

ELECTROPRODUCTION AND TRANSITION FORM FACTORS: on the road towards understanding baryon structure



"Nucleon and Resonance Structure with Hard Exclusive Processes"

29-31 mai 2017
IPN Orsay
Europe/Paris timezone



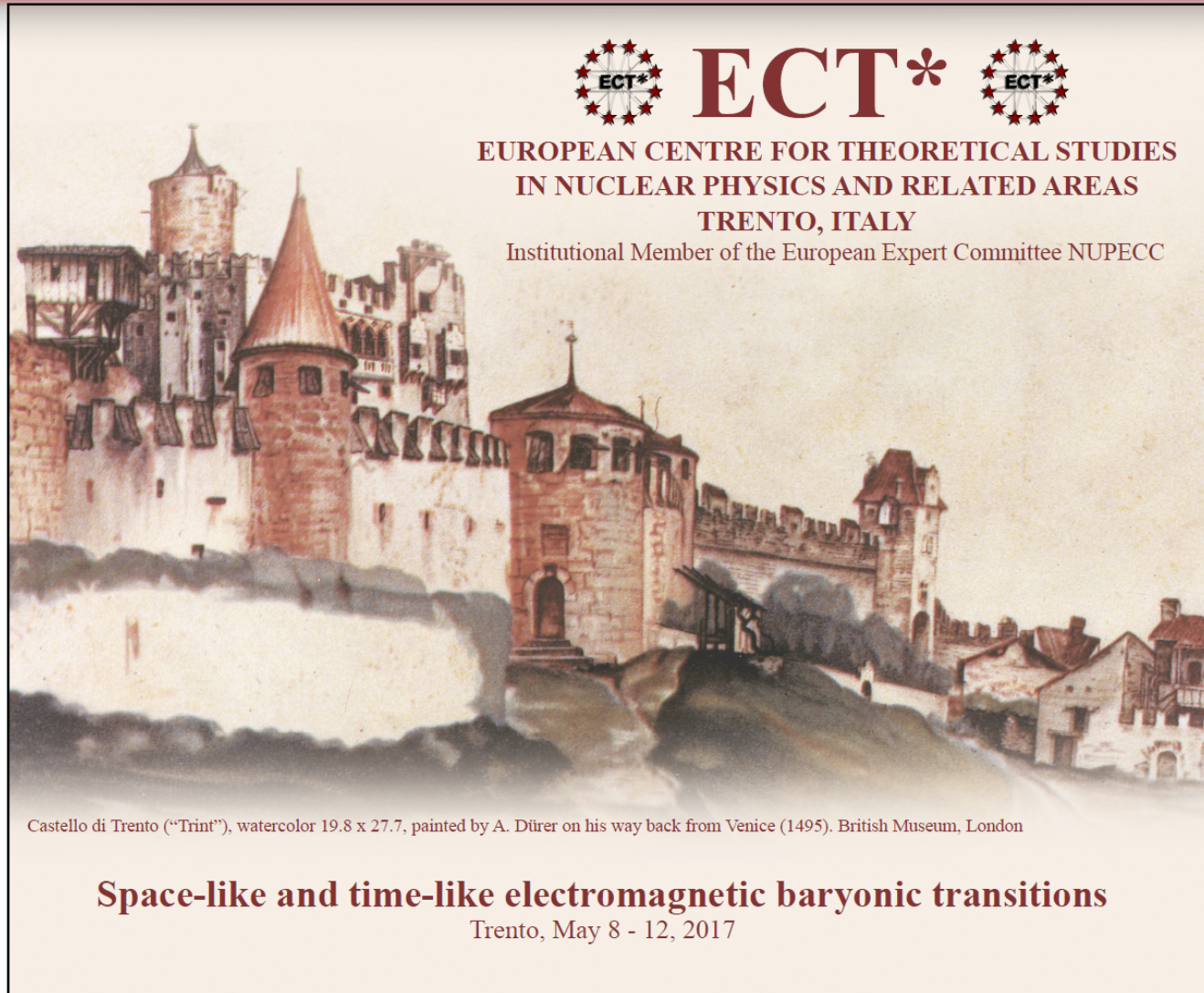
Philip Cole
Idaho State University

May 29, 2017





REPORT ON THE ECT* WORKSHOP: Space-like and time-like electromagnetic baryonic transitions

<https://indico.in2p3.fr/event/14330/overview>



The background of the slide features a watercolor painting of the Castello di Trento (Trin), a medieval fortress with multiple towers and a prominent red-roofed tower. The painting is in shades of brown, red, and white, with a soft, painterly style.

 **ECT*** 

**EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS
TRENTO, ITALY**

Institutional Member of the European Expert Committee NUPECC

Castello di Trento ("Trin"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495), British Museum, London

Space-like and time-like electromagnetic baryonic transitions
Trento, May 8 - 12, 2017

I would like to summarize the summary of the ECT* workshop

Our Research Vision

We sought to **bring together a representative sample of experimental, phenomenology, and theory groups**, who are working on the nucleon resonance problem.

- Discuss the direction on the study of understanding the underlying structure of nucleons in terms of the time-like and space-like electromagnetic baryon form factors and transitions;
- Delineate the spectrum of excited baryon states;
- Describe and detail how quarks are confined and acquire mass through the mechanism of dynamical chiral symmetry breaking.

Confirmed Speakers

- Daniele Binosi (ECT* Trento)
- Vladimir Braun (University of Regensburg)
- William Briscoe (George Washington University)
- Susanna Costanza (University of Pavia)
- Annalisa D'Angelo (University of Rome)
- Chaden Djalali (University of Iowa)
- Michael Döring (George Washington University)
- Christian Fischer (University of Giessen)
- Bengt Friman (TU Darmstadt)
- Tetyana Galatyuk (TU Darmstadt)
- Leonid Glozman (University of Graz)
- Ralf Gothe (University of South Carolina)
- Kyungseon Joo (University of Connecticut)
- Helmut Haberzettl (George Washington University)
- Kenneth Hicks (Ohio University)
- Hiroyuki Kamano (Osaka University)
- Eberhard Klempt (University of Bonn)
- Mikhail Krivoruchenko (ITEP, Moscow)

- Victor Nikonov (University of Bonn and PNPI, Gatchina)
- Teresa Peña (IST Lisbon)
- Vladimir Braun (University of Regensburg)
- Ralph Rapp (Texas A&M University)
- Hiroyuki Sako (JAEA)
- Piotr Salabura (Jagiellonian University in Krakow)
- Toru Sato (Osaka University)
- Hartmut Schmieden (University of Bonn)
- Federico Scozzi (IPN Orsay and TU Darmstadt)
- Kirill Semenov-Tyan-Shanskiy (*PNPI, Gatchina*)
- Igor Strakovsky (George Washington University)
- Joachim Stroth (Goethe University Frankfurt)
- Annika Thiel (University of Bonn)
- Lothar Tiator (University of Mainz)
- Ralf-Arno Tripolt (ECT* Trento)
- Jochen Wambach (TU Darmstadt and ECT* Trento)
- Qiang Zhao (IHEP-Beijing)

Reason for the Workshop

This ECT* workshop brought together several different experimental and theoretical communities, whose research spans the kinematical regimes in q^2 between the *space-like* and *time-like* regions

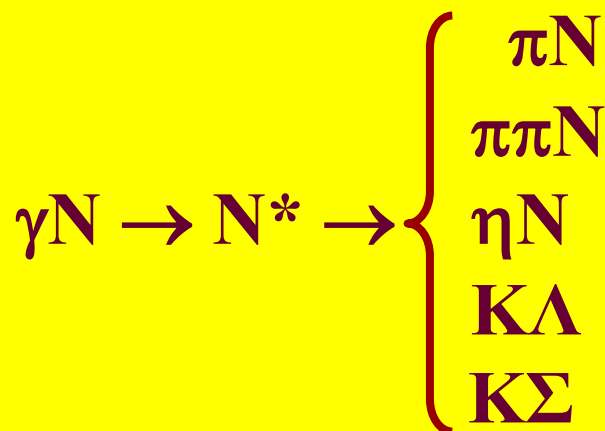
- $q^2 = 0$ [anchor point] photon-beam (unpolarized & linearly- and circularly polarized experiments (ELSA, JLab, LEPS, & MAMI))
- $q^2 > 0$ [time-like] meson-beam experiments (GSI and J-PARC)
proton-antiproton beam experiments (FAIR)
- $q^2 < 0$ [space-like] electron-beam experiments (JLab)

Following topics were covered

- Electromagnetic baryon excitations through meson electroproduction
- Theoretical approaches for baryon transition form factors in the *space-like* region
- Baryon spectroscopy from photoproduction and meson beam experiments
- Amplitude analysis and extraction of baryonic resonances properties
- Electromagnetic transitions through dilepton production
- Unified description of *space-like* and *time-like* baryon electromagnetic transitions
- Vector mesons in medium
- Prospects for future experimental studies

Coupled-channels picture of resonance excitation

[Motivation]

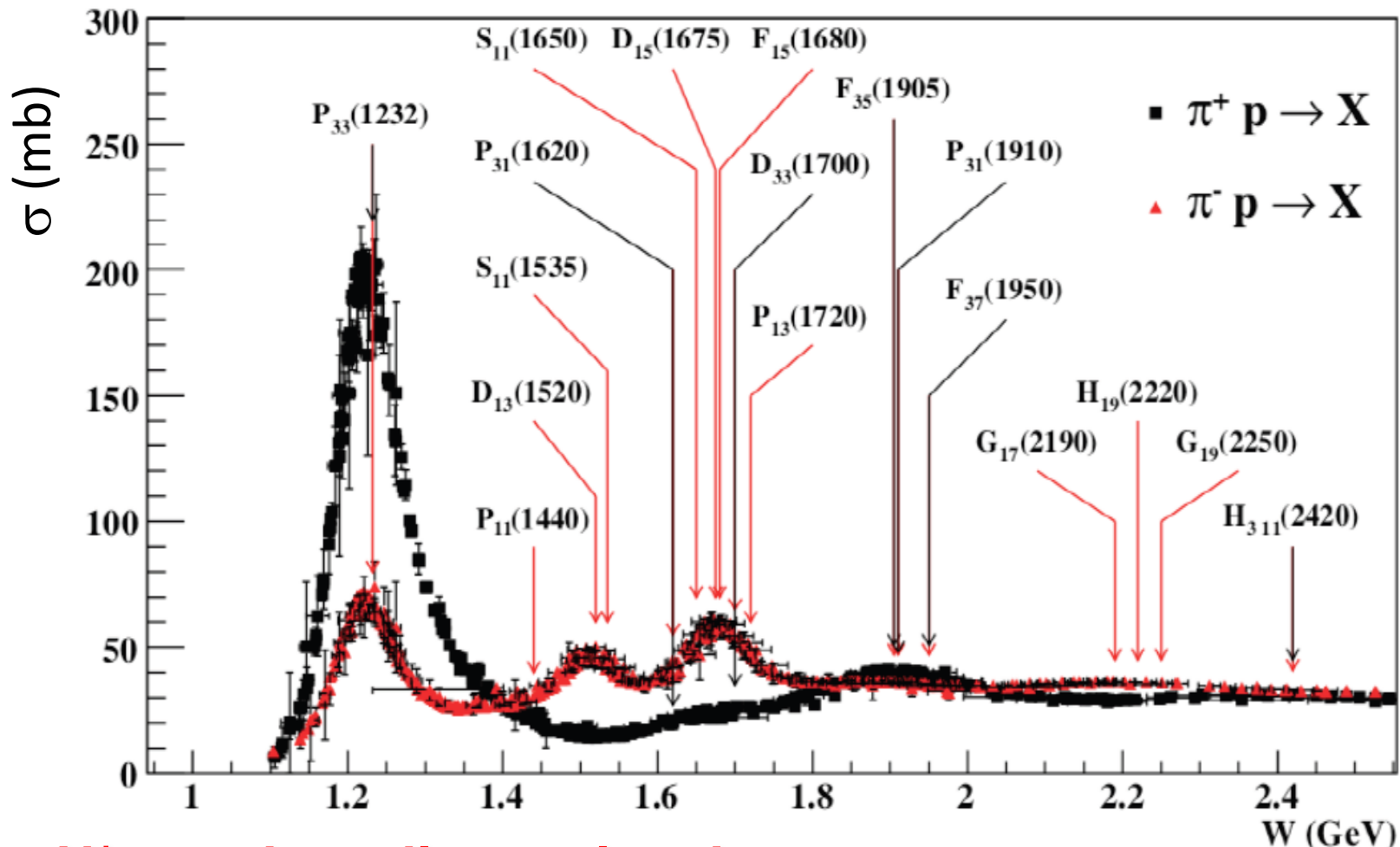


The **same** N^* resonance must be found in **different** reaction channels in a consistent way!

$$T =$$

$T_{\pi N \rightarrow \pi N}$	$T_{\eta N \rightarrow \pi N}$	$T_{\gamma N \rightarrow \pi N}$	$T_{\rho N \rightarrow \pi N}$	$T_{\sigma N \rightarrow \pi N}$	$T_{K\Lambda \rightarrow \pi N}$	$T_{K\Sigma \rightarrow \pi N}$
$T_{\pi N \rightarrow \eta N}$	$T_{\eta N \rightarrow \eta N}$	$T_{\gamma N \rightarrow \eta N}$	$T_{\rho N \rightarrow \eta N}$	$T_{\sigma N \rightarrow \eta N}$	$T_{K\Lambda \rightarrow \eta N}$	$T_{K\Sigma \rightarrow \eta N}$
$T_{\pi N \rightarrow \gamma N}$	$T_{\eta N \rightarrow \gamma N}$	$T_{\gamma N \rightarrow \gamma N}$	$T_{\rho N \rightarrow \gamma N}$	$T_{\sigma N \rightarrow \gamma N}$	$T_{K\Lambda \rightarrow \gamma N}$	$T_{K\Sigma \rightarrow \gamma N}$
$T_{\pi N \rightarrow \rho N}$	$T_{\eta N \rightarrow \rho N}$	$T_{\gamma N \rightarrow \rho N}$	$T_{\rho N \rightarrow \rho N}$	$T_{\sigma N \rightarrow \rho N}$	$T_{K\Lambda \rightarrow \rho N}$	$T_{K\Sigma \rightarrow \rho N}$
$T_{\pi N \rightarrow \sigma N}$	$T_{\eta N \rightarrow \sigma N}$	$T_{\gamma N \rightarrow \sigma N}$	$T_{\rho N \rightarrow \sigma N}$	$T_{\sigma N \rightarrow \sigma N}$	$T_{K\Lambda \rightarrow \sigma N}$	$T_{K\Sigma \rightarrow \sigma N}$
$T_{\pi N \rightarrow K\Lambda}$	$T_{\eta N \rightarrow K\Lambda}$	$T_{\gamma N \rightarrow K\Lambda}$	$T_{\rho N \rightarrow K\Lambda}$	$T_{\sigma N \rightarrow K\Lambda}$	$T_{K\Lambda \rightarrow K\Lambda}$	$T_{K\Sigma \rightarrow K\Lambda}$
$T_{\pi N \rightarrow K\Sigma}$	$T_{\eta N \rightarrow K\Sigma}$	$T_{\gamma N \rightarrow K\Sigma}$	$T_{\rho N \rightarrow K\Sigma}$	$T_{\sigma N \rightarrow K\Sigma}$	$T_{K\Lambda \rightarrow K\Sigma}$	$T_{K\Sigma \rightarrow K\Sigma}$

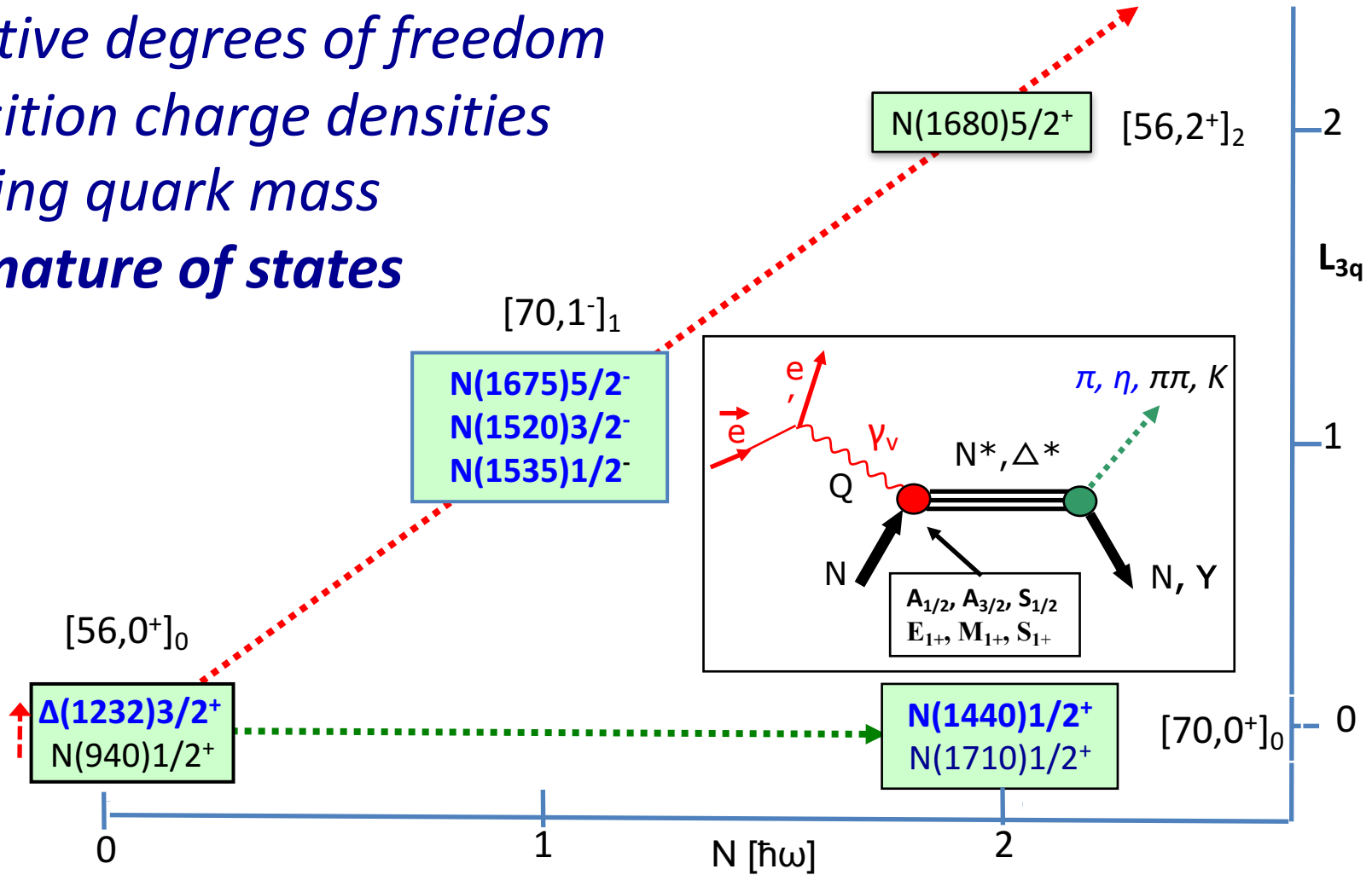
Baryon resonances (N^* s and Δ^* s)



- N^* s are broadly overlapping
- Hard to disentangle without polarization observables

Structure of excited baryons

- effective degrees of freedom
 - transition charge densities
 - running quark mass
- ⇒ nature of states



