Distribution Amplitudes of the Nucleon and the Roper Resonance

C. Mezrag

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In collaboration with:
C.D. Roberts and J. Segovia

Hadron Structure Gastronomy

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- Tourte Hadronique $\langle \mathrm{Hadron}|O|\mathrm{Hadron}\rangle$

  - PDF,
  - Form Factors,
  - GPDs, TMDs,
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- Tarte Hadronique $\langle \mathrm{Vacuum} | O | \mathrm{Hadron} \rangle$

  - Bethe-Salpeter and Faddeev wave functions,
  - Lightfront wave functions,
  - Parton Distribution Amplitudes.

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\Psi(x,k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

Nucleon Distribution Amplitudes

- 3 bodies matrix element:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle$$

- 3 bodies matrix element expanded at leading twist:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P\rangle = \frac{1}{4}\left[\left(\not{p}C\right)_{\alpha\beta}\left(\gamma_5 N^+\right)_\gamma V(z_i^-)\right.$$
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- Isospin symmetry:

$$2T(x_1,x_2,x_3) = \varphi(x_1,x_3,x_2) + \varphi(x_2,x_3,x_1)$$

Evolution and Asymptotic results

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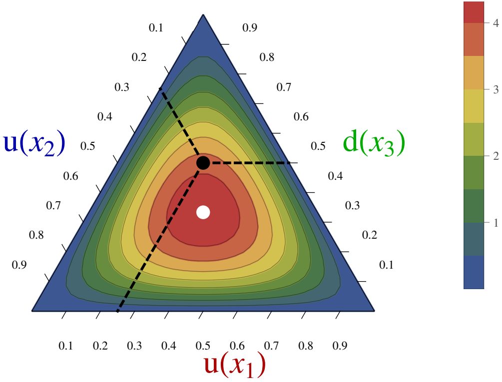
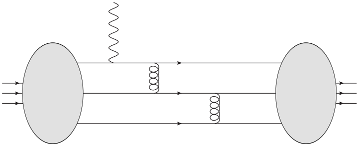
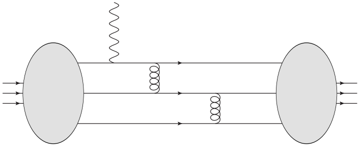
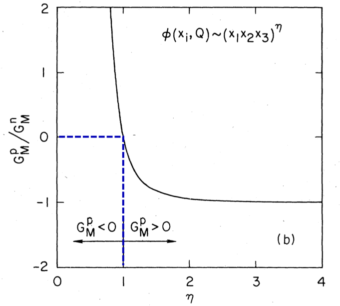
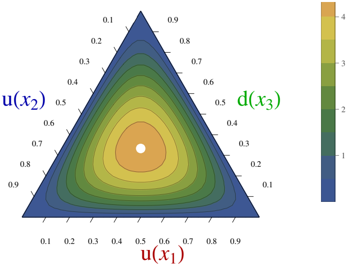


Form Factors: Nucleon case



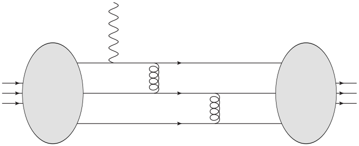


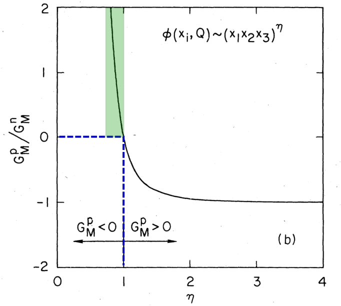




$\eta = 1$

S. Brodsky and G. Lepage, PRD 22, (1980)





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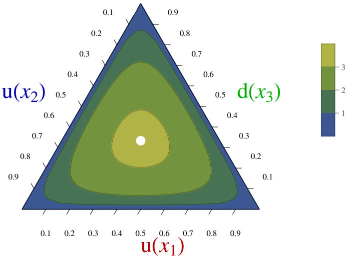

$\eta = 0.5$

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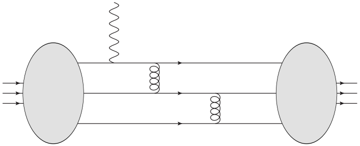

