Nonperturbative Aspects of Kaon Structure

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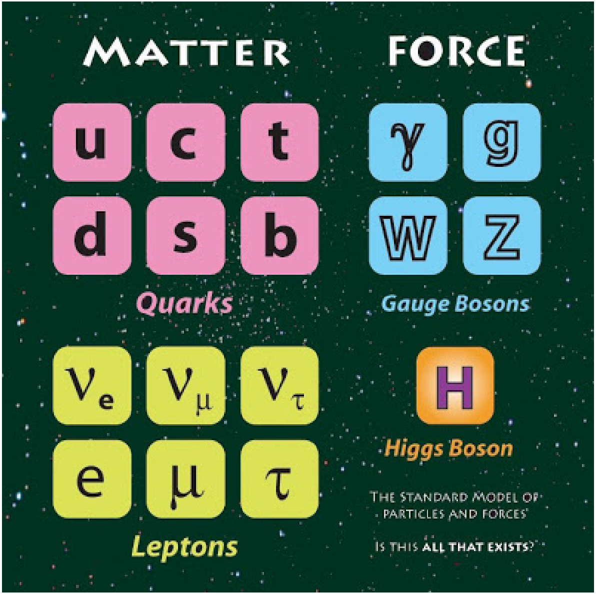
Nucleon and Resonance Structure with Hard Exclusive Processes

Asia Pacific Center for Theoretical Physics

Outline

Building Block of Matter



Lagrangian of the Standard Model

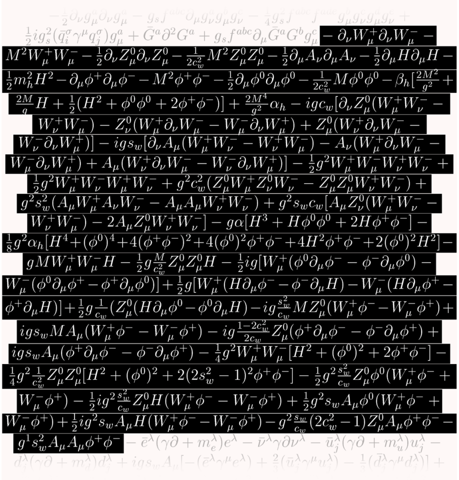
$\Rightarrow$ These three lines in the Standard Model are ultra-specific to the gluon, the boson that carries the strong force. Gluons come in eight types, interact among themselves and have what's called a color charge [1]

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4} g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$
$$\frac{1}{2} i g_s^2 (\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a \partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$
$$M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2} M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H -$$

[1] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model

$\Rightarrow$ Almost half of this equation is dedicated to explaining interactions between bosons, particularly W and Z bosons. Bosons are force-carrying particles, and there are four species of bosons that interact with other particles using three fundamental forces. Photons carry electromagnetism, gluons carry the strong force and W and Z bosons carry the weak force. The most recently discovered boson, the Higgs boson, is a bit different; its interactions appear in the next part of the equation [2]



[2] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model

$\Rightarrow$ This part of the equation describes how elementary matter particles interact with the weak force. According to this formulation, matter particles come in three generations, each with different masses. The weak force helps massive matter particles decay into less massive matter particles. This section also includes basic interactions with the Higgs field, from which some elementary particles receive their mass [3]

$$
\begin{aligned}
& W_\mu^- \phi^+) + \tfrac{1}{2} i g^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \tfrac{s_w}{c_w}(2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
& g^1 s_w^2 A_\mu A_\mu \phi^+ \phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda \gamma\partial \nu^\lambda - \bar{u}_j^\lambda(\gamma\partial + m_u^\lambda)u_j^\lambda - \\
& \bar{d}_j^\lambda(\gamma\partial + m_d^\lambda)d_j^\lambda + i g s_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}_j^\lambda\gamma^\mu u_j^\lambda) - \tfrac{1}{3}(\bar{d}_j^\lambda\gamma^\mu d_j^\lambda)] + \\
& \tfrac{ig}{4c_w} Z_\mu^0[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}_j^\lambda\gamma^\mu(\tfrac{4}{3}s_w^2 - \\
& 1 - \gamma^5)u_j^\lambda) + (\bar{d}_j^\lambda\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d_j^\lambda)] + \tfrac{ig}{2\sqrt{2}} W_\mu^+[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + \\
& (\bar{u}_j^\lambda\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d_j^\kappa)] + \tfrac{ig}{2\sqrt{2}} W_\mu^-[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger\gamma^\mu(1+ \\
& \gamma^5)u_j^\lambda)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] - \\
& \tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}_j^\lambda C_{\lambda\kappa}(1-\gamma^5)d_j^\kappa) +
\end{aligned}
$$

[3] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model

$\Rightarrow$ This part of the equation describes how matter particles interact with Higgs ghosts, virtual artifacts from the Higgs field [4]

$$(\bar{u}_j^\lambda \gamma^\mu (1+\gamma^5) C_{\lambda\kappa} d_j^\kappa)] + \frac{ig}{2\sqrt{2}} W_\mu^- [(\bar{e}^\lambda \gamma^\mu (1+\gamma^5)\nu^\lambda) + (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger \gamma^\mu (1+$$
$$\gamma^5) u_j^\lambda)] + \frac{ig}{2\sqrt{2}} \frac{m_e^\lambda}{M} [-\phi^+ (\bar{\nu}^\lambda (1-\gamma^5) e^\lambda) + \phi^- (\bar{e}^\lambda (1+\gamma^5)\nu^\lambda)] -$$
$$\frac{g}{2} \frac{m_e^\lambda}{M} [H(\bar{e}^\lambda e^\lambda) + i\phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}} \phi^+ [-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1-\gamma^5) d_j^\kappa) +$$
$$m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1+\gamma^5) d_j^\kappa] + \frac{ig}{2M\sqrt{2}} \phi^- [m_d^\lambda (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1+\gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\lambda C_{\lambda\kappa}^\dagger (1-$$
$$\gamma^5) u_j^\kappa] - \frac{g}{2} \frac{m_u^\lambda}{M} H(\bar{u}_j^\lambda u_j^\lambda) - \frac{g}{2} \frac{m_d^\lambda}{M} H(\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_u^\lambda}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) -$$
$$\frac{ig}{2} \frac{m_d^\lambda}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 -$$
$$\frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + igc_w W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \partial_\mu \bar{X}^+ X^0) + igs_w W_\mu^+ (\partial_\mu \bar{Y} X^- -$$

4 http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Lagrangian of the Standard Model

⇒ This last part of the equation includes more ghosts. These ones are called Faddeev-Popov ghosts, and they cancel out redundancies that occur in interactions through the weak force [5]

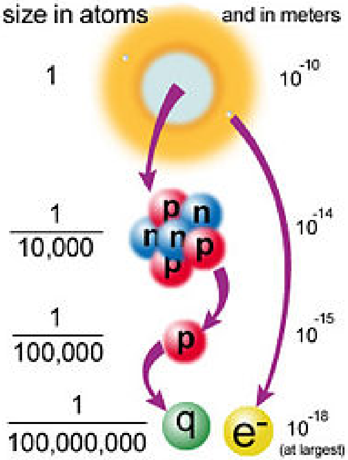
$$\gamma^5)u_j^\kappa] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar u_j^\lambda u_j^\lambda) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar d_j^\lambda d_j^\lambda) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u_j^\lambda\gamma^5 u_j^\lambda) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d_j^\lambda\gamma^5 d_j^\lambda) + \bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0(\partial^2 - \frac{M^2}{c_w^2})X^0 + \bar Y\partial^2 Y + igc_wW_\mu^+(\partial_\mu\bar X^0X^- - \partial_\mu\bar X^+X^0) + igs_wW_\mu^+(\partial_\mu\bar YX^- - \partial_\mu\bar X^+Y) + igc_wW_\mu^-(\partial_\mu\bar X^-X^0 - \partial_\mu\bar X^0X^+) + igs_wW_\mu^-(\partial_\mu\bar X^-Y - \partial_\mu\bar YX^+) + igc_wZ_\mu^0(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) - \frac{1}{2}gM[\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H] + \frac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \frac{1}{2}igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]$$

⇒ We concentrate on Quantum Chromodynamics (QCD)

[5] http://www.symmetrymagazine.org/article/the-deconstructed-standard-model-equation

Outline

Particle Size



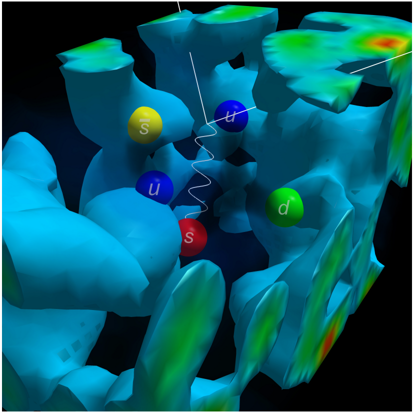
⇒ We focus on the interaction quarks inside proton/meson (1*fm*)

Quantum Chromodynamics (QCD)

QCD start out with almost massless quarks + massless gluons + gauge bosons, their interactions generate mass of the hadrons more than 95%.