I.G. Aznauryan et al., Analysis of $p(e, e'N\pi)$; V.I. Mokeev et al., Analysis of $p(e, e'\pi^+\pi^-)$

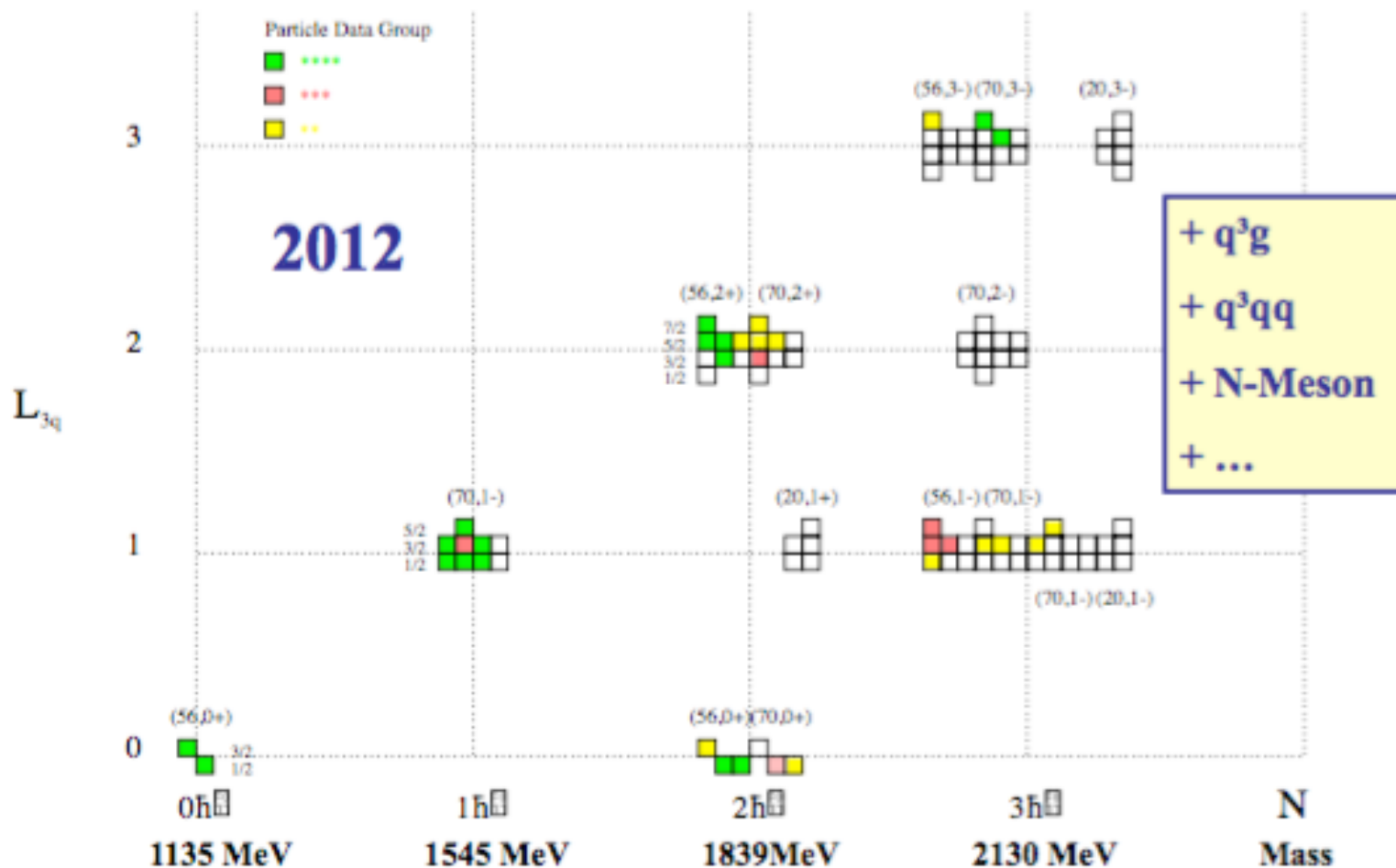
Evidence for New N^* in KY Final State

State $N(\text{mass})J^P$	PDG pre 2010	PDG 2016	$K\Lambda$	$K\Sigma$	$N\gamma$
$N(1710)1/2^+$	***	****	****	**	****
$N(1880)1/2^+$		**	**		**
$N(1895)1/2^-$		**	**	*	**
$N(1900)3/2^+$	**	***	***	**	***
$N(1875)3/2^-$		***	***	**	***
$N(2150)3/2^-$		**	**		**
$N(2000)5/2^+$	*	**	**	*	**
$N(2060)5/2^-$		**		**	**

Study these states in electroproduction and extend to higher masses

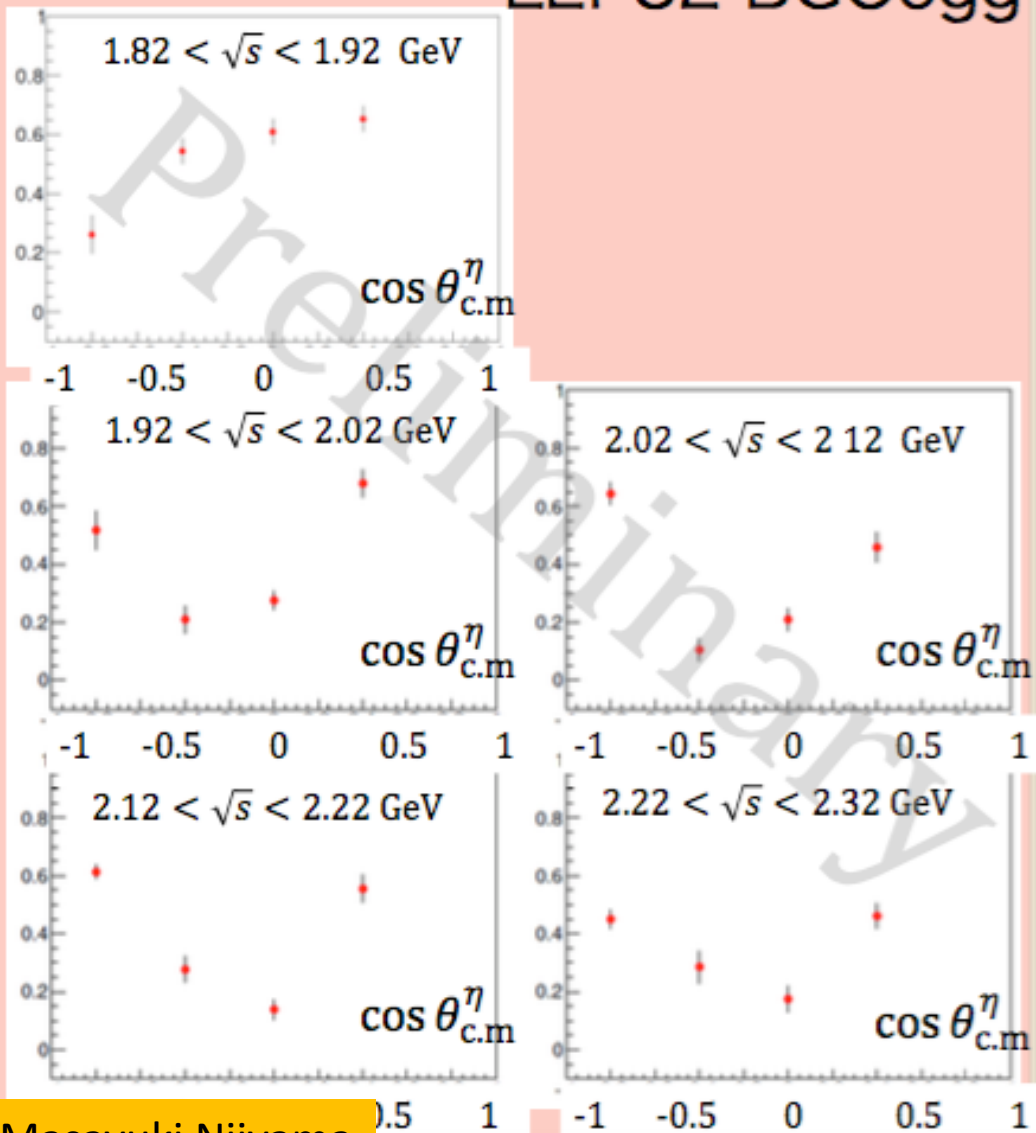
Quark Model Classification of N^*

BnGa energy-dependent coupled-channel PWA of CLAS $K^+\Lambda$ and other data



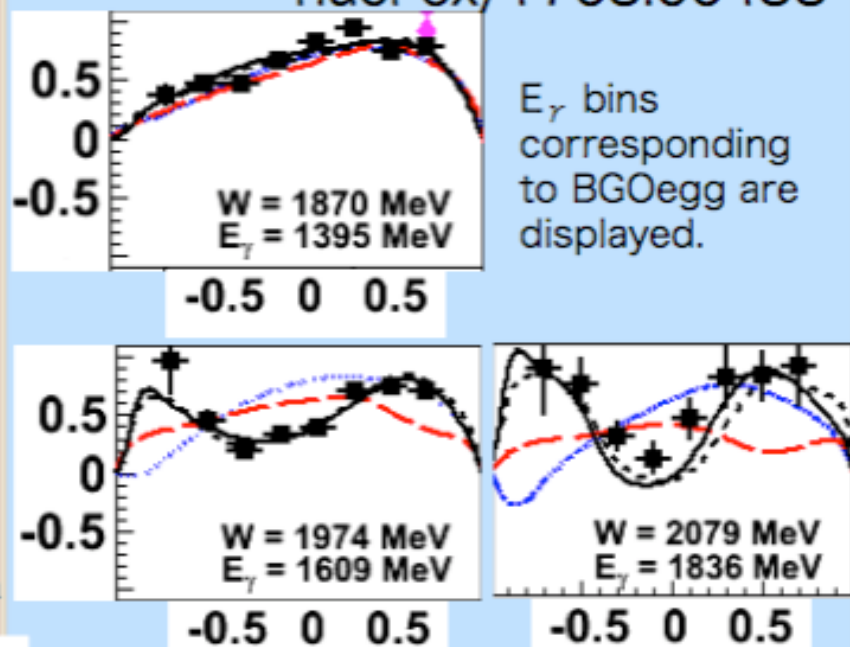
Preliminary results

LEPS2 BGOegg



J-Lab CLAS

nucl-ex/1703.00433



SAID

ETA-MAID

Julich-Bonn (solid)

Julich-Bonn w/o N(1900) (dashed)

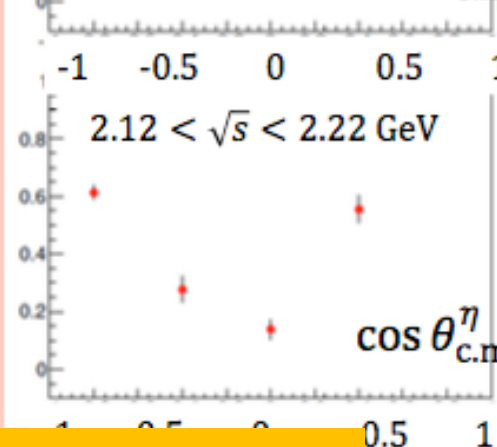
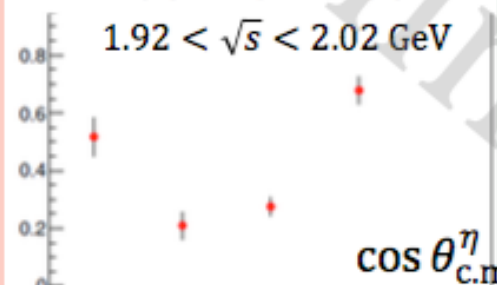
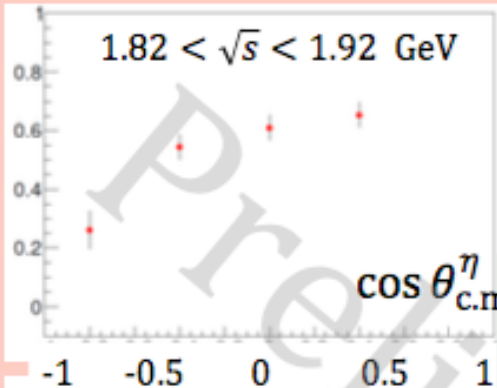
Supporting N(1895)1/2-,
N(1900)3/2+, N(2100)1/2+,
N(2120)3/2-

Preliminary results

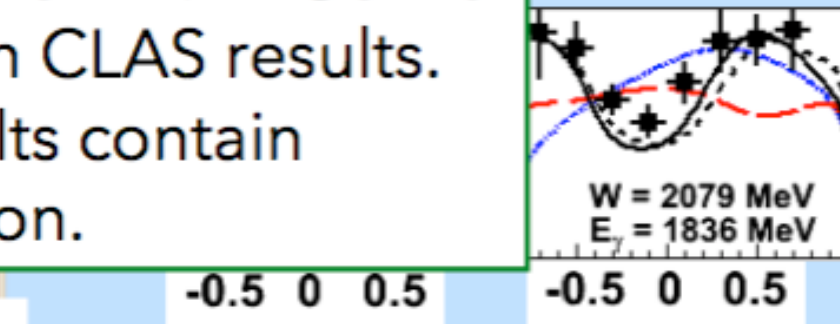
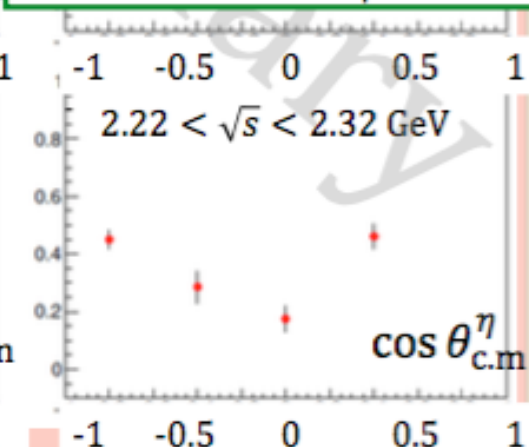
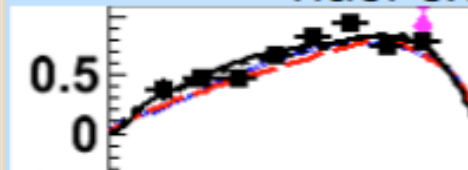
LEPS2 BGOegg

J-Lab CLAS

nucl-ex/1703.00433



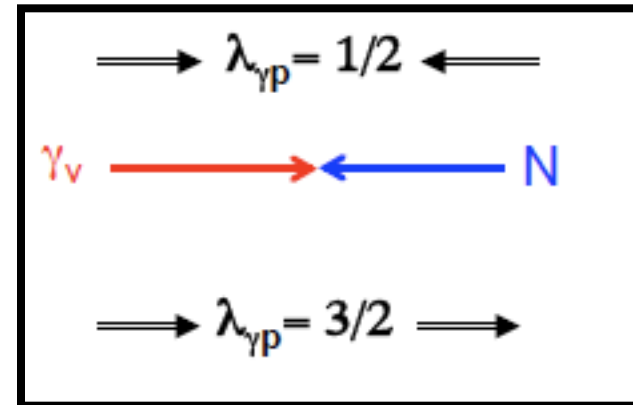
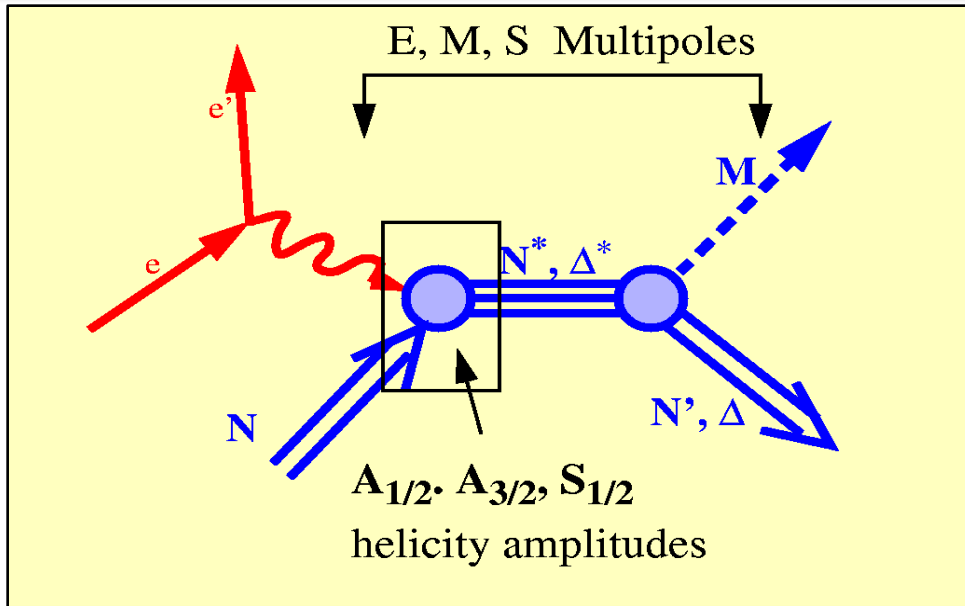
- Change of polar angle dependence from 1.92 GeV is consistent with CLAS results.
- BGOegg results contain higher E_{γ} region.



