Polarization in a thermal medium

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Based on: E. Speranza, A. Jaiswal, B.F. (to be published)

Space-like and time-like electromagnetic baryonic transitions

ECT* May 8-12, 2017

Polarization without polarization

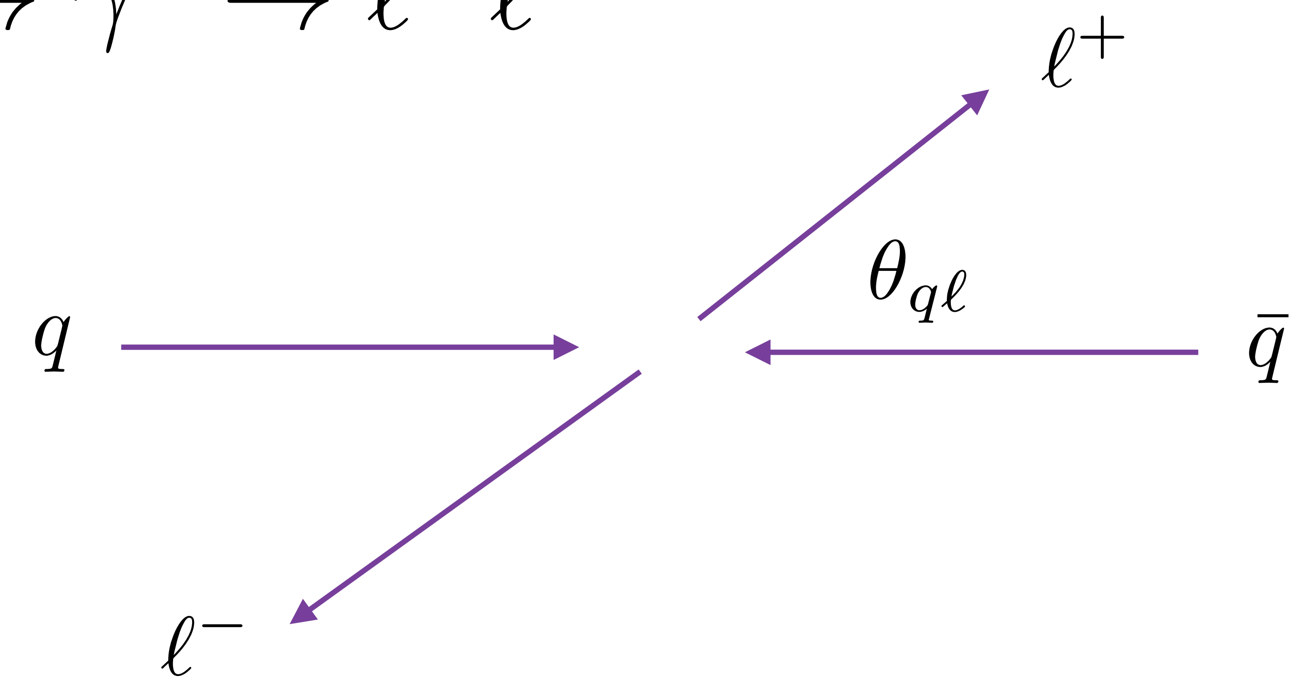
$$A + B \to X + \gamma^\star \to X + \ell^+\ell^-$$

- Polarization of virtual photon depends on the production mechanism
- Polarization of virtual photon is reflected in angular distribution of lepton pair
- Used to constrain production mechanism in elementary reactions (→ Miklos' talk)
- What about nucleus-nucleus collisions?

Drell-Yan process

$$q + \bar{q} \to \gamma^\star \to \ell^+ \ell^-$$

- In CM frame:



$$\frac{d\sigma}{d\Omega_\ell} \sim 1 + \cos^2 \theta_{q\ell}$$

- Photon polarized along beam axis (quantiz. axis)

$$\pi^+ + \pi^- \to \gamma^\star \to \ell^+ \ell^-$$

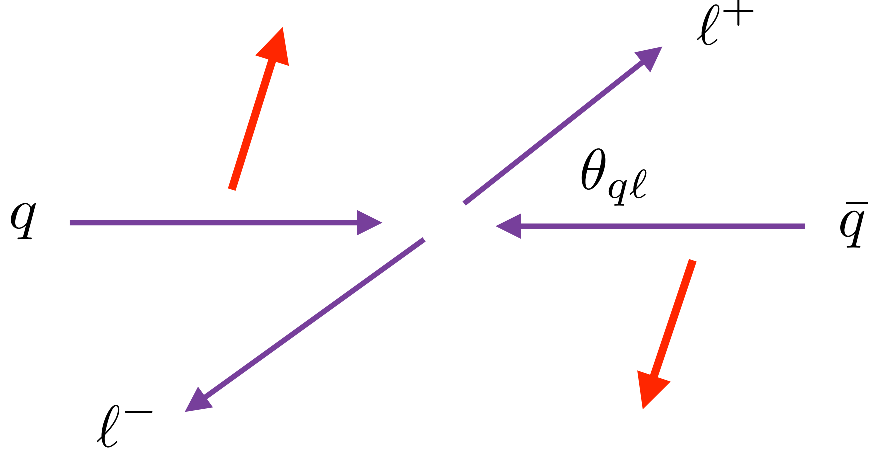
$$\frac{d\sigma}{d\Omega_\ell} \sim 1 - \cos^2 \theta_{q\ell}$$

Emission from a thermal system

- Thermal distribution of initial particles

$$\langle A \rangle = \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^{(4)}(q - p_1 - p_2) \times \frac{1}{e^{(p_1)/T} \pm 1} \frac{1}{e^{(p_2)/T} \pm 1} A$$

- Average over beam axis: polarization vanishes?



