



# Progenitors of Core Collapse SNe

Raphael HIRSCHI

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UChicago, UFrankfurt, ...

MESA: B. Paxton (KITP), F. X. Timmes, (UArizona, US)

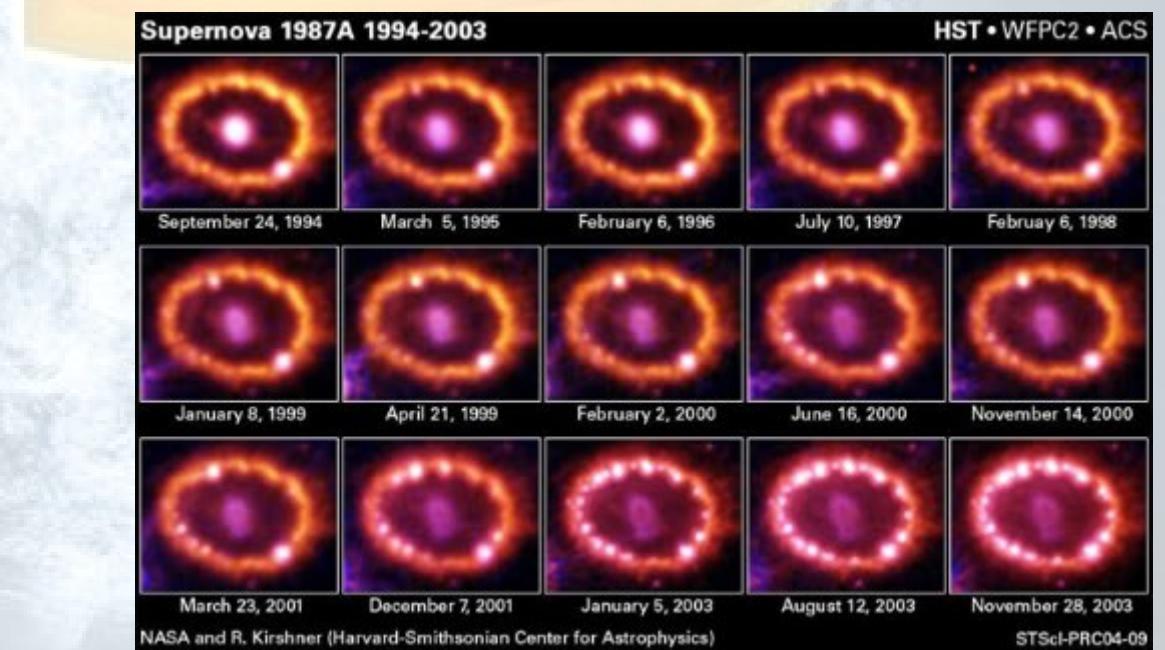
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HYDRO: C. Meakin, D. Arnett (UArizona), C. Georgy (GVA), M. Viallet (MPA),  
F. Roepke, P. Edelmann, S. Jones (HITS, D)

# *SN87A 30 Years Anniversary!*



- Exquisite Observations!
- Great memories!



# *Plan*

- Overview of link between MS evolution and SN types
- Effects of a few key physical ingredients
- 1D vs 3D modelling
- Convective boundaries
- Conclusions & outlook

# *Evolution of Surface Properties*

Main sequence:

hydrogen burning

After Main Sequence:

Helium burning

Low and intermediate-mass stars:

MS → RG → HB/RC → AGB → WD

Single massive stars:

Supergiant stage (red or blue)

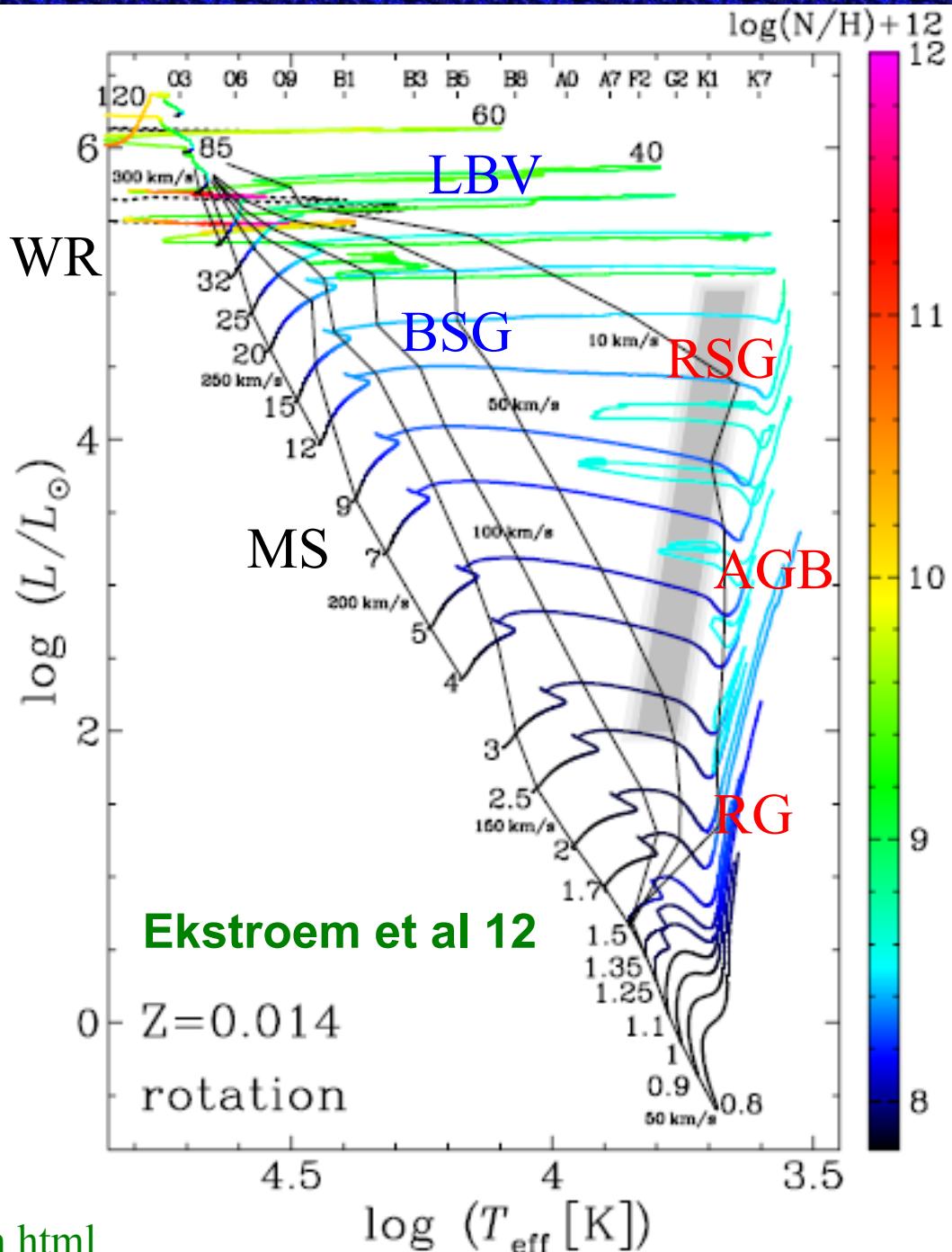
Wolf-Rayet (WR):  $M > 20\text{-}25 M_{\odot}$

WR without RSG:  $M > 40 M_{\odot}$

Advanced stages: C, Ne, O, Si

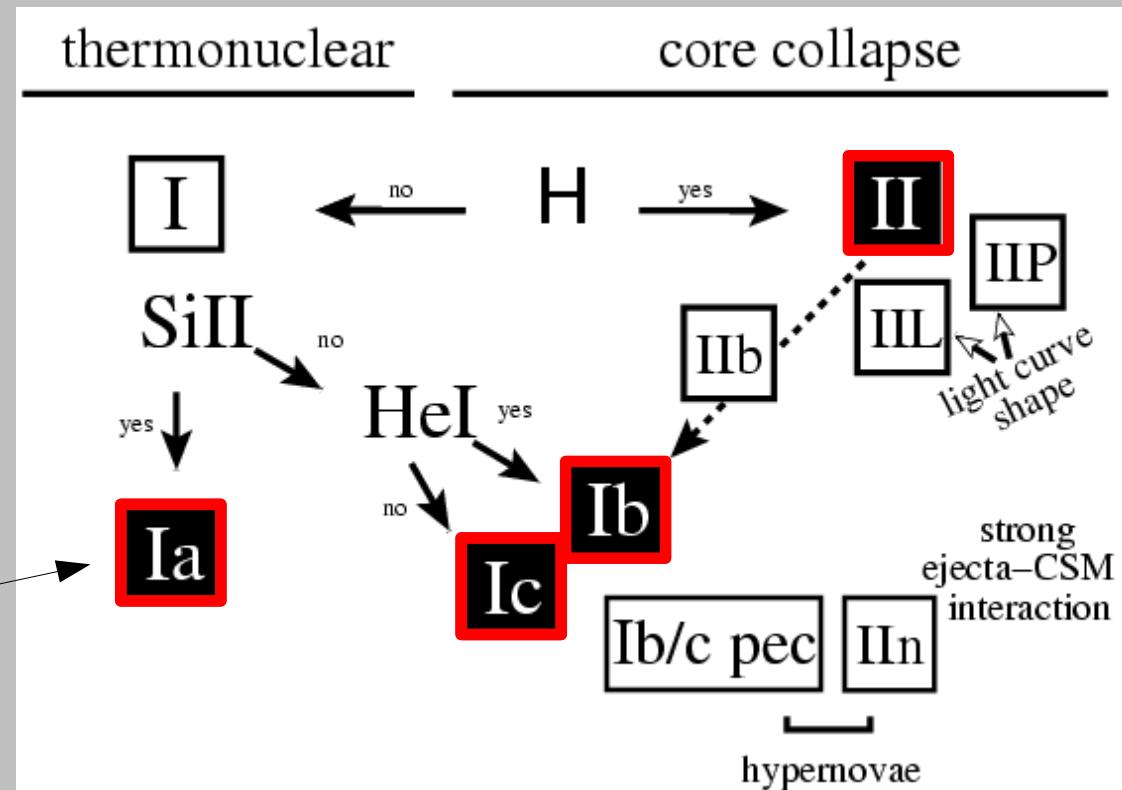
→ iron core → SN/NS/BH

<http://www.astro.keele.ac.uk/~hirschi/animation/anim.html>



# Supernova Explosion Types

Massive stars: → **SN II** (H envelope),  
**Ib** (no H), **Ic** (no H & He) ← WR



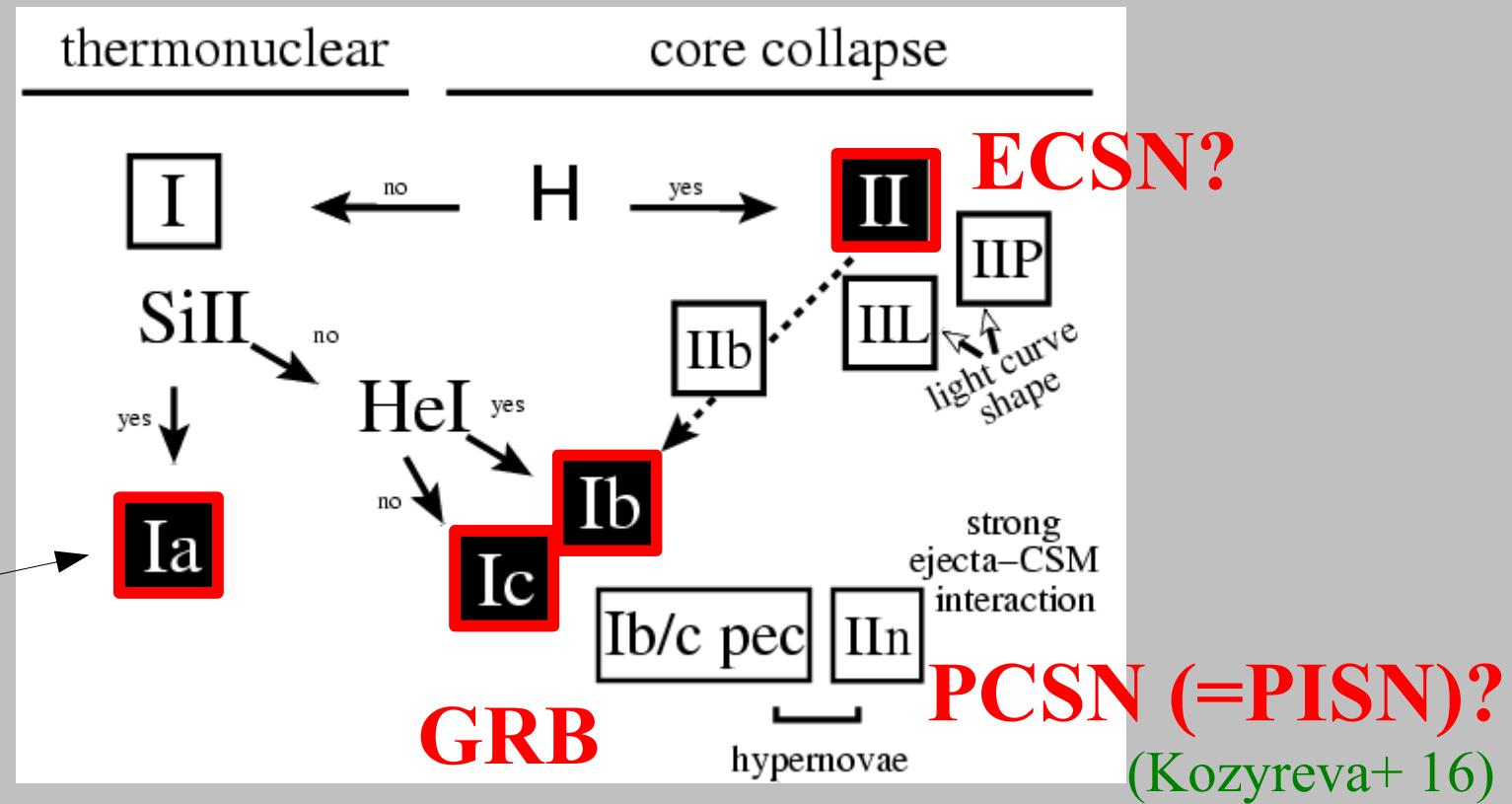
White dwarfs (WD):  
in binary systems  
Accretion →  
Chandrasekhar  
mass → SN **Ia**

(Turatto 03)

Binary interactions: → **SN II**: RSG → BSG (SN87A, e.g. Podsiadlowski+ 91)  
**Ib** (no H), **Ic** (no H & He) via mass transfer: smaller mass

# Supernova Explosion Types

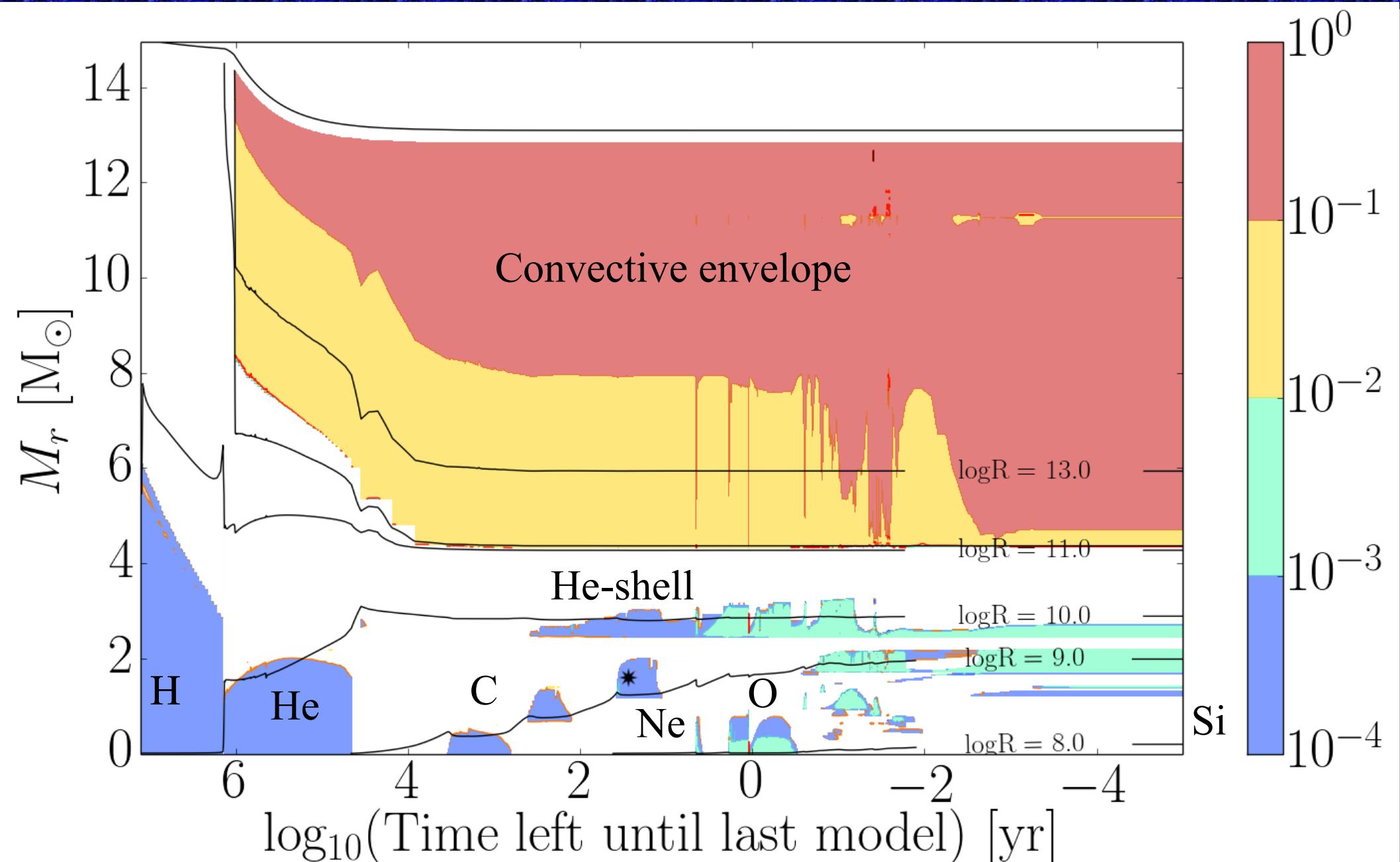
Massive stars: → **SN II** (H envelope),  
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White dwarfs (WD):  
in binary systems  
Accretion →  
Chandrasekhar  
mass → SN Ia

Binary interactions: → **SN II**: RSG → BSG (SN87A, e.g. Podsiadlowski+ 91)  
**Ib** (no H), **Ic** (no H & He) via mass transfer: smaller mass

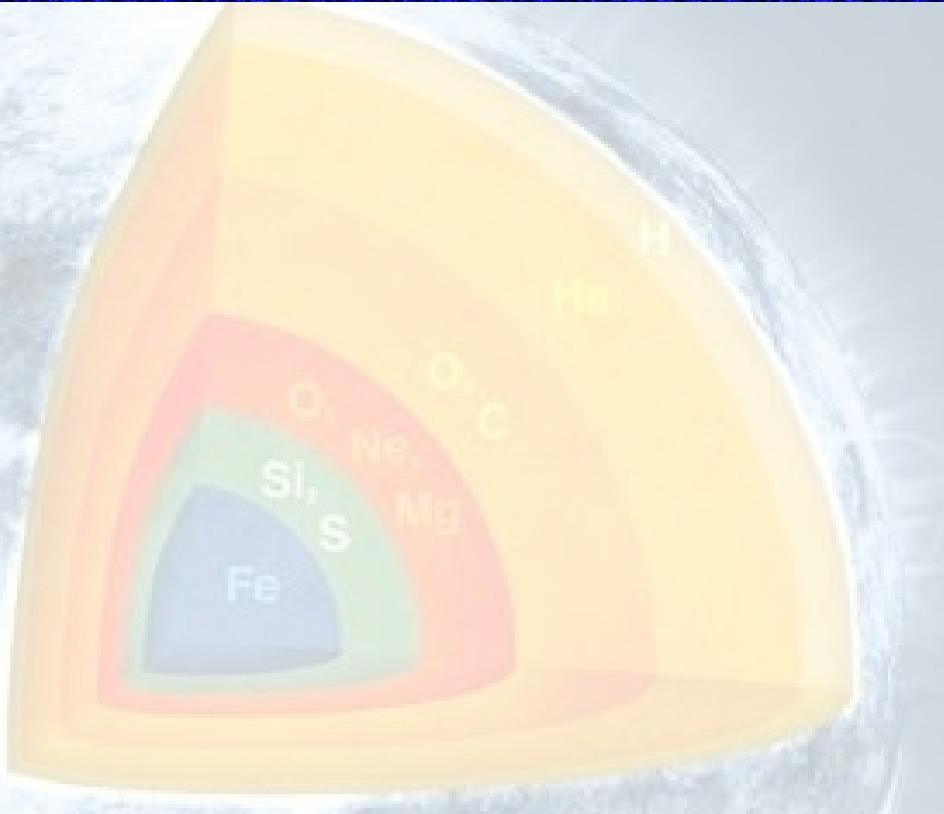
# *Evolution of Massive Stars*



Convection takes place during most burning stages

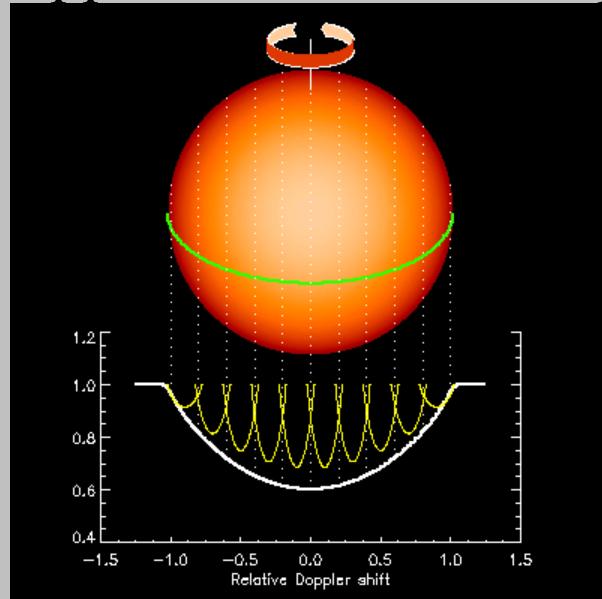
# *Physical Ingredients*

- Nuclear reactions
- Rotation
- Mass loss
- Convection
- Magnetic fields
- Binarity (following talks)
- Equation of state, opacities & neutrino losses  
including metallicity dependence

