Baryon spectrum and (transition) form factors from QCD's Faddeev equation

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with Gernot Eichmann, Helios Sanchis-Alepuz and Richard Williams

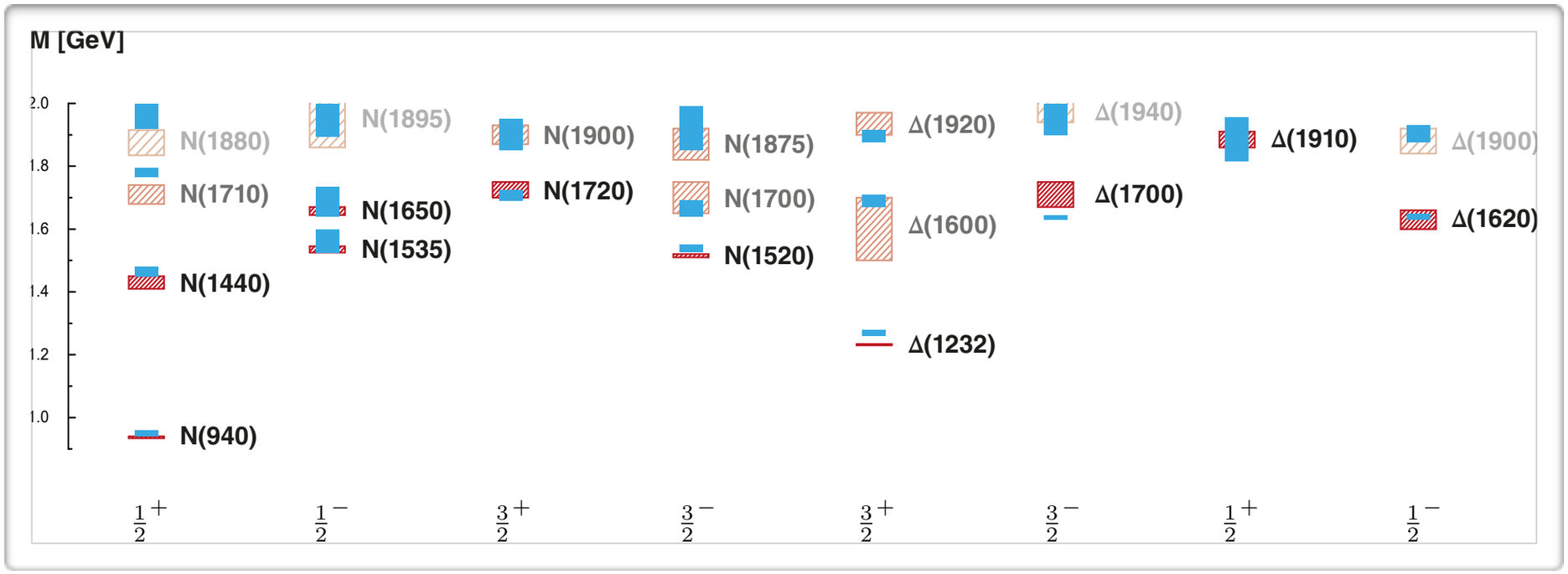
Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

Bundesministerium für Bildung und Forschung

HIC for FAIR Helmholtz International Center

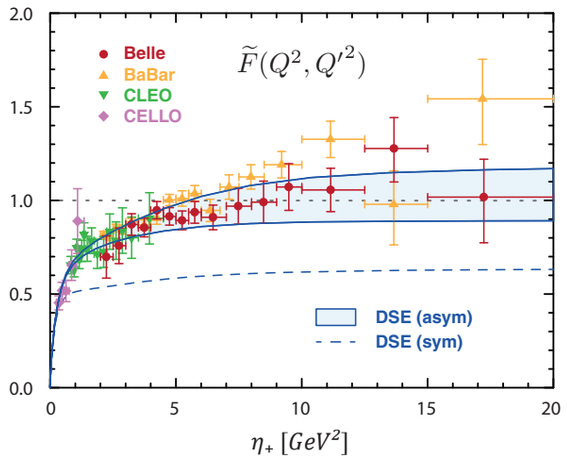
Take home messages

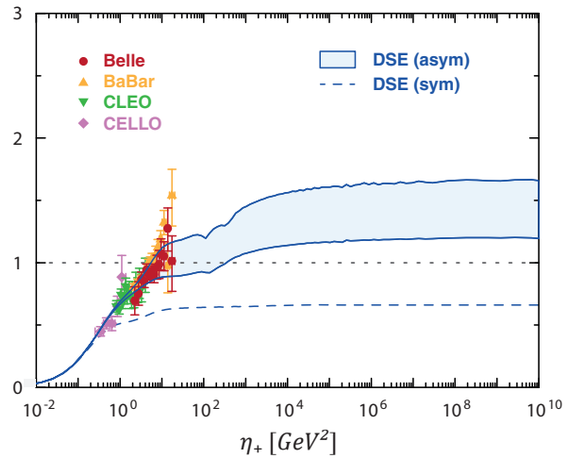
- Light baryon spectrum:



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

- Space- and time-like form factors: pion TFF as test-case





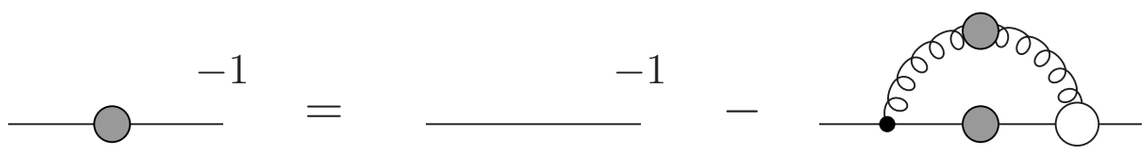
Weil, Eichmann, CF, Williams, arXiv:1704.05774

	our result	PDG
$\Gamma(\pi^0 \to e^+e^-\gamma)$	$9.11(4)\ \times 10^{-11}$ GeV	$9.06(18)\ \times 10^{-11}$ GeV
$\Gamma(\pi^0 \to e^+e^-e^+e^-)$	$2.63(1)\ \times 10^{-13}$ GeV	$2.58(12)\ \times 10^{-13}$ GeV
$\mathrm{B}(\pi^0 \to e^+e^-)$	$6.21(3)\ \times 10^{-8}$	$6.87(36)\ \times 10^{-8}$

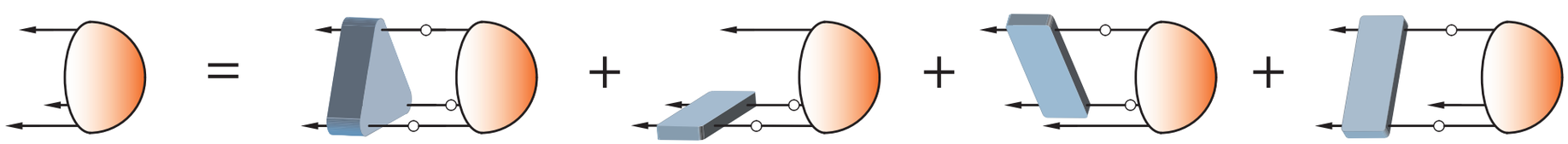
Eichmann, CF, Weil, Williams, arXiv:1704.06046

