

Insights into strong forces through GPDs

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Outline

- **Introduction**

hard-exclusive reactions \rightarrow GPDs(x, ξ, t)

\leftrightarrow tomography, Ji sum rule + more than that

$\underbrace{\hspace{10em}}_{\xi=0}$ $\underbrace{\hspace{10em}}_{\xi \neq 0}$

- **Energy-momentum tensor & D -term**

last unknown global property(!)

stress tensor, strong forces, stability

- **Applications**

pictorial: insights in strong forces

principle: test correctness of models & effective approaches

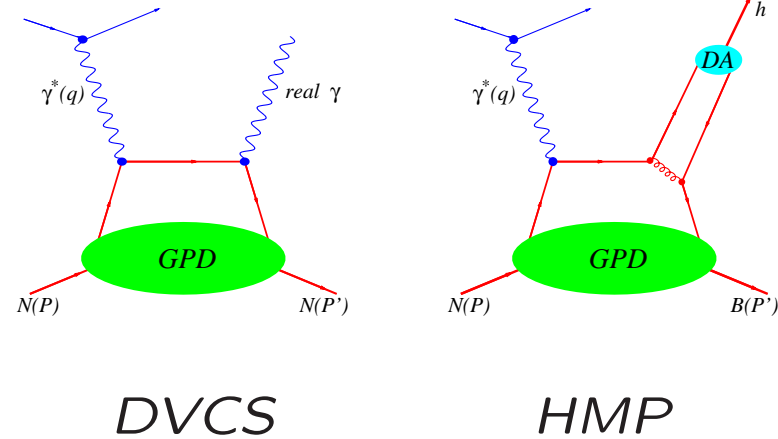
practical: hard-exclusive reactions at JLab \rightarrow $c\bar{c}$ pentaquark spectroscopy at LHCb

- **Outlook**

Introduction

- hard-exclusive reactions factorization, access to **GPDs**

Ji; Radyushkin; Collins, Frankfurt, Strikman



$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle N'(p') | \bar{\psi}_q(-\frac{\lambda n}{2}) n_\mu \gamma^\mu [-\frac{\lambda n}{2}, \frac{\lambda n}{2}] \psi_q(\frac{\lambda n}{2}) | N(p) \rangle$$

$$= \bar{u}(p') \left[n_\mu \gamma^\mu H^q(x, \xi, t) + \bar{u}(p') \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M_N} E^q(x, \xi, t) \right] u(p)$$

- one day: we will know the GPDs.
- what will we learn?

definitions for completeness:
 $\xi = (n \cdot \Delta) / (n \cdot P)$, $t = \Delta^2$
 $P = \frac{1}{2}(p' + p)$, $\Delta = p' - p$
 $n^2 = 0$, $n \cdot P = 2$, $k = xP$
 renormalization scale μ
 analog gluon GPDs

Will learn a lot!

- GPDs generalize form factors, PDFs

$$\int dx H^q(x, \xi, t) = F_1^q(t)$$

$$\lim_{\Delta \rightarrow 0} H^q(x, \xi, t) = f_1^q(x)$$

- use impact parameter space
→ tomography (Ralston, Pire, Burkardt)

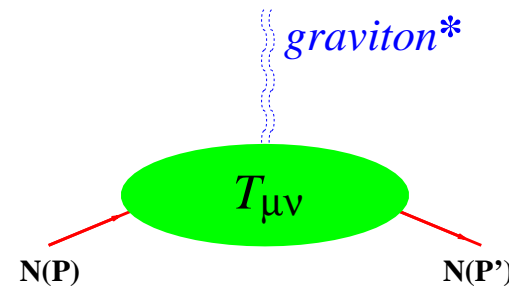
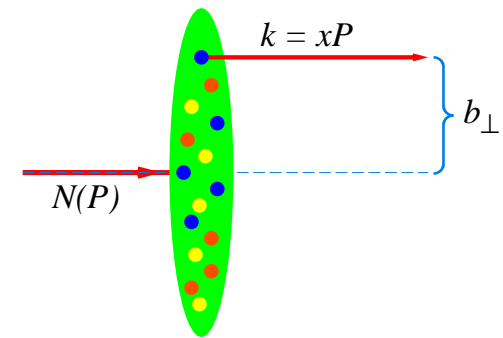
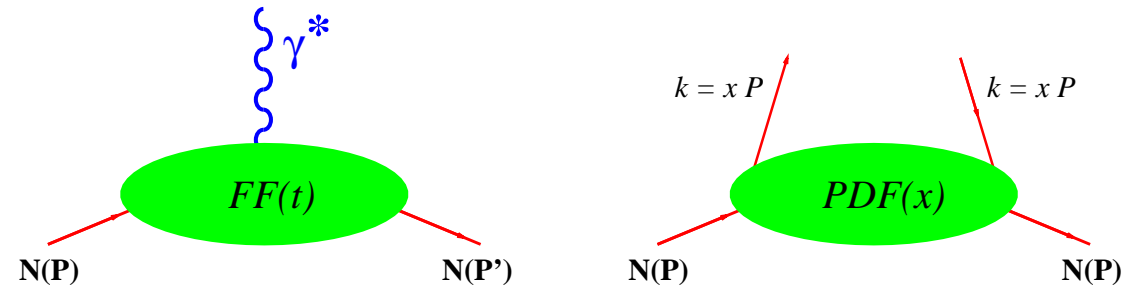
$$H^q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} \left[\lim_{\xi \rightarrow 0} H^q(x, \xi, t) \right] e^{i \Delta_\perp b_\perp}$$

- polynomiality → access to **gravitational form factors**

$$\int dx x H^q(x, \xi, t) = A^q(t) + \xi^2 D^q(t)$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - \xi^2 D^q(t)$$

- and **gravity** couples to **energy momentum tensor**
probably most fundamental quantity



Energy-momentum tensor (EMT)

- important: do you know introductory QFT text books which *do not* discuss EMT in first chapters?*
- generators of Poincaré group \rightarrow matrix elements \rightarrow mass, spin, D-term
(in introductory books: free fields, solvable, instructive) ! ! !?
- even if not solvable, studies of EMT insightful
prominent example: **Ji sum rule** PRL 78 (1997) 610

$$\int dx x \left(H^q(x, \xi, t) + E^q(x, \xi, t) \right) = A^q(t) + B^q(t) \xrightarrow{t \rightarrow 0} 2J^q(0)$$

notice: ξ -dependence
not explored, D -term is
“first” if $\xi \neq 0$ considered:
 $\int dx x H(x, \xi, t) = A(t) + \xi^2 D(t)$

* interestingly, advanced QFT books discuss EMT in later chapters: *trace anomaly*

$$\hat{T}_\mu^\mu \equiv \frac{\beta}{2g} F^{\mu\nu} F_{\mu\nu} + (1 + \gamma_m) \sum_q m_q \bar{\psi}_q \psi_q \quad \text{Adler, Collins, Duncan, PRD15 (1977) 1712;}$$