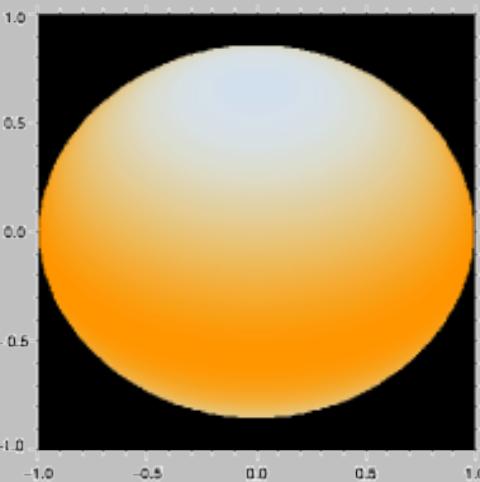


# *Rotational Effects on Surface*

Doppler-broadened line profile



$T_{\text{eff}}$  map (BMAD)

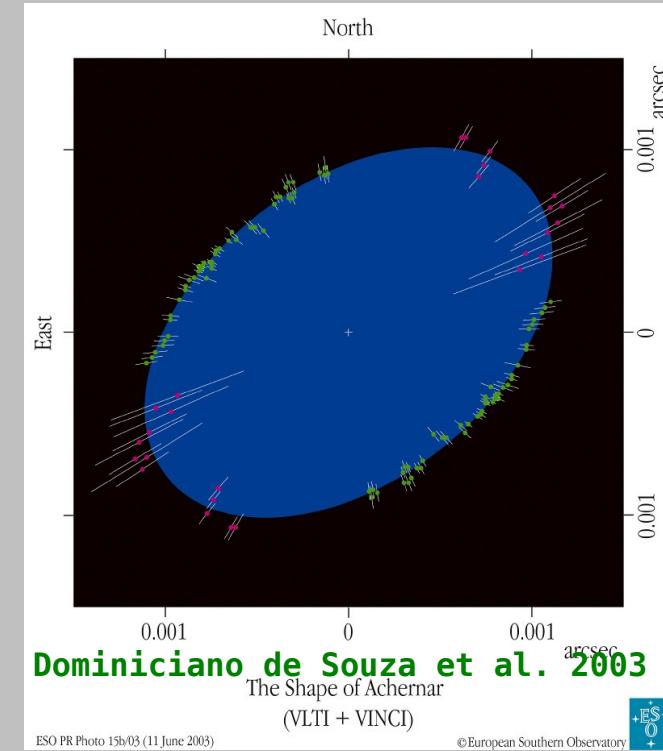
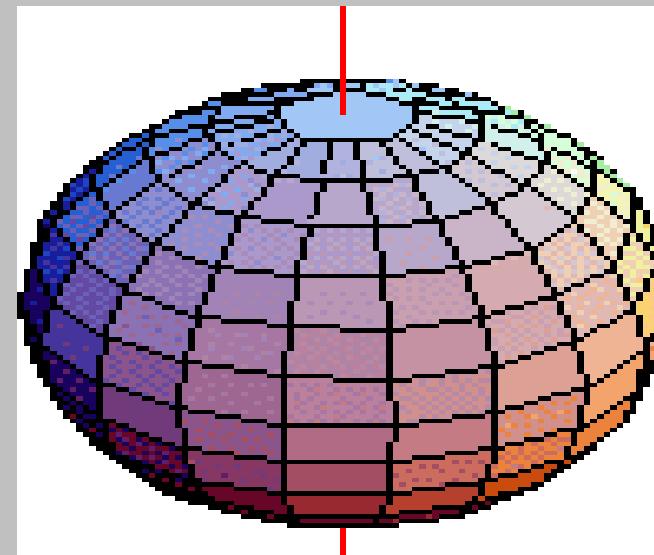


$T_{\text{max}} = 8499.9 \text{ K}$   
 $T_{\text{min}} = 6928.8 \text{ K}$

Inclination = 55.0°  
Rpolar/Req = 0.81

Domiciano de Souza et al. 2005  
Temperature (K)

Fast rotators  $\rightarrow$  oblate shape:



Domiciano de Souza et al. 2003

The Shape of Achernar  
(VLTI + VINCI)

ESO PR Photo 15b/03 (11 June 2003)



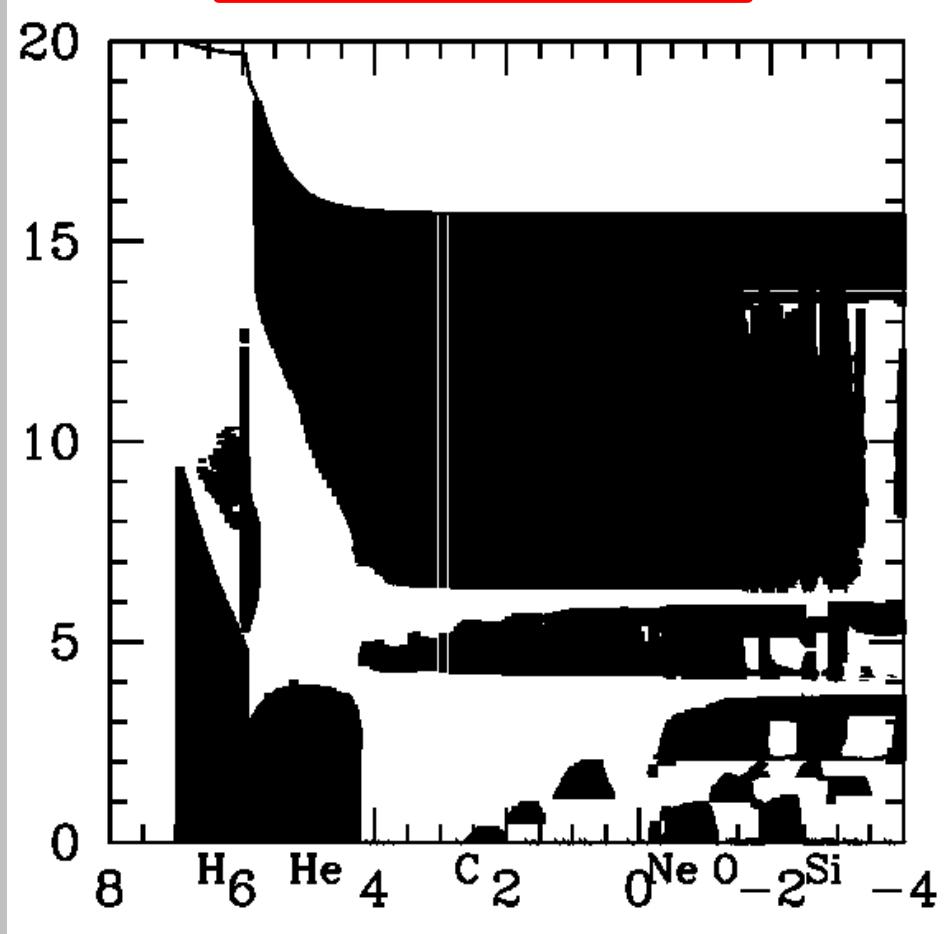
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← Altair: pole brighter than equator: Effect compatible with von-Zeipel theorem (1924)

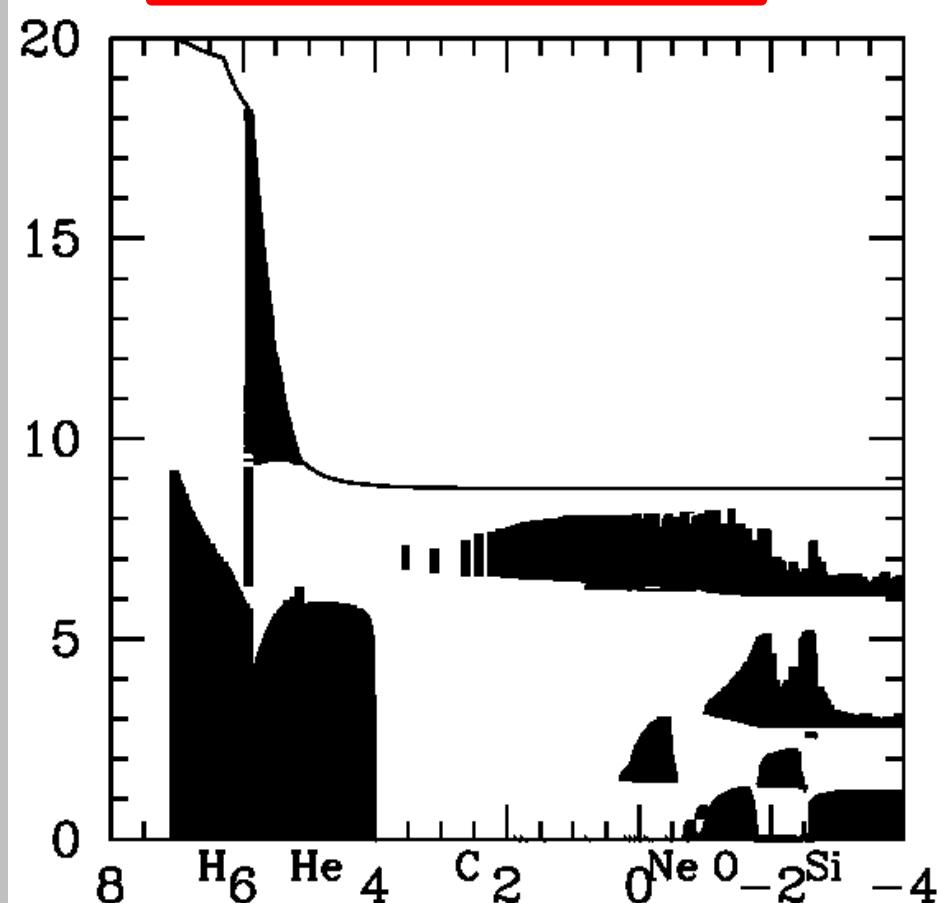
→ enhanced mass loss (+ anisotropic)

# Kippenhahn Diagrams, $\mathcal{M}_{ini} = 20 \mathcal{M}_o$ 10

$v_{ini} = 0 \text{ km/s}$



$v_{ini} = 300 \text{ km/s}$

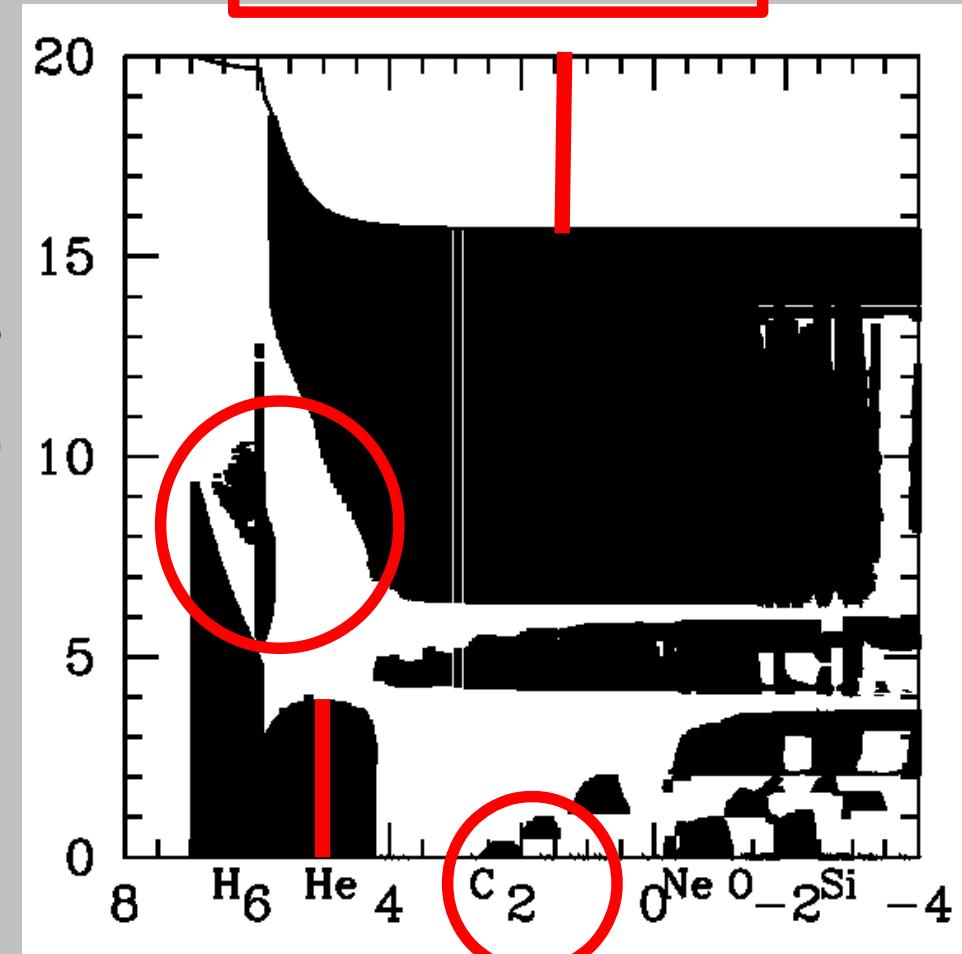


Log(time until core collapse) [yr]

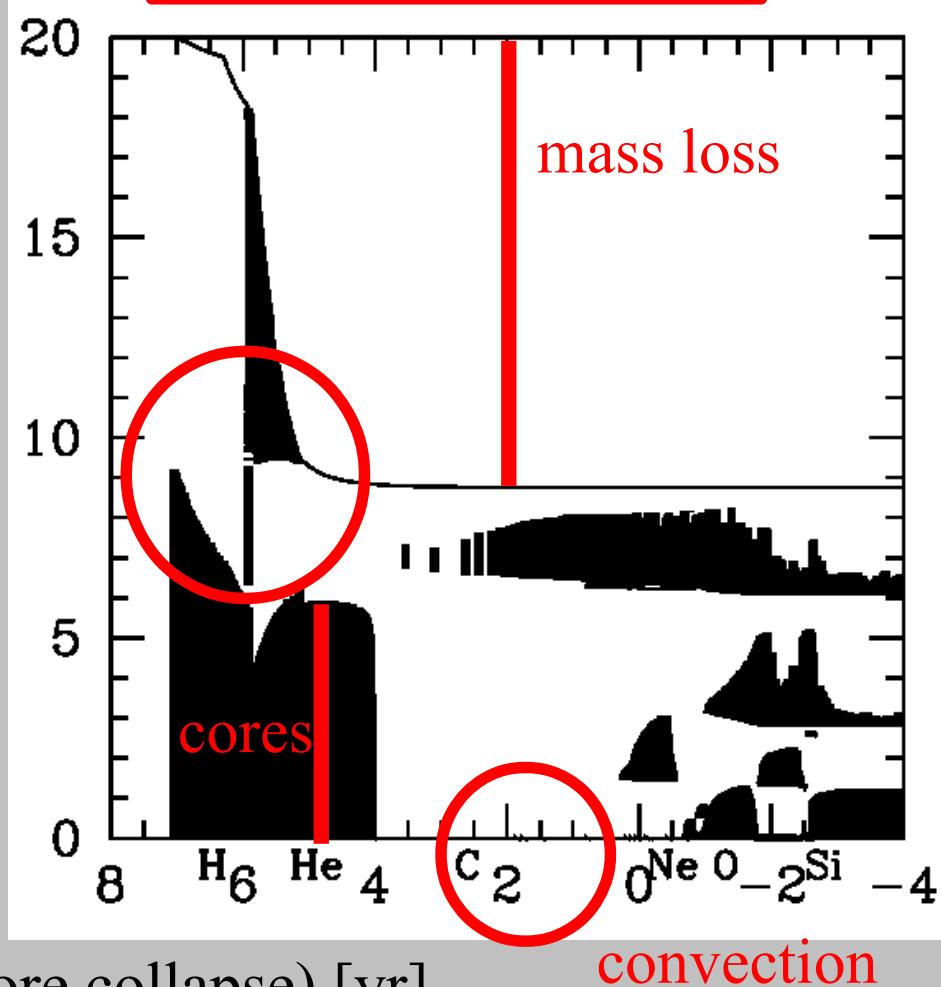
# Kippenhahn Diagrams, $\mathcal{M}_{ini} = 20 \mathcal{M}_o$

11

$v_{ini} = 0 \text{ km/s}$



$v_{ini} = 300 \text{ km/s}$



Log(time until core collapse) [yr]

# *Mass Loss: Types, Driving & Recipes*

Mass loss driving mechanism and prescriptions for different stages:

- O-type & “LBV” stars (bi-stab.): line-driven Vink et al 2000, 2001
- WR stars (clumping effect): line-driven Nugis & Lamers 2000, Gräfener & Hamann (2008)
- RSG: Pulsation/dust? de Jager et al 1988
- RG: Pulsation/dust? Reimers 1975,78, with  $\eta=\sim 0.5$
- AGB: Super winds? Dust Bloecker et al 1995, with  $\eta=\sim 0.05$
- LBV eruptions: continuous driven winds? Owocki et al
- ...

# *What changes at low Z?*

- Stars are **more compact**:  $R \sim R(Z_o)/4$  (lower opacities) at  $Z=10^{-8}$
- Rotation at low Z: stronger shear, weaker mer. circ.
- Mass loss weaker at low Z: → faster rotation

$$\dot{M}(Z) = \dot{M}(Z_o)(Z/Z_o)^\alpha$$

- $\alpha = 0.5-0.6$  (Kudritzki & Puls 00, Ku02)  
(Nugis & Lamers, Evans et al 05)
- $\alpha = 0.7-0.86$  (Vink et al 00,01,05)

$$Z(LMC) \sim Z_o/2.3 \Rightarrow \dot{M}/1.5 - \dot{M}/2$$

$$Z(SMC) \sim Z_o/7 \Rightarrow \dot{M}/2.6 - \dot{M}/5$$

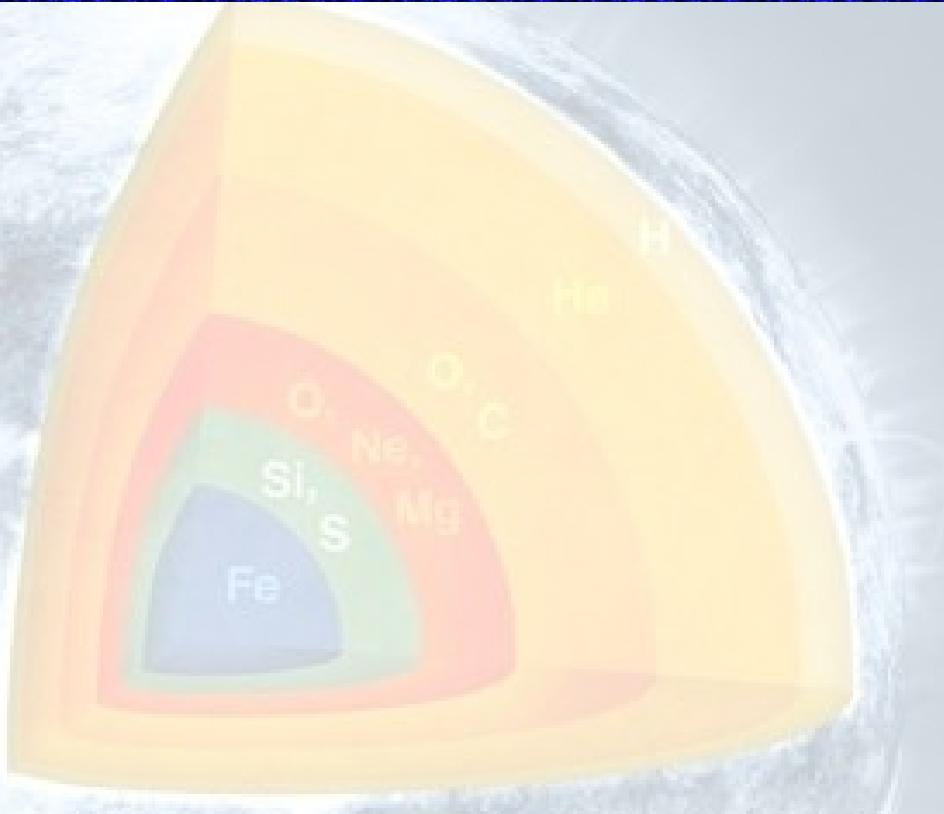
Mass loss at low Z still possible?