SAID
ETA-MAID
Julich-Bonn (solid)
Julich-Bonn w/o N(1900) (dashed)

Supporting N(1895)1/2-,
N(1900)3/2+, N(2100)1/2+,
N(2120)3/2-

Electroproduction



The helicity amplitudes are related to the matrix elements of the electromagnetic current via:

$$A_{1/2}: \langle N^*, S_z^* = +1/2 | \epsilon_\mu^{(+)} J_\mu^{\text{em}} | N, S_z = -1/2 \rangle$$

$$A_{3/2}: \langle N^*, S_z^* = +3/2 | \epsilon_\mu^{(+)} J_\mu^{\text{em}} | N, S_z = +1/2 \rangle$$

$$S_{1/2}: \langle N^*, S_z^* = +1/2 | \epsilon_\mu^{(0)} J_\mu^{\text{em}} | N, S_z = +1/2 \rangle$$

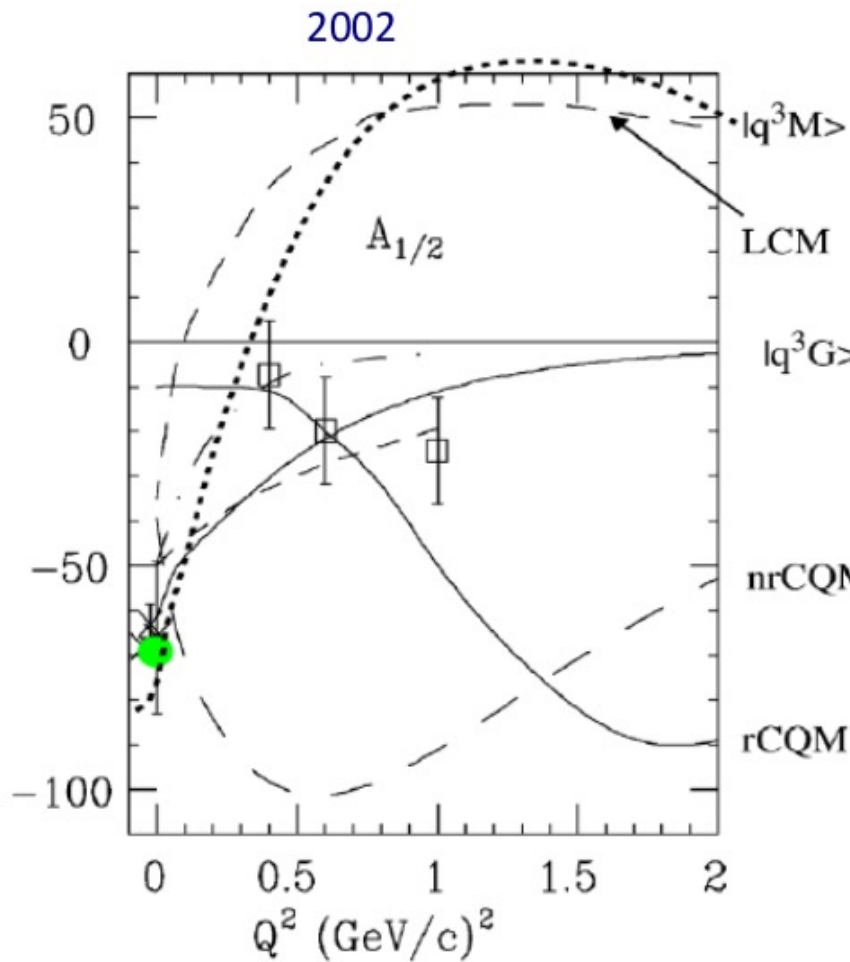
Transverse

- $A_{1/2}$
- $A_{3/2}$

Longitudinal

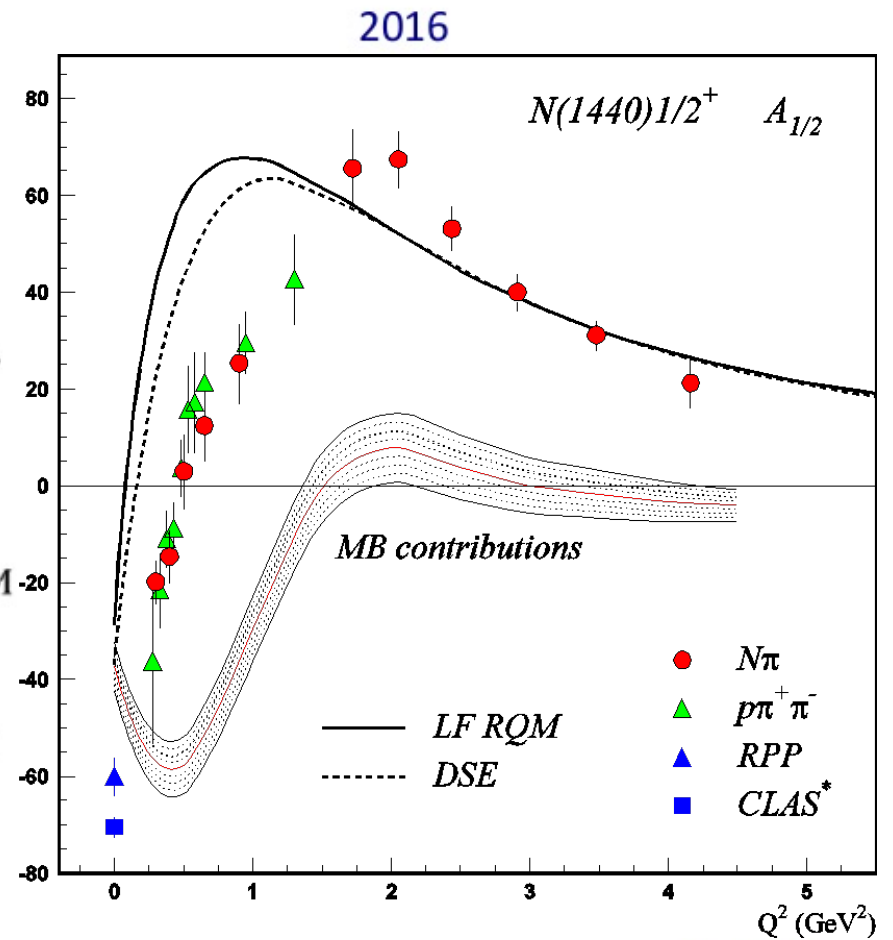
- $S_{1/2}$

Roper resonance in 2002 & 2016



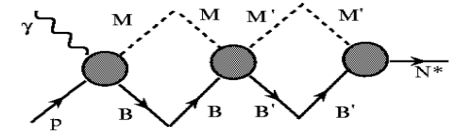
V. Burkert, *Baryons 2002*

DSE describe successfully the nucleon elastic and the transition $N \rightarrow \Delta(1232)3/2^+$, $N \rightarrow N(1535)1/2^-$ form factors with the same dressed quark mass function (J. Segovia, et al., PRL 115, 171801 (2015)).

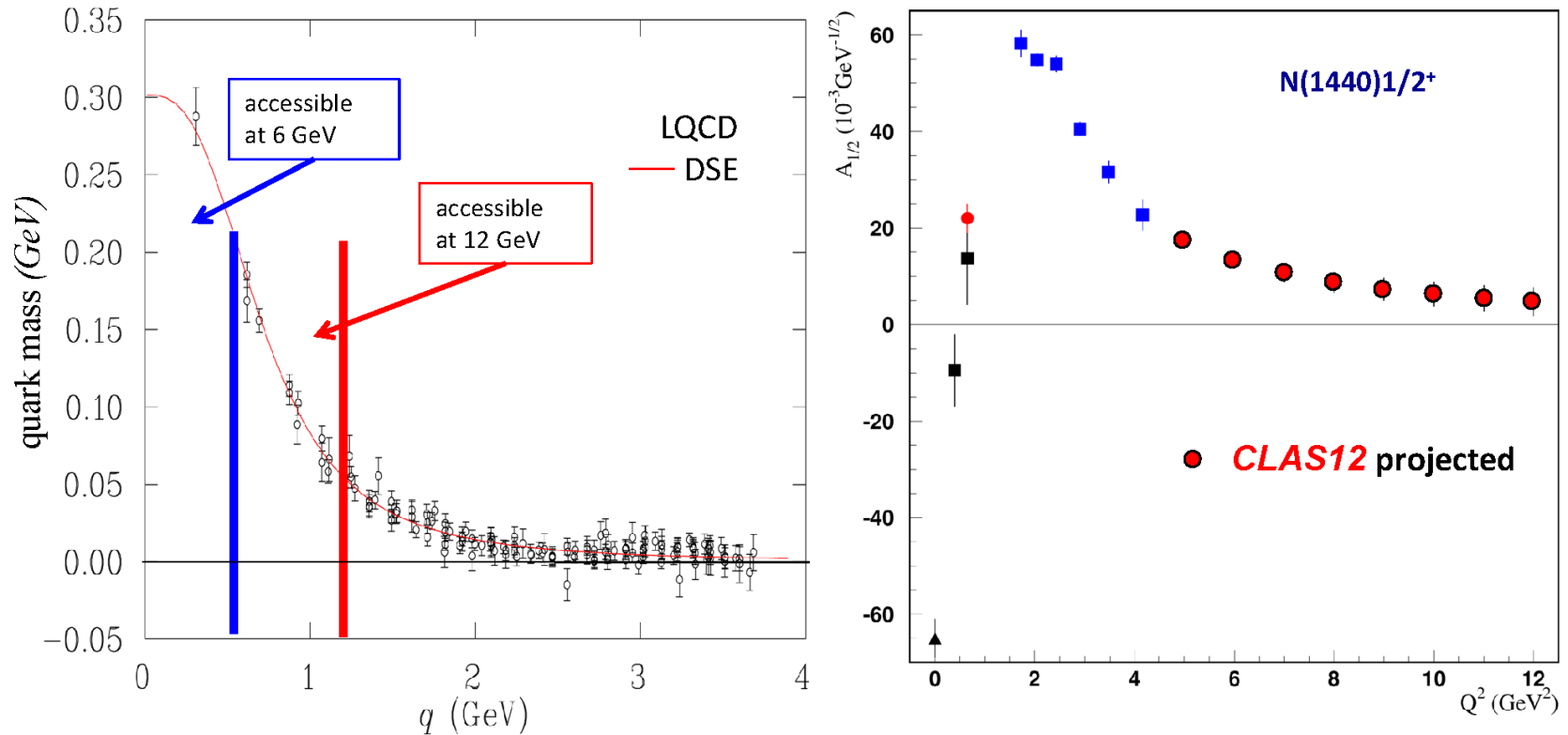


V. D. Burkert, *Baryons 2016*

The mechanisms of the meson-baryon dressing

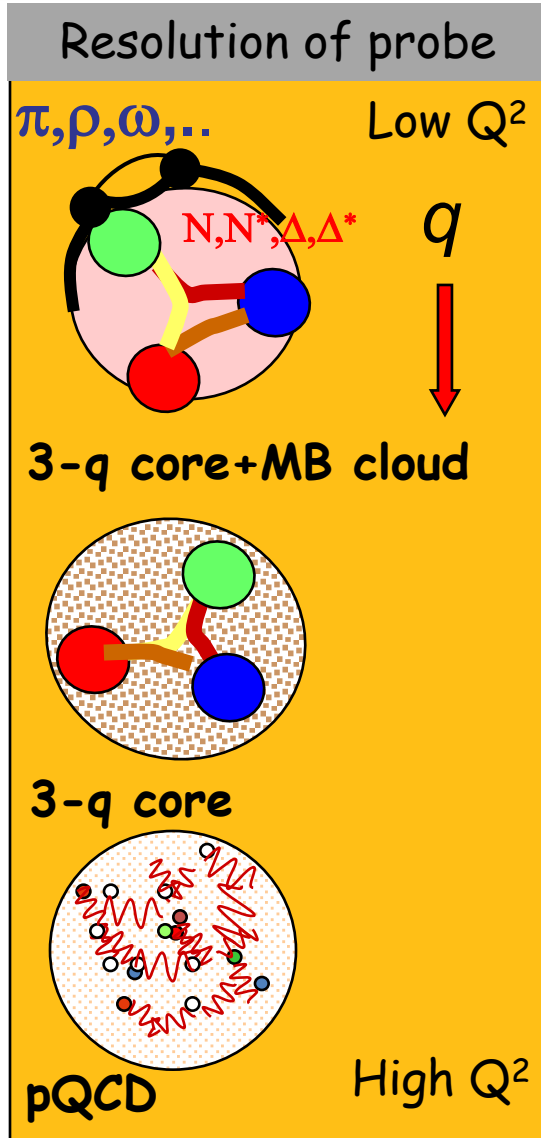


Probing the running quark mass with CLAS12



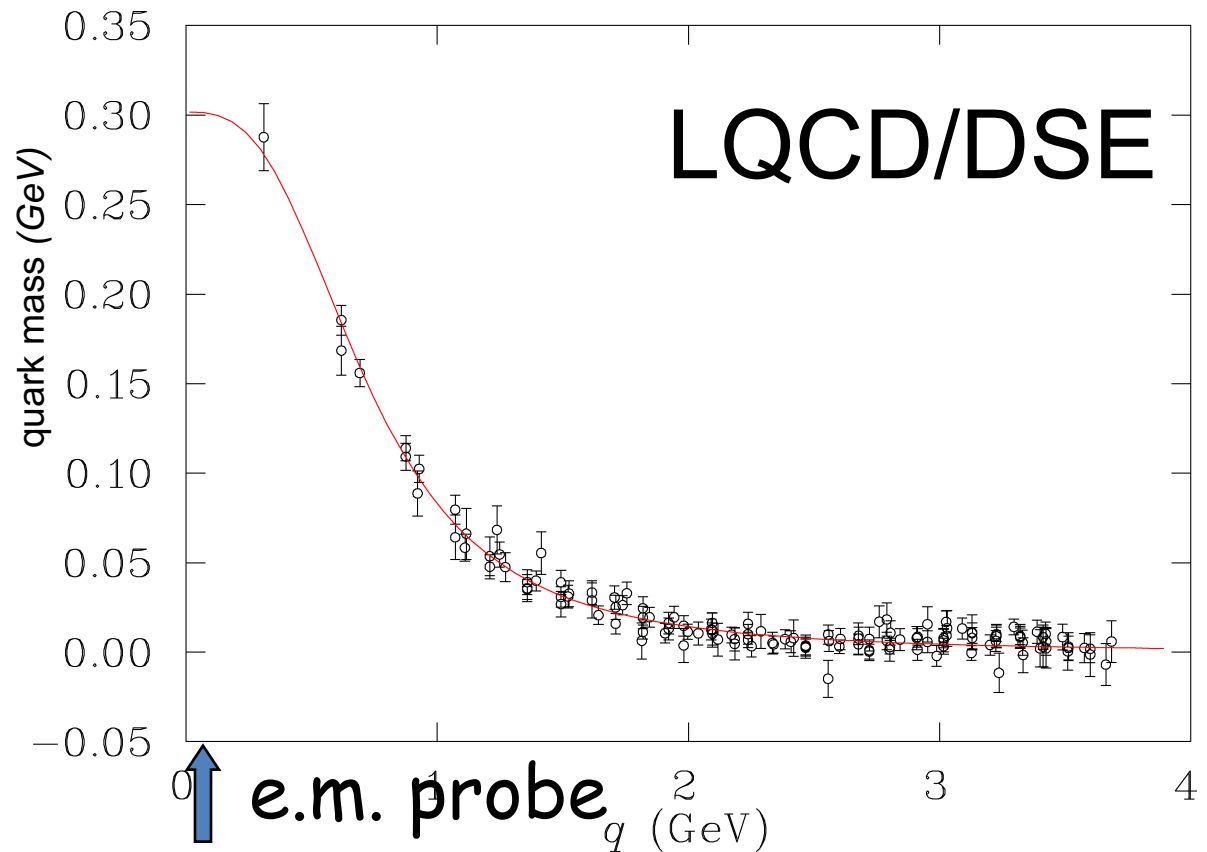
Nucleon resonance transitions amplitudes probe the quark mass function from constituent quarks to dressed quarks and elementary quarks.

One clear goal



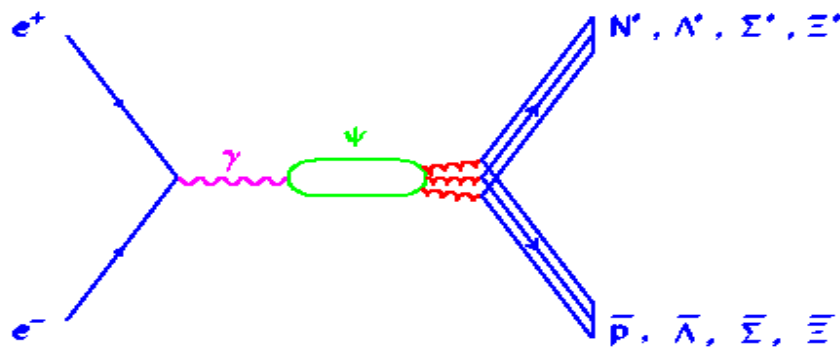
Allows us to address central the question:

What are the relevant degrees of freedom at varying distance scale?



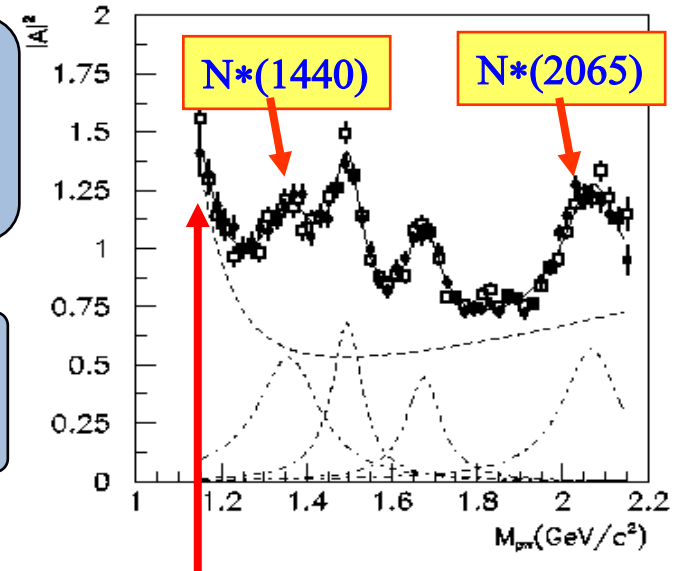
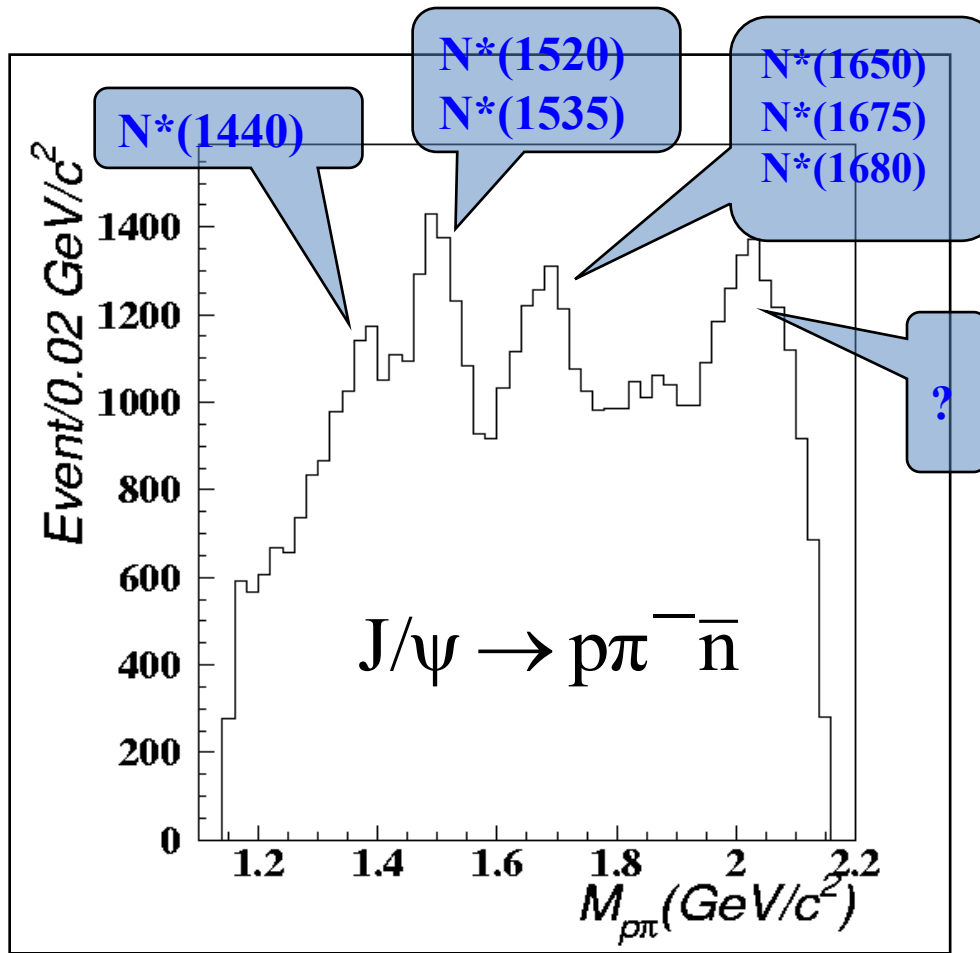
Baryon spectroscopy from J/ψ decays at BES/BEPC

$$J/\Psi \rightarrow \bar{B}B M \Rightarrow N^*, \Lambda^*, \Sigma^*, \Xi^*$$

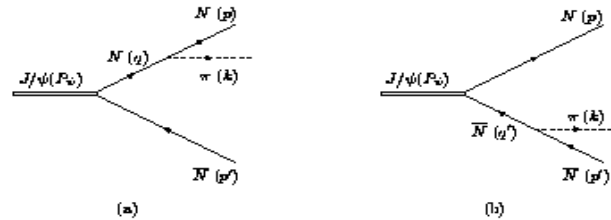


New mechanism for baryon production & an ideal isospin filter

Observation of Two New N^* Peaks in $J/\psi \rightarrow p\pi^- \bar{n}$ and $\bar{p}\pi^+ n$ Decays



Off-shell nucleon contribution

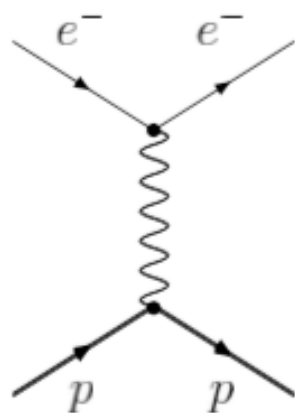


Nucleon-pole diagrams for $J/\psi \rightarrow \pi N\bar{N}$ decay.