Nielsen, NPB 120, 212 (1977); Collins, Duncan, Joglekar, PRD 16, 438 (1977)

definition of nucleon EMT form factors

$$\langle P' | \hat{T}_{\mu\nu}^{q,g} | P \rangle = \bar{u}(p') \left[\begin{aligned} & \mathbf{A}^{q,g}(t) \frac{\gamma_\mu P_\nu + \gamma_\nu P_\mu}{2} \\ & + \mathbf{B}^{q,g}(t) \frac{i(P_\mu \sigma_{\nu\rho} + P_\nu \sigma_{\mu\rho}) \Delta^\rho}{4M_N} \\ & + \mathbf{D}^{q,g}(t) \frac{\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2}{4M_N} \pm \bar{c}(t) g_{\mu\nu} \end{aligned} \right] u(p)$$

- $\hat{T}_{\mu\nu}^q$ and $\hat{T}_{\mu\nu}^g$ each gauge-invariant (not conserved)
- total EMT $\hat{T}_{\mu\nu} = \hat{T}_{\mu\nu}^q + \hat{T}_{\mu\nu}^g$ is conserved: $\partial_\mu \hat{T}^{\mu\nu} = 0$
- constraints: **mass** $\Leftrightarrow A^q(0) + A^g(0) = 1$ (100% of nucleon momentum carried by quarks + gluons)
spin $\Leftrightarrow B^q(0) + B^g(0) = 0$ (i.e. $J^q + J^g = \frac{1}{2}$ nucleon spin due to quarks + gluons)*
- property: **D-term** $\Leftrightarrow D^q(0) + D^g(0) \equiv \mathbf{D} \rightarrow$ *fundamental quantity!*
but unconstrained!
Unknown!

* also expressed as: vanishing of total gravitomagnetic moment

notation: $A^q(t) = M_2^q(t)$, $A^q(t) + B^q(t) = 2J^q(t)$, $D^q(t) = \frac{4}{5}d_1^q(t) = \frac{1}{4}C^q(t)$ or $C^q(t)$

Last global unknown: How do we learn about nucleon?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em: $\partial_\mu J_{\text{em}}^\mu = 0 \quad \langle N' | J_{\text{em}}^\mu | N \rangle \longrightarrow Q, \mu, \dots$

weak: PCAC $\quad \langle N' | J_{\text{weak}}^\mu | N \rangle \longrightarrow g_A, g_p, \dots$

gravity: $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0 \quad \langle N' | T_{\text{grav}}^{\mu\nu} | N \rangle \longrightarrow M, J, D, \dots$

| | | | |
|--------------------------|---------------------|---|--------------------------------------------|
| basic global properties: | Q_{prot} | = | $1.602176487(40) \times 10^{-19} \text{C}$ |
| | μ_{prot} | = | $2.792847356(23) \mu_N$ |
| | g_A | = | $1.2694(28)$ |
| | g_p | = | $8.06(0.55)$ |
| | M | = | $938.272013(23) \text{ MeV}$ |
| | J | = | $\frac{1}{2}$ |
| | D | = | ? |

and more: t -dependence
parton structure, etc

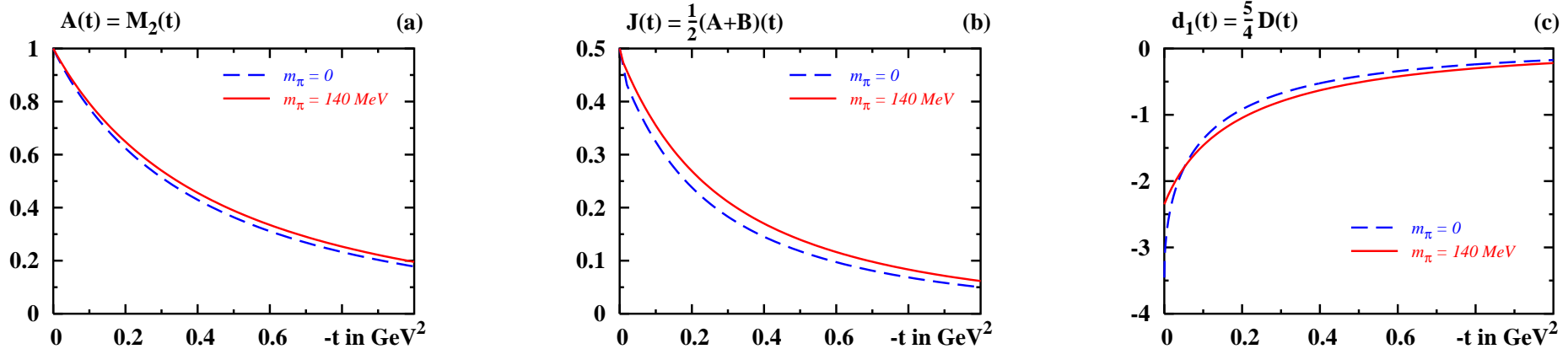
...
...

$\hookrightarrow D = \text{"last" global unknown}$
which value does it have?
what does it mean?

EMT form factors & D -term of nucleon:

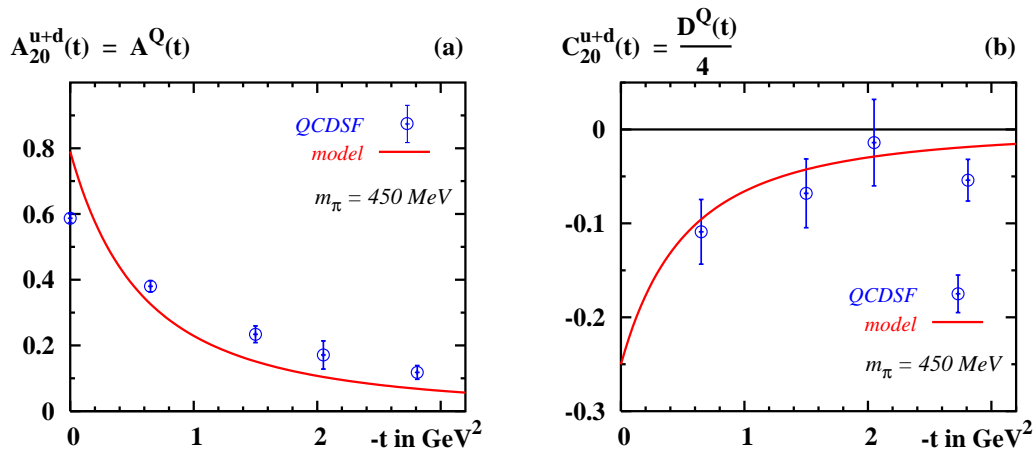
- nature: **unknown!**

- model: e.g. chiral quark soliton model, Goeke et al, PRD75 (2007) 094021



well-tested model, many nucleon properties vs data within 30% ✓

- lattice: QCDSF Collaboration, Gockeler et al, PRL92 (2004) 042002 & hep-ph/0312104 → test models



lattice QCD, bag model, Skyrme model,
chiral quark soliton model: $D_{\text{nucleon}} < 0$

other particles:
nuclei, pions, photons, Q -balls, Q -cloud, Higgs(!)
have also negative D -terms! (in theory!)