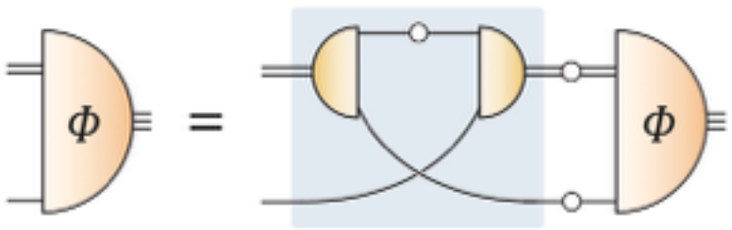
Overview

1. Introduction - quarks, gluons and mesons



2. Baryon spectrum - light and strange





3. Pion TFF, baryon form factors and decays

Baryon spectroscopy from QCD

- Underlying QCD forces
  - two-body vs. three-body → Δ vs Y - configuration
  - confinement → Regge trajectories ?!
  - spin structure → (Hyper)-Fine structure
  - meson cloud effects → GB-exchange vs QCD
  - heavy/heavy-light systems → Flavor dependence
- 'Missing resonances' → 3-quark vs. quark-diquark
- Coupled-channel effects

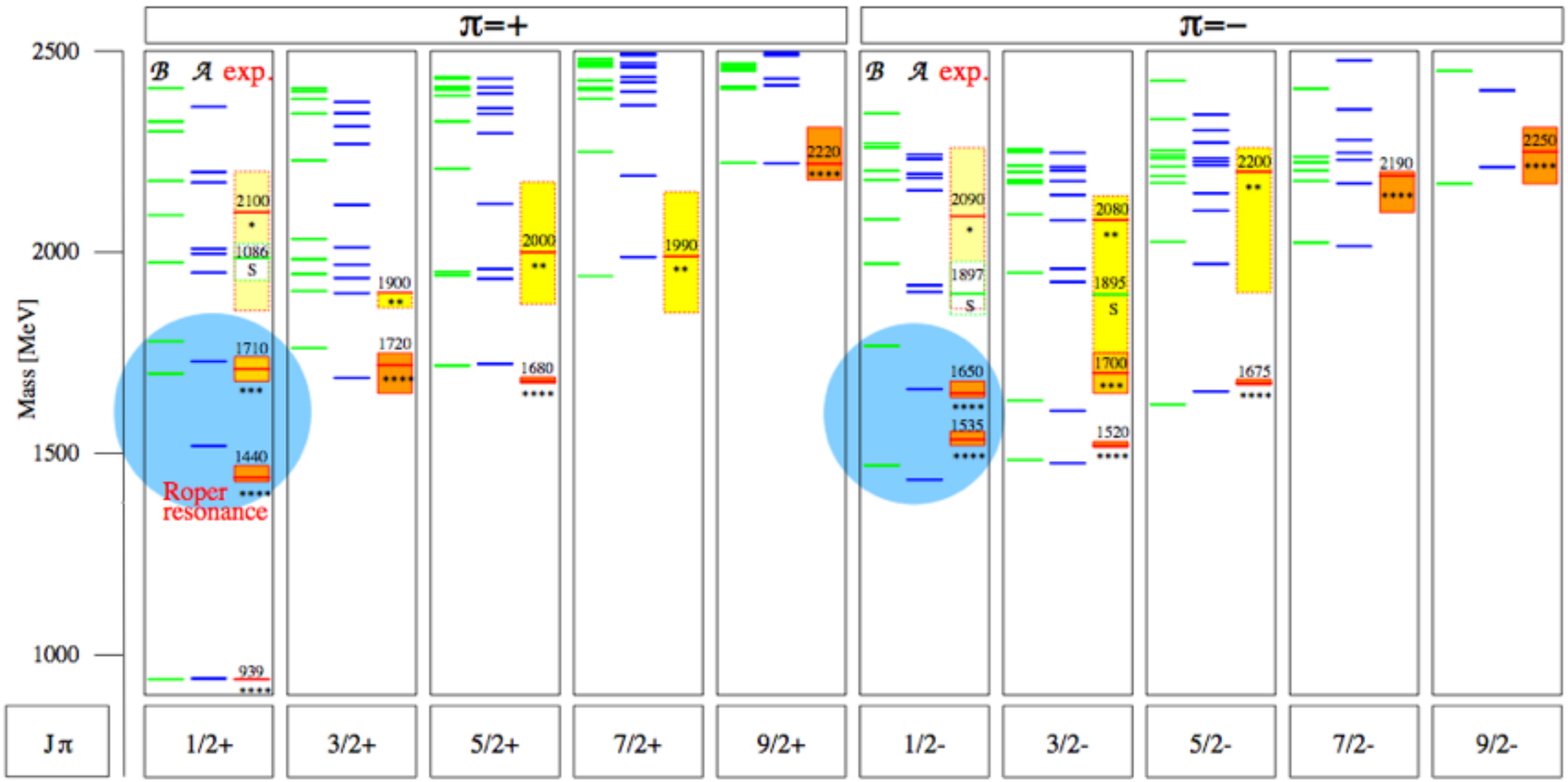
Strategies to deal with this situation:

Nonperturbative QCD: Lattice, Functional methods

Effective theories with hadronic dof

Klemt, Richard, Rev.Mod.Phys. 82 (2010) 1095

Light baryon spectrum - quark model

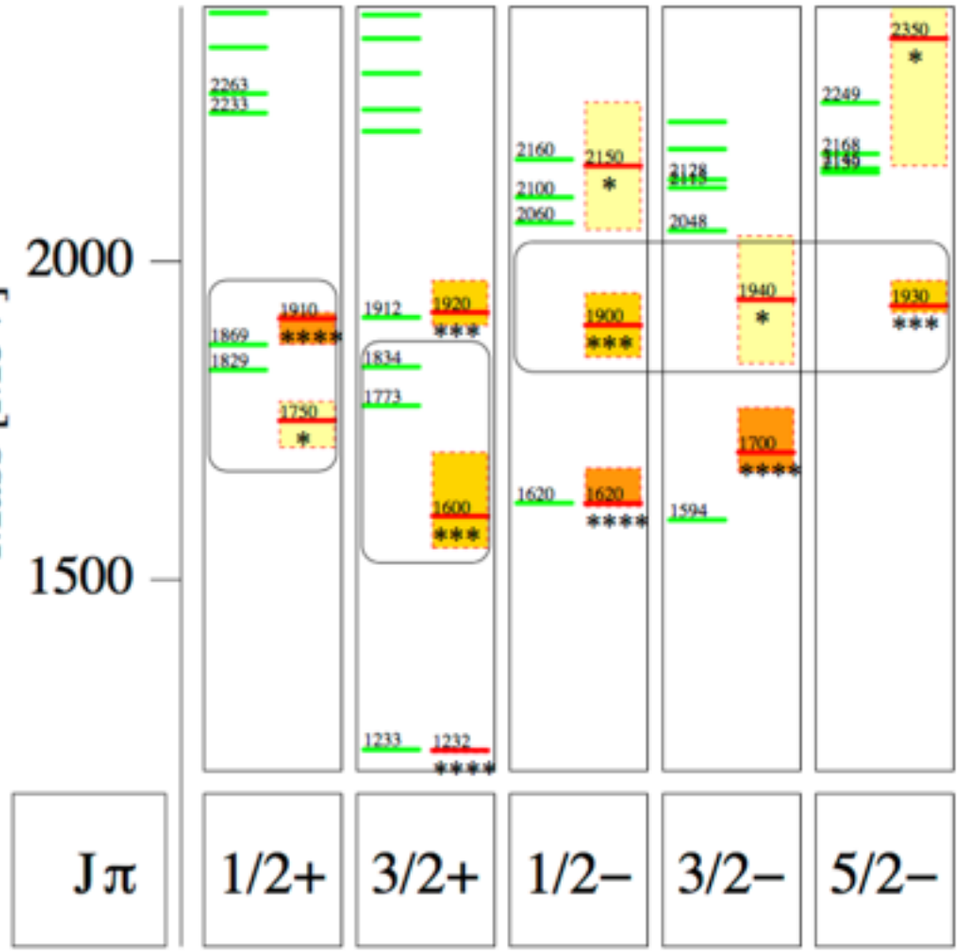


Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances' - three-body vs. quark-diquark
- level ordering:

$$N\frac{1}{2}^{\pm} \quad \text{vs.} \quad \Lambda\frac{1}{2}^{\pm}$$

Strange baryon spectrum quark model

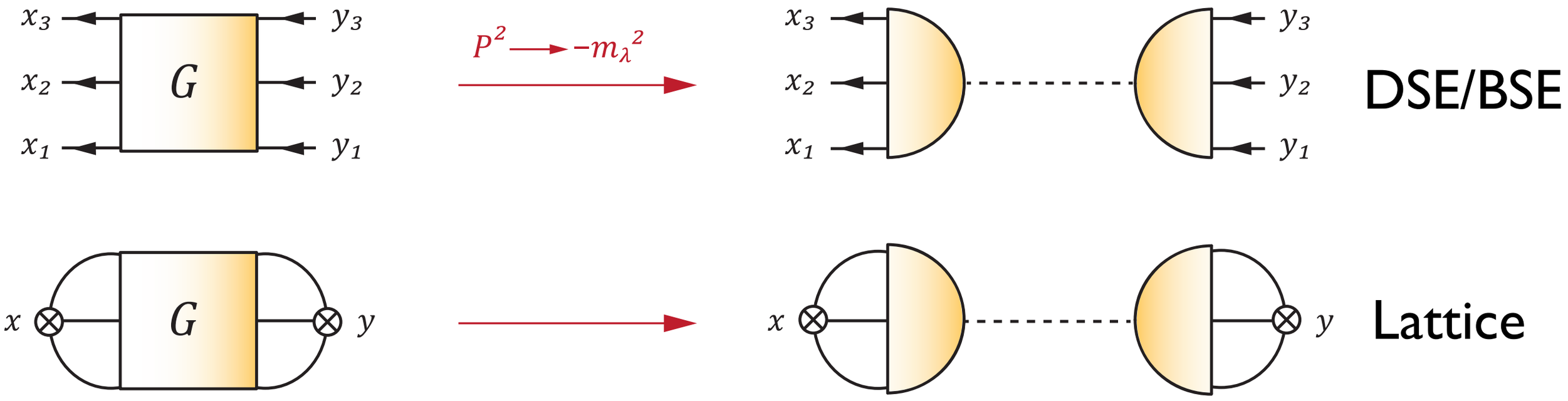


- light, strange and heavy spectrum probe QCD physics at different scales
- need flavor dependent QCD forces to explain spectrum
- models: parametrization via exchange of Goldstone-bosons

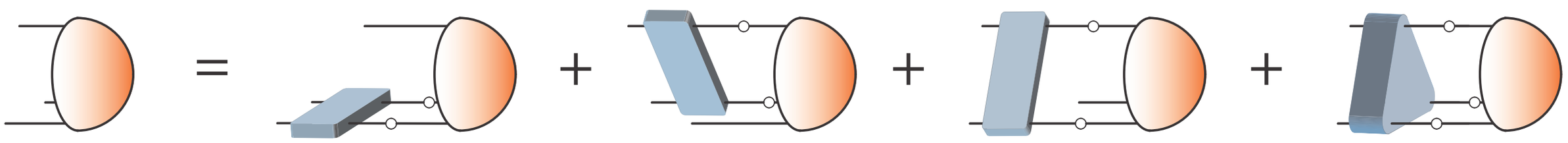
Ronniger, Metsch, EPJA 47 (2011) 162
see also Glozmann, Riska, Plessars et al.

- flavor dependent forces should be determined from QCD

Extracting spectra from correlators



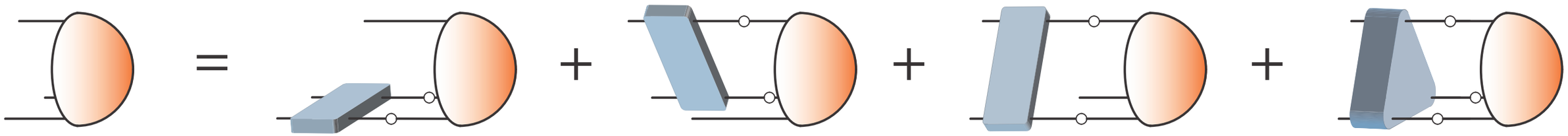
BSE for baryons (derived from equation of motion for G)



- exact equation for baryon ‘wave function’

Diquark-Quark approximation

BSE for baryons (derived from equation of motion for G)



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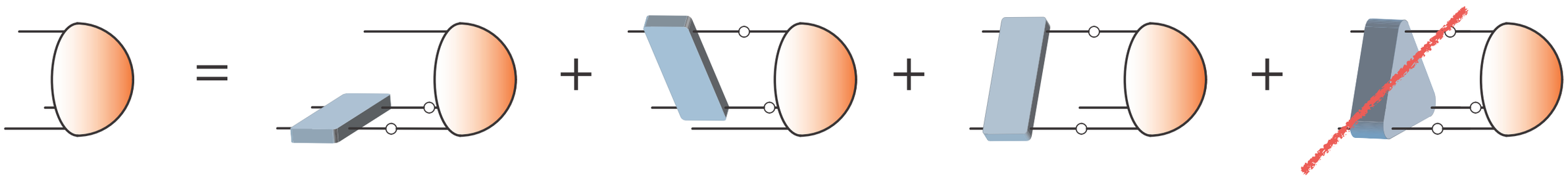

