Chiral odd GPDs in the MIT Bag Model

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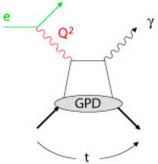
Outlook

- Properties of chiral-even GPDs
- Properties of chiral-odd GPDs
- Chiral-even GPDs in Bag Model
- Chiral-odd GPDs in Bag Model
- Model Comparisons
- Large-N_c expansion in Bag Model
- Conclusions

Properties of chiral-even GPDs

- They appear, for example in DVCS processes



- There are four chiral-even GPDs $H, \tilde{H}, E, \tilde{E}$ at leading twist, parametrized in the following form [Diehl '03]

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H^q\,\gamma^+ + E^q\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}\right]u(p,\lambda),$$
$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\,\psi(\tfrac{1}{2}z)\,|p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[\tilde{H}^q\,\gamma^+\gamma_5 + \tilde{E}^q\,\frac{\gamma_5\Delta^+}{2m}\right]u(p,\lambda),$$

where $P = \frac{1}{2}(p+p'), \Delta = p' - p$ and $\xi = \frac{p^+ - p'^+}{p^+ + p'^+}$

Properties of chiral-even GPDs

- In order to obtain chiral-even GPDs from these equations, we need 2 linearly independent equations for each case; project on different spin components
- In the forward limit $\Delta \to 0$, H^q and $\tilde{H}^q$ reduce to quark density and quark helicity distributions respectively;

$$H^q(x,0,0) \to f_1(x)$$
$$\tilde{H}^q(x,0,0) \to g_1(x)$$

- It follows from the time reversal invariance that the chiral-even GPDs $H, \tilde{H}, E, \tilde{E}$ are invariant under the transformation $\xi \to -\xi$, i.e

$$GPD(x,\xi,t) = GPD(x,-\xi,t) \quad \text{for } GPD = H, \tilde{H}, E, \tilde{E}$$

Properties of chiral-even GPDs

- First moments of chiral-even GPDs are

$$\int_{-1}^{1} \left\{H, E, \tilde{H}, \tilde{E}\right\}(x, \xi, t)dx = F_1(t), F_2(t), G_A(t), G_P(t)$$

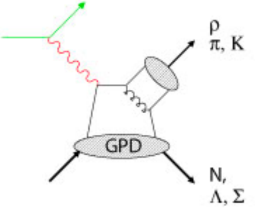
- Polynomiality: In fact first moments of GPDs are special case of a more general theory. Integrals of $x^n \times$ GPD over x are even polynomials of ξ maximum at the order of $n+1$

$$\int_{-1}^{1} x^N \times (GPD)dx = a_0(t) + a_2(t)\xi^2 + \ldots + a_{N+1}(t)\xi^{N+1}$$

- So, chiral-even GPDs are relatively well constrained (...compared to chiral-odd sector)

Properties of chiral-odd GPDs

- They appear in exclusive meson production processes



- There are four chiral-odd GPDs $H_T, \tilde{H}_T, E_T, \tilde{E}_T$ at leading twist

$$\frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \bar{\psi}(-\tfrac{1}{2}z)\, i\sigma^{+i}\, \psi(\tfrac{1}{2}z)\, |p,\lambda\rangle\Big|_{z^+=0,\, \mathbf{z}_T=0}$$
$$= \frac{1}{2P^+}\bar{u}(p',\lambda')\left[H_T^q\, i\sigma^{+i} + \tilde{H}_T^q\, \frac{P^+\Delta^i - \Delta^+P^i}{m^2} + E_T^q\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q\, \frac{\gamma^+P^i - P^+\gamma^i}{m}\right] u(p,\lambda).$$

where $i = 1, 2$ is the transversity index [Diehl '03]

Properties of chiral-odd GPDs

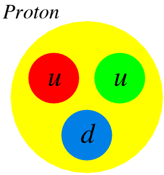
- In the forward limit $\Delta \to 0$, H_T reduces to transversity PDF; $H_T(x,0,0) \to h_1(x)$
- It follows from the time reversal invariance that the GPDs $H_T, \tilde{H}_T, E_T$ are invariant under the transformation $\xi \to -\xi$. Whereas $\tilde{E}_T$ is subject to sign change, i.e.

$$
\begin{aligned}
GPD(x,\xi,t) =& GPD(x,-\xi,t) \qquad \text{for } GPD = H_T, \tilde{H}_T, E_T \\
GPD(x,\xi,t) =& -GPD(x,-\xi,t) \quad \text{for } GPD = \tilde{E}_T
\end{aligned}
$$

