Why local gauge invariance is necessary

Helmut Haberzettl

Institute for Nuclear Studies and Department of Physics

THE GEORGE WASHINGTON UNIVERSITY

WASHINGTON, DC

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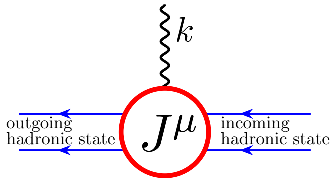
Outline

- Preliminaries
- Gauge invariance basics
- Implementation of local gauge invariance
- A practical example
- Summary

Setup: Currents

$J^\mu \; =$ 

Photon may be real or virtual, incoming or outgoing

All hadrons on-shell:

$$k_\mu J^\mu = 0 \qquad \text{(necessary condition)} \qquad \Rightarrow \qquad \text{gauge invariance}$$

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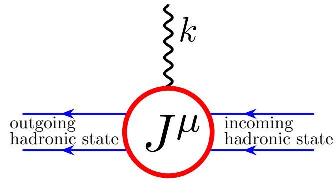

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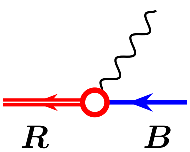
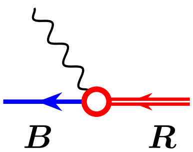
Transition currents:

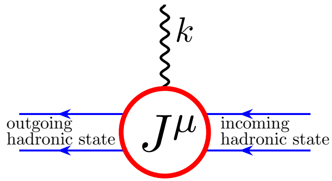




B: Baryon
R: Resonance

Setup: Currents

J^μ =



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Transition currents:

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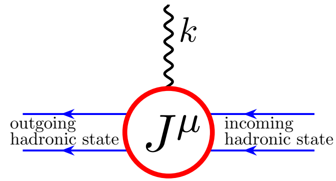
Current operators for transition currents must necessarily be manifestly transverse.
Hence, gauge-invariance condition is trivially satisfied for on- and off-shell baryons.

$J^\mu =$ 

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All hadrons on-shell:

$k_\mu J^\mu = 0$ (necessary condition) $\Rightarrow$ gauge invariance

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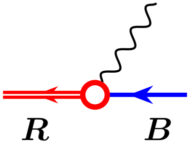

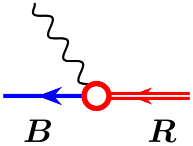

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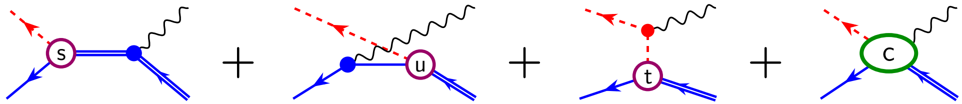
Why bother?

Production Current

$\longleftarrow$ time $\longrightarrow$

$M^\mu =$



s-channel + u-channel + t-channel + interaction current

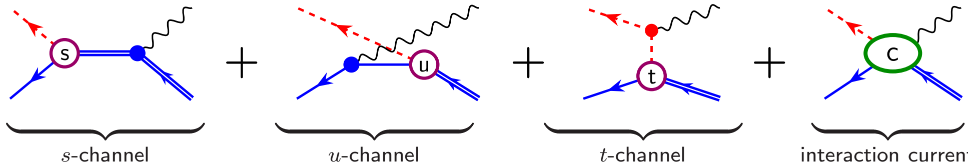
$$= F_s\, S_i\, J_i^\mu \;+\; J_f^\mu\, S_f\, F_u \;+\; J_m^\mu\, \Delta_m\, F_t \;+\; M^\mu_{\text{int}}$$

Generic expressions; summations over all possible intermediate states implied

Production Current

← time —

$$M^\mu =$$



$$= F_s\, S_i\, J_i^\mu + J_f^\mu\, S_f\, F_u + J_m^\mu\, \Delta_m\, F_t + M_{\mathrm{int}}^\mu$$

Generic expressions; summations over all possible intermediate states implied

- s-channel term contains transition-current contributions

  Diagrams apply to spacelike process; for timelike process, read diagrams in time-reversed order