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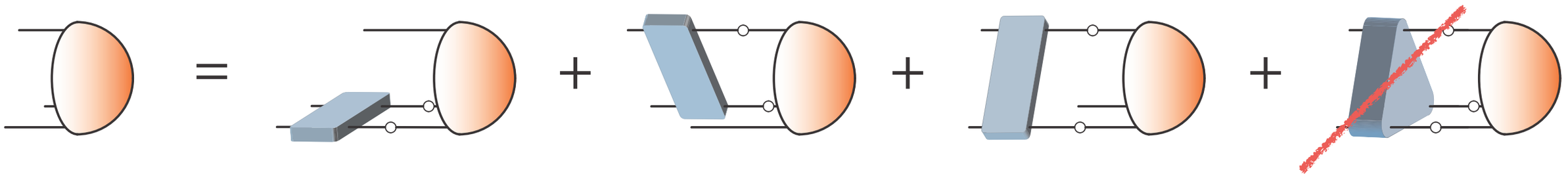
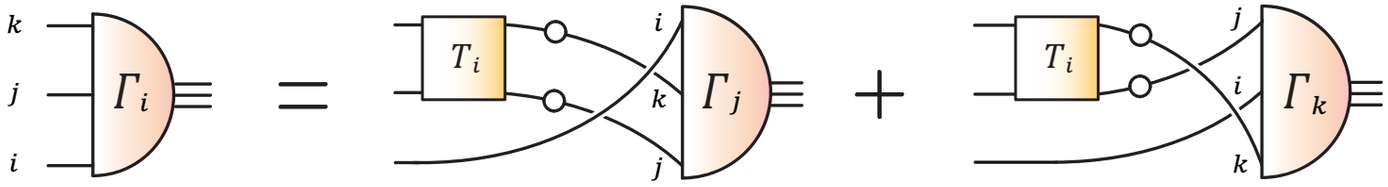
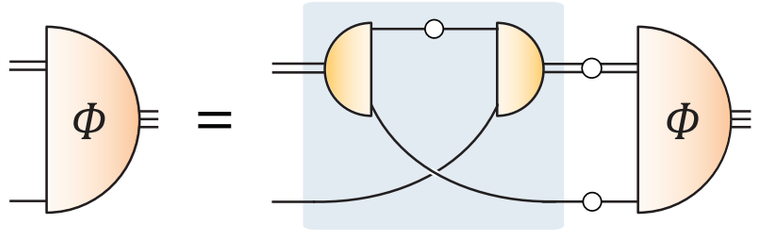


Faddeev equation (no three-body forces)



Diquark-quark



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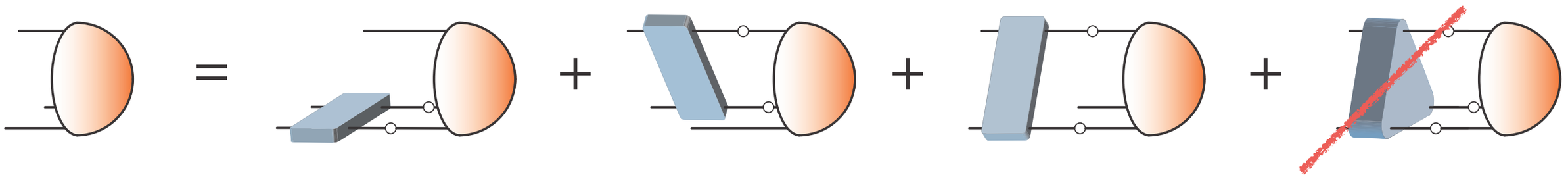

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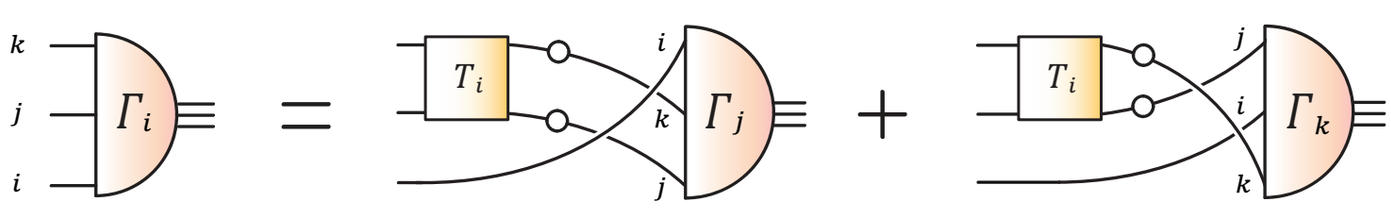

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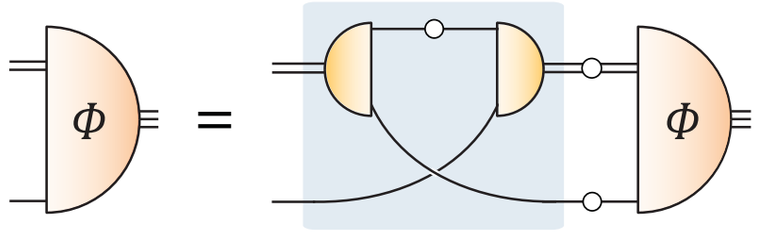
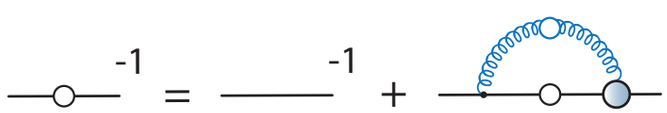
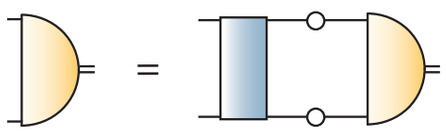
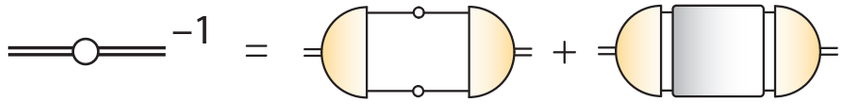








- Input in both cases: quark propagator and interaction

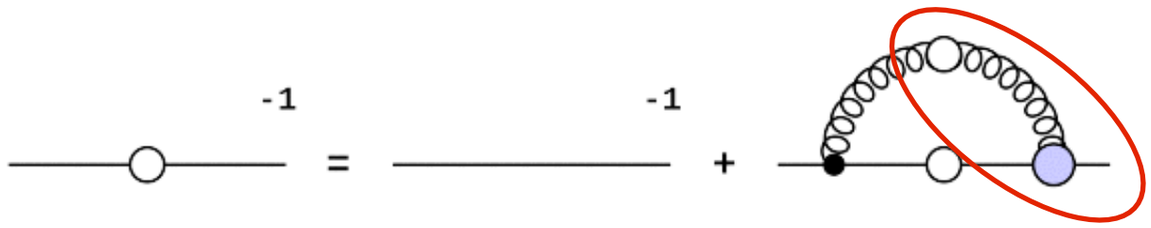
The DSE for the quark propagator

Approximations:

I) NJL/contact model:

II) Quark-diquark model: Ansatz for quark prop
(and diquark wave function)

III) Rainbow-ladder:



IV) Beyond rainbow-ladder:

- solve DSEs for quarks, gluons and quark-gluon vertex

Sanchis-Alepuz, Williams, PLB 749 (2015) 592
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein
Binosi, Chang, Papavassiliou, Qin, Roberts PRD95 (2017) 031501 and refs. therein

