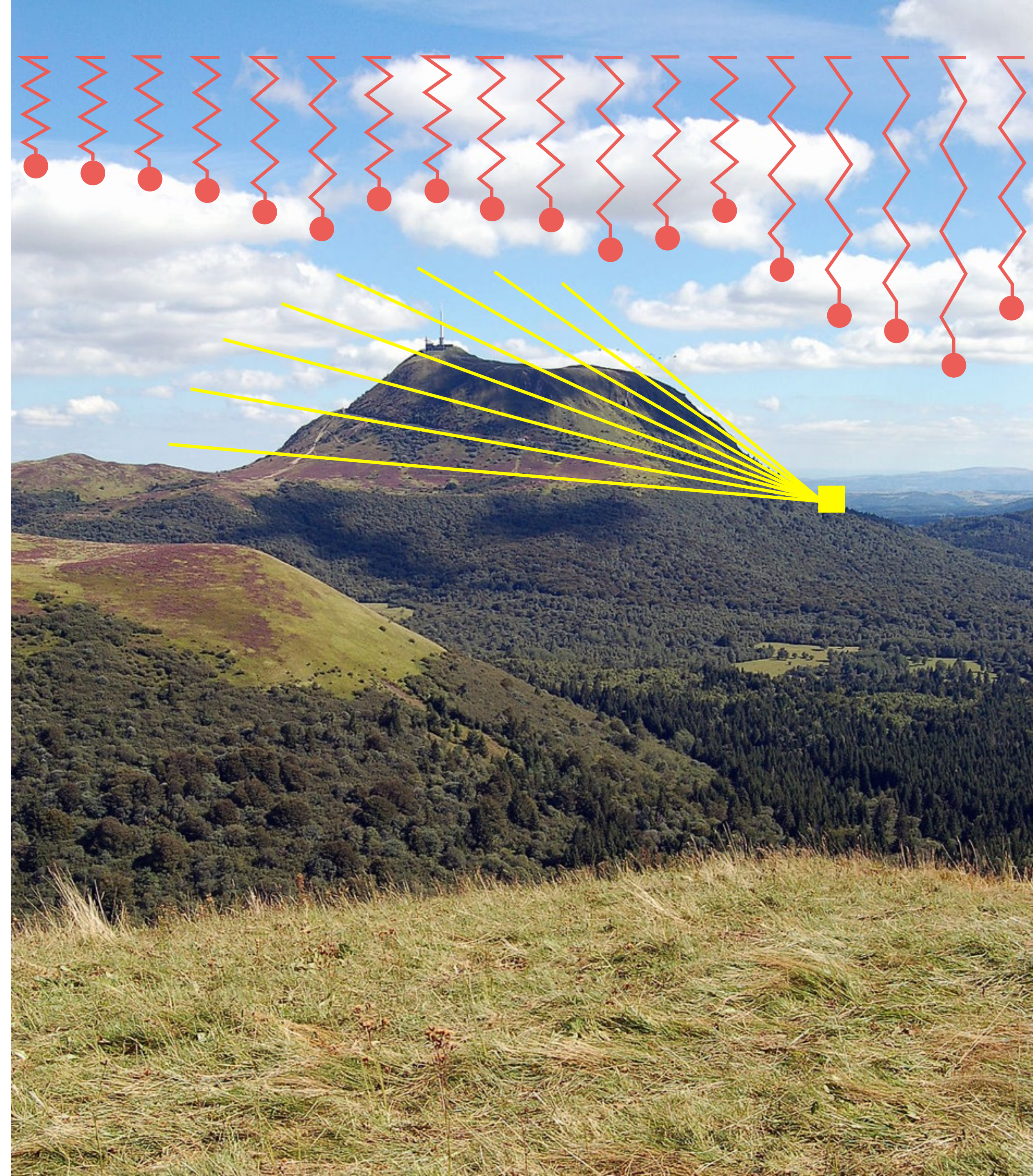


Imaging of the puy de Dôme volcano from gravimetric and muographic data

Anne Barnoud

LPC - Clermont-Ferrand - 09/06/17

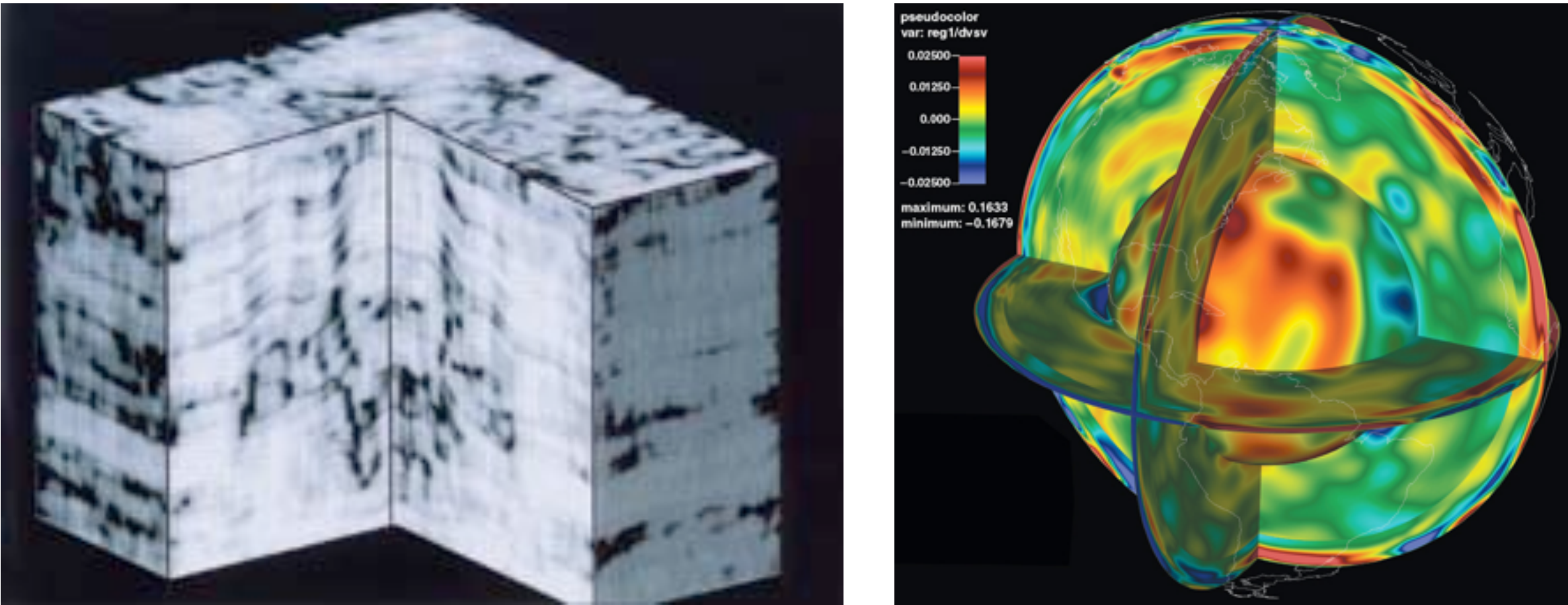


- Introduction on geophysical imaging
- Puy de Dôme
- Gravimetry
 - Principle of the method
 - Inversion approach
 - Inversion of the puy de Dôme data
- Muography
 - Principle of the method
 - Muography of the puy de Dôme
- Joint inversion

Medical imaging to see inside the human body



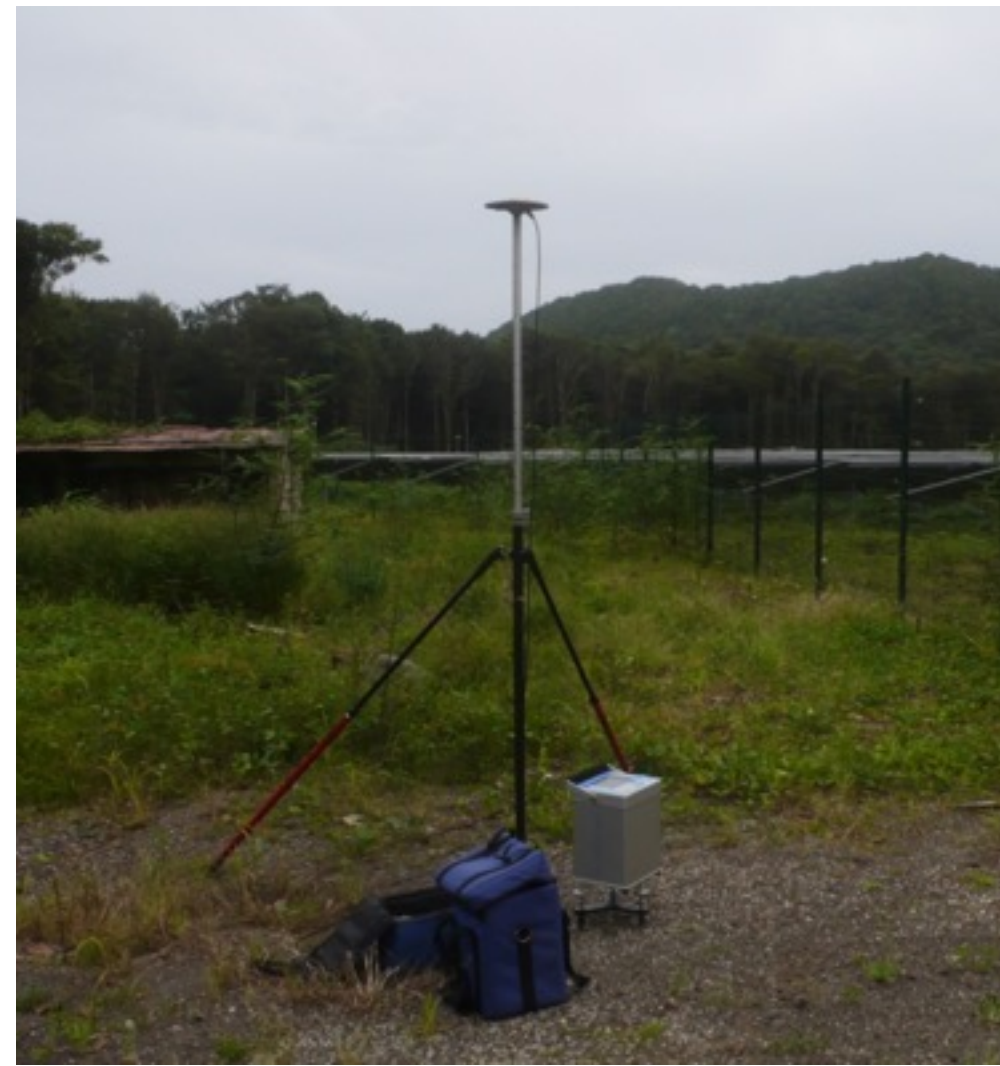
Geophysical imaging to see inside the Earth



Non destructive sounding of the Earth's subsurface, from local to global scale

Some geophysical methods

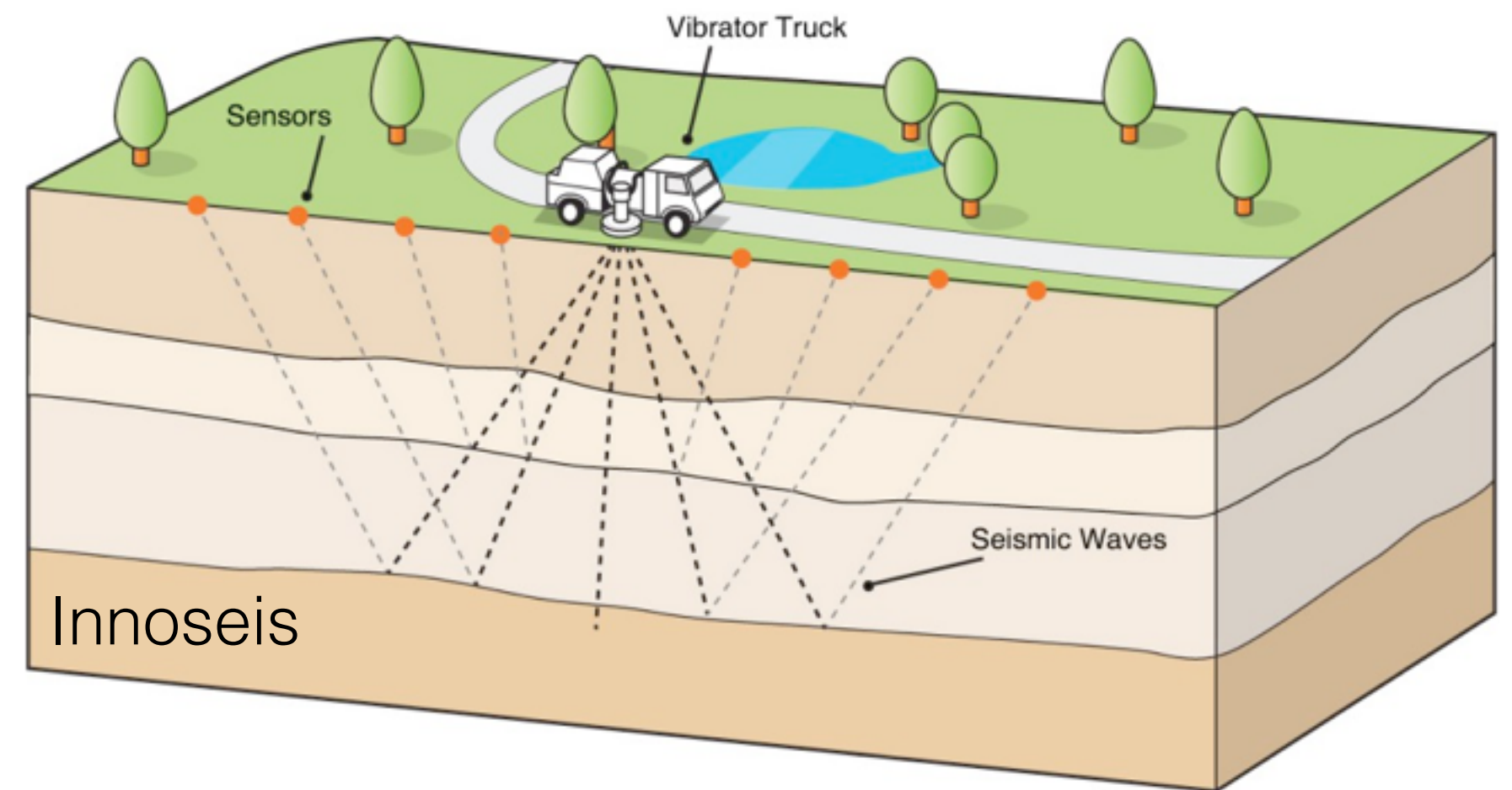
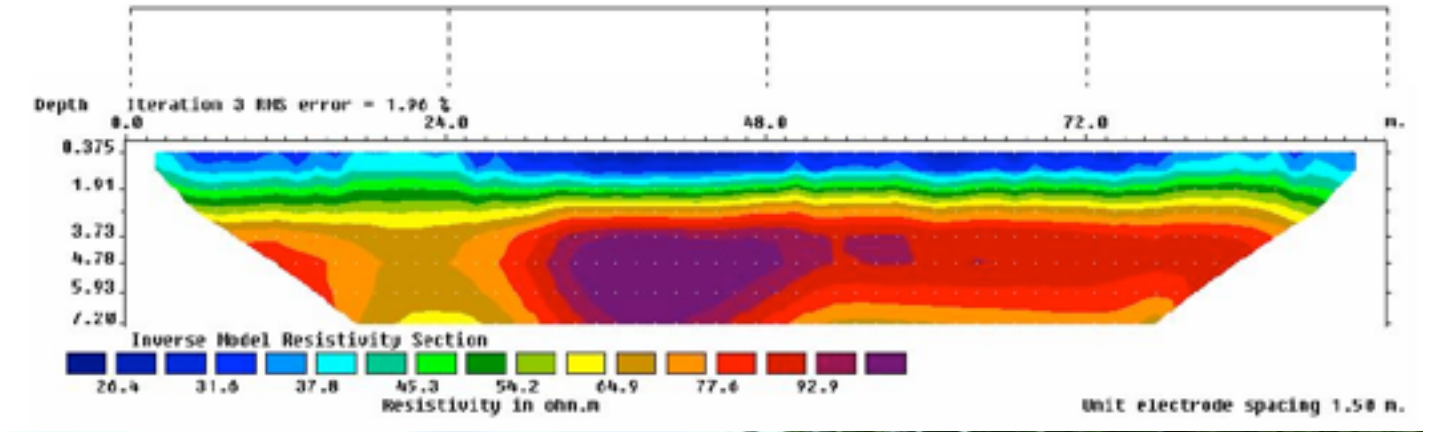
Gravimetry



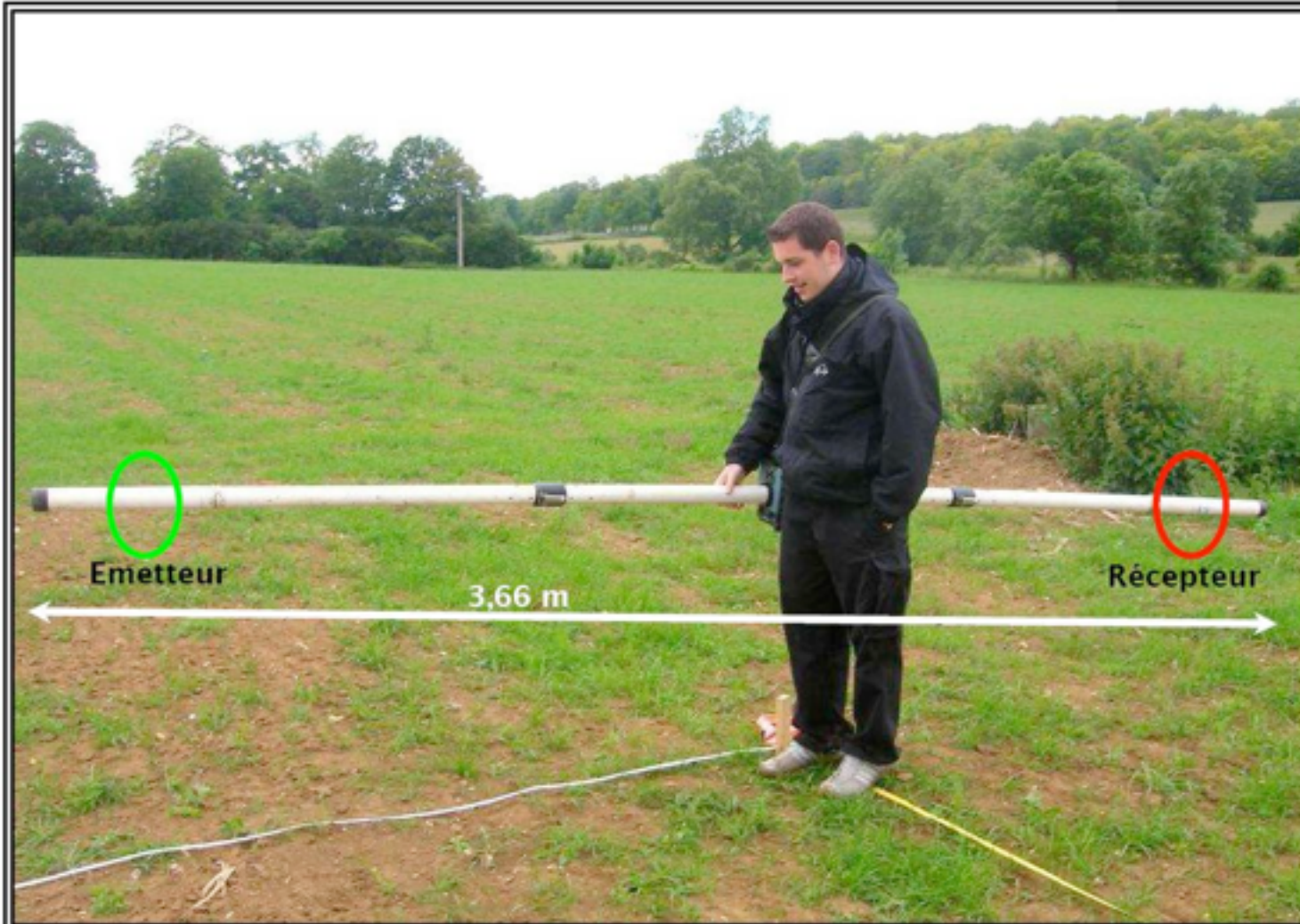
Heliborne magnetic and electromagnetic



Electrical resistivity

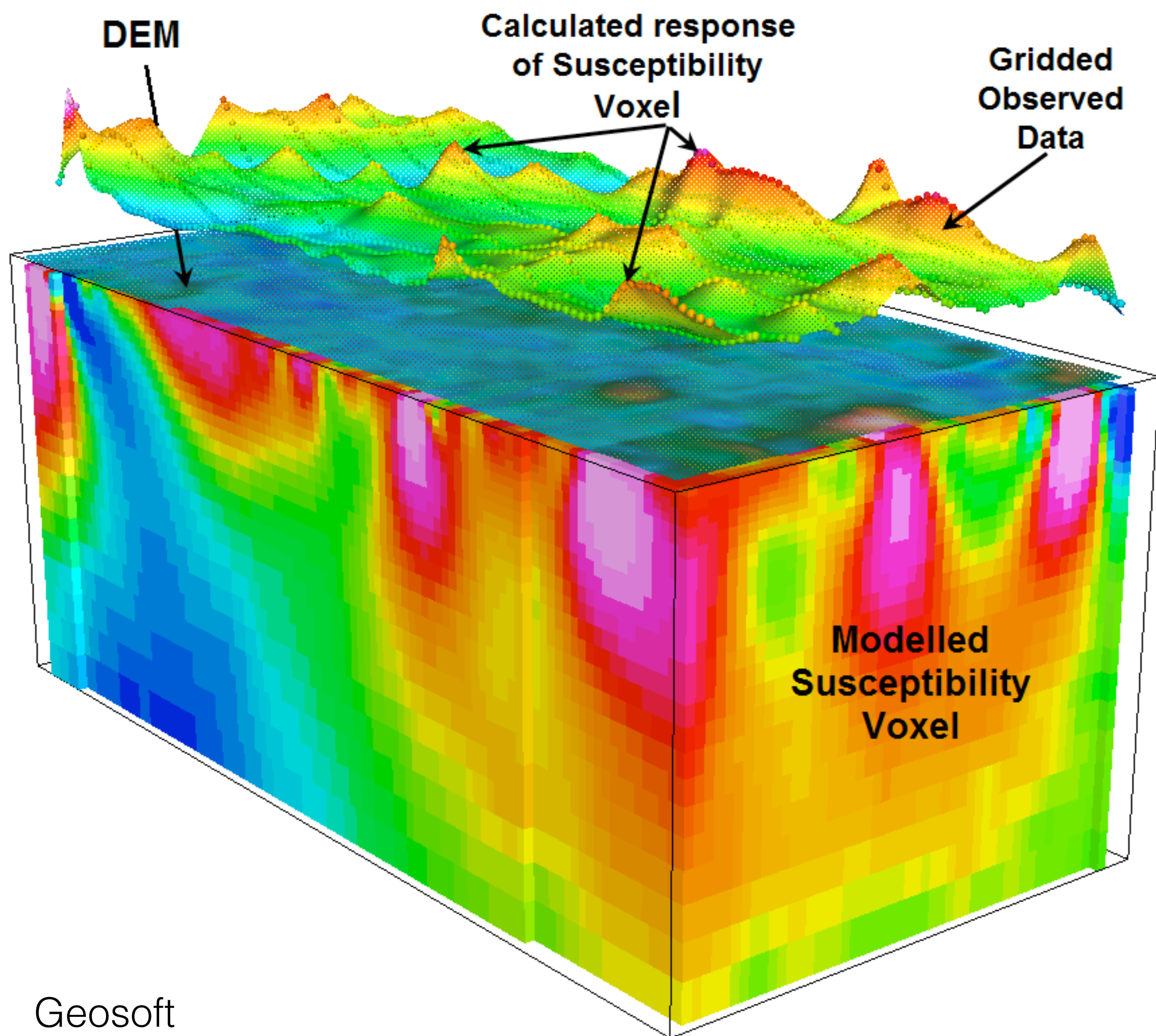


Seismic methods



Electromagnetism

Inversion in geophysics



Geosoft

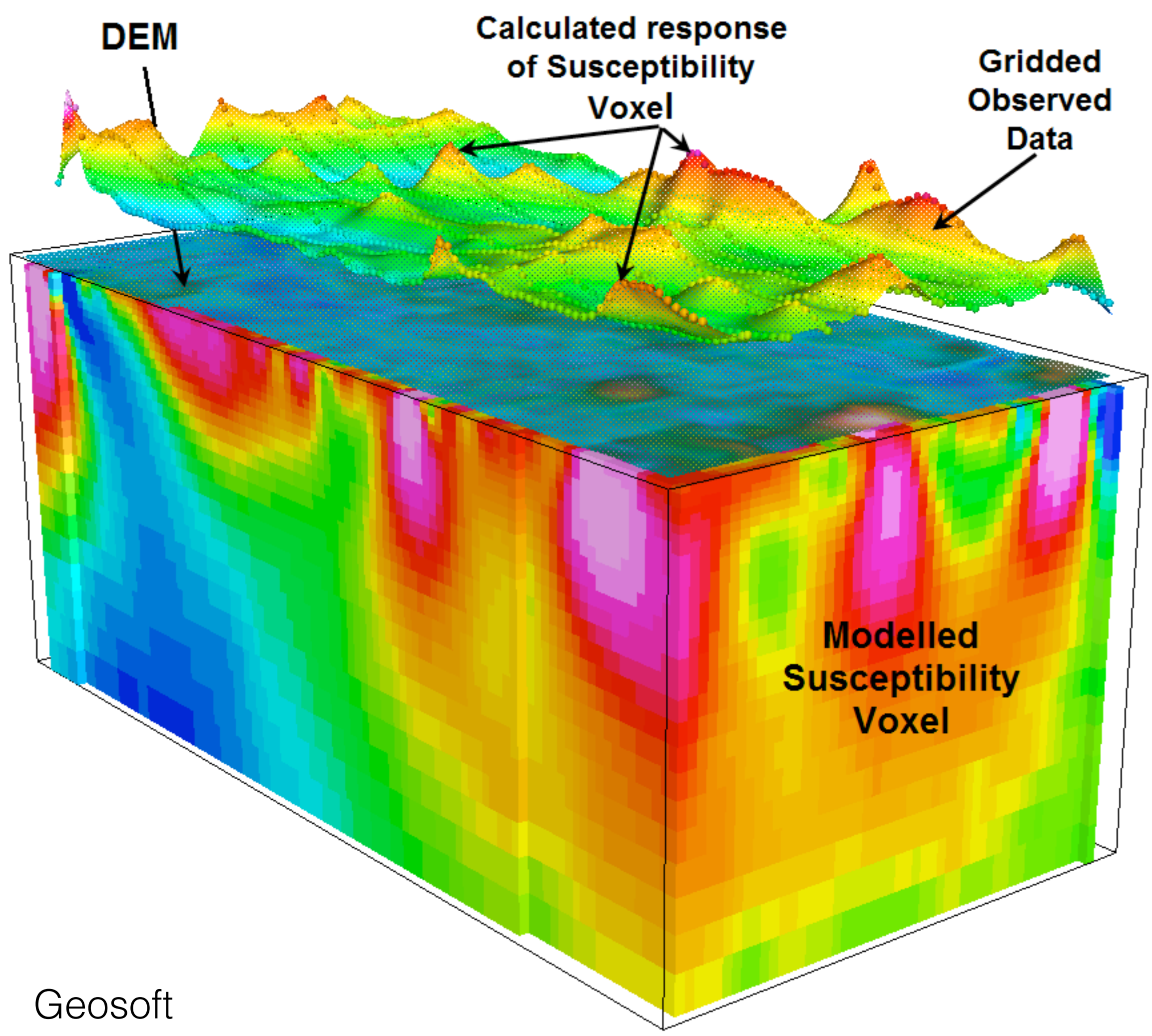
Data, eg.:

- seismic wave travel times
- gravitational field of the Earth

Model, eg.:

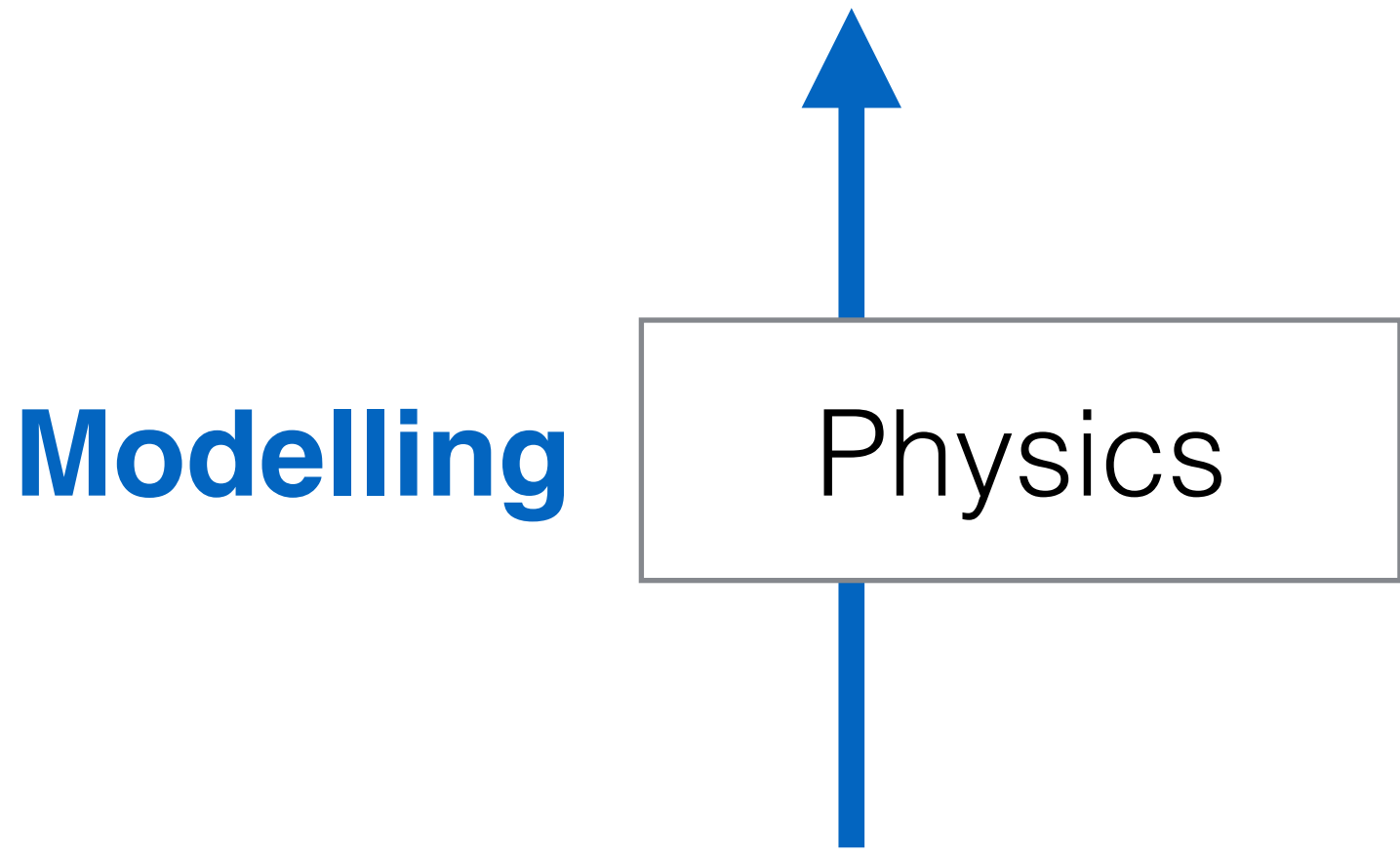
- seismic velocity and density
- density

$$g(m) = d$$



Data, eg.:

- seismic wave travel times
- gravitational field of the Earth



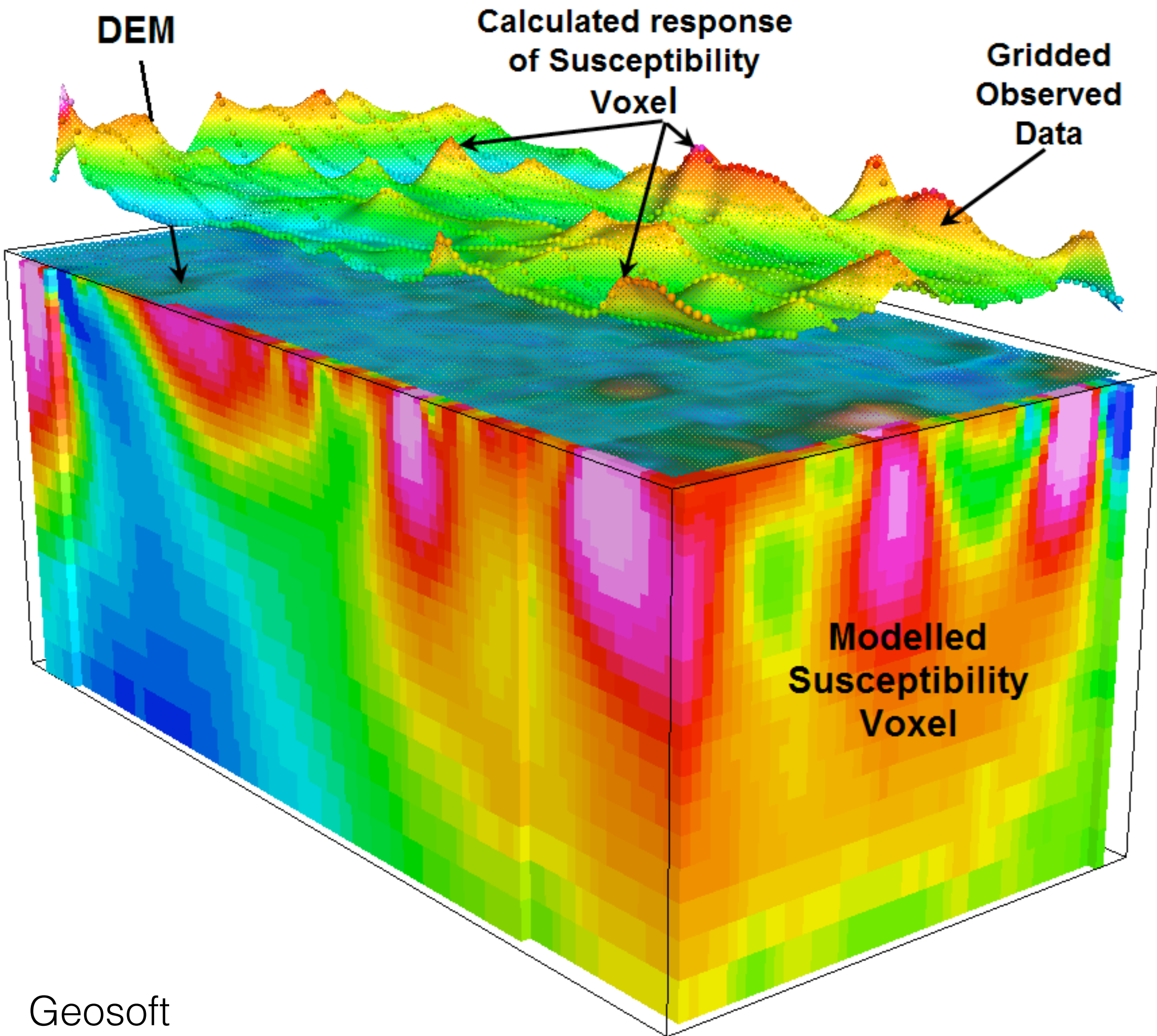
Model, eg.:

- seismic velocity and density
- density

Geosoft

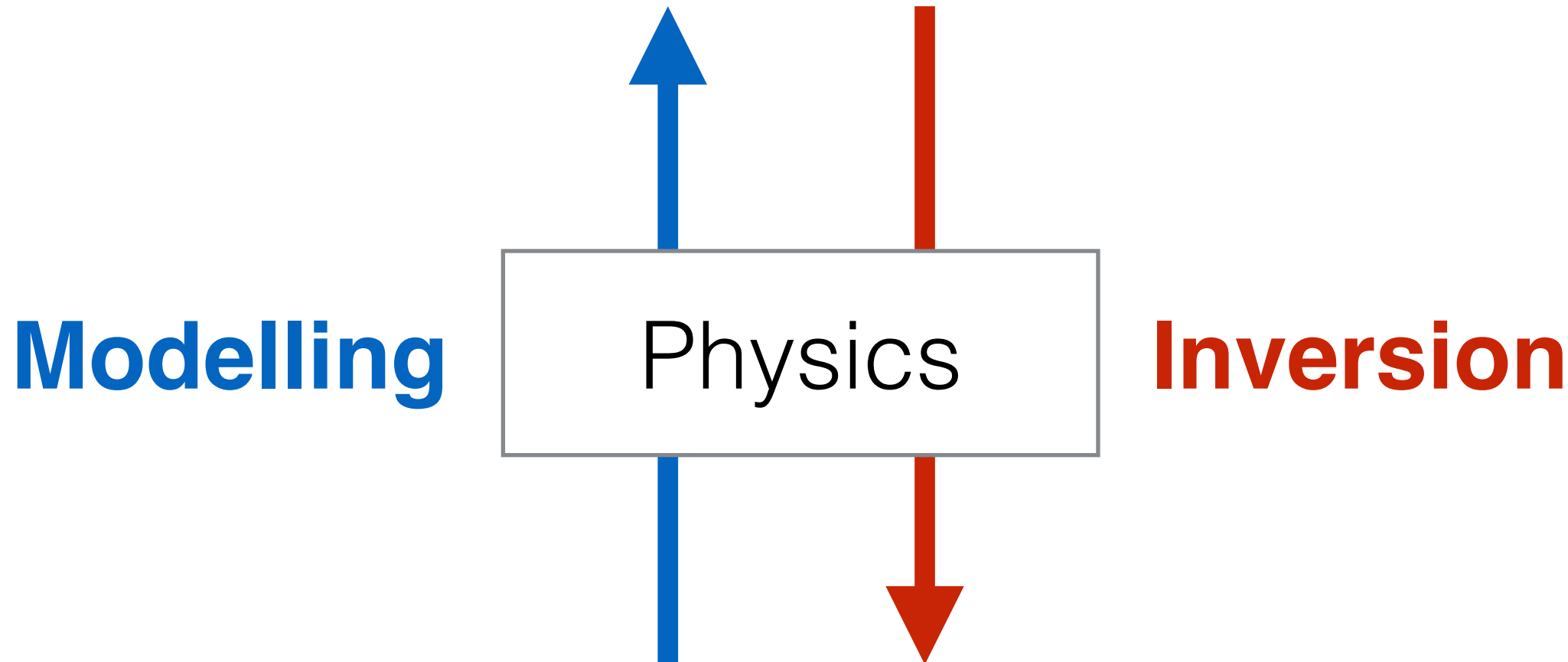
Inversion in geophysics

$$g(m) = d$$



Data, eg.:

- seismic wave travel times
- gravitational field of the Earth



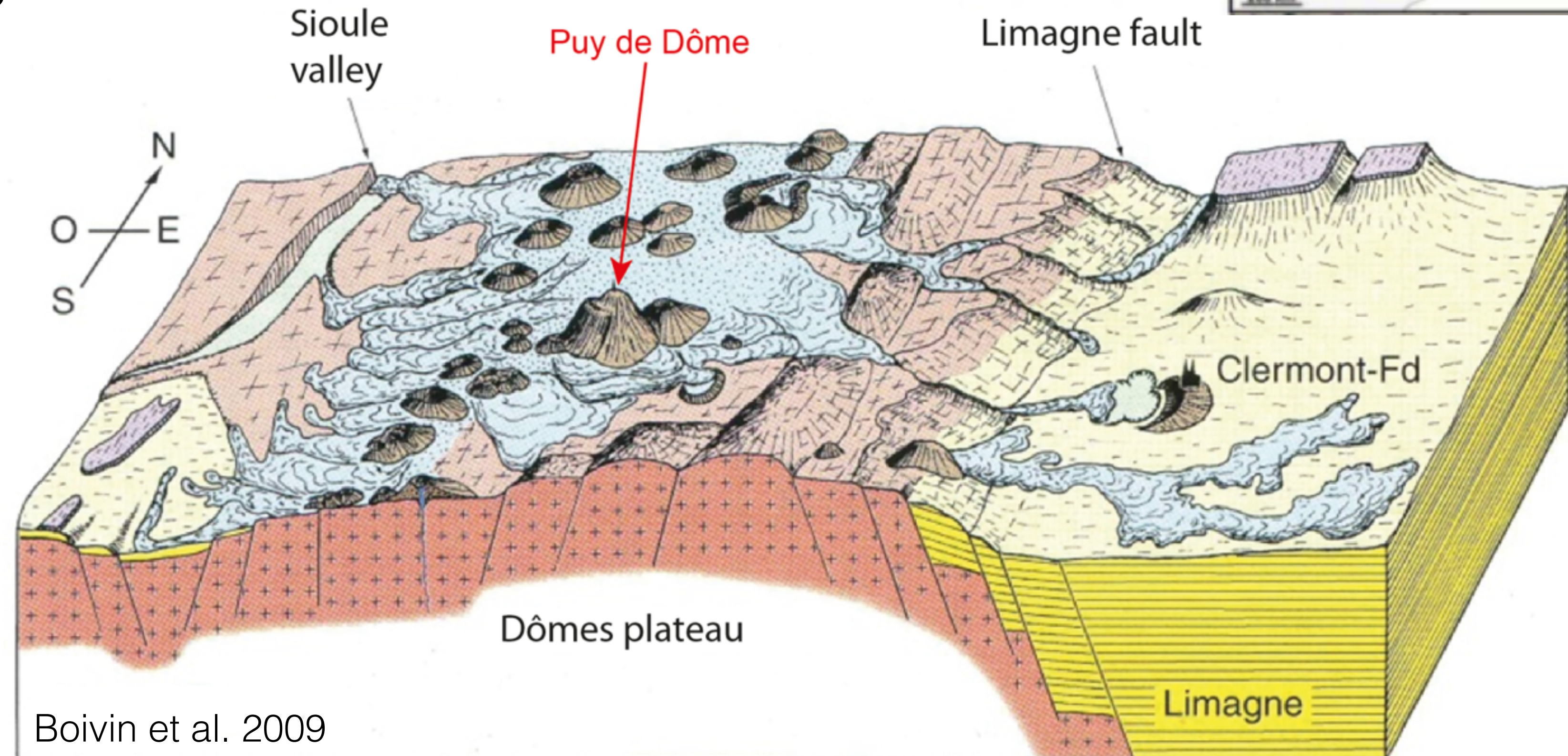
Model, eg.:

- seismic velocity and density
- density

Geosoft

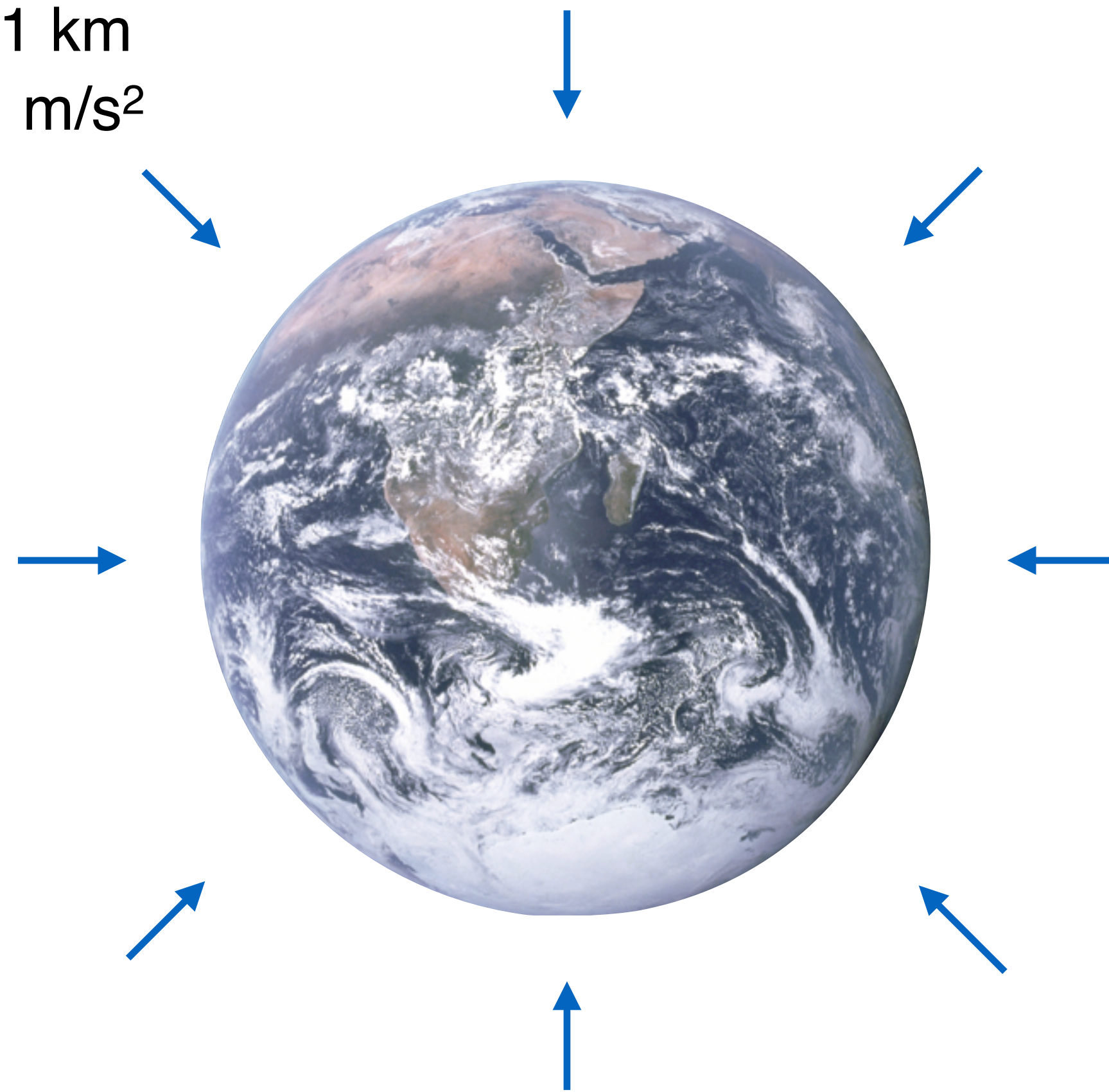
Puy de Dôme volcano

- located in the French Massif Central ancient volcanic zone
- lies on a hercynian basement
- about 11000 year old
- dome **400 m high** and **1.8 km wide**
- formed by two distinct extrusions
- hydrothermal alteration
- **isolated** from neighbor edifices



Spherical Earth

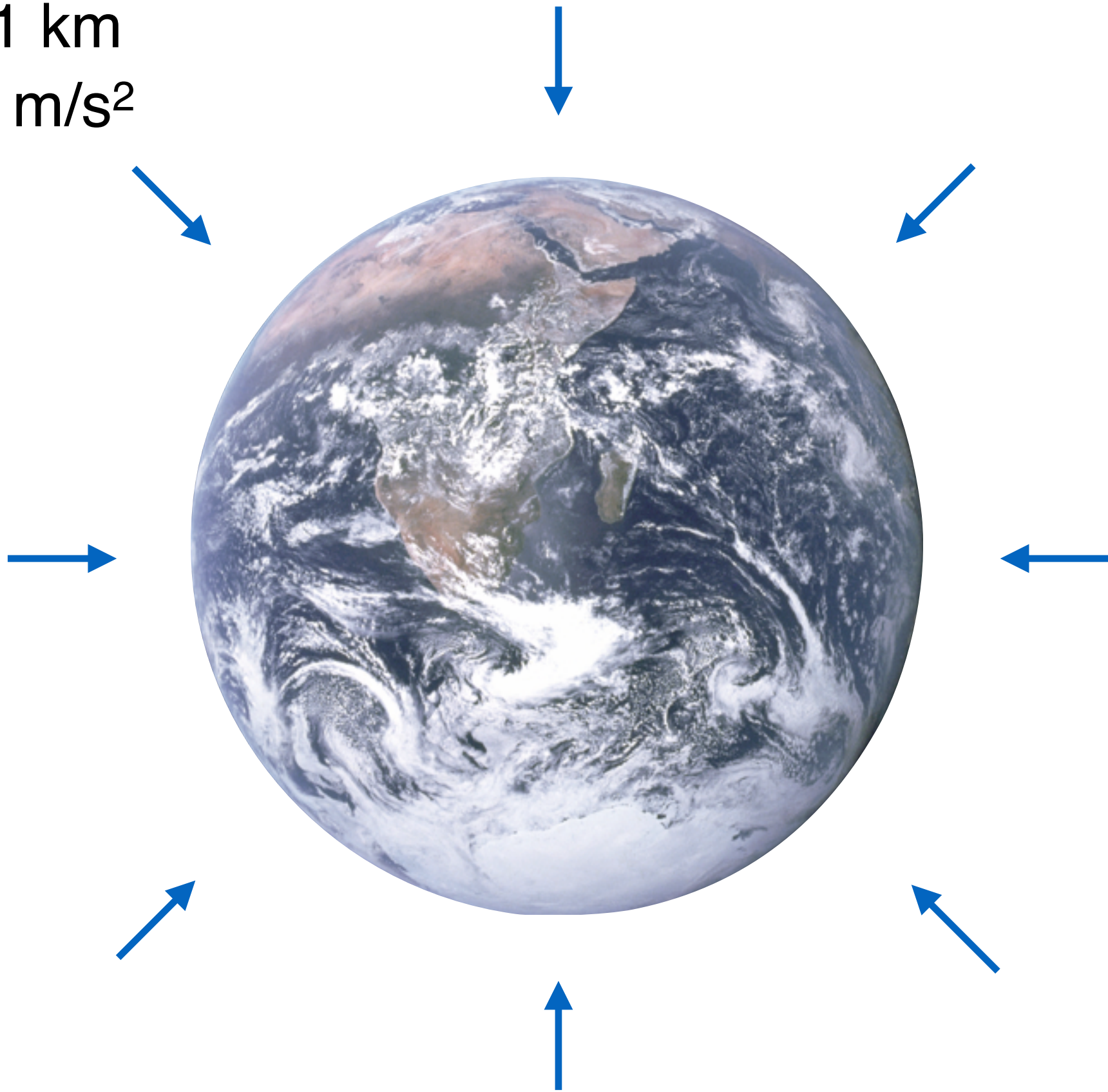
$R = 6371 \text{ km}$
 $g = 9,81 \text{ m/s}^2$