P. Hoyer, PLB 187, 162

$$\frac{1}{4\pi} \int d\Omega_q \cos^2 \theta_{q\ell} = \frac{1}{3}$$

- More detailed study:

E. Bratkovskaya, PLB 348, 283

Polarization of spin-1 particle

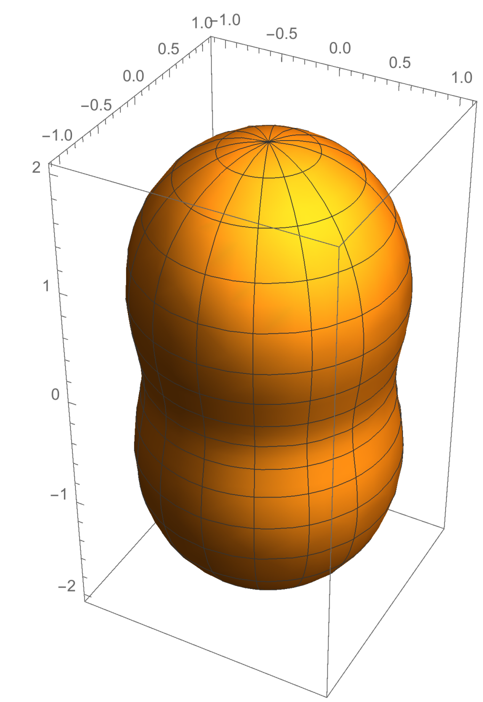
- Three spin states $(\vec{p} = |\vec{p}|\vec{e}_z)$

$$\epsilon(\vec{p}, \lambda = +1) = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}$$

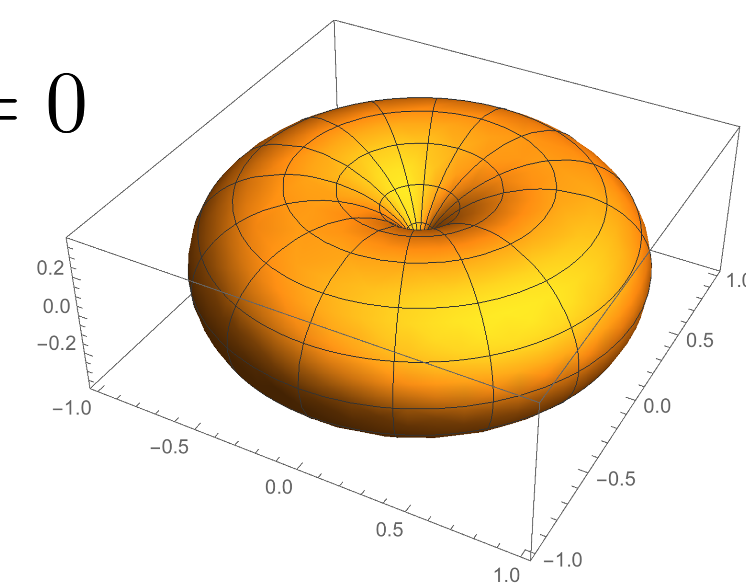
$$\epsilon(\vec{p}, \lambda = 0) = \frac{1}{m} \begin{pmatrix} |\vec{p}| \\ 0 \\ 0 \\ p_0 \end{pmatrix}$$

$$\epsilon(\vec{p}, \lambda = -1) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -i \\ 0 \end{pmatrix}$$

$\lambda = \pm 1$



$\lambda = 0$



- Decay of a polarized spin-1 particle

$$\Gamma(\lambda = \pm 1) \sim 1 + \cos^2\theta_\ell \qquad \Gamma(\lambda = 0) \sim 2(1 - \cos^2\theta_\ell)$$

Spin-density matrix

- State of ensemble specified by density matrix $\rho_{\lambda\lambda'}$

- Pure state $\quad | \Psi \rangle = \sum_{\lambda} c_{\lambda} | \lambda \rangle$

- Expectation value of operator $\hat{O}$

$$\langle \hat{O} \rangle_{\Psi} \equiv \langle \Psi | \hat{O} | \Psi \rangle = \sum_{\lambda\lambda'} c^{*}_{\lambda'} O_{\lambda'\lambda} c_{\lambda} \qquad O_{\lambda'\lambda} = \langle \lambda' | \hat{O} | \lambda \rangle$$

- Mixed state: incoherent mixture of $\quad | \Psi^{(i)} \rangle$

$$\langle \hat{O} \rangle = \sum_{i} p_i \, \langle \hat{O} \rangle_{\Psi^{(i)}} = \sum_{\lambda'\lambda} O_{\lambda'\lambda} \, \rho_{\lambda\lambda'} \qquad \rho_{\lambda\lambda'} = \sum_{i} p_i \, c^{(i)}_{\lambda} \, c^{*(i)}_{\lambda'}$$

$$= \mathrm{Tr}(O \, \rho)$$

Spin-½ particle

- 2 x 2 hermitian matrix $$\rho = \frac{1}{2}(\mathbf{1} + \vec{\mathcal{P}} \cdot \vec{\sigma})$$
- Spin polarization vector $$\vec{\mathcal{P}} = \langle \vec{\sigma} \rangle = \mathrm{Tr}(\rho\, \vec{\sigma})$$
- Pure state: $\vec{\mathcal{P}} \cdot \vec{\mathcal{P}} = 1$
- Mixed state: $\vec{\mathcal{P}} \cdot \vec{\mathcal{P}} \leq 1$

$$\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix}$$

- Diagonalize:

$$\rho = \frac{1}{2}(\mathbf{1} + (p_+ - p_-)\sigma_z)$$

$$N_+ = p_+ N$$

$$N_- = p_- N$$

Spin-1 particle

- Spin matrix

$$S_x = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad S_y = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad S_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

- ρ hermitian, trace=1, 3 x 3 matrix: 8 parameters

$$\rho = \frac{1}{3}\left(1 + \frac{3}{2}\vec{\mathcal{P}}\cdot\vec{S} + \sqrt{\frac{3}{2}}T_{ij}(S_iS_j + S_jS_i)\right)$$

- Vector polarization: $\vec{\mathcal{P}}$ (3 parameters)

- Tensor polarization: T_{ij} (5 parameters) $\sum_i T_{ii} = 0$

$$\vec{\mathcal{P}} = \langle\vec{S}\rangle \qquad T_{ij} = \frac{1}{2}\sqrt{\frac{3}{2}}\left(\langle S_iS_j + S_jS_i\rangle - \frac{4}{3}\delta_{ij}\right)$$

In Ensemble

- Spin-density matrix can be diagonalized

$$\rho = \frac{1}{3}\begin{pmatrix} 1+\frac{3}{2}\mathcal{P}_z+\sqrt{\frac{3}{2}}T_{zz} & 0 & 0 \\ 0 & 1-\sqrt{6}T_{zz} & 0 \\ 0 & 0 & 1-\frac{3}{2}\mathcal{P}_z+\sqrt{\frac{3}{2}}T_{zz} \end{pmatrix}$$

- In unpolarized system $\mathcal{P}_z = 0$, but often $T_{zz} \neq 0$, i.e., no vector but tensor polarization!