DSE/Faddeev landscape

	Quark-diquark			Three-quark		
	I) Contact interaction	II) QCD-based model	III) DSE (RL)	III) RL	IV) bRL	IV) bRL + 3q
N, Δ masses	√	√	√	√	√	...
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	...		
Roper	√	√		...		
$N \to N^*\gamma$	√	√		...		
$N^*(1535), \ldots$	...	...		...	...	
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	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

see talk of Ralf Gothe

Eichmann, N*-Workshop, Trento 2015

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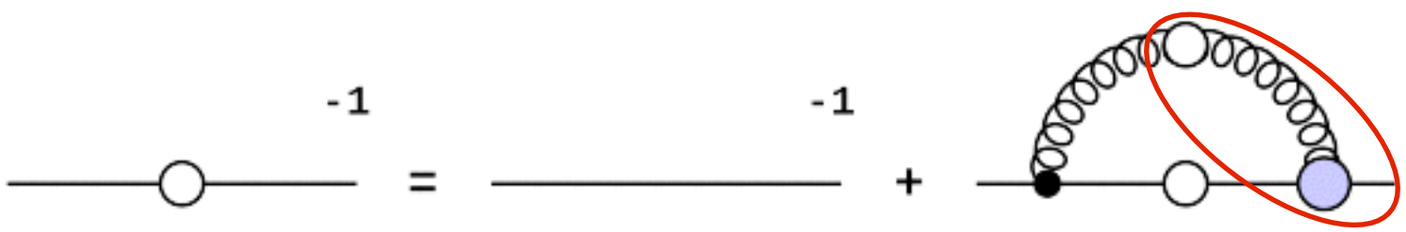
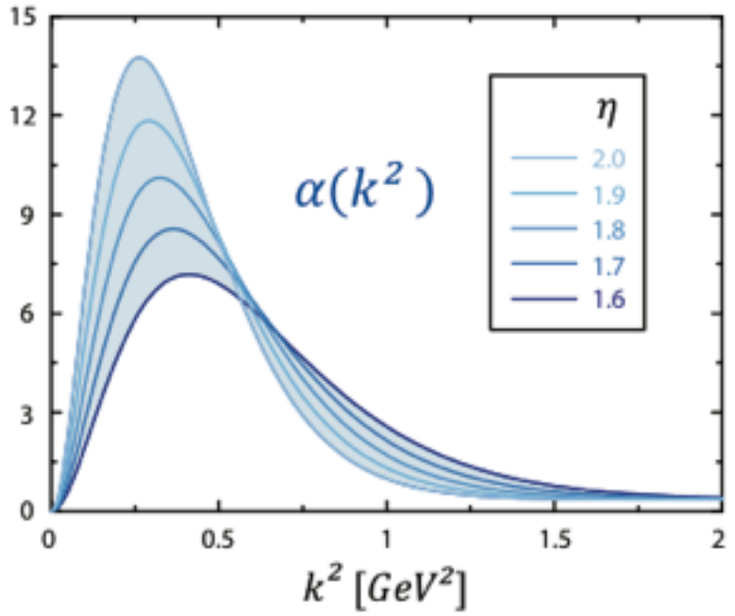
Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

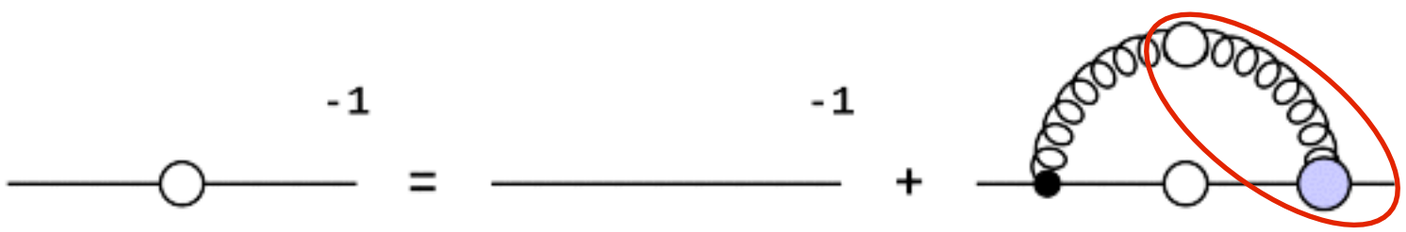
Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results

Rainbow-ladder model for quark-gluon interaction



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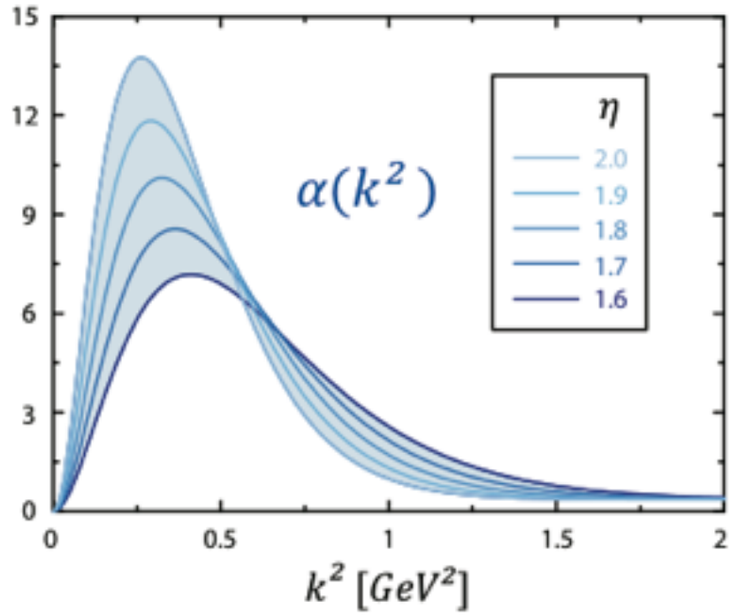

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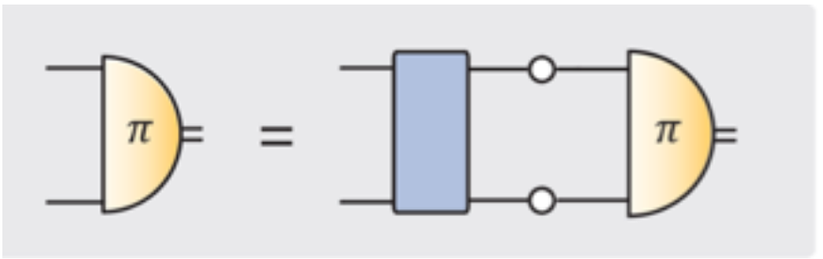

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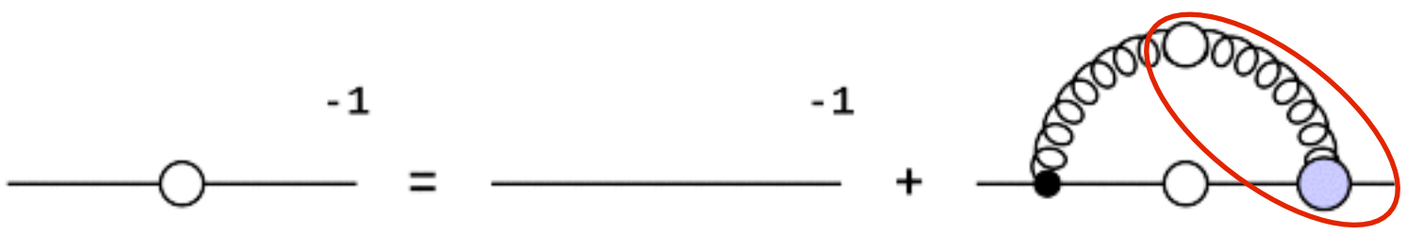