One day we will know $D(t)$ from **experiment!**
what will we learn?

interpretation of Fourier transforms of form factors as 3D-densities

- Breit frame $\Delta^\mu = (0, \vec{\Delta})$ and $t = -\vec{\Delta}^2$

- analog to electric form factor $G_E(\vec{\Delta}^2) = \int d^3\vec{r} \rho_E(\vec{r}) e^{i\vec{\Delta}\vec{r}} \rightarrow$ charge distribution
Sachs, PR126 (1962) 2256

$$\hookrightarrow Q = \int d^3\vec{r} \rho_E(\vec{r})$$

- static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle \rightarrow$ mechanical properties of nucleon
M.V.Polyakov, PLB 555 (2003) 57

$$\hookrightarrow M_N = \int d^3\vec{r} T_{00}(\vec{r})$$

limitations of 3D densities (\exists in contrast to 2D \leftrightarrow tomography)

well known since earliest days (Sachs, 1962)

comprehensive studies, e.g. by

- Belitsky & Radyushkin, Phys. Rept. 418, 1 (2005), Sec. 2.2.2
- X.-D. Ji, PLB254 (1991) 456 (Skyrme model, not dramatic)
- G. Miller, PRC80 (2009) 045210 (toy model, dramatic)

No doubt: mathematical operation is well-defined

The question: is the concept justified? Answer:

yes of course, modulo **corrections!**

how large are these corrections?

are corrections large? Look at simplest framework

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \Phi)(\partial^\mu \Phi) - \frac{1}{2} m^2 \Phi^2 \quad \text{free neutral elementary point-like scalar particle}$$

(“Higgs” modulo standard model corrections)

evaluate EMT:

$$\langle \vec{p}' | \hat{T}^{\mu\nu}(x) | \vec{p} \rangle = e^{i(p'-p)x} \frac{1}{2} \left\{ P^\mu P^\nu A(t) + \left(\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2 \right) D(t) \right\}, \quad A(t) = -D(t) = 1$$

compute **energy density**

$$\mathbf{T}_{00}(\vec{r}) \equiv m^2 \int \frac{d^3\Delta}{E(2\pi)^3} e^{i\vec{\Delta}\vec{r}} \left[A(t) - \frac{t}{4m^2} (A(t) + D(t)) \right] \quad \text{in Breit frame } E = E' = \sqrt{m^2 + (\vec{\Delta}/2)^2}$$

$$\stackrel{\text{here}}{=} \frac{m}{\sqrt{1 - \vec{\nabla}^2/(4m^2)}} \delta^{(3)}(\vec{r})$$

reproduces correctly $\int d^3r \mathbf{T}_{00}(\vec{r}) \stackrel{!}{=} m$

but yields $\langle r_E^2 \rangle = \frac{1}{m} \int d^3r r^2 \mathbf{T}_{00}(\vec{r}) \stackrel{??}{=} \frac{3}{4m^2}$

mean square radius $\neq 0$ for point-like particle???

effect of relativistic “recoil” corrections

bad concept for point-like particle

However ...

- take **heavy mass limit** to recover “correct” description

$$\mathbf{T}_{00}(\vec{r}) \longrightarrow m \delta^{(3)}(\vec{r}) \quad \text{for } m \rightarrow \text{large} \dots \quad \text{large with respect to what?}$$

- let’s give particle a **finite size R** (i.e. “smear out” δ -function, such that reduces to $\delta^{(3)}(\vec{r})$ for $R \rightarrow 0$)

$$\mathbf{T}_{00}(\vec{r})_{\text{true}} \stackrel{\text{e.g.}}{=} m \frac{e^{-r^2/R^2}}{\pi^{3/2} R^3} \quad \text{“true energy density”}$$

$$\rightarrow \langle r_E^2 \rangle = \langle r_E^2 \rangle_{\text{true}} \left(1 + \delta_{\text{corr}} \right) \simeq \langle r_E^2 \rangle_{\text{true}} \quad \text{with } \delta_{\text{corr}} \equiv \frac{1}{2m^2 R^2} \ll 1 \quad \langle r_E^2 \rangle_{\text{true}} = \frac{3}{2} R^2 \text{ for Gaussian}$$

- **nuclei** ($m_A \simeq m_N A$, $R_A = R_0 A^{1/3}$, $R_0 \sim 1.3 \text{ fm}$) $\rightarrow \delta_{\text{corr}} \sim 0.16 A^{-8/3} \lesssim 4 \times 10^{-3}$ (${}^4\text{He}$, ...)

- **for nucleon** ($m_p \sim 938 \text{ MeV}$, $R_p \sim 0.875 \text{ fm}$) $\rightarrow \delta_{\text{corr}} \sim 0.03 \ll 1$ ^{!?} small enough!?

- **large- N_c nucleon** ($m_N \sim N_c$, $R_N \sim N_c^0$) $\rightarrow \delta_{\text{corr}} \sim \frac{1}{N_c^2} \ll 1$ ^{!!} small!!! Why?

remark: $\frac{1}{N_c}$ is the only available (known) small parameter in QCD at **all** energies

(large- N powerful theoretical method, much more general than QCD)

- **interpretation as 3D-densities** (M.V.Polyakov, PLB 555 (2003) 57)

Breit frame with $\Delta^\mu = (0, \vec{\Delta})$: static EMT $T_{\mu\nu}(\vec{r}, \vec{s}) = \int \frac{d^3\vec{\Delta}}{2E(2\pi)^3} e^{i\vec{\Delta}\cdot\vec{r}} \langle P' | \hat{T}_{\mu\nu} | P \rangle$

interpretation in terms of 3D-densities subject to corrections, BUT all formulae correct!

$$\int d^3r T_{00}(\vec{r}) = M_N \quad \text{known}$$

$$\int d^3r \varepsilon^{ijk} s_i r_j T_{0k}(\vec{r}) = \frac{1}{2} \quad \text{known}$$

$$-\frac{5M_N}{8} \int d^3r \left(r^i r^j - \frac{r^2}{3} \delta^{ij} \right) T_{ij}(\vec{r}) \equiv D \quad \text{new!}$$

with: $T_{ij}(\vec{r}) = \mathbf{s}(\mathbf{r}) \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) + \mathbf{p}(\mathbf{r}) \delta_{ij} \quad \text{stress tensor}$