- First moments of the chiral-odd GPDs are the nucleon's tensor form factors, i.e.

$$
\begin{aligned}
\int_{-1}^{1} \left\{ H_T, \tilde{H}_T, E_T \right\}(x,\xi,t)dx =& H_T(t), \tilde{H}_T(t), E_T(t) \\
\int_{-1}^{1} \left\{ \tilde{E}_T \right\}(x,\xi,t)dx =& 0
\end{aligned}
$$

Bag Model



- Quarks are constrained inside of a finite size "bag"
- Quarks are free inside the bag (Asymptotic freedom), however are subject to sharp boundary conditions on the surface to implement the confinement.
- Evaluate the correlators by using this model in order to calculate chiral-odd GPDs.
- Bag model has been used to obtain the first estimations for chiral-even GPDs [Ji, Melnitchouk, Song '97]

Chiral-even GPDs in Bag Model

- The momentum space wave function in the bag is given by

$$\varphi(\vec{k}) = \sqrt{4\pi} N R^3 \begin{pmatrix} t_0(k)\chi_m \\ \vec{\sigma}\cdot\hat{k} \quad t_1(k)\chi_m \end{pmatrix}$$

  where

$$t_0(k) = \frac{j_0(w_0)\cos(kR) - j_0(kR)\cos(w_0)}{w_0^2 - \vec{k}^2 R^2}$$

$$t_1(k) = \frac{j_0(kR) j_1(w_0) kR - j_0(w_0) j_1(kR) w_0}{w_0^2 - \vec{k}^2 R^2}$$

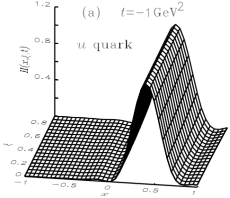
- Use this wave function to evaluate the correlators on the left hand side;

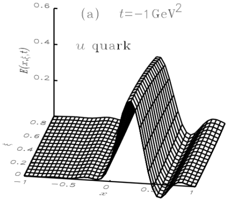
$$\varphi^\dagger(k') S(\Lambda_{-\vec{v}}) \gamma^0 \Gamma S(\Lambda_{\vec{v}}) \varphi(k)$$

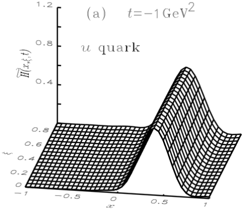
  where $\Gamma = \gamma^+$ or $\gamma^+\gamma_5$ and $S(\Lambda_{\vec{v}})$ is the Lorentz boost transformation

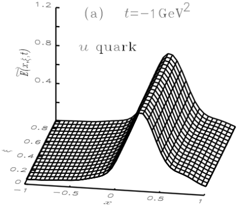
Chiral-even GPDs in Bag Model

- Bag Model estimations of chiral-even GPDs for up quark at $t = -1 Gev^2$ [Ji, Melnitchouk,Song '97]









Chiral-odd GPDs in Bag Model

- Same type of analysis could be implemented on the chiral-odd sector, with $\Gamma = i\sigma^{+i}$
- We work in the Breit frame

$$p_\mu = (\bar{m}; -\vec{\Delta}/2) \qquad p'_\mu = (\bar{m}; \vec{\Delta}/2)$$

where $\bar{m}^2 = P^2 = m^2 - t/4$.

- Again, evaluate the correlators on the l.h.s
- We have 2 equation (for $i = 1, 2$) and 4 unknowns; project on different spin components to obtain 4 equations and 4 unknowns

- Bag Model (thick line) vs. Light Front Constituent Model (dashed line) [Pasquini, Pincetti,Boffi '05]

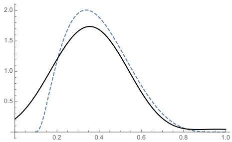


Figure: H_T^u for $\xi = 0.1, t = -0.2 GeV^2$

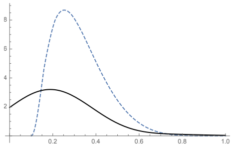


Figure: E_T^u for $\xi = 0.1, t = -0.2 GeV^2$

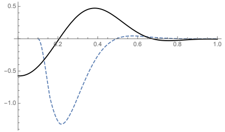


Figure: $\tilde{H}_T^u$ for $\xi = 0.1, t = -0.2 GeV^2$

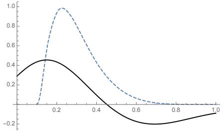
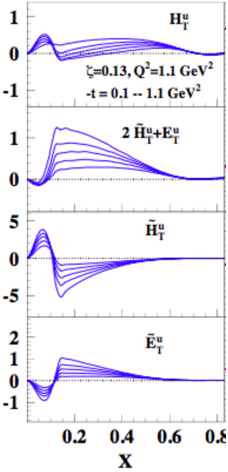