- Entire production current must be gauge invariant

  Without it, wrong background contribution for extraction of form factors

Production Current

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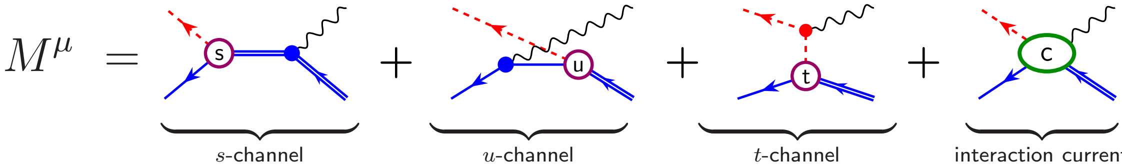

$$= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M^\mu_{\text{int}}$$

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Diagrams apply to spacelike process; for timelike process, read diagrams in time-reversed order

- Entire production current must be gauge invariant

Without it, wrong background contribution for extraction of form factors

- Any approximation will likely destroy gauge invariance
- For a microscopic theory, it is not sufficient to fix $k_\mu M^\mu = 0$ on shell

Next. . .

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- Implementation of local gauge invariance
- A practical example
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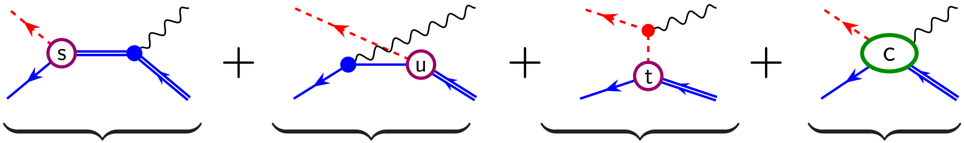
Gauge Invariance

$M^\mu =$



$$= F_s\, S_i\, J_i^\mu \; + \; J_f^\mu\, S_f\, F_u \; + \; J_m^\mu\, \Delta_m\, F_t \; + \; M^\mu_{\text{int}}$$

Global gauge invariance

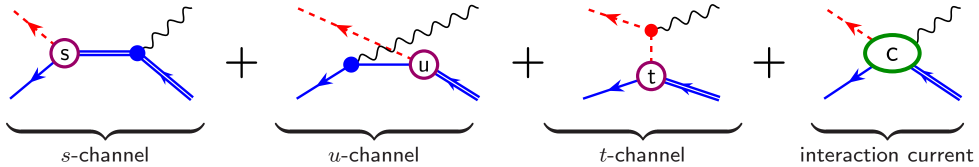
$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \to \Phi\, \mathrm{e}^{-i\Lambda}$$

Gauge Invariance

$$M^\mu =$$



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conserved current $\Rightarrow$ implies charge conservation

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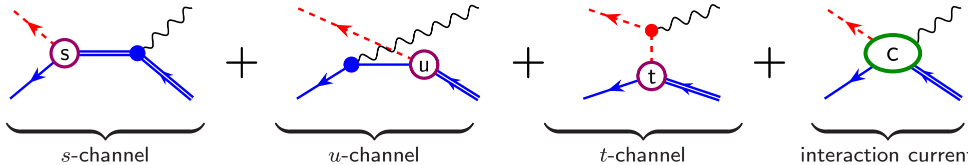

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Global gauge invariance

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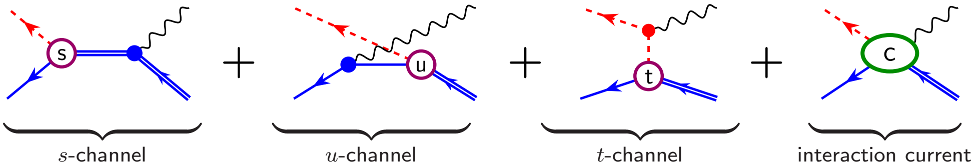
Fixing global gauge invariance does not mean internal damage is fixed as well