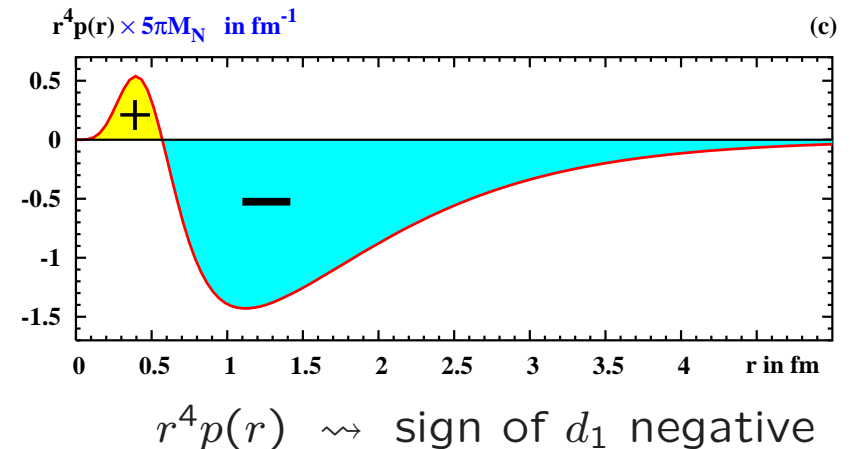
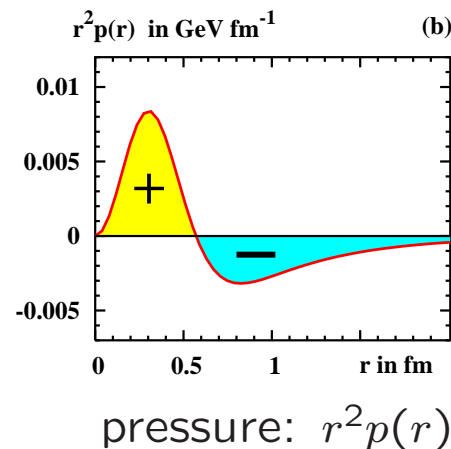
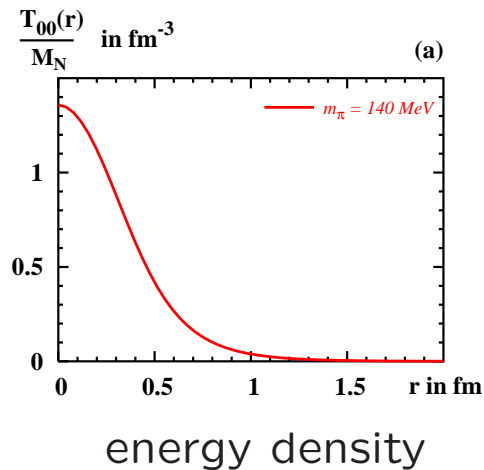
$\left. \begin{array}{l} \mathbf{s}(\mathbf{r}) \text{ related to distribution of } \textit{shear forces} \\ \mathbf{p}(\mathbf{r}) \text{ distribution of } \textit{pressure} \text{ inside hadron} \end{array} \right\} \longrightarrow \text{“mechanical properties”}$

- **relation to stability:** EMT conservation $\Leftrightarrow \partial^\mu \hat{T}_{\mu\nu} = 0 \Leftrightarrow \nabla^i T_{ij}(\vec{r}) = 0$

\hookrightarrow necessary condition for stability $\int_0^\infty dr r^2 p(r) = 0$ (von Laue, 1911)

$$D = -\frac{16\pi}{15} M_N \int_0^\infty dr r^4 s(r) = 4\pi M_N \int_0^\infty dr r^4 p(r) \quad \hookrightarrow \text{shows how internal forces balance}$$

- lessons from model



$$T_{00}(0) = 1.70 \text{ GeV}/\text{fm}^3 \approx 3 \times 10^{15} \rho(\text{H}_2\text{O}) \approx 13 \times (\text{nuclear density})$$

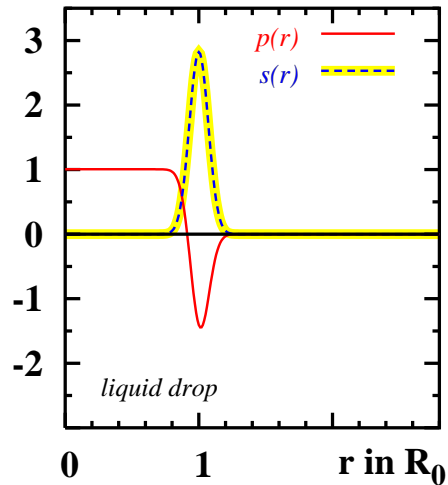
$$p(0) = 0.23 \text{ GeV}/\text{fm}^3 \approx 4 \times 10^{34} \text{ N}/\text{m}^2 \gtrsim 100 \times (\text{pressure in center of neutron star})$$

in chiral quark soliton model (Goeke et al, PRD75 (2007) 094021)

... how does it look like in QCD? *Would be fascinating to know!*

- intuition on shear forces and pressure

$p(r)$ & $s(r)$ in units of p_0



liquid drop

radius R_0

inside pressure p_0

surface tension $\gamma = \frac{1}{2}p_0R_0$

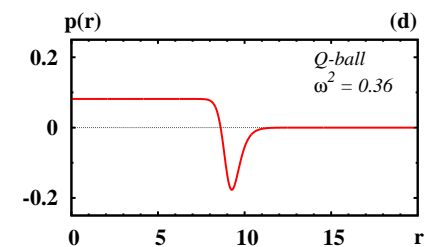
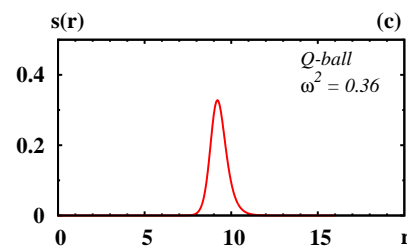
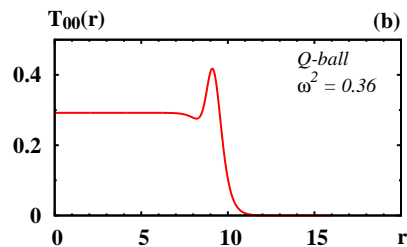
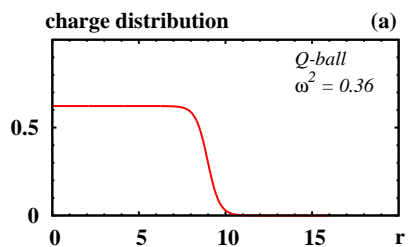
$$s(r) = \gamma \delta(r - R_0)$$

$$p(r) = p_0 \Theta(R_0 - r) - \frac{1}{3}p_0R_0 \delta(r - R_0)$$

application: liquid drop model of nucleus

(M.V.Polyakov, PLB 555 (2003) 57)

realized in field theoretical Q -ball system



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi^*)(\partial^\mu \Phi) - V \text{ with U(1) global symm., } V = A(\Phi^*\Phi) - B(\Phi^*\Phi)^2 + C(\Phi^*\Phi)^3, \quad \Phi(t, \vec{r}) = e^{i\omega t} \phi(r)$$