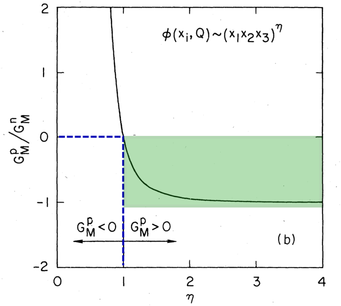

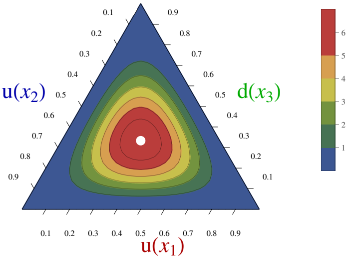

$\eta = 2$

S. Brodsky and G. Lepage, PRD 22, (1980)

Some previous studies of DA

- QCD Sum Rules
  - V. Chernyak and I. Zhitnitsky, Nucl. Phys. B 246 (1984)
- Relativistic quark model
  - Z. Dziembowski, PRD 37 (1988)
- Scalar diquark clustering
  - Z. Dziembowski and J. Franklin, PRD 42 (1990)
- Phenomenological fit
  - J. Bolz and P. Kroll, Z. Phys. A 356 (1996)
- Lightcone quark model
  - B. Pasquini *et al.*, PRD 80 (2009)
- Lightcone sum rules
  - I. Anikin *et al.*, PRD 88 (2013)
- Lattice Mellin moment computation
  - G. Bali *et al.*, JHEP 2016 02

Faddeev Equation and diquark picture

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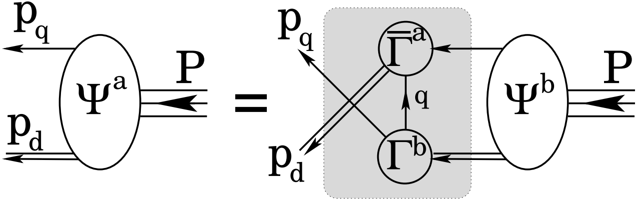

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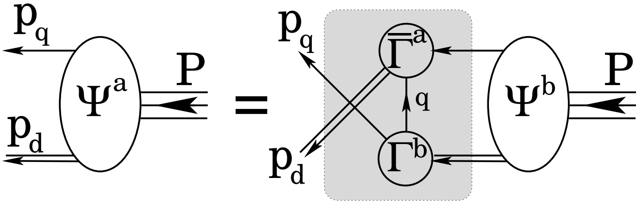

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
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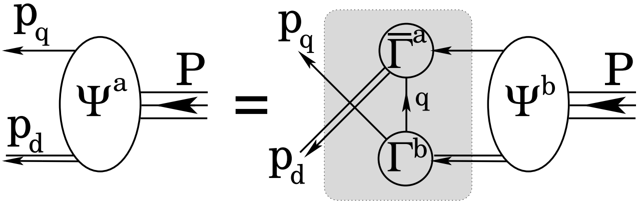

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- Can we understand the nucleon DA in terms of quark-diquarks correlations?

- Operator point of view for every DA (and at every twist):

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1)C\not{n}u_\downarrow^j(z_2)\right)\not{n}d_\uparrow^k(z_3)|P,\lambda\rangle \to \varphi(x_i) \to O_\varphi,$$

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Braun *et al.*, Nucl.Phys. B589 (2000)

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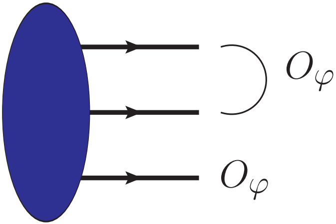

Nucleon DA and DSE

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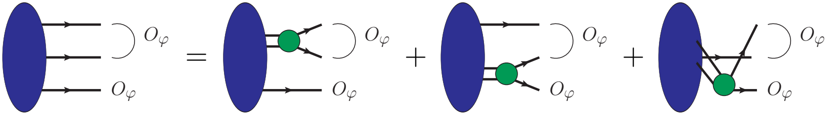

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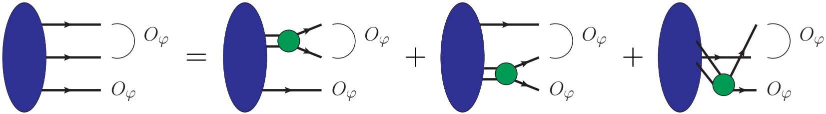

- The operator then selects the relevant component of the wave function.

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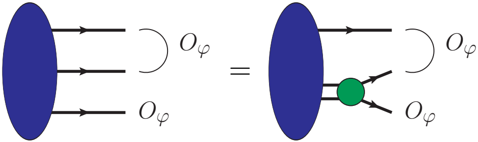

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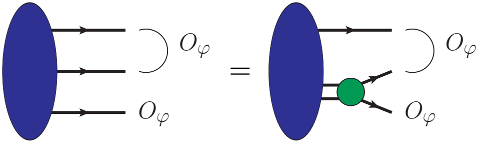

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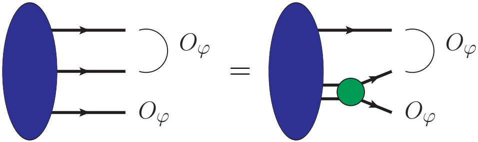

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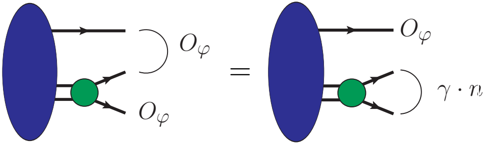