Earth shape and gravity field

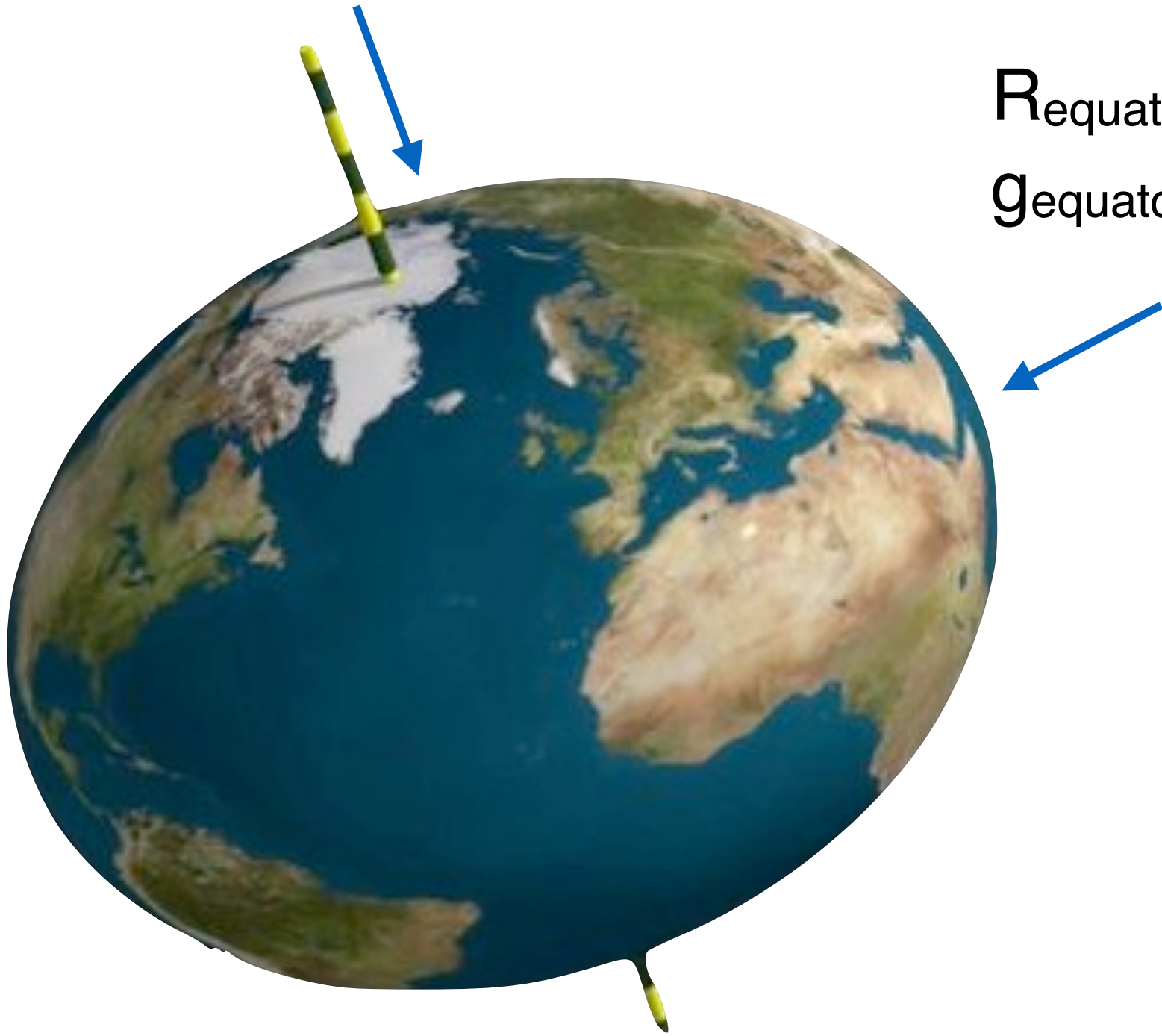
Spherical Earth

$R = 6371 \text{ km}$
 $g = 9,81 \text{ m/s}^2$



Ellipsoidal Earth

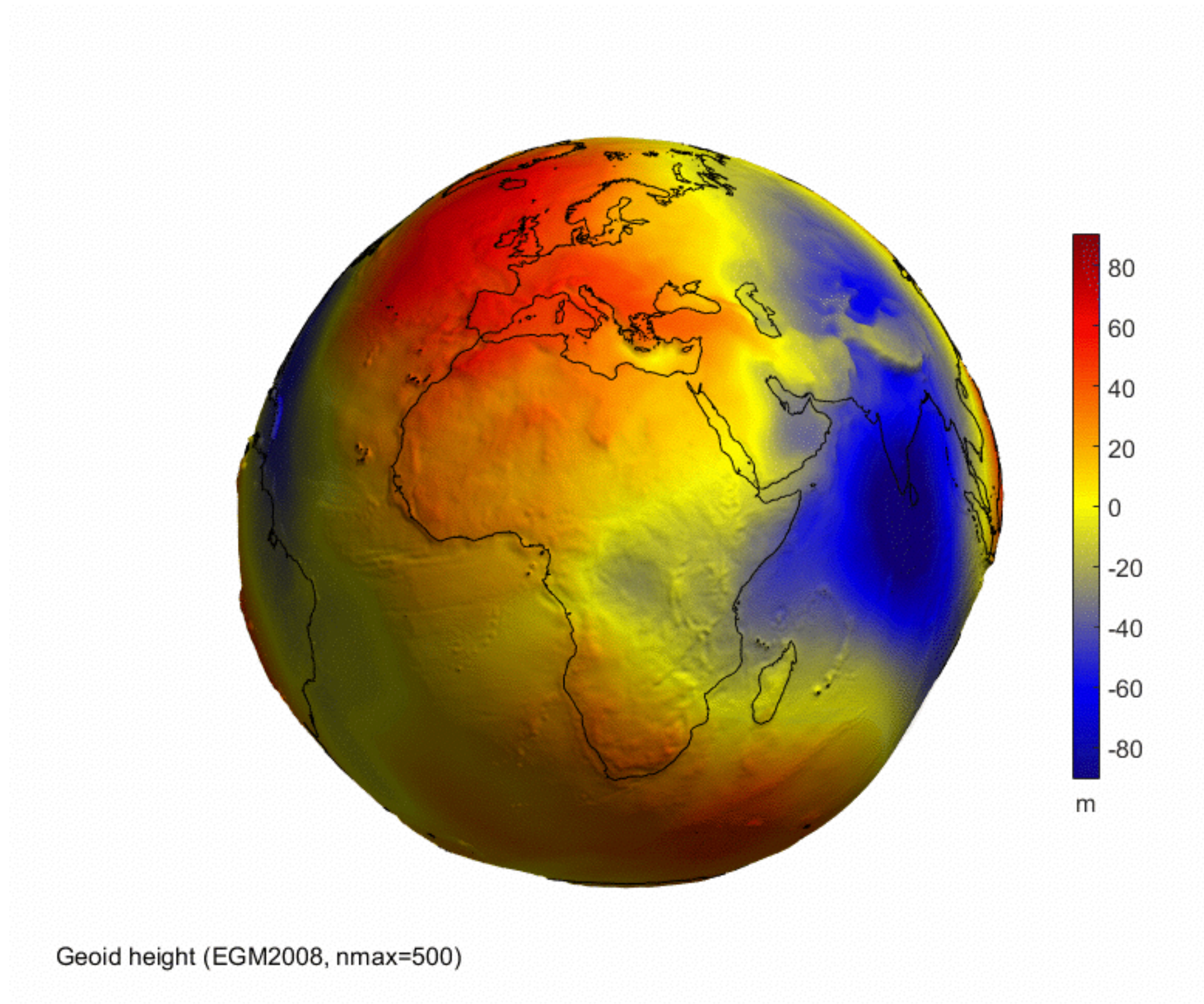
$R_{\text{poles}} = 6357 \text{ km}$
 $g_{\text{poles}} = 9,832 \text{ m/s}^2$



$R_{\text{equator}} = 6378 \text{ km}$
 $g_{\text{equator}} = 9,780 \text{ m/s}^2$

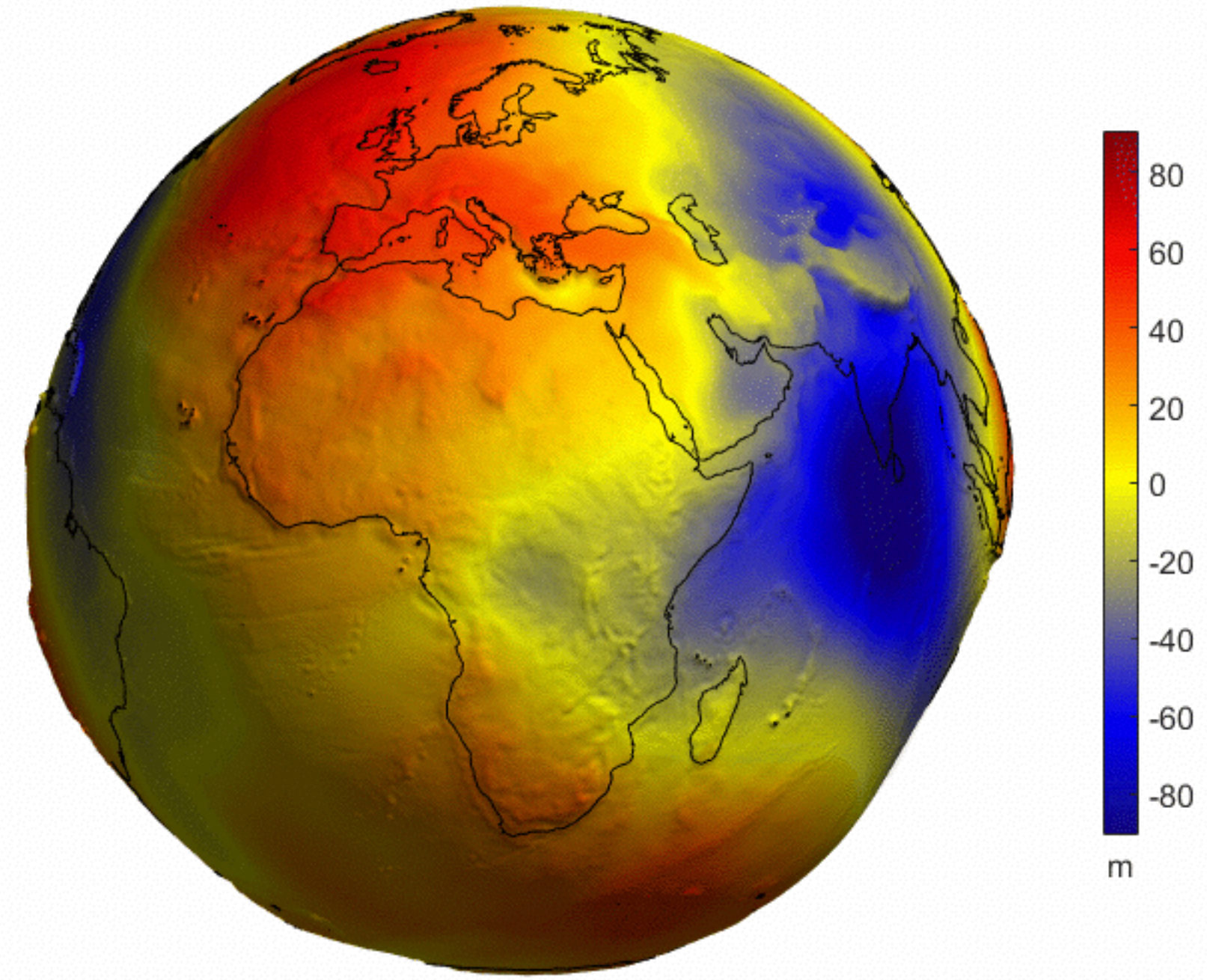
$dg = 0,05 \text{ m/s}^2$

Geoid (equipotential surface following the averaged sea level)



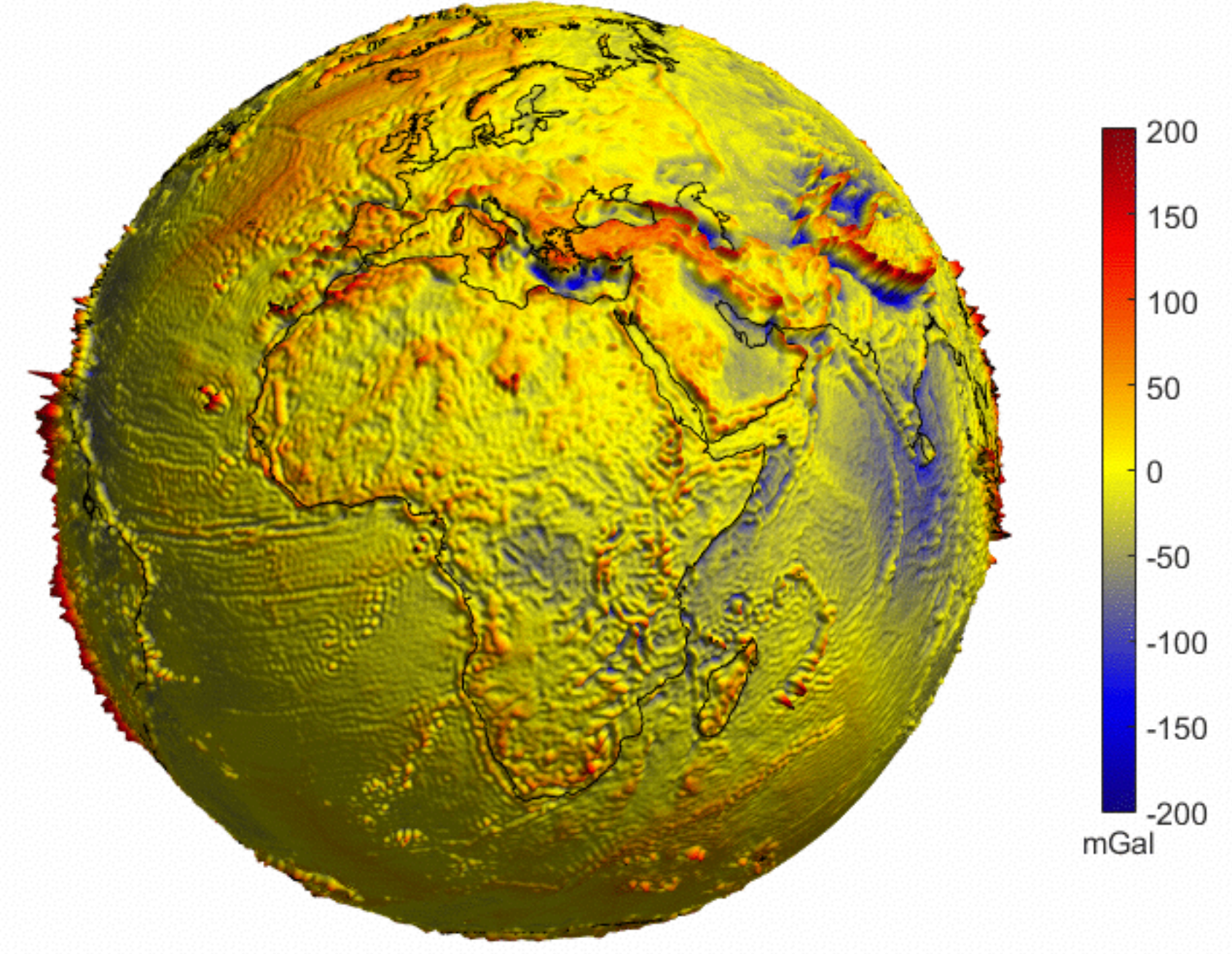
Earth shape and gravity field

Geoid (equipotential surface following the averaged sea level)



Geoid height (EGM2008, nmax=500)

Gravity field variations due to density variations

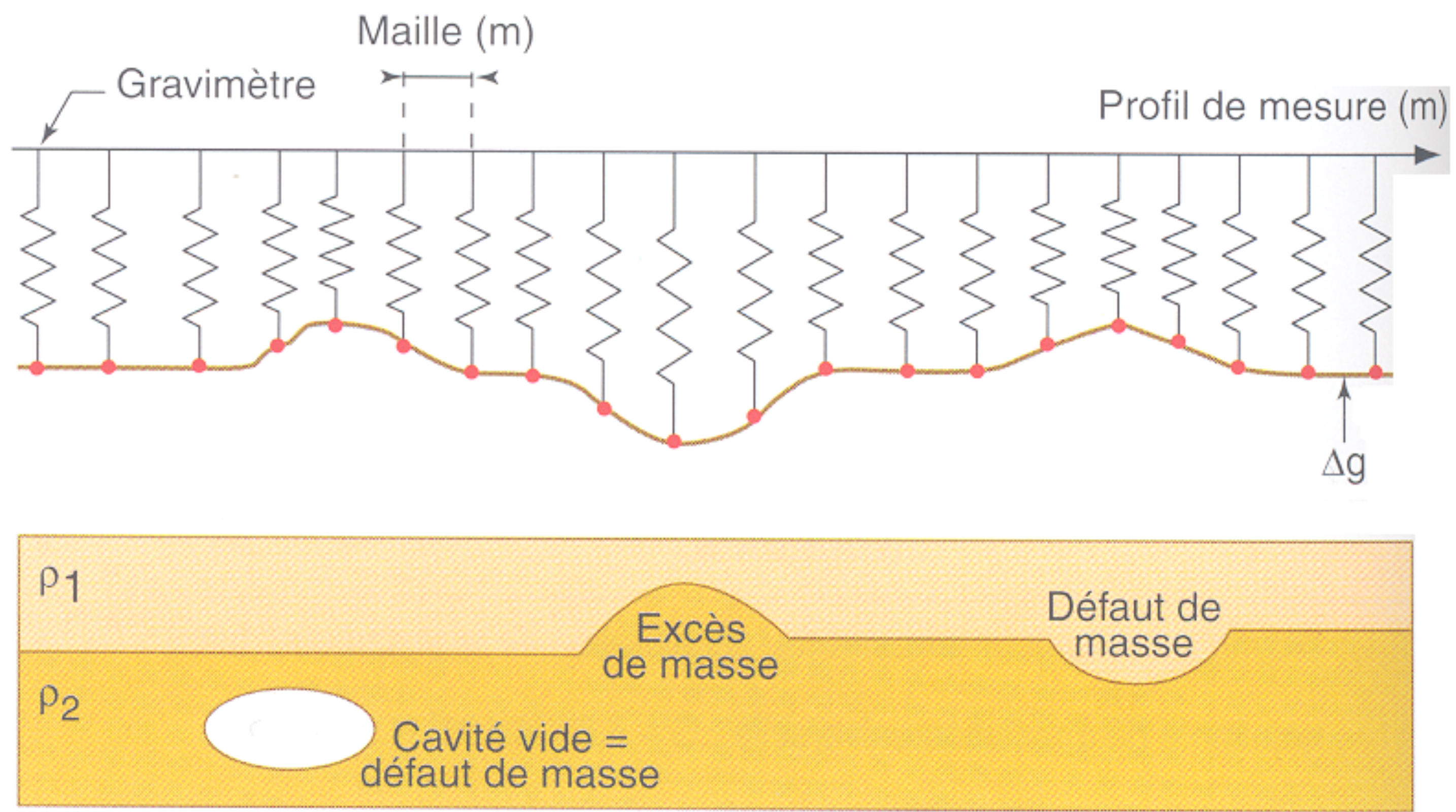


Gravity disturbance (EGM2008, nmax=500)

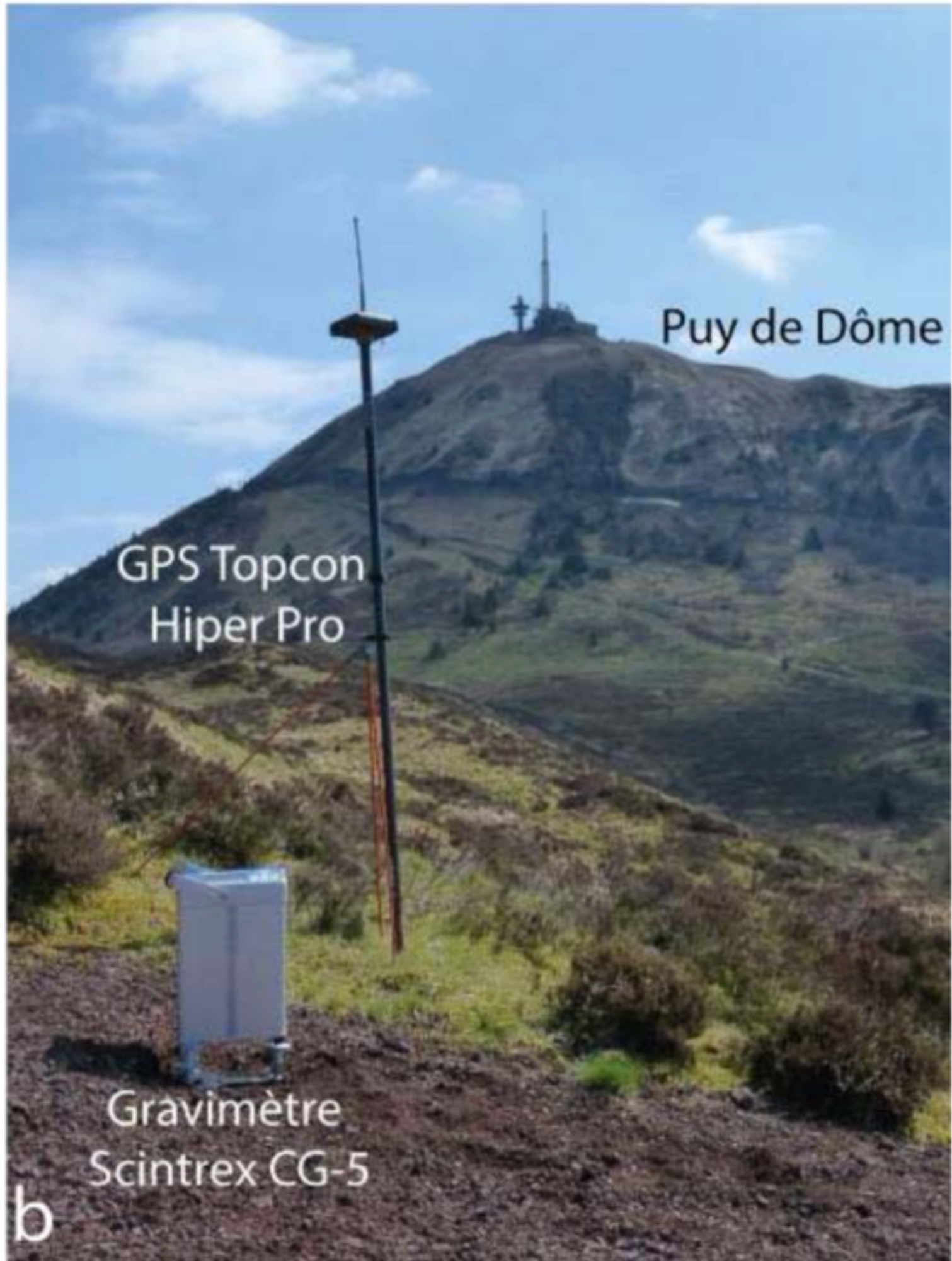
$dg = 0,004 \text{ m/s}^2 = 400 \text{ mGal}$

Bezdek and Sebera 2013

Local gravimetric variations



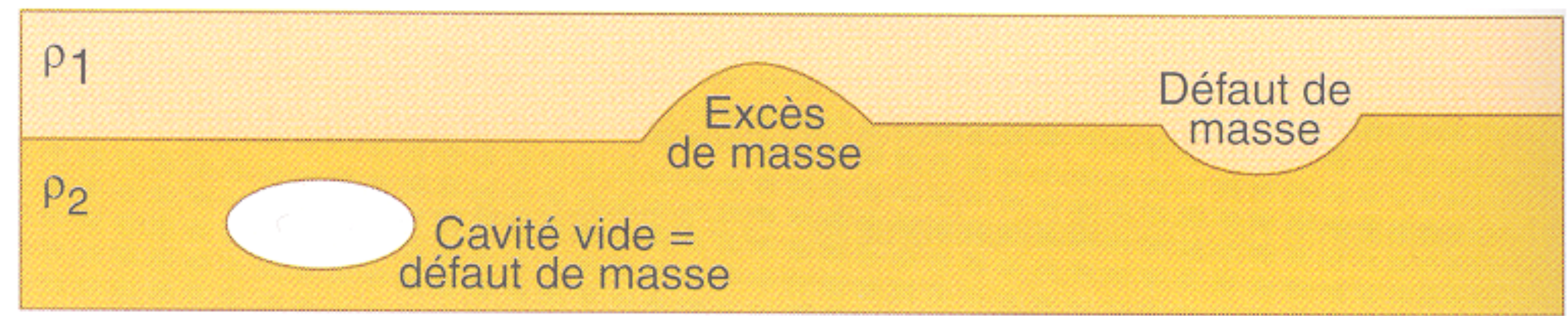
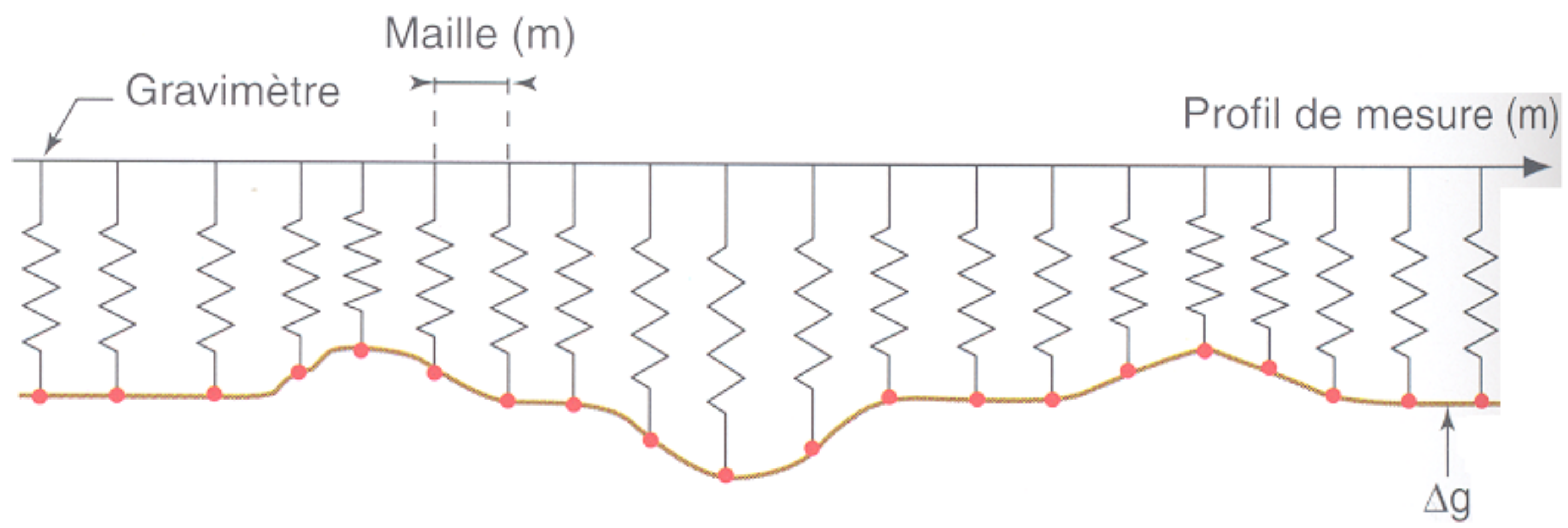
LCPC 2004