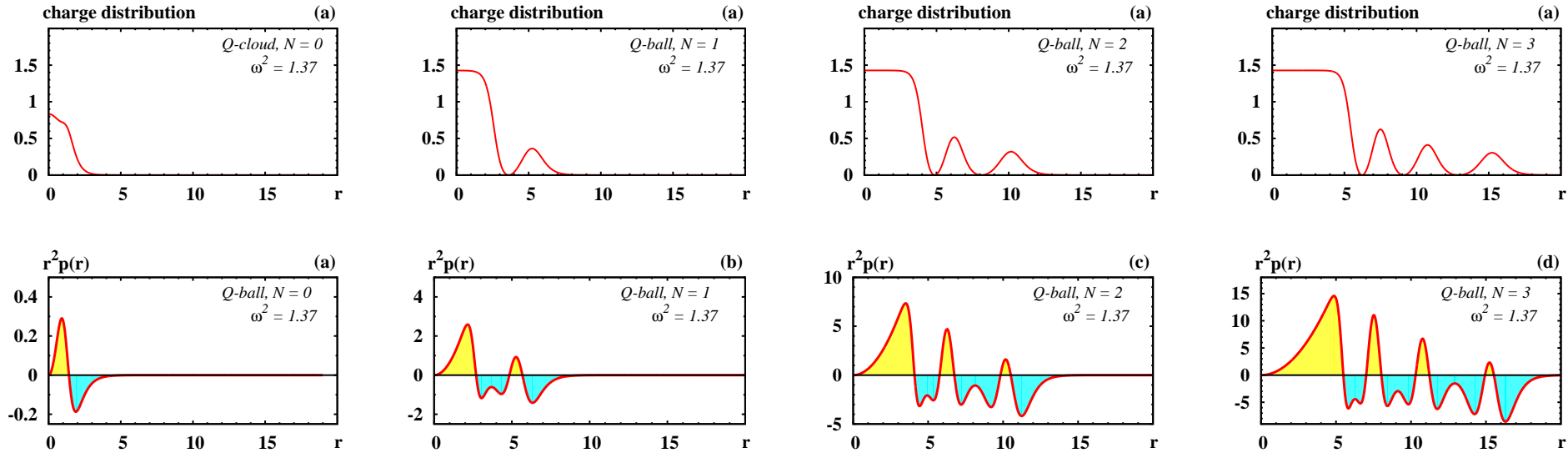
S. R. Coleman, NPB262 (1985) 263, 269 (1986) 744E; M. Mai, PS PRD86 (2012) 076001

to satisfy $\int_0^\infty dr r^2 p(r) = 0 \rightarrow p(r)$ must have a zero! Could it have more zeros?

N^{th} radial excitations of Q -balls

$N = 0$ ground state,
 $N = 1$ first excited state, etc

Mai, PS PRD86 (2012) 096002
 charge density exhibits N shells
 $p(r)$ exhibits $(2N + 1)$ zeros



$N > 0$ radial excitations all unstable

→ decay to ground state Q -balls of smaller total energy and same total charge

nevertheless $\int_0^{\infty} dr r^2 p(r) = 0$ always valid

→ necessary (not sufficient) condition \Leftrightarrow (local vs global minimum of action)

D -term always negative!

→ is it a theorem? → for Q -balls yes Mai, PS PRD86 (2012) 076001

→ rigorous proof that $d_1 < 0$ for hadrons in QCD and other particles still awaiting

so far all D -terms negative: pions, nucleons, nuclei, nucleons in nuclear matter, photons, Q -balls, Q -clouds*

(Q -cloud: most extreme instability(!); parametric limit where Q -balls dissociate in free quanta; still $D < 0$)

Application I: investigating forces

prominent property of proton:
life time $\tau_{\text{prot}} > 2.1 \times 10^{29}$ years!

question: how do strong forces
balance to produce stability?

- answer in **model**: strong cancellation of **repulsive forces** due to quark core, and **attractive forces** from pion cloud
- answer in **QCD**: we do not know nice pictures, attractive insights underexplored propaganda(?)

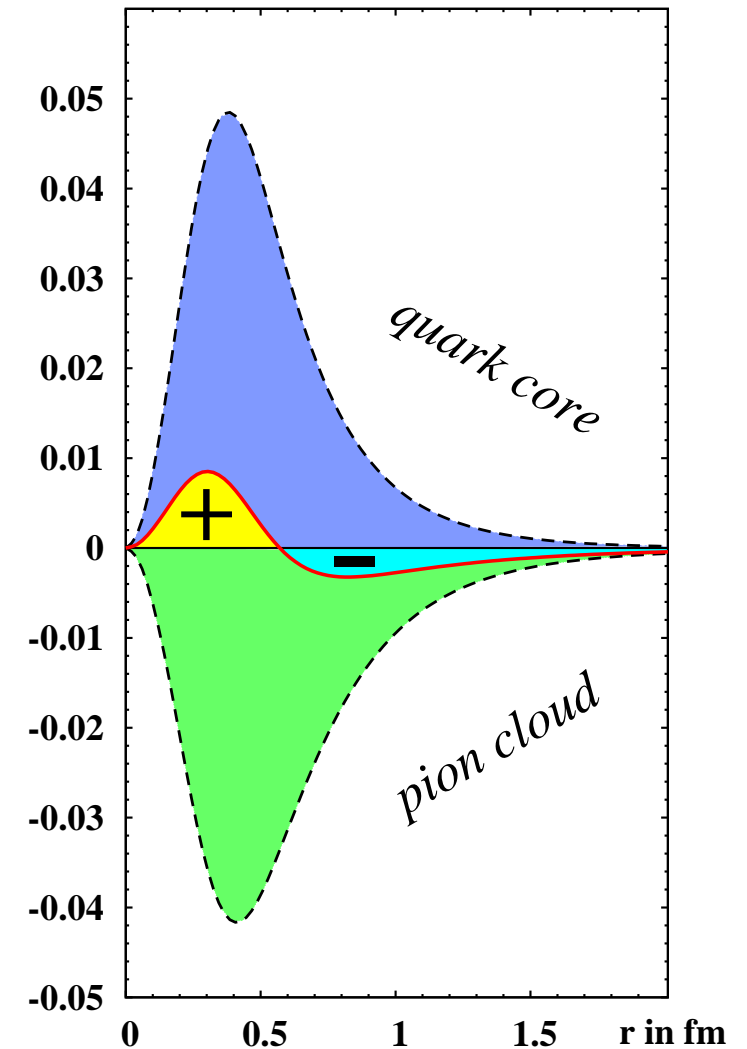
be aware: same for neutron,
 $\tau_{\text{neut}} = 14 \text{ min } 40 \text{ sec} \gg 10^{-23} \text{ sec}$
and even the same picture for Δ ...
 $\tau_{\Delta} \sim 10^{-23} \text{ sec} \rightarrow$ necessary condition!

- as motivation for GPD program: okay

... *but is there any practical use of that?*

answer: in principle yes!

$r^2 p(r)$ in GeV fm^{-1}



in chiral quark soliton model

chiral symmetry breaking ✓

realization of QCD in large- N_c ✓

built on instanton vacuum calculus ✓

not bad, but after all a model ...