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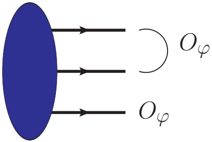

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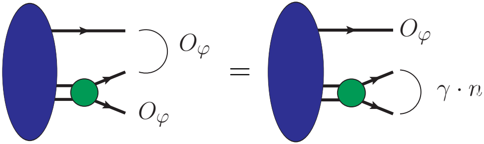

We recognise the leading twist DA of a scalar diquark

AV Contributions

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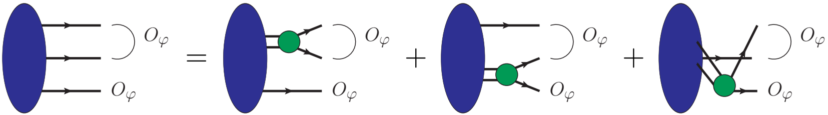

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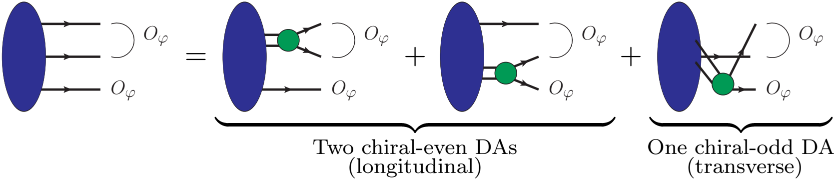

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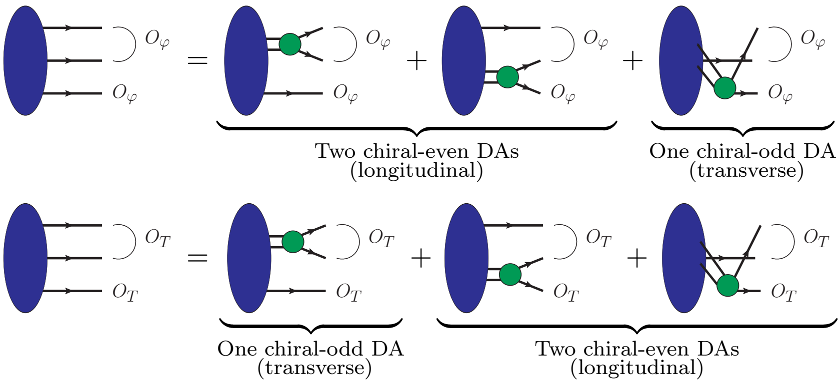

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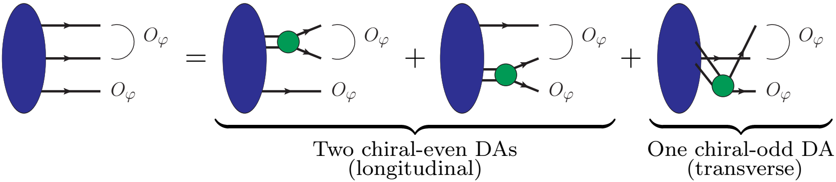

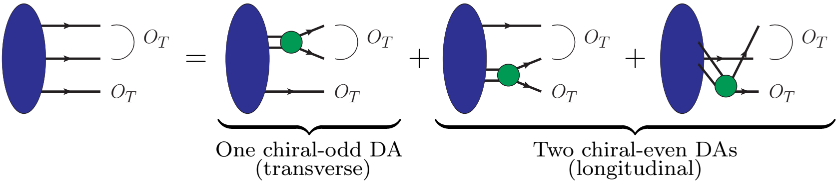

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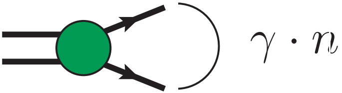
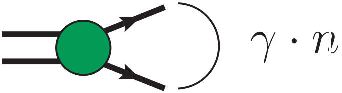
Modeling the Diquarks

Scalar diquark I: the point-like case



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$



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- Bethe-Salpeter amplitude (1 out of 4 structures):

$$\Gamma^{0+}_{\mathrm{PL}}(q, K) = i\gamma_5 C \mathcal{N}^{0+}$$

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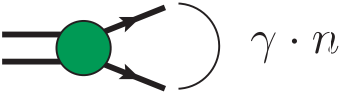

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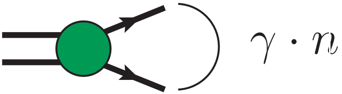
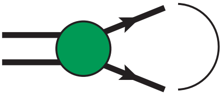
- This point-like case leads to a flat DA:

$$\phi_{\mathrm{PL}}(x) = 1$$



- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$



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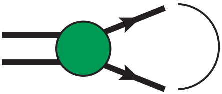
- Bethe-Salpeter amplitude (1 out of 4 structures):

$$\Gamma^{0+}_{\mathrm{PL}}(q,K) = i\gamma_5 C \mathcal{N}^{0+} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]}$$