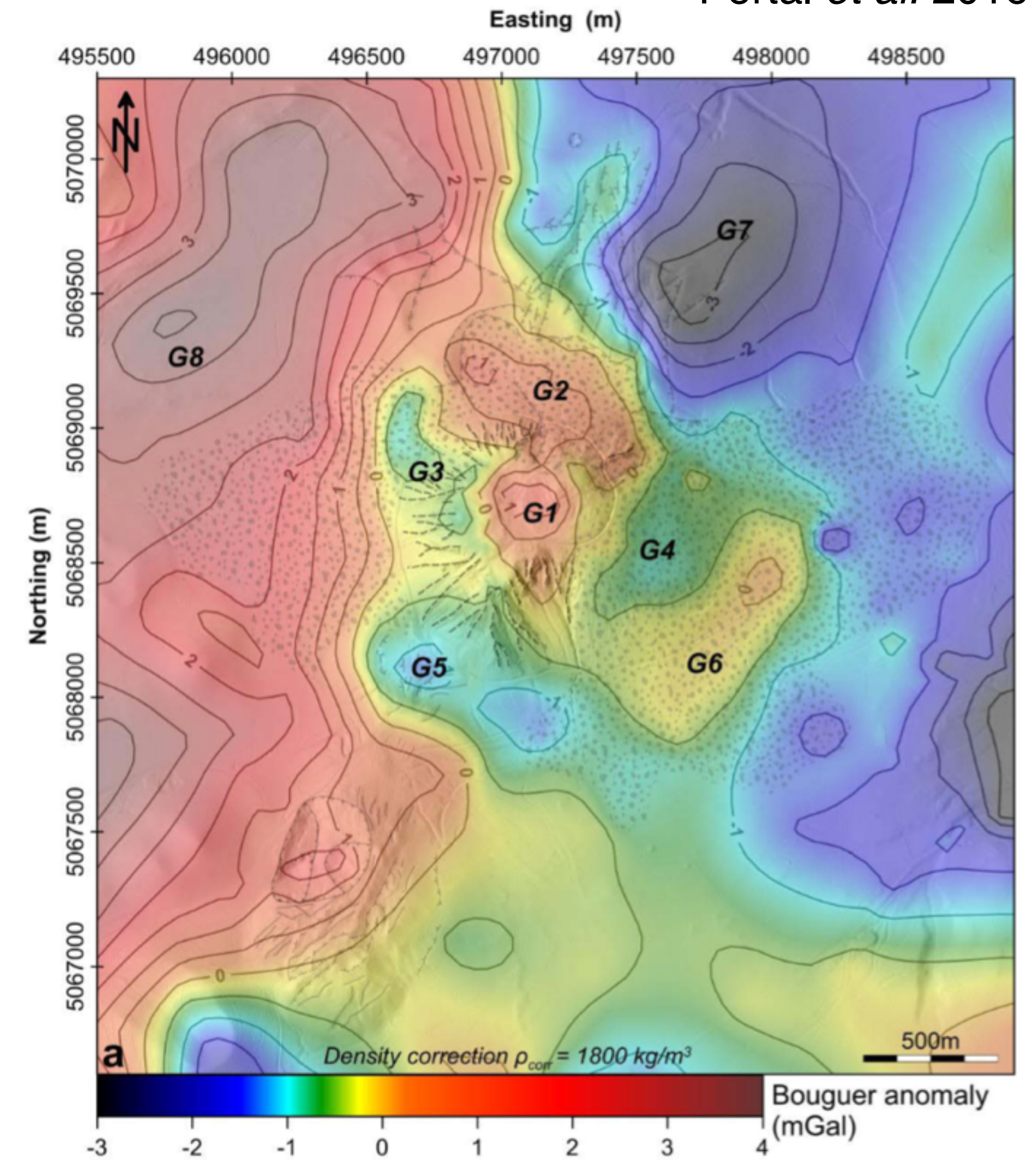
Portal 2015

Local gravimetric variations

Portal *et al.* 2016

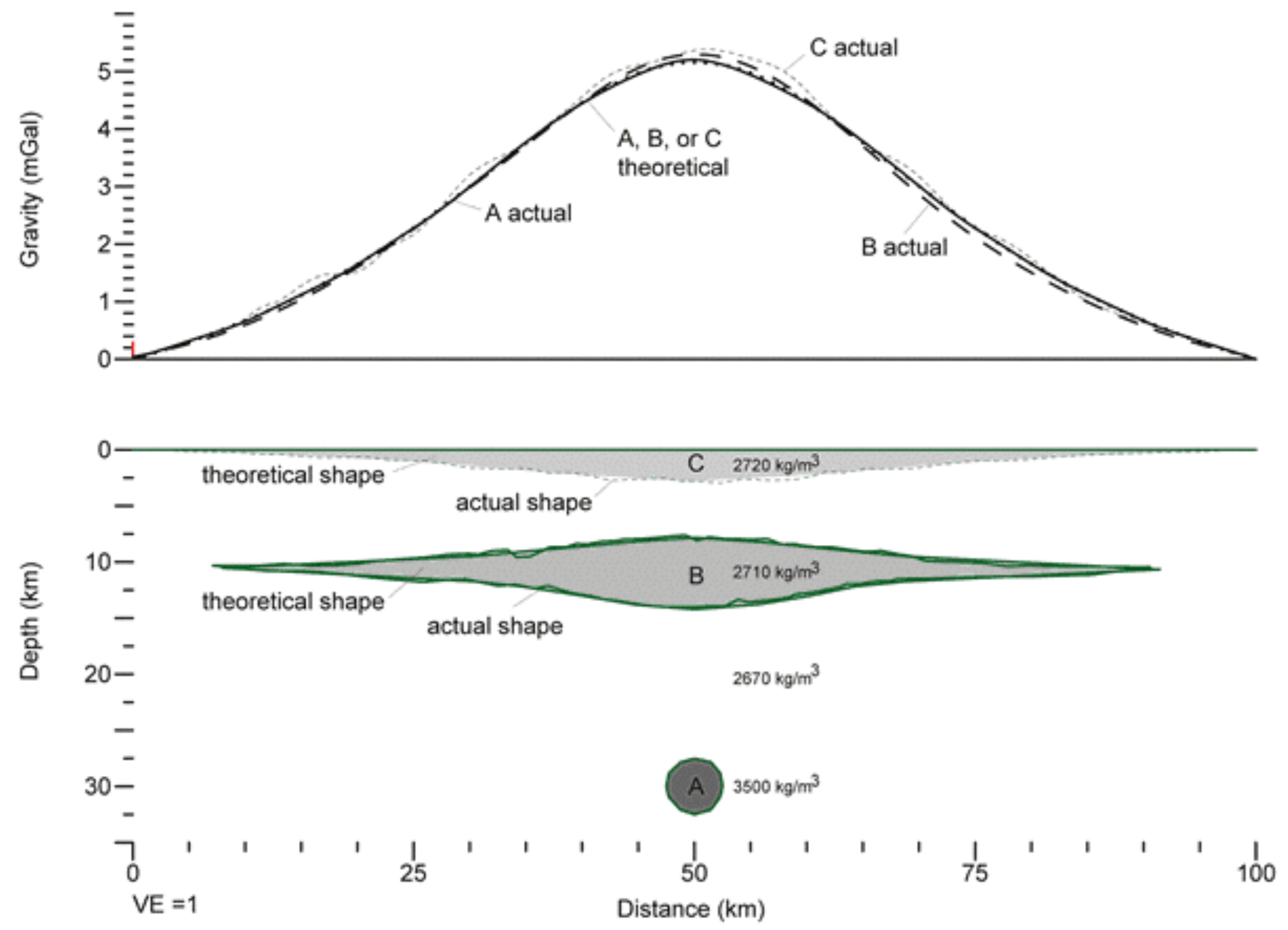


LCPC 2004



$dg \sim 0,00001 \text{ m/s}^2 = 1 \text{ mGal}$

Non-uniquicity of the gravimetric method



Saltus & Blakely 2011

Linear gravimetric inverse problem

- Modelling
 - Nodes of density
 - Topography taken into account
- Linear problem

sensitivity matrix (derivatives) → \mathbf{G}

densities → ρ

gravimetric data → d

$$\mathbf{G}\rho = d$$

- Ill-posed

- Function to minimize: Bayesian regularization (Tarantola & Valette 1982)

$$\mathbf{G}\boldsymbol{\rho} = \mathbf{d}$$

$$\phi(\tilde{\boldsymbol{\rho}}) = (\mathbf{d} - \mathbf{G}\tilde{\boldsymbol{\rho}})^t \mathbf{C}_D^{-1} (\mathbf{d} - \mathbf{G}\tilde{\boldsymbol{\rho}}) + (\tilde{\boldsymbol{\rho}} - \boldsymbol{\rho}_{prior})^t \mathbf{C}_P^{-1} (\tilde{\boldsymbol{\rho}} - \boldsymbol{\rho}_{prior})$$

$$\mathbf{C}_{D,ii} = \sigma_{d,i}$$

$$\begin{bmatrix} 1/\sigma_d^2 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_d^2 \end{bmatrix}$$

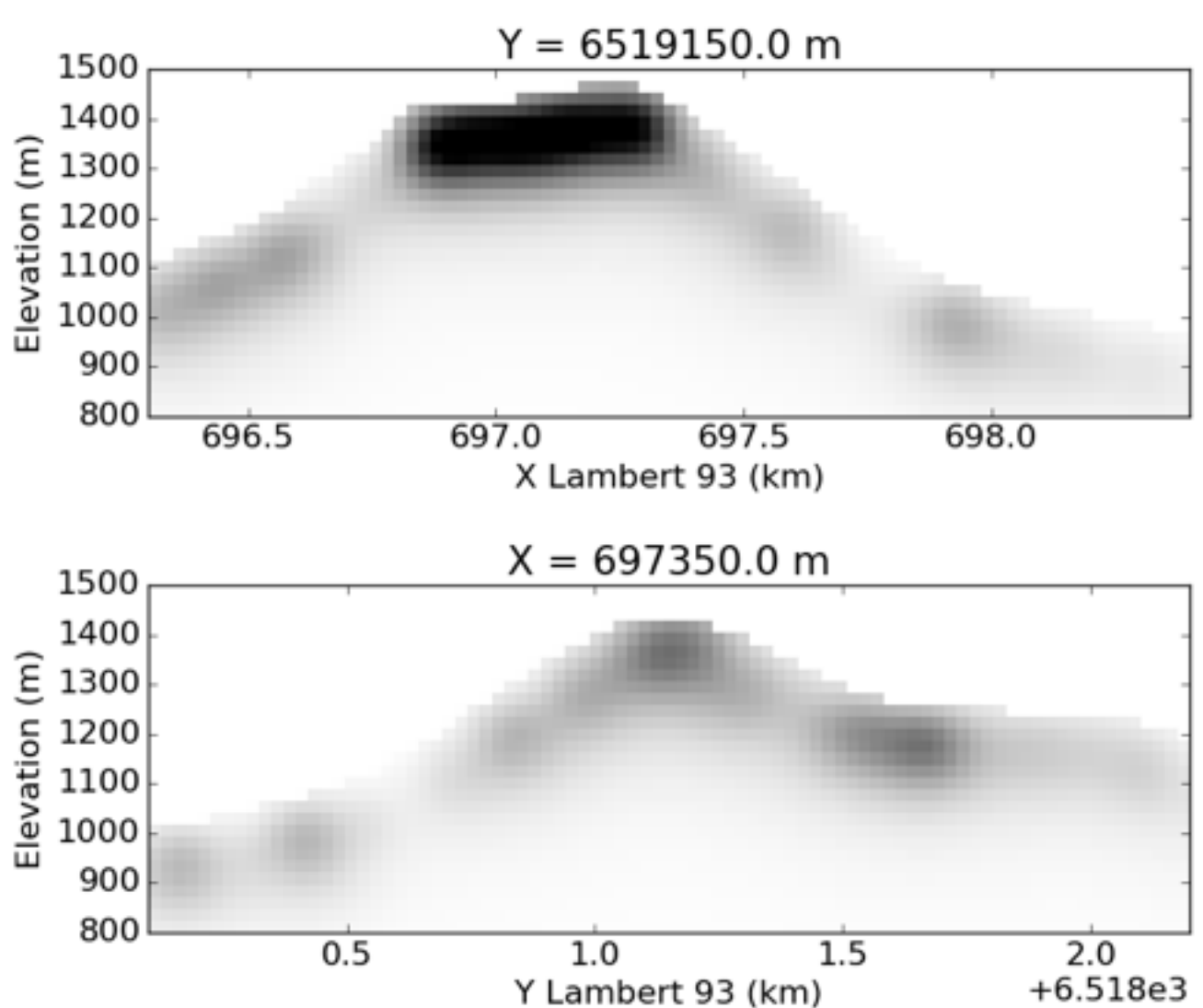
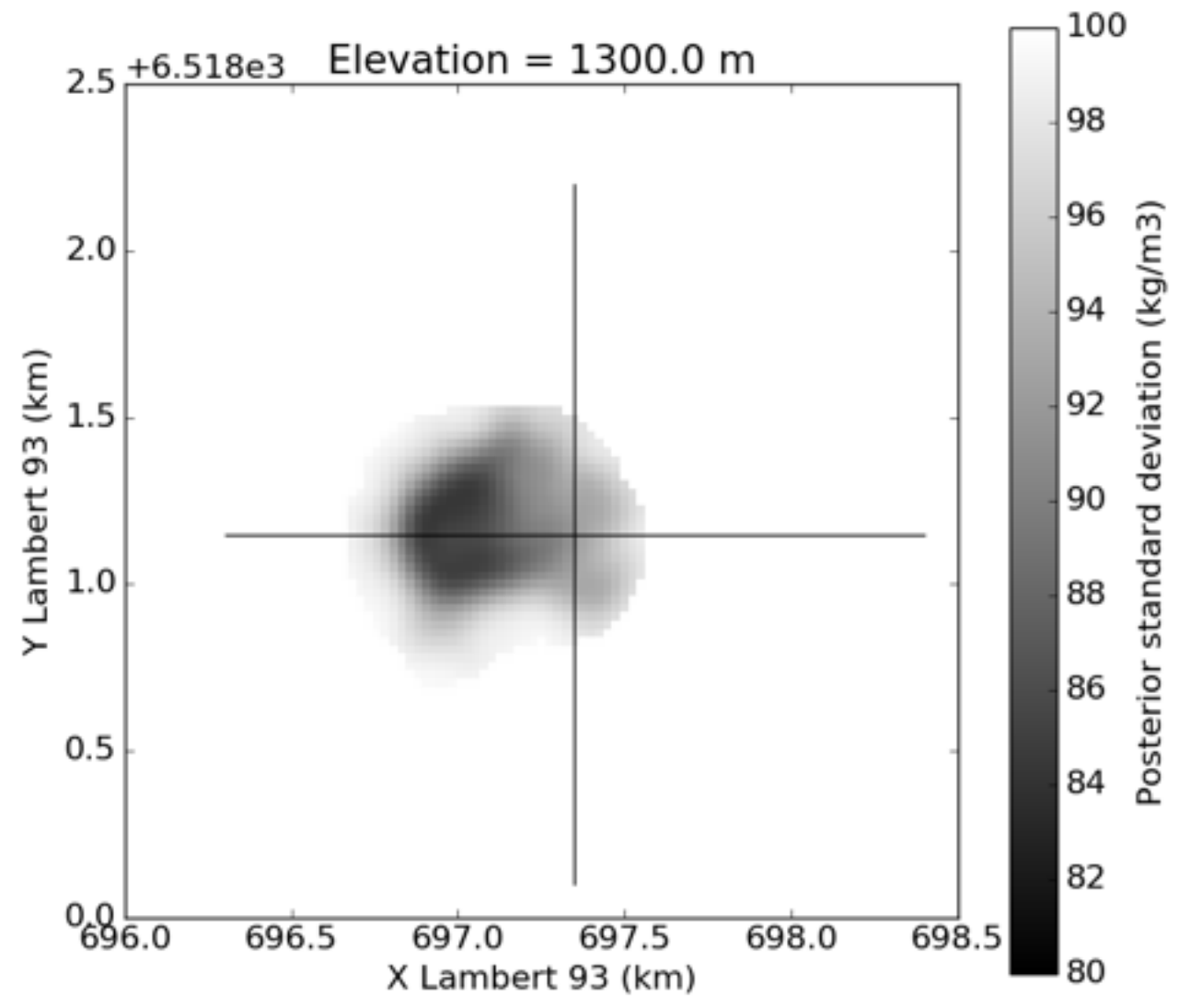
- data error

$$\mathbf{C}_{P,ij} = \sigma_{\rho,ij}^2 \exp^{-\frac{D_{ij}^2}{\lambda^2}}$$

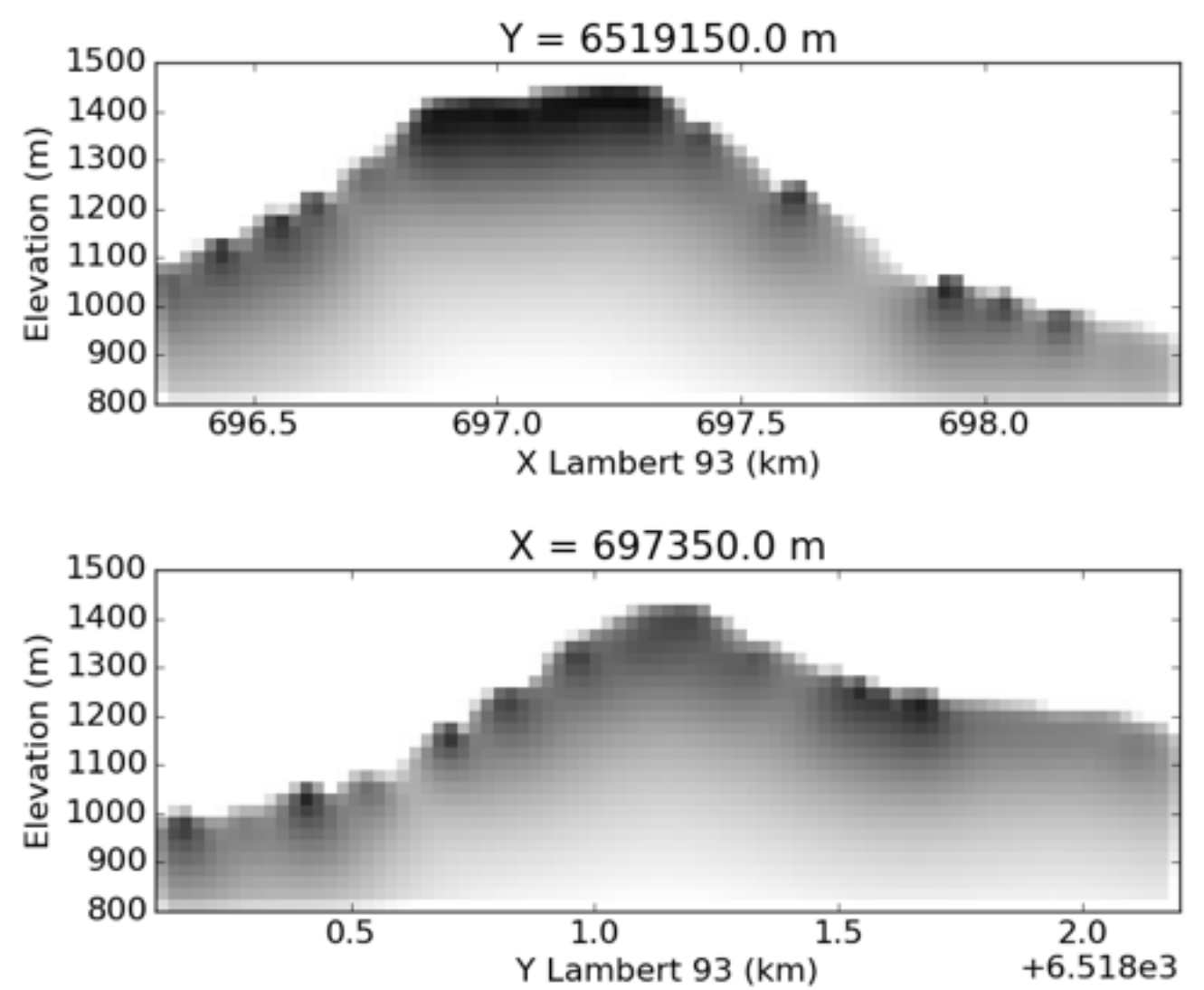
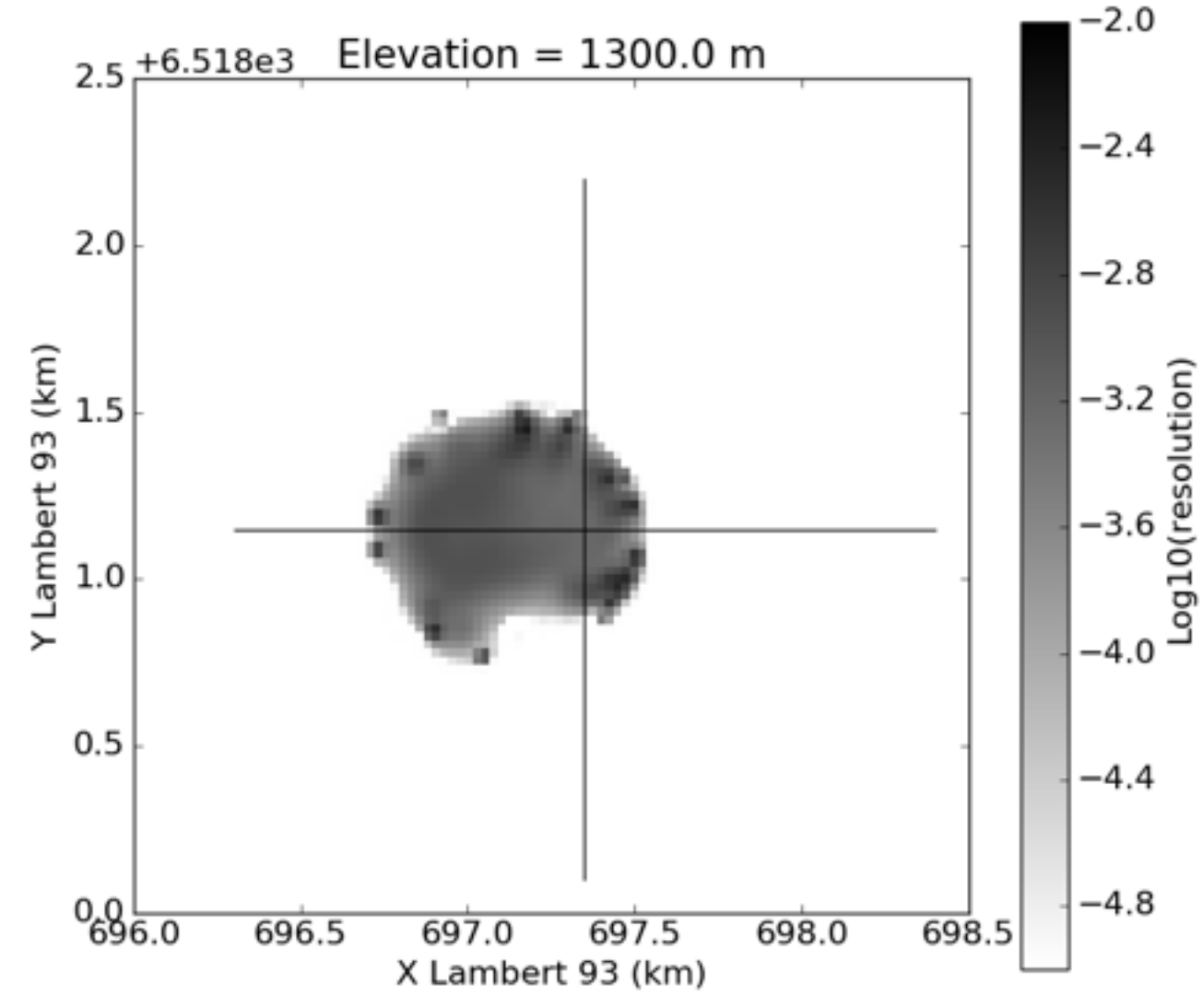
$$\begin{bmatrix} 1/\sigma_\rho^2 & \dots & \dots \\ \dots & \ddots & \dots \\ \dots & \dots & 1/\sigma_\rho^2 \end{bmatrix}$$