- ⇒ Short distance scales ($r < 0.1 fm$) ⇒ QCD is a theory of weakly coupled quark and gluon ⇒ PERTURBATIVE QCD applicable
- ⇒ Low energy limit, at momentum below 1 GeV ($r > 1 fm$) ⇒ QCD is governed by quark (color) confinement and dynamical breaking of chiral symmetry ⇒ NON-PERTURBATIVE QCD applicable

STRUCTURE of THE HADRONS $\Longleftrightarrow$ QCD

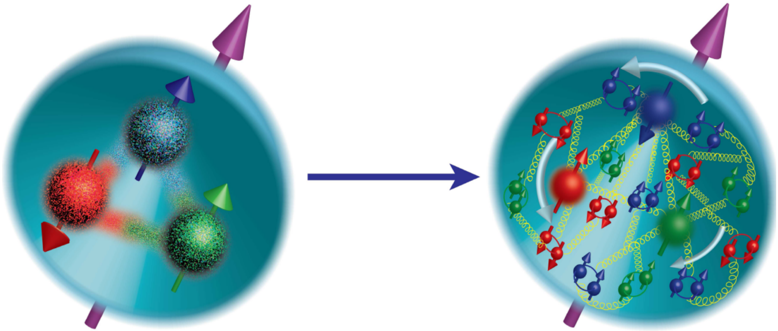


Courtesy : Derek Leinweber, CCSM, University of Adelaide

Outline

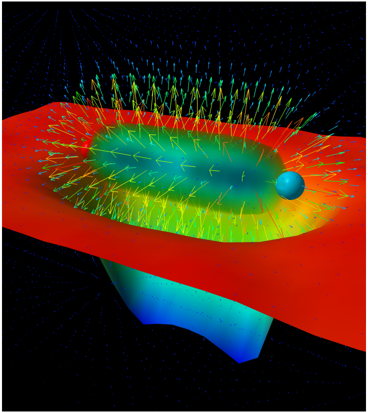
Hadrons : Proton

⇒ proton ($qqq + gluons + sea\ quarks\ (intrinsic)$)



HADRONS : MESON

⇒ Focus on MESON ($\bar{q}q + gluons + sea\ quarks\ (intrinsic)$)

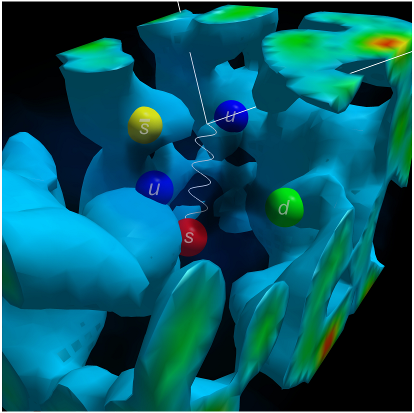


Courtesy : Derek Leinweber, CCSM, University of Adelaide

Motivation : PDF and FF in the Pion and Kaon

- QCD, as underlying theory of strong interaction, is unable to directly predict structure of hadrons. The solution:
  - Lattice QCD
  - QCD inspired models (mimicking features of QCD) such as NJL model, DSE model, QCD Sum rules and Chiral effective models
- To understand the structure of strongly interacting matter, PDF and FF are of fundamental importance and provide complementary information
- Kaon structure is simpler than the nucleon, but not so simple. In fact, we do not really understand the structure of the kaon
- A great opportunity to gain useful information of the dynamics of quarks to use it to understand the quark dynamics in the nucleon, which is more complicated system
- From experimental side, next experimental data for the pion and kaon will be coming soon from JLAB (expected 2017), J-PARC as well as COMPASS

PION AND KAON PROPERTIES IN THE CONFINING NJL MODEL



Courtesy : Derek Leinweber, CCSM, University of Adelaide

Pion and Kaon in the confining NJL Model

The three flavor NJL Lagrangian [6]– containing only four fermion interactions

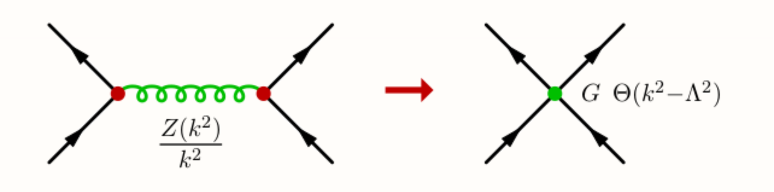
$$\mathcal{L}_{NJL} = \bar{\psi}[i\partial\!\!\!/ - \hat{m}_q]\psi + G_\pi \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\psi)^2 + (\bar{\psi}\lambda_a\gamma_5\psi)^2\right] - G_\rho \sum_{a=0}^{8}\left[(\bar{\psi}\lambda_a\gamma^\mu\psi)^2 + (\bar{\psi}\lambda_a\gamma^\mu\gamma_5\psi)^2\right] \quad (1)$$

- $\psi = (u, d, s)$ denotes the quark field with the flavor components
- G_π and G_ρ are four-fermion coupling constants
- $\lambda_1, \cdots, \lambda_8$ are Gell-Mann matrices in flavor space and $\lambda_0 \equiv \sqrt{\frac{2}{3}}\mathbb{1}$
- $\hat{m}_q = \mathrm{diag}(m_u, m_d, m_s)$ denotes the current quark matrix

[6] PH, Ian Cloet, and A. Thomas, PRC94 (2016), 035201

Pion and Kaon in the confining NJL Model

- In the NJL model, the gluon exchange is replaced by four-fermion contact interaction

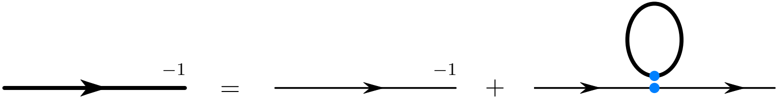


- NJL model has a lack of confinement (it can be simply seen quark propagator has a pole)
- Therefore we regularize using the proper time regularization. Here we introduce Λ_{IR} cut-off parameter to simulate confinement

$$\frac{1}{D^n} = \frac{1}{(n-1)!}\int_0^\infty d\tau\tau^{n-1}e^{-\tau D} \rightarrow \frac{1}{(n-1)!}\int_{\frac{1}{\Lambda_{UV}^2}}^{\frac{1}{\Lambda_{IR}^2}} d\tau\tau^{n-1}e^{-\tau D} \quad (2)$$

$\Rightarrow$ Λ_{IR} eliminates the possibility the hadron decay into quarks

- NJL Gap Equation

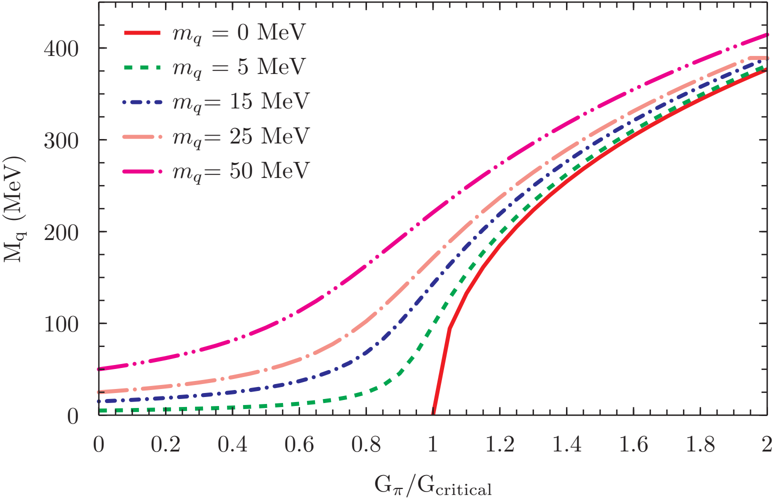


$$
\begin{aligned}
M_q &= m_q + 48 i G_\pi M_q \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 - M_q^2 + i\epsilon} \\
&= m_q + M_q \frac{3G_\pi}{\pi^2} \int d\tau \frac{e^{-\tau M_q^2}}{\tau^2} \\
&= m_q - 2G_\pi \langle \bar{\psi}\psi \rangle \qquad (3)
\end{aligned}
$$

- Chiral condensates is defined by $\langle \bar{\psi}\psi \rangle = -\int \frac{d^4p}{(2\pi)^4} Tr[iS(p)]$
- Mass is generated through interaction vacuum $\rightarrow \langle \bar{\psi}\psi \rangle \neq 0$

NJL Gap Equation

The NJL Gap equation in the confining NJL model result [7]

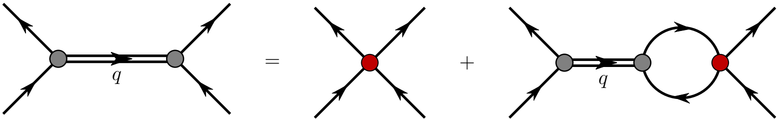


[7] PH, Ian Cloet, and A. Thomas, PRC94 (2016), 035201

NJL Gap Equation

- In the chiral limit ($m_q \sim 0$), the constituent quark mass has non-trivial solution where $M \neq 0$, provided $G_\pi > G_{critical}$. This corresponds to DCSB and the Nambu-Goldstone phase. The Nambu-Goldstone phase occurs when the chiral has been dynamically broken
- when $m_q = 0$, the critical coupling has a form $G_{critical} = \frac{\pi^2}{3(\Lambda_{UV}^2 - \Lambda_{IR}^2}$
- From gap equation figure, we also clearly that when $M \neq m_q$ then $\langle \bar{\psi}\psi \rangle \neq 0$. This indicates that the dynamical mass generation is also associated with the generation of a non-zero chiral condensate.
- The chiral condensate are zero when $G_\pi < G_{critical}$. This is well know as the Wigner-Weyl phase