Figure: $\tilde{E}_T^u$ for $\xi = 0.1, t = -0.2 GeV^2$

Reggeized Diquark Model

- Reggeized diquark model estimations [Goldstein, Hernandez,Liuti '13]
- The node on $\tilde{E}_T^u$ can also be seen in this model



Bag Model Estimations

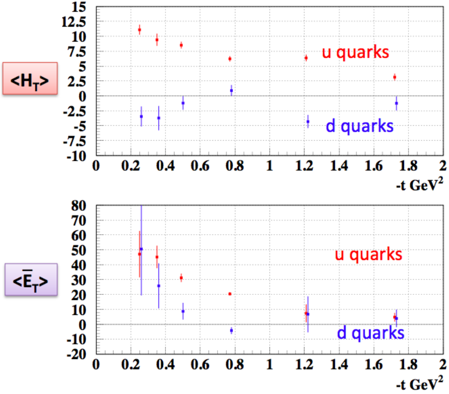


Figure: Preliminary [Kubarovsky '15], talk given at EMP and Short Range Hadron Structure. $Q^2 = 1.8GeV^2$

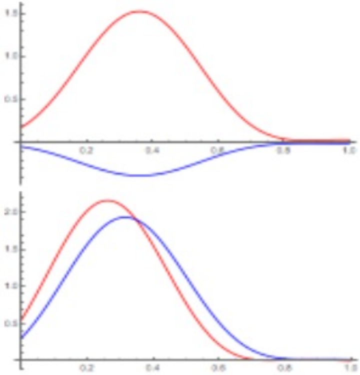


Figure: H_T and $\bar{E}_T = 2\tilde{H}_T + E_T$ for $\xi = 0.22, t = -0.3GeV^2$. $Q^2 < 1GeV^2$

Large-N_c expansion

- Usually once we can not solve a problem exactly, we use perturbation theory; anharmonic oscillator in QM, ϕ^4 theory in QFT, ect.
- In QCD, the coupling constant g is high at low energies. Hence is not a good expansion parameter.
- The only known expansion parameter valid in all regions in QCD is $\frac{1}{N}$ obtained by generalizing the $SU(3) \rightarrow SU(N)$ [t'Hooft '74]
- Still many phenomenological insights can be gained by studying the Large-N_c limit

Large-N_c expansion in Bag Model

- In Large-N_c limit

$$m, \sim N_c$$
$$\xi \sim N_c^{-1}$$
$$t \sim N_c^0$$

- In Bag Model, we have the following scaling properties

$$H_T^{u-d} \sim N_c^2$$
$$H_T^{u+d} \sim N_c$$
$$\bar{E}_T^{u-d} \sim N_c^2$$
$$\bar{E}_T^{u+d} \sim N_c^3$$

Dominance of flavor-nonsinglet H_T^{u-d} over flavor singlet H_T^{u+d} and dominance of flavor singlet $\bar{E}_T^{u+d}$ over flavor-nonsinglet $\bar{E}_T^{u-d}$

Large-N_c expansion in Bag Model

- We have a qualitative understanding of the data (π^0 and η production) in the Large-N_c framework
- Different signs of H_T^u and H_T^d and the same signs of $\bar{E}_T^u$ and $\bar{E}_T^d$ confirms the leading order behavior
- Those scaling behaviors were first given by [Schweitzer, Weiss '16] and are confirmed in Bag model

Conclusions

- The first model estimation of chiral-even GPDs (work done by Ji et. al.) are extended to chiral-odd sector
- The model is applicable at low scale in valence-x region
- The model satisfies important properties: like polynomiality, the sum rule $\int dx \tilde{E}^q(x, \xi, t) = 0$ (not all models satisfy it)
- Flavor structure dictated by quark model SU(4)-spin flavor symmetry, in qualitative agreement with JLab data, lattice QCD, analysis by Goloskokov and Kroll