N* Electron-production (CLAS/JLab) and dilepton production (HADES/GSI) data complement each other.

The knowledge gained from the CLAS/JLAB data can be used to constrain the interpretation of HADES/GSI data.

Space-like $q^2 > 0$

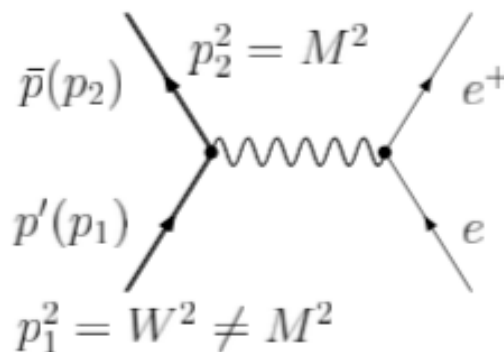


$$q^2 \leq 0$$

Most world data
is CLAS data

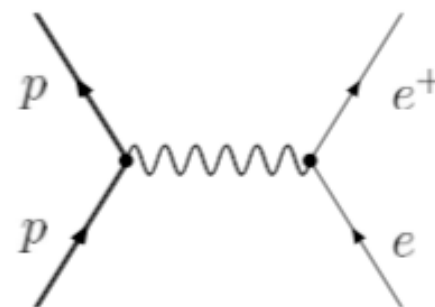
Time-like $q^2 > 0$

Unphysical



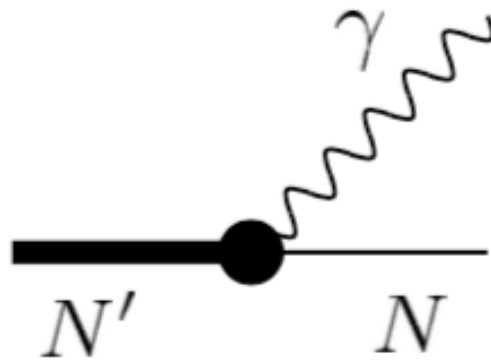
$$4m_e^2 \leq q^2 \leq 4M^2$$

Physical



$$q^2 \geq 4M^2$$

HADES, BES III, FAIR



R rest frame

$$P_R = (W, 0, 0, 0); \quad P_N = (E_N, 0, 0, -|\mathbf{q}|); \quad q = (\omega, 0, 0, |\mathbf{q}|)$$

Timelike $q^2 > 0$

$$\omega = \frac{W^2 - M^2 + q^2}{2W}$$

$$|\mathbf{q}|^2 = \frac{[(W + M) - q^2][(W - M)^2 - q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 - q^2}{2W}$$

$$\text{TL: } q^2 \leq (W - M)^2$$

Near $q = 0$

Spacelike $-q^2 = Q^2 > 0$

$$\omega = \frac{W^2 - M^2 - Q^2}{2W}$$

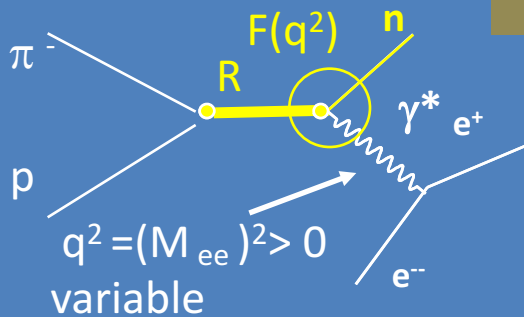
$$|\mathbf{q}|^2 = \frac{[(W + M) + Q^2][(W - M)^2 + Q^2]}{4W^2}$$

$$E_N = \frac{W^2 + M^2 + Q^2}{2W}$$

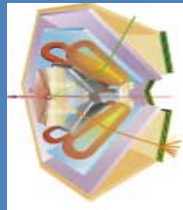
$$W \geq M$$

Electromagnetic baryonic transitions in time-like and space-like regions: towards a global picture?

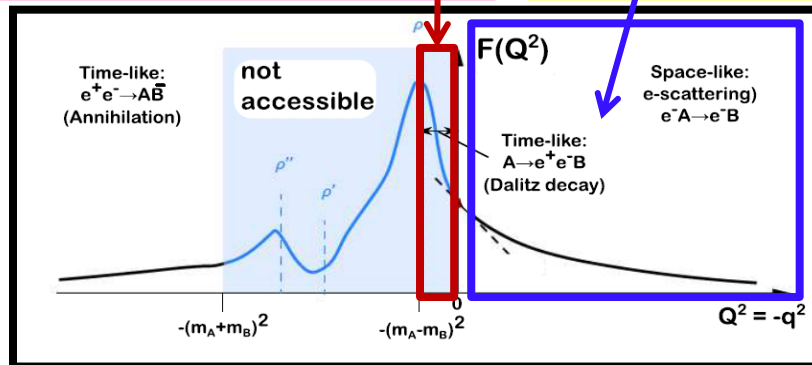
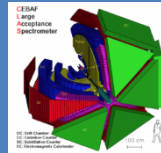
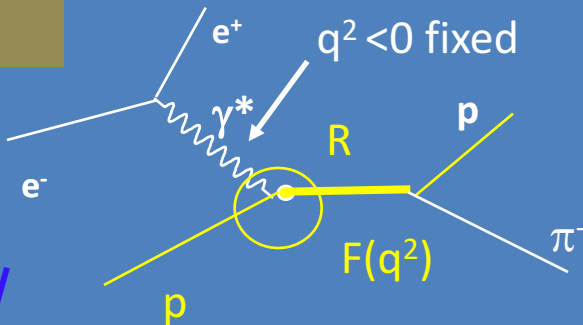
Time-Like electromagnetic form factors
preliminary studies with HADES/GSI



Inverse pion electroproduction

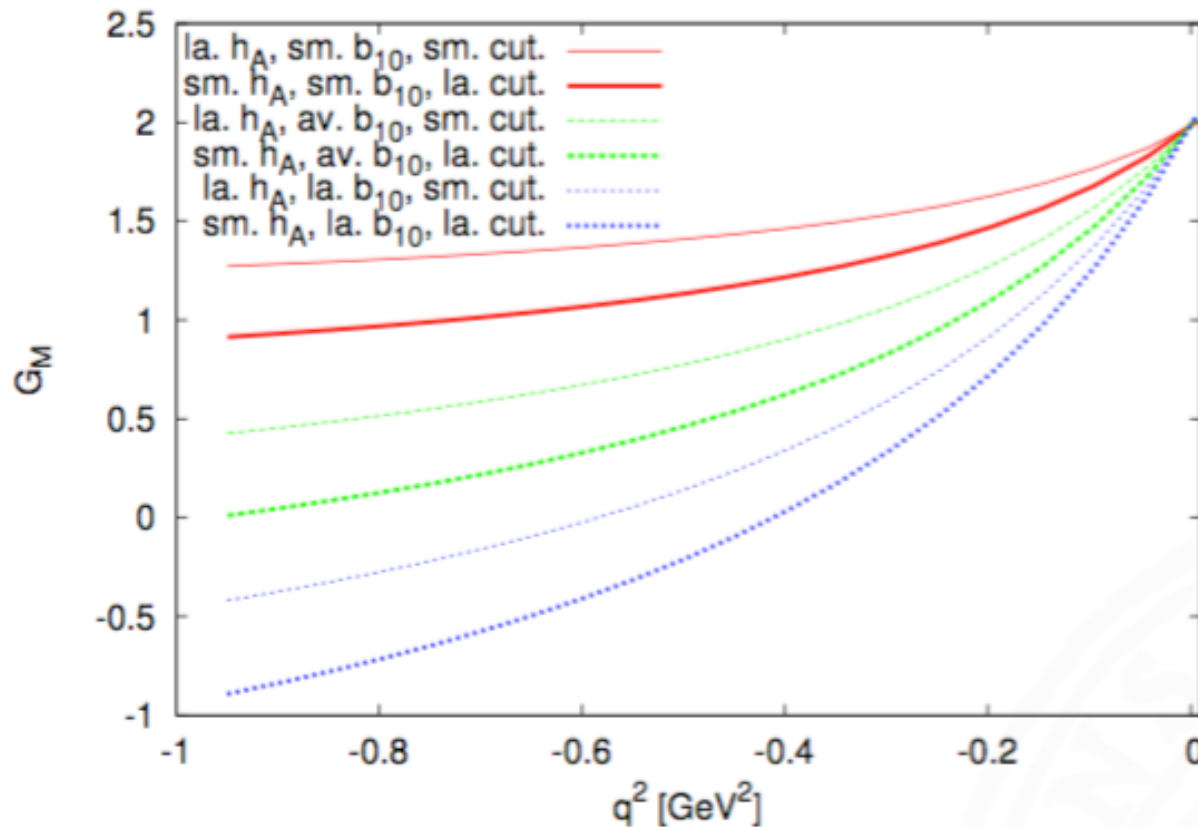


Space-Like electromagnetic form factors
Precise data from JLab/CLAS up to $-q^2=4 \text{ GeV}^2$



- Theoretical tools: Dispersion Relations, Dyson-Schwinger, Vector Dominance, Constituent Quarks ?**

Magnetic transition form factor $\Sigma\text{-}\Lambda$



- large uncertainty
- ↪ directly related to uncertainty in NLO low-energy constant b_{10}
- ↪ can be determined from measuring magnetic transition radius

Time-like form factors

- Time-like FF's are complex:
 - $G_E(q^2) = |G_E(q^2)| \cdot e^{i\Phi_E}$
 - $G_M(q^2) = |G_M(q^2)| \cdot e^{i\Phi_M}$
 - $\Delta\Phi = \Phi_M - \Phi_E =$ relative phase between G_E and G_M
- The phase between G_E and G_M – polarization effects on the final state even when the initial state is unpolarized.

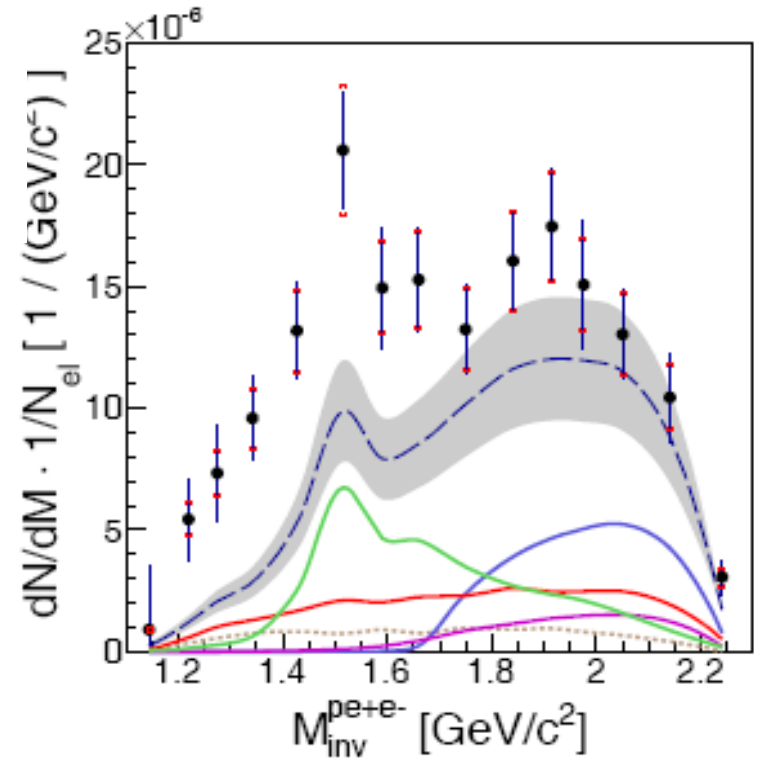
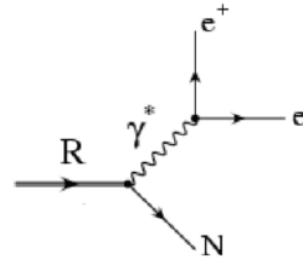
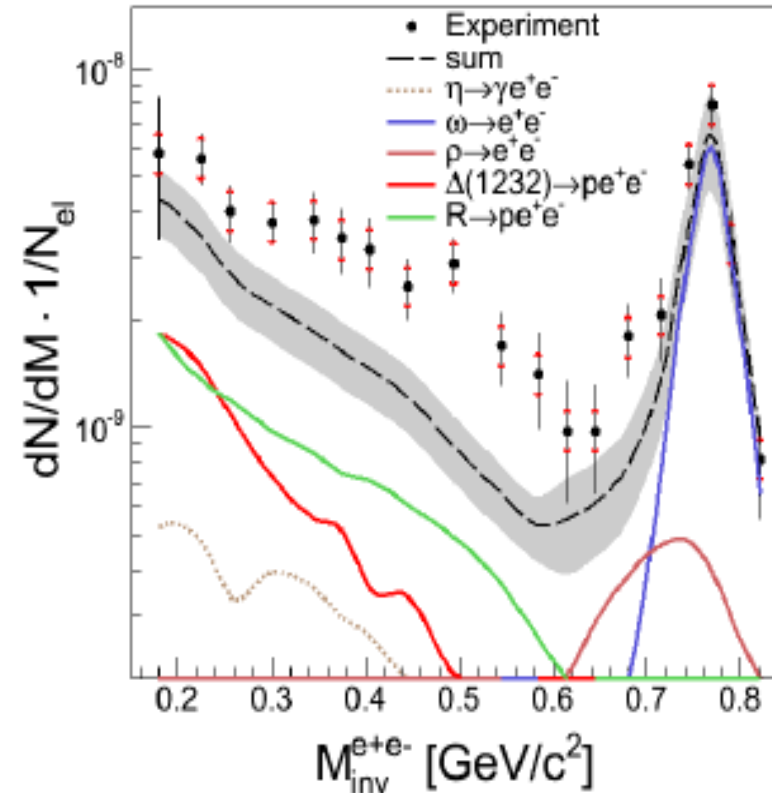
BES III

The logo for the PANDA experiment, featuring the word "panda" in a stylized, lowercase font. The letter "p" is enclosed in a rounded rectangle. Above the "a" are three small colored squares (blue, red, green). Below the "a" is a yellow horizontal bar.

$p+p \rightarrow ppe^+e^-$ @ 3.5 GeV

HADES coll. EPJA50(2014) 82

„QED” cocktail :



- several contributing resonances : $N^*(1520)$, $N^*(1720)$, $\Delta(1620)$, $\Delta(1905)$,...
- ☞ Excess above „QED” cocktail . Seems to originate from $N^*(1520)$ region

Unpolarized decay distribution:

$$W^0(\cos \theta, \phi, \rho_{\alpha\beta}^0) = \frac{3}{4\pi} \left(\frac{1}{2} \sin^2 \theta + \frac{1}{2} (3 \cos^2 \theta - 1) \rho_{00}^0 - \sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos \phi - \rho_{1-1}^0 \sin^2 \theta \cos 2\phi \right),$$

Linearly-polarized decay distribution:

$$W^1(\cos \theta, \phi, \rho_{\alpha\beta}^1) = \frac{3}{4\pi} (\rho_{11}^1 \sin^2 \theta + \rho_{00}^1 \cos^2 \theta - \sqrt{2} \operatorname{Re} \rho_{10}^1 \sin 2\theta \cos \phi - \rho_{1-1}^1 \sin^2 \theta \cos 2\phi),$$

$$\left\{ \begin{aligned} \rho_{ik}^0 &= \frac{1}{A} \sum_{\lambda\lambda_2\lambda_1} H_{\lambda v_i \lambda_2, \lambda\lambda_1} H_{\lambda v_k \lambda_2, \lambda\lambda_1}^* \\ \rho_{ik}^1 &= \frac{1}{A} \sum_{\lambda\lambda_2\lambda_1} H_{\lambda v_i \lambda_2, -\lambda\lambda_1} H_{\lambda v_k \lambda_2, \lambda\lambda_1}^* \\ \rho_{ik}^2 &= \frac{i}{A} \sum_{\lambda\lambda_2\lambda_1} \lambda H_{\lambda v_i \lambda_2, -\lambda\lambda_1} H_{\lambda v_k \lambda_2, \lambda\lambda_1}^* \\ \rho_{ik}^3 &= \frac{i}{A} \sum_{\lambda\lambda_2\lambda_1} \lambda H_{\lambda v_i \lambda_2, \lambda\lambda_1} H_{\lambda v_k \lambda_2, \lambda\lambda_1}^* \end{aligned} \right.$$

e.g. The polarized beam asymmetry:

$$\Sigma \equiv \frac{\sigma_{\perp} - \sigma_{\parallel}}{\sigma_{\perp} + \sigma_{\parallel}} = \frac{2\rho_{11}^1 + \rho_{00}^1}{2\rho_{11}^0 + \rho_{00}^0}$$

$$\begin{aligned} \epsilon_{\perp} &= \hat{y} = i(\epsilon_{\gamma+} + \epsilon_{\gamma-})/\sqrt{2} \\ \epsilon_{\parallel} &= \hat{x} = -(\epsilon_{\gamma+} - \epsilon_{\gamma-})/\sqrt{2} \end{aligned} \quad \Rightarrow \quad \left\{ \begin{aligned} \sigma_{\perp} \\ \sigma_{\parallel} \end{aligned} \right.$$

Zhao, Al-Khalili & Cole, PRC71, 054004 (2005); Pichowsky, Savkli & Tabakin, PRC53, 593 (1996)

Polarization density matrices

- ▶ The leptonic density matrix is explicitly known:

$$\rho_{\lambda'\lambda}^{\text{lep}} = 4|\mathbf{k}_1|^2 \begin{pmatrix} 1 + \cos^2 \theta_e + \alpha & \sqrt{2} \cos \theta_e \sin \theta_e e^{i\phi_e} & \sin^2 \theta_e e^{2i\phi_e} \\ \sqrt{2} \cos \theta_e \sin \theta_e e^{-i\phi_e} & 2(1 - \cos^2 \theta_e) + \alpha & \sqrt{2} \cos \theta_e \sin \theta_e e^{i\phi_e} \\ \sin^2 \theta_e e^{-2i\phi_e} & \sqrt{2} \cos \theta_e \sin \theta_e e^{-i\phi_e} & 1 + \cos^2 \theta_e + \alpha \end{pmatrix}$$

where $\alpha = 2m_e^2/|\mathbf{k}_1|^2$ (neglect in the following)