Bethe Salpeter Equation of the pion and kaon



- In the NJL model, T-matrix is given by

$$T(q) \;=\; \mathcal{K} + \int \frac{d^4k}{(2\pi)^4} \mathcal{K} S(q+k) T(q) S(k)$$

- The solution to the BSE in the pion and kaon

$$T_\alpha(q)_{ab,cd} = [\gamma_5 \lambda_\alpha]_{ab}\, t_\alpha(q) \left[\gamma_t \lambda_\alpha^\dagger\right] \quad (4)$$

- The reduced t-matrix in this channel take a form

$$t_\alpha(q) \;=\; \frac{-2i\,G_\pi}{1 + 2G_\pi \Pi_\pi(q^2)}$$

$$t_\beta^{\mu\nu}(q) \;=\; \frac{-2i\,G_\rho}{1 + 2G_\rho \Pi_\beta(q^2)} \left(g^{\mu\nu} + 2G_\rho \Pi_\beta(q^2) \frac{q^\mu q^\nu}{q^2} \right) \quad (5)$$

Bethe Salpeter Equation of the pion and kaon

- The bubble diagrams appearing read

$$\begin{aligned}\Pi_\pi(q^2) &= 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_l(k+q)\right],\\ \Pi_K(q^2) &= 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma_5 S_l(k)\gamma_5 S_s(k+q)\right],\\ \Pi_\nu^{aa}(q^2) &= 6i\int\frac{d^4k}{(2\pi)^4}Tr_D\left[\gamma^\mu S_a(k)\gamma^\nu S_a(k+q)\right]\end{aligned} \quad (6)$$

- The kaon and pion masses is given by the pole of the t-matrix

$$\begin{aligned}1+2G_\pi\Pi_\pi(k^2=m_\pi^2) &= 0\\ 1+2G_\pi\Pi_K(k^2=m_K^2) &= 0\end{aligned} \quad (7)$$

Pion and Kaon Masses

The meson masses are defined by the pole in the corresponding t-matrix and therefore the kaon and pion masses are given by

$$\begin{aligned} m_\pi^2 &= \left[\frac{m}{M_l}\right] \frac{2}{G_\pi \mathcal{I}_{ll}(m_\pi^2} \\ m_K^2 &= \left[\frac{m_s}{M_s} + \frac{m}{M_l}\right] \frac{1}{G_\pi \mathcal{I}_{ls}(m_K^2} + (M_s - M_l)^2 \end{aligned} \tag{8}$$

where $\mathcal{I}_{ll}$ and $\mathcal{I}_{ls}$ in the proper time regularization scheme are defined by

$$\mathcal{I}_{ab}(k^2) = \frac{3}{\pi^2} \int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(x(x-1)k^2 + xM_b^2 + (1-x)M_a^2)} \tag{9}$$

The meson-quark-quark coupling constants and Pion and Kaon Decay Constants

The residue at a pole in the $\bar{q}q$ $t-$matrix defines the effective meson-quark -quark coupling constants:

$$
\begin{aligned}
Z_\pi(q^2) &= -\frac{\partial \Pi_\pi(q^2)}{\partial q^2} \mid_{q^2=m_\pi^2} \\
Z_K(q^2) &= -\frac{\partial \Pi_K(q^2)}{\partial q^2} \mid_{q^2=m_K^2} \\
Z_\rho(q^2) &= -\frac{\partial \Pi_\rho(q^2)}{\partial q^2} \mid_{q^2=m_\rho^2}
\end{aligned} \tag{10}
$$

Pion and kaon decay constant in the proper time regularization is given by

$$
\begin{aligned}
f_\pi &= \frac{N_C\sqrt{Z_\pi}M}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+M^2)} \\
f_K &= \frac{N_C\sqrt{Z_K}}{4\pi^2}[(1-x)M_2 + xM_1]\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(k^2(x^2-x)+xM_2^2-(x-1)M_1^2)}
\end{aligned} \tag{11}
$$

The NJL Parameters Result

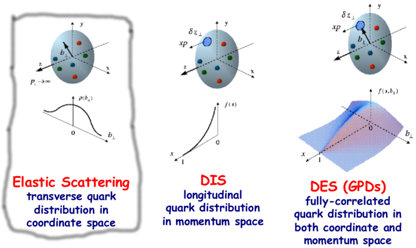
The parameters of our NJL model are:

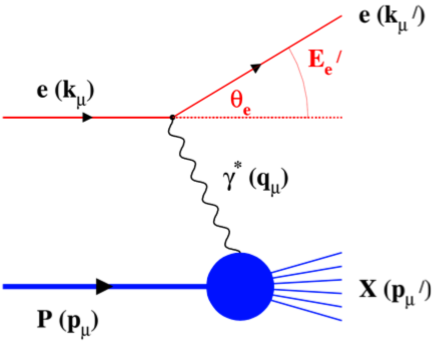
1. The Coupling constants in the NJL Lagrangian G_π and G_ρ
2. The regularization parameters, Λ_{IR} and Λ_{UV}. In QCD the confinement scale is set by Λ_{QCD} and therefore we fix $\Lambda_{IR} = 240$ MeV and choose the dressed light quark mass as $M_l = 400$ MeV
3. The u/d and s dressed quark masses (current quark masses)
4. The remaining parameters are then fit to the physical pion ($m_\pi = 140$ MeV), kaon ($m_K = 495$ MeV and ρ ($m_\rho = 770$ MeV) masses, together with the pion decay constant ($f_\pi = 93$ MeV)
5. This gives $G_\pi = 19.04\ \mathrm{GeV}^-2$, $G_\rho = 11.04\ \mathrm{GeV}^{-2}$, $\Lambda_{UV} = 645$ MeV, and $M_s = 611$ MeV. Note that for the ϕ mass we obtain $m_\phi = 1001$ MeV. Results for Z_α, f_α and the quark condensates:

Z_π	Z_K	Z_ρ	Z_ω	Z_ϕ	f_K	$\langle\bar{u}u\rangle^{1/3}$	$\langle\bar{s}s\rangle^{1/3}$
17.85	20.89	8.44	8.44	13.02	0.097	- 0.215	- 0.191

Outline

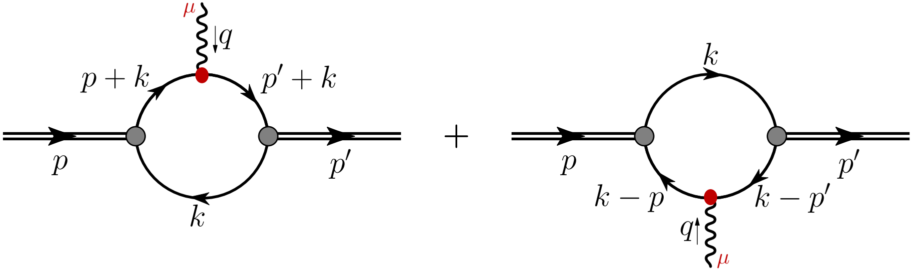
FORM FACTOR IN THE CONFINING NJL MODEL





Form Factor in NJL model

Diagrammatic representation of the electromagnetic current for the pion and kaon [8]



$\Rightarrow$ Feynman diagram for quark [left] and for the anti quark [right]

[8] PH, Ian Cloet, and A. Thomas, PRC94 (2016), 035201

Form Factor in NJL model

The matrix element of the electromagnetic current for a pseudoscalar mesons reads

$$J^{\mu}_{\alpha}(p', p) \;=\; \left(p'^{\mu} + p^{\mu}\right) F_{\alpha}(Q^2), \qquad \alpha = \pi, K \tag{12}$$

where p and $p^{'}$ denote the initial and final four momentum of the state, $q^2 = (p^{'} - P)^2 = -Q^2$ and $F_{\alpha}(Q^2)$ is the pion or kaon form factor. The pseudoscalar meson form factor in the NJL model are given by the sum of the two Feynman diagrams, which are respectively given by

$$\begin{aligned} j^{\mu}_{1,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda^{\dagger}_{\alpha} S(p^{'} + k) \hat{Q} \gamma^{\mu} S(p + k) \gamma_5 \lambda_{\alpha} S(k)\right] \\ j^{\mu}_{2,\alpha}(p^{'}, p) &= iZ_{\alpha} \int \frac{d^4k}{(2\pi)^4} Tr\left[\gamma_5 \lambda_{\alpha} S(k - p) \hat{Q} \gamma^{\mu} S(k - p^{'}) \gamma_5 \lambda^{\dagger}_{\alpha} S(k)\right] \end{aligned} \tag{13}$$

where the Tr is over Dirac, color and flavor indices. The index α labels the state and the λ_{α} are the corresponding flavor matrices