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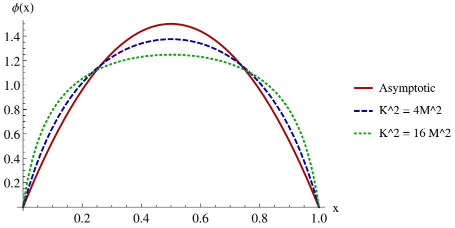
- The Nakanishi case leads to a non trivial DA:

$$\phi(x) = 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar DA behaviour

$$\phi(x) \propto 1 - \frac{M^2}{K^2} \frac{\ln\left[1 + \frac{K^2}{M^2} x(1-x)\right]}{x(1-x)}$$

Scalar diquark



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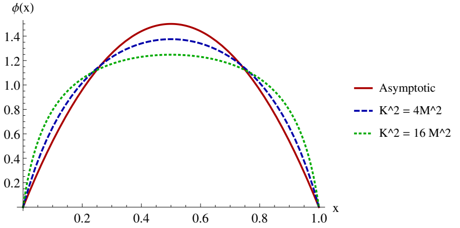
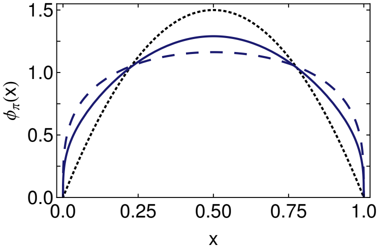
Scalar diquark



Pion

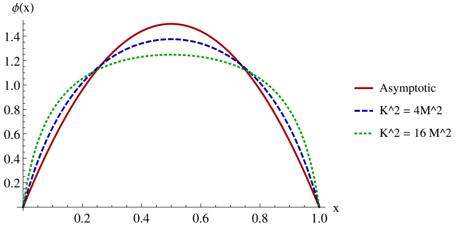


Pion figure from L. Chang *et al.*, PRL 110 (2013)

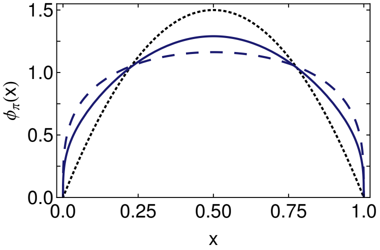
Scalar DA behaviour

$$\phi(x) \propto 1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}$$

Scalar diquark



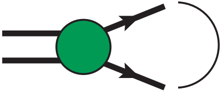
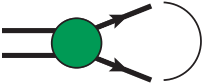
Pion



Pion figure from L. Chang *et al.*, PRL 110 (2013)

This extended version of the DA seems promising!

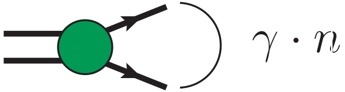
AV diquark DA

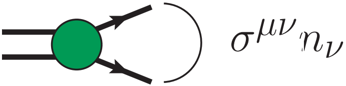
 $\gamma \cdot n$  $\sigma^{\mu\nu} n_\nu$

- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

AV diquark DA





- Quark propagator:

$$S(q) = \frac{-i\not{q} + M}{q^2 + M^2}$$

- Bethe-Salpeter amplitude (2 out of 8 structures):

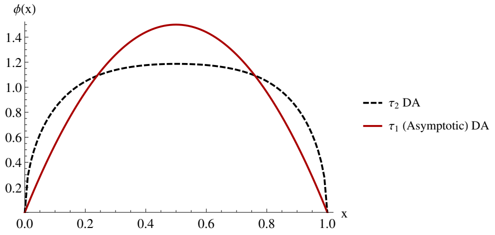
$$\Gamma^{\mu}_{\mathrm{PL}}(q,K) = (\mathcal{N}_1 \tau_1^\mu + \mathcal{N}_2 \tau_2^\mu)\, C \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(q - \frac{1-z}{2}K\right)^2 + \Lambda_q^2\right]}$$

$$\tau_1^\mu = i\left(\gamma^\mu - K^\mu \frac{\not{K}}{K^2}\right) \to \text{Chiral even}$$

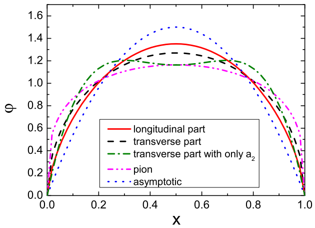
$$\tau_2^\mu = \frac{K\cdot q}{\sqrt{q^2 (K-q)^2}\sqrt{K^2}} \left(-i\tau_1^\mu \not{q} + i\not{q}\tau_1^\mu\right) \to \text{Chiral odd}$$