Gauge Invariance

$M^\mu =$



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Local gauge invariance

$$\Phi \to \Phi\, \mathrm{e}^{-i\lambda(x)}$$

Gauge Invariance

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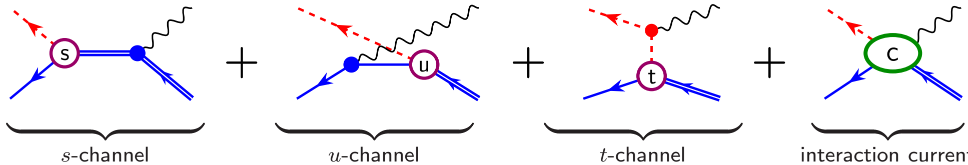

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Generalized Ward-Takahashi identities (gWTI)

$$k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

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similarly for baryons

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off-shell relations

Gauge Invariance

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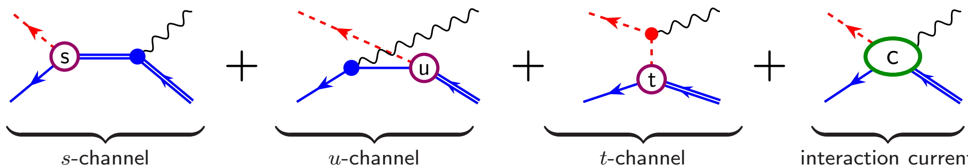

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local gauge invariance $\Rightarrow$ implies existence of e.m. field

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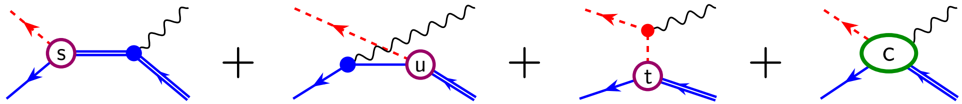

s-channel | u-channel | t-channel | interaction current

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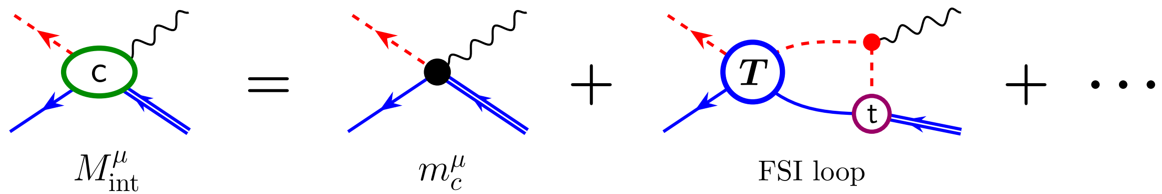
local gauge invariance $\Rightarrow$ implies existence of e.m. field

Without gWTI underlying e.m. field is damaged

Implementation: Reorganize interaction current



- Interaction current contains all contributions from final-state interactions

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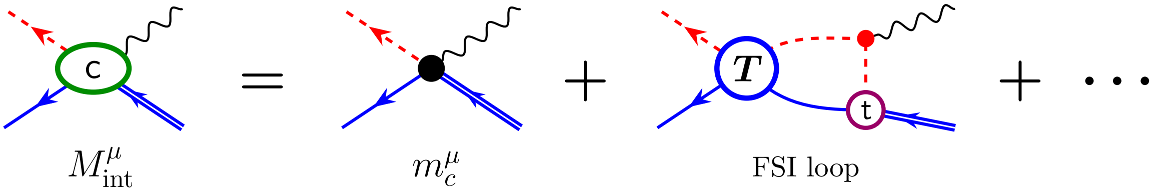

- Interaction current contains all contributions from final-state interactions

- Decompose FSI loops into longitudinal and transverse contributions

For details, see HH, Nakayama, Krewald, PRC74, 045202 (2006); HH, Huang, Nakayama, PRC83, 065502 (2011); HH, Wang, He, PRC92, 055503 (2015)

- Impose generalized Ward-Takahashi identities to longitudinal part

Generalized Ward-Takahashi Identities

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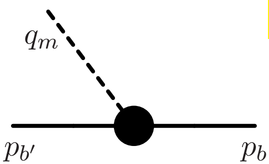

Hadronic vertex

$$F(p_{b'}, p_b) = \boldsymbol{G}(q_m)\, \boldsymbol{\tau}\, f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

This phenomenologically motivated ansatz
can be improved in a systematic way

Generalized Ward-Takahashi Identities

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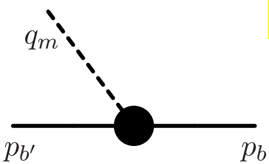