Form Factor in NJL model

We will focus on the quark sector and total form factors for π^+, K^+ and K^0, we find

$$
\begin{aligned}
F_{\pi^+}^{(bare)}(Q^2) &= (e_u - e_d) f_\pi^{ll}(Q^2) \\
F_{K^+}^{(bare)}(Q^2) &= e_u f_K^{ls}(Q^2) - e_s f_K^{sl}(Q^2) \\
F_{K^0}^{(bare)}(Q^2) &= e_d f_K^{ls}(Q^2) - e_s f_K^{sl}(Q^2)
\end{aligned} \tag{14}
$$

The results are denoted as "*bare*" because the quark-photon vertex is elementary result, that is, $\Lambda_{\gamma q}^{\mu(bare)} = \hat{Q}\gamma^\mu$. The quark-sector form factors for a hadron α are defined by

$$
F_\alpha(Q^2) = e_u F_\alpha^u(Q^2) + edF_\alpha^d(Q^2) + e_s F_\alpha^s(Q^2) + \cdots \tag{15}
$$

therefore the "*bare*" pseudoscalar meson quark-sector form factors are easily read from the total form factor equation above

Form Factor in NJL model

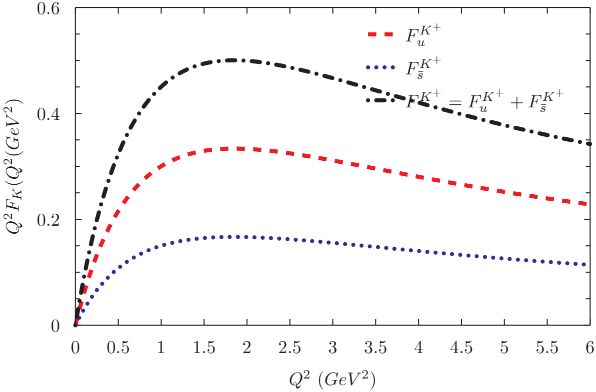
The first superscript on the body form factors, $f_\alpha^{ab}(Q^2)$, indicates the struck quark and the second the spectator, where

$$
\begin{aligned}
f_\alpha^{ab}(Q^2) &= \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} e^{-\tau(M_a^2+x(1-x)Q^2)} \\
&+ \frac{3Z_\alpha}{4\pi^2}\int_0^1 dx \int_0^{1-x} dz \int d\tau \\
&\times \; e^{-\tau((x+z)(x+z-1)m_\alpha^2+(x+z)M_a^2+(1-x-z)M_b^2+xzQ^2)} \\
&\times \; \left[(x+z)m_\alpha^2 + (M_a - M_b)^2(X+Z) + 2M_b(M_a - M_b)\right]
\end{aligned}
\tag{16}
$$

⇒ Importantly, these expression satisfy charge conservation.

Elastic Form Factor Results

Results for the kaon form factor – from the bare quark-photon vertex



⇒ We find that our results are not excellent agreement with the perturbative QCD that predicts $F_K(Q^2) \sim \frac{\alpha_S(Q^2)}{Q^2}$.

Form Factor in NJL model : Modify the Quark-Photon vertex

⇒ The limit $Q^2 >> m_\alpha^2$ of the body form factors can be obtained by noting the Feynman parameter domains which dominate the integrals giving

$$\begin{aligned} Q^2 f_\alpha^{ab}(Q^2) &= \frac{3Z_\alpha}{2\pi^2}\int \frac{d\tau}{\tau^2} e^{-\tau M_a^2} + \frac{3Z_\alpha}{2\pi^2} M_b(M_a - M_b)\int \frac{d\tau}{\tau} e^{-\tau M_b^2} \\ &\times \left[\gamma_E + \log(M_b^2\tau) + \log(Q^2/M_b^2)\right] \end{aligned} \tag{17}$$

Therefore the form factors receive log correction at large Q^2 only if $M_a \neq M_b$

⇒ In general the quark-photon vertex is not elementary ($\Lambda_{\gamma q}^{\mu(bare)} = \hat{Q}\gamma^\mu$) but instead **dressed**, with this dressing given by the inhomogeneous BSE. The general solution for the dressed quark-photon vertex for a quark of flavor q has the form

$$\Lambda_{\gamma Q}^{\mu}(p', p) = e_q\gamma^\mu + \left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2}\right) F_Q(Q^2) \to \gamma^\mu F_{1Q}(Q^2) \tag{18}$$

Form Factor in NJL model

where the final result is used because the $\frac{q^\mu \not{q}}{q^2}$ term cannot contribute to a hadron electromagnetic current because of current conservation
The dressed u, d and s quarks are expressed by

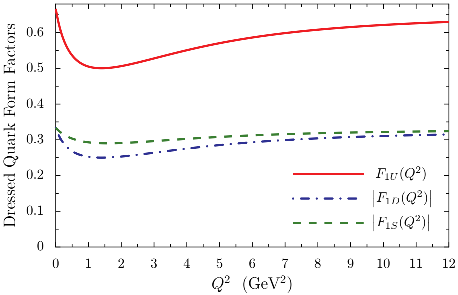
$$\begin{aligned} F_{1U/D}(Q^2) &= e_{u/d}\frac{1}{1+2G_\rho \Pi_v^{ll}(Q^2)} \\ F_{1S}(Q^2) &= e_s\frac{1}{1+2G_\rho \Pi_v^{ss}(Q^2)} \end{aligned} \tag{19}$$

where the explicit form of the bubble diagram is

$$\Pi_v^{qq}(Q^2) = \frac{3Q^2}{\pi^2}\int_0^1 dx \int \frac{d\tau}{\tau} x(1-x) e^{-\tau\left[M_q^2 + x(1-x)Q^2\right]} \tag{20}$$

Form Factor in NJL model

The dressed quark form factors obtained as solutions to the inhomogeneous BSE:



$\Rightarrow$ In the limit $Q^2 \to \infty$ these form factors reduce to the elementary quark charges, as expected because of asymptotic freedom in QCD. For small Q^2 these results are similar to expectations from vector meson dominance, where the dressed u and d quarks are dressed by ρ and ω mesons and the dressed s quark by ϕ meson.

Form Factor in NJL model

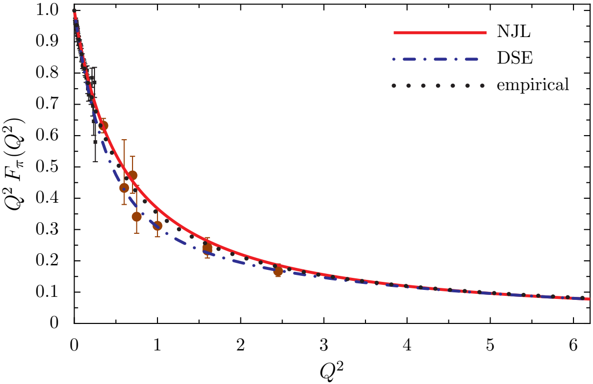
The complete results for the pseudoscalar meson form factor – with a dressed quark-photon vertex – read

$$\begin{aligned} F_{\pi^+}(Q^2) &= \left[F_{1U}(Q^2) - F_{1D}(Q^2)\right] f_\pi^{ll}(Q^2) \\ F_{K^+}(Q^2) &= F_{1U}(Q^2) f_K^{ls}(Q^2) - F_{1S}(Q^2) f_K^{sl}(Q^2) \\ F_{K^0}(Q^2) &= F_{1D}(Q^2) f_K^{ls}(Q^2) - F_{1S}(Q^2) f_K^{sl}(Q^2) \end{aligned} \tag{21}$$

$\Rightarrow$ $f_\pi^{ll}(Q^2)$, $f_K^{sl}(Q^2)$ and $f_K^{sl}(Q^2)$ are the same as the *BARE* form factor expressions

Elastic Form Factor Results

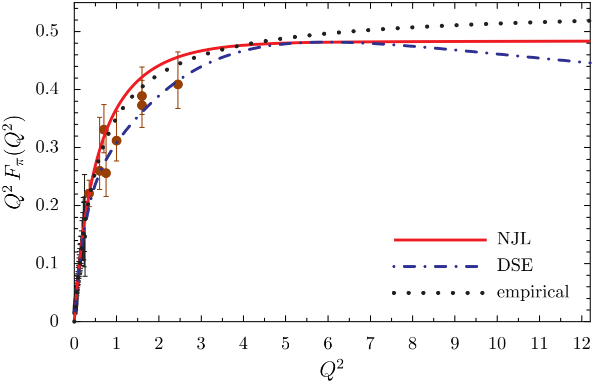
Results for the pion form factor – the dressed quark-photon vertex



⇒ We find that excellent agreement with existing data and the modest difference with the DSE result for $Q^2 \leq 6\ \mathrm{GeV}^2$

Elastic Pion Form Factor Results

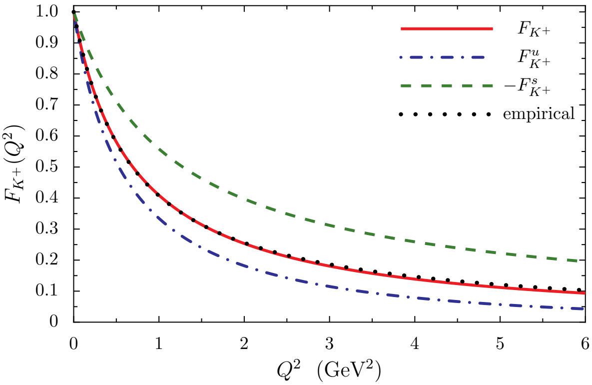
Results for $Q^2 F_\pi(Q^2)$



⇒ Our result for $Q^2 F_\pi(Q^2)$ is very similar to the empirical monopole result but begins to plateau for $Q^2 \geq 6\ \mathrm{GeV}^2$, where $Q^2 F_\pi(Q^2) \sim 0.49$. This maximum is almost identical to that obtaining using the DSEs, which is not surprising because in both approaches it is driven by DCSB.