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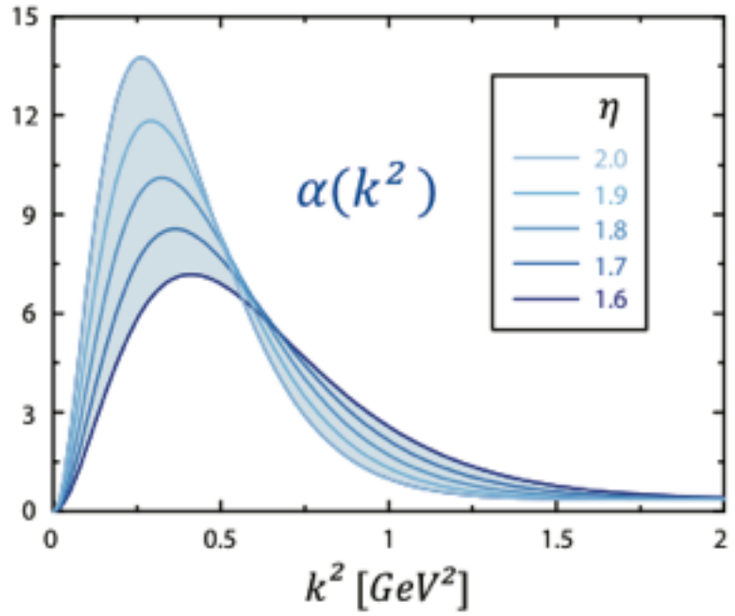

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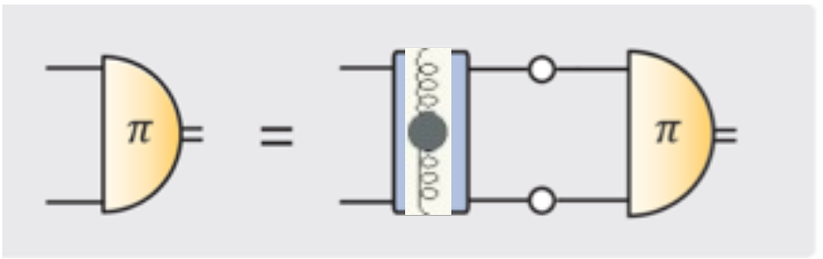
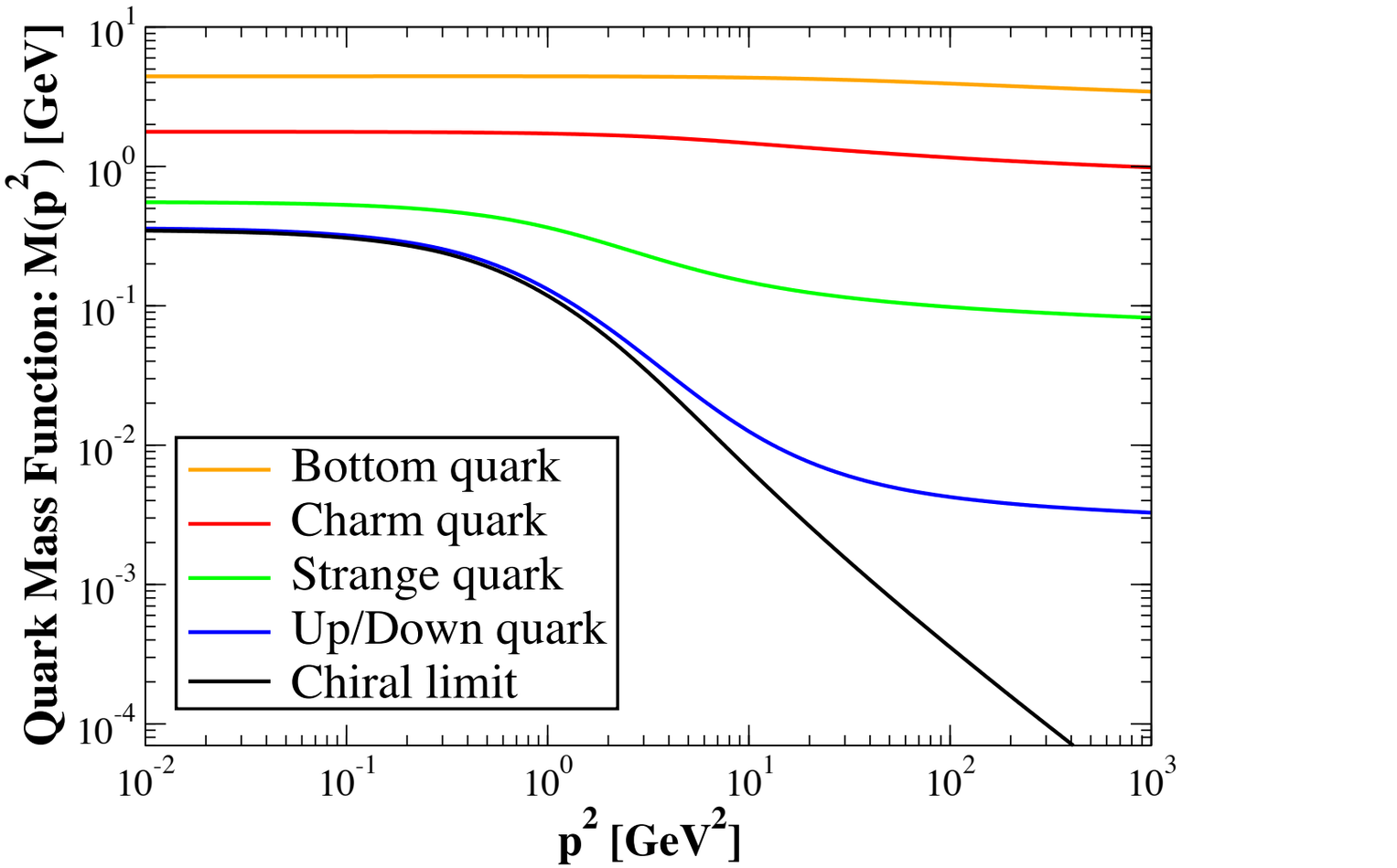
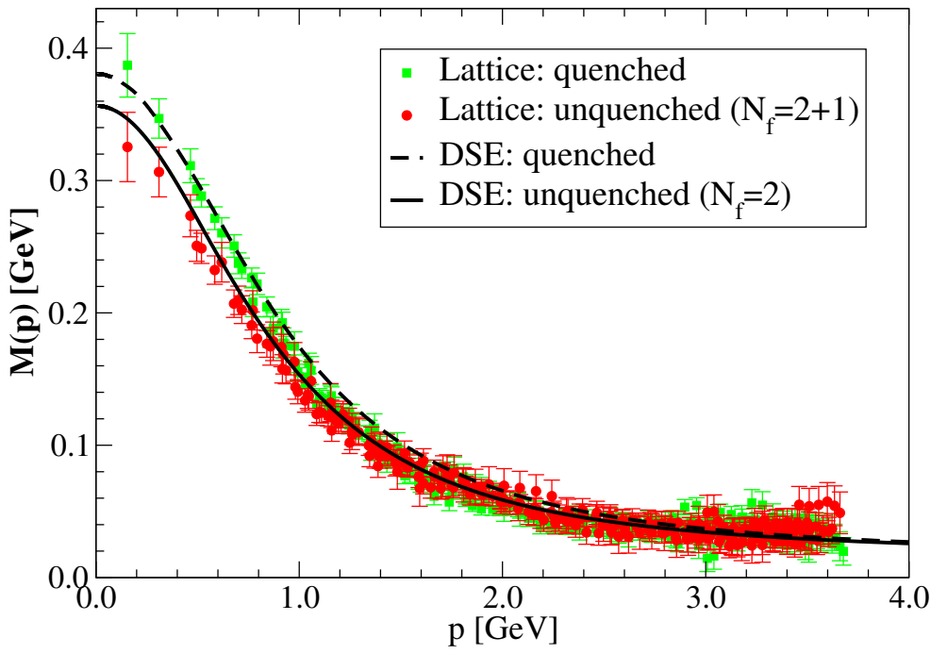