- a priori density standard deviation
- spatial correlation length

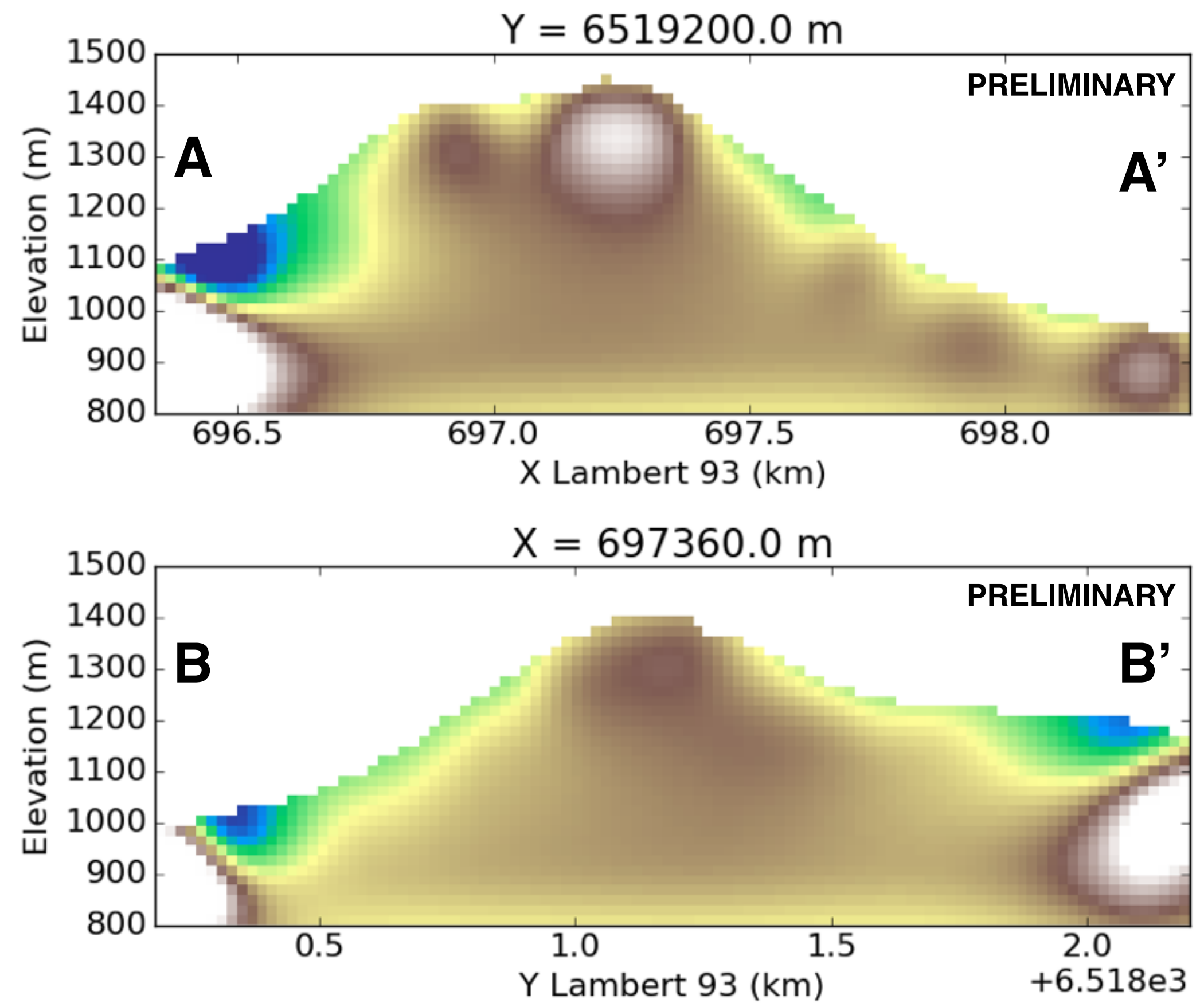
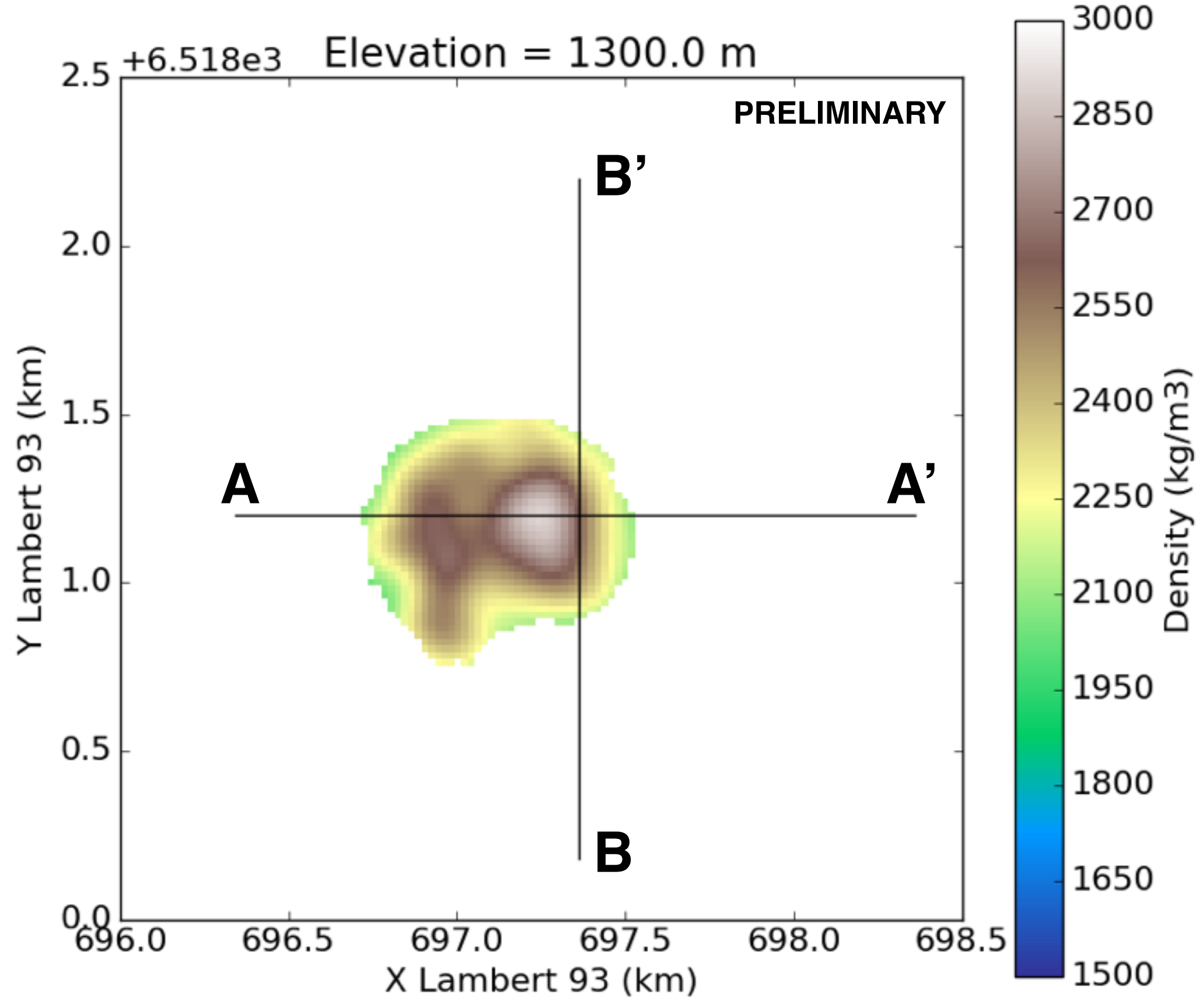
Resolving power of the gravimetric inversion: Cas of the puy de Dôme



- weak vertical resolution
- good lateral resolution at shallow depth
- lateral resolution length increasing with depth



Result of the inversion of the puy de Dôme gravimetric data



Gravimetric data RMS = 0.70 mGal



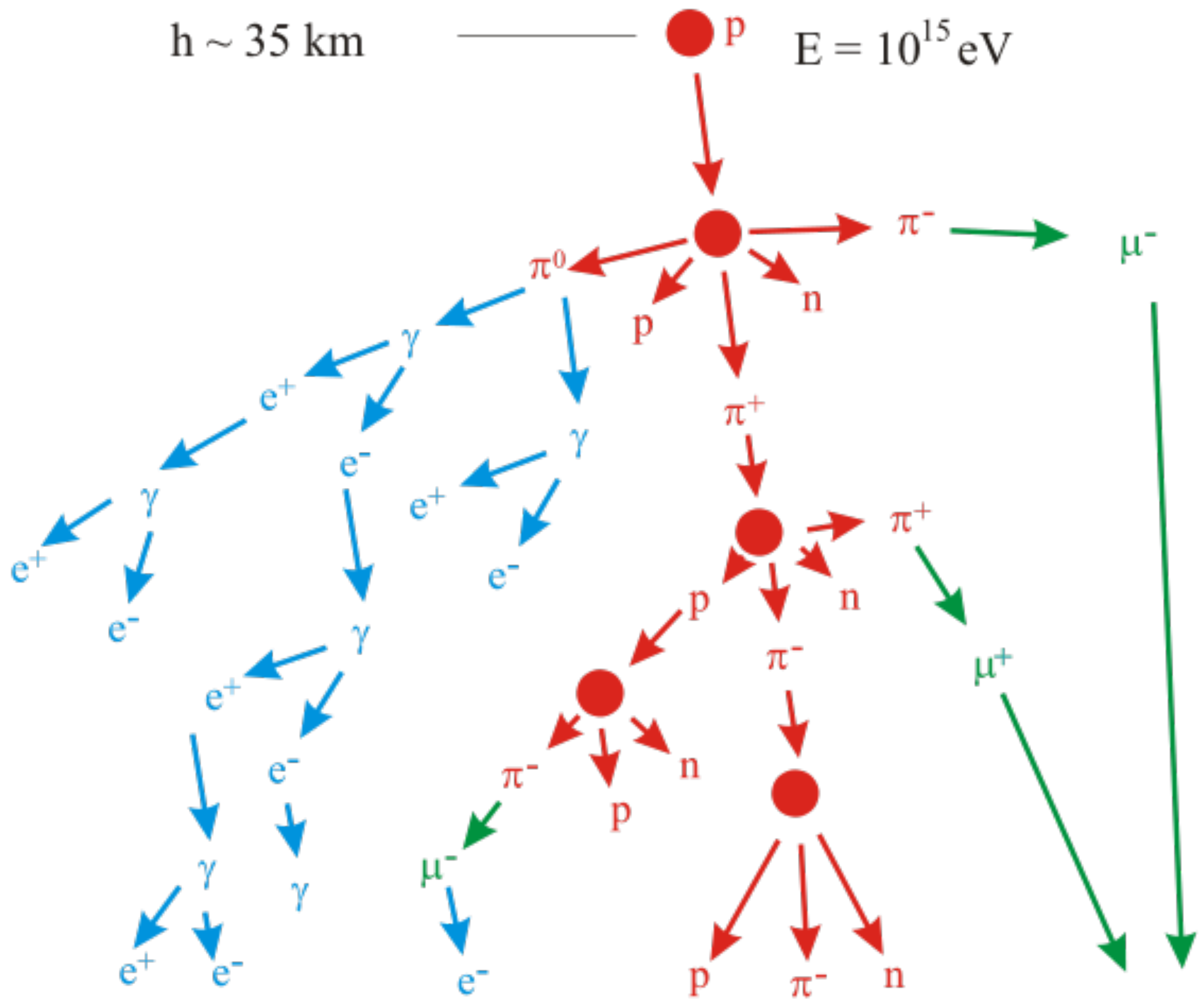
What is muography?

Idea: radiography with muons
-> 2D images of averaged density



Muons

- come from the interaction of **cosmic rays** with the atmosphere (free!)
- similar to electron, 200 times heavier
- large energy spectrum 100 MeV -> PeV
- **interact with matter in a stochastic way** (depending on their energies and medium density)
- at high energy **cross several kms of matter** before decaying

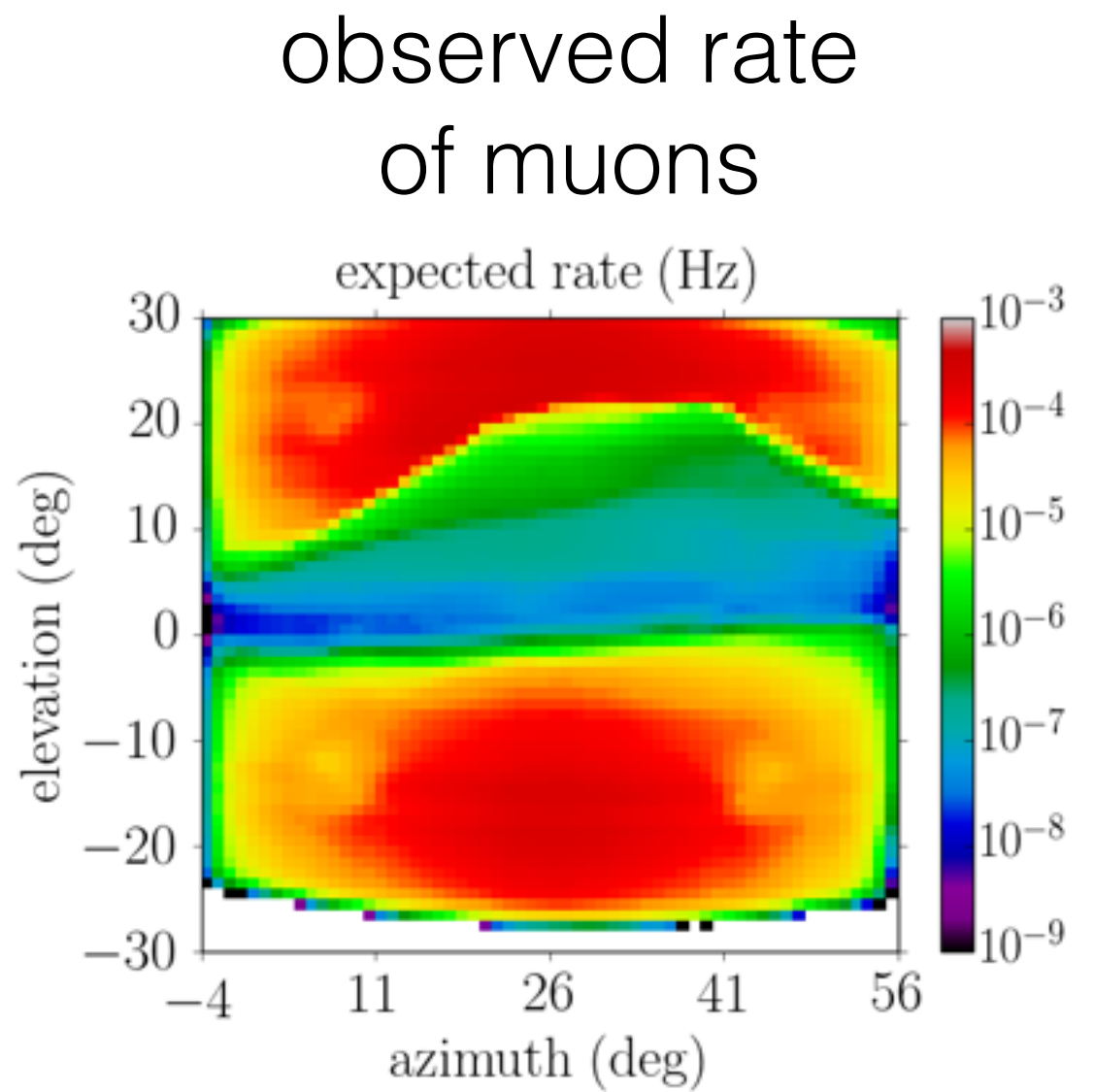
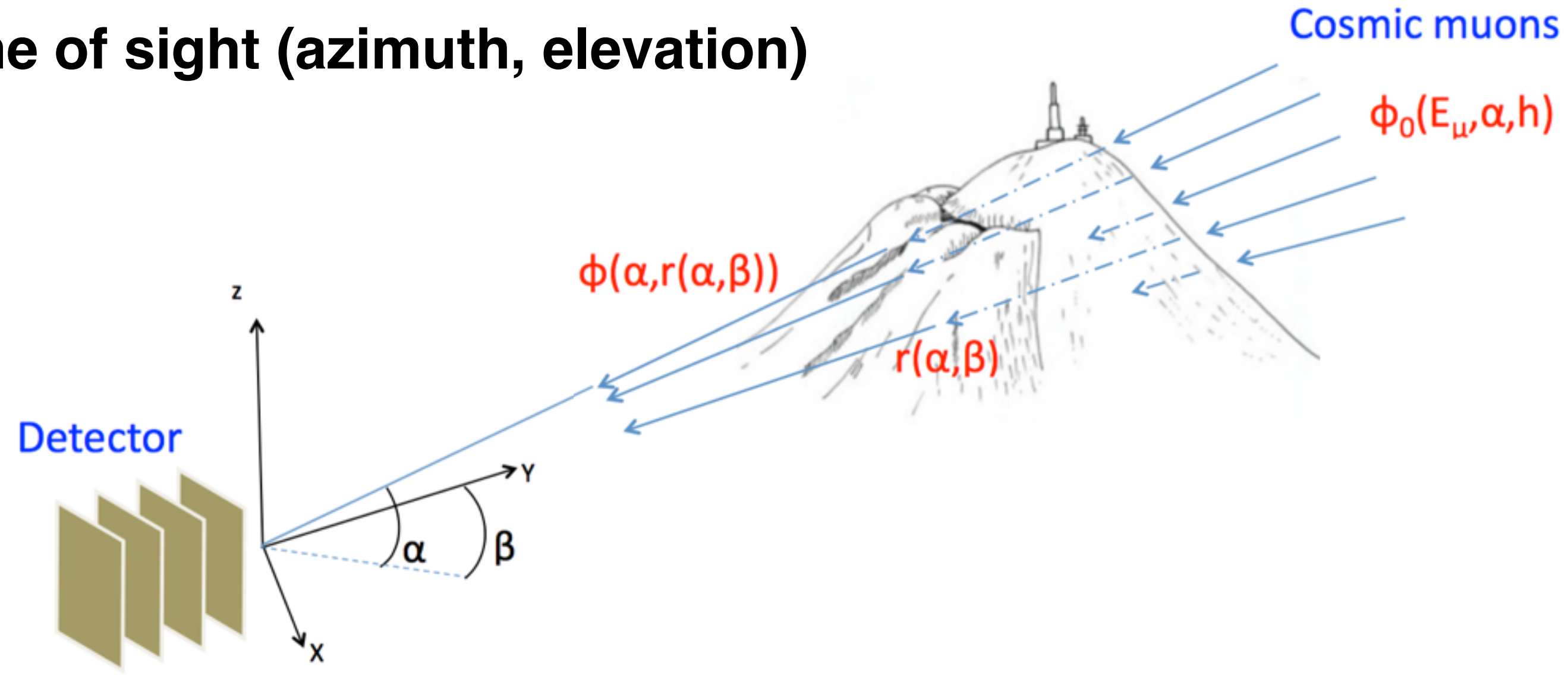


At sea level: $\sim 1 \text{ muon/cm}^2/\text{min}$
i.e. $\sim 1 \text{ muon/s}$ crosses one's hand

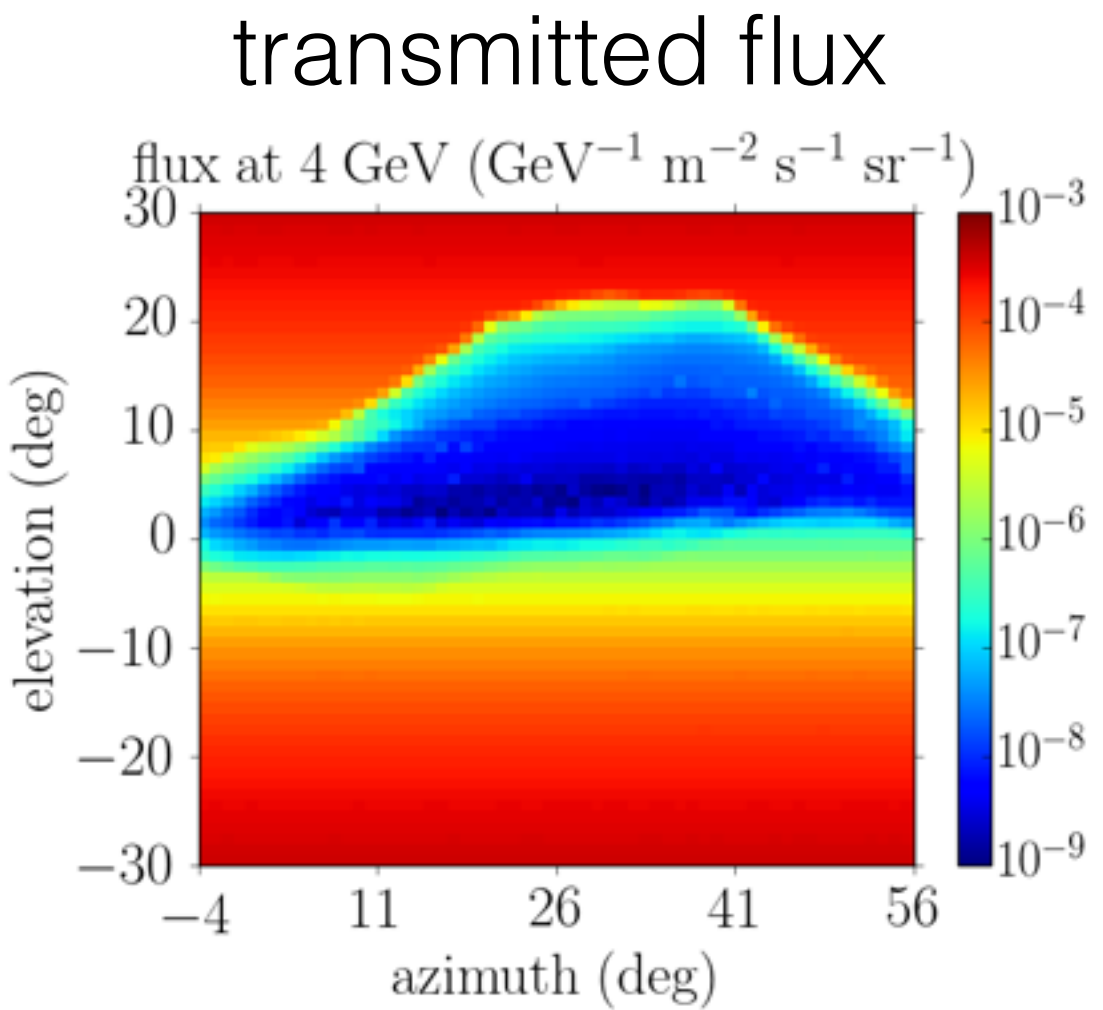
Principle of transmission muography

Muon telescope: counts the number of muons along **line of sight (azimuth, elevation)**

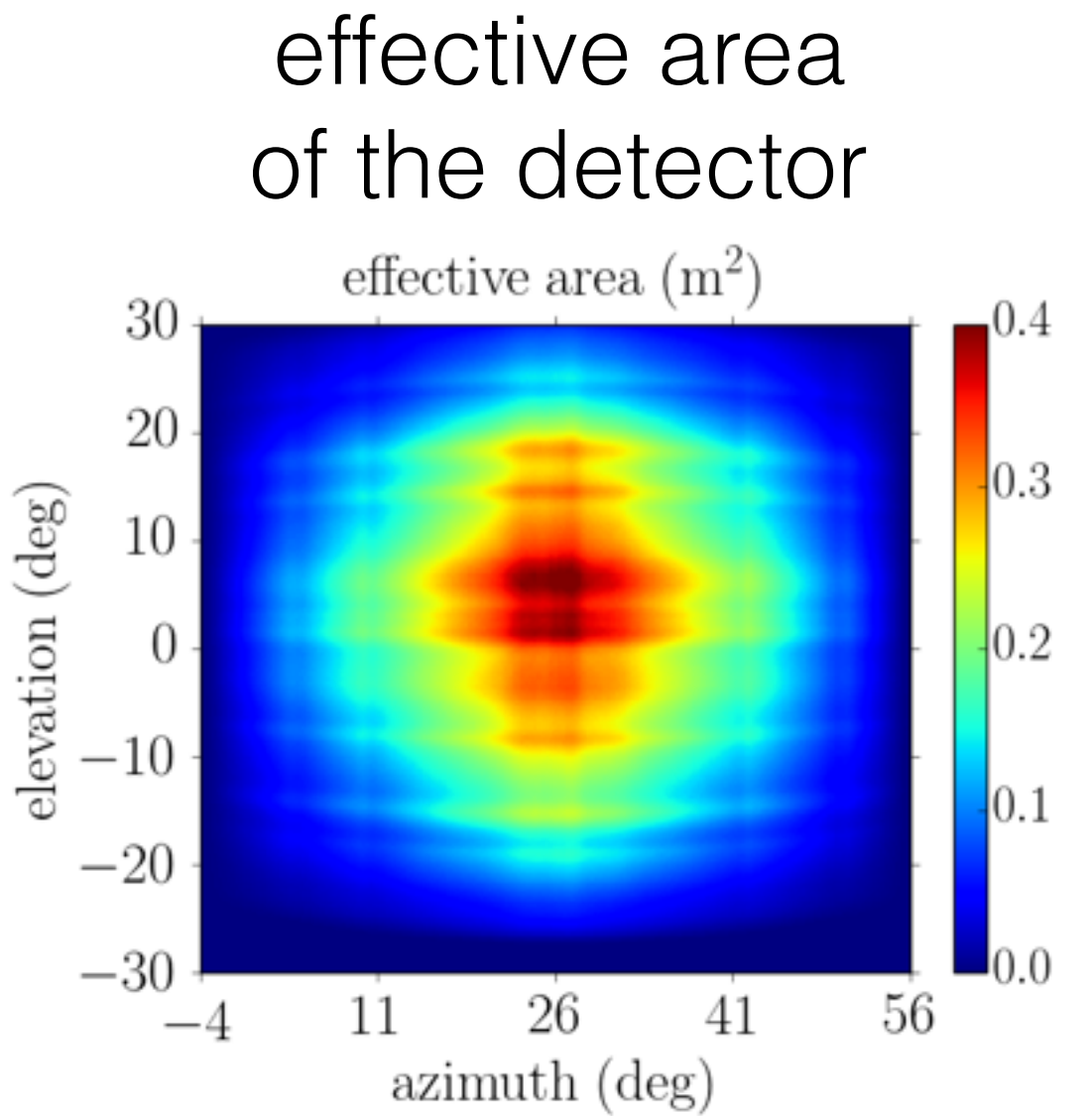
- A uniform density is assumed along the line of sight to compute the transmitted flux.
- For each line of sight, the observed rate is matched to the best density hypothesis.