RSG (and LBV?): no Z-dep.; CNO? (Van Loon 05, Owocky et al)

Mechanical mass loss ← critical rotation/ Eddington limit  
(e.g. Hirschi 2007, Ekstroem et al 2008, Yoon et al 2012)

# *Physical Ingredients*

- Nuclear reactions
- Rotation
- Mass loss
- Convection
- Magnetic fields
- Binarity (following talks)
- Equation of state, opacities & neutrino losses  
including metallicity dependence



## Advantages:

- model fluid instabilities (e.g. Rayleigh-Taylor)
- modeling 3D processes
- model diffusive and advective processes

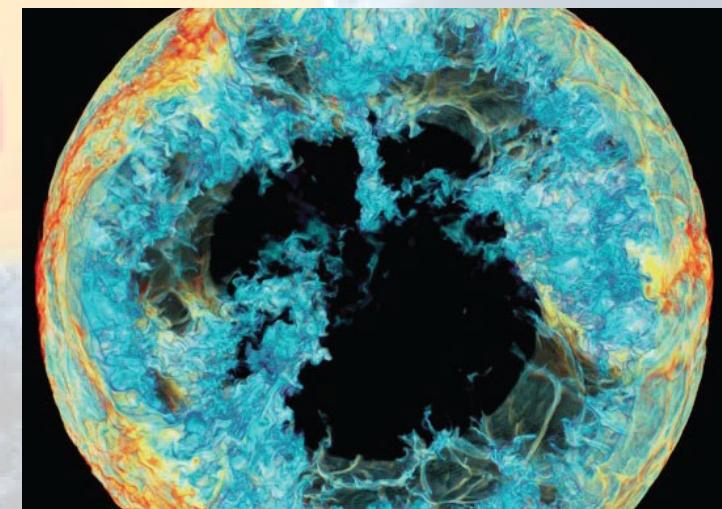
## Disadvantages:

- resolution dependent?
- initial condition dependent?
- computational cost
- limited to dynamical timescales ( $t_{\text{conv}} \sim 1\text{ s} - \text{days}$ )

# 3D stellar models

## What's missing?

- full star or lifetime simulations
- Large scale (LES) and small scale (DNS) cannot be followed simultaneously



Herwig, Woodward et al 2013

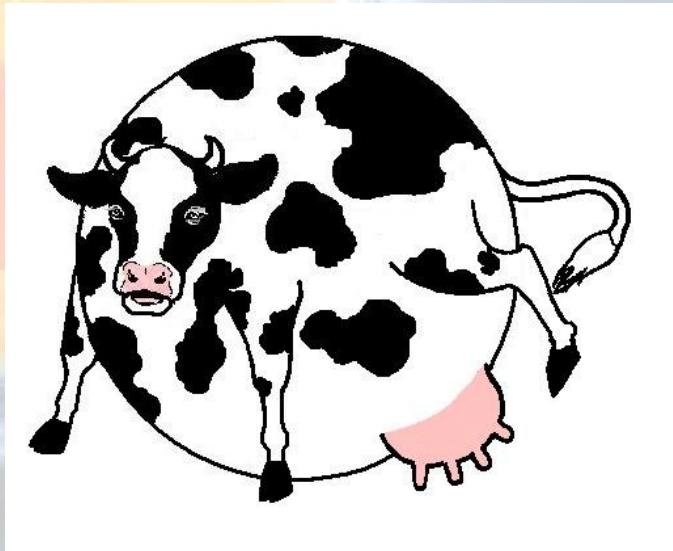
## Advantages:

- model entire evolution ( $\Delta t \sim 10^3$  yrs)
- compare to observations
- progenitor models
- large grids (M, Z)

## Disadvantages:

- parametrized physics (e.g. convection)
- missing multi-D processes
- incapable of modelling turbulence

# 1D stellar models



## What's missing?

- self-consistent physical descriptions of mass loss, **convection**, **rotation**, magnetic fields, opacity, binarity

# *Convection: Current Implementation in 1D Codes*

## Multi-D processes:

Major contributor to turbulent mixing

Turbulent entrainment at convective boundaries

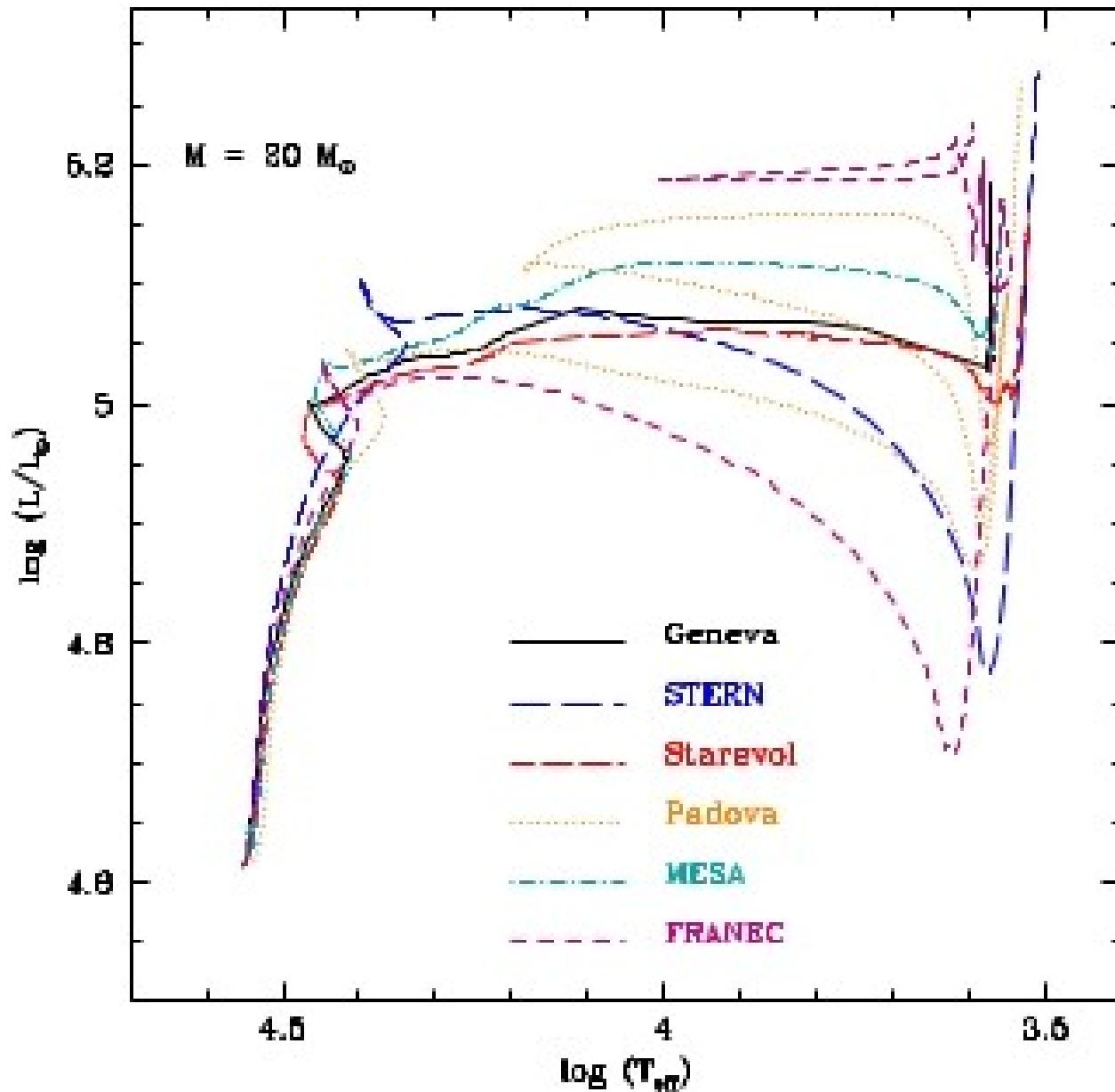
Internal gravity waves

## 1D prescriptions:

- Energy transport in convective zone: **mixing length theory (MLT)** *Bohm-Vitense (1957, 58)*, or updates, e.g. FST: *Canuto & Mazitelli (1991)*
- Boundary location: Schwarzschild criterion OR Ledoux (+semi-convection)
- Convective boundary mixing (CBM, also composition dependent)

# *1D Model Uncertainties*

*Martins and Palacios (2013)*



Different prescriptions for convective mixing and free parameters **strongly affect** post-MS evolution.

See also Jones et al 2015,  
MNRAS, 447, 3115

# 1D Model Uncertainties: Complex Convective History

**Detailed convective shell history affects fate of models: strong/weak/failed explosions!!!**

Sukhbold & Woosley, 2014ApJ...783...10S

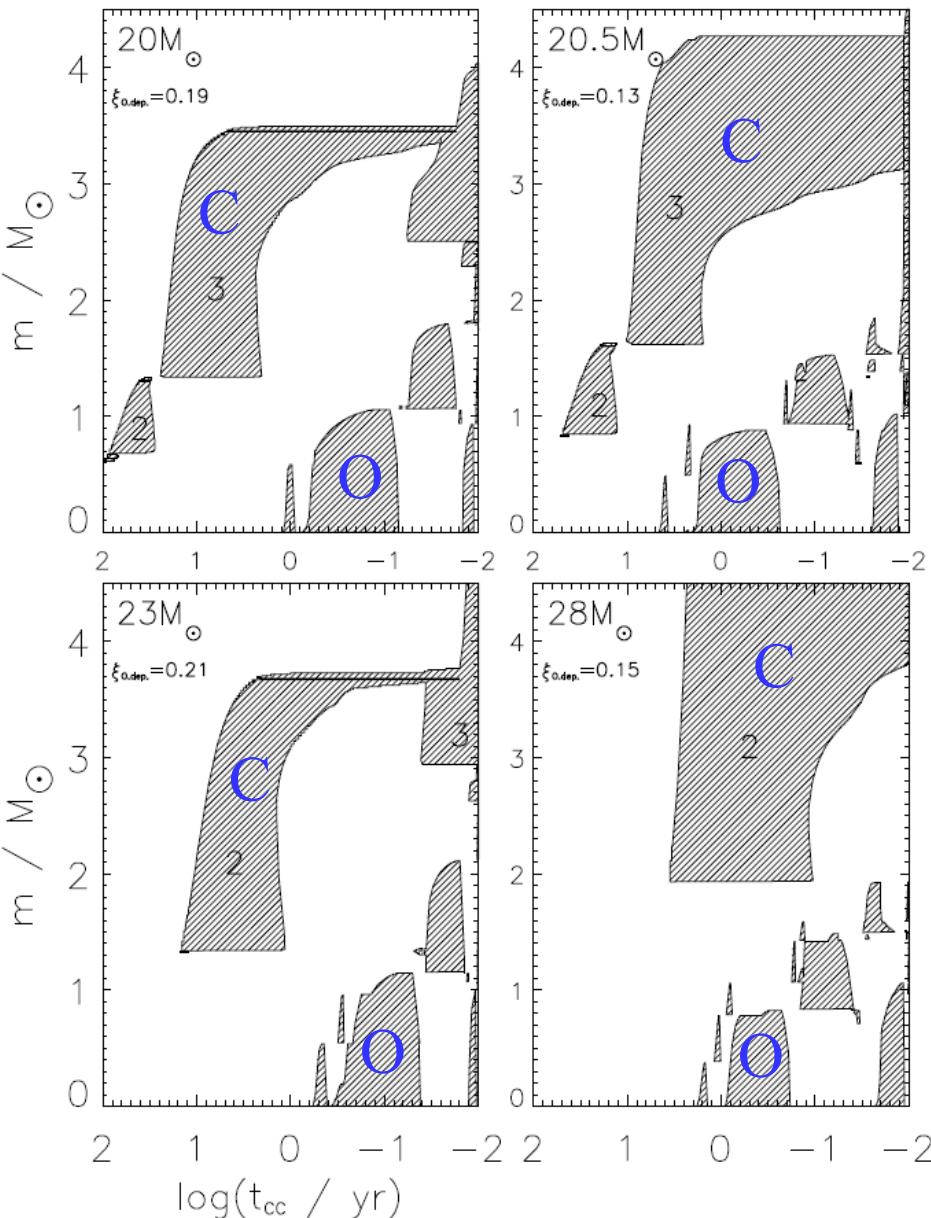


FIG. 13.— Convective history of four models showing the major

Sukhbold, Ertl et al, 2016ApJ...821...38S,  
Ugliano et al 2012, Ertl et al 2015

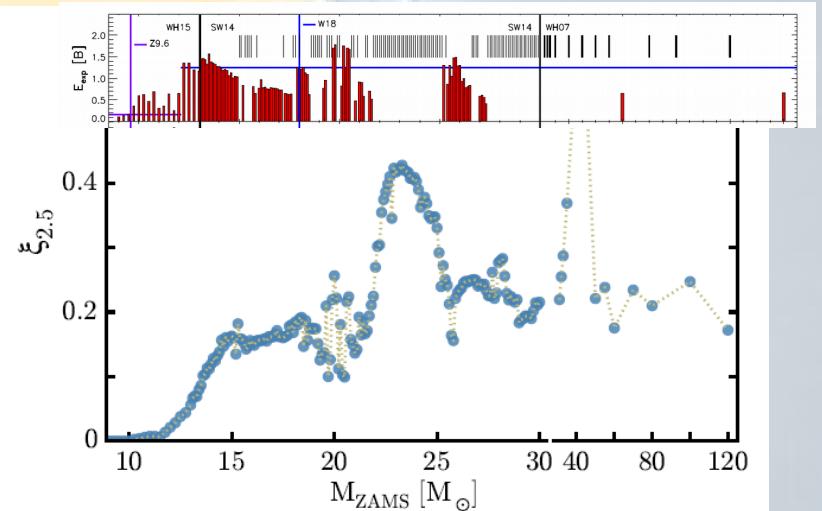


FIG. 1.— The compactness parameter,  $\xi_{2.5}$ , (eq. (1); O'Connor & Ott 2011) characterizing the inner  $2.5 M_\odot$  of the presupernova star is shown as a function of zero-age main sequence (ZAMS) mass for all 200 models between  $9.0$  and  $120 M_\odot$ . The compactness

**Non-monotonic behaviour!**

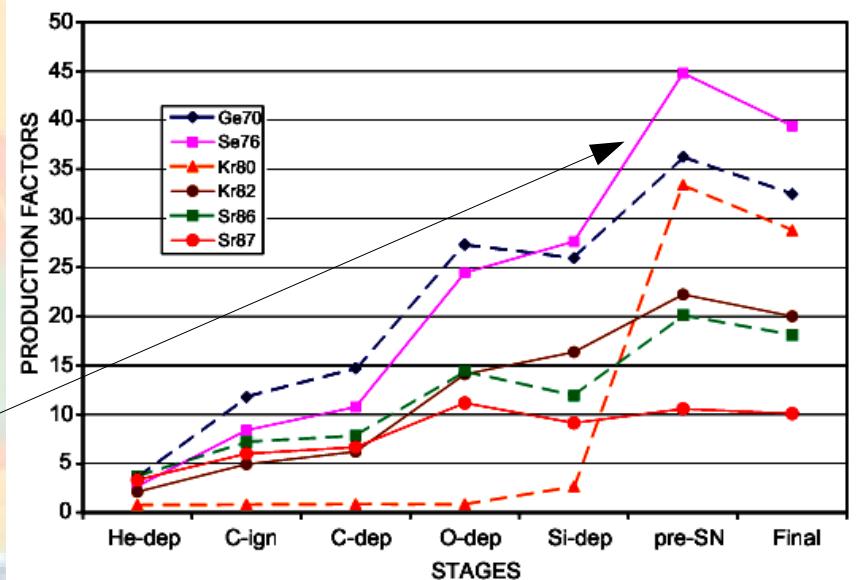
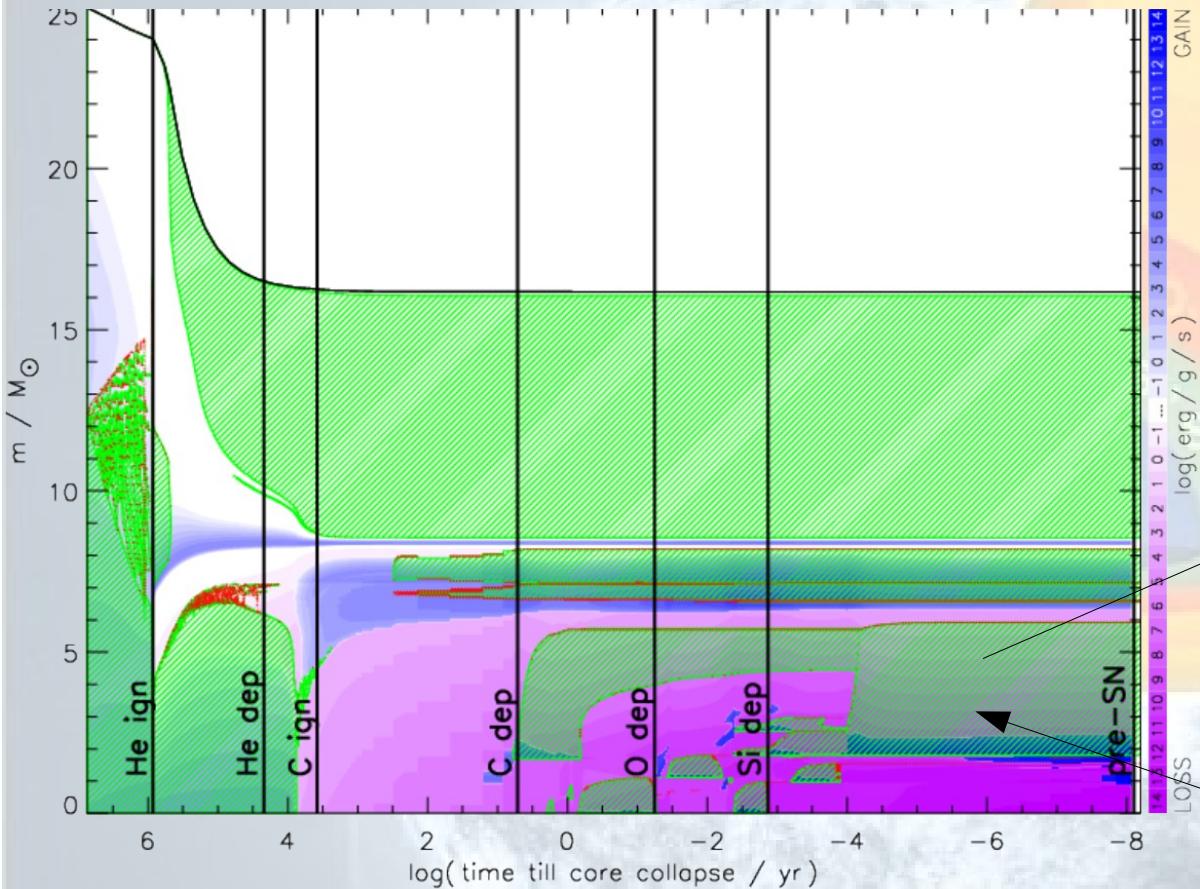
We are particularly interested in how the “explodability” of the presupernova models and their observable properties correlate with their “compactness” (Fig. 1; O’Connor & Ott 2011)

$$\xi_M = \frac{M/M_\odot}{R(M)/1000 \text{ km}} \Big|_{t_{\text{bounce}}}, \quad (1)$$

and other measures of presupernova core structure (§ 3.1.3; Ertl et al. (2015)). Using a standard central engine in presupernova models of variable compactness, a significant correlation in outcome is found (§ 4). As pre-

# 1D Model Uncertainties: Possible Shell Mergers

Tur, Heger et al 07/09/10



C/Ne/O shell mergers

Rauscher, Heger and Woosley 2002: "Interesting and unusual nucleosynthetic results are found for one particular 20M model as a result of its special stellar structure."