- In general vector and tensor polarization axes can be different.

Lepton angular distribution

- Polarization state of virtual photon reflected in lepton angular distribution

- General form (parity conserved)

$$\frac{d\Gamma}{d^4q\, d\Omega_e} \propto \mathcal{N}\Big(1 + \lambda_\theta \cos^2\theta_e + \lambda_\phi \sin^2\theta_e \cos 2\phi_e + \lambda_{\theta\phi} \sin 2\theta_e \cos\phi_e + \lambda_\phi^\perp \sin^2\theta_e \sin 2\phi_e + \lambda_{\theta\phi}^\perp \sin 2\theta_e \sin\phi_e\Big)$$

- Angles defined in photon rest frame

- Different frames related through rotation: Helicity, Collins-Soper, ….

Reference frames

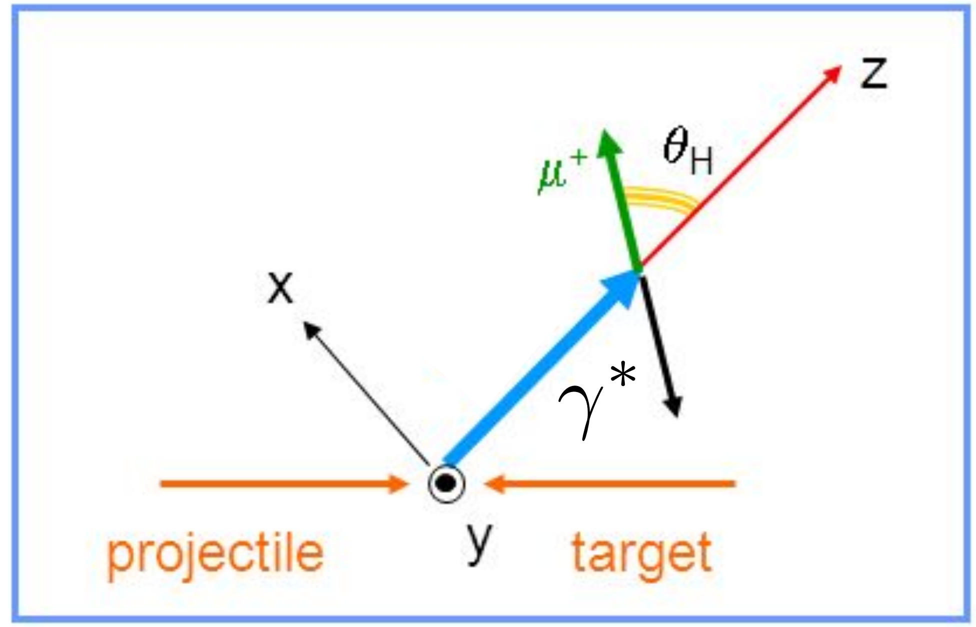
- Helicity frame

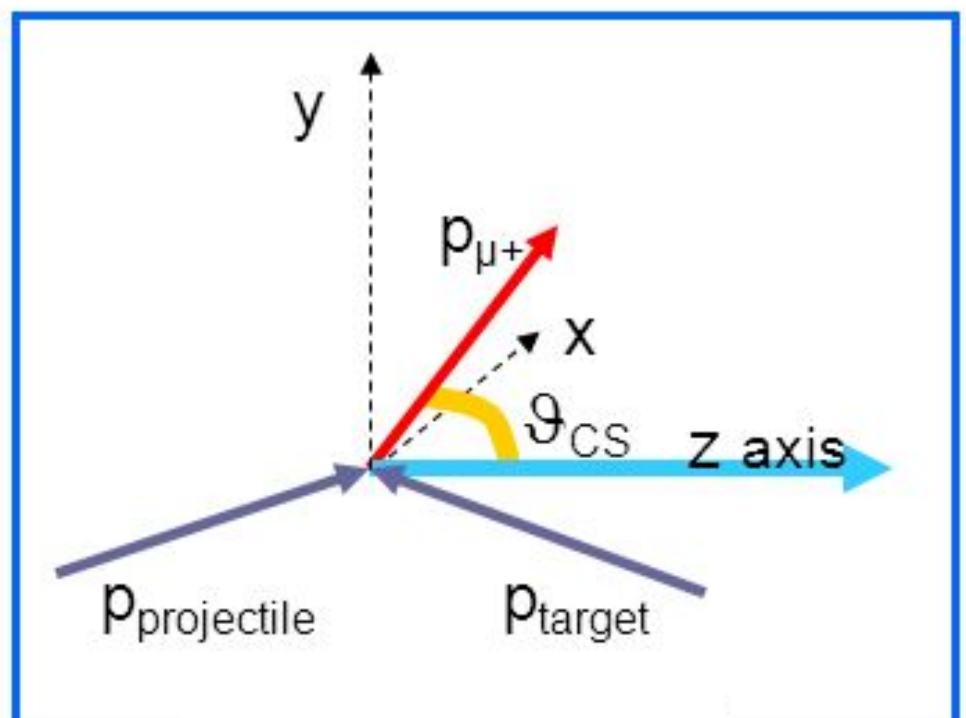
  Photon direction
  in target-projectile
  CM-frame

Rotation

- Collins-Soper frame

  Bisector of angle
  between “beam”
  and “target”