Comparison with the ρ meson

AV diquark



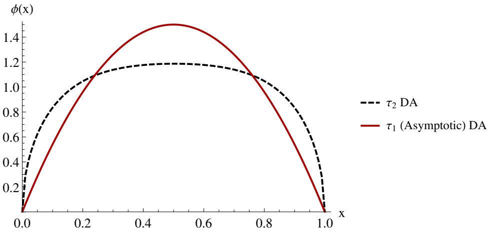
ρ meson



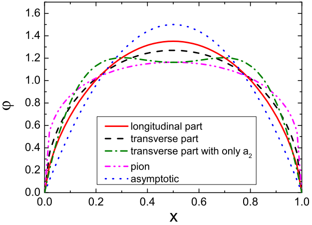
ρ figure from F. Gao *et al.*, PRD 90 (2014)

Comparison with the ρ meson

AV diquark



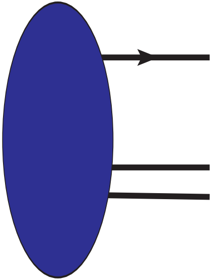
ρ meson



ρ figure from F. Gao *et al.*, PRD 90 (2014)

- Same "shape ordering" $\rightarrow \phi_\perp$ is flatter in both cases.
- Farther apart compared to the ρ meson case.

Modeling the Faddeev Amplitude



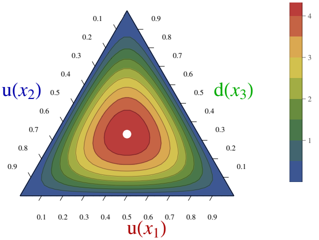
- Scalar case:

$$s_1(K,P) = \mathcal{N}_1 \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

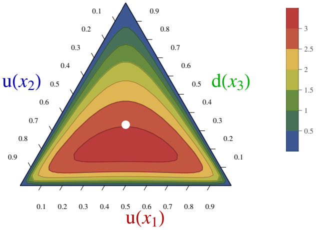
- AV case (2 out of 6 structures):

$$A^\mu(K,P) = \left(\gamma_5\gamma^\mu - i\gamma_5\hat{P}^\mu\right) \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]}$$