$$= \int$$



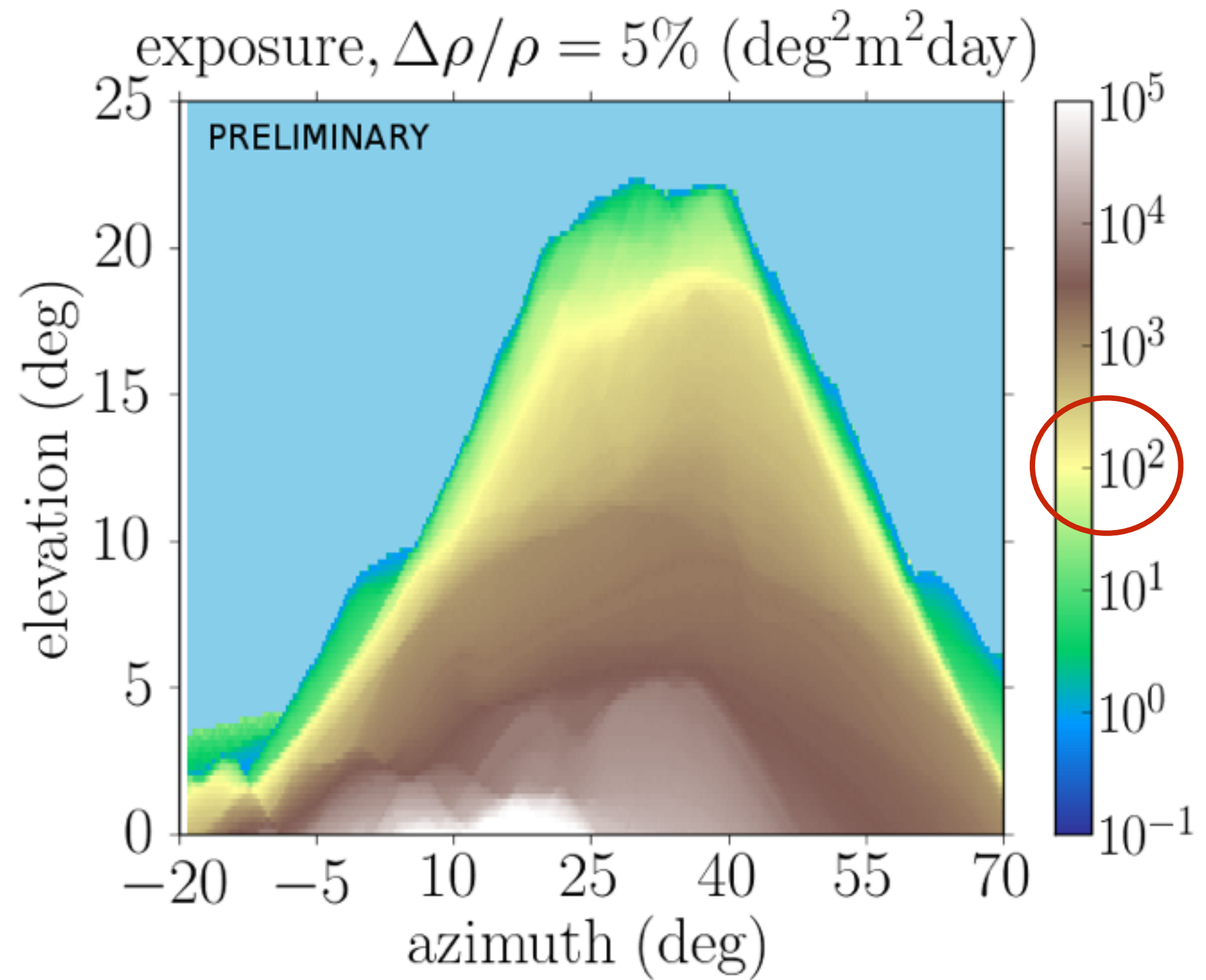
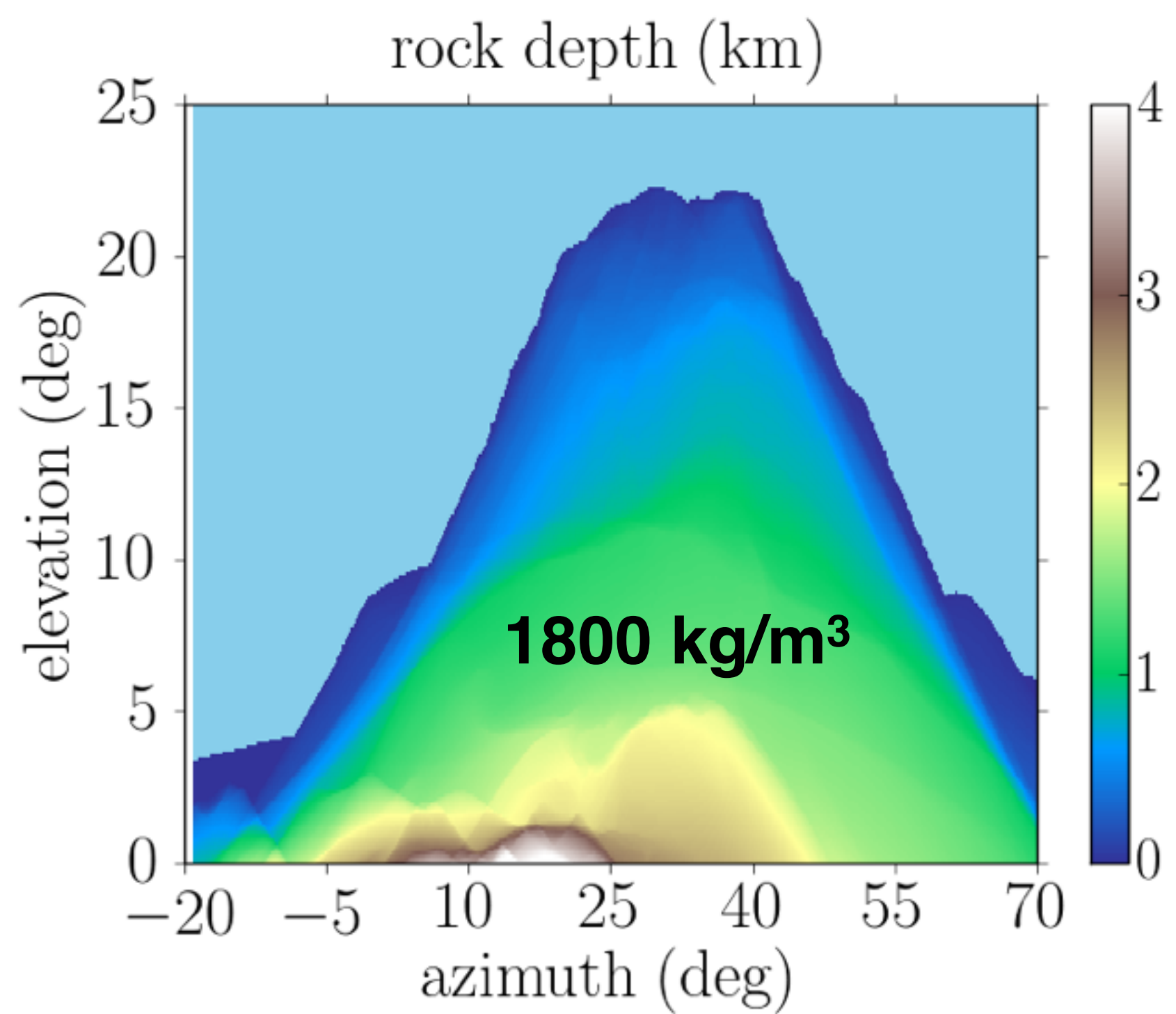
$$\times$$



$$dE$$

What can be imaged with muography?

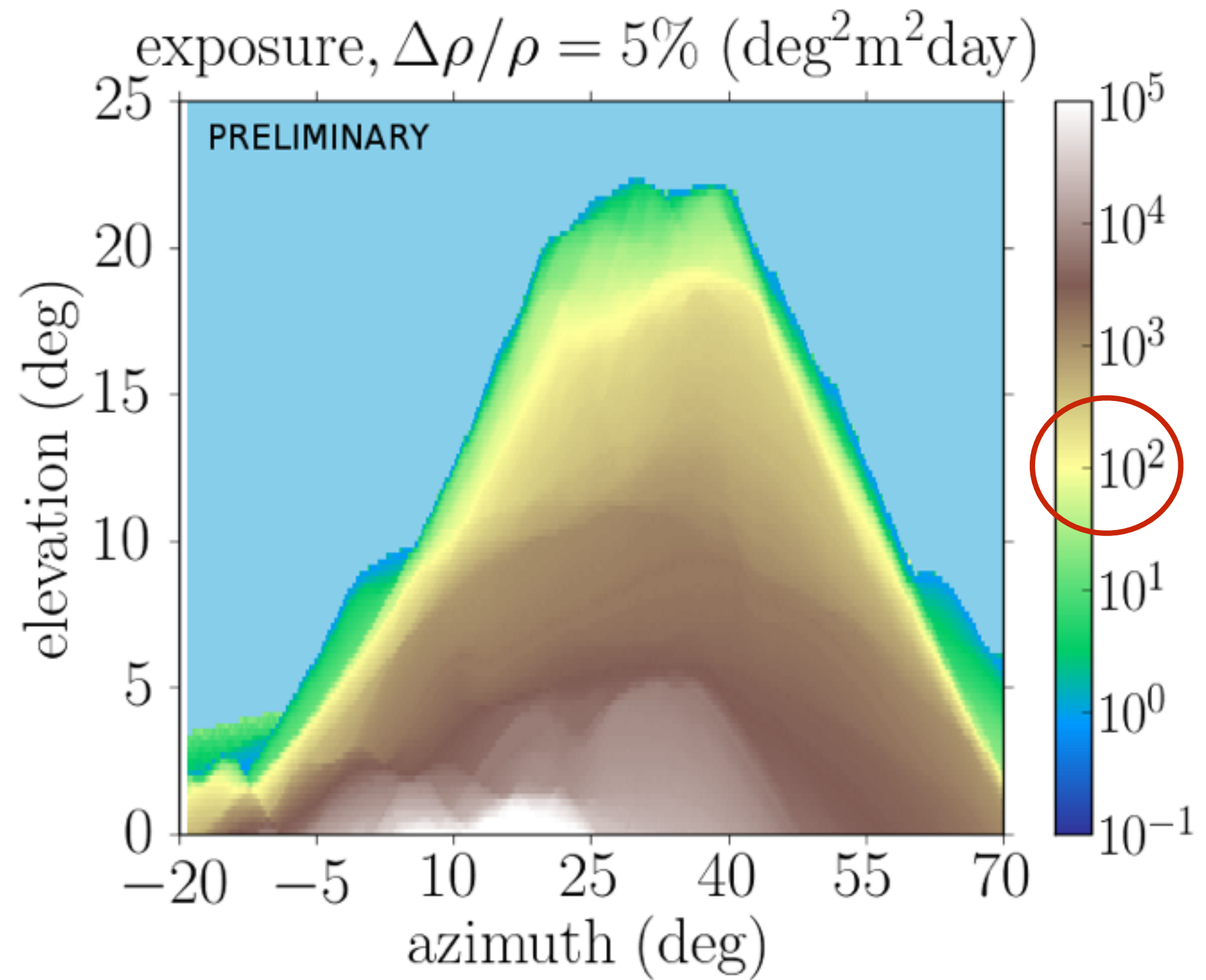
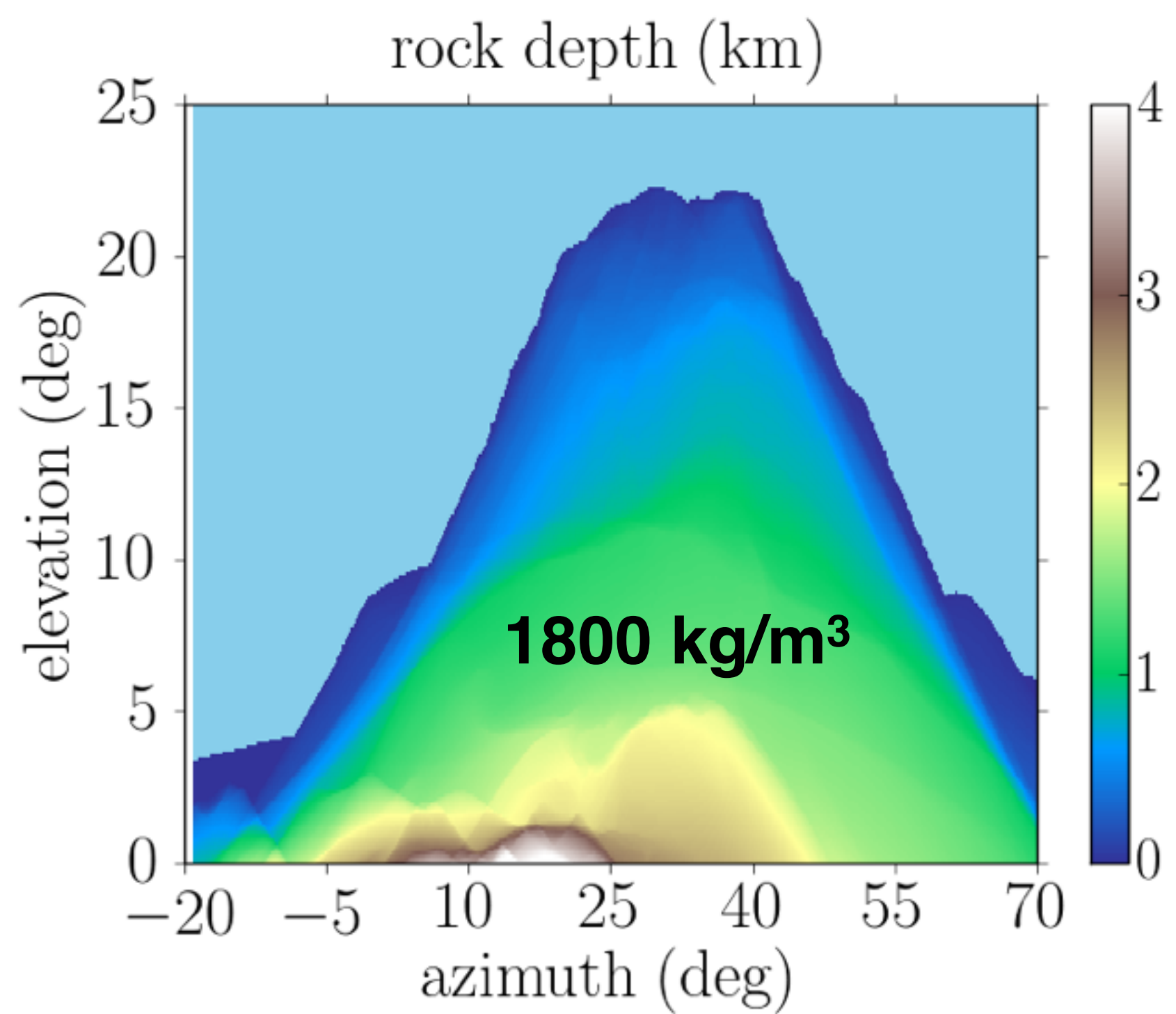
- **Exposure:** number of days to reach a precision on density of 5% in a $1^\circ \times 1^\circ$ solid angle and with a detector of 1 m^2



100 days for (5 %, 1°x1°, 1 m²)

What can be imaged with muography?

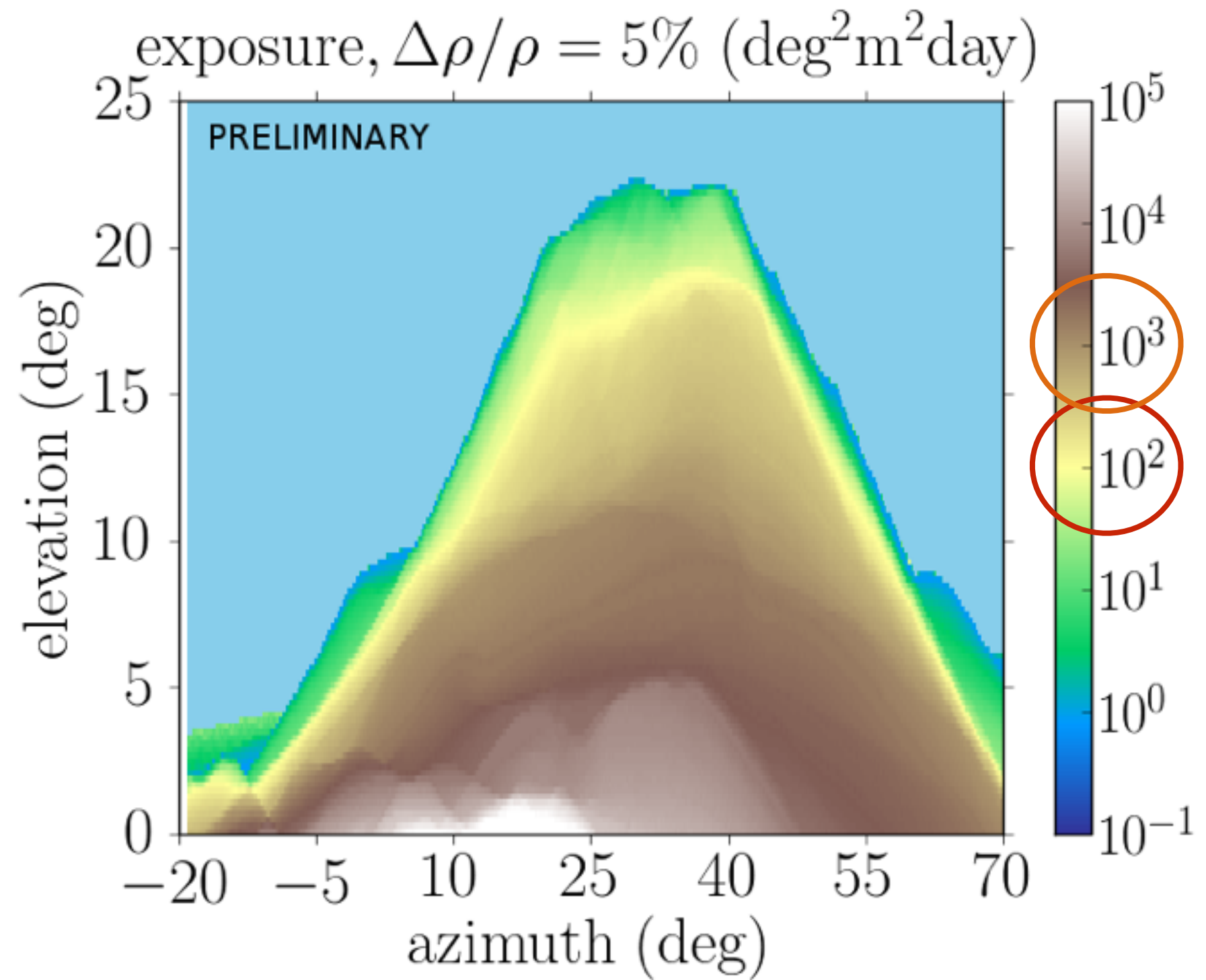
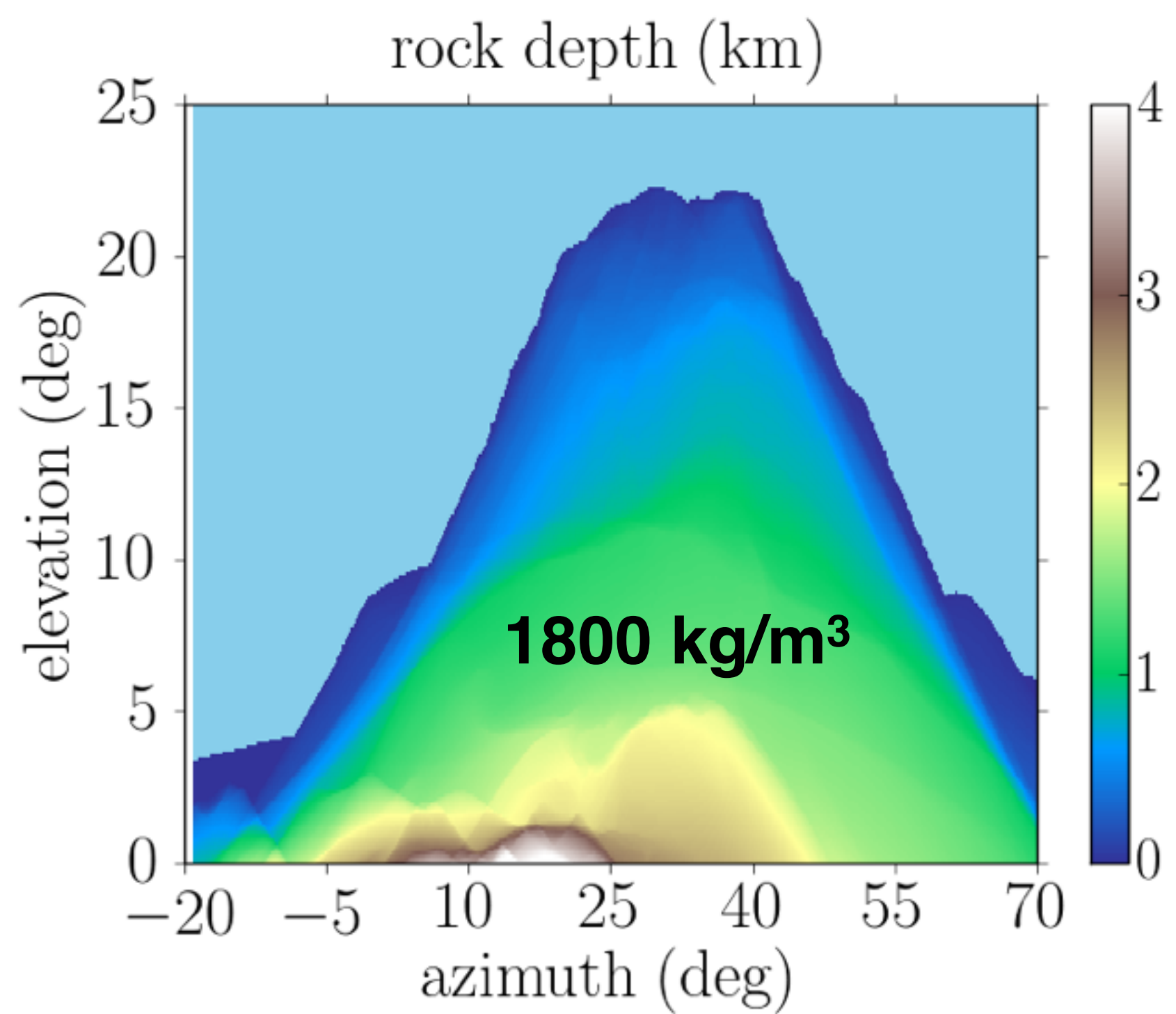
- **Exposure:** number of days to reach a precision on density of 5% in a $1^\circ \times 1^\circ$ solid angle and with a detector of 1 m^2



100 days for (5 %, $1^\circ \times 1^\circ$, 1 m^2)
10 days for (5 %, $1^\circ \times 1^\circ$, 10 m^2)

What can be imaged with muography?

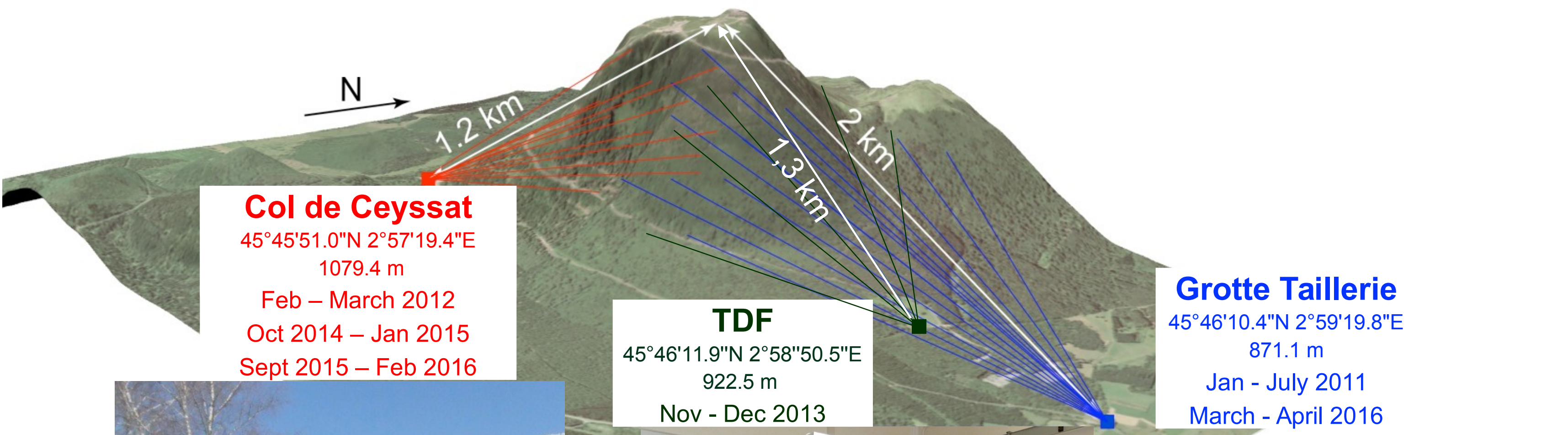
- **Exposure:** number of days to reach a precision on density of 5% in a 1°x1° solid angle and with a detector of 1 m²



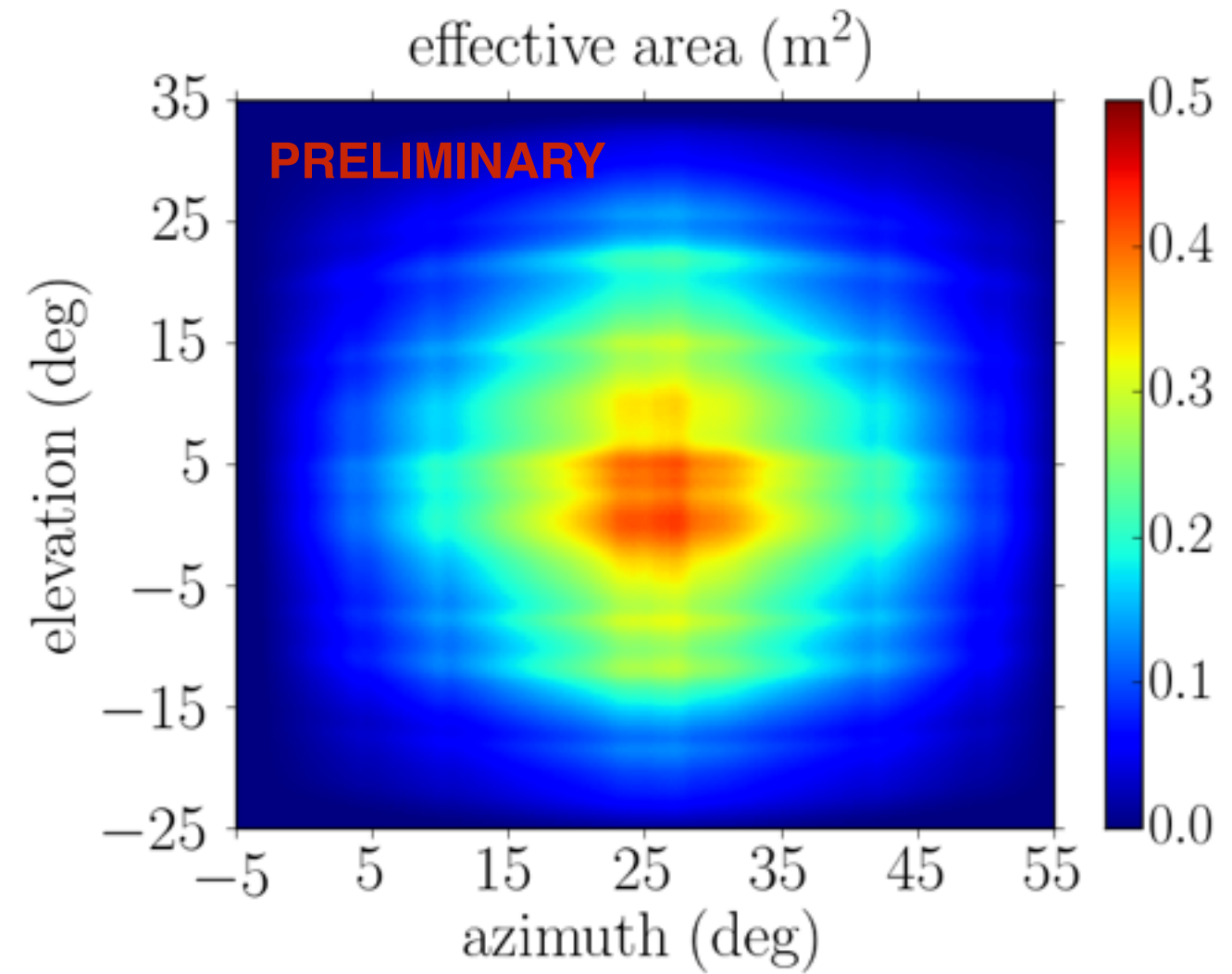
- 100 days for (5 %, 3°x3°, 1 m²)
- 100 days for (5 %, 1°x1°, 1 m²)
- 10 days for (5 %, 1°x1°, 10 m²)