In γ^* rest-frame

Emission from thermal system

- Consider Drell-Yan photon with $q^\mu = (q_0, 0, 0, q_z)$

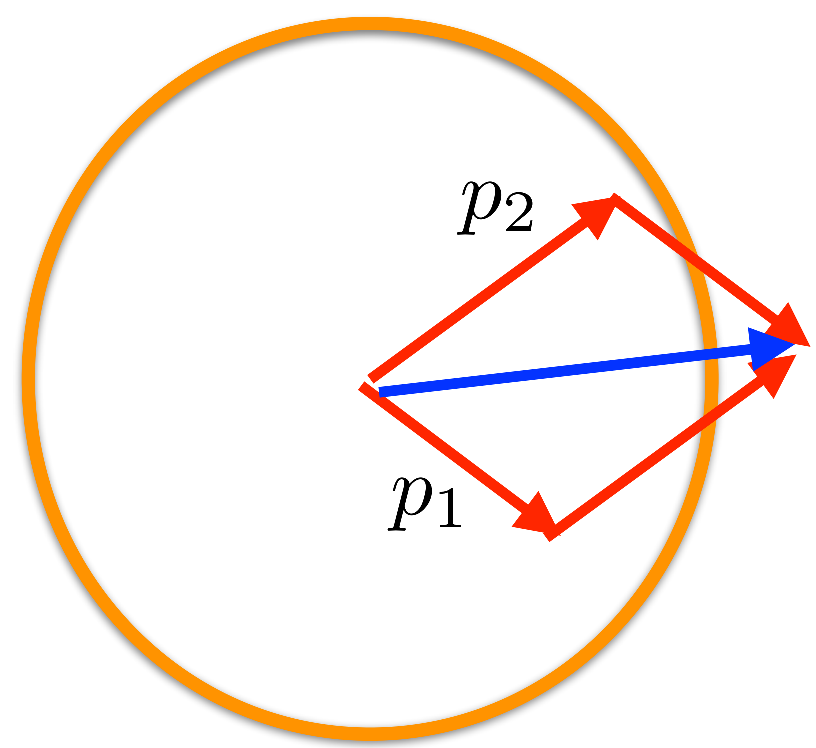
$$q + \bar{q} \to \gamma^\star \to \ell^+ \ell^- \qquad M^2 = q_0^2 - q_z^2$$

- Quark distribution function $$f(p) = \frac{1}{e^{(u\cdot p)/T} + 1}$$

- In fluid rest frame $u^\mu = (1, 0, 0, 0)$
  - spherically symmetric distribution
- Photon momentum “breaks” spherical symmetry azimuthal symmetry persists

→ tensor polarized photon

$$q^\mu = p_1^\mu + p_2^\mu$$

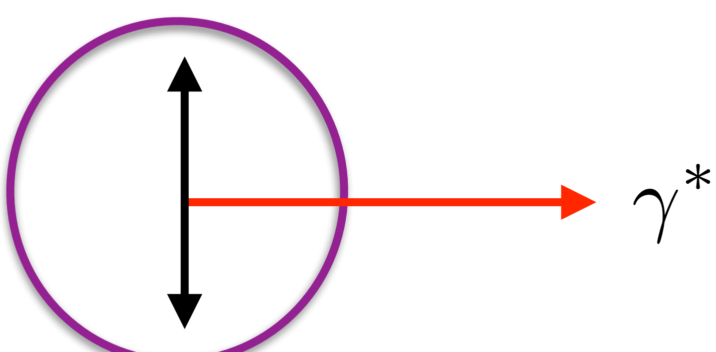


Boltzmann limit

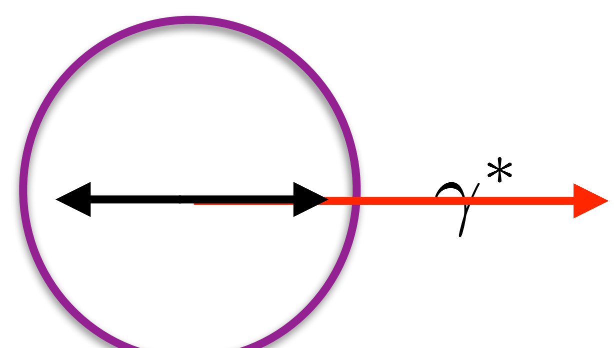
$$\frac{1}{e^{(u\cdot p_1)/T} \pm 1} \frac{1}{e^{(u\cdot p_2)/T} \pm 1} \to e^{-u\cdot(p_1+p_2)/T} = e^{-u\cdot q/T}$$

- Independent of direction of quark momenta
  - Yields unpolarized photons
- Non-zero polarization for $0 < |\vec{q}| < \infty$
- Thermal polarization is a quantum statistical effect!