Shell mergers also affect compactness

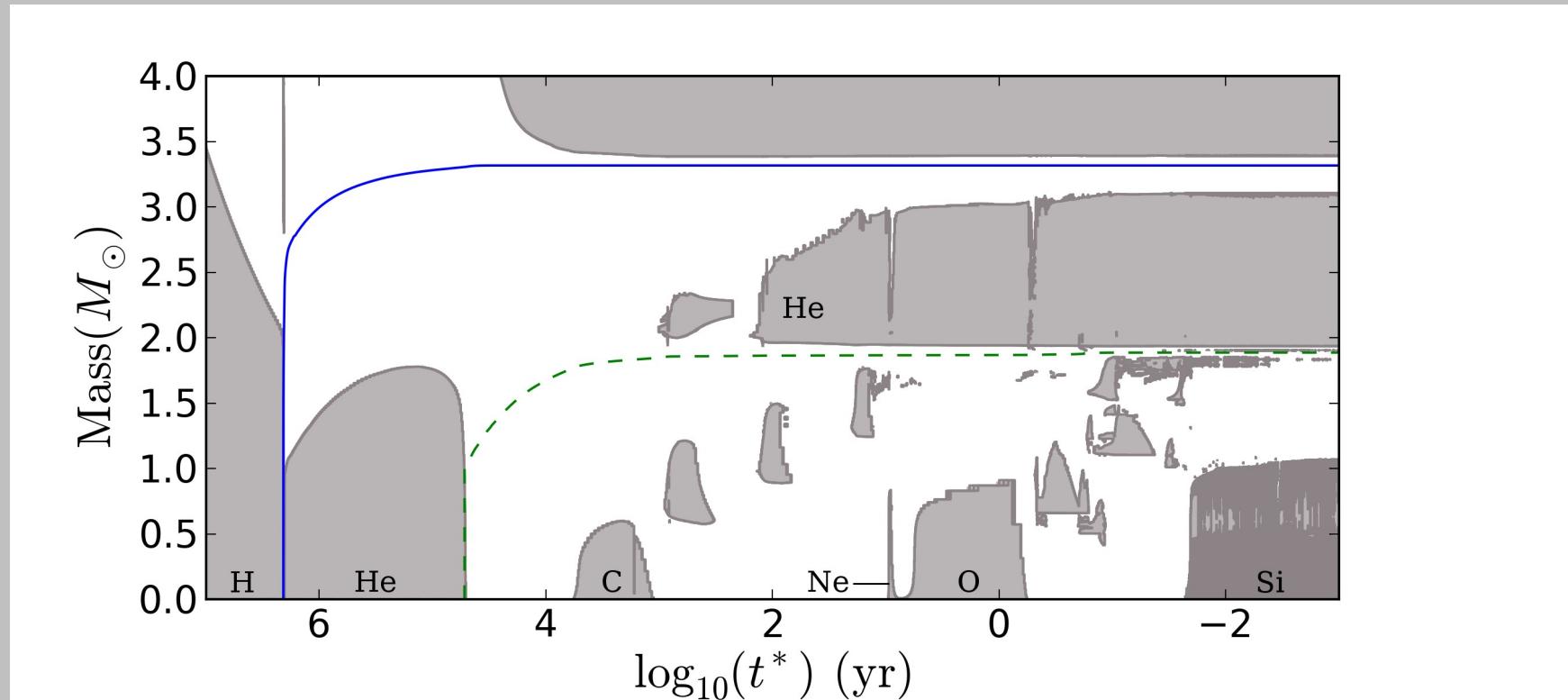
**Convection physics uncertainties affect fate of models: strong/weak/failed explosions!!!**

# Fate of Least-Massive MS: ECSN/Fe-CCSN?

7-15 M<sub>o</sub> models ← MESA stellar evolution code: <http://mesa.sourceforge.net/>

Paxton et al 10,12

12 M<sub>o</sub> is a typical massive star:



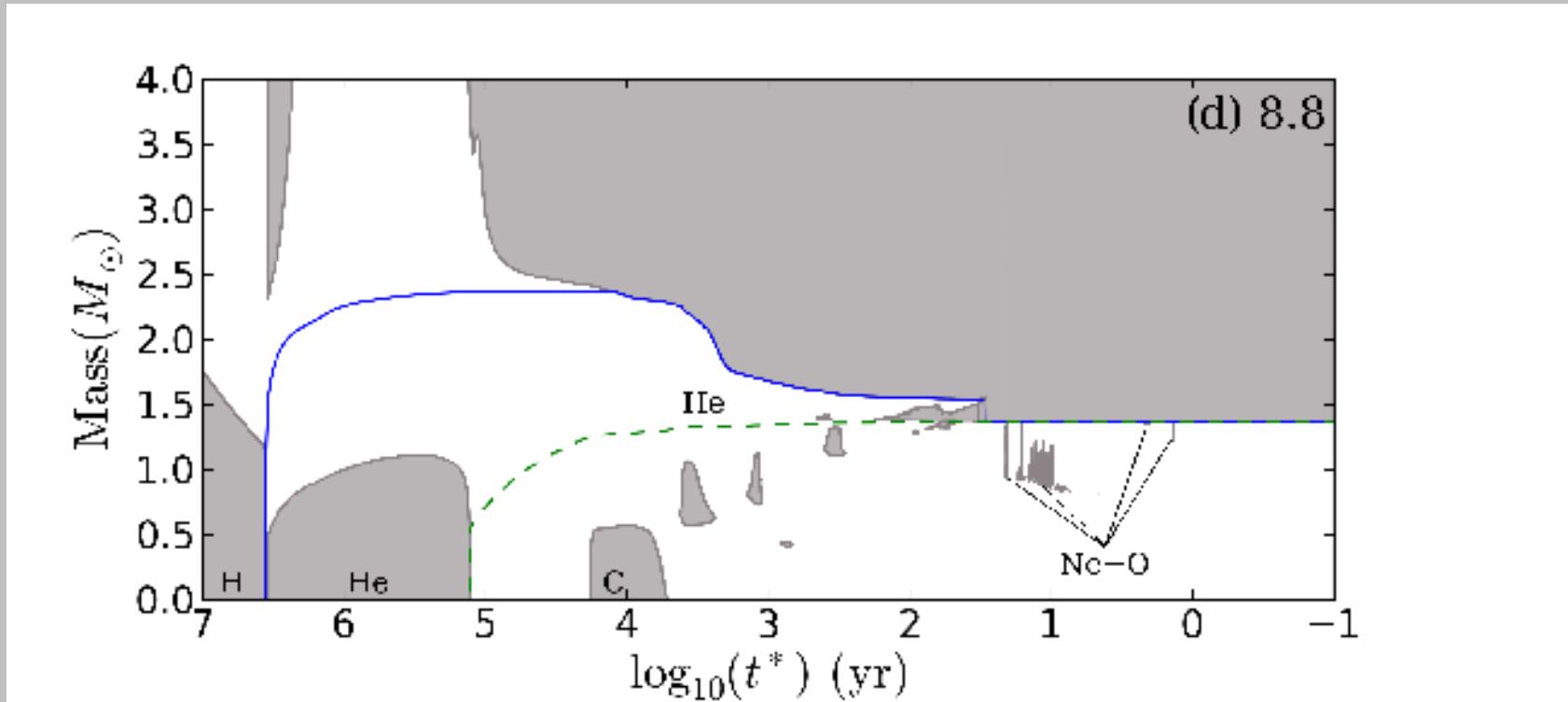
All burning stages ignited centrally. Fate: Fe-CCSN

Jones et al. (2013), ApJ 772, 150;

see also Mueller et al 12, Umeda et al 12, Takahashi et al 13

# Fate of Least-Massive MS: ECSN/Fe-CCSN?

8.8  $M_{\odot}$  failed massive star:



Ne-b. starts off-centre but does not reach the centre.  
MESA → Oxygen deflagration

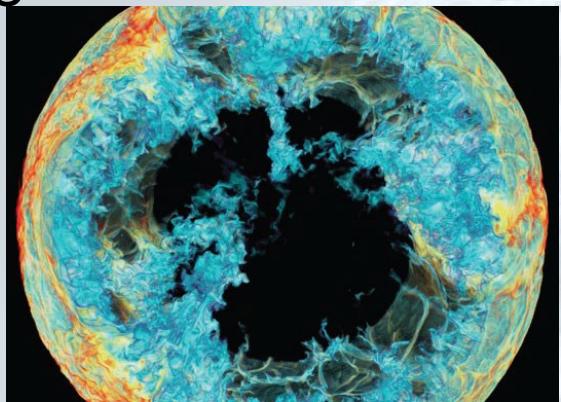
Jones et al. (2013), ApJ 772, 150  
See also Nomoto 84: case 2.6  
Timmes et al 92, 94  
Eldridge & Tout 04  
Heger et al ...

Fate: ECSN!

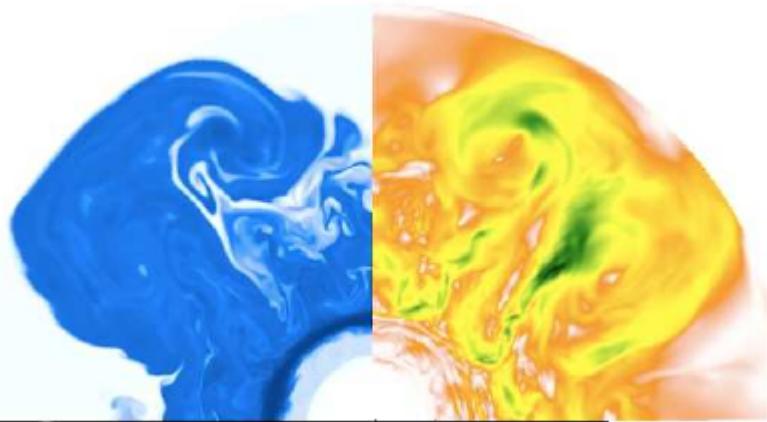
Key uncertainties: convective boundary mixing, mass loss

# Way Forward: 1 to 3 to 1D link

Targetted 3D simulations

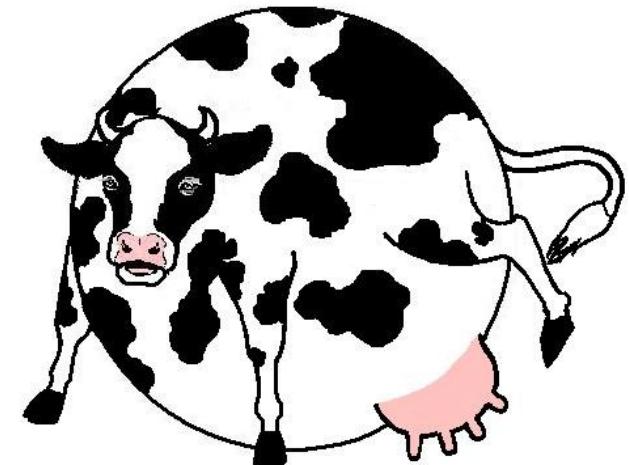


Herwig et al 06, Herwig, Woodward et al 2013

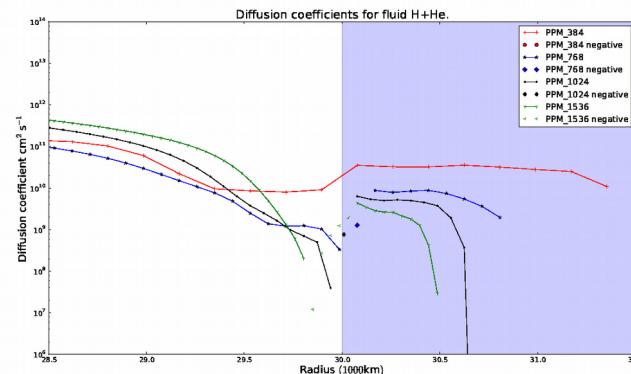


e.g. Arnett & Meakin 2011, ...  
Mocak et al 2011,  
Viallet et al 2013, ...

Uncertainties in 1D



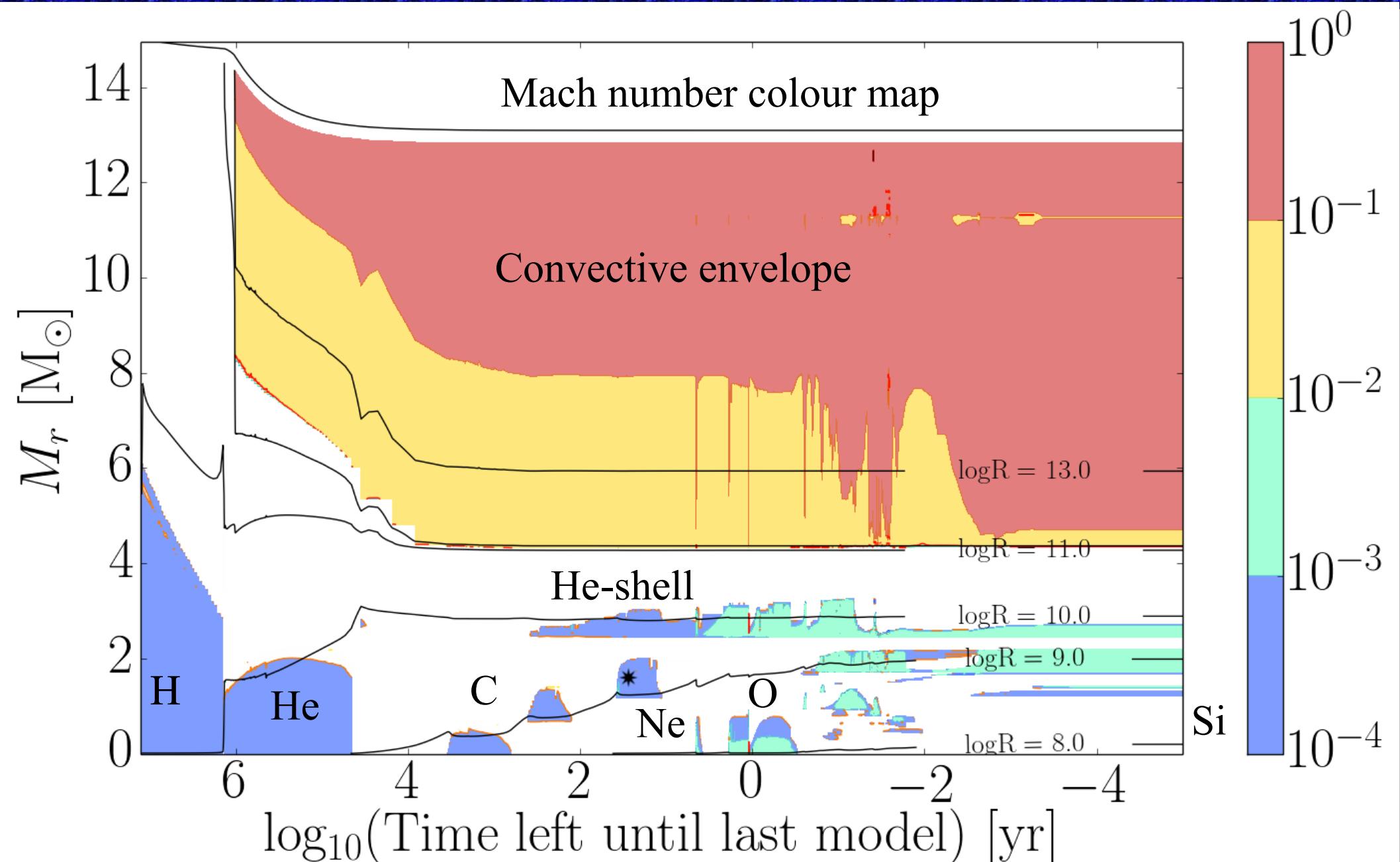
+asteroseismology



Meakin et al 09 ; Bennett et al (thesis), Jones et al 16

→ Determine effective coefficient / improve theoretical prescriptions

# *Where to Start?*



Convection takes place during most burning stages

# Priority List + Existing Efforts

\* Convective boundary mixing during core hydrogen burning: (Meakin & Arnett 07ApJ667, Gilet 13)

- +: many constraints (HRD, astero, ...)
- -: difficult to model due to important thermal/radiative effects
- -: long time-scale
- 

\* Silicon burning:

- +: important to determine impact on SNe of multi-D structure in progenitor (Couch et al 2015a,b, Mueller & Janka aph1409.4783, Mueller et al ArXiv1605.01393)
- +: possible shell mergers occurring after core Si-burning (e.g. Tur et al 2009ApJ702.1068; Sukhbold & Woosley 2014ApJ783.105) strongly affect core compactness
- +: radiative effects small/negl.
- -: ~  $10^9$  CPU hours needed for full silicon burning phase will be ok soon;
- -: might be affected by convective shell history
- 

\* AGB thermal pulses/H-ingestion:

- +: already doable (e.g. Herwig et al 2014ApJ729.3, 2011ApJ727.89, Mocak et al 2010A&A520.114, Woodward et al 2015)
- +: thermal/radiative effects not dominant
- ?: applicable to other phases?
- 

\* Oxygen shell: (Meakin & Arnett 2007ApJ667.448/665.448, Viallet et al 2013ApJ769.1, Jones et al ArXiv1605.03766)

- +: similar to silicon burning but smaller reaction network needed
- -: might be affected by convective shell history
- 

\* Carbon shell: (PhD A. Cristini)

- +: not affected by prior shell history
- +: first stage for which thermal effects become negligible
- 

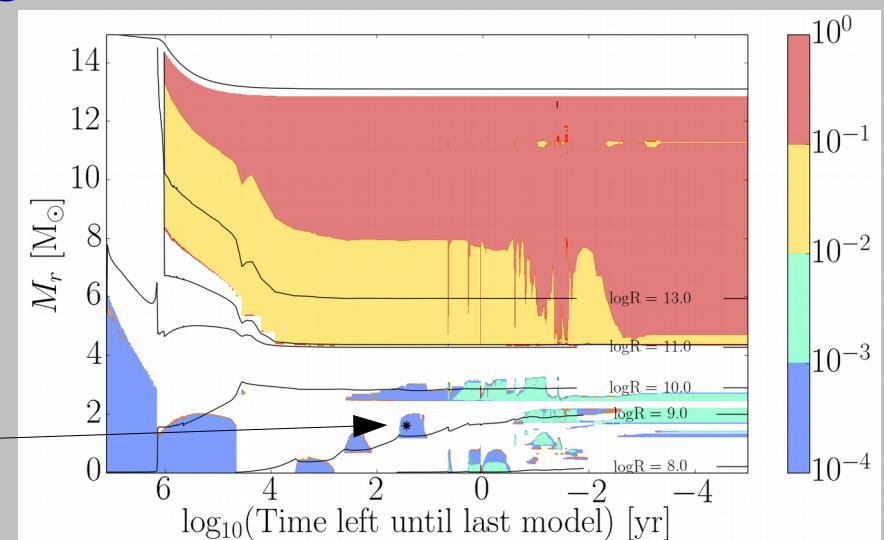
\* Envelope of RSG (e.g. Viallet et al. 2013, Chiavassa et al 2009-2013),

\* Solar-type stars (e.g. Magic et al. 2013A&A557.26, ...)

# *Where to Start? Carbon burning shell*

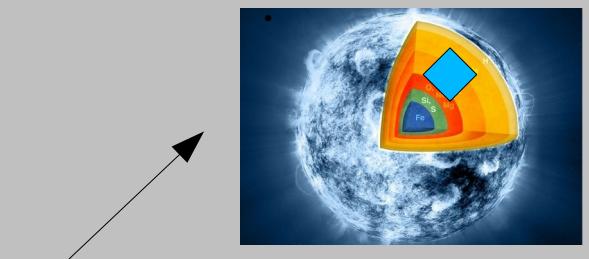
- “Simple” convective history before C-shell
- Cooling dominated by neutrinos: 1) radiative diffusion can be neglected, 2) “fast” timescale
- O-shell done before *Meakin & Arnett 2007-*  
*important*
- H/He burning lifetime much longer + *radiative effects*
- Si-burning: complex  
reaction network

C-shell



# *C-shell Setup & Approximations*

- PROMPI code Meakin, Arnett et al 2007-...
- Initial conditions provided by stellar model from GENEC:  
 $15M_{\odot}$ , non-rotating at solar metallicity (see previous slide)
- “Box in a star” (plane-parallel) simulation using Cartesian co-ordinates
- Parameterised gravitational acceleration and  $^{12}\text{C}+^{12}\text{C}$  energy generation rate  
(energy rate boosted by a factor of 1000 for parameter study)
- Radiative diffusion neglected
- Turbulence initiated through random low-amplitude perturbations in temperature and density
- Constant abundance of  $^{12}\text{C}$  fuel over simulation time
- 4 resolutions: lrez:  $128^3$ , mrez:  $256^3$ , hrez:  $512^3$ , vhrez:  $1024^3$

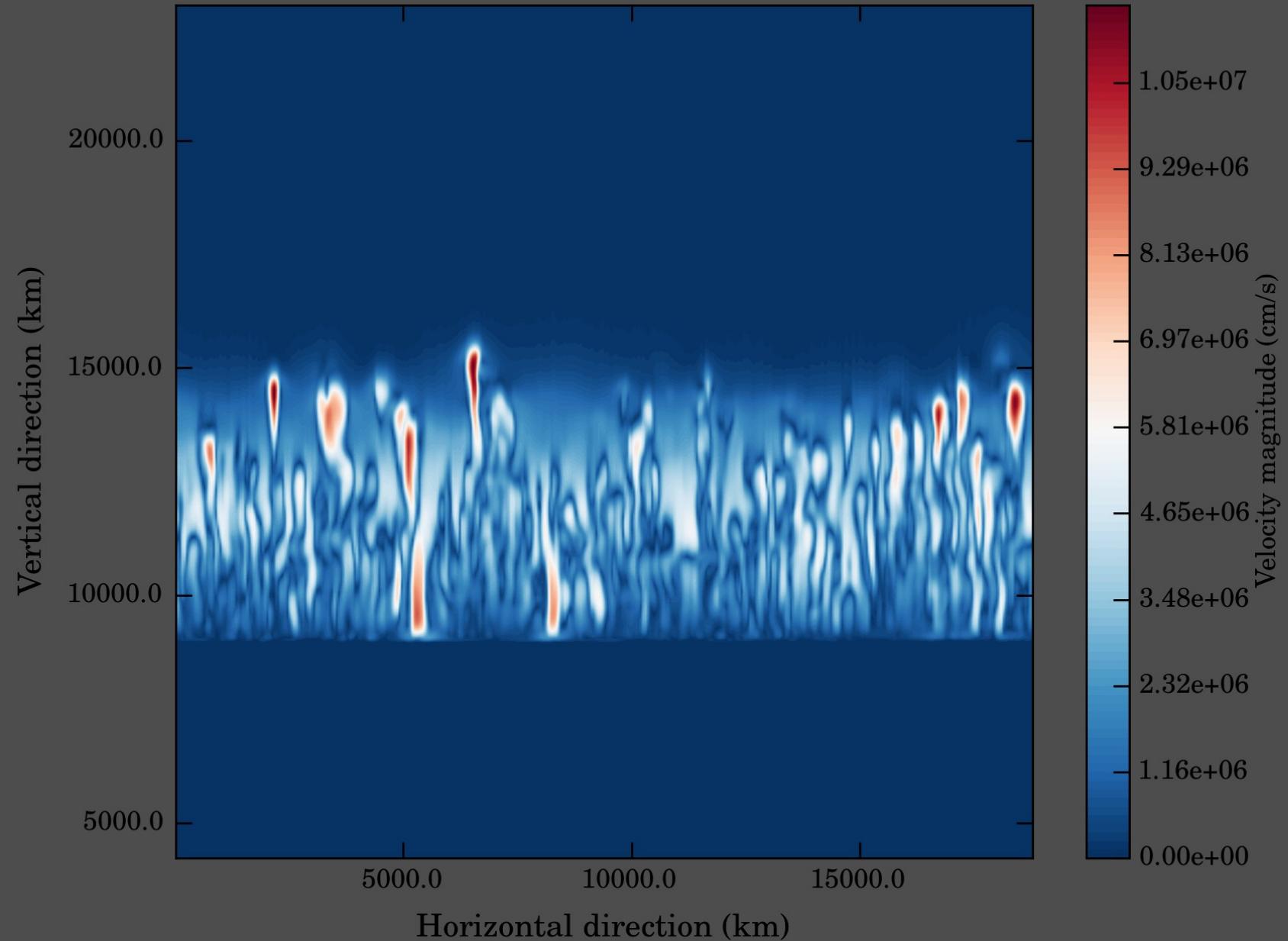


# *C-shell Simulations*

$384^3$  res.