Elastic Kaon Form Factor Results

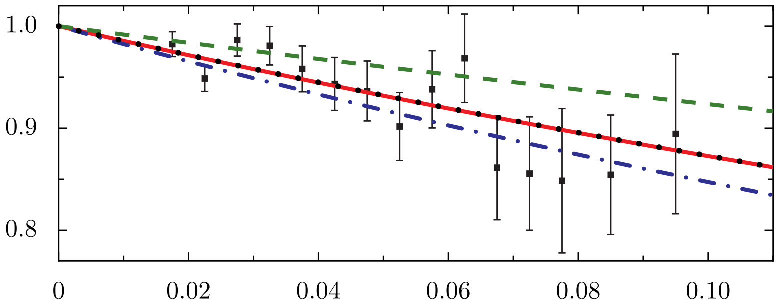
Results for the K^+ form factor and the quark sector components – each including effects from the dressed quark-photon vertex



$\Rightarrow$ we find an excellent agreement with the data and the empirical monopole $F_K(Q^2) = \left[1 + \frac{Q^2}{\Lambda_K^2}\right]^{-1}$ determined by the charge radius.

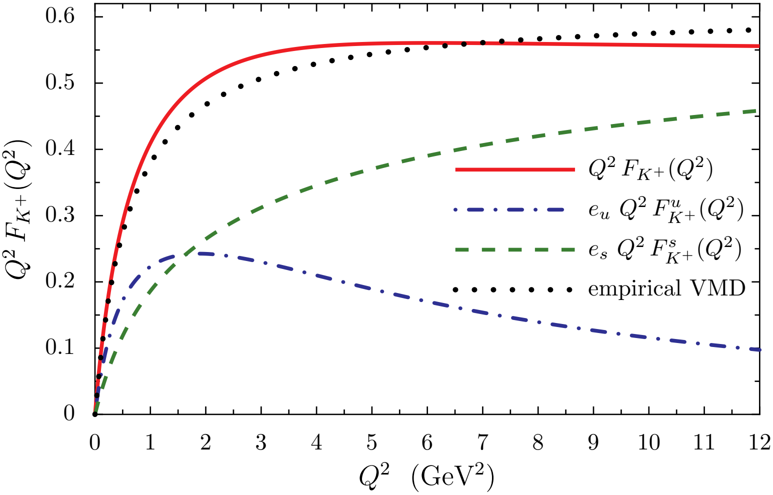
Elastic Kaon Form Factor Results

Results for the K^+ form factor – each including effects from the dressed quark-photon vertex – compare with the available experimental data



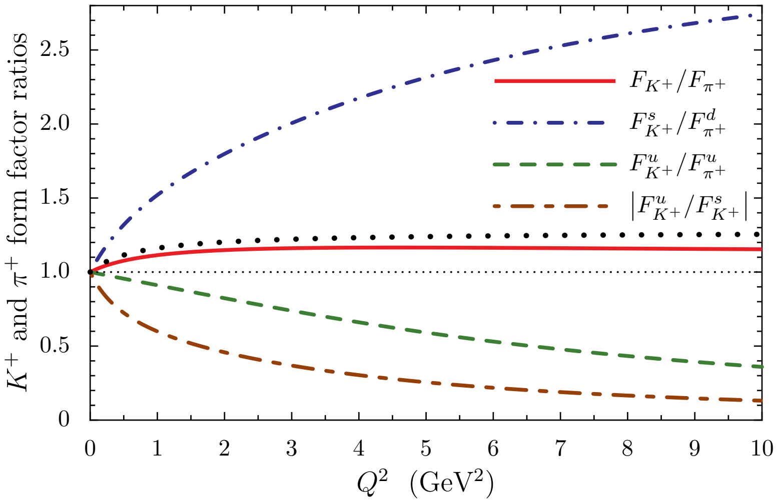
Elastic Kaon Form Factor Results

Results for the $Q^2 F_{K^+}(Q^2)$ form factor and the quark sector components – each including effects from the dressed quark-photon vertex



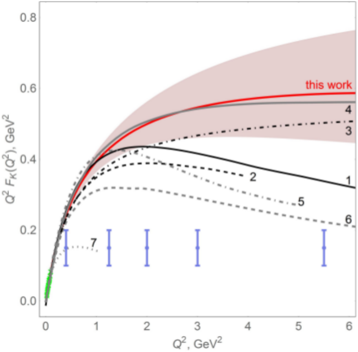
Elastic Kaon Form Factor Results

Results for the form factor and the quark sector components ratio



Elastic Kaon Form Factor Results

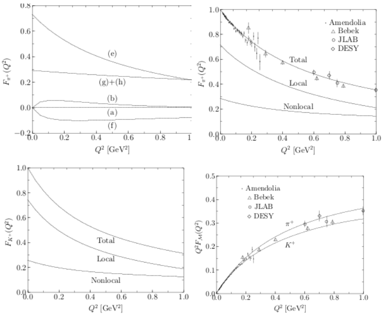
Recent paper in arxiv : 1601.06405v1 [hep-ph]. Our model compare with other model calculations.



⇒ The three form of the relativistic kinematic model (1 [full black]), DSE model (2 [dashed black]), NJL model (3 [dot-dashed black]), Our model (4 [full gray]), DSE model (5 [dot-dashed gray]), Front light model (7 [dotted gray])

Elastic Kaon Form Factor Results

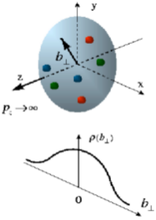
A form factor result in the nonlocal chiral quark model (NLChQM) in paper : Seung-il Nam and Hyun-Chul Kim, Phys. Rev. D **77** (2008)



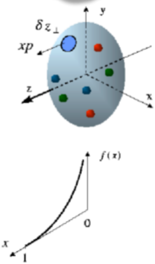
$\Rightarrow$ The nonlocal contributions turn out to be crucial to reproduce the experimental data. They found that $\langle r^2\rangle_{\pi^+} = 0.455\mathrm{fm}^2$ for the pion and $\langle r^2\rangle_{K^+} = 0.537\mathrm{fm}^2$.

Outline

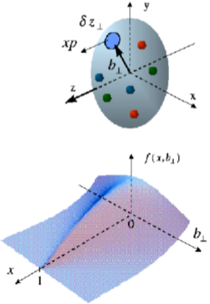
PARTON DISTRIBUTION FUNCTIONS IN THE CONFINING NJL MODEL



Elastic Scattering
transverse quark distribution in coordinate space



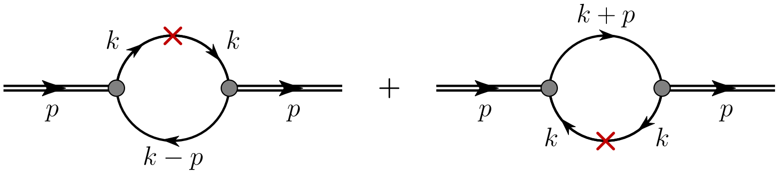
DIS
longitudinal quark distribution in momentum space



DES (GPDs)
fully-correlated quark distribution in both coordinate and momentum space

Parton distribution functions

The valence quark distribution functions of the pion or kaon are given by the two Feynman diagrams



The operator insertion $\gamma^+\delta\left(k^+ - xp^+\right)\hat{P}_q$, where $\hat{P}_q$ is the projection operator for quarks of flavor q:

$$\begin{aligned} \hat{P}_{u/d} &= \frac{1}{2}\left(\frac{2}{3}\mathbb{1} \pm \lambda_3 + \frac{1}{\sqrt{3}}\lambda_8\right) \\ \hat{P}_s &= \frac{1}{3}\mathbb{1} - \frac{1}{\sqrt{3}}\lambda_8 \end{aligned} \tag{22}$$

Parton distribution functions

The valence quark and anti-quark distributions in the pion or kaon are given by

$$\begin{aligned} q_\alpha(x) &= iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ - xp^+\right) \\ &\times Tr\left[\gamma_5 \lambda_\alpha^\dagger S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha S(k-p)\right] \\ \bar{q}_\alpha(x) &= -iZ_\alpha \int \frac{d^4k}{(2\pi)^4} \delta\left(k^+ + xp^+\right) \\ &\times Tr\left[\gamma_5 \lambda_\alpha S(k) \gamma^+ \hat{P}_q S(k) \gamma_5 \lambda_\alpha^\dagger S(k+p)\right] \end{aligned} \quad (23)$$

To evaluate these expression we first take the moments

$$\mathcal{A}_n = \int_0^1 dx x^{n-1} q(x) \quad (24)$$

where $n = 1, 2, 3, \cdots$ is an integer.

