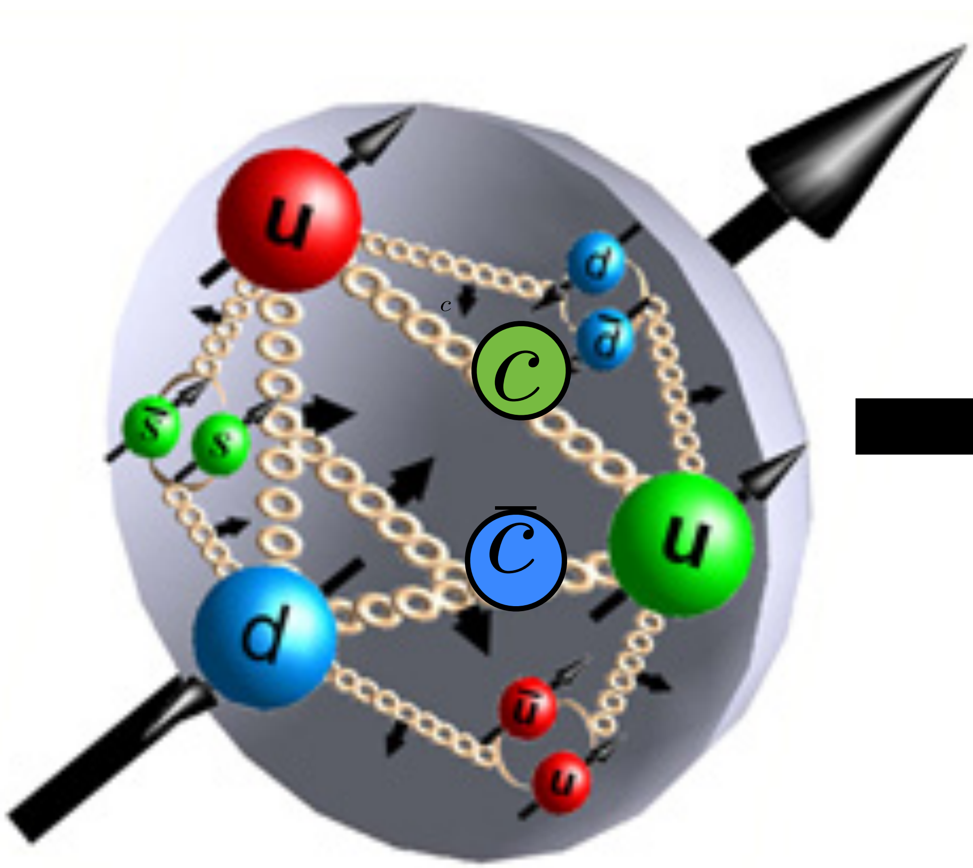
$\lim N_C \to 0$ at fixed $\alpha = C_F \alpha_s, n_\ell = n_F / C_F$

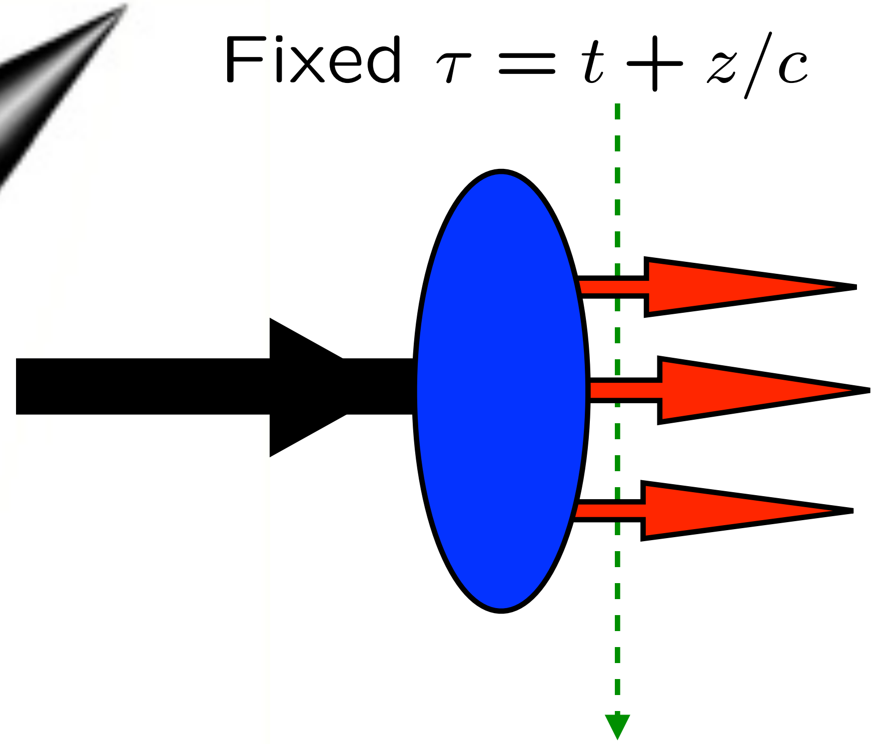
QCD $\to$ Abelian Gauge Theory

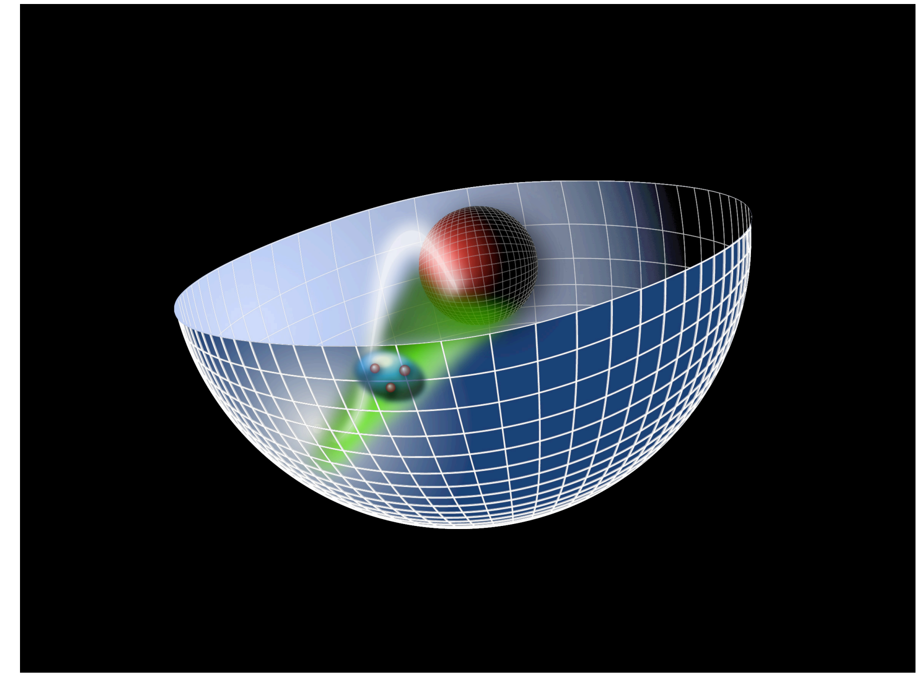
Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED

Supersymmetric Properties of Hadron Physics and Predictions for Exclusive Processes from Light-Front Holography and Superconformal Algebra







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