Gas Velocity:  $\|v\|$ , t = 300.1 s (dump 31)

run

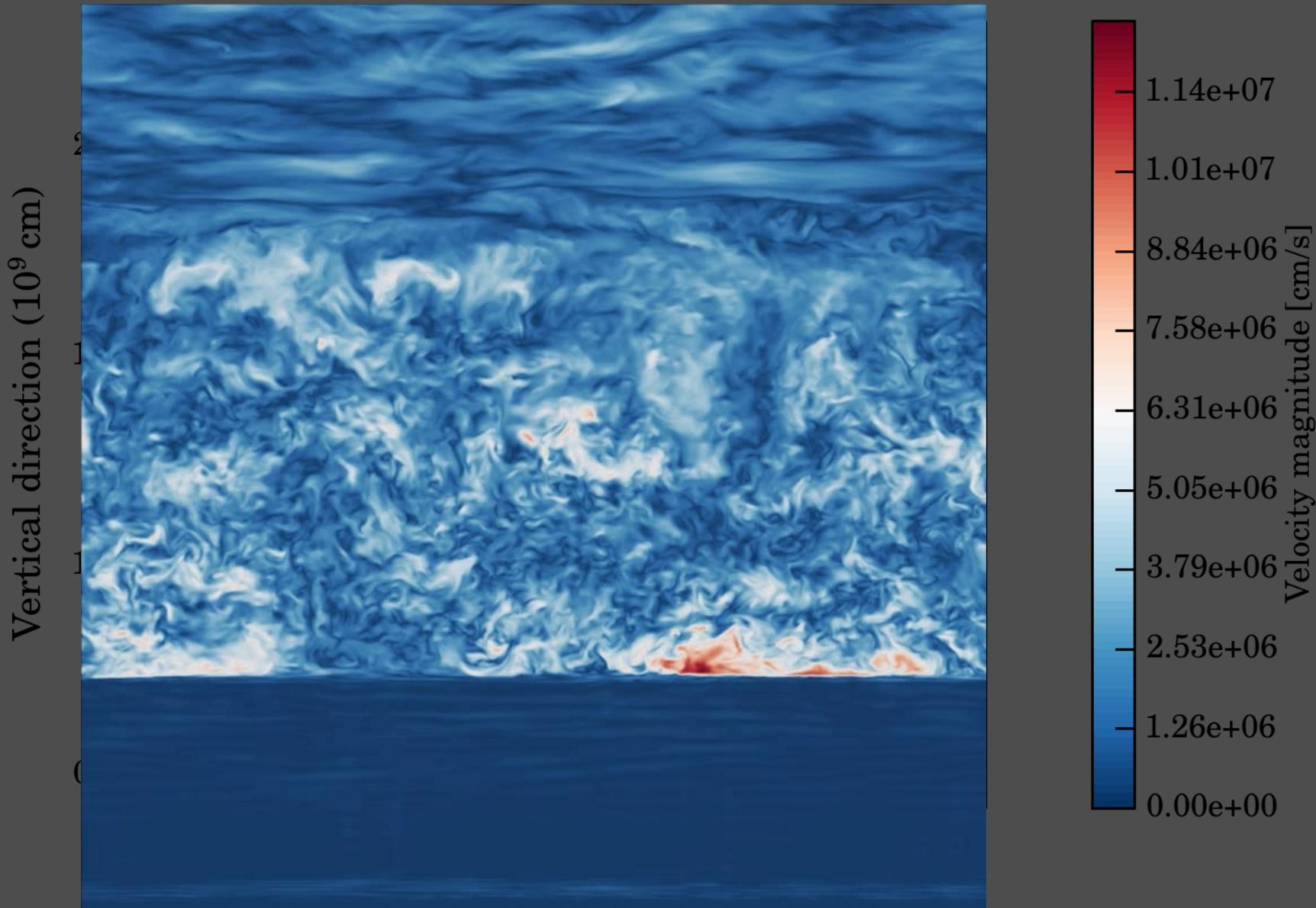


Cristini et al  
in 2017

# *C-shell Simulations: |v| movie*

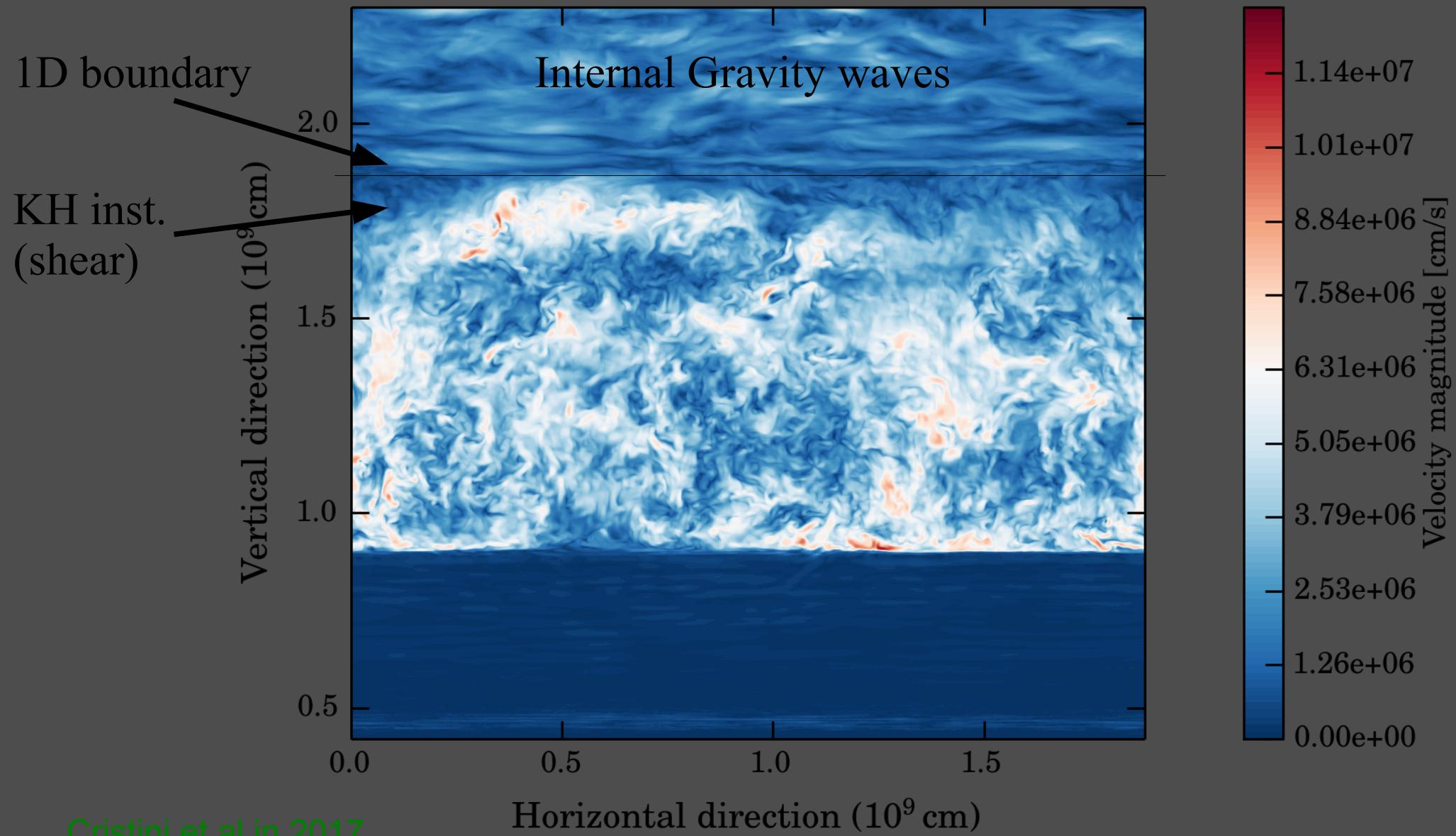
Cristini et al in 2017

Gas Velocity  $\|v\|$

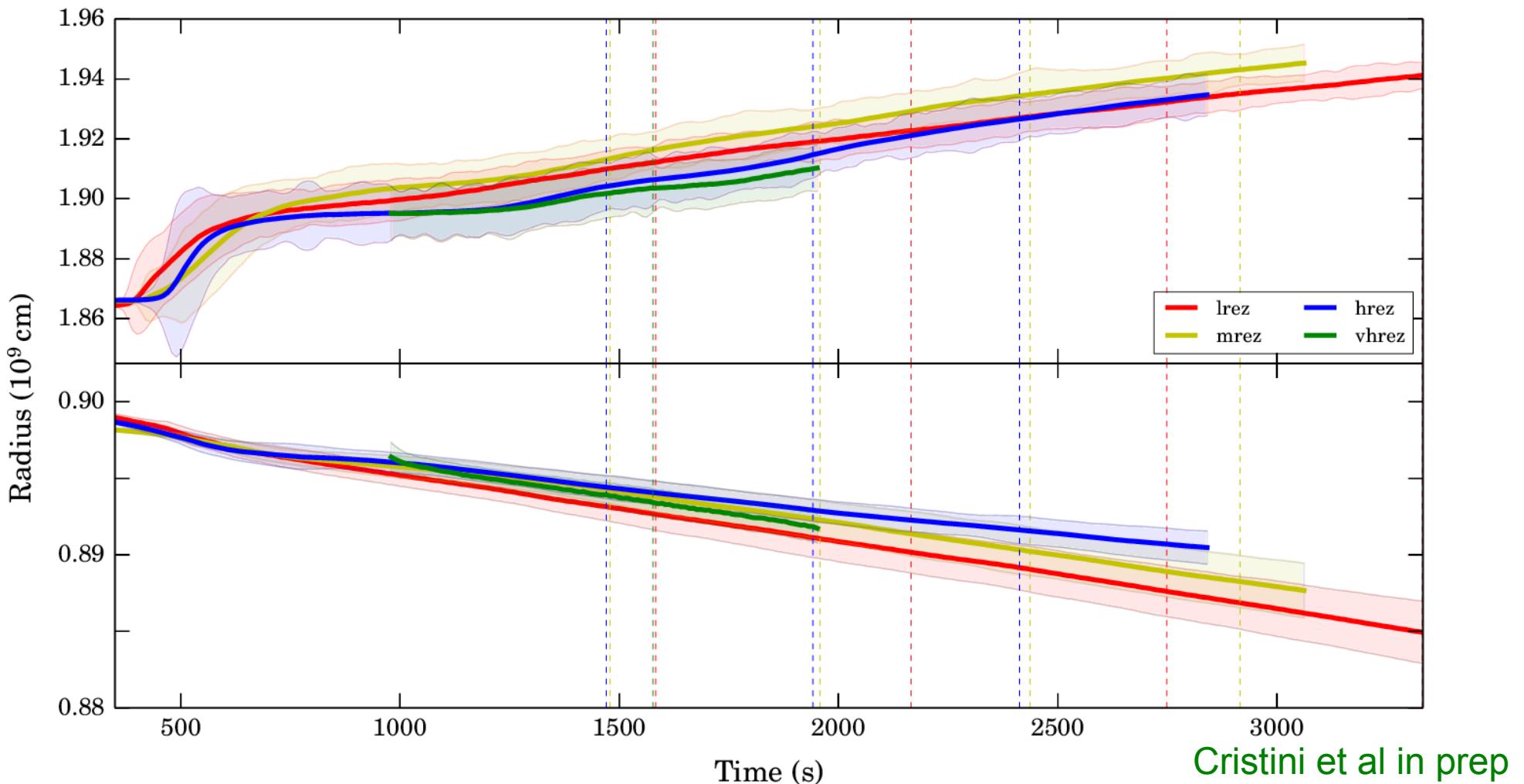


# *C-shell Simulations*

Snapshot from  $1024^3$  resolution run: Gas Velocity  $\|v\|$



# *Boundary Entrainment*

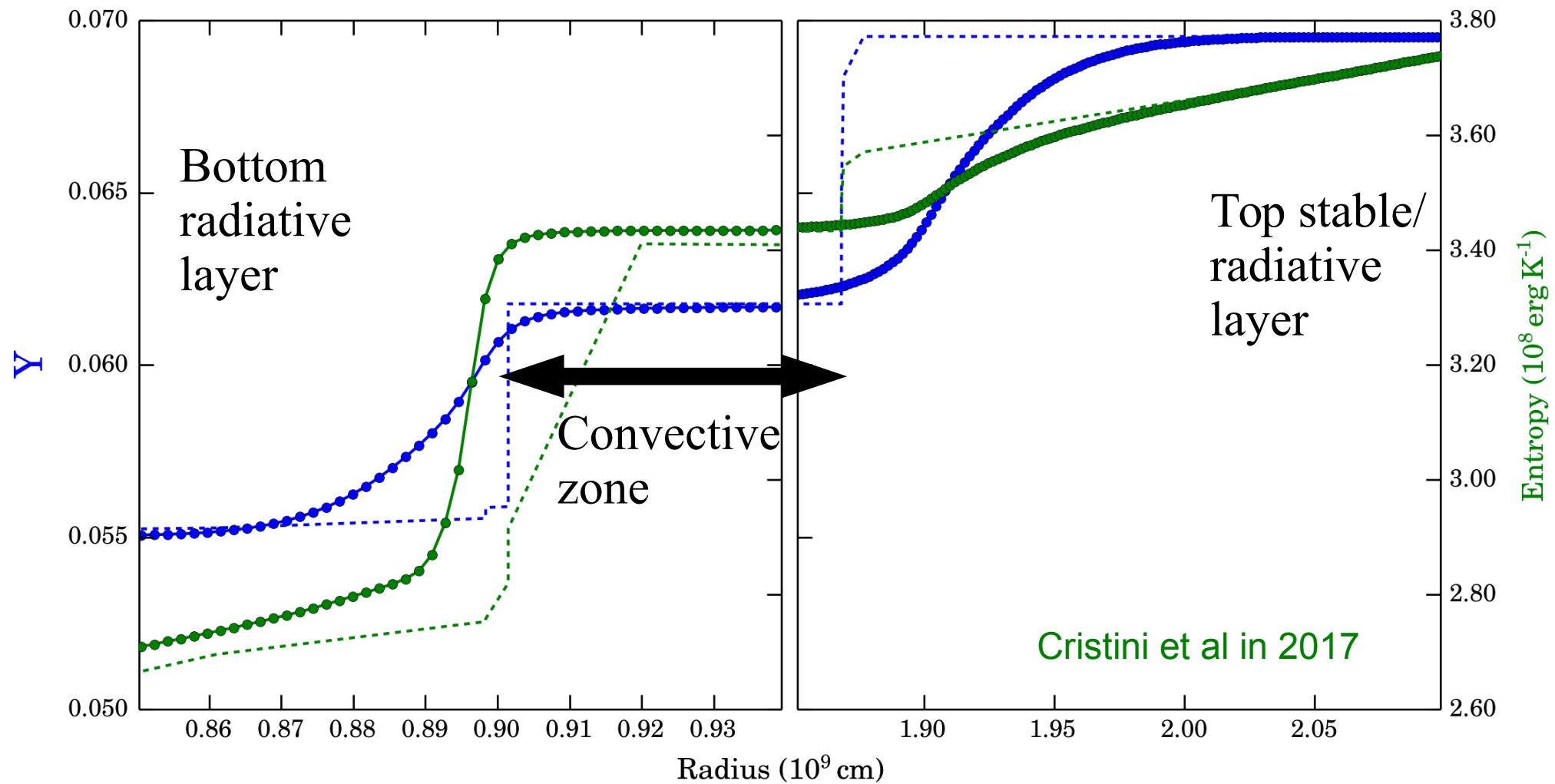


Cristini et al in prep

Top:  $u_e \sim 20,000 \text{ cm/s}$ ; Bottom:  $u_e \sim 3,000 \text{ cm/s}$ . Rescaled for  $\epsilon_{\text{burn}}$  boosting (1/1000)  
→ In 1 year, top:  $\Delta R \sim 6 \times 10^8 \text{ cm}$ , bottom:  $\Delta R \sim 10^8 \text{ cm}$ : large but reasonable

Consistent with oxygen-shell results and entrainment law.

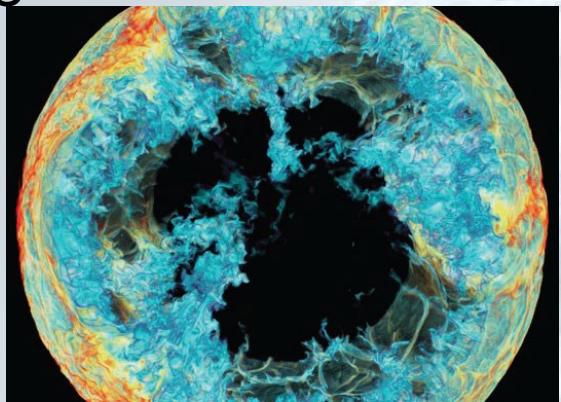
# $3\mathcal{D}$ versus $1\mathcal{D}$



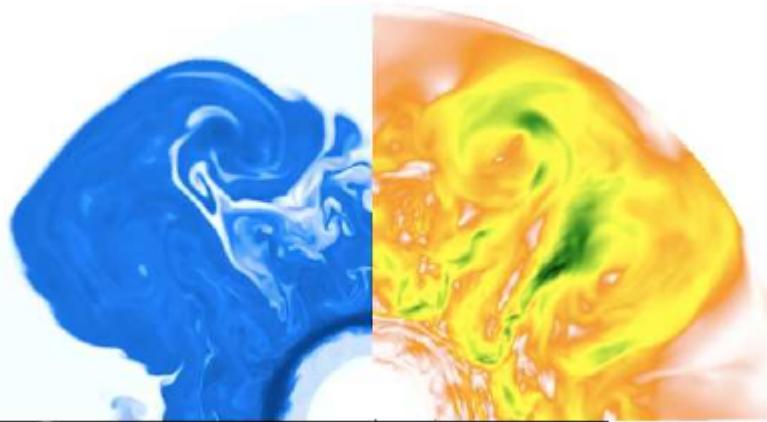
- Improved prescriptions for CBM needed!

# Way Forward: 1 to 3 to 1D link

Targetted 3D simulations

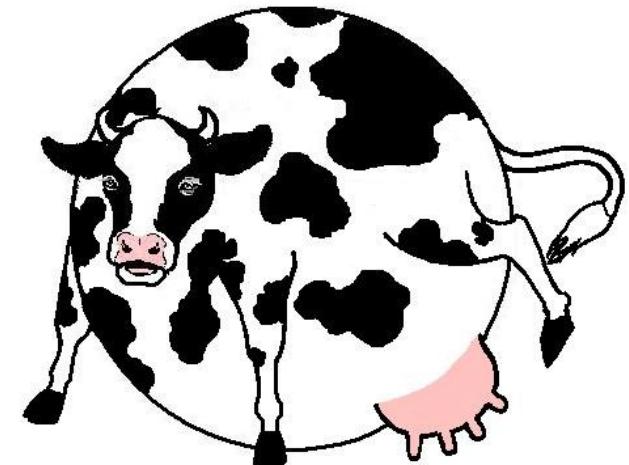


Herwig et al 06, Herwig, Woodward et al 2013

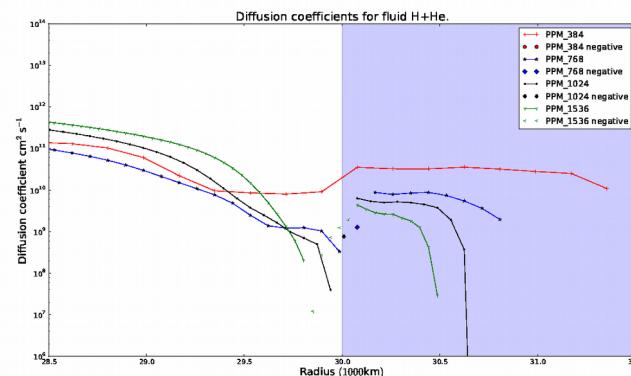


e.g. Arnett & Meakin 2011, ...  
Mocak et al 2011,  
Viallet et al 2013, ...