Muography of puy de Dôme volcano - Acquisition and track reconstruction

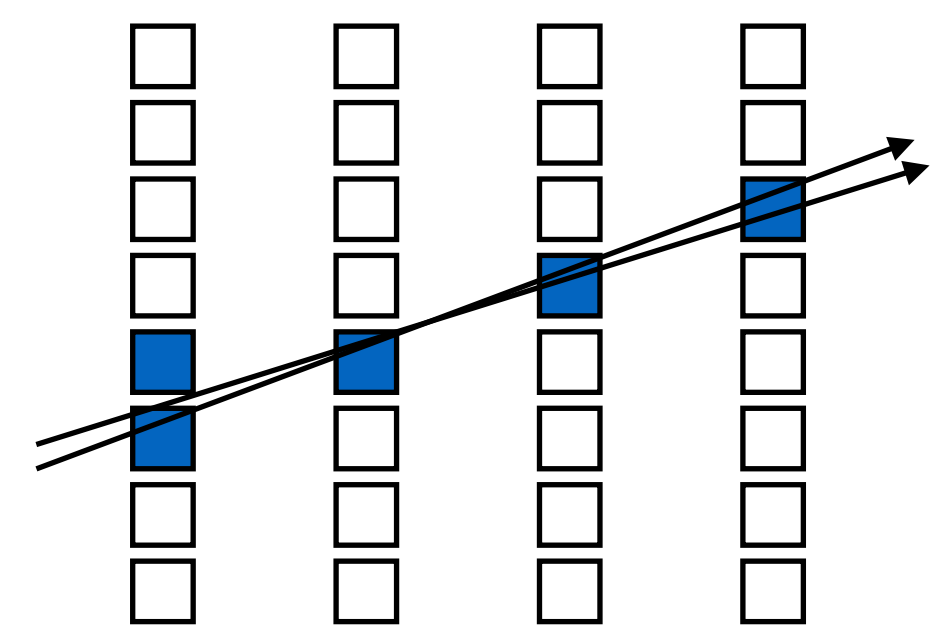
- Col de Ceysat campaign 2015-2016: equivalent to **100 days of data**



$$S_{\text{eff}} = S_{\text{det}} \epsilon_{\text{det}} A_{\text{geom}} \epsilon_{\text{illum}}$$

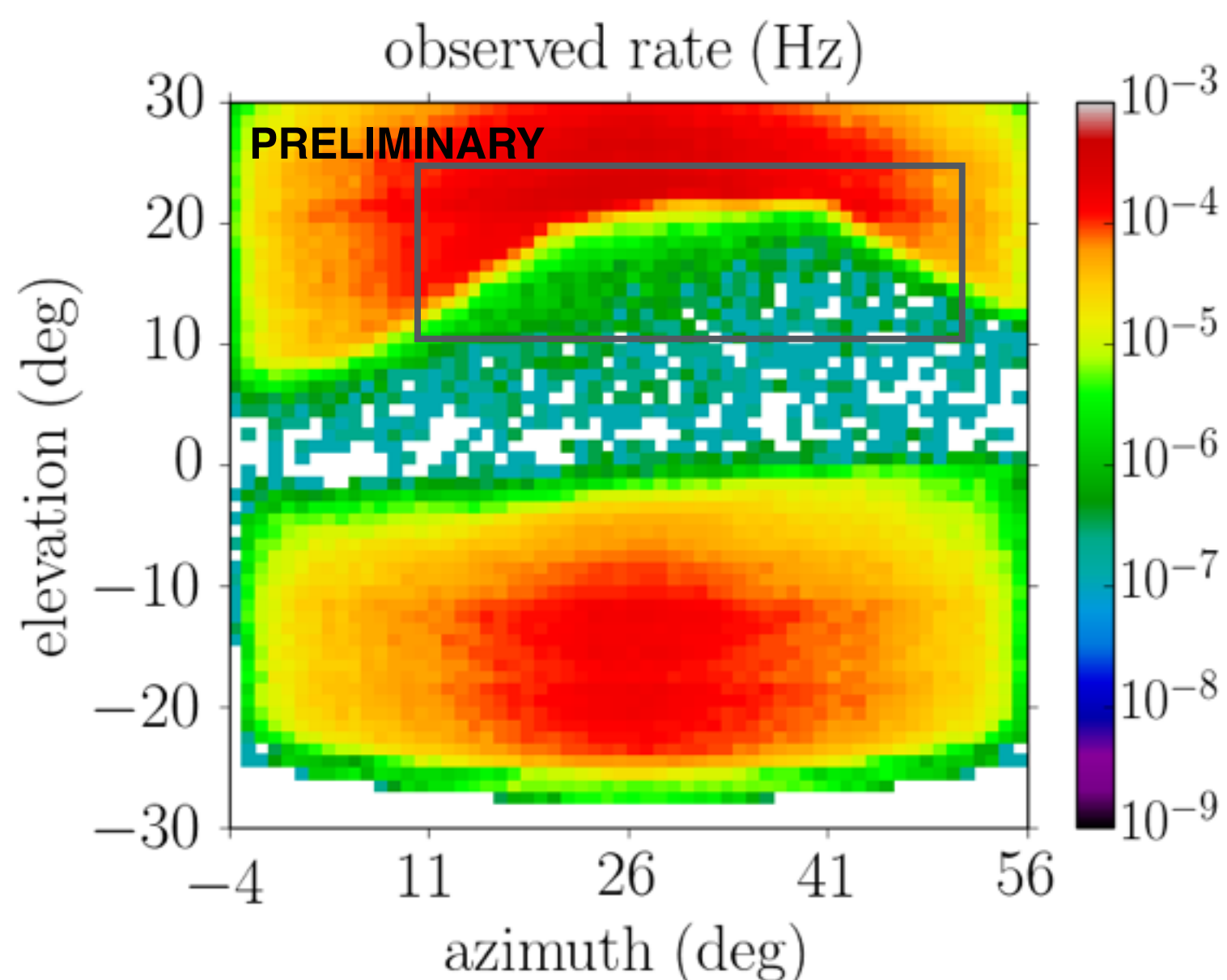


4 layers of gas resistive plate chambers (GRPCs)

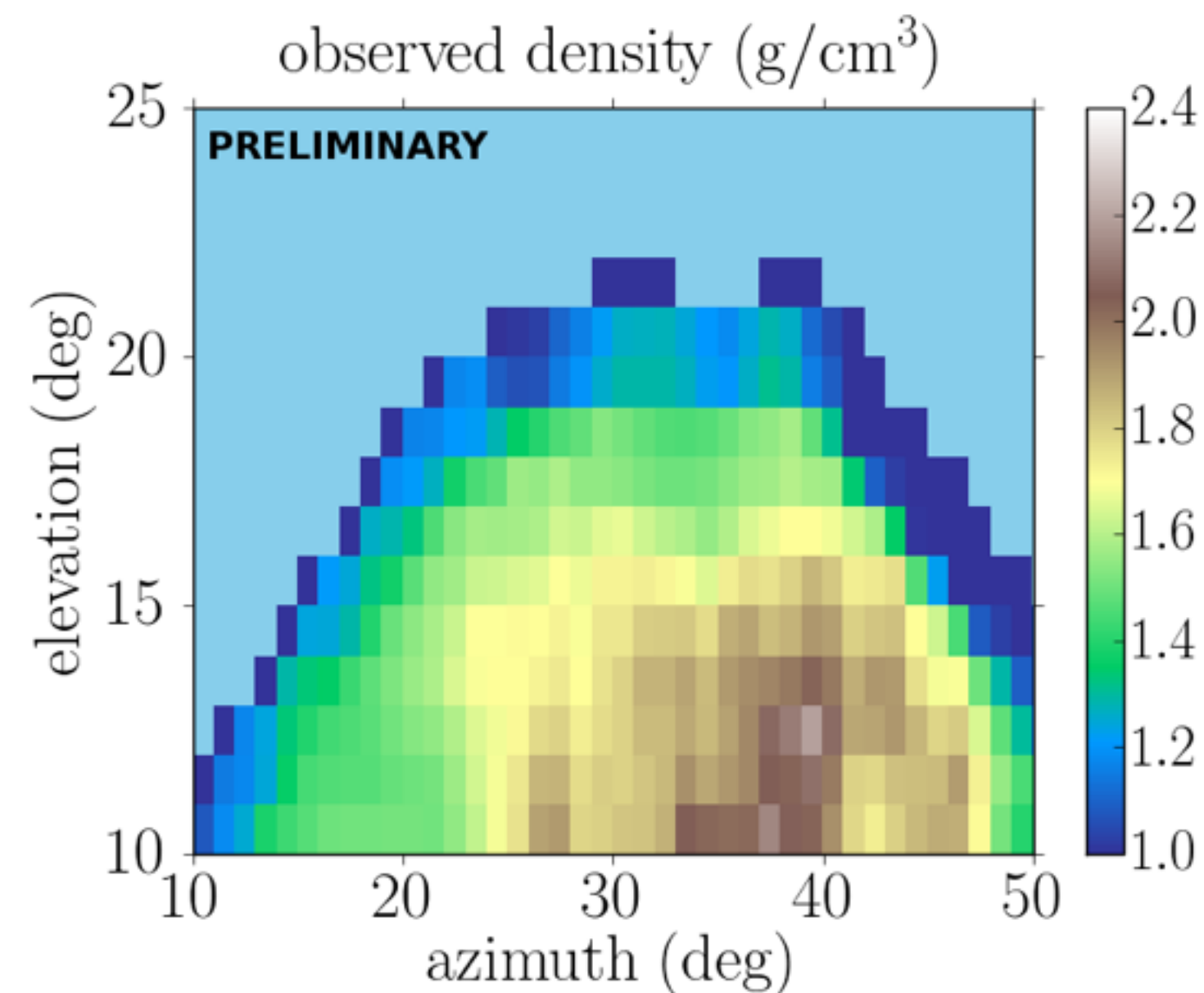
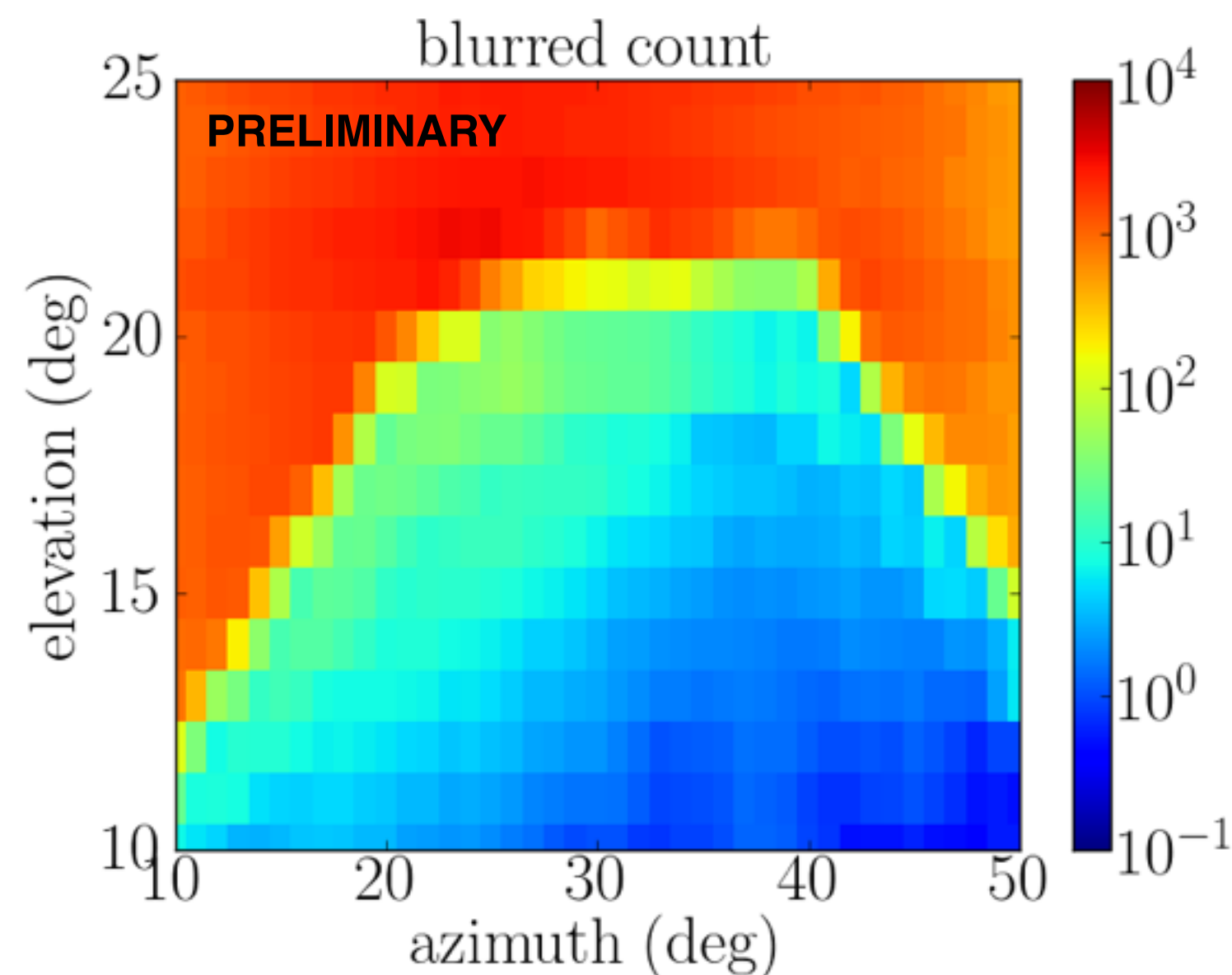
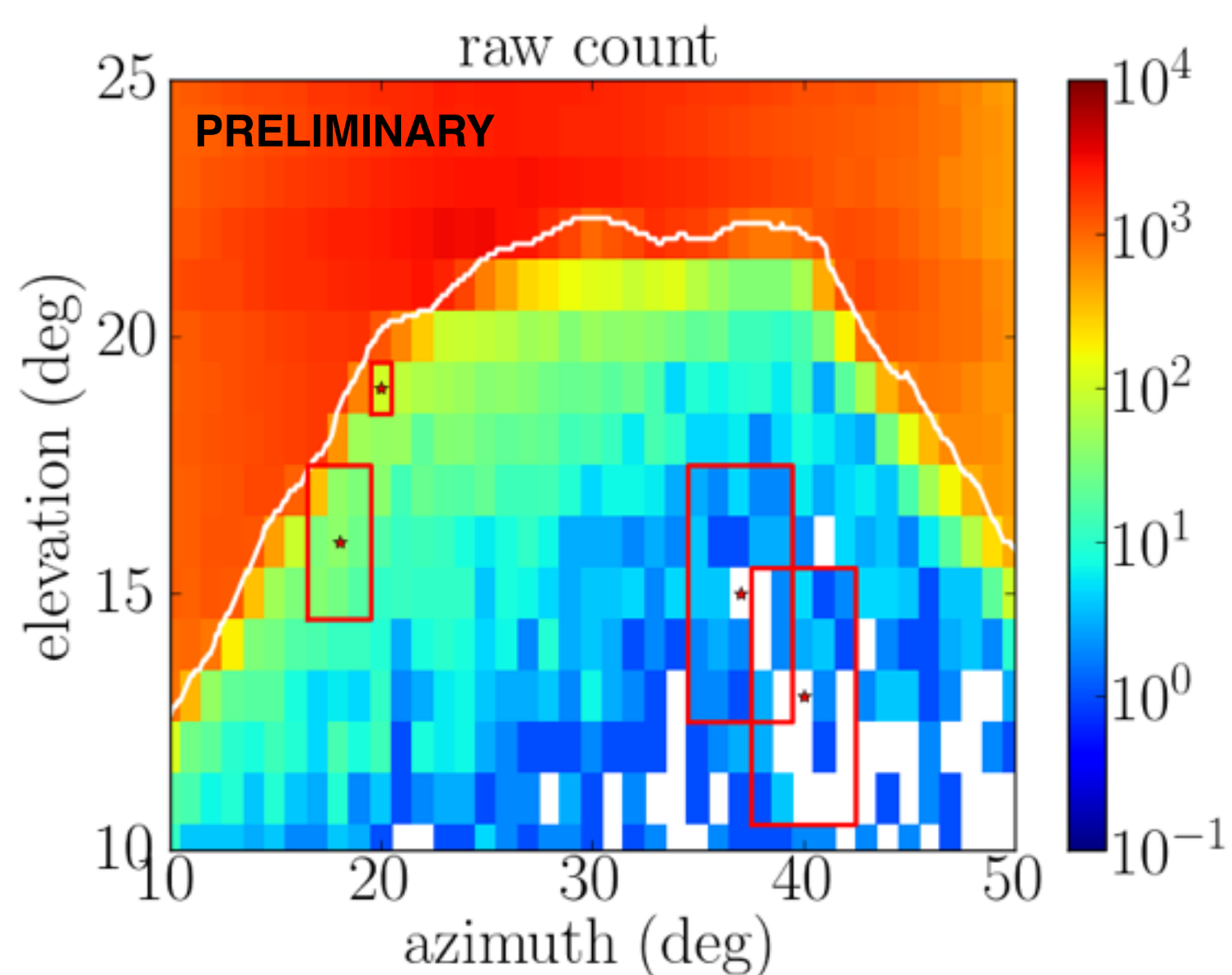


Track reconstruction with measurement uncertainties

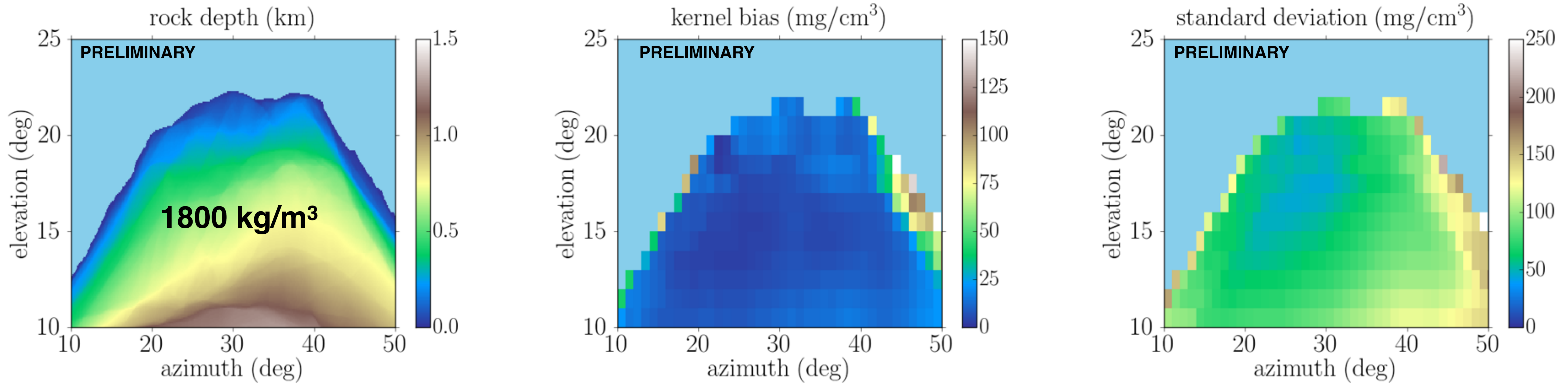
Muography of puy de Dôme volcano - Density reconstruction



- In practice, the rate of transmitted muons is low.
- A **kernel method** (blurring) is used to equalize statistical uncertainties. The angular resolution is degraded at large rock depths where rates are low.
- A bootstrap procedure allows to estimate the resulting **statistical uncertainty** and the **bias**, assuming a uniform density distribution.



Crosscheck for a uniform density model of 1800 kg/m³



- Few degrees from the border:
 - bias negligible ($\sim 10\text{-}20 \text{ kg/m}^3$)
 - statistical uncertainties below $\sim 100 \text{ kg/m}^3$
- Close to the rock border: mixing of transmitted and free sky flux increases bias and uncertainties
- Small rock depths: muon flux weakly sensitive to density

Motivation for the joint inversion of gravimetric and muographic data

- Both methods are sensitive to **density**.
- Muography provides 2D images of density averaged along given directions.
- Gravimetry allows for 3D reconstruction of density variations through inversion.

Gravimetry

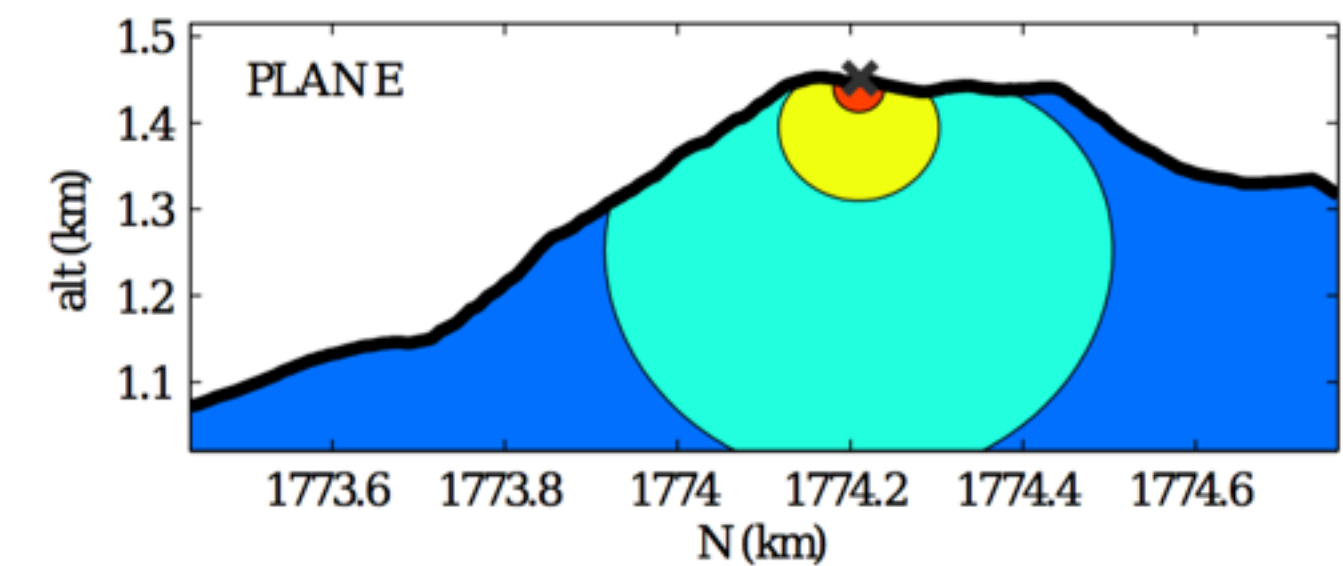
- Advantages: good resolution of shallow structures and lateral variations
- Limitations: rapid decrease of resolution with depth, non unicity of the method

Muography

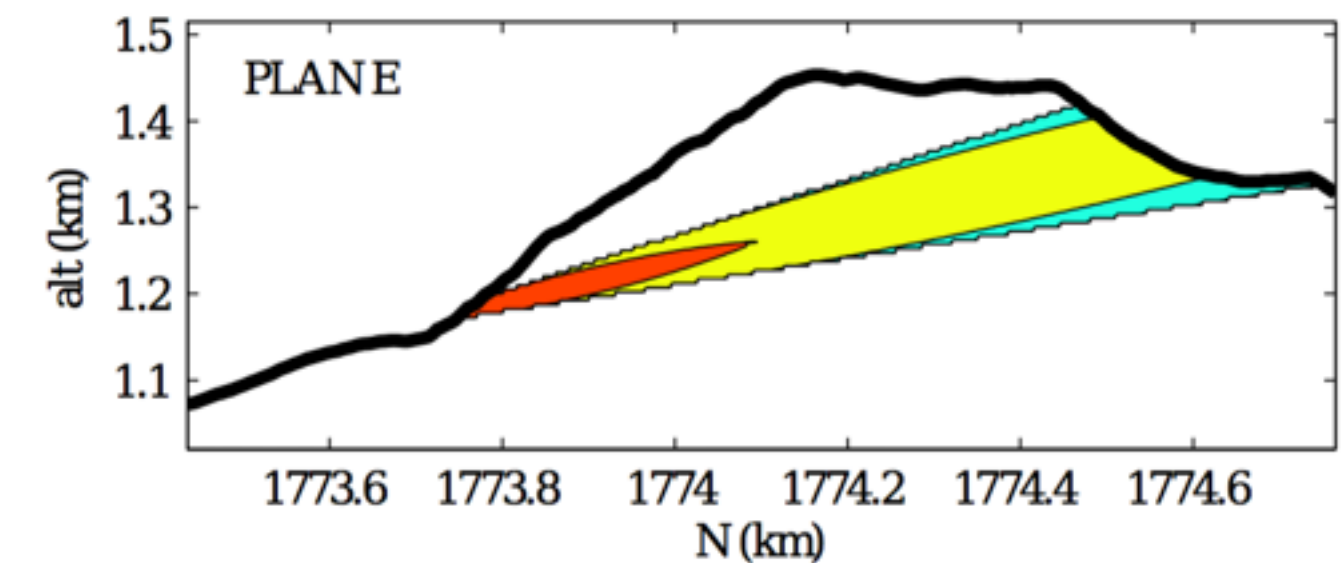
- Advantages: localized measurement of density along lines of sight, high resolution in space and density
- Limitations: less sensitive close to the surface: smaller attenuation due to smaller rock depths, muon scattering...
- Resolution depending on aperture and time of exposure

Examples: Nishiyama *et al.* 2014, Jourde *et al.* 2015

Jourde *et al.* 2015



(1) gravimetry acquisition kernel, \mathcal{G}



(2) tomography acquisition kernel, \mathcal{M}



Formulation of the joint inversion problem

$$\mathbf{A} \boldsymbol{\rho} = \mathbf{d}$$
$$\begin{bmatrix} \mathbf{G} \\ \mathbf{M} \end{bmatrix} \begin{bmatrix} \boldsymbol{\rho} \end{bmatrix} = \begin{bmatrix} \mathbf{g} \\ \mathbf{q} \end{bmatrix}$$