Quark mass: flavor dependence

Typical solution:

$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$





CF, Nickel, Williams, EPJ C 60 (2009) 47

- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $\langle \bar{\Psi}\Psi \rangle \approx (250\,\mathrm{MeV})^3$

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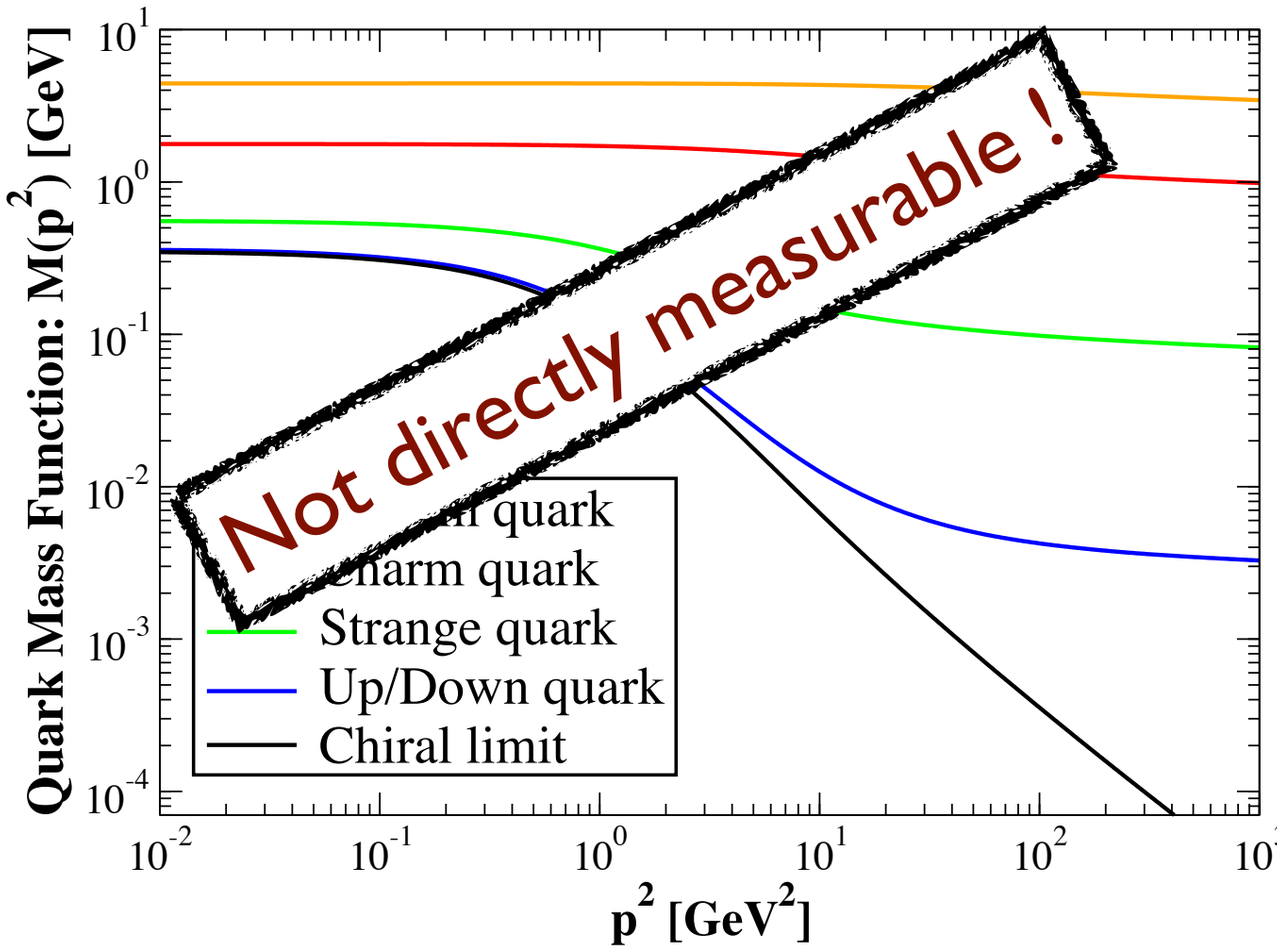