Parton distribution functions

Using the Ward-like identity $S(k)\gamma^+ S(k) = \frac{-\partial S(k)}{\partial k_+}$ and introducing the Feynman parameterization, the quark and anti-quark distributions can then be straightforwardly determined. For the valence quark and anti-quark distributions of the K^+ we find:

$$
\begin{aligned}
q_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2+xM_s^2+(1-x)M_l^2\right]} \\
&\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2-(m_l-M_s)^2\right]\right] \\
\bar{q}_K(x) &= \frac{3Z_K}{4\pi^2}\int d\tau e^{-\tau\left[x(x-1)m_K^2+xM_l^2+(1-x)M_s^2\right]} \\
&\times \left[\frac{1}{\tau}x(1-x)\left[m_K^2-(m_l-M_s)^2\right]\right] \qquad (25)
\end{aligned}
$$

$\Rightarrow$ Results for the π^+ are obtained by $M_s \to M_l$ and $Z_K \to Z_\pi$, giving the result $u_\pi(x) = \bar{d}_\pi(x)$

Parton distribution functions

The quark distributions satisfy the baryon number and momentum sum rules, which for the K^+ read:

$$\int_0^1 dx \left[u_{K^+}(x) - \bar{u}_{K^+}(x)\right] = \int_0^1 \left[\bar{s}_{K^+}(x) - s_{k^+}(x)\right] = 1 \qquad (26)$$

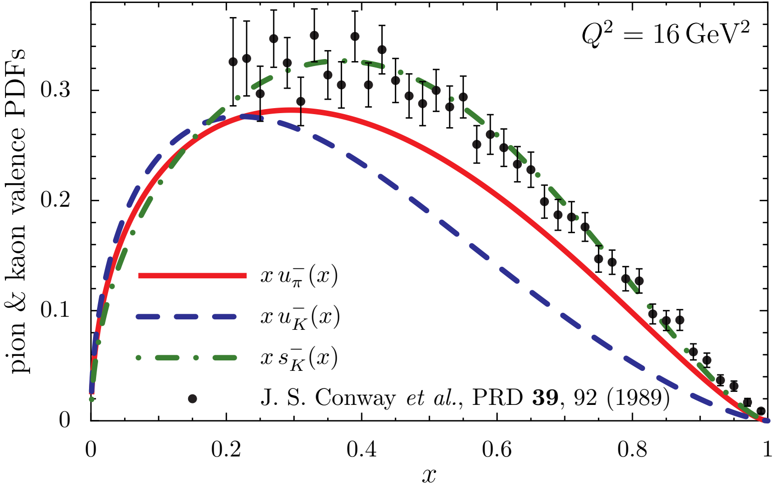
for the number sum rules and at the model scale the momentum sum rules is given by

$$\int_0^1 dxx \left[u_{K^+}(x) + \bar{u}_{K^+}(x) + \bar{s}_{K^+}(x) + s_{k^+}(x)\right] = 1 \qquad (27)$$

Analogous results holds for the remaining kaons and the pions.

Kaon PDF Results

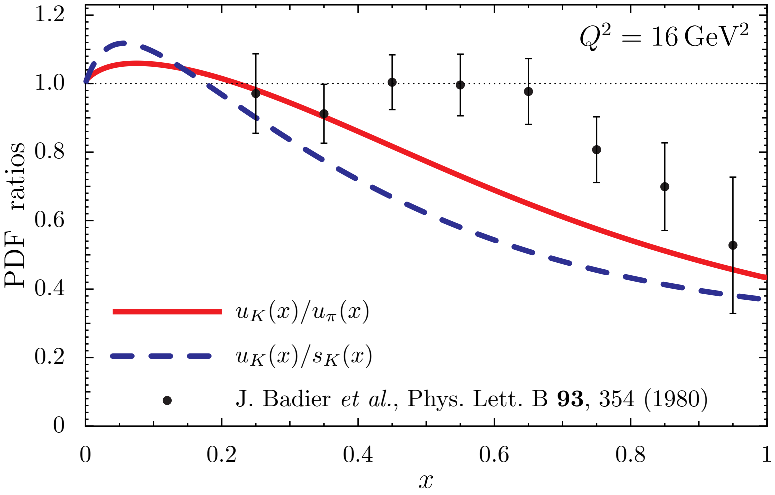
Results for the valence quark distributions of the π^+ and K^+, evolved from the model scale using NLO DGLAP equations.



⇒ We find that our results is an excellent agreement with the data

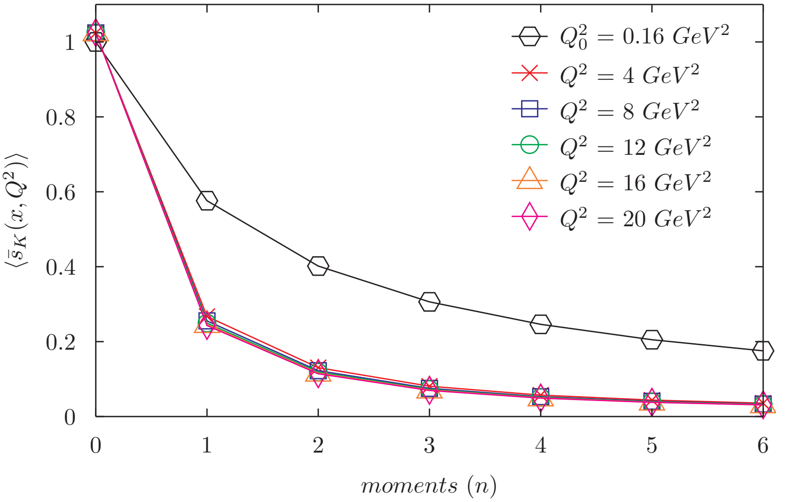
Kaon PDF Results

The ratio of the u quark distribution in the K^+ to the u quark distribution in the π^+, after NLO evolution to $Q^2 = 16\ \mathrm{GeV}^2$