Uncertainties in 1D



+asteroseismology



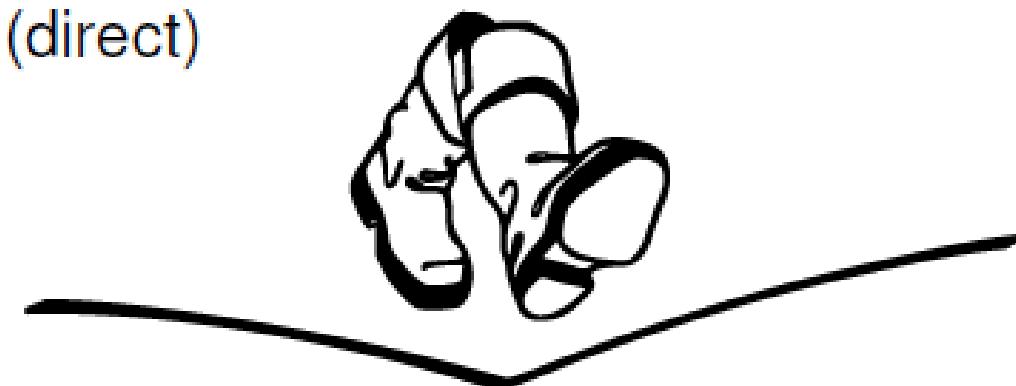
Meakin et al 09 ; Bennett et al (thesis), Jones et al 16

→ Determine effective coefficient / improve theoretical prescriptions

# SLH (Seven-League Hydro) Code

F. Miczek, F. K. Röpke, P. V. F. Edelmann

- ▶ solves the compressible Euler equations in 1-, 2-, 3-D
- ▶ explicit and implicit time integration
- ▶ flux preconditioning to ensure correct behavior at low Mach numbers
- ▶ other low Mach number schemes (e.g. AUSM<sup>+</sup>-up)
- ▶ works for low and high Mach numbers on the same grid
- ▶ hybrid (MPI, OpenMP) parallelization (scaling up to 100 000 cores)
- ▶ several solvers for the linear system:  
BiCGSTAB, GMRES, Multigrid, (direct)
- ▶ arbitrary curvilinear meshes
- ▶ radiation in the diffusion limit
- ▶ general equation of state
- ▶ general nuclear reaction network



*Seven-League Hydro*

# *Conclusions & Outlook*

- General link between MS and SN type understood
- BUT
- Physical ingredients still uncertain: convection, rotation, mass loss, B-fields, binarity
- 1D to 3D to 1D work underway for convection and rotation
- Priority list established: **large effort needed!**

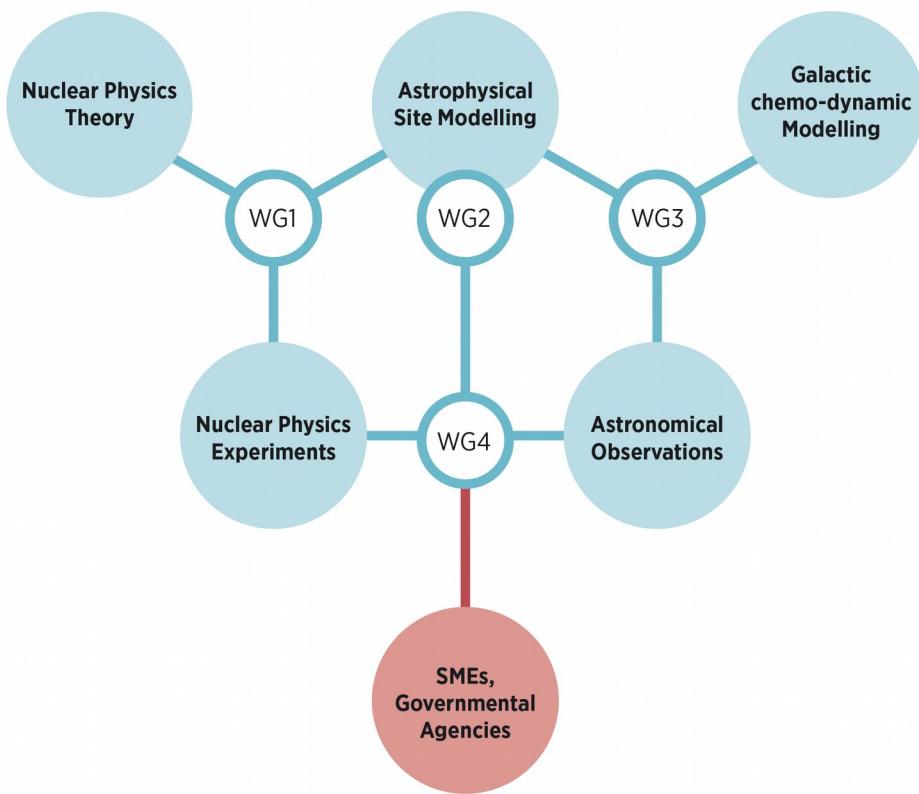
# *ChETEC COST Action (2017-2021)*

## **Chemical Elements as Tracers of the Evolution of the Cosmos**

A network to bring European research, science and business together to further our understanding of the early universe



Keele  
University



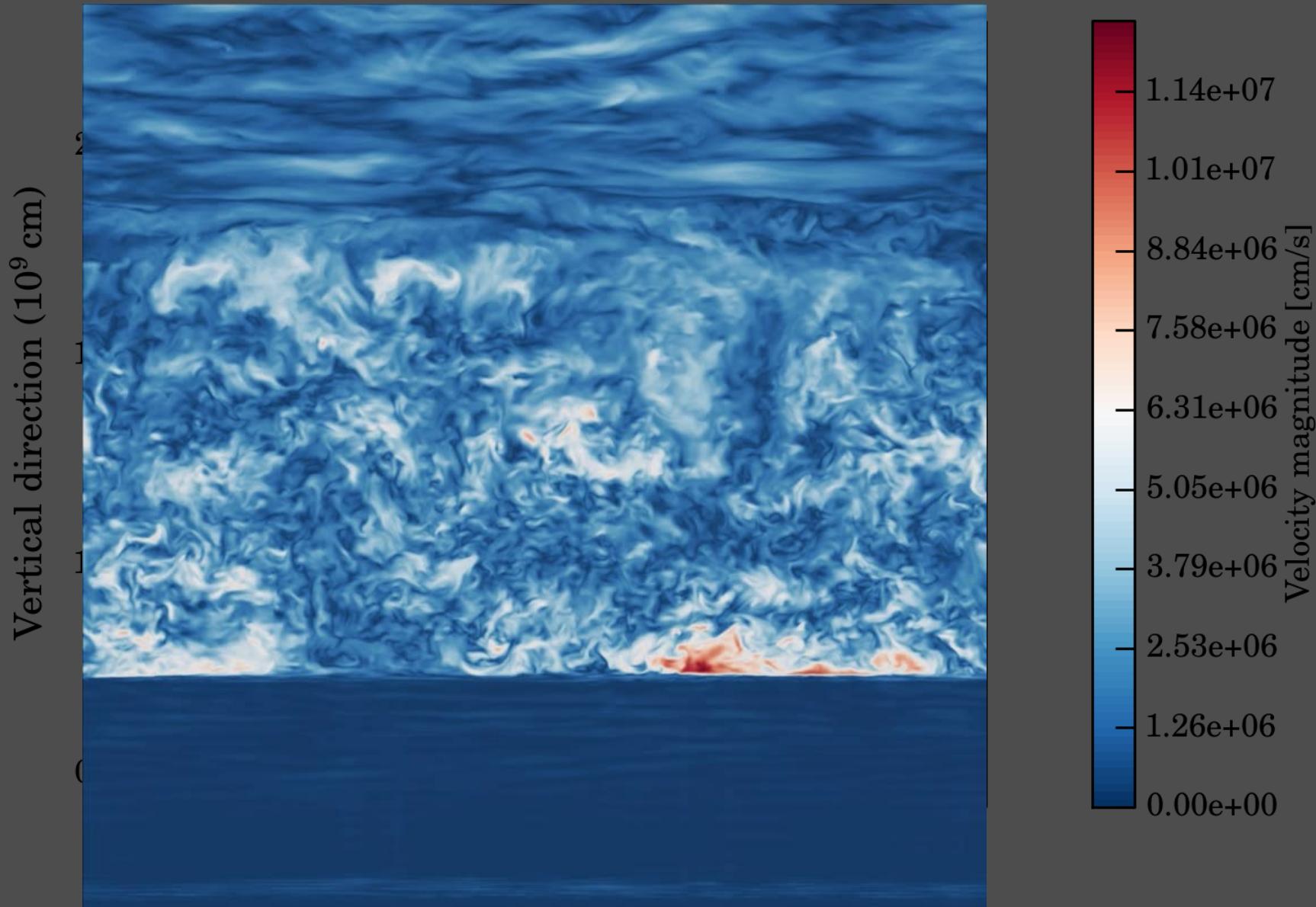
~100 scientists across 25-30 European countries to coordinate research efforts in Nuclear Astrophysics

[http://www.cost.eu/COST\\_Actions/ca/CA16117](http://www.cost.eu/COST_Actions/ca/CA16117)

# *C-shell Simulations: |v| movie*

Cristini et al in 2017

Gas Velocity  $\|v\|$



# *Recent Papers/Reviews*

## Reviews:

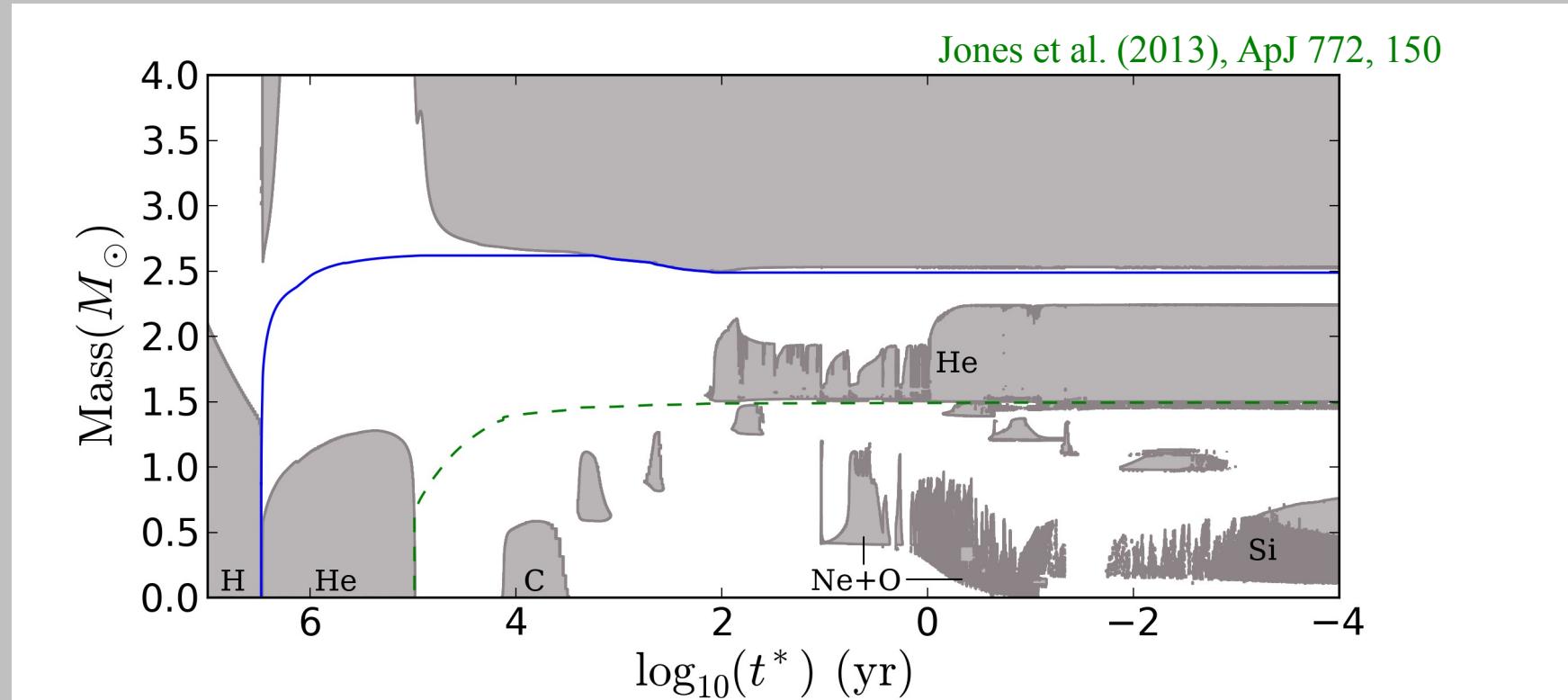
- Umeda, Yoshida and Takahashi, "Massive Star Evolution and Nucleosynthesis -Lower End of Fe-Core Collapse Supernova Progenitors and Remnant Neutron Star Mass Distribution", 2012arXiv1207.5297U, Accepted for publication in Progress of Theoretical and Experimental Physics
- Langer, "Pre-Supernova Evolution of Massive Single and Binary Stars", ARAA, 2012, astroph-1206.5443
- Maeder and Meynet, "Rotating massive stars: From first stars to gamma ray bursts", 2012RvMP...84...25M
- Woosley, Heger and Weaver,"The evolution and explosion of massive stars", 2002RvMP...74.1015W

## Textbooks:

- R. Kippenhahn & A. Weigert, Stellar Structure and Evolution, 1990, Springer-Verlag, ISBN 3-540-50211-4
- A. Maeder, Physics, Formation and Evolution of Rotating Stars, 2009, Springer-Verlag, ISBN 978-3-540-76948-4

# Fate of Least-Massive MS: ECSN/Fe-CCSN?

$9.5 M_{\odot}$  still a massive star:



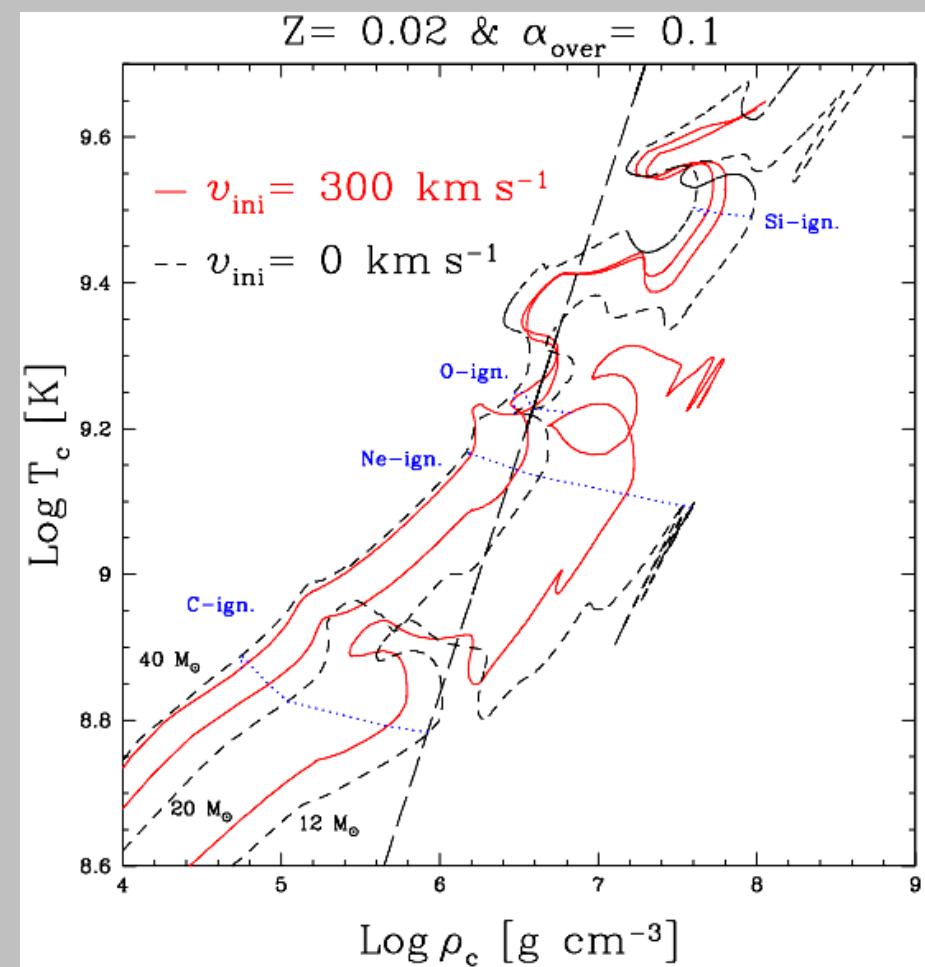
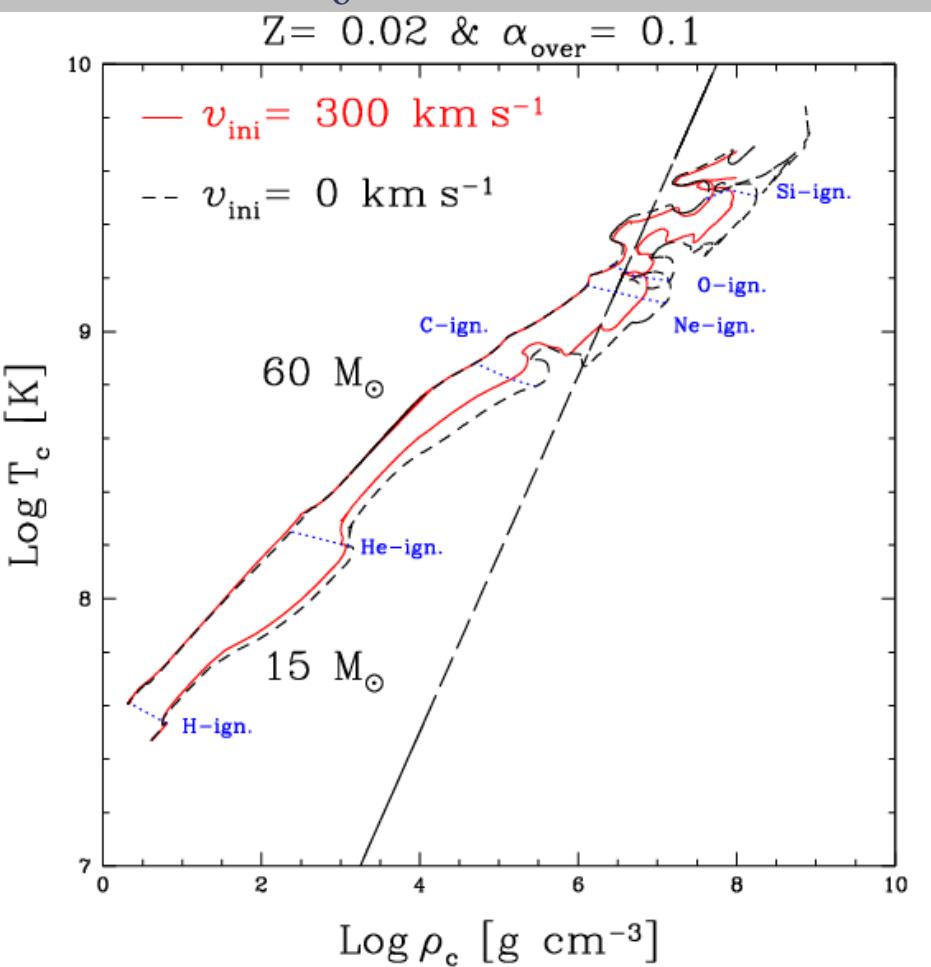
Ne-Si burning stages ignited off-centre. Fate: still Fe-CCSN

Simulations include 114-isotope network!

# Massive Stars

$M < \sim 20 M_{\odot}$ : Rotational mixing dominates  $\rightarrow$  bigger cores

$M > \sim 30 M_{\odot}$ : mass loss dominates  $\rightarrow$   $\sim$  or smaller cores

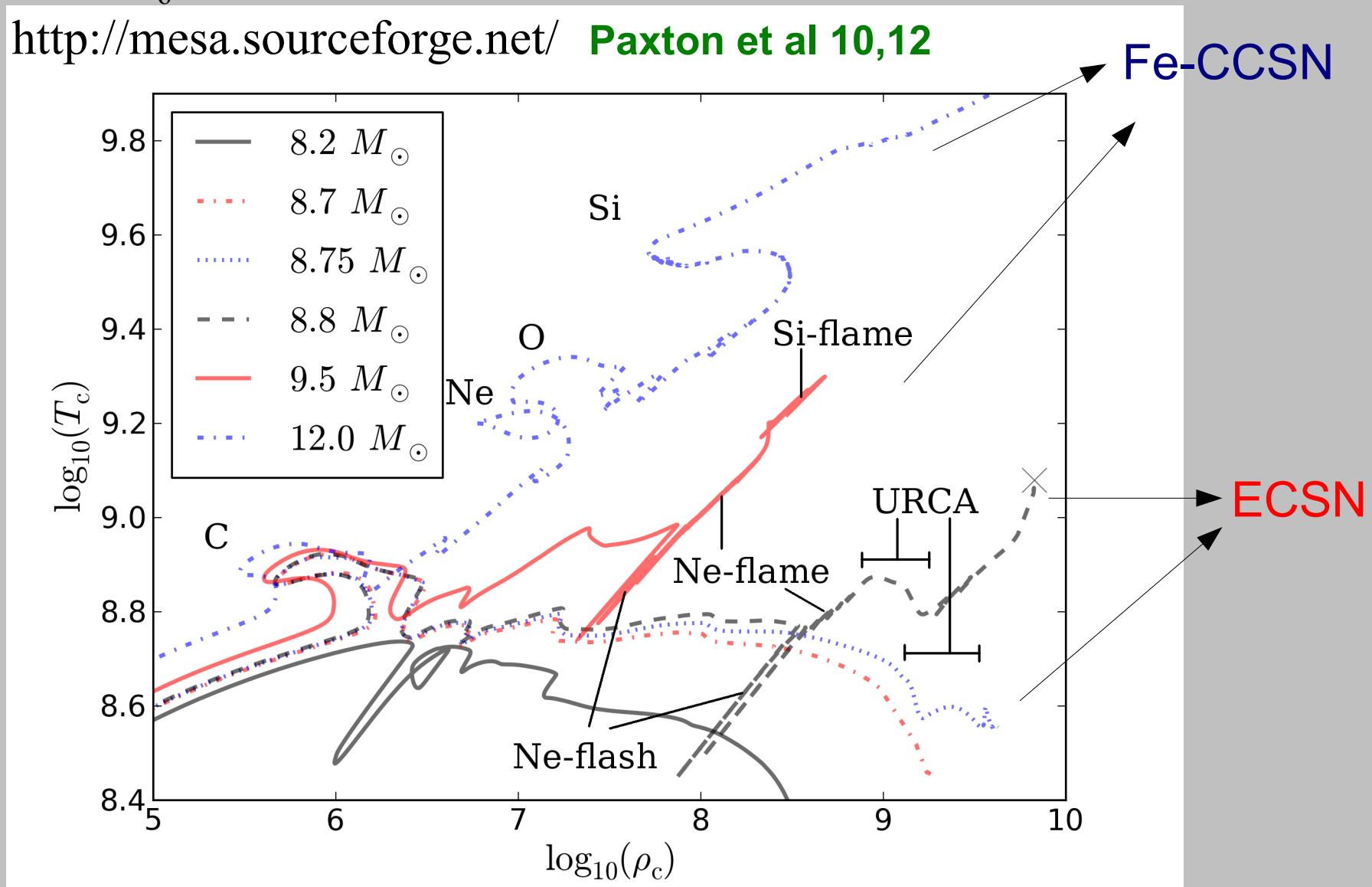


CO-core mass & C/O ratio: key parameters that determine evolution during late stages but things are complicated

# Fate of Least-Massive MS: ECSN/Fe-CCSN?

7-15  $M_{\odot}$  models  $\leftarrow$  MESA stellar evolution code:

<http://mesa.sourceforge.net/> **Paxton et al 10,12**

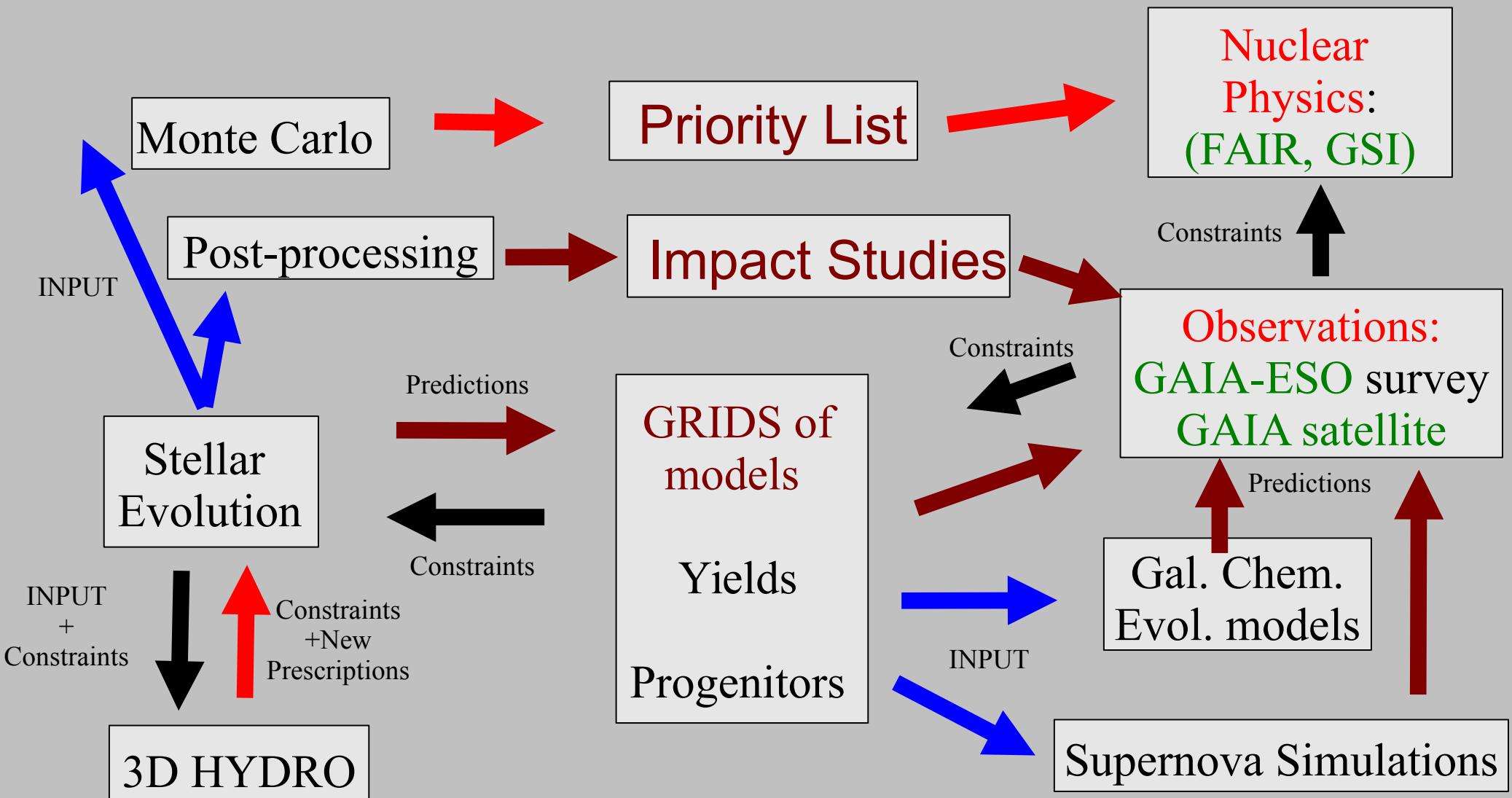


Both SAGB and failed massive stars may produce ECSN

# *Stellar Hydrodynamics Nucleosynthesis & Evolution (SHYNE) Project*

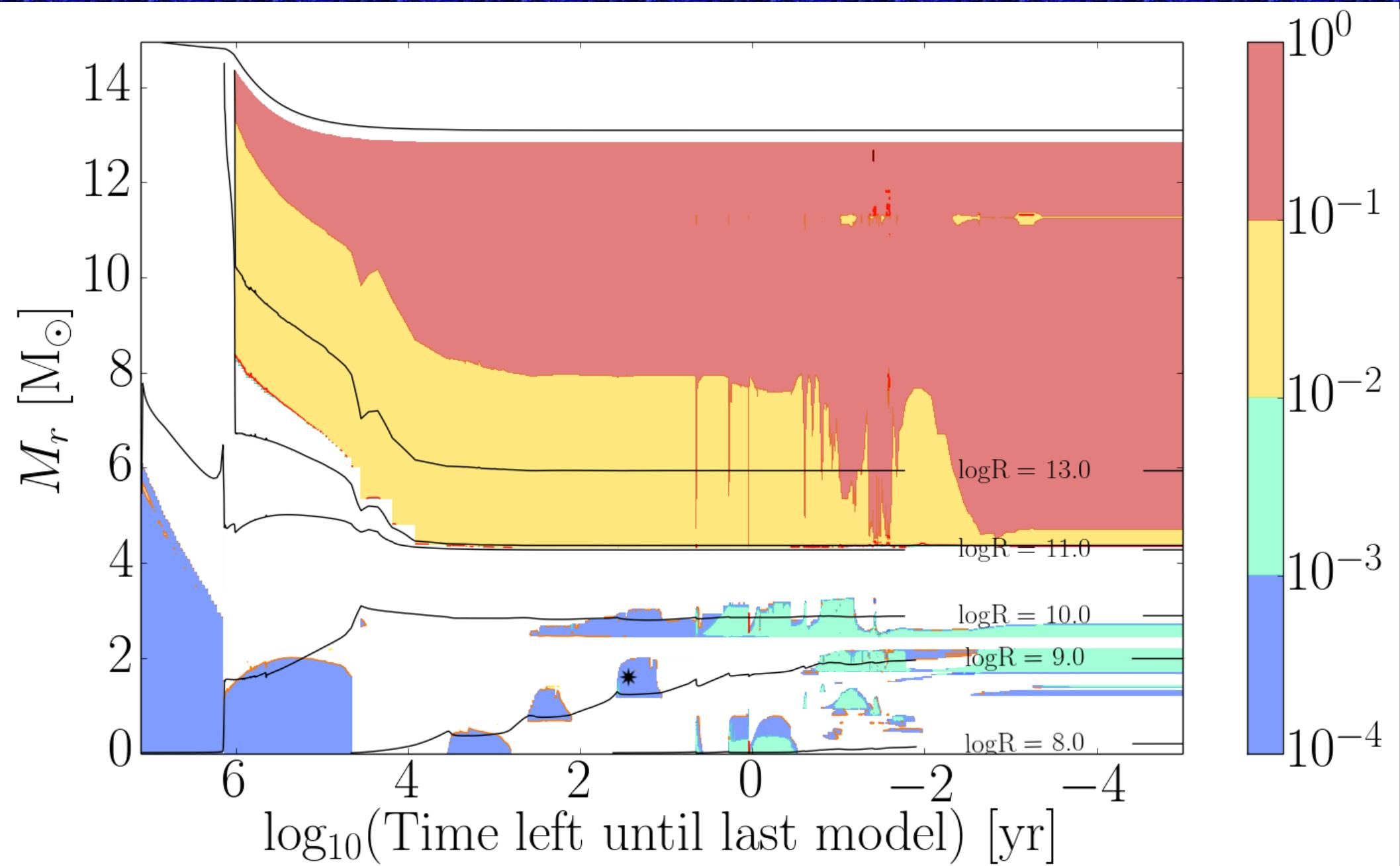
ERC Starting grant: 2012-2017

*TOOL SUITE* → *DATASETS* → *IMPACT*



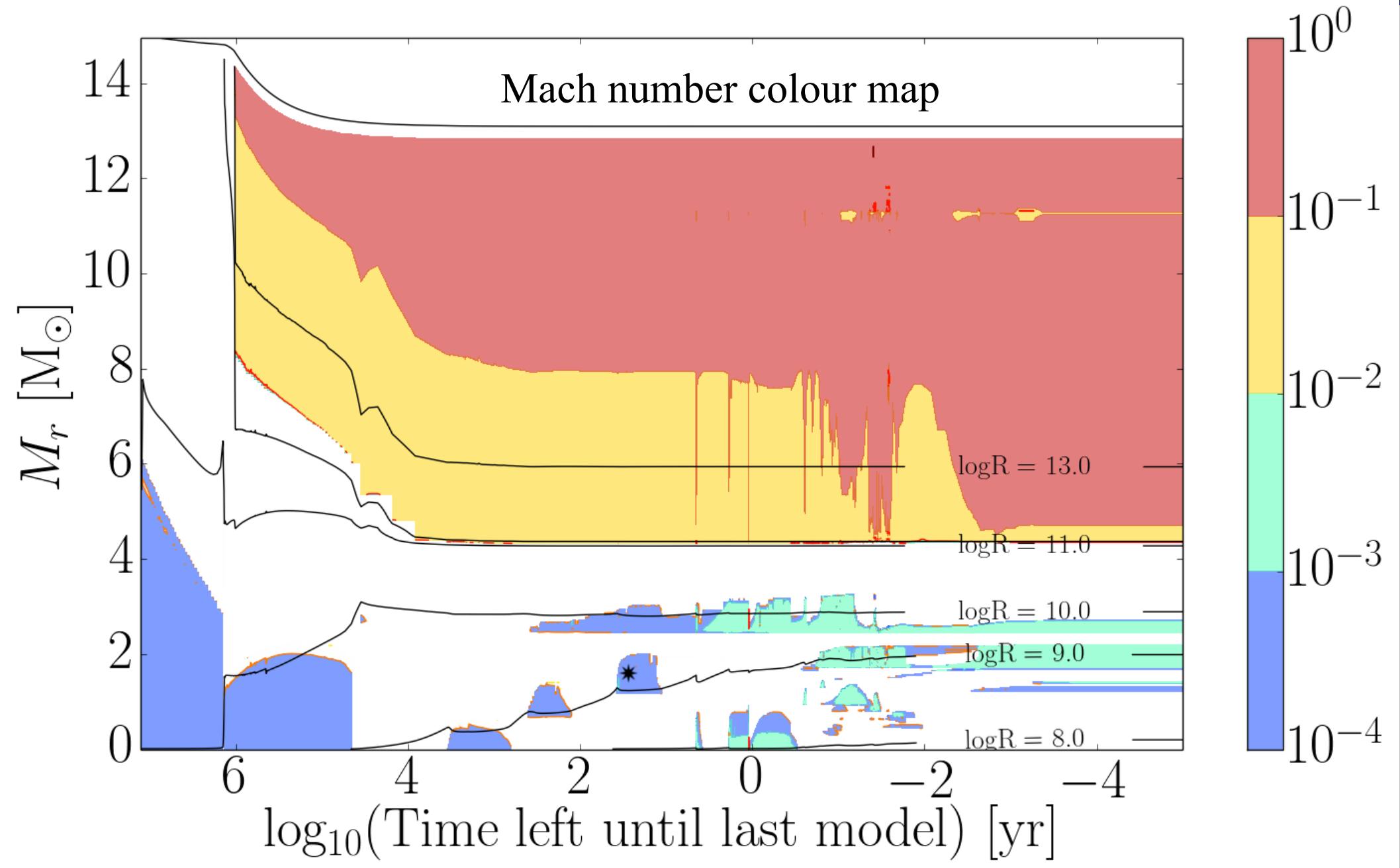
- Efficient pipeline: nuclear/hydro/astro

# *Next Steps? Carbon Burning*



Run simulation at later point during C-burning shell + vary Lum

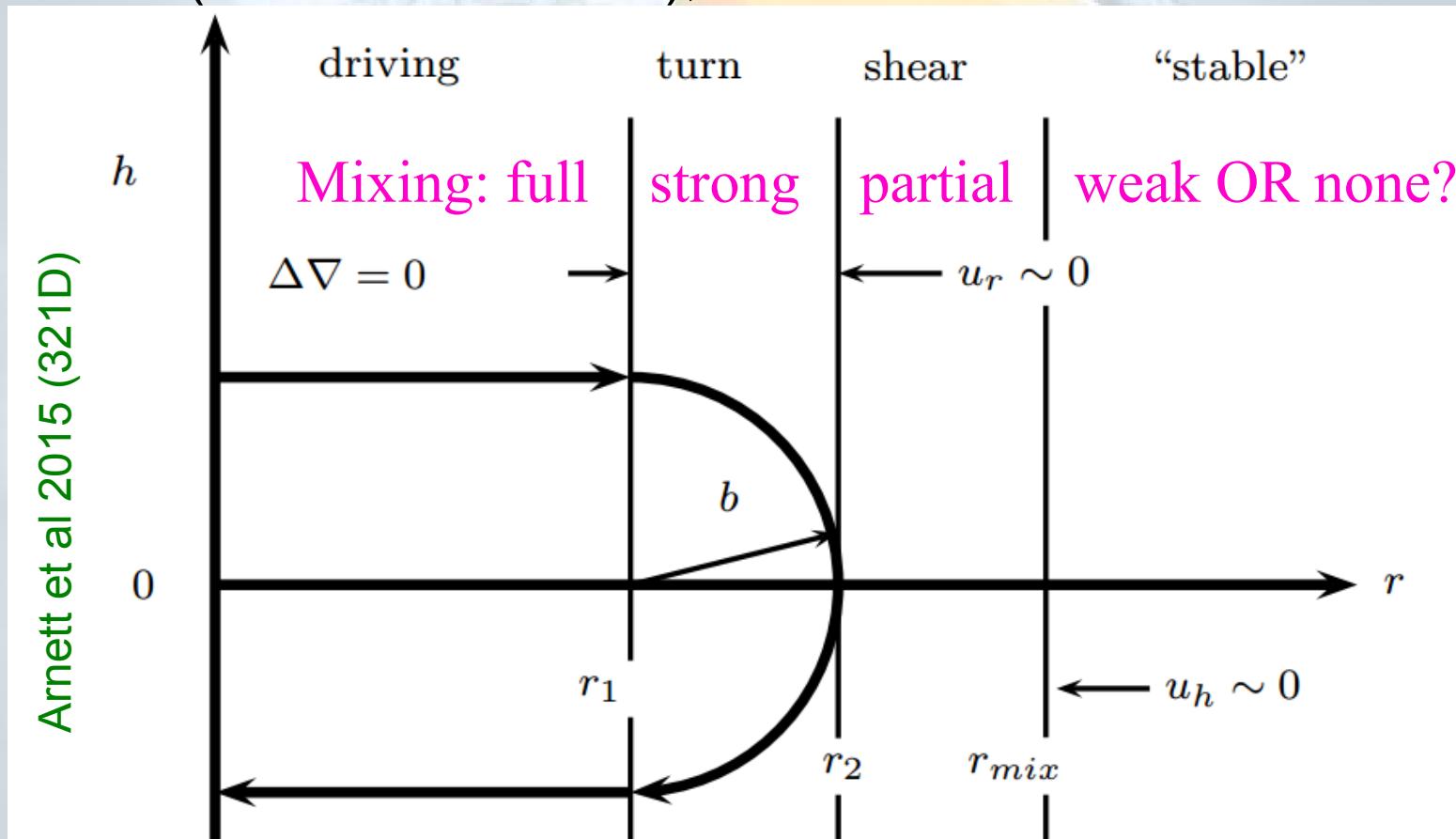
# *Next Steps? Other Phases*



Low-Mach scheme better for H, He phases!

# 321D – MLT Replacement + New CBM Prescriptions

Long term: MLT-replacement theory (Arnett et al 2015), RANS implementation (Mocak et al 2015), Arnett et al 2016



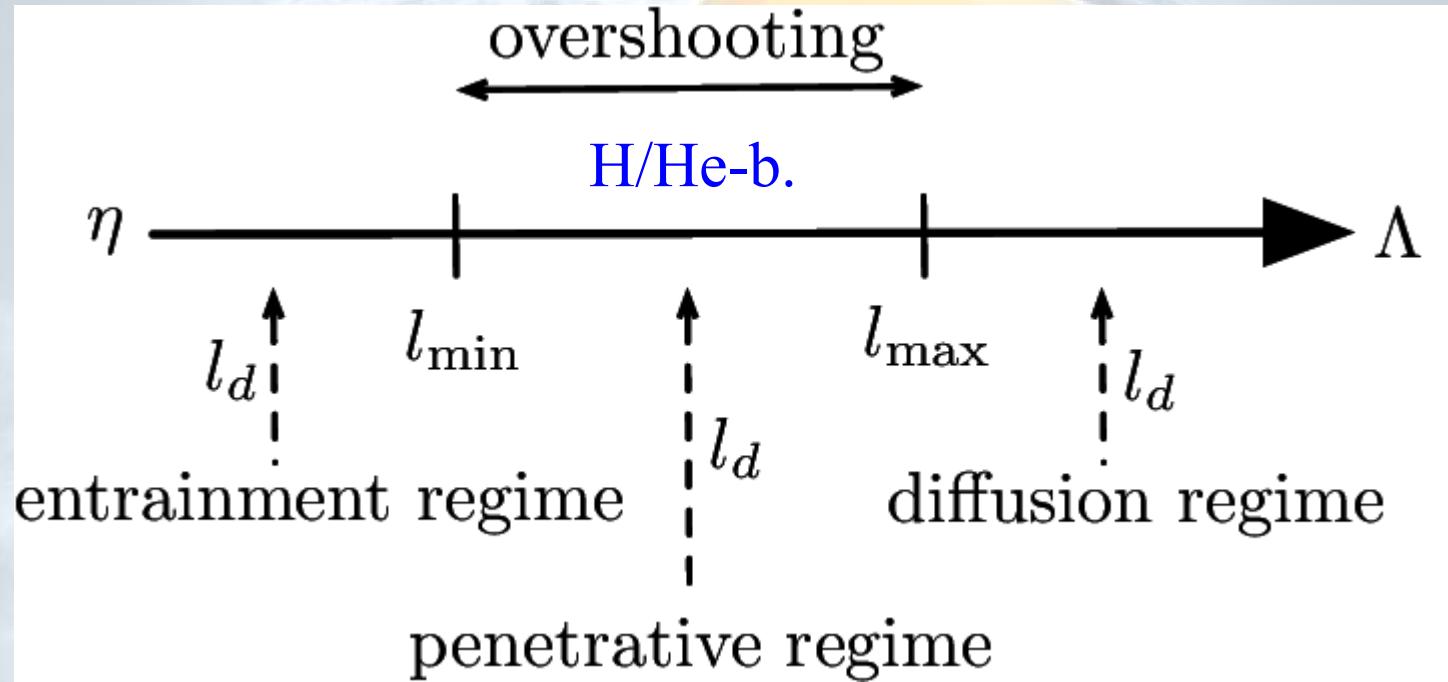
**Figure 5.** Simplified schematic of a convective boundary. The length  $b$  corresponds to the radius of curvature needed to reverse (contain) the flow ( $u_r \rightarrow -u_r$ ). The centrifugal acceleration is provided by pressure fluctuations (see text). The boundaries oscillate due to surface waves. The radial direction is denoted by  $r$  and the transverse by  $h$ . Orientation is for the top of a convection zone; the bottom may be described by appropriate reversals.

# *Importance of Thermal Effects*

Viallet et al 2015

Advanced  
phases:  
O/Si-b.

Stellar  
surface

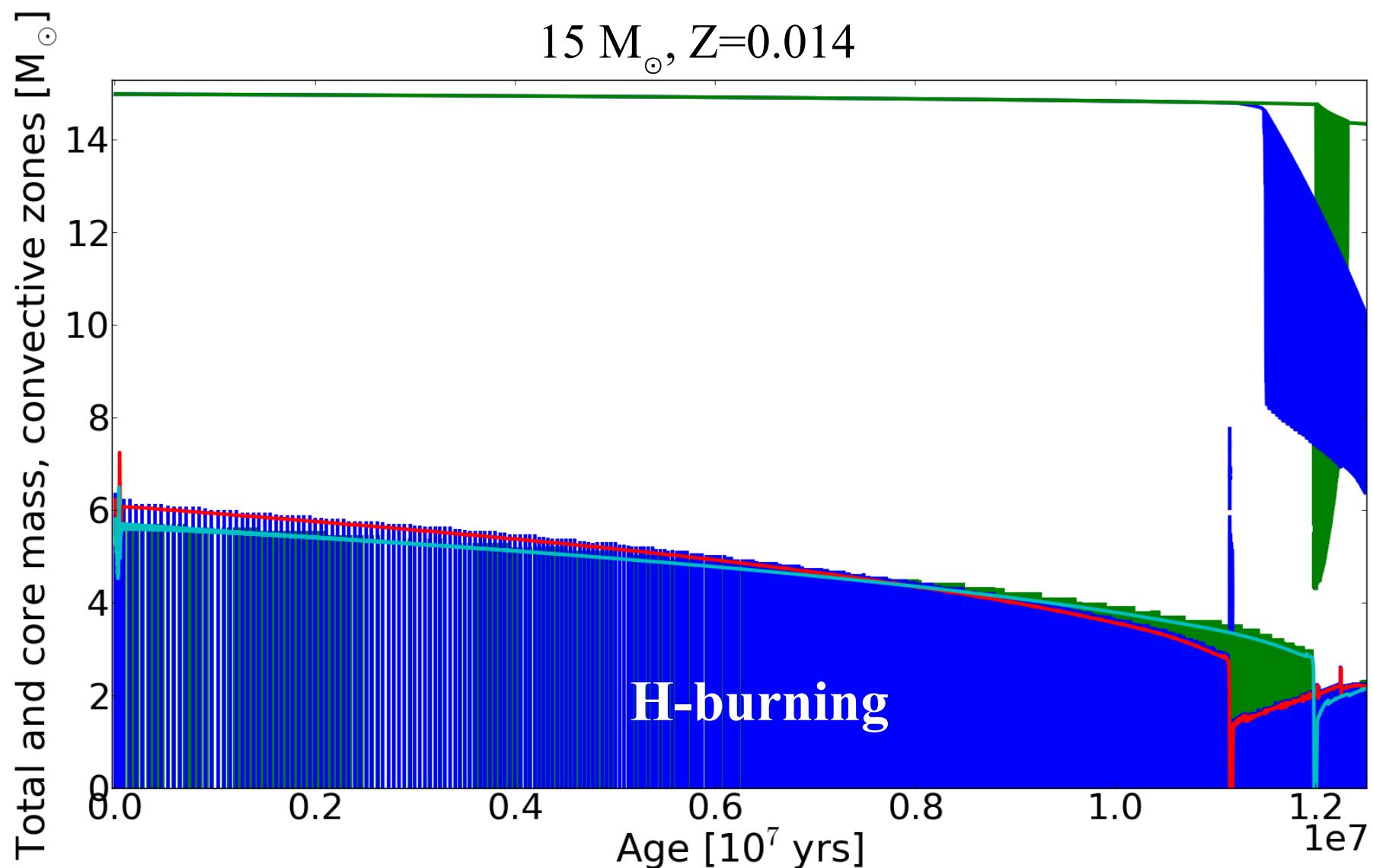


*Thermal effects determine which prescription/implementation to adopt:*

Penetrative vs exp-D vs Entrainment CBM?