Fermions

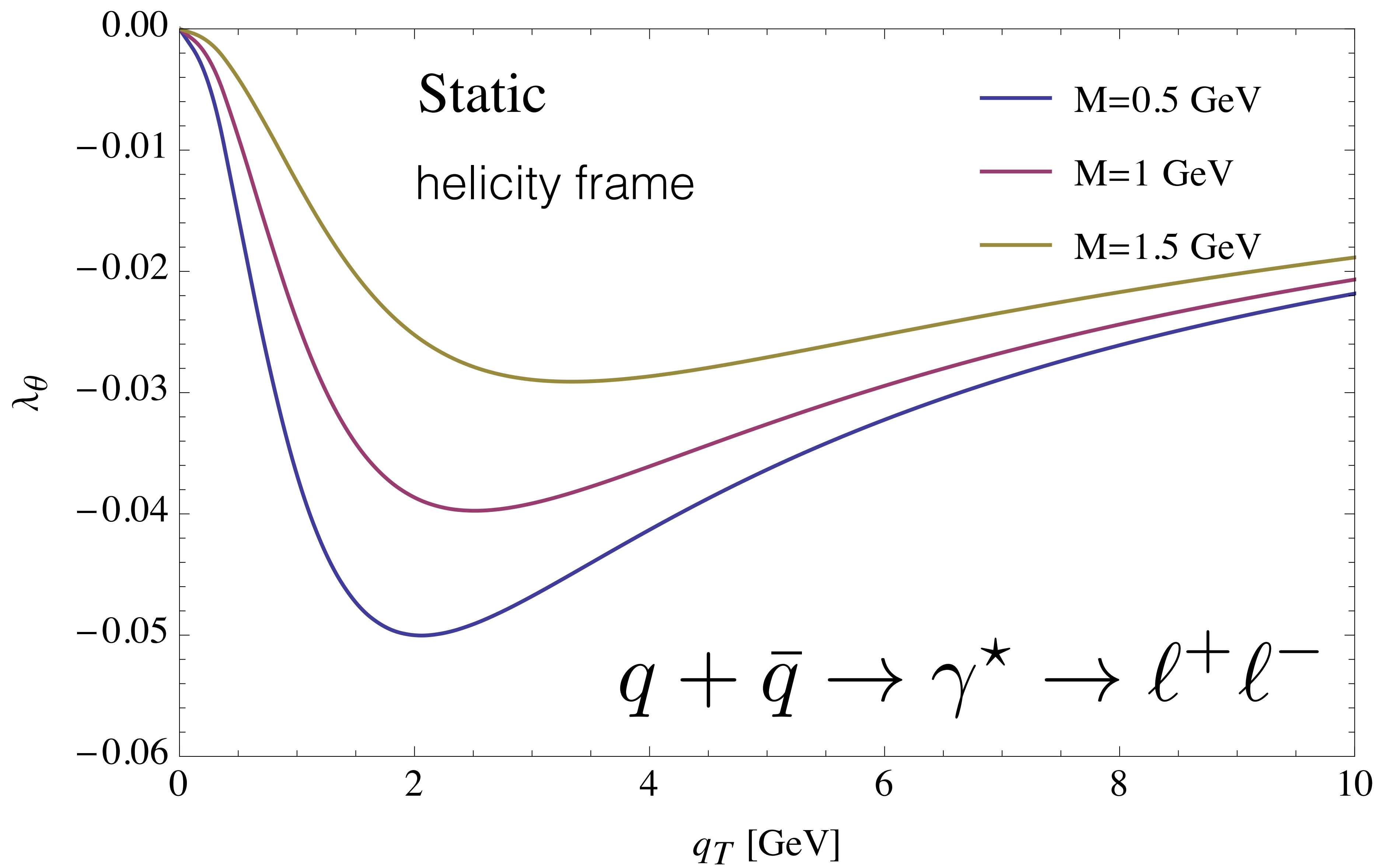


Bosons



Anisotropy coefficients

- Static homogeneous thermal system



- λ_θ only non-zero coefficient

Effect of hydrodynamic flow

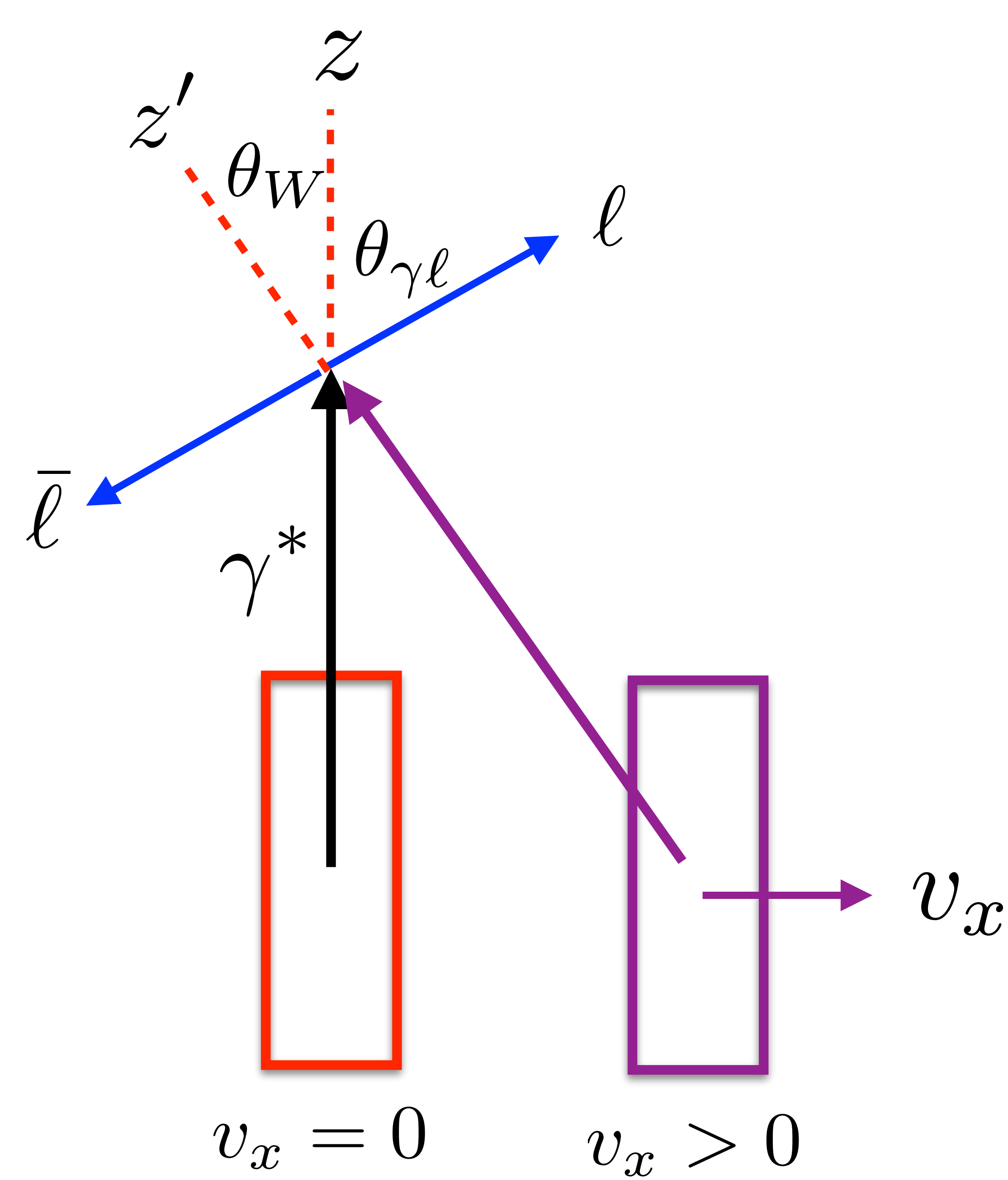
- Bjorken flow along x-axis $(c_s^2 = 1/3)$

$$v_x = x/t$$

$$T(\tau) \sim \tau^{-1/3}$$

$$\tau = \sqrt{t^2 - x^2}$$

- Photon emitted from moving cell: tensor polarized along z'-axis
  Wick helicity rotation
- Transform polarization to helicity frame (z).



Anisotropy parameters

- Tensor polarization along z':
  axial symmetry about z lost!