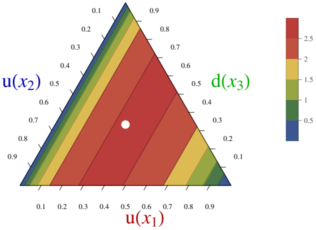
Results in the scalar channel



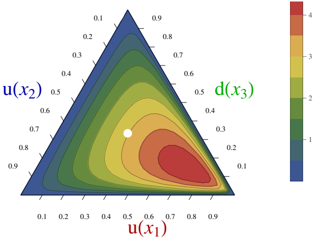
Asymptotic DA



Extended case: T

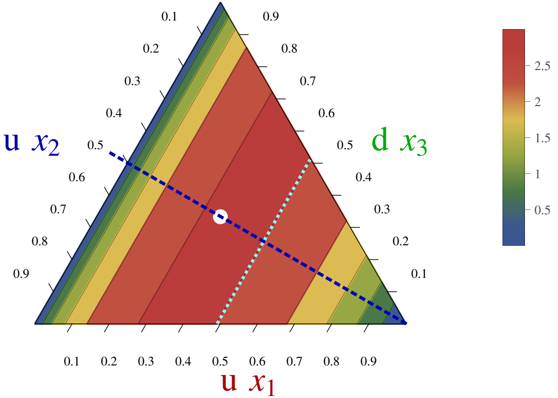


Point-like case: φ



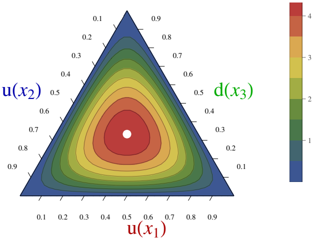
Extended case: φ

Results in the scalar channel

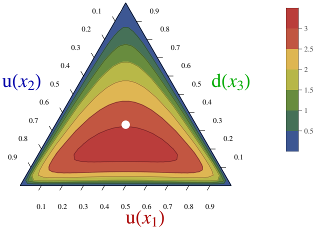


Point-like case: φ

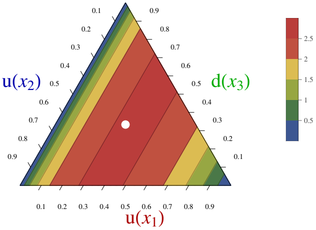
Results in the scalar channel



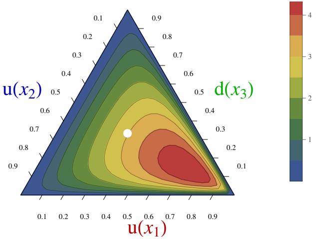
Asymptotic DA



Extended case: T

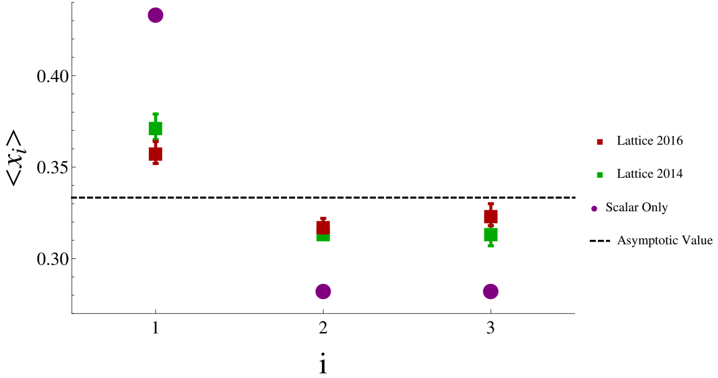


Point-like case: φ



Extended case: φ

$$< x_i >_\varphi = \int \mathcal{D}x \; x_i \varphi(x_1, x_2, x_3)$$

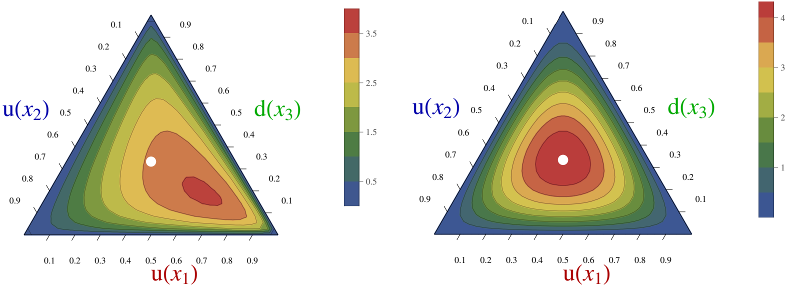


Lattice data from V.Braun *et al*, PRD 89 (2014)

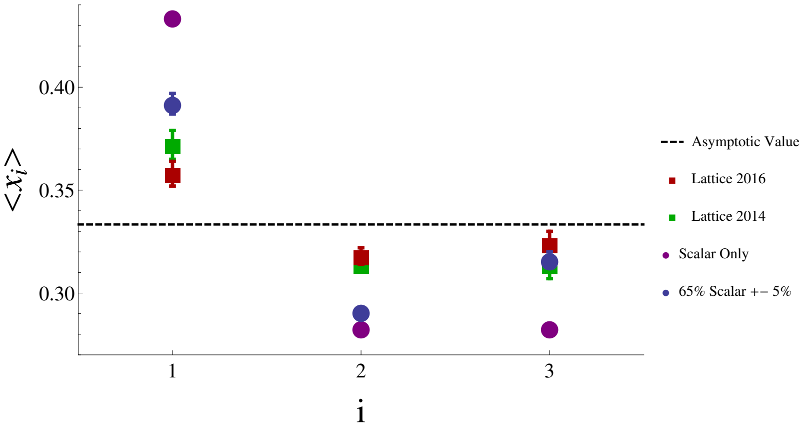
G. Bali *et al.*, JHEP 2016 02

Complete results for φ

- We use the prediction from the Faddeev equation to weight the scalar and AV contributions 65/35:



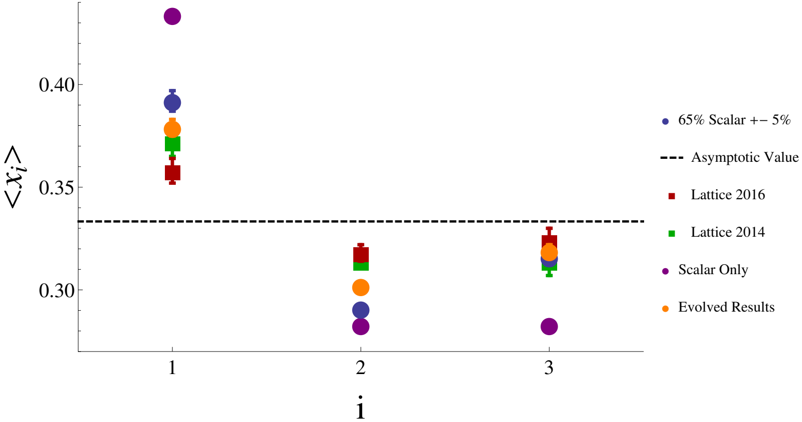
Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

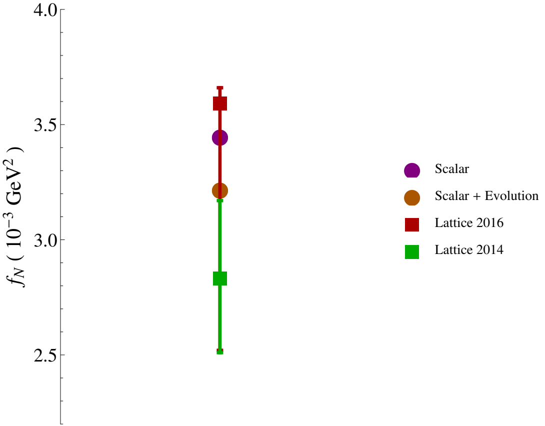
G. Bali *et al.*, JHEP 2016 02

Comparison with lattice II



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02



Lattice data from V.Braun *et al*, PRD 89 (2014)