# *Back to 1D*

Penetrative vs exp-D CBM: prescription choice affects results

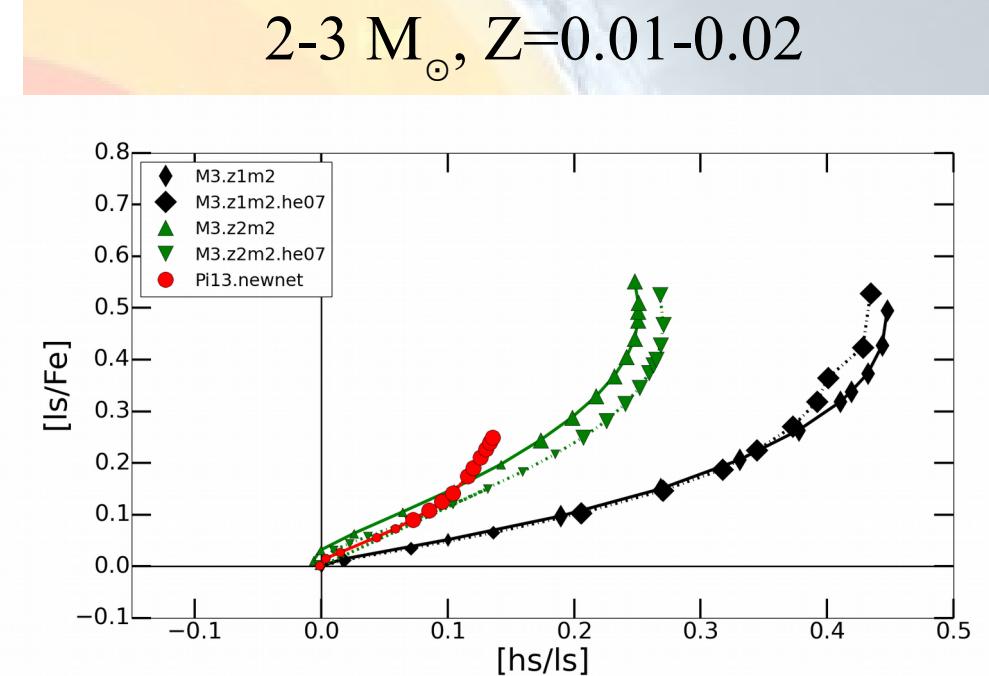
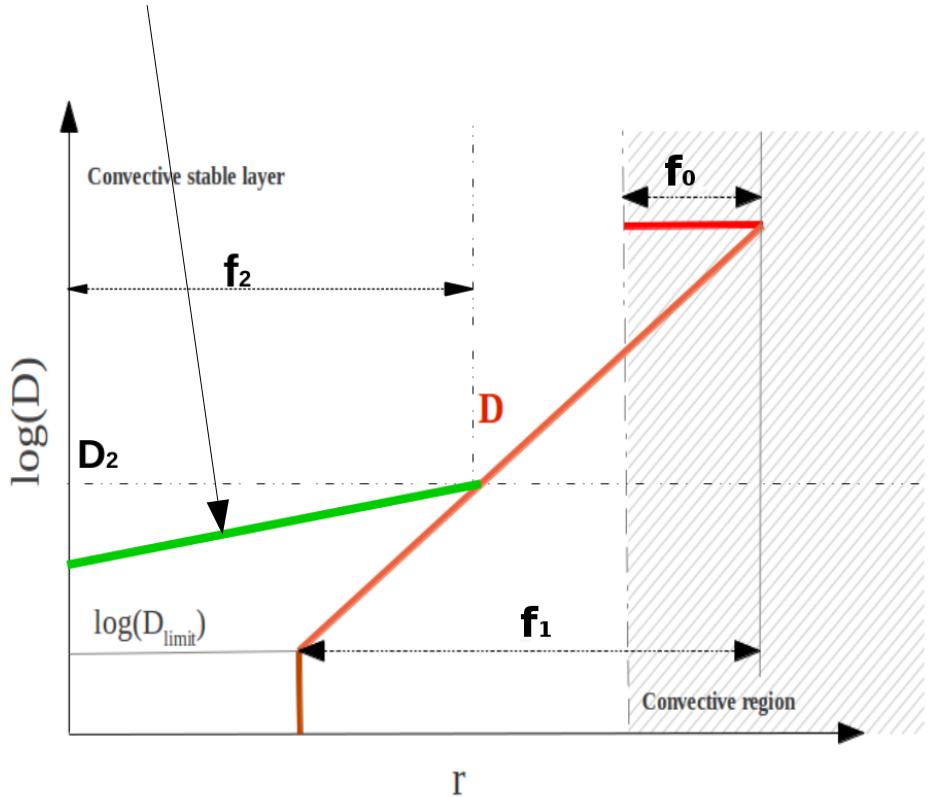


# Back to 1D: CBM in AGB Stars (*NuGrid* project)

## Internal gravity wave (IGW)

Battino,...,Hirschi et al ApJ 2016

## driven mixing



- 1) CBM (first f) plays a key role both for the C13 pocket via CBM below CE (needed for TDU) and for the c12 & o16 abundances in the intershell via CBM below TPs
- 2) IGW (second f) plays a key role for the C13 pocket (not so much for mixing below the TPs)  
Study of the effects of rotation and B-field underway (den Hartogh, Hirschi, Herwig et al in prep)

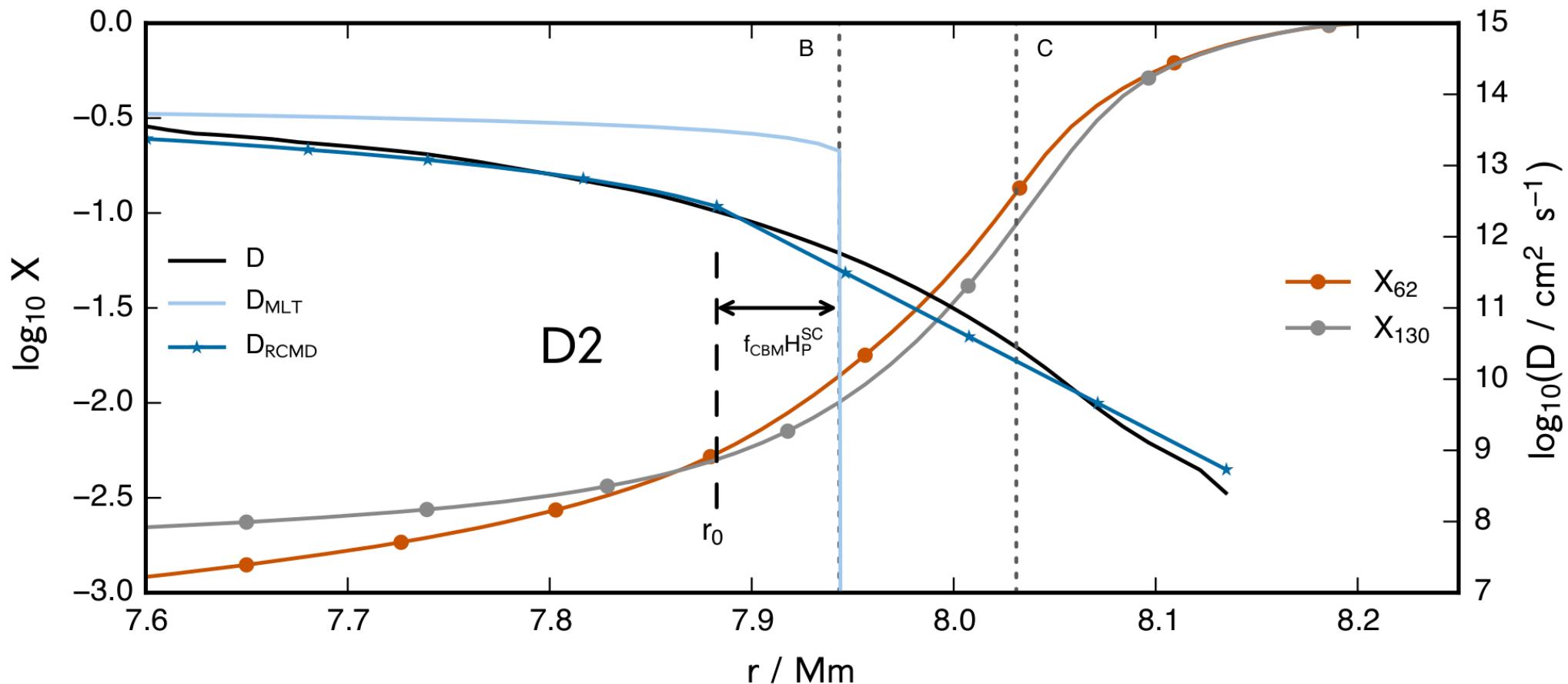
# MIXING IN STARS

1D MIXING MODEL

$$f_{\text{CBM}} = 0.03$$

$$\frac{1}{3} V_{\text{MLT}} \times \min(\ell, r_0 - r)$$

$$D(r) = D(r_0) \times \exp \left\{ -\frac{2(r - r_0)}{f_{\text{CBM}} H_P(r_0)} \right\}$$

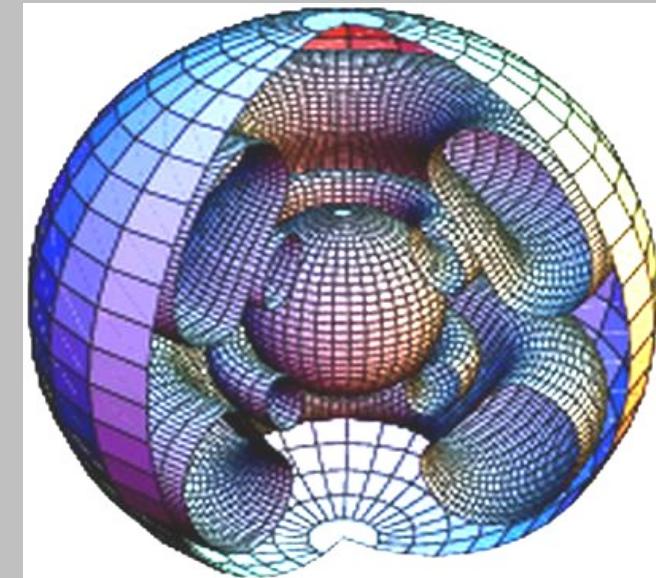


# *Rotation-Induced Transport*

Zahn 1992: strong horizontal turbulence

Transport of angular momentum:

$$\rho \frac{d}{dt} (r^2 \bar{\Omega})_{Mr} = \underbrace{\frac{1}{5r^2} \frac{\partial}{\partial r} (\rho r^4 \bar{\Omega} U(r))}_{\text{advection term}} + \underbrace{\frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho D r^4 \frac{\partial \bar{\Omega}}{\partial r} \right)}_{\text{diffusion term}}$$

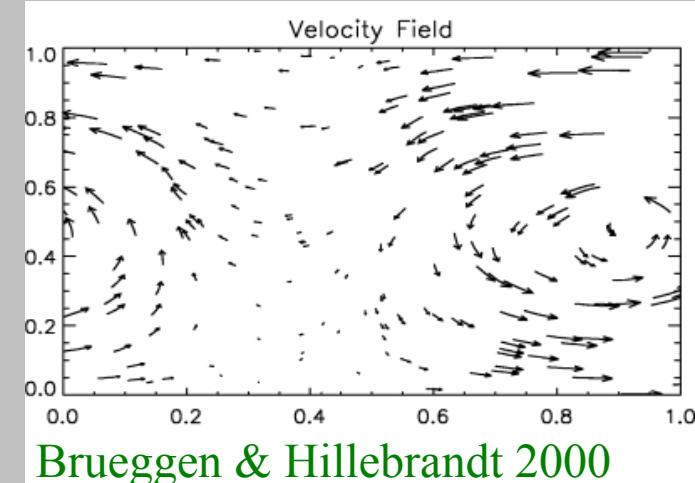


Meynet & Maeder 2000

Transport of chemical elements:

$$\rho \frac{dX_i}{dt} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( \rho r^2 [D + D_{eff}] \frac{\partial X_i}{\partial r} \right) + \left( \frac{dX_i}{dt} \right)_{nucl}$$

Shear instabilities



Brueggen & Hillebrandt 2000

D: diffusion coeff. due to various transport mechanisms (convection, shear)

D<sub>eff</sub>: diffusion coeff. due to meridional circulation + horizontal turbulence

## 2.3. Dynamical shear

The criterion for stability against dynamical shear instability is the Richardson criterion:

$$Ri = \frac{N^2}{(\partial U / \partial z)^2} > \frac{1}{4} = Ri_c, \quad (1)$$

Hirschi et al 2004

where  $U$  is the horizontal velocity,  $z$  the vertical coordinate and  $N^2$  the Brunt-Väisälä frequency:

$$N^2 = \frac{g\delta}{H_P} [\nabla_{ad} - \nabla + \frac{\varphi}{\delta} \nabla_\mu] \quad (2)$$

### 2.3.1. The recipe

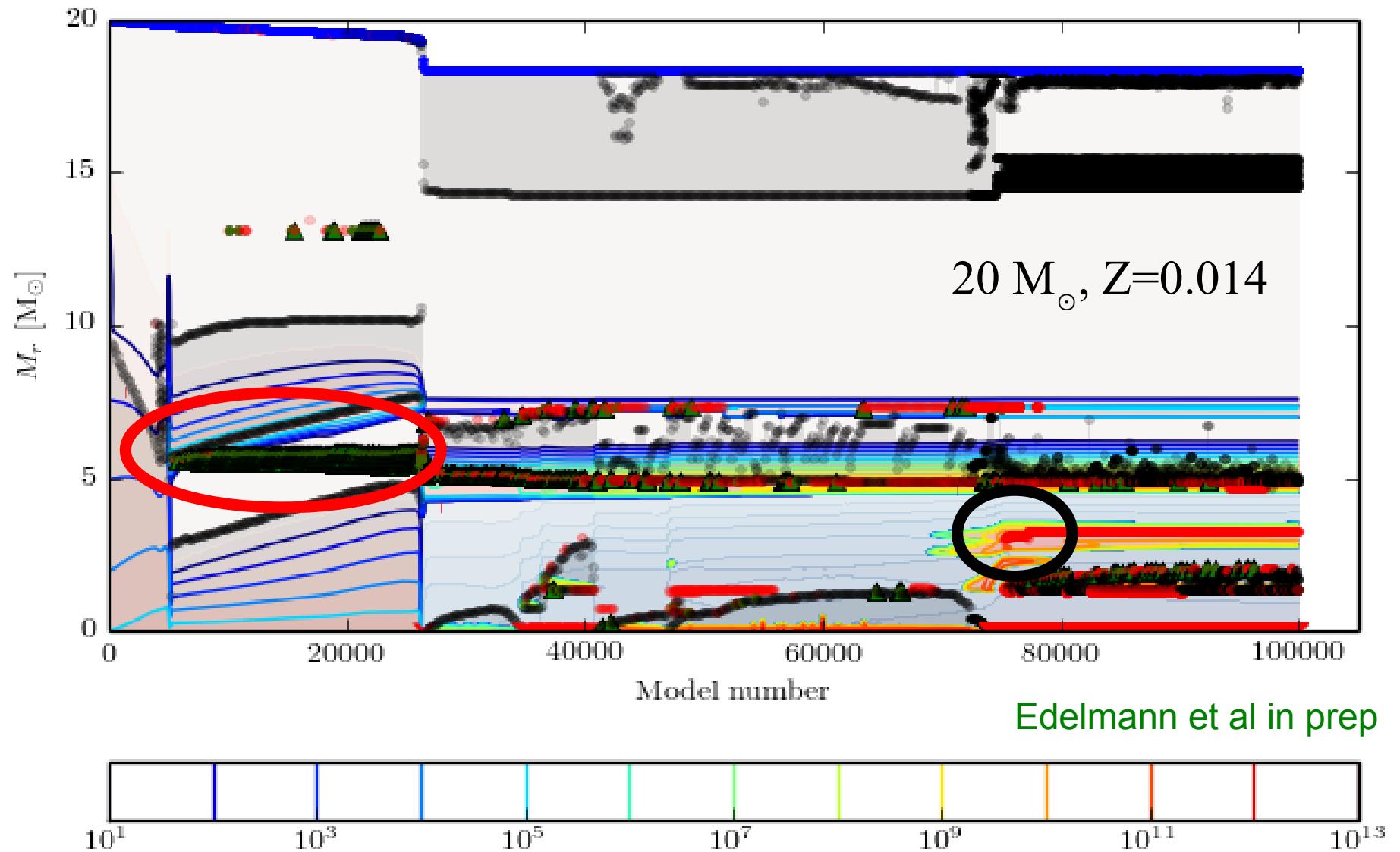
The following dynamical shear coefficient is used, as suggested by J.-P. Zahn (priv. comm.):

$$D = \frac{1}{3} v l = \frac{1}{3} \frac{v}{l} l^2 = \frac{1}{3} r \frac{d\Omega}{dr} \Delta r^2 = \frac{1}{3} r \Delta\Omega \Delta r \quad (5)$$

where  $r$  is the mean radius of the zone where the instability occurs,  $\Delta\Omega$  is the variation of  $\Omega$  over this zone and  $\Delta r$  is the extent of the zone. The zone is the reunion of

# *Dynamical Shear*

Energy generation contours (red/color: heating, blue: cooling, grey: convection)



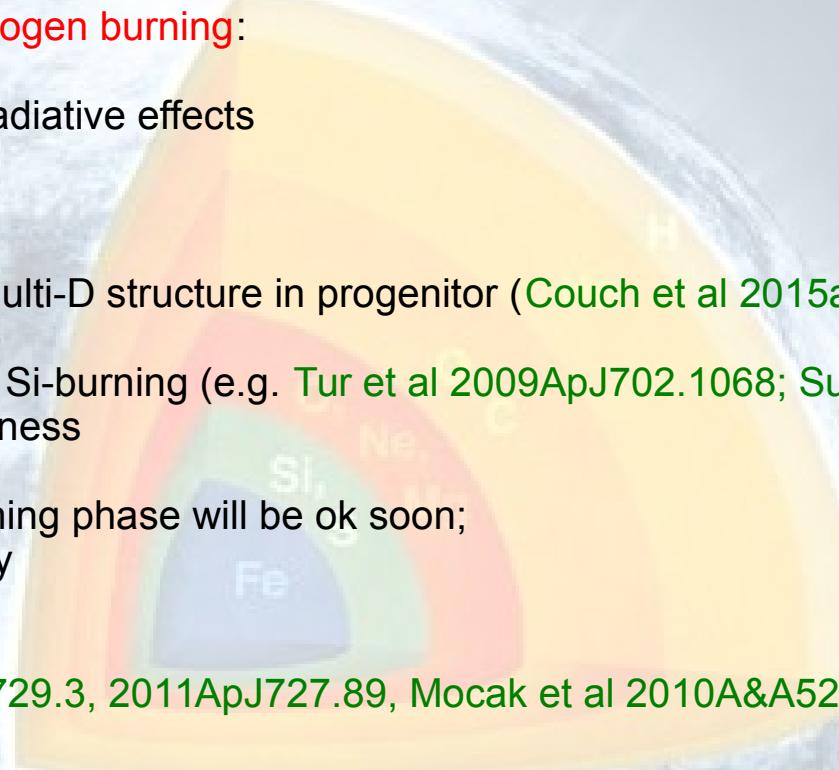
# Priority List

## \* Convective boundary mixing during core hydrogen burning:

- +: many constraints (HRD, astero, ...)
- -: difficult to model due to important thermal/radiative effects
- -: long time-scale
- 

## \* Silicon burning:

- +: important to determine impact on SNe of multi-D structure in progenitor (Couch et al 2015a,b, Mueller & Janka [aph1409.4783](#), Mueller et al [ArXiv1605.01393](#))
- +: possible shell mergers occurring after core Si-burning (e.g. Tur et al 2009ApJ702.1068; Sukhbold & Woosley 2014ApJ783.105) strongly affect core compactness
- +: radiative effects small/negl.
- -: ~  $10^9$  CPU hours needed for full silicon burning phase will be ok soon;
- -: might be affected by convective shell history
- 



## \* AGB thermal pulses/H-ingestion:

- +: already doable (e.g. Herwig et al 2014ApJ729.3, 2011ApJ727.89, Mocak et al 2010A&A520.114, Woodward et al 2015)
- +: thermal/radiative effects not dominant
- ?: applicable to other phases?
- 

## \* Oxygen shell: (Meakin & Arnett 2007ApJ667.448/665.448, Viallet et al 2013ApJ769.1, Jones et al [ArXiv1605.03766](#))

- +: similar to silicon burning but smaller reaction network needed
- -: might be affected by convective shell history
- 

## \* Carbon shell: (PhD A. Cristini)

- +: not affected by prior shell history
- +: first stage for which thermal effects become negligible
- 

## \* Envelope of RSG (e.g. Viallet et al. 2013, Chiavassa et al 2009-2013),

- \* Solar-type stars (e.g. Magic et al. 2013A&A557.26, ...)