Goeke et al, PRD75 (2007)

Application II: test models or effective theories

Whether you like 3D densities or not:

- you can evaluate EMT form factor: $D(t)$
- take Fourier transform $\rightarrow p(r)$
(do not call it pressure, if you do not like it)

- check that $\int_0^{\infty} dr r^2 p(r) = 0$ (must be due to $\partial_\mu T^{\mu\nu} = 0$)

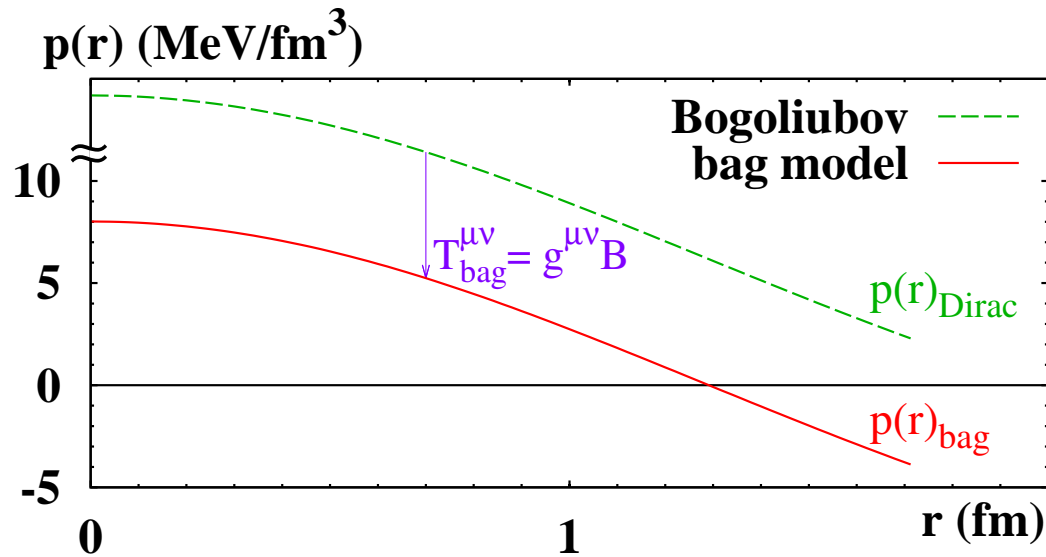
if true: consistent model, nucleon exists

(e.g. chiral quark soliton model, bag model, Skyrme model LO, ...)

if not: something is screwed up, no nucleon exists

(e.g. Bogolyubov model, spectator model, Skyrme model NLO, ...)

- depending on answer use model (with due care)
or improve/repair (if you know how)
or discard



Bogoliubov model (1967)

Thomas & Weise,
 "Structure of Nucleon,"
 Sec. 8.4.1

Bogoliubov: $M_N = \frac{3\omega_0}{R_{\text{bag}}}$ where R_{bag} fixed by hand

bag model: $M_N = \frac{3\omega_0}{R_{\text{bag}}} + \frac{4\pi}{3} R_{\text{bag}}^3 B \rightarrow R_{\text{bag}}$ from $\delta M_N = 0 \Leftrightarrow \int_0^{R_{\text{bag}}} dr r^2 p(r) = 0$

try to minimize M_N in Bogoliubov model $\Rightarrow R_{\text{bag}} \rightarrow \infty$ due to $p(r)_{\text{Dirac}} > 0$ **Explosion!!**

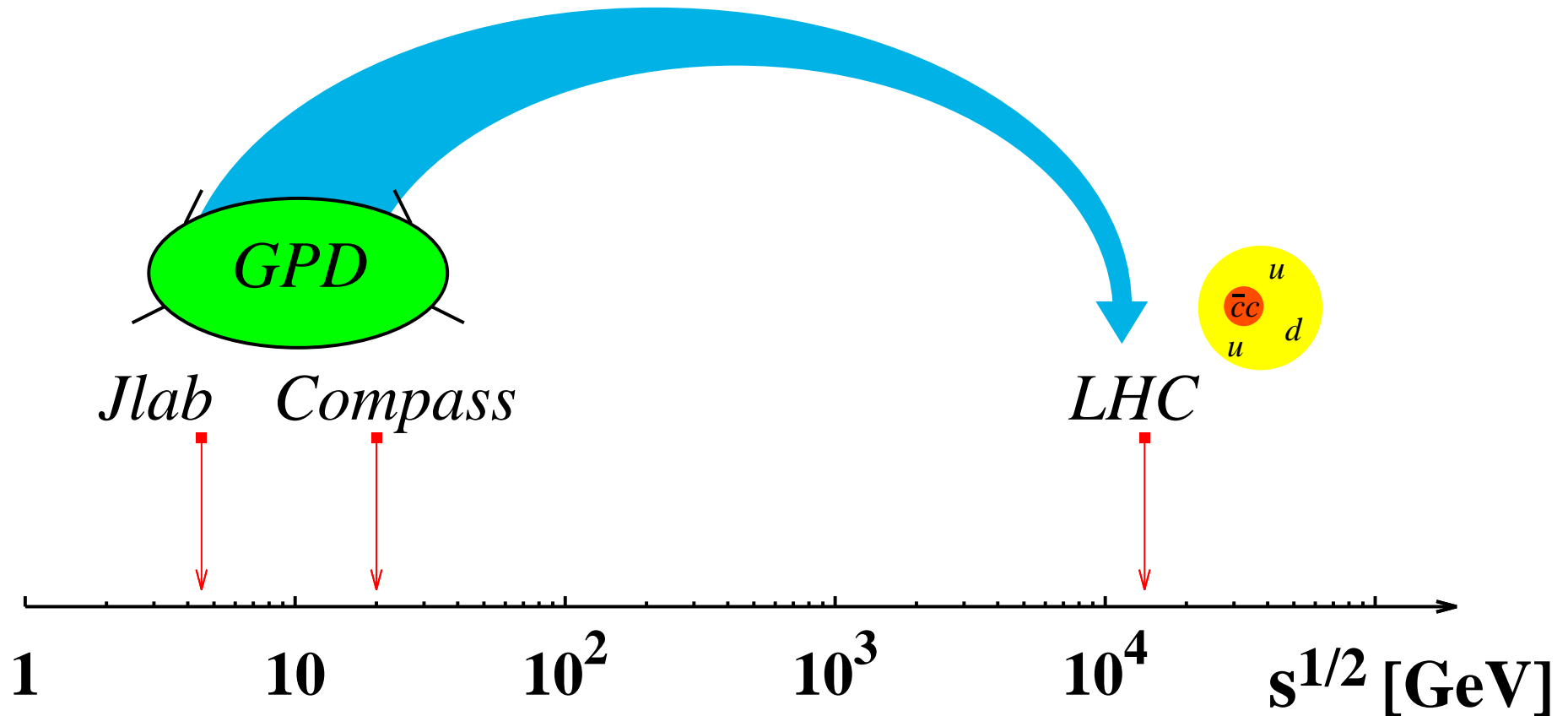
Notice that D_{Bogo} positive! But unphysical system. Model discarded!

(has done its job, paved the way for bag model)

Why bag model in 21 century? Because still insightful!

And enthusiastic undergraduate students can handle it!