G. Bali *et al.*, JHEP 2016 02

Evolution

- Evolution equations are known but not the general eigenvectors.
- We have computed the 36 first eigenvectors and eigenvalues to evolve the Nucleon DA.
- Polynomial fit on $(\nu_1(Q^2), \nu_2(Q^2), \omega_{N,n}(x_i, Q^2))$:

$$\varphi(x_i, Q^2) = 120 x_1^{\nu_1(Q^2)-1/2}(x_2 x_3)^{\nu_2(Q^2)-1/2} \times \left(\omega_0(Q^2) + \sum_{N=1}^{2}\sum_{n=0}^{N} \omega_{N,n}(Q^2)\Omega_{N,n}^{1,(2,3)}(x_i, \nu_1(Q^2), \nu_2(Q^2))\right),$$

- Idea: Fit the approriate basis to get a small number of relevant moments.
- At original scale, we found $(\nu_1 \simeq 1.3, \nu_2 \simeq 1.05)$, close to what we get for the mesons.

The Roper Resonance

Extension to the Roper

- Everything done before can actually be extended to the Roper case.

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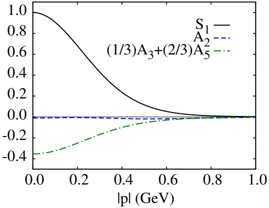
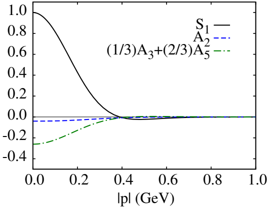
- In particular in the Chebychev moments:





figures from J. Segovia *et al.*,PRL 115 (2015)

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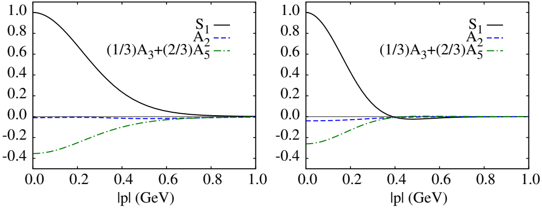
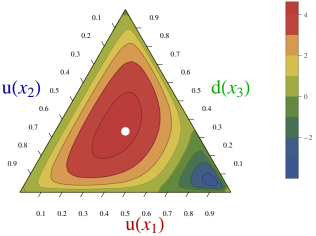


figures from J. Segovia *et al.*,PRL 115 (2015)

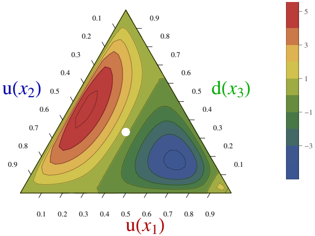
- This behaviour can be obtained by adding a zero in the Faddeev amplitude through:

$$\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_N^2\right]} \to \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)(z-\kappa)}{\left[\left(K - \frac{1-z}{2}P\right)^2 + \Lambda_R^2\right]}$$

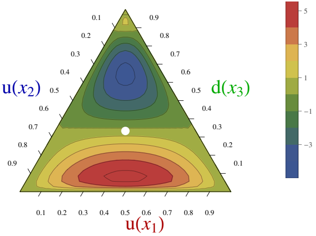
Scalar and AV components



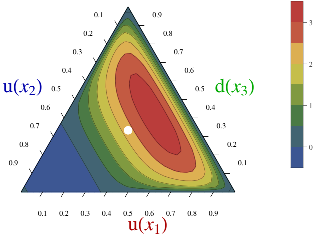
Scalar



AV Long.

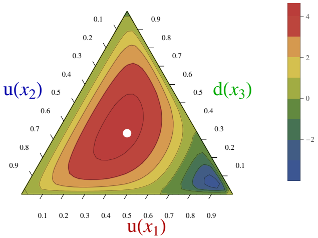


AV Long.

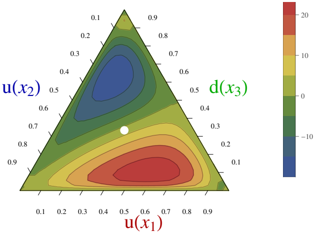


AV Trans.