- ▶ This gives the angular distribution of e^+ and e^- in the virtual photon rest frame:

$$\begin{aligned} \sum_{\text{pol}} |\mathcal{M}|^2 &\propto (1 + \cos^2 \theta_e)(\rho_{-1,-1}^{\text{had}} + \rho_{1,1}^{\text{had}}) + 2(1 - \cos^2 \theta_e)\rho_{0,0}^{\text{had}} \\ &\quad + \sin^2 \theta_e (e^{2i\phi_e} \rho_{-1,1}^{\text{had}} + e^{-2i\phi_e} \rho_{1,-1}^{\text{had}}) \\ &\quad + \sqrt{2} \cos \theta_e \sin \theta_e \left[e^{i\phi_e} (\rho_{-1,0}^{\text{had}} + \rho_{0,1}^{\text{had}}) + e^{-i\phi_e} (\rho_{1,0}^{\text{had}} + \rho_{0,-1}^{\text{had}}) \right] \end{aligned}$$

- ▶ c.f.:

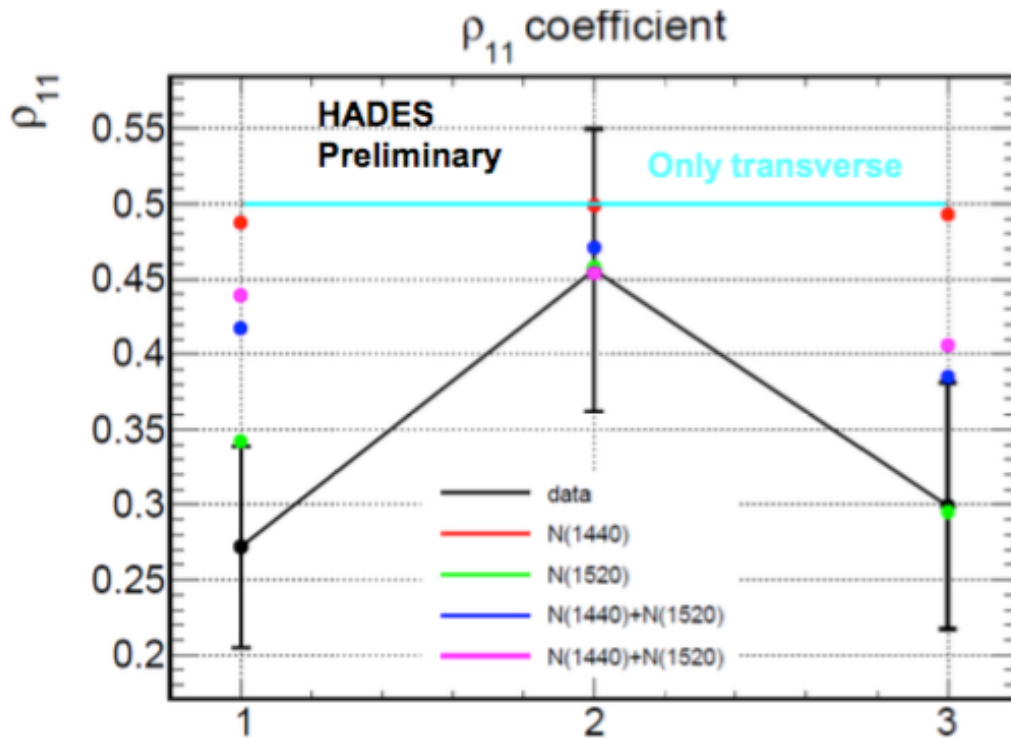
$$\frac{d\sigma}{dM d \cos \theta_{\gamma^*} d \cos \theta_e} \propto \Sigma_{\perp} (1 + \cos^2 \theta_e) + \Sigma_{\parallel} (1 - \cos^2 \theta_e)$$

$$\Rightarrow \Sigma_{\perp} = \rho_{-1,-1}^{\text{had}} + \rho_{1,1}^{\text{had}}, \quad \text{and} \quad \Sigma_{\parallel} = 2\rho_{0,0}^{\text{had}}$$

Model predictions

Comparison with data

- Comparison of density matrix coefficients extracted from the data and in the microscopic model in the same M_{ee} and θ_{γ^*} ranges



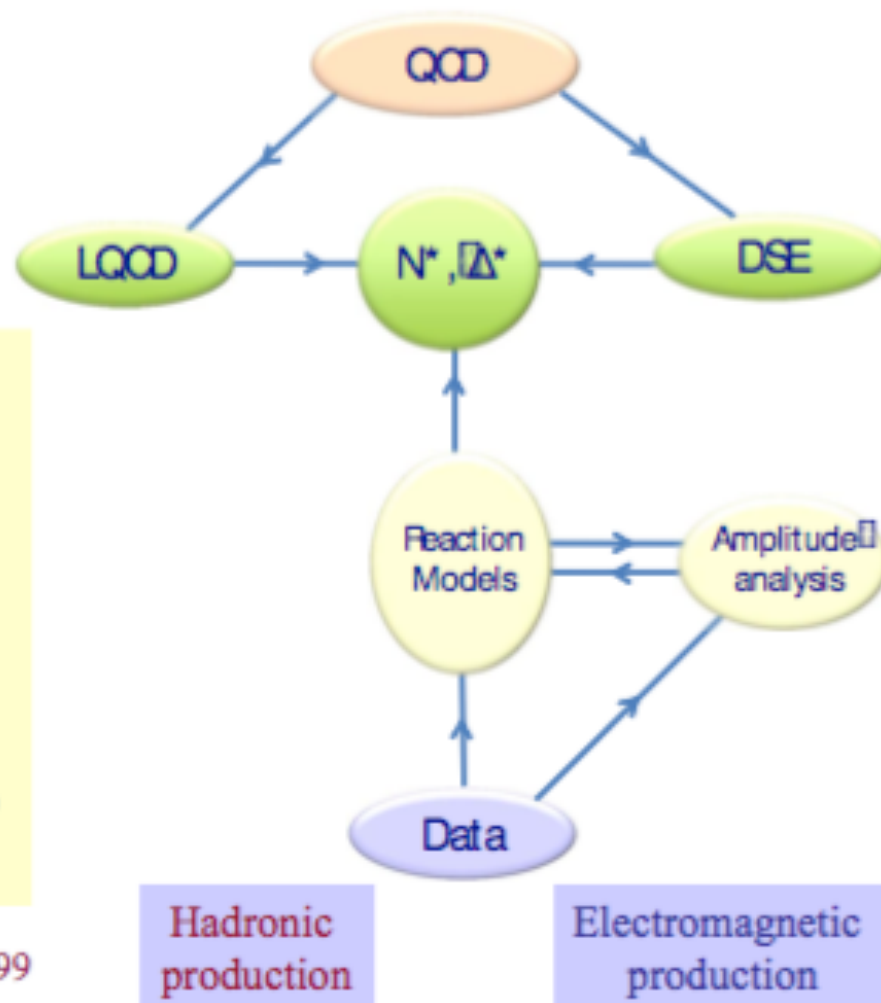
- 1: $-1 < \cos \theta_{cm} < 0$
- 2: $0 < \cos \theta_{cm} < 0.5$
- 3: $0.5 < \cos \theta_{cm} < 1$

Data-Driven Data Analyses

Consistent Results



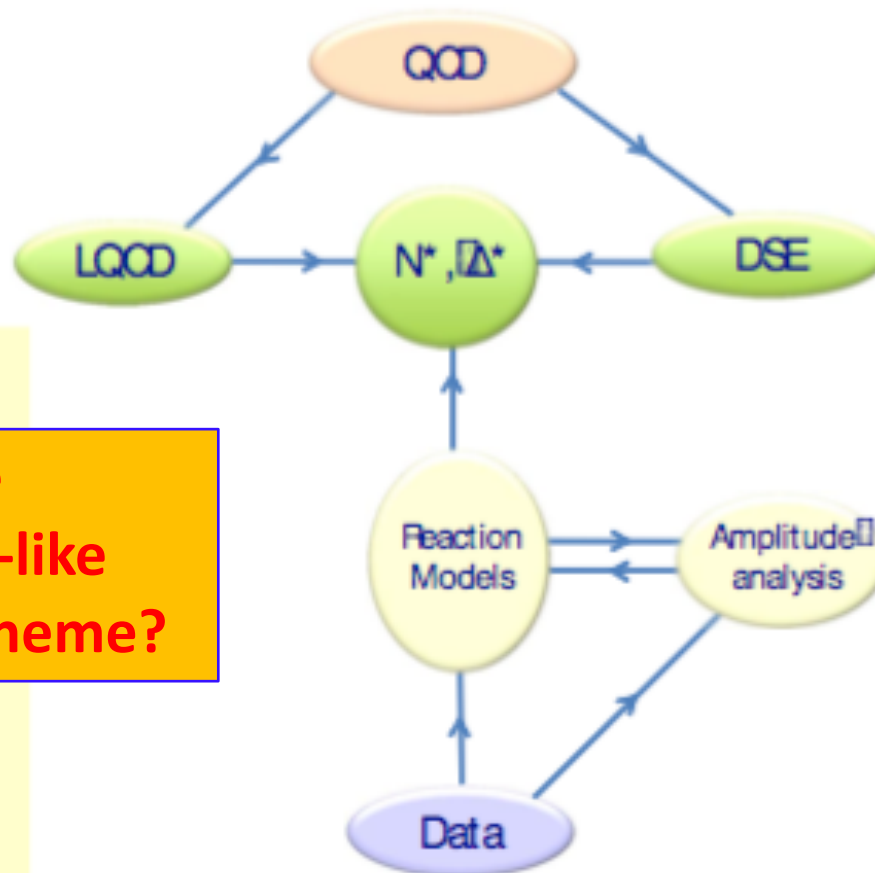
- Single meson production:
Unitary Isobar Model (UIM)
Fixed- t Dispersion Relations (DR)
- Double pion production:
Unitarized Isobar Model (JM)
- Coupled-Channel Approaches:
EBAC \Rightarrow Argonne-Osaka
JAW \Rightarrow Jülich-Athens-Washington \Rightarrow JüBo
BoGa \Rightarrow Bonn-Gatchina



Int. J. Mod. Phys. E, Vol. 22, 1330015 (2013) 1-99

Data-Driven Data Analyses

Consistent Results



Where does the analysis of the time-like regime fit in to this scheme?

➤ Single meson production:

Unitary Isobar Model (UIB)

Fixed

➤ Do

Unit

➤ Coupled-Channel Approaches.

EBAC \Rightarrow Argonne-Osaka

JAW \Rightarrow Jülich-Athens-Washington \Rightarrow JüBo

BoGa \Rightarrow Bonn-Gatchina

Int. J. Mod. Phys. E, Vol. 22, 1330015 (2013) 1-99

Hadronic production

Electromagnetic production

IN THIS SUMMARY OF OUR WORKSHOP FROM A VERY BIASED VIEW OF A SL EXPERIMENTALIST

I have said almost nothing or absolutely nothing on

- Dyson-Schwinger Equation approaches
- Covariant Spectator Theory approach
- PWA approaches
- Phenomenological approaches
- Pion form factors across the q^2 divide
- Spectral functions
- VDM as commonly understood....
- Lattice QCD
- And probably a host of other very important topics

This leads us to...

Discussion Focus and Ultimate Goals

1. Establish the nucleon excitation spectrum and reaction models with emphasis on the high-mass region and gluonic excitations;
2. Measure space-like and time-like baryonic transition form factors, and thereby quantify the role of the active degrees of freedom in the nucleon excitation spectrum;
3. Pin down the dressed-quark mass as a function of quark momentum, which will critically deepen our understanding of mass generation dynamics and emergence of quark-gluon confinement.
4. Provide the analysis tools to enable comparisons of future lattice QCD simulations with experimental results.

White Paper

Physics Opportunities with Meson Beams

[William J. Briscoe](#) (GW), [Michael Döring](#) (GW), [Helmut Haberzettl](#) (GW), [D. Mark Manley](#) (KSU), [Megumi Naruki](#) (Kyoto Univ.), [Igor I. Strakovsky](#) (GW), [Eric S. Swanson](#) (Univ. of Pittsburgh)

(Submitted on 26 Mar 2015)

Over the past two decades, meson photo- and electro-production data of unprecedented quality and quantity have been measured at electromagnetic facilities worldwide. By contrast, the meson-beam data for the same hadronic final states are mostly outdated and largely of poor quality, or even nonexistent, and thus provide inadequate input to help interpret, analyze, and exploit the full potential of the new electromagnetic data. To reap the full benefit of the high-precision electromagnetic data, new high-statistics data from measurements with meson beams, with good angle and energy coverage for a wide range of reactions, are critically needed to advance our knowledge in baryon and meson spectroscopy and other related areas of hadron physics. To address this situation, a state-of-the-art meson-beam facility needs to be constructed. The present paper summarizes unresolved issues in hadron physics and outlines the vast opportunities and advances that only become possible with such a facility.

Such as

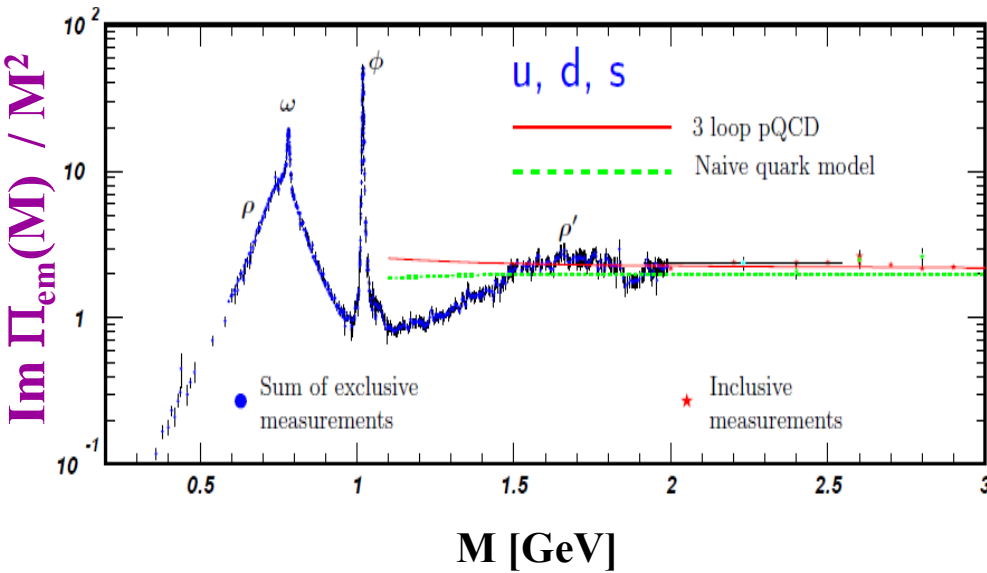


135 endorsers from 77 labs worldwide

Change in Degrees of Freedom

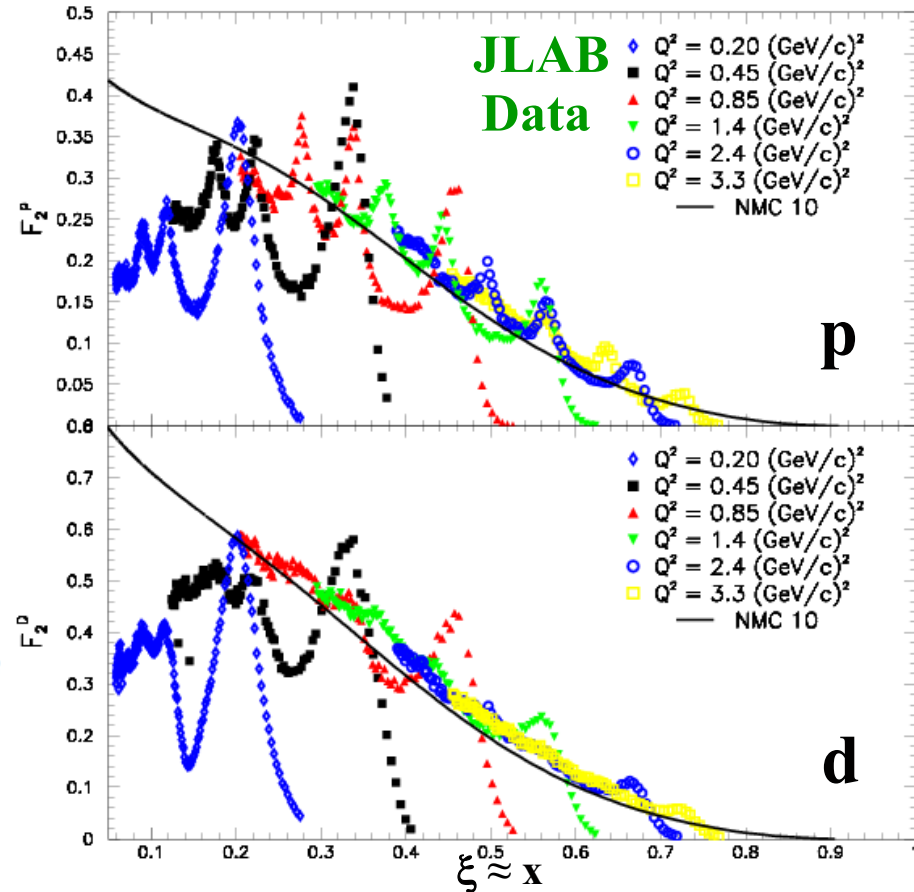
- As function of q^2

Timelike: $e^+e^- \rightarrow \text{hadrons}$



- $Q^2_{\text{dual}} \sim 2.5\text{-}3 \text{ GeV}^2$
- depends on channel?

Spacelike: F_2 -Structure Funct.



- average \rightarrow Quark-Hadron Duality
- lower onset- Q^2 in nuclei?

Some Questions to Ponder

1. How to compare Helicity Amplitudes between SL and TL?
2. Can the data in the SL region afford constraints for those in the TL regime? (e.g. Covariant Spectator Theory, Teresa Peña).
3. What is the relationship between the density matrix elements for SL \rightarrow TL? Again do they offer any constraints on the Helicity Amplitudes between the SL and TL regimes?
4. Will there be scaling in $q^2 > 0$ and $q^2 < 0$ (i.e. $Q^2 > 0$)?
5. Can we find a consistent *ab initio* approach for the QCD d.o.f. in determining the SL and TL transition FFs?
6. What role does the MB Cloud play? (again for SL and TL)? And how to separate? [Through comparing to other models?]
7. What are the relevant d.o.f. as a function of q^2 for the SL and TL regimes?

N^{*}STAR 2017

- ✓ Baryon spectrum through meson photoproduction
- ✓ Baryon resonances in experiments with hadron beams and in the e^+e^- collisions
- ✓ Baryon resonances in ion collisions and their role in cosmology
- ✓ Baryon structure through meson electroproduction, transition form factors, and time-like form factors
- ✓ Amplitude analyses and baryon parameter extraction
- ✓ Baryon spectrum and structure from first principles of QCD
- ✓ Advances in the modeling of baryon spectrum and structure
- ✓ Facilities and future projects
- ✓ Other topics related to N^* physics

There will be a dedicated session on space-like and time-like transition form factors.