Application III: amazing!



from hard-exclusive reactions at JLab, COMPASS ...

... to spectroscopy of \bar{c} -pentaquarks at LHCb

not usual hadrons, not just any exotic hadron

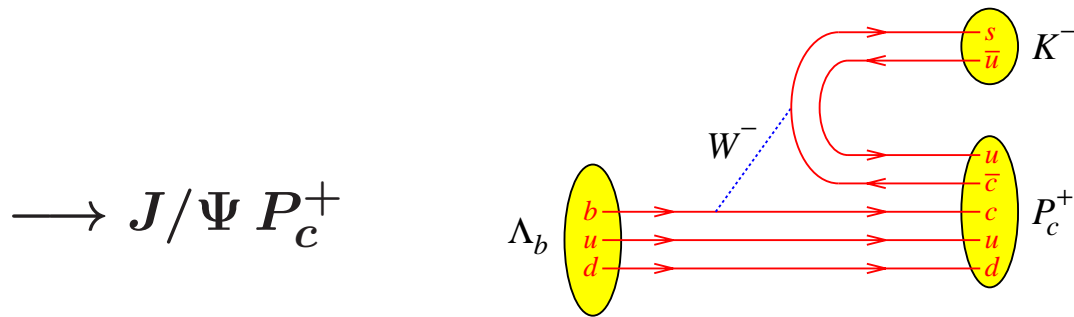
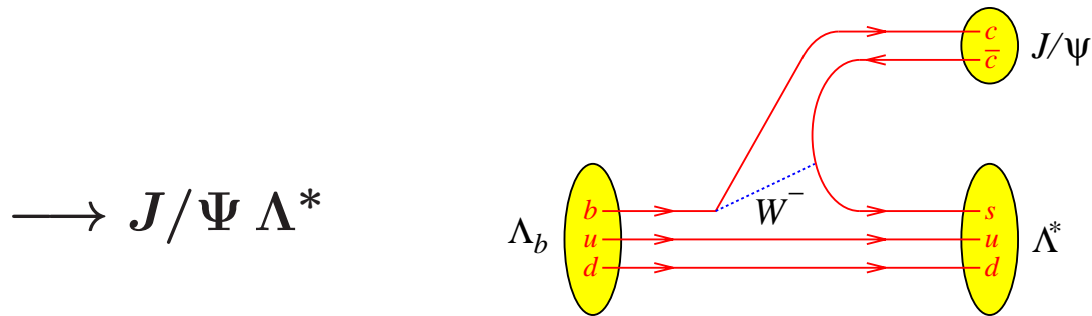
only \bar{c} -baryon bound states \rightarrow rich enough!

- discovery of charmonium pentaquarks in Λ_b^0 decays at LHCb

Aaij et al. PRL 115, 072001 (2015)

$\Lambda_b^0 \longrightarrow J/\Psi p K^-$ seen

Λ_b^0 $m = 5.6 \text{ GeV}, \tau = 1.5 \text{ ps}$
 J/Ψ $m = 3.1 \text{ GeV}, \Gamma = 93 \text{ keV}, \Gamma_{\mu^+\mu^-} = 6 \%$
 Λ^* $m = 1.4 \text{ GeV or more}, \Lambda^* \rightarrow K^- p \text{ in } 10^{-23} \text{ s}$



| state | m [MeV] | Γ [MeV] | Γ_{rel} | mode | J^P |
|---------------|---------------------|---------------------|----------------------------|------------|----------------------------------------|
| $P_c^+(4380)$ | $4380 \pm 8 \pm 29$ | $205 \pm 18 \pm 86$ | $(4.1 \pm 0.5 \pm 1.1) \%$ | $J/\psi p$ | $\frac{3}{2}^-$ or $\frac{5}{2}^+$ |
| $P_c^+(4450)$ | $4450 \pm 2 \pm 3$ | $39 \pm 5 \pm 19$ | $(8.4 \pm 0.7 \pm 4.2) \%$ | $J/\psi p$ | $\frac{3}{2}^{\pm}$ or $\frac{3}{2}^-$ |

Appealing approach to new pentaquarks

Eides, Petrov, Polyakov, PRD93, 054039 (2016)

Perevalova, Polyakov, PS, PRD94, 054024 (2016)

- theoretical approach**

$R_{J/\psi} \ll R_N \Rightarrow$ non-relativistic multipole expansion Gottfried, PRL 40 (1978) 598
 baryon-quarkonium interaction dominated by 2 virtual chromoelectric dipole gluons

$$V_{\text{eff}} = -\frac{1}{2} \alpha \vec{E}^2 \quad \text{Voloshin, Yad. Fiz. 36, 247 (1982)}$$

- chromoelectric polarizability**

$$\alpha(1S) \approx 0.2 \text{ GeV}^{-3} \text{ (pert),}$$

$$\alpha(2S) \approx 12 \text{ GeV}^{-3} \text{ (pert),}$$

$$\alpha(2S \rightarrow 1S) \approx \begin{cases} -0.6 \text{ GeV}^{-3} \text{ (pert),} \\ \pm 2 \text{ GeV}^{-3} \text{ (pheno),} \end{cases}$$

in heavy quark mass limit & large- N_c limit

\rightsquigarrow “perturbative result” Peskin, NPB 156 (1979) 365

value for $2S \rightarrow 1S$ transition from

phenomenological analysis of $\psi' \rightarrow J/\psi \pi \pi$ data

Voloshin, Prog. Part. Nucl. Phys. 61 (2008) 455

- chromoelectric field strength:**

$$\vec{E}^2 = g^2 \left(\frac{8\pi^2}{bg^2} T^\mu{}_\mu + T_{00}^G \right)$$

$b = \frac{11}{3} N_c - \frac{2}{3} N_F$ leading coeff. of β -function

g = strong coupling at low (nucleon) scale $\lesssim 1 \text{ GeV}$

g_s = strong coupling at scale of heavy quark ($g_s \neq g$)

$T_{00}^G = \xi T_{00}$ with ξ = fractional contributions of gluon to M_N

$T^\mu{}_\mu = T^{00} - T^{ii}$, stress tensor $T^{ij} = \left(\frac{r^i r^j}{r} - \frac{1}{3} \delta^{ij} \right) s(r) + \delta^{ij} p(r)$

- universal effective potential**

$$V_{\text{eff}} = -\frac{1}{2} \alpha \frac{8\pi^2}{b} \frac{g^2}{g_s^2} \left[\nu T_{00}(r) + 3p(r) \right], \quad \nu = 1 + \xi_s \frac{b g_s^2}{8\pi^2}$$

$\nu \approx 1.5$ estimate by Eides et al, op. cit.