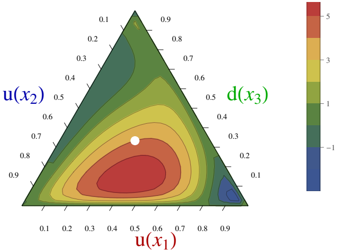
Complete results for φ_R



100 % Scalar

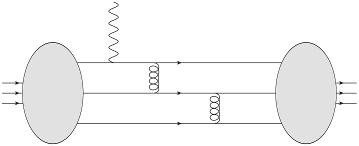


100% AV



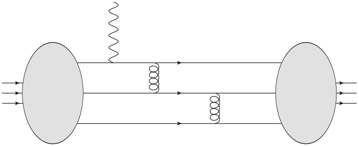
65% Scalar


The road
To the Form Factors



$$F_1(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \Big[\varphi(x_i, \widetilde{Q}_x^2) H_\varphi(x_i, y_i, Q^2) \varphi(y_i, \widetilde{Q}_y^2) + T(x_i, \widetilde{Q}_x^2) H_T(x_i, y_i, Q^2) T(y_i, \widetilde{Q}_y^2) \Big]$$

Form Factors



$$F_1(Q^2) = \mathcal{N} \int [\mathrm{d}x_i][\mathrm{d}y_i] \Big[\varphi(x_i, \widetilde{Q}_x^2) H_\varphi(x_i, y_i, Q^2) \varphi(y_i, \widetilde{Q}_y^2) + T(x_i, \widetilde{Q}_x^2) H_T(x_i, y_i, Q^2) T(y_i, \widetilde{Q}_y^2) \Big]$$

- Kernel well known since more than 30 years...
- ...but different groups have argue different choices for the treatment of scales:
  - for the DA : $\varphi(Q^2), \varphi((min(x_i) \times Q)^2)...$,
  - for the strong coupling constant : $\alpha_S(Q^2), \alpha_s(< x_i > Q^2), \alpha_s^{\mathrm{reg}}(g(x_i, y_j)Q^2)$

- The form factor is only the first Mellin Moment of GPDs and GDAs.
- The perturbative formula have been generalised to GPDs at large t and GDAs at large s for mesons and baryons.

M. Diehl *et al.*, PRD 61, (2000) 074029
C. Vogt, PRD 64, (2001), 057501
P. Hoodboy *et al.*, PRL 92 (2004) 012003
B. Pire *et al.*, PLB 639, (2006) 642-651

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Can we use our DA models to get relevant information on GPDs and GDAs for mesons and baryons?

Summary

- Both nucleon DAs φ and T can be described using a quark-diquark approximation.
- We show how the diquark types and diquarks polarisations were selected.
- The comparison with lattice computation explains how the different diquarks contribute to the total DAs, and the respective sensitivity of the latter to the AV-diquarks.
- The comparison with the lattice data is encouraging.
- It is possible to extend the work on the nucleon to the Roper case.
- In the Roper case, the results of individual diquarks contributions seem to be consistent with a $n = 1$ excited state.
- Working on an Evolution code.
- Computations of the Form Factors are in progress.

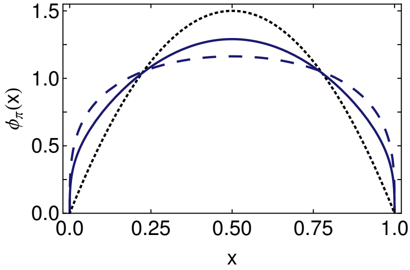
Outlooks

- The use of the numerical solutions of the DSE-Faddeev Equation will certainly modify the previous results, and improve our understanding of the physics of the nucleon.
- Transition form factors.
- Large-t GPDs / Large-s GDAs
- Other resonances, like the $N(1535)$, on which Lattice QCD shows surprising results.

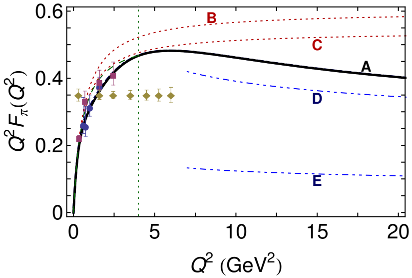
Thank you for your attention

Back up slides

$$\phi_{As}(x) = 6x(1-x)$$



L. Chang *et al.* (2013)



L. Chang *et al.* (2013)

- Broad DSE pion DA is much more consistent with the form factor than the asymptotic one.
- The scale when the asymptotic DA become relevant is huge.

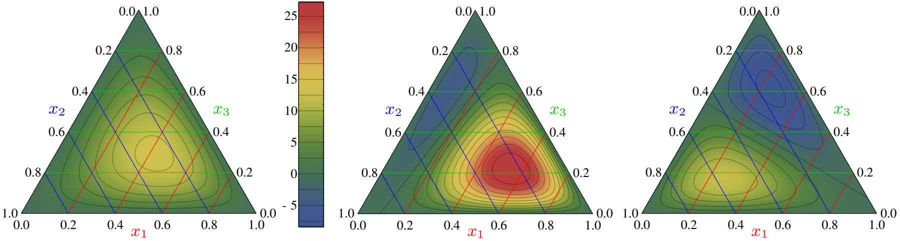


Figure from V. Braun *et la.*,Phys. Rev. **D89**, 094511 (2014)