August 20-23, 2017

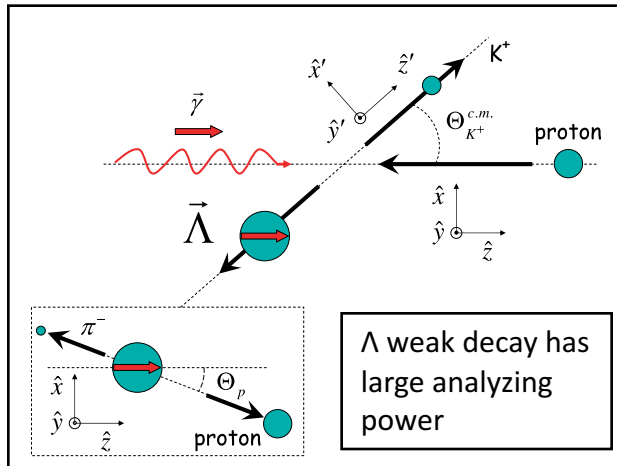
at the University of South Carolina, Columbia, SC

<http://nstar2017.physics.sc.edu/>

ThintKo Shukriya Buznyg TapadhLeat Kösönöm Murakoze aDank Grazi Nouari Grazie
 Blagodaram AsanteSana WaadMahadsantahay TapadhLeibh Takk Enkosi Bedankt Zikomo Mesi Chokrane Kiitos
 Matondo Mercé TesekkurEderim Rahmat Waita Rahmat Danyavaad Trugarez Aabhar BarakAllahufik Danyavaad Mesi Chokrane Kiitos
 Taiku TesekkurEderim Mammun Welalin TangioTumas Tanen Suliya Danyavaad BarakAllahufik Mesi Chokrane Kiitos
 Dakujem Betra Mamiin KeYalaboha Tanemirt Vinaka Tenki Gracias Barkal KurreSumanga GratiasAgimus
 Terimakasih Nizzik aji Aguyje Grandmercé Danyavadalu Sulpay Arigato Hvala Mahalo Gracinas DankluWel KamSahHamnida Camon Gracie
 NajsTuke Merdzi Akun Niringrazzjak Bayarlalaa Obrigado Saha Multumesc Camon Gracie
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 KamSahHamnida Camon Gracie
 Gracie

Additional Slides

Complete photoproduction experiments



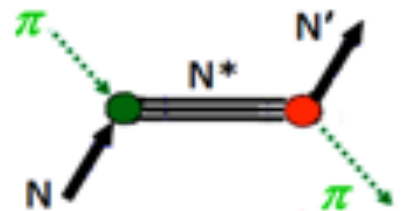
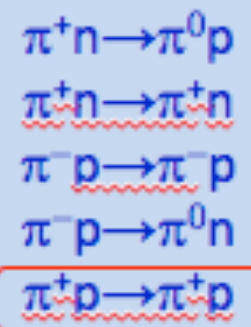
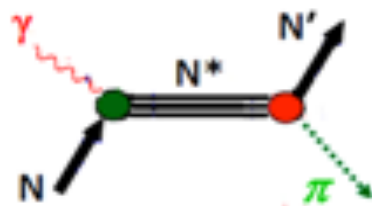
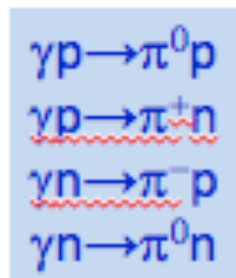
- Process described by **4** complex amplitudes
- **8** well-chosen measurements are needed to determine amplitude.
- Up to **16** observables measured directly
- **3** inferred from double polarization observables
- **13** inferred from triple polarization observables

Beam (P^γ)	Target (P^T)			Recoil (P^R)			Target (P^T) + Recoil (P^R)								
	x	y	z	x'	y'	z'	x'	x'	x'	y'	y'	y'	z'	z'	z'
unpolarized $d\sigma_0$	\hat{T}			\hat{P}			$\hat{T}_{x'}$		$\hat{L}_{x'}$	$\hat{\Sigma}$		$\hat{T}_{z'}$		$\hat{L}_{z'}$	
$P_L^\gamma \sin(2\phi_\gamma)$	\hat{H}		\hat{G}	$\hat{O}_{x'}$		$\hat{O}_{z'}$		$\hat{C}_{z'}$		\hat{E}		\hat{F}		$-\hat{C}_{x'}$	
$P_L^\gamma \cos(2\phi_\gamma)$	$-\hat{\Sigma}$		$-\hat{P}$			$-\hat{T}$	$-\hat{L}_{z'}$		$\hat{T}_{z'}$	$-d\sigma_0$		$\hat{L}_{x'}$		$-\hat{T}_{x'}$	
circular P_c^γ	\hat{F}		$-\hat{E}$	$\hat{C}_{x'}$		$\hat{C}_{z'}$		$-\hat{O}_{z'}$		\hat{G}		$-\hat{H}$		$\hat{O}_{x'}$	


A. Sandorfi, S. Hoblit, H. Kamano, T.-S.H. Lee, J.Phys. 38 (2011) 053001

Status of Data for Specific Reactions

- Measurements of final states involving single pseudoscalar meson & spin-1/2 baryon are particularly interesting due to simple interpretation.
- The reactions involving πN channels include:



- Only $\pi^+ p \rightarrow \pi^+ p$ corresponds to isospin 3/2 while rest of reactions is mixture of isospins 1/2 & 3/2.

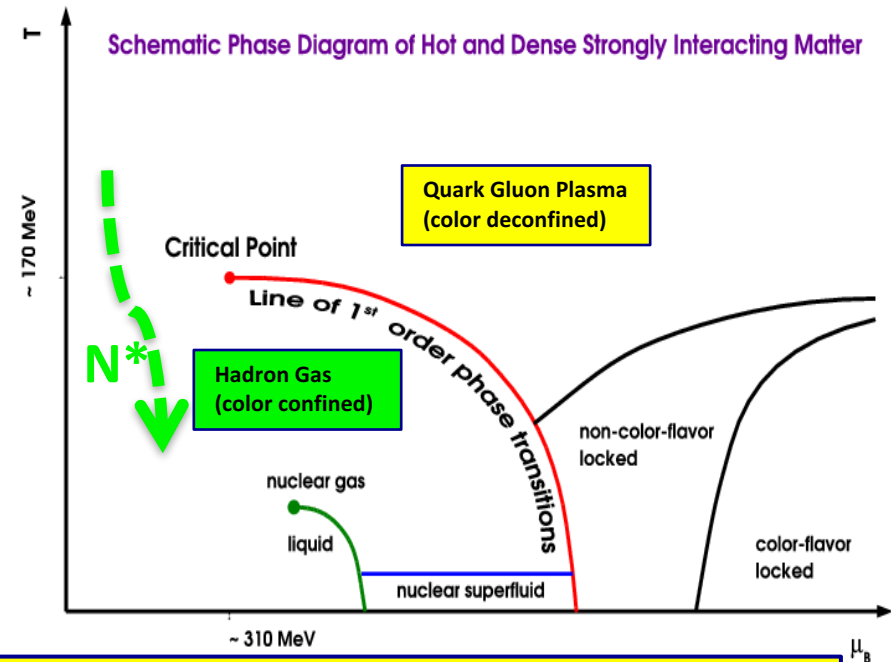
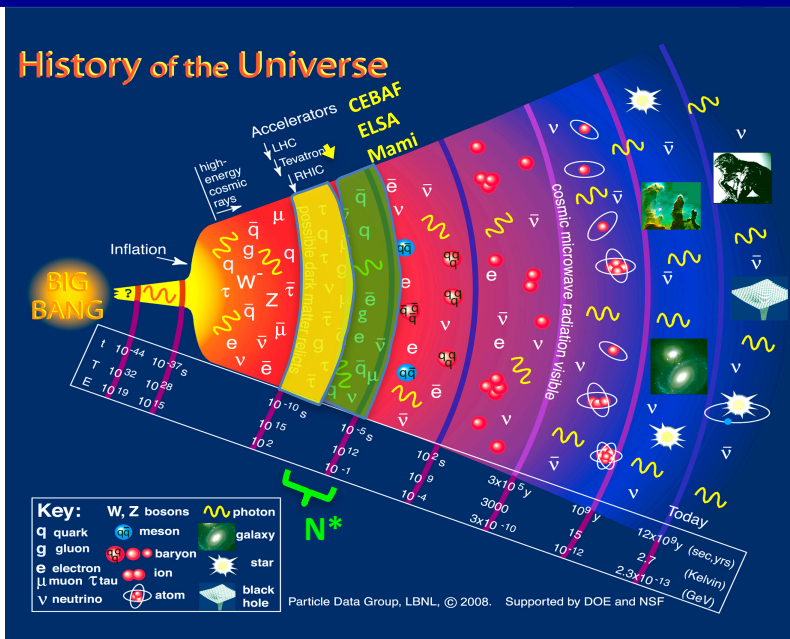
 Available data for πN elastic scattering are still **incomplete**.

- Measurements of **P**, **A**, & **R** observables (limited number of data available) are needed to construct truly unbiased **PW amplitudes**.



πN elastic scattering data have allowed establishment of **4^*** resonances.

N* in the History of the Universe



Dramatic events occur in the microsecond old Universe.

- The transition from the QGP to the baryon phase is dominated by excited baryons. A quantitative description requires more states than found to date => **missing baryons**.
- During the transition the quarks acquire **dynamical mass** and the **confinement of color** occurs.

$$J^\mu = \text{outgoing hadronic state} \left[\text{---} \circ J^\mu \text{---} \right] \text{incoming hadronic state}$$

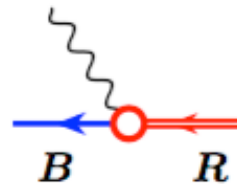
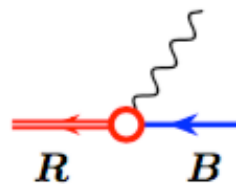
Photon may be real or virtual,
incoming or outgoing

All hadrons on-shell:

$$k_\mu J^\mu = 0 \quad (\text{necessary condition}) \quad \Rightarrow \quad \text{gauge invariance}$$



Transition currents:



B: Baryon
R: Resonance

Production Current

← time →

$$\begin{aligned}
 M^\mu &= \underbrace{\text{[s-channel diagram]}}_{s\text{-channel}} + \underbrace{\text{[u-channel diagram]}}_{u\text{-channel}} + \underbrace{\text{[t-channel diagram]}}_{t\text{-channel}} + \underbrace{\text{[interaction current diagram]}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

Generic expressions; summations over all possible intermediate states implied

- **s-channel term contains transition-current contributions**

Diagrams apply to spacelike process; for timelike process, read diagrams in time-reversed order

- **Entire production current must be gauge invariant**

Without it, wrong background contribution for extraction of form factors

- **Any approximation will likely destroy gauge invariance**

- **For a microscopic theory, it is **not** sufficient to fix $k_\mu M^\mu = 0$ on shell**



Introduction: the quark model (ctd.)

$$SU(6) : 6 \otimes 6 \otimes 6 = 56_S \oplus 70_M \oplus 70_M \oplus 20_A .$$

$$56 = {}^4 10 \oplus {}^2 8, \quad 70 = {}^2 10 \oplus {}^4 8 \oplus {}^2 8 \oplus {}^2 1, \quad 20 = {}^2 8 \oplus {}^4 1.$$

$\Delta(1620)1/2^-$	$\Delta(1700)3/2^-$	${}^2 10, L=1, S=1/2$
$N(1650)1/2^-$	$N(1700)3/2^-$	${}^4 8, L=1, S=3/2$
$N(1535)1/2^-$	$N(1520)3/2^-$	${}^2 8, L=1, S=1/2$

The prediction of the correct number of negative-parity states was one of great successes of the quark model!

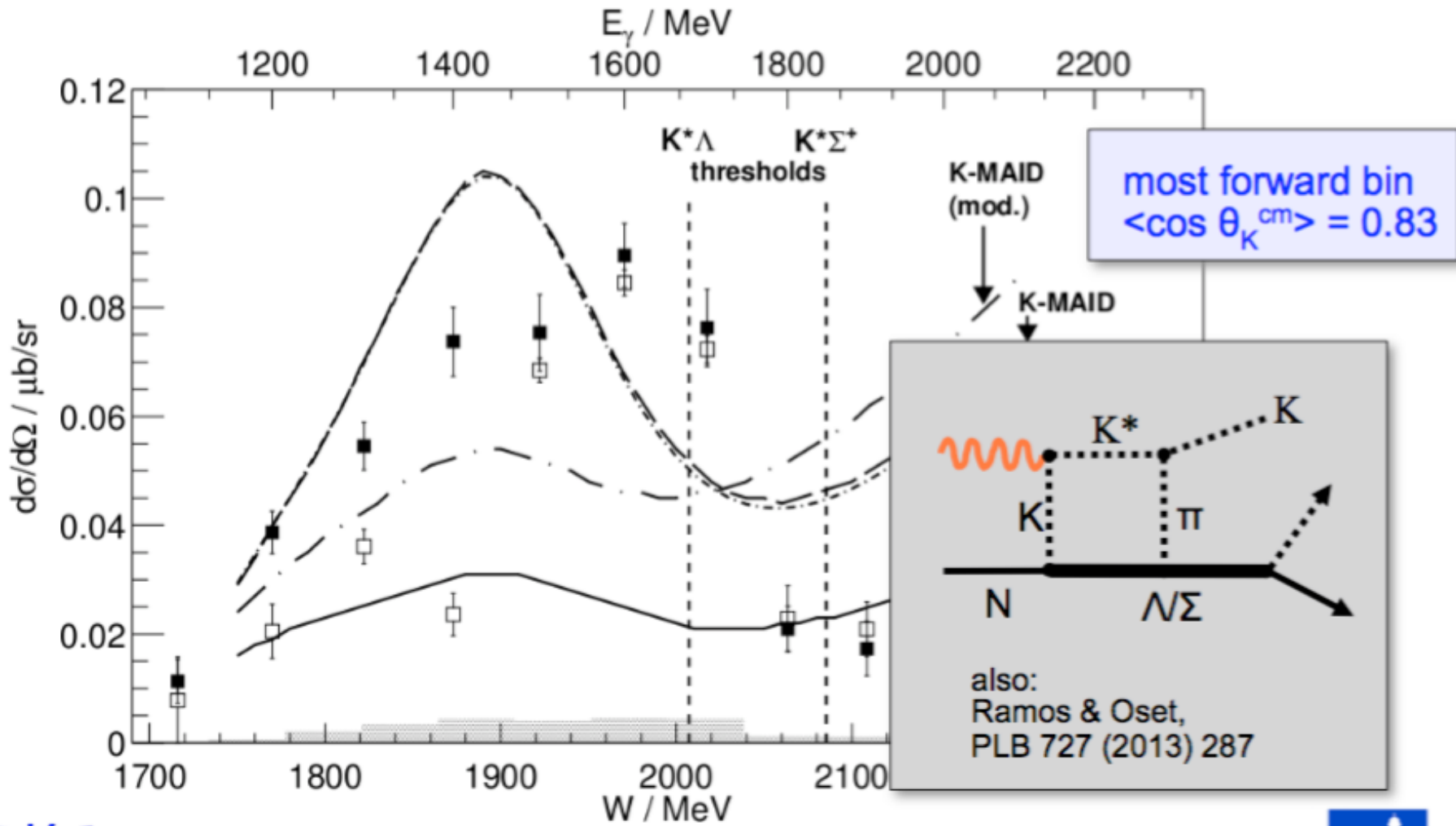
$\Delta(1910)1/2^+$	$\Delta(1920)3/2^+$	$\Delta(1905)5/2^+$	$\Delta(1950)7/2^+$	${}^4 10, L=2, S=3/2$
	$N(1720)3/2^+$	$N(1680)5/2^+$		${}^2 8, L=2, S=3/2$
	$\Delta(\text{xxx})3/2^+$	$\Delta(2000)5/2^+$		${}^2 10, L=2, S=1/2$
$N(1880)1/2^+$	$N(1900)3/2^+$	$N(1860)5/2^+$	$N(1990)7/2^+$	${}^4 8, L=2, S=3/2$
(BnGa)	$N(1960)3/2^+$	$N(2000)5/2^+$		${}^2 8, L=2, S=1/2$
$N(\text{xxx})1/2^+$	$N(\text{xxx})3/2^+$	20-plet!		${}^2 8, \ell_1=1, \ell_2=1, L=1, S=1/2$
$\Delta(1750)1/2^+$				${}^2 10, L=0, N=1, S=1/2$
$N(1710)1/2^+$	$N(1740)3/2^+$	(CLAS)		${}^2 8 ({}^4 8), L=0, N=1, S=1/2 (3/2)$
	$\Delta(1600)3/2^+$			${}^4 10, L=0, N=1, S=3/2$
$N(1440)1/2^+$				${}^2 8, L=0, N=1, S=1/2$

1. Missing resonances (ctd.)

New resonances!

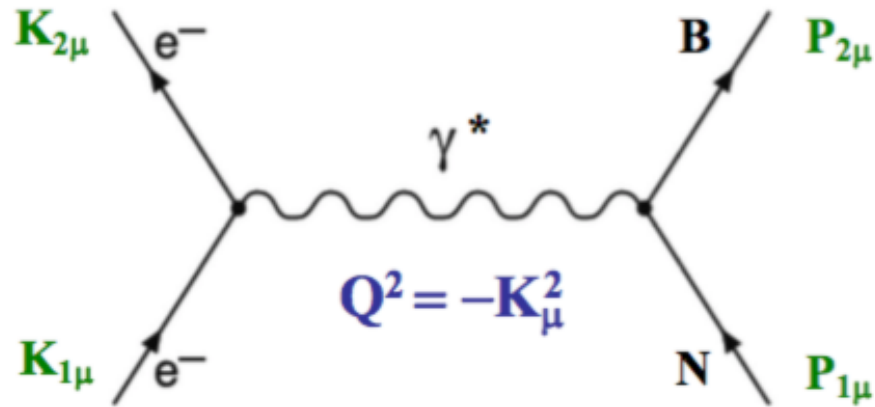
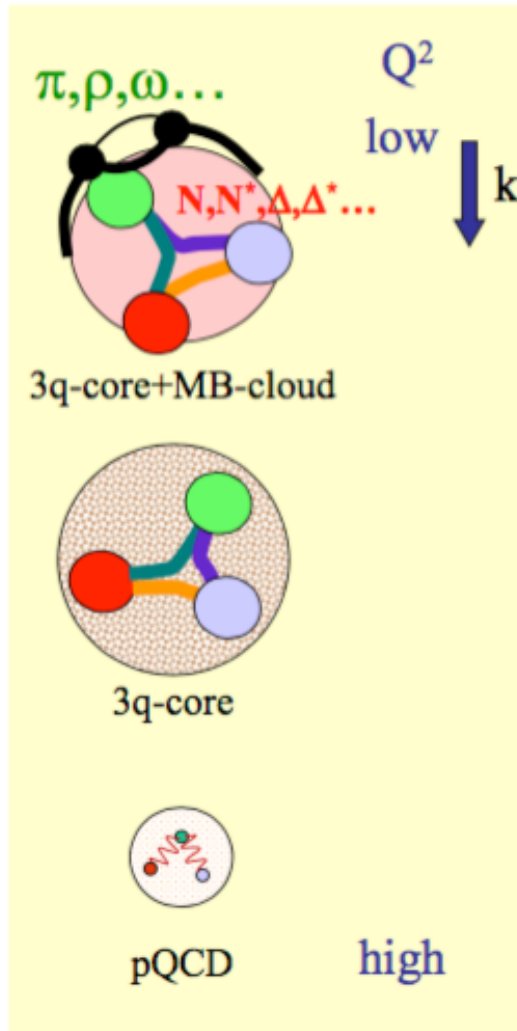
		RPP 2010	our analyses	RPP	GWU'06 (SAID)
New evidence from new analyses	N(1700)3/2 ⁻	***	***	***	no evidence
	N(1710)1/2 ⁺	***	***	***	no evidence
	N(1860)5/2 ⁺		*	**	
	N(1875)3/2 ⁻		***	***	no evidence
	N(1880)1/2 ⁺		** *	**	no evidence
	N(1895)1/2 ⁻		** ** *	**	no evidence
	N(1900)3/2 ⁺	**	*** *	***	no evidence
	N(1990)7/2 ⁺	**	**	**	no evidence
	N(2000)5/2 ⁺	**	**	**	no evidence
	N(2060)5/2 ⁻		***	**	no evidence
	N(2150)3/2 ⁻		**	**	no evidence
	Δ(1900)1/2 ⁻	*	*	**	no evidence
	Δ(1920)3/2 ⁺	***	***	***	no evidence
Δ(1940)3/2 ⁻	*	**	**	no evidence	

Too many resonances for quark-diquark models!