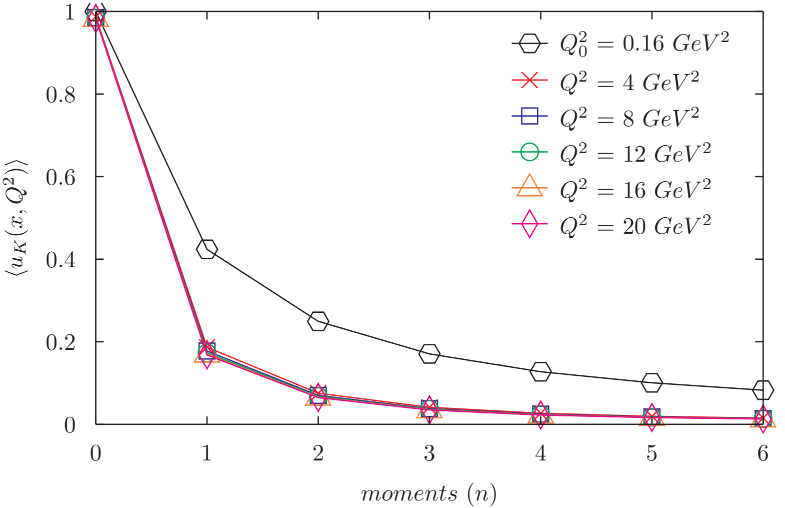
Kaon PDF Results

Moments PDF of the s quark in the Kaon



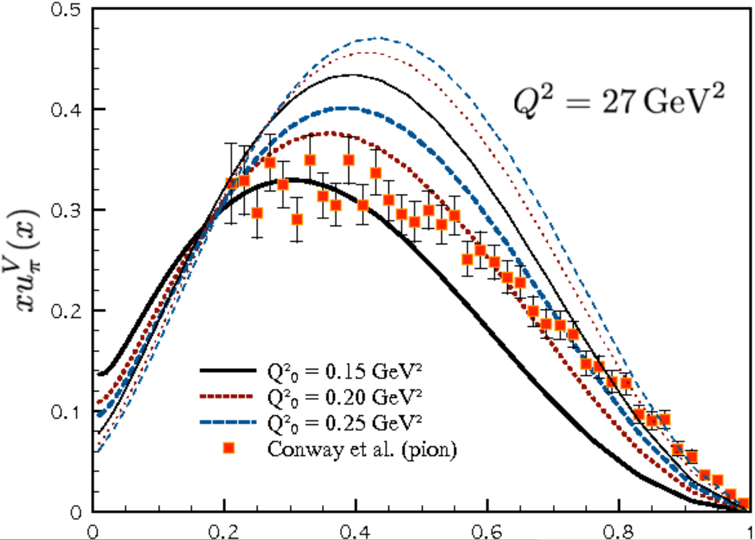
Kaon PDF Results

Moment PDF of the u quark in the kaon



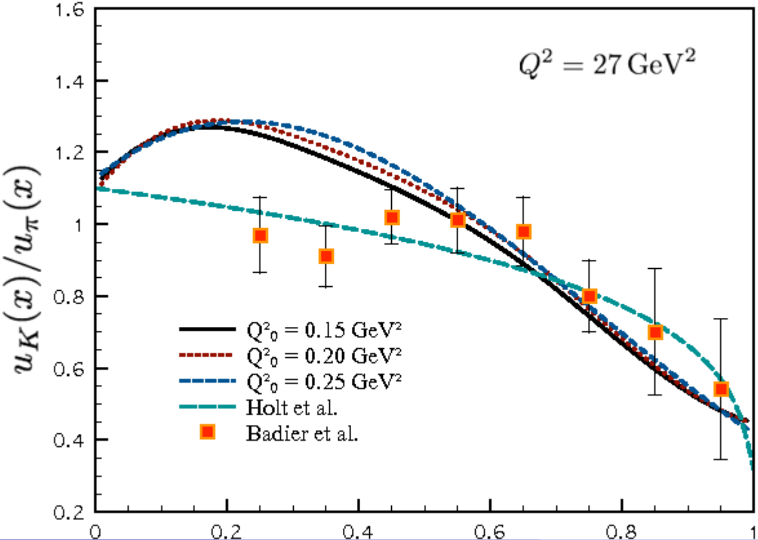
Kaon PDF Results from NLChQM

PDF of the pion and kaon in the nonlocal chiral quark model [Seung-il Nam, Phys. Rev. D **86**,074005 (2012)]



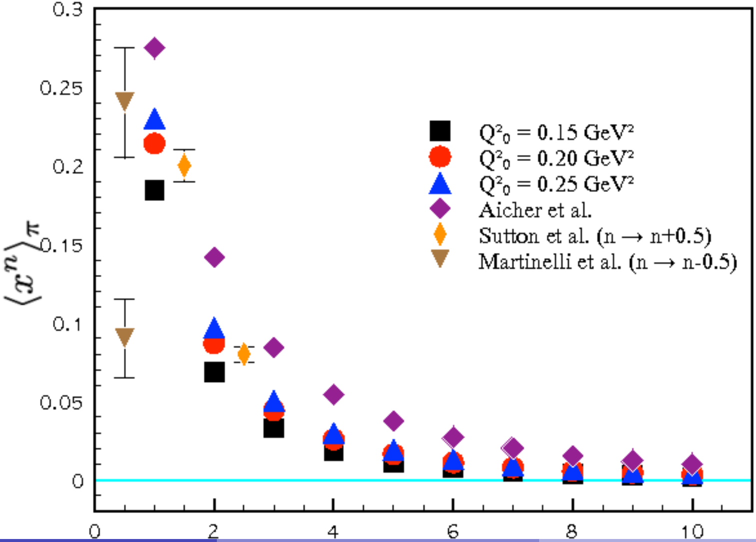
Kaon PDF Results from NLChQM

A ratio of the PDF of the pion and kaon in the nonlocal chiral quark model [Seung-il Nam, Phys. Rev. D **86**,074005 (2012)]



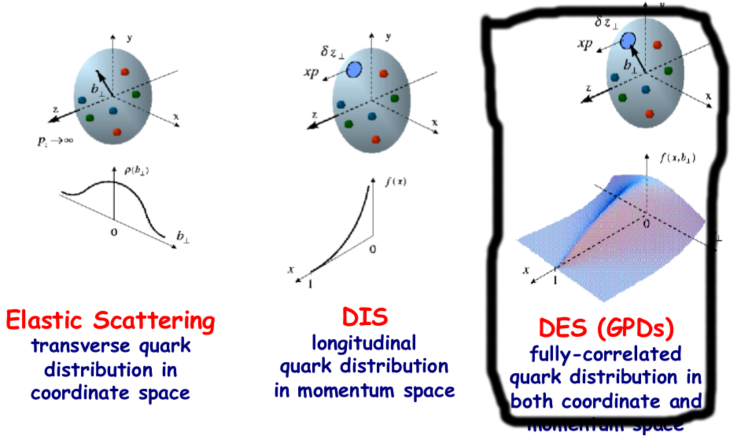
Kaon PDF Results from NLChQM

The moment of the PDF of the pion and kaon in the nonlocal chiral quark model [Seung-il Nam, Phys. Rev. D **86**,074005 (2012)]



Outline

GPD OF THE PION AND KAON IN THE NONLOCAL CHIRAL QUARK MODEL



Introduction

The pion GPD in the asymmetric notation is defined as follows:

$$H^{ab}_{\pi}(x,\zeta,t) = \int \frac{dz^-}{4\pi} \exp^{ixp^+z^-}, \times \langle \pi^b(p+q) \mid \bar{q}(0)\not{n} T q(z) \mid \pi^a(p)\rangle_{z^+=Z^{\perp}=0}, \quad (28)$$

We define the longitudinal moment fraction x, skewness ζ and virtuality $q^2 = -Q^2$ in the asymmetric notations as

$$x = \frac{k^+}{p^+}, \qquad \zeta = \frac{-q^+}{p^+}, \qquad t = q^2 = -2p.q. \quad (29)$$

Being equivalently, the symmetric notations are

$$X = \frac{x - \frac{\xi}{2}}{1 - \frac{\xi}{2}} \in [-1,1], \quad \zeta = \frac{\xi}{2-\xi} \in [-1,1], \quad t = q^2 = -2p.q. \quad (30)$$

where $0 \leq \zeta$ and the x variable $-1+\zeta \leq x \leq 1$ is defined in the *asymmetric* notation, a and b are isospin indices for the pion. T is the isospin matrix equal 1 for isoscalar and τ_3 for the isovector. ψ is thequark field and z is the light cone coordinate. The two isospin projections are equal to

$$\begin{aligned}
\delta_{ab}\mathcal{H}^{I=0}(x,\zeta,t) &= \int \frac{dz^-}{4\pi} \exp^{ixp^+z^-} \\
&\times \langle \pi^b(p+q) \mid \bar{\psi}(0)\gamma.n\psi(z) \mid \pi^a(p)\rangle \mid_{z^+=0,z^\perp=0}, \\
i\epsilon_{3ab}\mathcal{H}^{I=1}(x,\zeta,t) &= \int \frac{dz^-}{4\pi} \exp^{ixp^+z^-} \\
&\times \langle \pi^b(p+q) \mid \bar{\psi}(0)\gamma.n\psi(z)\tau_3 \mid \pi^a(p)\rangle \mid_{z^+=0,z^\perp=0} .
\end{aligned} \tag{31}$$

where $\gamma.n = \not{n}$, $n^2 = 0$, $p.n = 1$ and $q.n = -\zeta$.

Introduction

From the term combinations of Eq.(31), the quark and antiquark pion GDP can be defined as

$$\begin{aligned}\mathcal{H}_q(x,\zeta,t) &= \frac{1}{2}\left(\mathcal{H}_{I=0}(x,\zeta,t)+\mathcal{H}_{I=1}(x,\zeta,t)\right)\\ \mathcal{H}_{\bar{q}}(x,\zeta,t) &= \frac{1}{2}\left(\mathcal{H}_{I=0}(x,\zeta,t)-\mathcal{H}_{I=1}(x,\zeta,t)\right). \qquad (32)\end{aligned}$$

where

- $\mathcal{H}_q(x,\zeta,t)$ has support $x \in [0,1]$
- $\mathcal{H}_{\bar{q}}(x,\zeta,t)$ supports $x \in [-1+\zeta,\zeta]$
- The range $x \in [0,\zeta]$ is called the Efremov-Radyuskin-Brodsky-Lepage (ERBL) region
- The range $x \in [-1+\zeta,0]$ and $x \in [\zeta,1]$ are the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) regions, where the nomenclature refers to the QCD evolution

Introduction

In the symmetric notation, one introduces

$$\xi = \frac{\zeta}{2-\zeta}, \qquad X = \frac{x-\frac{\zeta}{2}}{1-\frac{\zeta}{2}} \tag{33}$$

where $0 \leq \xi \leq 1$ and $-1 \leq X \leq 1$. Then

$$\begin{aligned} H^{I=0}(X,\xi,t) &= \mathcal{H}^{I=0}\left(\frac{\xi+X}{\xi+1}, \frac{2\xi}{\xi+1}, t\right), \\ H^{I=1}(X,\xi,t) &= \mathcal{H}^{I=1}\left(\frac{\xi+X}{\xi+1}, \frac{2\xi}{\xi+1}, t\right) \end{aligned} \tag{34}$$

with symmetry properties about the $X = 0$,

$$\begin{aligned} H^{I=0}(X,\xi,t) &= -H^{I=0}(-X,\xi,t), \\ H^{I=1}(X,\xi,t) &= H^{I=1}(-X,\xi,t). \end{aligned} \tag{35}$$

Introduction

The following sum rules hold

$$\int_{-1}^{1} dXH^{I=1}(X,\xi,t) = 2F_V(t) \;\; (\mathit{Electric\ Charge\ Conservation}),$$
$$\int_{-1}^{1} dXXH^{I=0}(X,\xi,t) = \theta_2(t)\xi^2\theta_1(t) \;\; (\mathit{Momentum\ Sum\ Rule}). \quad (36)$$

where

- $F_V(t)$ is the electromagnetic form factor
- $\theta_1(t)$ and $\theta_2(t)$ are gravitational formfactors of the pion, which satisfy the low energy theorem $\theta_1(0) = \theta_2(0)$

$$\mathcal{H}^{I=0,1}(X,\xi=0,t=0) = q(X) \quad (\mathit{for} \;\; X \leq 0) \qquad (37)$$

where relating to the pion's forward diagonal parton distribution functions (PDFs), $q(X)$.