← gravimetric data

← densities averaged along lines of sight



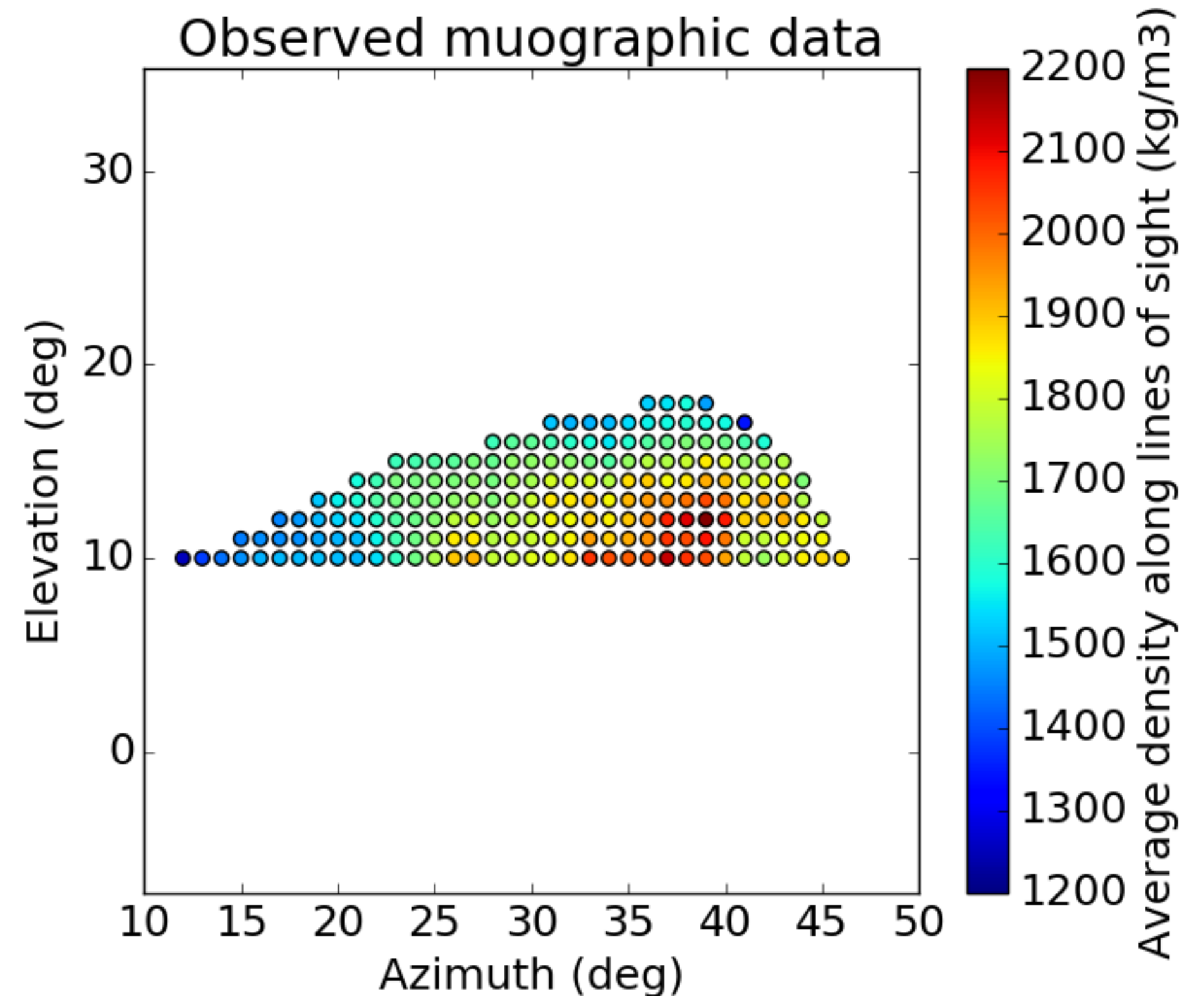
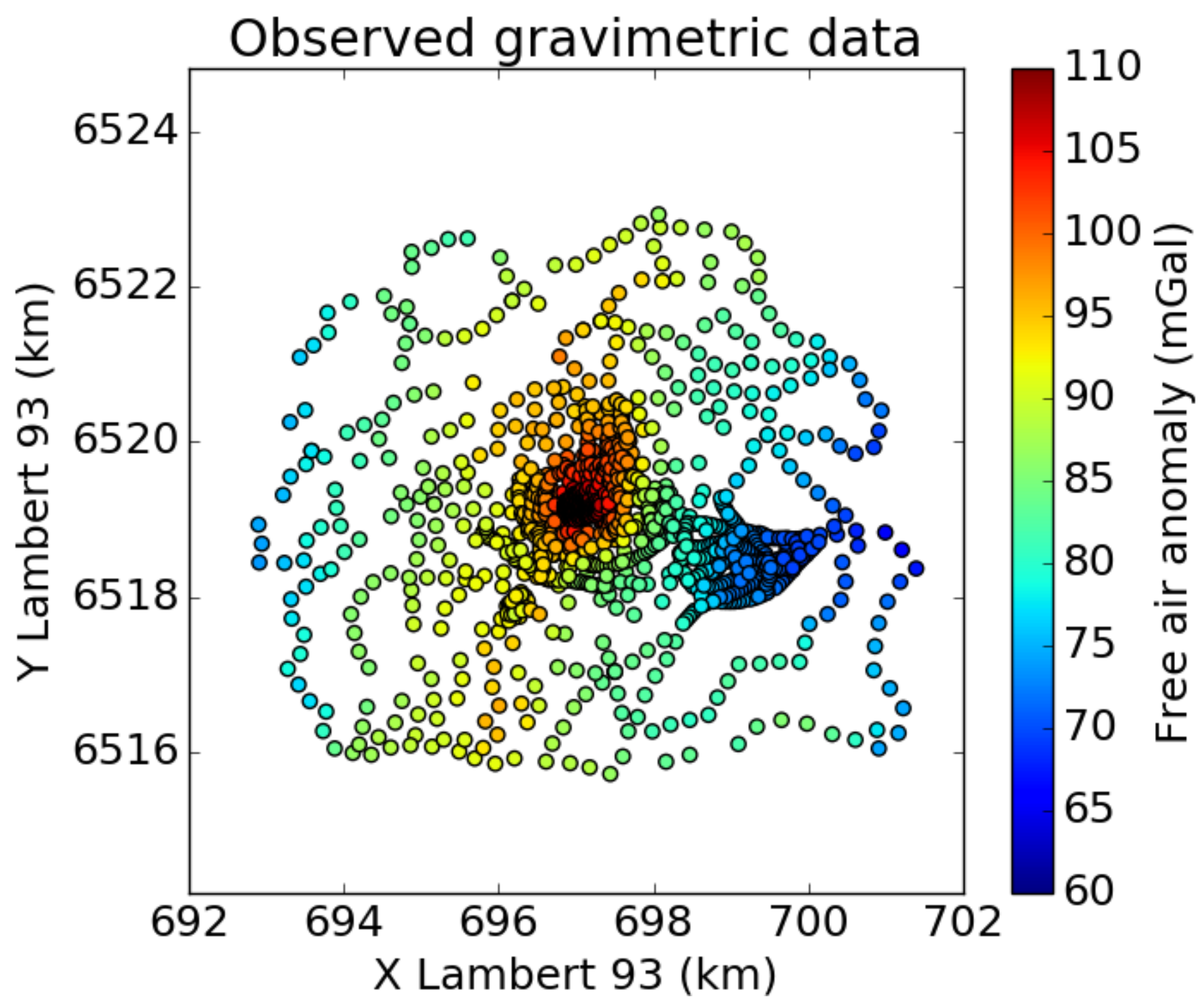
densities at the nodes
of the investigated volume

We minimize:

$$\phi(\boldsymbol{\rho}) = \|\mathbf{d} - \mathbf{A}\boldsymbol{\rho}\|_{\mathbf{D}}^2 + \|\boldsymbol{\rho} - \boldsymbol{\rho}_{prior}\|_{\mathbf{P}}^2$$
$$= (\mathbf{d} - \mathbf{A}\boldsymbol{\rho})^t \mathbf{C}_D^{-1} (\mathbf{d} - \mathbf{A}\boldsymbol{\rho}) + (\boldsymbol{\rho} - \boldsymbol{\rho}_{prior})^t \mathbf{C}_P^{-1} (\boldsymbol{\rho} - \boldsymbol{\rho}_{prior})$$

Joint inversion: Data from the puy de Dôme volcano

- Data for the inversion



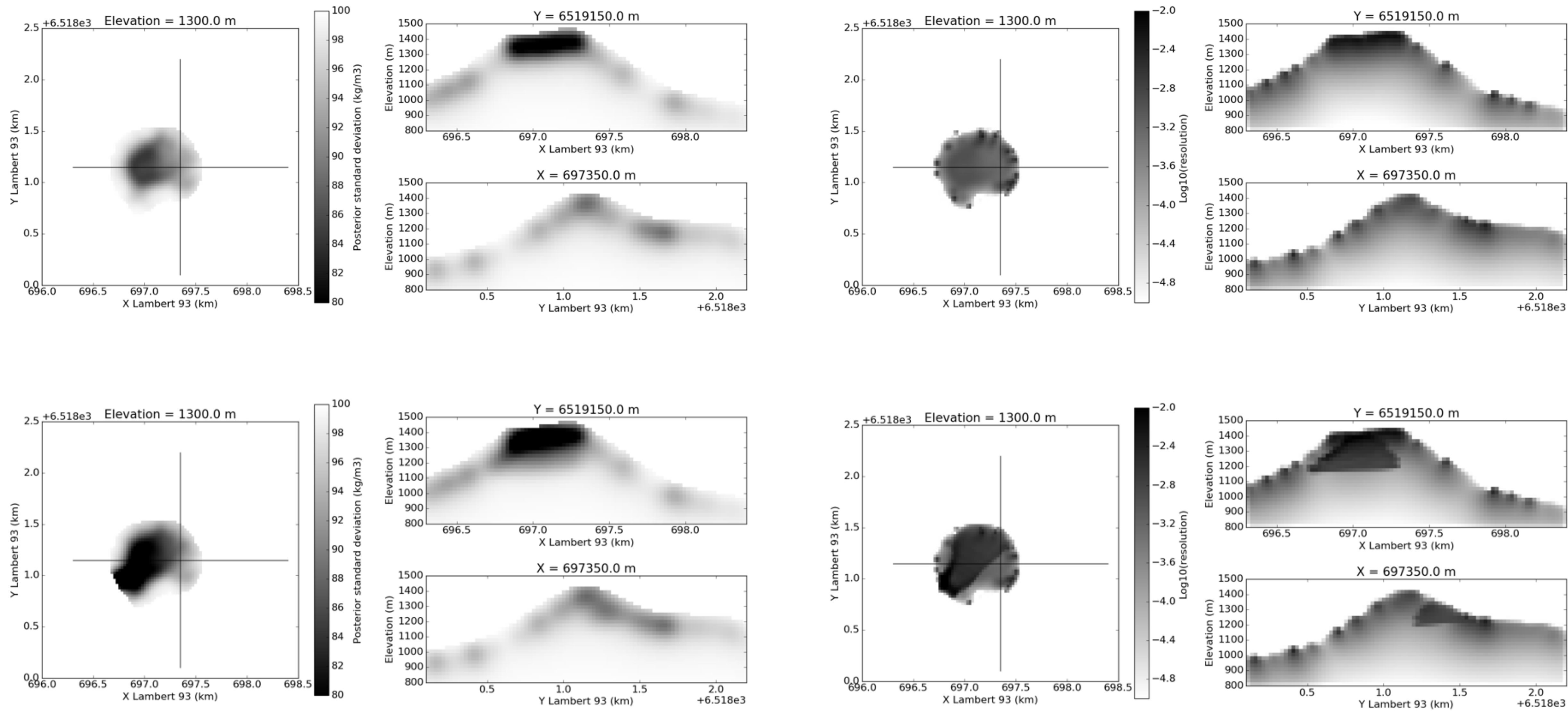
Data from Portal *et al.* 2016

Posterior density standard deviation (kg/m³)

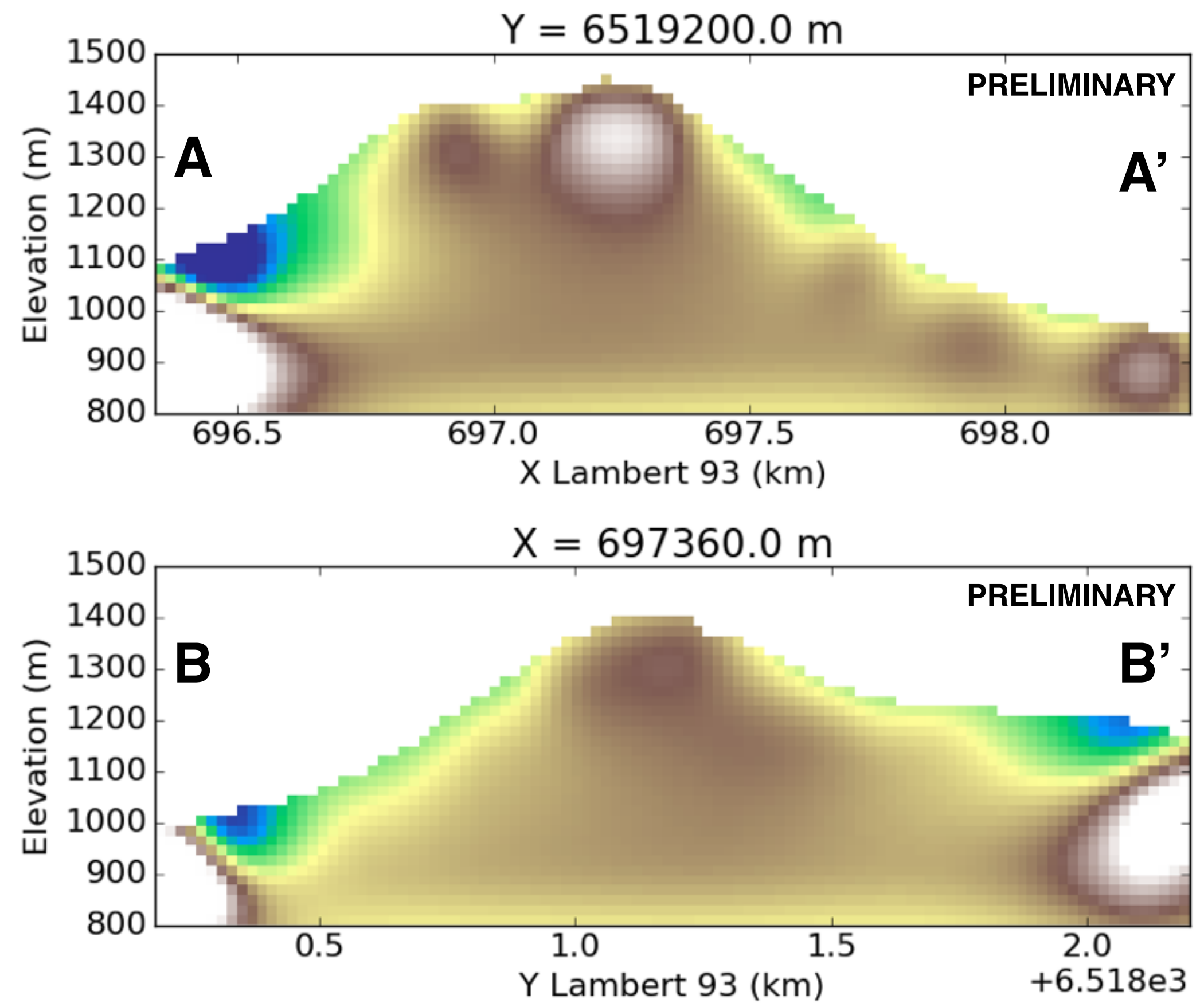
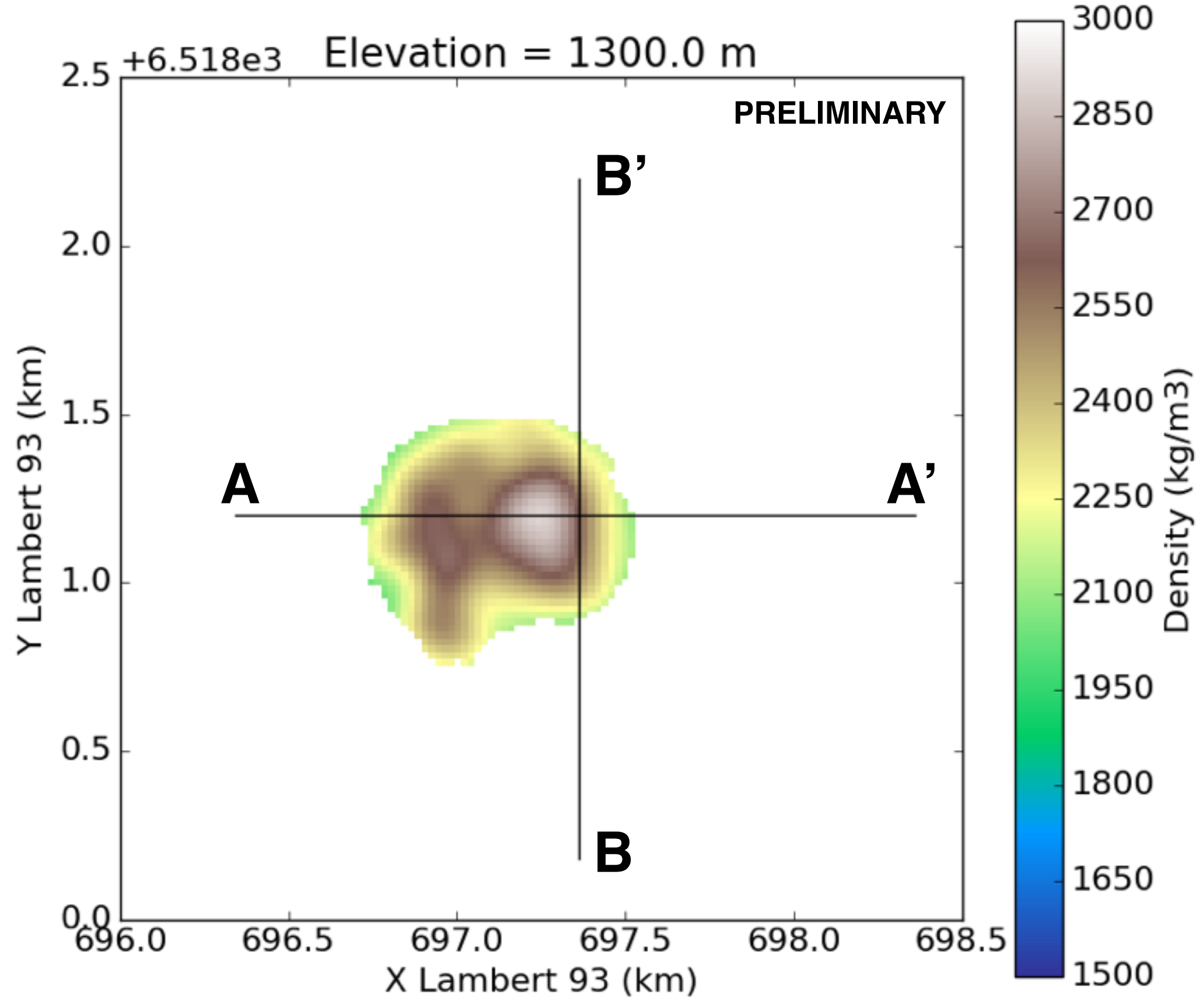
Log Resolution

Gravimetry only

Gravimetry and muography

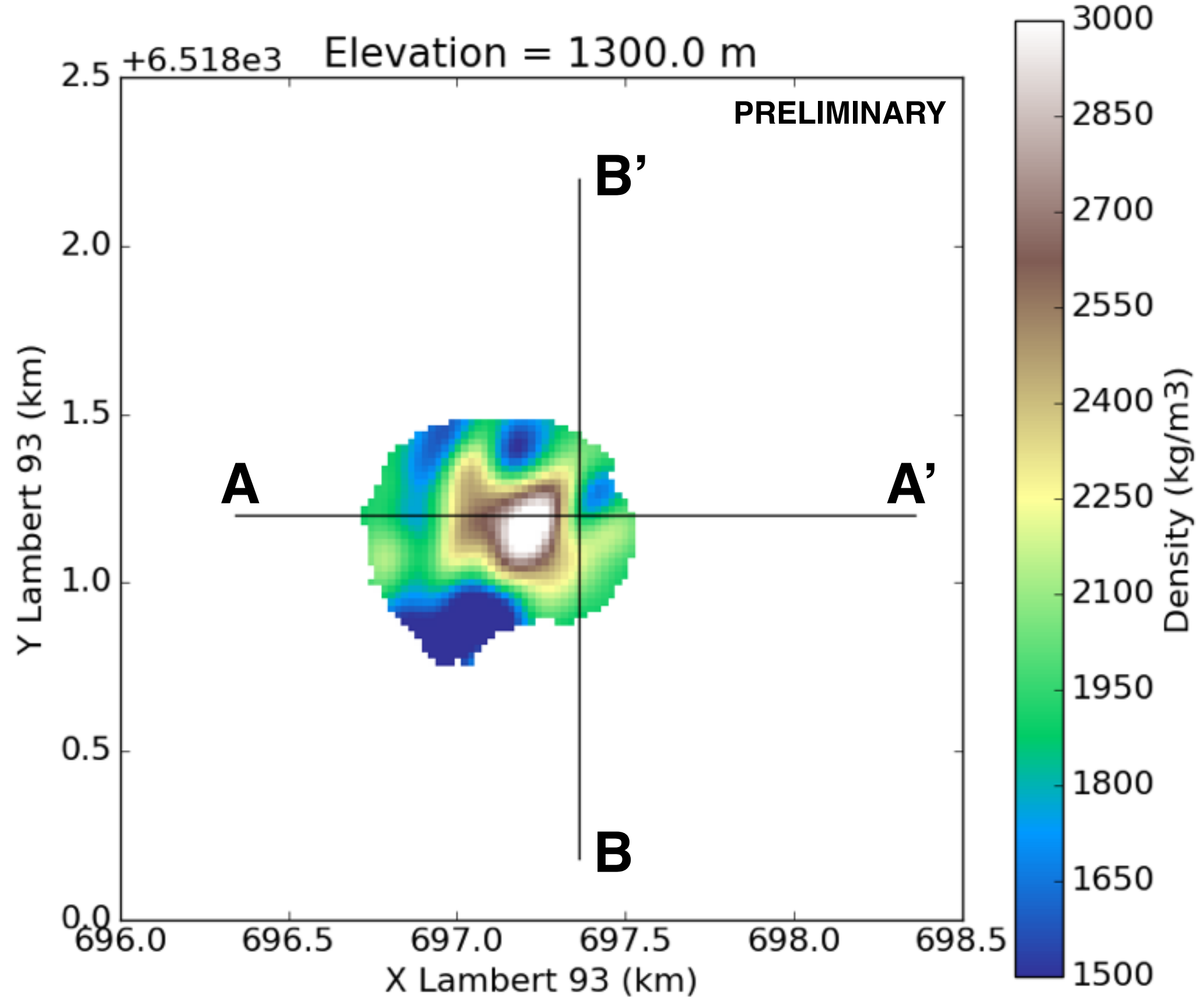


Result of the inversion of gravimetric data only

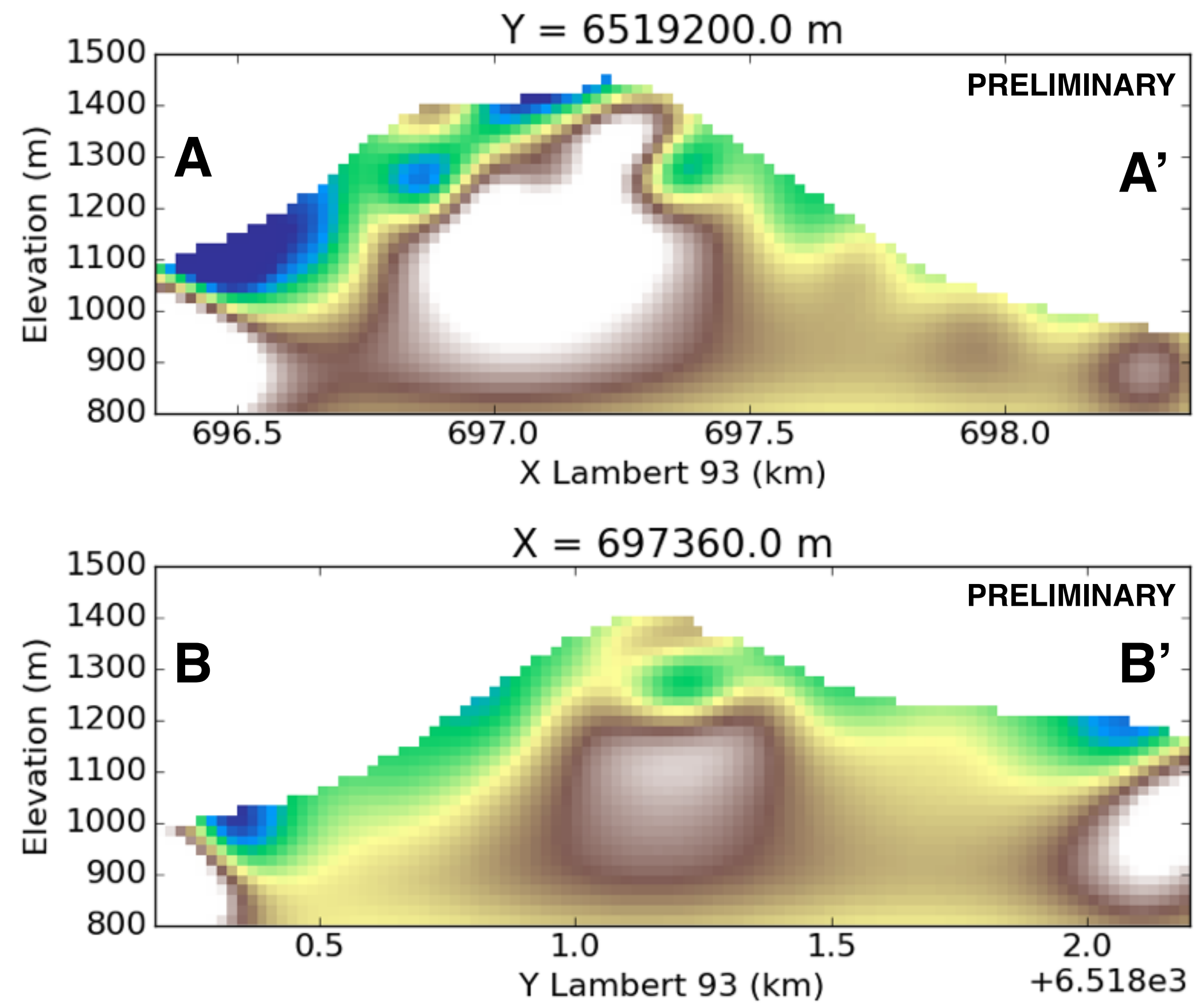


Gravimetric data RMS = 0.70 mGal

Result of the joint inversion of gravimetric and muographic data



Gravimetric data RMS = 0.70 mGal



Muographic data RMS = 40 kg/m³

Conclusion

- promising preliminary results for the muography and the combined inversion
- joint inversion improves resolution compared to gravimetric only inversion, equally fitting the data
- ongoing improvements
 - muography: muon scattering taken into account, refined description of instrumental response, background...
 - inversion: synthetic tests, systematic estimation of regularization parameters
 - muon tomography with several view points
- density imaging of active volcanoes (Vesuvius and Stromboli)





Thank you for your attention!
Questions?