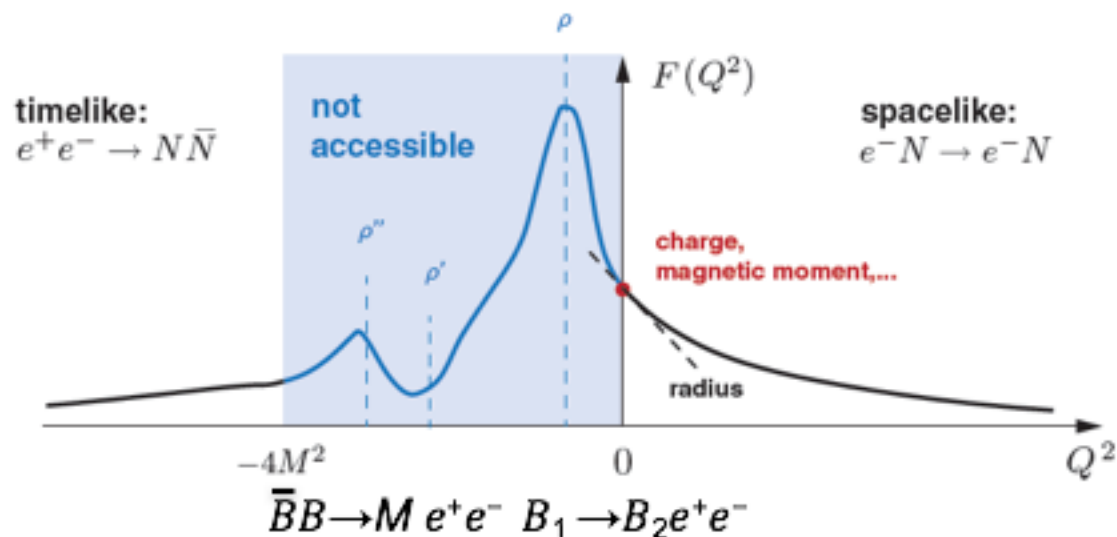
Hadron Structure with Electromagnetic Probes

- Study the structure of the nucleon spectrum in the domain where dressed quarks are the major active degree of freedom.
- Explore the formation of excited nucleon states in interactions of dressed quarks and their emergence from QCD.



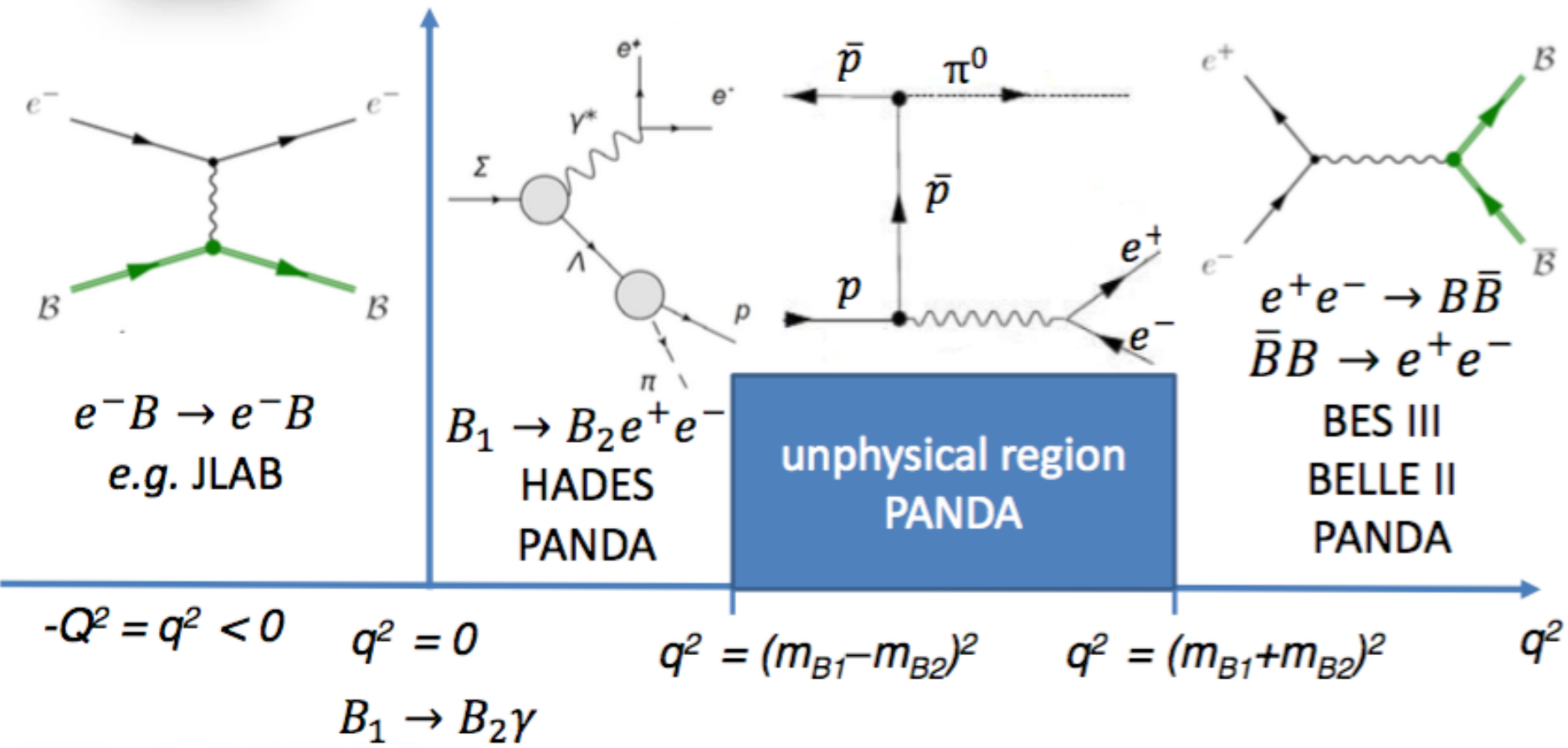
Radiative Hyperon Decays

- Timelike ($G_M(q^2)$, $G_E(q^2)$) complementary to Spacelike
- Low energy TL complementary to high energy TL (e.g. BES-III, CLEO)



Jim Ritman

Space-like vs. time-like FF's



Electromagnetic form factors of hyperons

to large extent
terra incognita

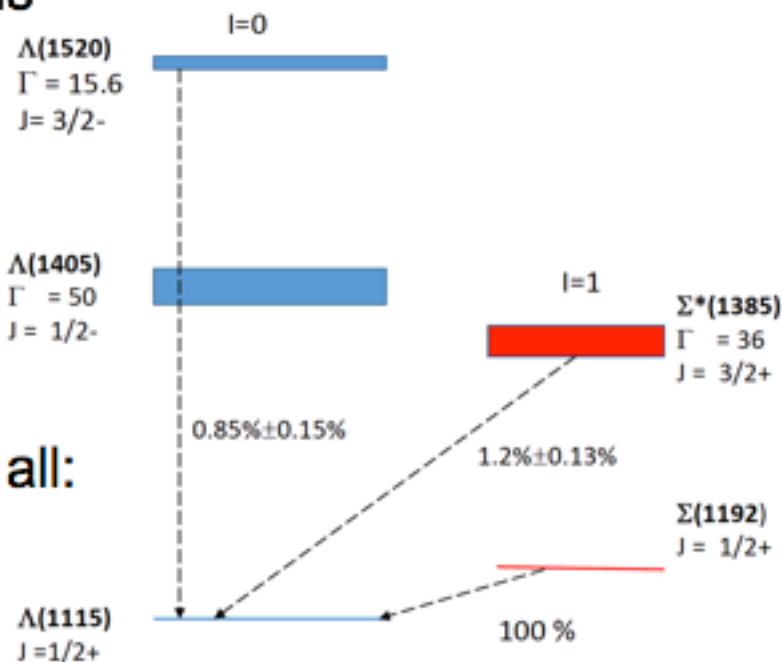
hic sunt dracones



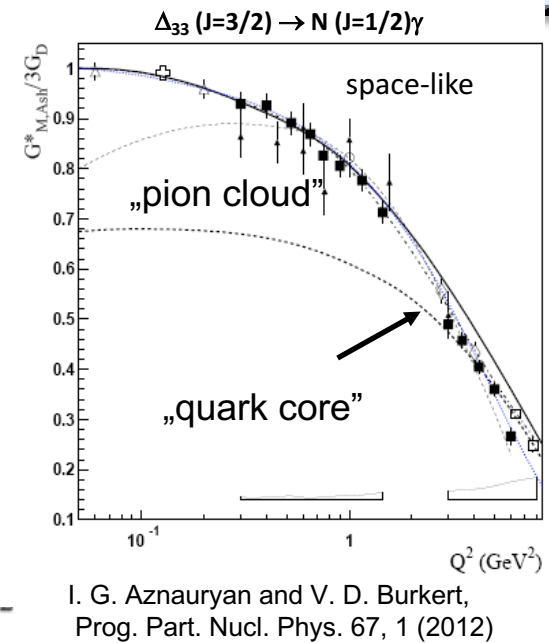
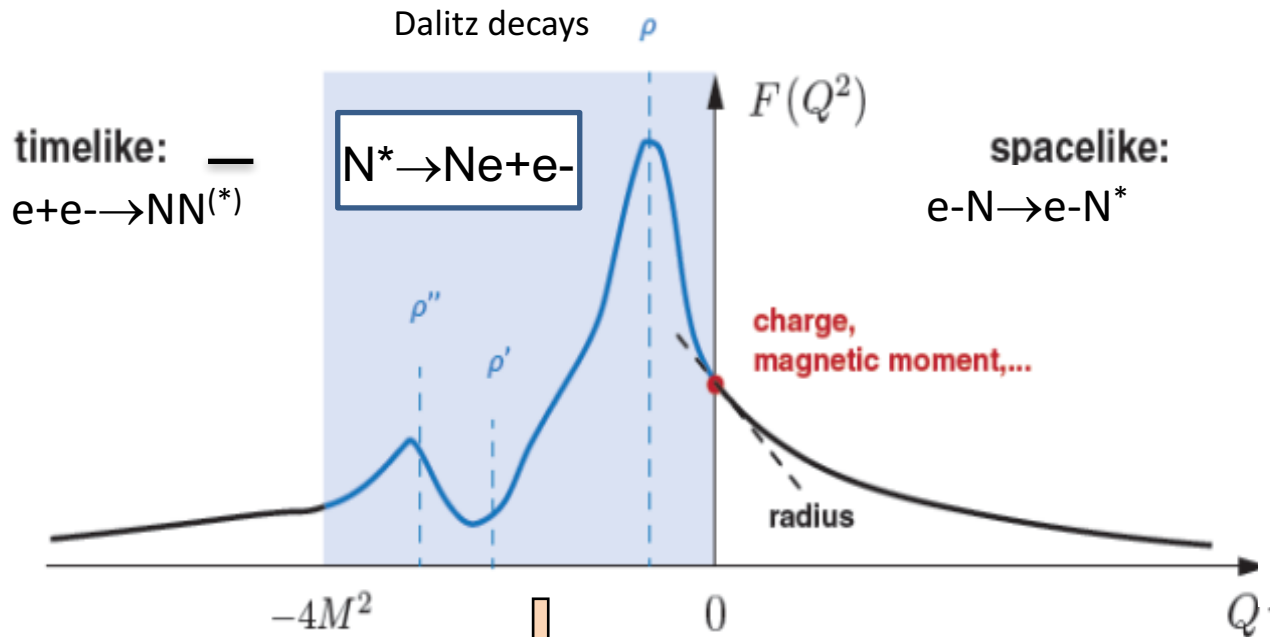
- electron-hyperon scattering complicated
- ↪ instead:
 - reactions $e^+ e^- \rightarrow \text{hyperon anti-hyperon } (Y_1 \bar{Y}_2) \rightsquigarrow \text{BESIII}$
 - ↪ form factors and transition form factors for **large** time-like $q^2 > (m_{Y_1} + m_{Y_2})^2$
(time-like means $q^2 > 0$, i.e. energy transfer $>$ momentum transfer)
 - decays $Y_1 \rightarrow Y_2 e^+ e^- \rightsquigarrow \text{HADES+PANDA}$
 - ↪ transition form factors for **small** time-like $q^2 < (m_{Y_1} - m_{Y_2})^2$

Radiative Hyperon Decays

- Decays to real and virtual photons sensitive to baryon structure:
- $\Sigma \rightarrow \Lambda \gamma$: test of CP violation
- Hyperon $\rightarrow \Lambda e e$ not measured at all: strong effects of vector mesons expected



Electromagnetic structure of baryons



$$d\Gamma(R \rightarrow N l^+ l^-) = \frac{\alpha}{3\pi} d\Gamma(R \rightarrow N \gamma^*) \left[1 - \frac{4m_l^2}{q^2}\right]^{1/2} \left[1 + \frac{2m_l^2}{q^2}\right] \frac{dq^2}{q^2}$$

for example for $J=1/2$

$$\Gamma(R \rightarrow N \gamma^*) = H(m, M) \left(2|G_{M/E(M)}^\pm|^2 + \frac{M}{m} |G_C(M)|^2 \right)$$

„QED”

$R \rightarrow N \gamma^*$: em. Transition Form Factors

$R (J \geq 3/2)$: (3) $G_M (q^2)$, $G_E (q^2)$, $G_C (q^2)$

$R (J=1/2)$: (2) $G_{M/E} (q^2)$, $G_C (q^2)$

.. or covariant eTFF

M. I. Krivoruchenko, et. al
Annals Phys. 296, 299 (2002)

The Decay Angular Distribution

The complete angular distribution $W(\cos\theta, \phi, \Phi)$ is given by,

$$W(\cos\theta, \phi, \Phi) = W^0(\cos\theta, \phi, \rho_{\alpha\beta}^0) - P_\gamma \cos 2\Phi W^1(\cos\theta, \phi, \rho_{\alpha\beta}^1) - P_\gamma \sin 2\Phi W^2(\cos\theta, \phi, \rho_{\alpha\beta}^2),$$

where

$$W^0(\cos\theta, \phi, \rho_{\alpha\beta}^0) = \frac{3}{4\pi} \left[\frac{1}{2} \sin^2\theta + \frac{1}{2} (3 \cos^2\theta - 1) \rho_{00}^0 - \sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos\phi - \rho_{1-1}^0 \sin^2\theta \cos 2\phi \right]$$

$$W^1(\cos\theta, \phi, \rho_{\alpha\beta}^1) = \frac{3}{4\pi} \left[\rho_{11}^1 \sin^2\theta + \rho_{00}^1 \cos^2\theta - \sqrt{2} \operatorname{Re} \rho_{10}^1 \sin 2\theta \cos\phi - \rho_{1-1}^1 \sin^2\theta \cos 2\phi \right]$$

$$W^2(\cos\theta, \phi, \rho_{\alpha\beta}^2) = \frac{3}{4\pi} \left[\sqrt{2} \operatorname{Im} \rho_{10}^2 \sin 2\theta \sin\phi + \operatorname{Im} \rho_{1-1}^2 \sin^2\theta \sin 2\phi \right],$$

and θ and ϕ are the polar and azimuthal angles of the normal to decay plane of $\omega \rightarrow \pi^+\pi^-\pi^0$ wrt to the quantization axis, and P_γ is the degree of linear polarization.

By using *linearly polarized* photons, we are sensitive to six additional density matrix elements, providing a total of nine constraints for extracting the helicity amplitudes.

Towards a new extraction of N^* couplings in the 2π channel and pioneering studies of time-like electromagnetic transitions (HADES)

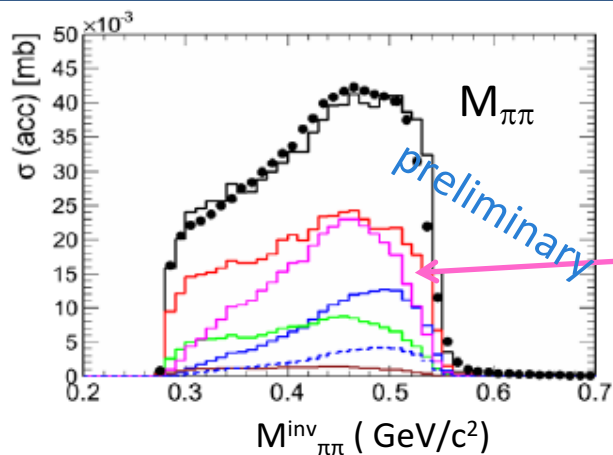
Goals: New data for baryon spectroscopy

- **Hadronic channels ($\pi\pi$)** : Partial Wave Analysis with **Bonn-Gatchina model (A. Sarantsev)**
- 4 data samples from HADES ($\pi^-p \rightarrow n\pi^+\pi^-$ / $\pi^-p \rightarrow \pi^-\pi^0 p$) + photon and pion database
→ e.g. **$N(1520)$ branching ratio to $\Delta\pi$, ρN , σN**
- **Electromagnetic channels (e^+e^-)** *Very first information for e.m. transitions in time-like region*

Partial Wave Analysis $\pi^-p \rightarrow n\pi^+\pi^-$

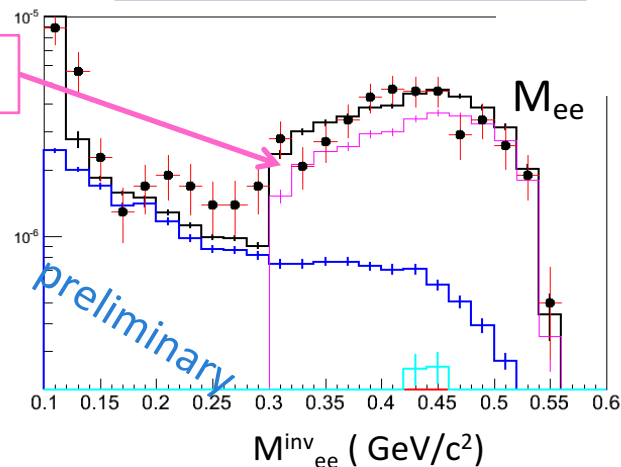
$\sqrt{s}=1.49 \text{ GeV}/c^2$

Quasi-free $\pi^-p \rightarrow ne^+e^-$



$\rho \rightarrow e^+e^-$

$\rho \rightarrow \pi^+\pi^-$
($I=1$)