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Interaction-current Ansatz:

$$\boxed{M_{\text{int}}^\mu = m_c^\mu\, f_t(t) + \boldsymbol{G}(q) C^\mu + T_{\text{int}}^\mu} \qquad k_\mu T_{\text{int}}^\mu \equiv 0$$

Generalized Ward-Takahashi Identities

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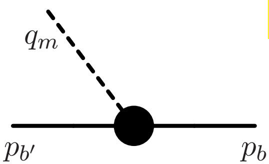

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$\Rightarrow$ Determine C^μ such that (3) is true

Auxiliary Contact Current C^μ

HH, Nakayama, Krewald, PRC74, 045202 (2006)

Non-singular

$$
\begin{aligned}
C^\mu = & -e_m(2q-k)^\mu \frac{f_t - 1}{t - M_m^2}\left(\delta_s f_s + \delta_u f_u - \delta_s\delta_u f_s f_u\right) \\
& - e_f(2p'-k)^\mu \frac{f_u - 1}{u - M_f^2}\left(\delta_t f_t + \delta_s f_s - \delta_t\delta_s f_t f_s\right) \\
& - e_i(2p+k)^\mu \frac{f_s - 1}{s - M_i^2}\left(\delta_u f_u + \delta_t f_t - \delta_u\delta_t f_u f_t\right)
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\end{aligned}
$$

where

$$
\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \qquad x = s, u, t
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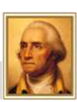
Charge conservation: $Q_m \boldsymbol{\tau} + Q_f \boldsymbol{\tau} - \boldsymbol{\tau} Q_i = e_m + e_f - e_i = 0$



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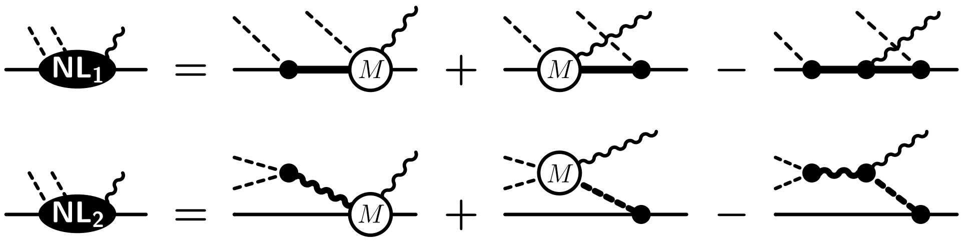
Charge conservation: $Q_m\boldsymbol{\tau} + Q_f\boldsymbol{\tau} - \boldsymbol{\tau}Q_i = e_m + e_f - e_i = 0$

Four-divergence: $k_\mu C^\mu = e_m f_t + e_f f_u - e_i f_s$

ensures correct four-divergence for M^μ_{int}

Practical Relevance of Local Gauge Invariance

Example: Two-pion production at the no-loop level



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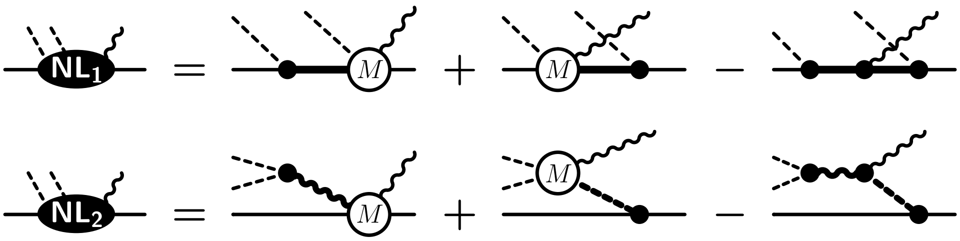

Without gWTI, this amplitude will not be gauge invariant

Summary

- Global gauge invariance is necessary but not sufficient to provide consistent implementation of the electromagnetic field throughout the interaction volume
- Correct dynamical basis provided by generalized Ward-Takahashi identities as they follow from local gauge invariance
- Results obtained within microscopic reaction models that do not obey local gauge invariance need to be viewed with caution
- Applications show that contributions from gauge-fixing contact current can be substantial

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Thank you!