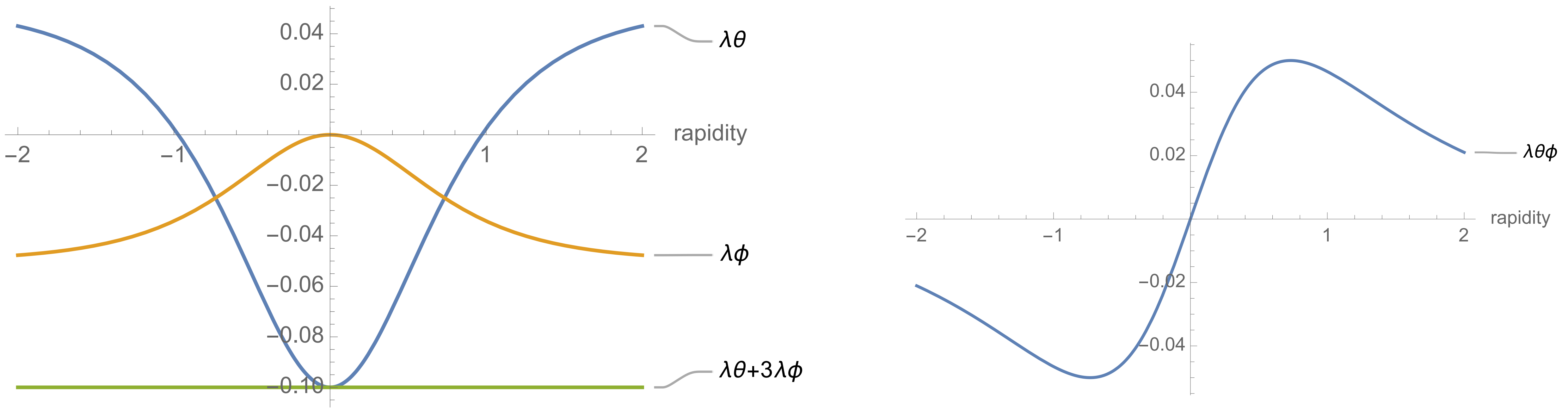
$$\frac{d\Gamma}{d^4q d\Omega_e} \propto \mathcal{N}\Big(1 + \lambda_\theta \cos^2\theta_e + \lambda_\phi \sin^2\theta_e \cos 2\phi_e + \lambda_{\theta\phi} \sin 2\theta_e \cos\phi_e + \lambda_\phi^\perp \sin^2\theta_e \sin 2\phi_e + \lambda_{\theta\phi}^\perp \sin 2\theta_e \sin\phi_e\Big)$$

- $\lambda_\theta, \lambda_\phi, \lambda_{\theta\phi} \neq 0$, $\lambda_\phi^\perp, \lambda_{\theta\phi}^\perp = 0$ (odd under $\phi \to -\phi$)

- $\lambda_{\theta\phi}$ odd under $v_x \to -v_x$

Polarization with Bjorken flow

- Anisotropy parameters from moving fluid cell



- Integrate over space-time evolution

$$\frac{dN}{d^2q_T dM^2 dy} = \pi R_A^2 \int_{\tau_i}^{\tau_f} d\tau\, \tau \int_{-\infty}^{\infty} d\eta \left(\frac{1}{2} \frac{dN}{d^4x d^4q} \right)$$

Drell-Yan

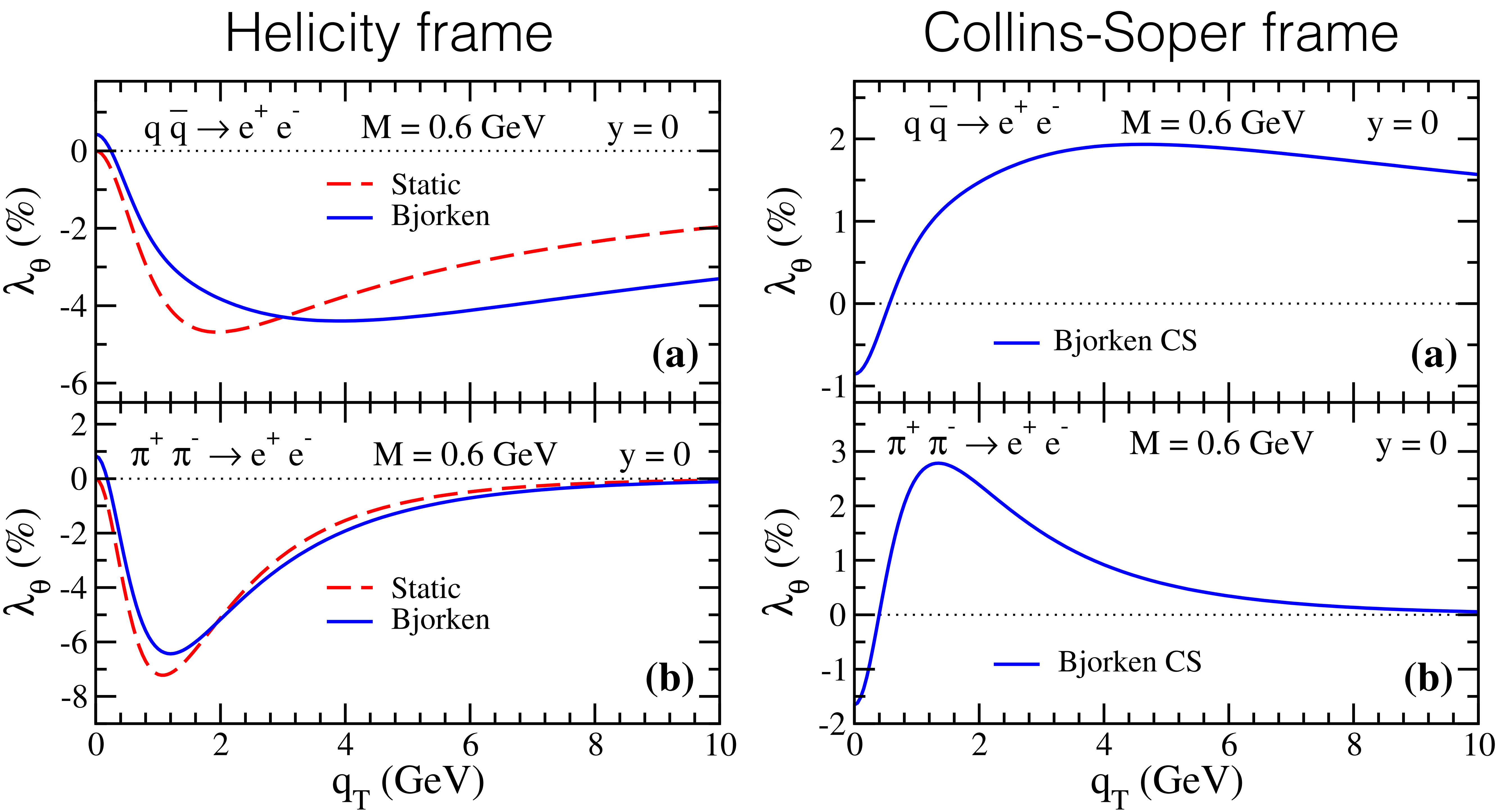
$$T_i = 500 \text{ MeV}$$
$$T_f = 160 \text{ MeV}$$