Nonlocal Chiral Quark Model

In the effective chiral action $\mathcal{S}_{eff}[\mathcal{M}^{\alpha}, V, m_q]$ for the light quark flavor SU(2) sector is Wick rotated into Minskowski space,

$$\begin{aligned} \mathcal{S}_{eff} &= iSp_{\gamma,c,f}\ln[i\not{D} - m_q - \sqrt{M(D^2)}U_5\sqrt{M(D^2)}], \\ &= iSp\ln[i\not{D}_{5M}]. \end{aligned} \tag{38}$$

Where

- The covariant derivative, $i\not{D} \equiv i\partial_\mu + A_\mu$, where A_μ is the photon field
- $Sp_{\gamma,c,f}$ stand for the functional trace i.e. $\int d^4x Tr_{\gamma,c,f}\langle x \mid \cdots \mid x\rangle$
- The current quark mass matrix m_q is given by $diag[m_u, m_d]$
- Considering the gauge boson interacting with the quarks and pesudocalar (PS) mesons, it is necessary to pay attention on the vector current conversation
- Hence we take the covariant derivative even in the effective quark mass $M(D^2)$, since the nonlocal interactions break the gauge invariance

Nonlocal Chiral Quark Model

The nonlinear field of the pion is defined as

$$
\begin{aligned}
U_5 &= U(x)\gamma_5\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2}, \\
&= \exp\left(i\gamma_5\frac{\tau.\pi}{F_\pi}\right), \\
&= 1 + \frac{i}{F_\pi}\gamma_5(\tau.\pi) - \frac{1}{2F_\pi^2}(\tau.\pi)^2 + \cdots
\end{aligned}
\tag{39}
$$

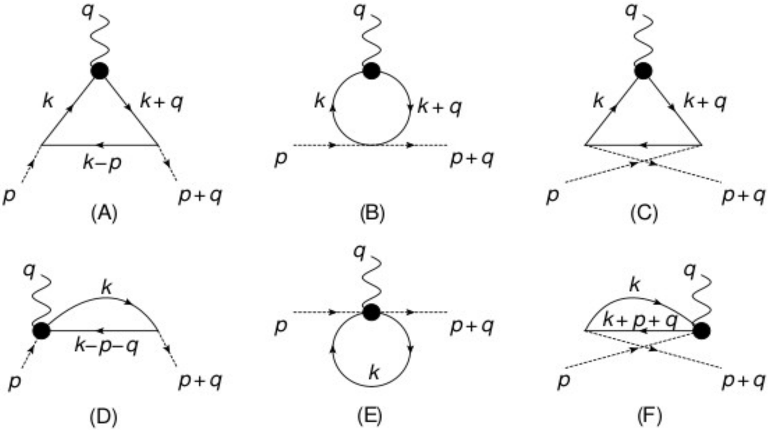
The pseudoscalar meson field π are defined explicitly as

$$
\tau.\pi = \begin{pmatrix} \sqrt{2}\pi^+ & \pi^0 \\ \pi^0 & \sqrt{2}\pi^- \end{pmatrix}
\tag{40}
$$

where τ^a stands for the Pauli matrix, satisfying $Tr[\tau^\alpha, \tau^\beta] = \frac{\delta_{\alpha\beta}}{2}$ and $F_\pi = 93.2\ \mathrm{MeV}$.

Nonlocal Chiral Quark Model

Relevant Feynman diagrams generated from NLChQM Model, satisfying the Ward-Takahashi identity. The dotted, solid and wavy lines indicate the pion, quark and photon, respectively. The black blobs denote the local and / or nonlocal photon interaction vertices.



Nonlocal Chiral Quark Model

In order to compute the matrix element, $\langle \pi^b(p+q) \mid \bar{q}(0)\not{n} T q(z) \mid \pi^a(p)\rangle \mid_{z^+=z^\perp=0}$, one need three operator and that can be obtained from Effective Chiral Action (EChA) by performing the functional derivatives over the photon, A_μ and pion fields, π^q and π^b,

$$\begin{aligned} \frac{\delta \mathcal{S}_{eff}}{\delta A_\mu \delta \pi^a \delta \pi^b} &= -iSp\left[\frac{1}{i\not{D}_{5M}}\sqrt{M}U_5^b\sqrt{M}\frac{1}{i\not{\partial}_{5M}}\sqrt{M}U_5^a\sqrt{M}\frac{1}{i\not{\partial}_{5M}}K_\mu\right], \\ &- iSp\left[\frac{1}{i\not{\partial}_{5M}}\sqrt{M}U_5^{ab}\sqrt{M}\frac{1}{i\not{\partial}_{5M}}K_\mu\right], \\ &- iSp\left[-\frac{1}{i\not{\partial}_{5M}}\sqrt{M}U_5^a\sqrt{M}\frac{1}{i\not{\partial}_{5M}}\sqrt{M}U_5^b\sqrt{M}\frac{1}{i\not{\partial}_{5M}}K_\mu\right], \\ &- iSp\left[\frac{1}{i\not{\partial}_{5M}}\sqrt{M}U_5^a\sqrt{M}\frac{1}{i\not{\partial}_{5M}}Z_\mu\right] \; [continue], \qquad (41) \end{aligned}$$

where $K_\mu = (\gamma_\mu + \sqrt{M_\mu}U_5\sqrt{M} - \sqrt{M}U_5\sqrt{M_\mu})$,
$Z_\mu = (\sqrt{M_\mu}U_5^b\sqrt{M} - \sqrt{M}U_5^b\sqrt{M_\mu})$

Nonlocal Chiral Quark Model

[continue from next page]

$$
\begin{aligned}
&+ \quad iSp\left[-\frac{1}{i\not{\partial}_{5M}}\sqrt{M}U_5^b\sqrt{M}\frac{1}{i\not{\partial}_{5M}}(\sqrt{M_\mu}U_5^a\sqrt{M}-\sqrt{M}U_5^a\sqrt{M_\mu})\right], \\
&+ \quad iSp\left[\frac{1}{i\not{\partial}_{5M}}(\sqrt{M_\mu}U_5^{ab}\sqrt{M}-\sqrt{M}U_5^{ab}\sqrt{M_\mu})\right]. \qquad (42)
\end{aligned}
$$

where $\sqrt{M_\mu(\partial^2)} \equiv \frac{\delta\sqrt{M(D^2)}}{\delta A_\mu}\,|_{A=0}$ and $U_5^a = i\gamma_5\frac{\tau^a}{F_\pi}U_5$, $U_5^{ab} = -\frac{\tau^a\tau^b}{F_\pi^2}U_5$.

- Based on Eq. (41) and (42), we have fifteen Feynman diagrams for the pion GPD in total as in Fig. 69.
- The black blobs in Fig. 69 denote the local and / or nonlocal photon interaction vertices

GPD of the pion

Putting the operator in Eq. (42) into the pion GPD in Eq. (28), we obtain the following expressions for the pion GPD with local (L) and nonlocal (NL) photon vertices :

$$
\begin{aligned}
H^L_{(A)}(x,\zeta,t) &= -\frac{4iN_C}{F_\pi^2}\int\frac{d^4k}{(2\pi)^4}\delta\left(x-\frac{k^+}{p^+}\right) \\
&\times \frac{\sqrt{M_aM_c^2M_b}}{[k^2-M_a^2+i\epsilon][(k-p)^2-M_c^2+i\epsilon][(k+q)^2-M_b^2+i\epsilon]} \\
&\times [M_aM_b-k^2-k.q-\zeta(M_aM_b-k^2+k.p) \\
&+ x(M_aM_c+M_aM_b-M_cM_bb-k^2+2k.q-\frac{t}{2})], \qquad (43)
\end{aligned}
$$

Outline

Conclusion and Outlook

- We have calculated the form factor and parton distribution of the kaon and pion in the confining NJL model
  $\Rightarrow$ Our results on FF and PDF of the kaon and pion are excellent agreement with the available experimental data
- We have calculate the form factor and parton distribution function of the pion and kaon by including nonlocal interaction (condiser the momentum dependence of the effective quark mass) in the NLChQM model
  $\Rightarrow$ The results of the FF and PDF in the NLChQM model is qualitatively agreement with the empirical data and theoretical prediction.
- On-going work, we are calculating the GPD of the pion for complete relevant diagrams in the NLChQM model
  $\Rightarrow$ STILL IN PROGRESS..

THANK YOU FOR ATTENTION THANK YOUR FOR ORGANIZER

Any questions or comments ??