Novikov & Shifman, Z.Phys.C8, 43 (1981);

X. D. Ji, Phys. Rev. Lett. 74, 1071 (1995)

- **in future GPDs can help:** GPDs \Rightarrow EMT form factors \Rightarrow EMT densities \Rightarrow universal potential V_{eff} for quarkonium-baryon interaction!
- **currently:** e.g. chiral quark soliton model, Skyrme model
- **compute quarkonium-nucleon bound state**

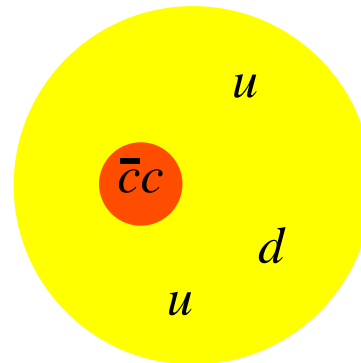
$$\text{solve } \left(-\frac{\vec{\nabla}^2}{2\mu} + V_{\text{eff}}(r) \right) \psi = E_{\text{bind}} \psi$$

$\mu =$ reduced quarkonium-baryon mass

- **results:**

nucleon and J/ψ form no bound state

nucleon and $\psi(2S)$ form 2 bound states with nearly degenerate masses around 4450 MeV in $L = 0$ channel, $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ if $\alpha(2S) \approx 17 \text{ GeV}^{-3}$ (consistent with guideline from pert. calc.)
robust! Largely model-independent



- **decay of $P_c(4450)$**

cannot decay directly to $\psi(2S)$ and nucleon, as $M_{\psi(2S)} + M_N > 4450 \text{ MeV}$

instead transition $(2S) \rightarrow (1S)$ governed by the same V_{eff}
but with small $\alpha(2S \rightarrow 1S)$ transition polarizability
 \Rightarrow it “takes time”

after the transition is “completed,”
prompt decay to $J/\psi + \text{nucleon}$
(observed final states)

estimated width is tens of MeV \rightarrow compatible with data!

- first found in χ QSM (Eides et al)
confirmed in Skyrme model (Perevalova et al)

- prediction: also Δ and $\psi(2S)$ form isospin- $\frac{3}{2}$ bound state

mass = 4.5 GeV, $\Gamma_{\Delta\bar{c}c} \sim 60$ MeV

negative parity ($l = 0$), spin $\frac{1}{2} \leq J \leq \frac{5}{2}$

(states degenerate in heavy quark limit)

possibly also a broad isospin- $\frac{3}{2}$ resonance

mass = 4.9 GeV, $\Gamma_{\Delta\bar{c}c} \sim 150$ MeV

positive parity ($l = 1$), spin $\frac{1}{2} \leq J \leq \frac{7}{2}$

decays $\rightarrow J/\psi + \underbrace{\text{nucleon} + \text{pion}}_{\Delta\text{-resonance}}$

hyperon-quarkonium bound states are also possible! (Perevalova et al)

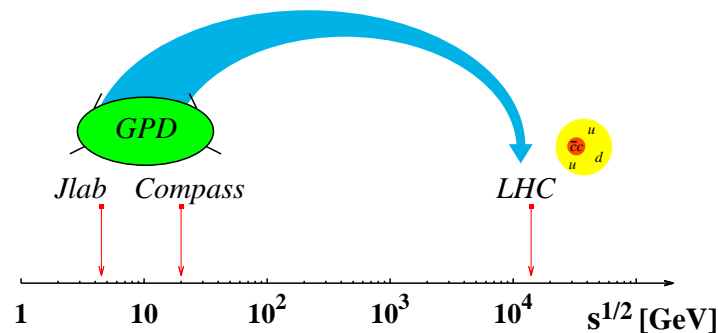
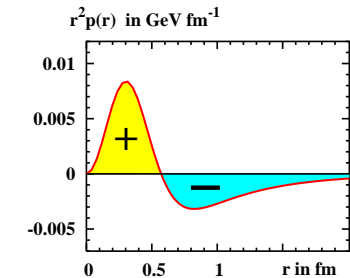
- what about $P_c^+(4380)$?

cannot be charmonium-nucleon bound state

broader, more possibilities, threshold cusp effect

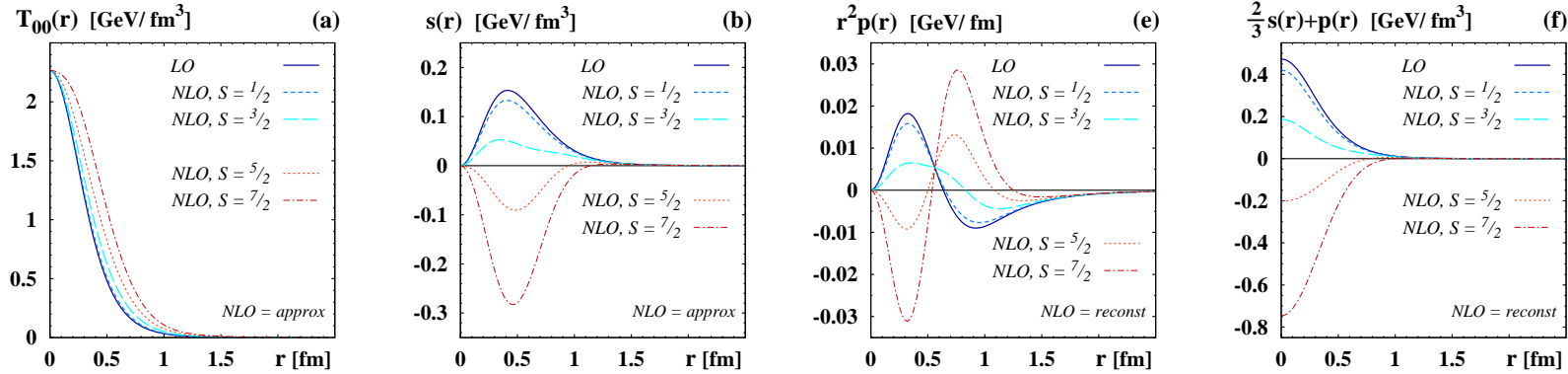
Summary & Outlook

- GPDs \leftrightarrow form factors of **energy momentum tensor**
mass decomposition, spin decomposition, and D -term!
- **D-term**: last unknown global property, related to forces
attractive and physically appealing \rightarrow “motivation”
- recent development: knowledge of EMT densities
 \rightarrow **quarkonium-baryon interaction** V_{eff}
- naturally explains properties of P_c^+ (4450) observed at LHCb
rich potential, new predictions, ongoing work



Thank you!

support slide



EMT densities from the Skyrme model as functions of r . The LO results are valid for any $S = I$ in the large- N_c limit. The estimates of NLO corrections in the $1/N_c$ -expansion are shown for states with the quantum numbers $S = I = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$.

The Figures show:

- (a) energy density $T_{00}(r)$,
- (b) shear forces $s(r)$,
- (e) $r^2 p(r)$ with NLO corrections reconstructed from $s(r)$.
- (f) the local stability criterion $\frac{2}{3}s(r) + p(r) > 0$ (if it holds)

States with the exotic quantum numbers $S = I \geq 5/2$ do not satisfy $\frac{2}{3}s(r) + p(r) > 0$ **and have a positive D -term!! That's why they do not exist!**
(Perevalova, Polyakov, PS, PRD94, 054024)