Pion annihilation

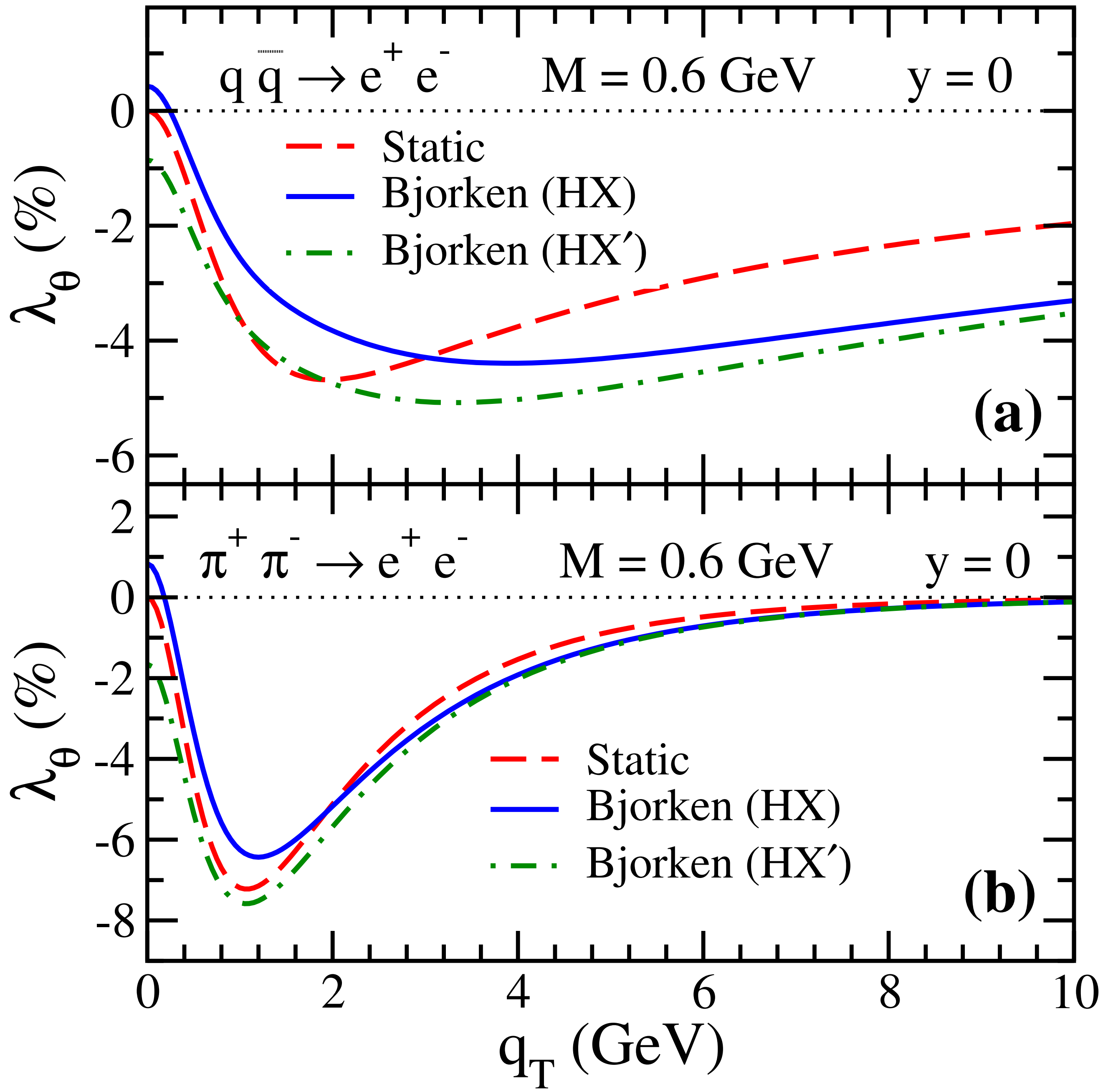
$$T_i = 160 \text{ MeV}$$
$$T_f = 120 \text{ MeV}$$

Results (Bjorken)

- Photon rapidity $y = 0$ (polarization not boost invariant)

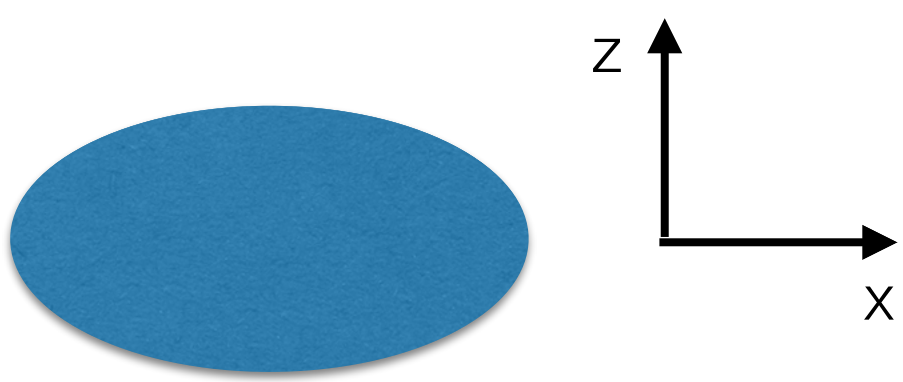


Results ($\lambda_\theta + 3\lambda_\phi$)