Total

$\pi^0 \rightarrow e^+e^-\gamma$

$\eta \rightarrow e^+e^-\gamma$

$N^0(1520) \rightarrow ne^+e^-$

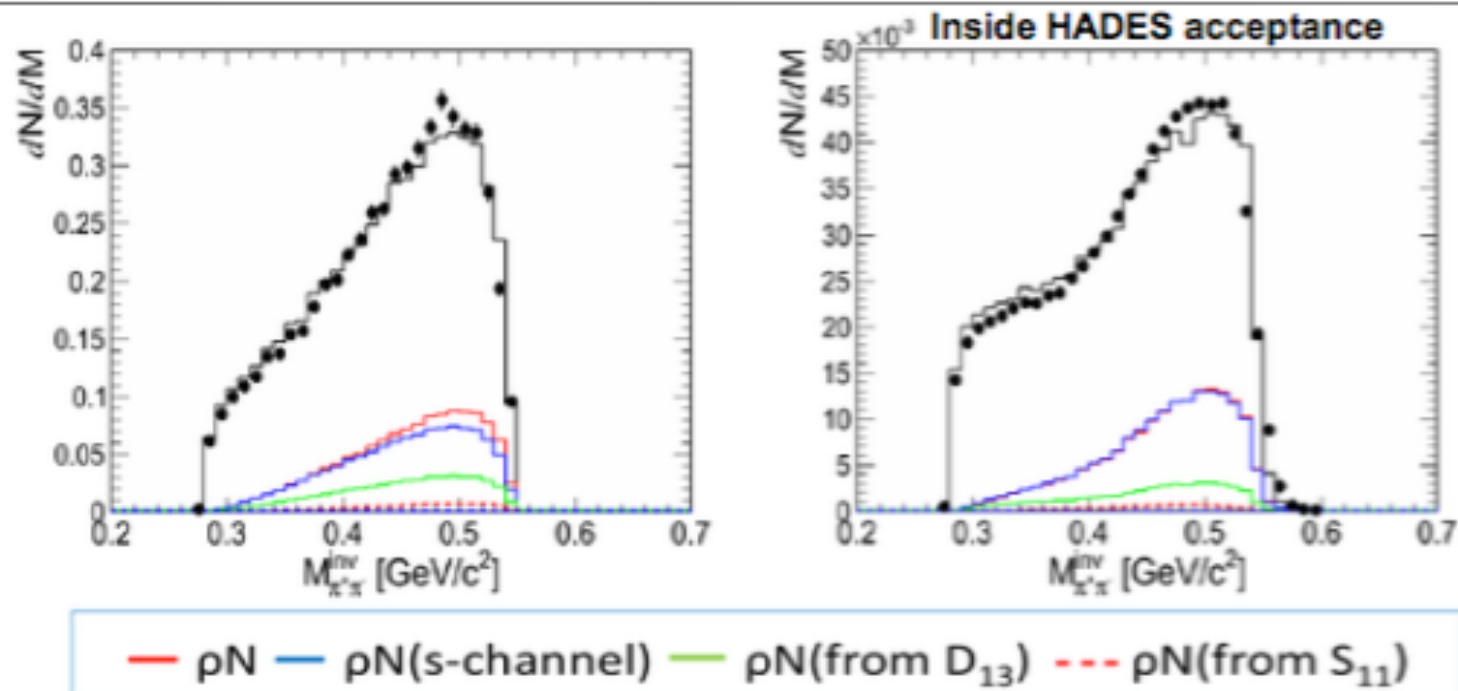
$\Delta^0(1232) \rightarrow ne^+e^-$

Impact on:

- Space-like transition form factors extracted from $ep \rightarrow e'N\pi\pi$ (CLAS data)
- understand the role of ρ meson in time-like e.m. baryonic transitions (HADES data)
- medium effects (ρ coupling to baryon resonances)

PWA $\pi^+\pi^-$ inv. mass ρ contribution

$\pi p \rightarrow \pi^+\pi^- n$ at 0.69 GeV/c



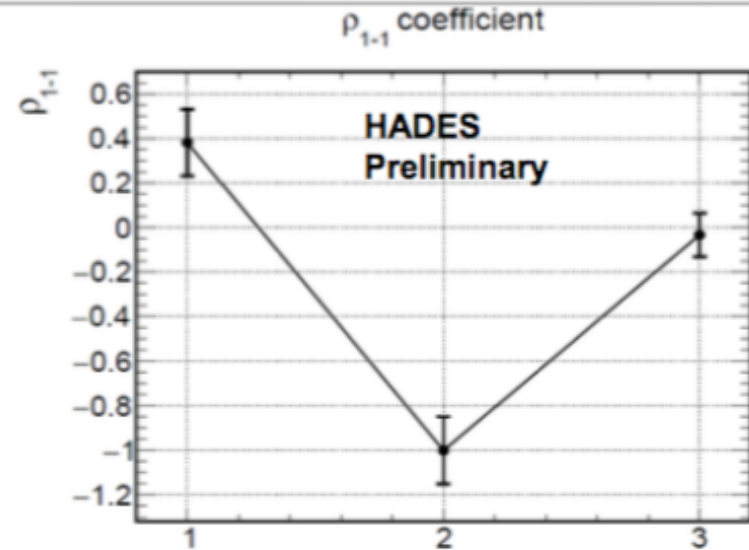
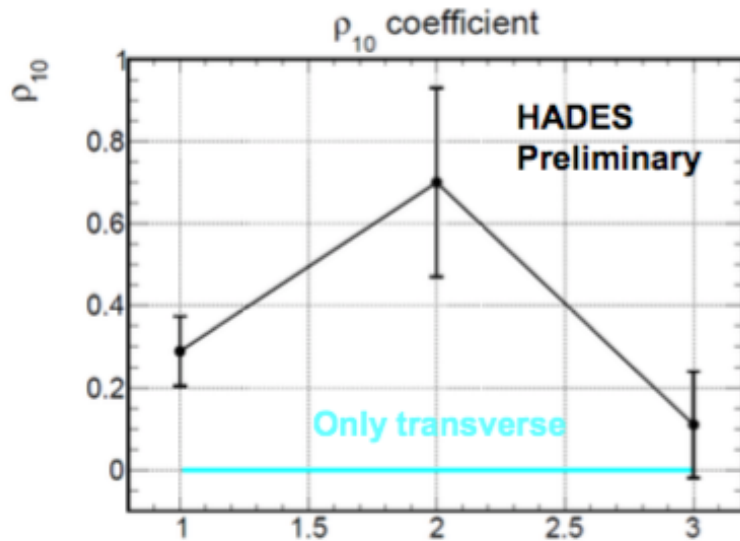
$N(1520)D_{13}$ coupling to ρN : 12 %

Total ρN : 1.3 mb

- Dominated by s-channel
- Resonant $D_{13}(1520)$ production
- Strong interferences between $1/2^-$ states with isospin $1/2$ and $3/2$

Model predictions

Comparison with data



- Model independent statements: transverse photons give $\rho_{11}=1/2$, $\rho_{10}=0$
- Data indicate significant contribution of longitudinal virtual photons, especially for $\cos \theta_{cm}$ in $[-1,0]$ and $[0.5,1]$.
- Consistent with pure contribution of $N(1520)$
- Points to a too large $N(1440)$ contribution (also supported by PWA of $\pi^-p \rightarrow n\pi^+\pi^-$ channel)
- Effects of non-resonant terms to be studied

What are the Density Matrix Elements?

Density matrix elements, are related to the *helicity amplitudes*.

$$(\rho^0, \rho^n) = \mathbf{H} \left(\frac{1}{2} \mathbf{I}, \frac{1}{2} \sigma^n \right) \mathbf{H}^\dagger \quad n = 1, 2, 3$$

where \mathbf{H} is the helicity amplitude matrix, and σ^n are the Pauli spin matrices.

The *density matrix elements* can be written as a sum of bilinear combinations of the *helicity amplitudes*. (The summation over the nucleon spins is implied). For the unpolarized case:

$$\rho_{\lambda_p \lambda'_p}^0 = \sum_{\lambda_\gamma \lambda''_\gamma} H_{\lambda_p \lambda_\gamma} H_{\lambda_p \lambda''_\gamma}^*, \quad \lambda_\gamma, \lambda''_\gamma = 1, -1$$

Some Examples

ρ_{00}^0 measures the intensity of the helicity flip by one unit at the $\gamma\omega$ vertex, i.e., $\rho_{00}^0 \sim |H_{01}|^2 + |H_{0-1}|^2$.

ρ_{1-1}^0 measures the interference of nonflip and double-flip amplitudes, i.e., $\rho_{1-1}^0 \sim H_{11}H_{-11}^* + H_{1-1}H_{-1-1}^*$.

For unnormalized density matrix elements:

$$\begin{aligned}\rho_{11} &= \frac{H_{xx} + H_{yy}}{2} & \rho_{00} &= H_{zz} \\ \rho_{10} &= \frac{1}{\sqrt{2}} [\mathbf{Re}(-H_{xz} - iH_{yz})] = \frac{1}{\sqrt{2}} [\mathbf{Re}(-H_{xz}) + \mathbf{Im}H_{yz}] \\ \rho_{1-1} &= \frac{H_{yy} - H_{xx}}{2}\end{aligned}$$

For the electroproduction:

$$\begin{aligned}\frac{d\sigma_T}{d\Omega_\pi} &= \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \frac{H_{xx} + H_{yy}}{2} & \frac{d\sigma_L}{d\Omega_\pi} &= \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} H_{zz} \\ \frac{d\sigma_{TT}}{d\Omega_\pi} &= \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \frac{H_{xx} - H_{yy}}{2} & \frac{d\sigma_{TL}}{d\Omega_\pi} &= \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} (-\mathbf{Re}H_{xz}) \\ & & \frac{d\sigma_{TL'}}{d\Omega_\pi} &= \frac{|\vec{k}_{\pi N}|}{|\vec{k}_{\gamma^* N}|} \mathbf{Im}H_{yz}\end{aligned}$$

Lepton angular distributions

E. Speranza, B. Friman, M. Zetyeni
Physics Letters B 764 (2017) 282–288

A.Sarantsev to be published

Spin density
matrix

$$\sum_{\text{pol}} |\mathcal{M}|^2 = \sum_{\lambda, \lambda'} \rho_{\lambda, \lambda'}^{\text{had}} \rho_{\lambda', \lambda}^{\text{lep}}$$

$$\rho_{\lambda', \lambda}^{\text{had}} = \frac{e^2}{q^4} \epsilon^\mu(k, \lambda') H_{\mu\nu} \epsilon^\nu(k, \lambda)^* \quad \text{hadron decay to } \gamma^*$$

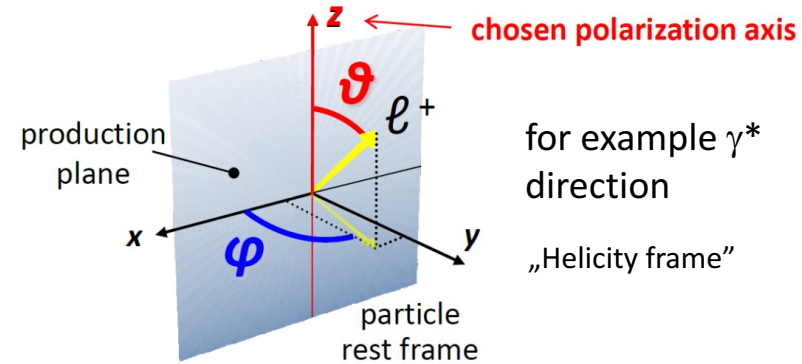
$$\rho_{\lambda', \lambda}^{\text{lep}} = \epsilon^\mu(k, \lambda') l_{\mu\nu} \epsilon^\nu(k, \lambda)^* \quad \text{QED: } \gamma^* \text{ decay to } e^+e^-$$

$$\rho^{\text{dec}} = \frac{1}{2} |\mathbf{k}|^2 \begin{pmatrix} 1 + \cos^2 \theta & \sqrt{2} \sin \theta \cos \theta e^{i\phi} & \sin^2 \theta e^{2i\phi} \\ \sqrt{2} \sin \theta \cos \theta e^{-i\phi} & 2 \sin^2 \theta & -\sqrt{2} \sin \theta \cos \theta e^{i\phi} \\ \sin^2 \theta e^{-2i\phi} & -\sqrt{2} \sin \theta \cos \theta e^{-i\phi} & 1 + \cos^2 \theta \end{pmatrix}$$

$\mu(k, -1) = \frac{1}{\sqrt{2}}(0, 1, -i, 0)$, transverse
 $\epsilon^\mu(k, 0) = (0, 0, 0, 1)$, longitudinal
 $\mu(k, +1) = -\frac{1}{\sqrt{2}}(0, 1, i, 0)$, transverse

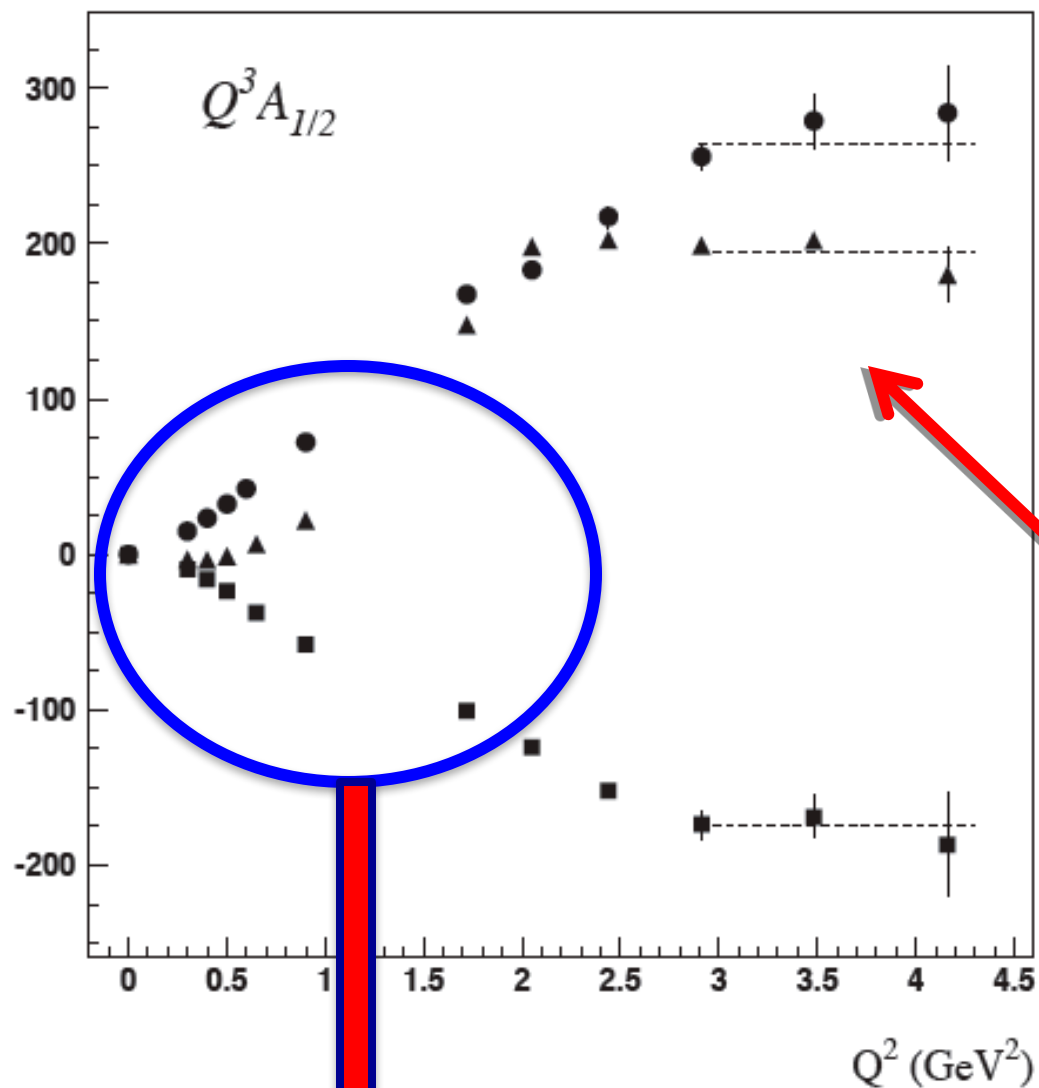
for example for $\Delta \rightarrow \text{Ne} + e^-$ at small q^2 $|M|^2 \sim 1 + B \cos^2 \theta, B \sim 1$

Lepton emission
angles



photon polarisation vectors

$A_{1/2}$ Electrocouplings



▲ $P_{11}(1440)$

■ $D_{13}(1520)$

● $S_{11}(1535)$

scaled by Q^3

improved empirical parametrization

example of a fruitful ECT* discussion

at ECT* Workshop, Trento, October, 12-16, 2015

Nucleon Resonances: From Photoproduction to High Photon Virtualities

L.T. Few-Body Syst 57 (2016) 1087 - problem with Siegert Theorem discussed

G. Ramalho, Phys. Lett. B 759 (2016) 126 - problem solved for S11(1535)

G. Ramalho, Phys. Rev. D93 (2016) 113012 - problem solved for P33(1232) and D13(1520)

modified MAID-SG parametrization for S11(1535):

$$A_{1/2} = a_0 \left(1 + a_1 Q^2 \right) e^{-a_4 Q^2}, \quad (19)$$

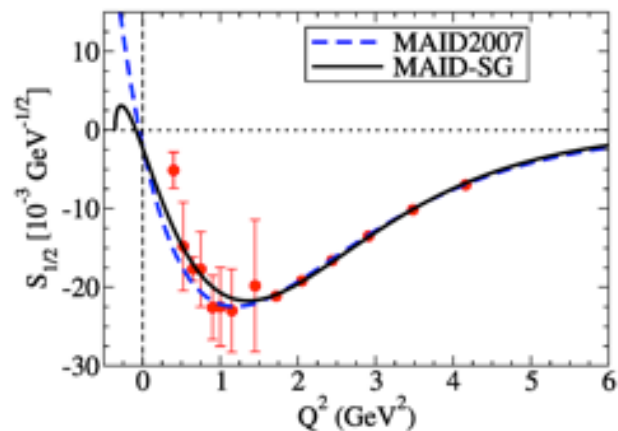
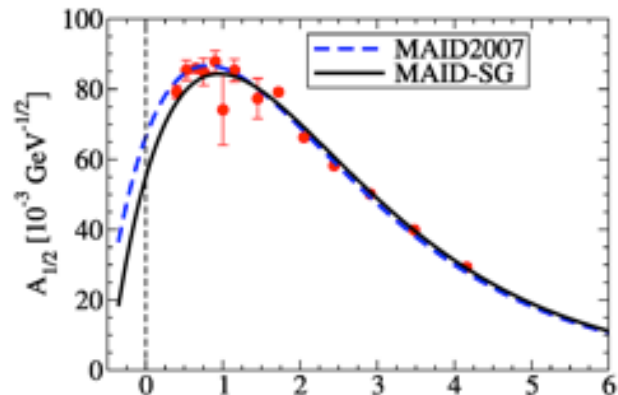
$$S_{1/2} = \frac{2M_R |\mathbf{q}|}{Q_+^2} s'_0 \left(1 + s_1 Q^2 + s_2 Q^4 \right) e^{-s_4 Q^2}, \quad (20)$$

$$|\mathbf{q}| = \frac{\sqrt{Q_+^2 Q_-^2}}{2M_R}, \quad \text{where } Q_{\pm}^2 = (M_R \pm M)^2 + Q^2.$$

improved empirical parametrization for $N(1535)1/2^- S_{11}$

G. Ramalho, Phys. Lett. B 759 (2016) 126

helicity form factors: $A_{1/2}(Q^2)$, $S_{1/2}(Q^2)$



Sachs form factors: $G_E(Q^2)$, $G_C(Q^2)$