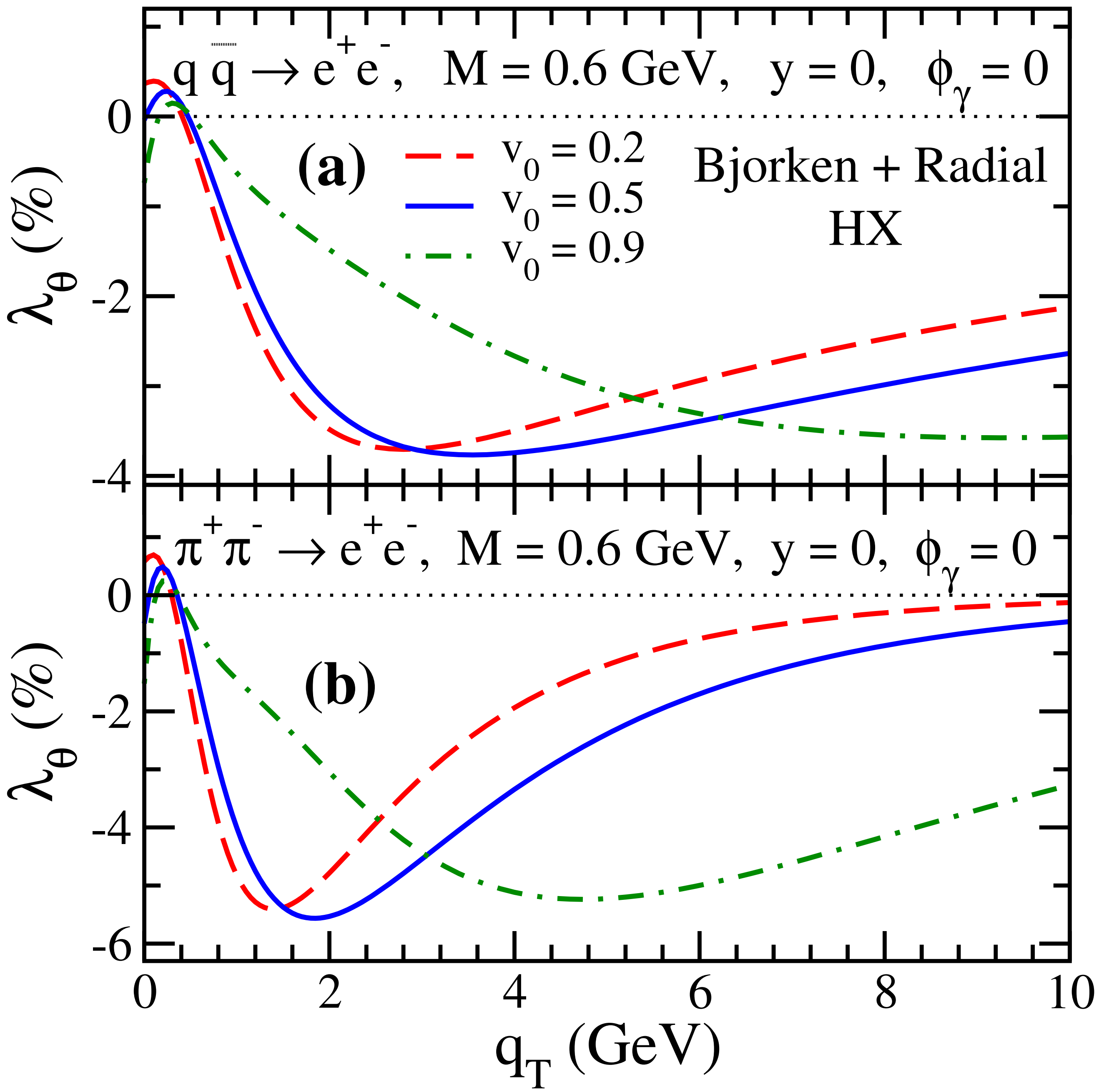


- Local thermal equilibrium + longitudinal flow ≃ anisotropic momentum distribution (Baym + Hatsuda + Strickland)

- "Effective" momentum distribution (schematic)

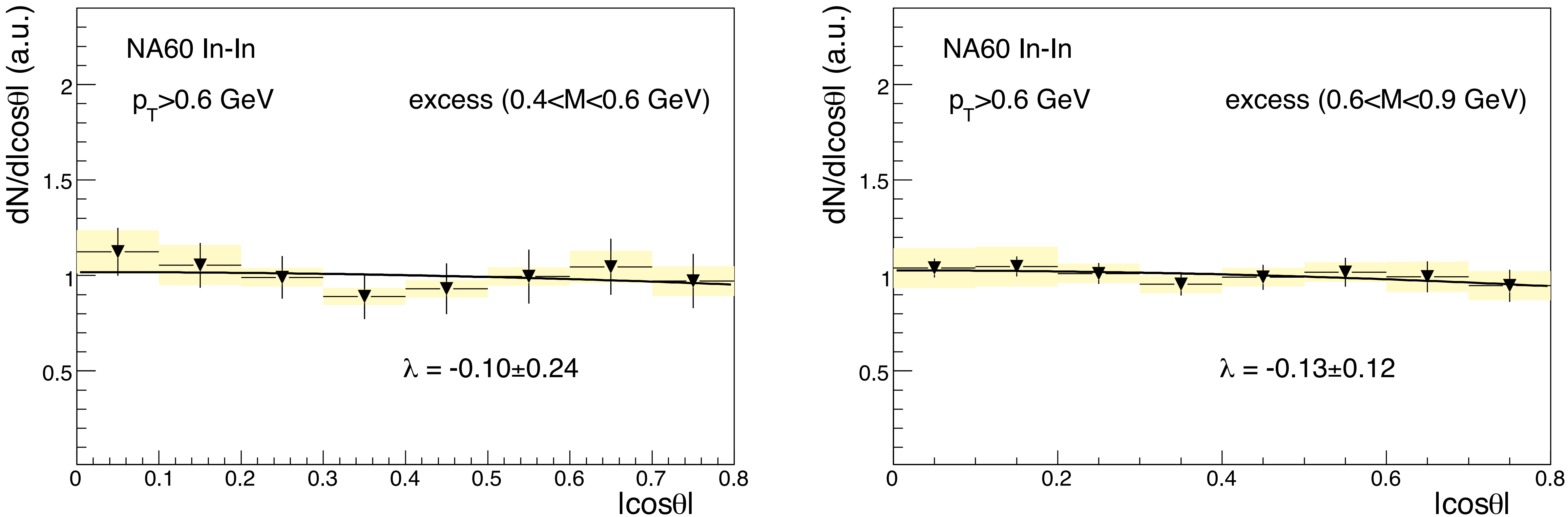


Results (Bjorken + radial flow)



- In general
  $\lambda_\theta, \lambda_\phi, \lambda_{\theta\phi} \neq 0$
- Shape of λ_θ vs. q_T depends on flow velocity

NA-60 data



R. Arnaldi *et al.* PRL 102, 222301

- Analyzed in Collins-Soper frame
- Consistent with $\lambda = 0$
- Need more statistics, better in helicity frame (?)

Summary + Outlook

- Virtual photons from thermal source are polarized
- Collective flow modifies shape of λ vs. $q_\perp$
- Small effect, consistent with NA60 (CS vs. HX?)
- Non-equilibrium (deformed momentum distribution)

  Stronger effect? Baym + Hatsuda + Strickland

- Polarization due to magnetic fields & rotation?

  Two directions:
  photon momentum + polarization axis
  expect richer structure