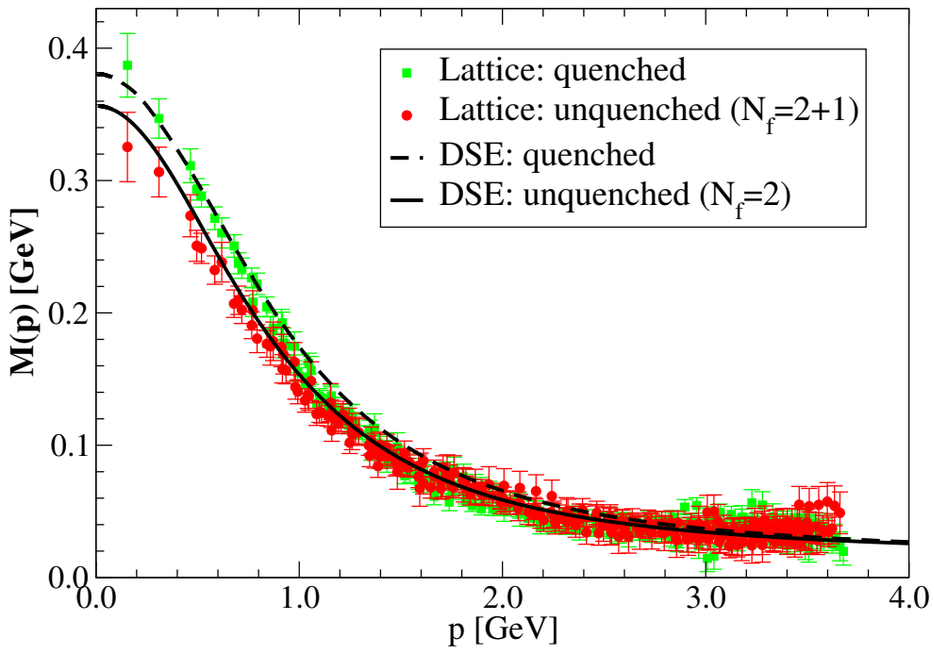

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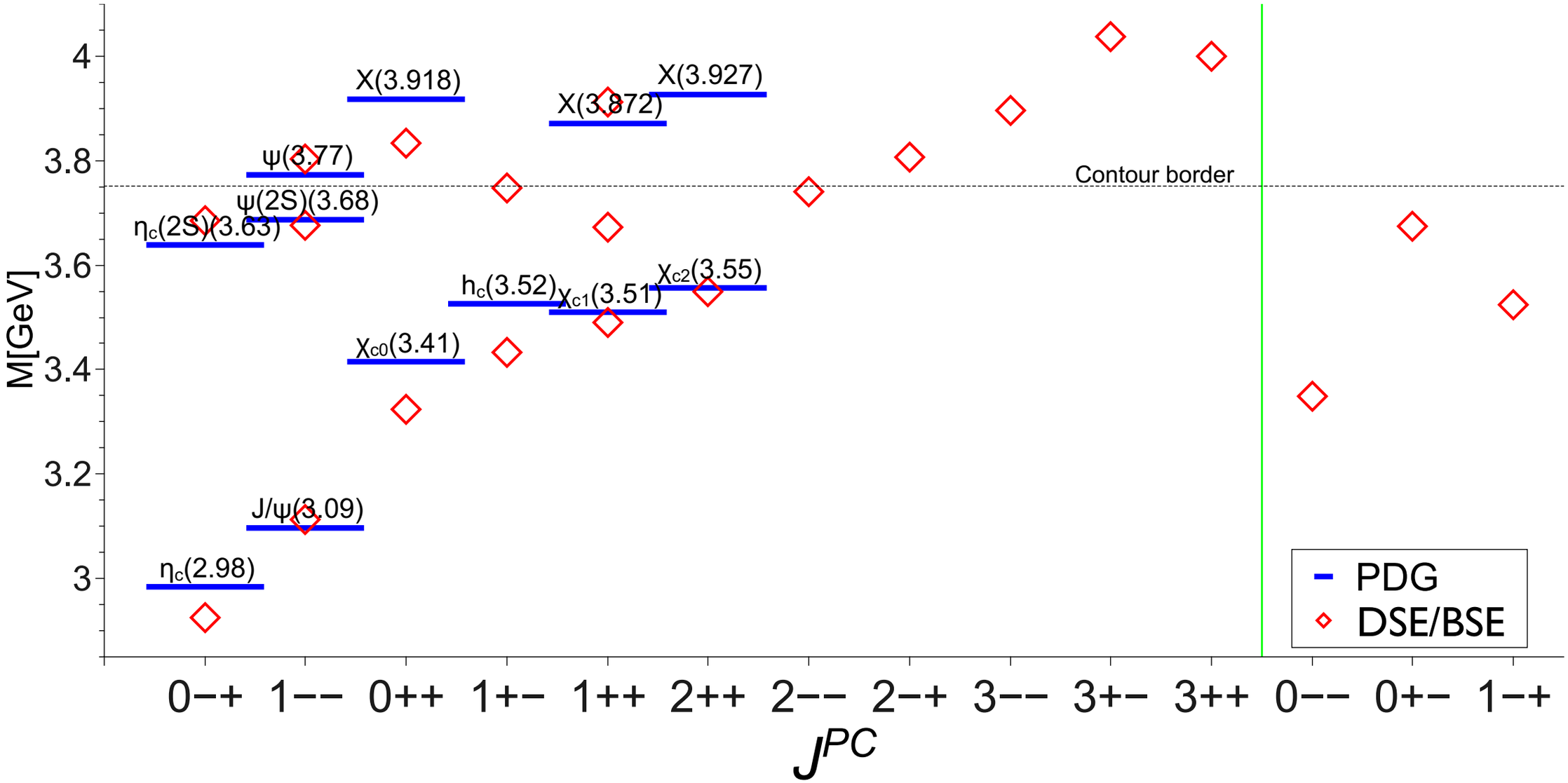
Charmonium spectrum



- good channels: $1^{--}, 2^{++}, 3^{--}, \ldots$
- acceptable channels: 0^{-+}
- clear deficiencies in other channels: missing spin-structure
- excited states fine ! (in good channels)

CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

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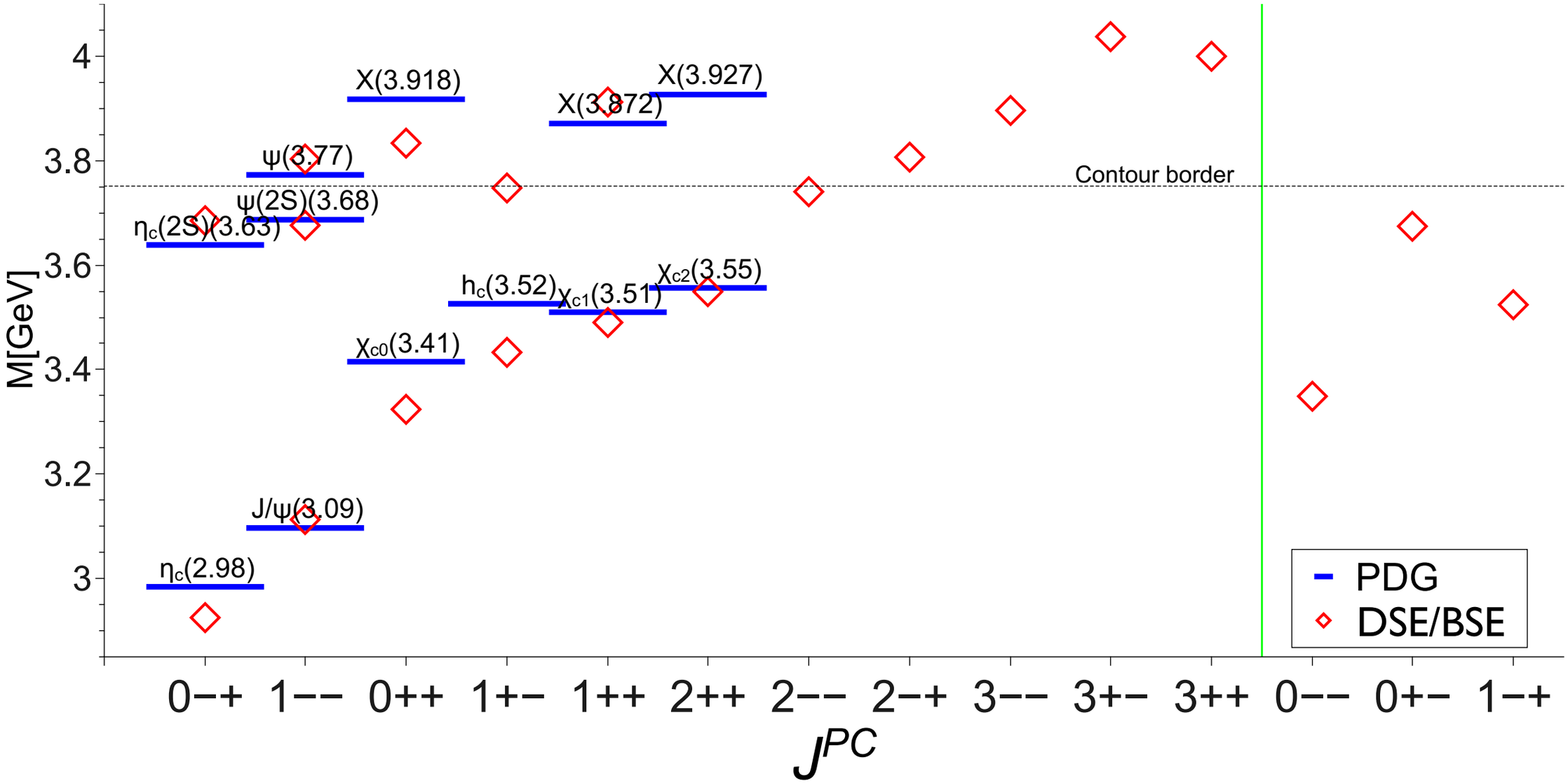

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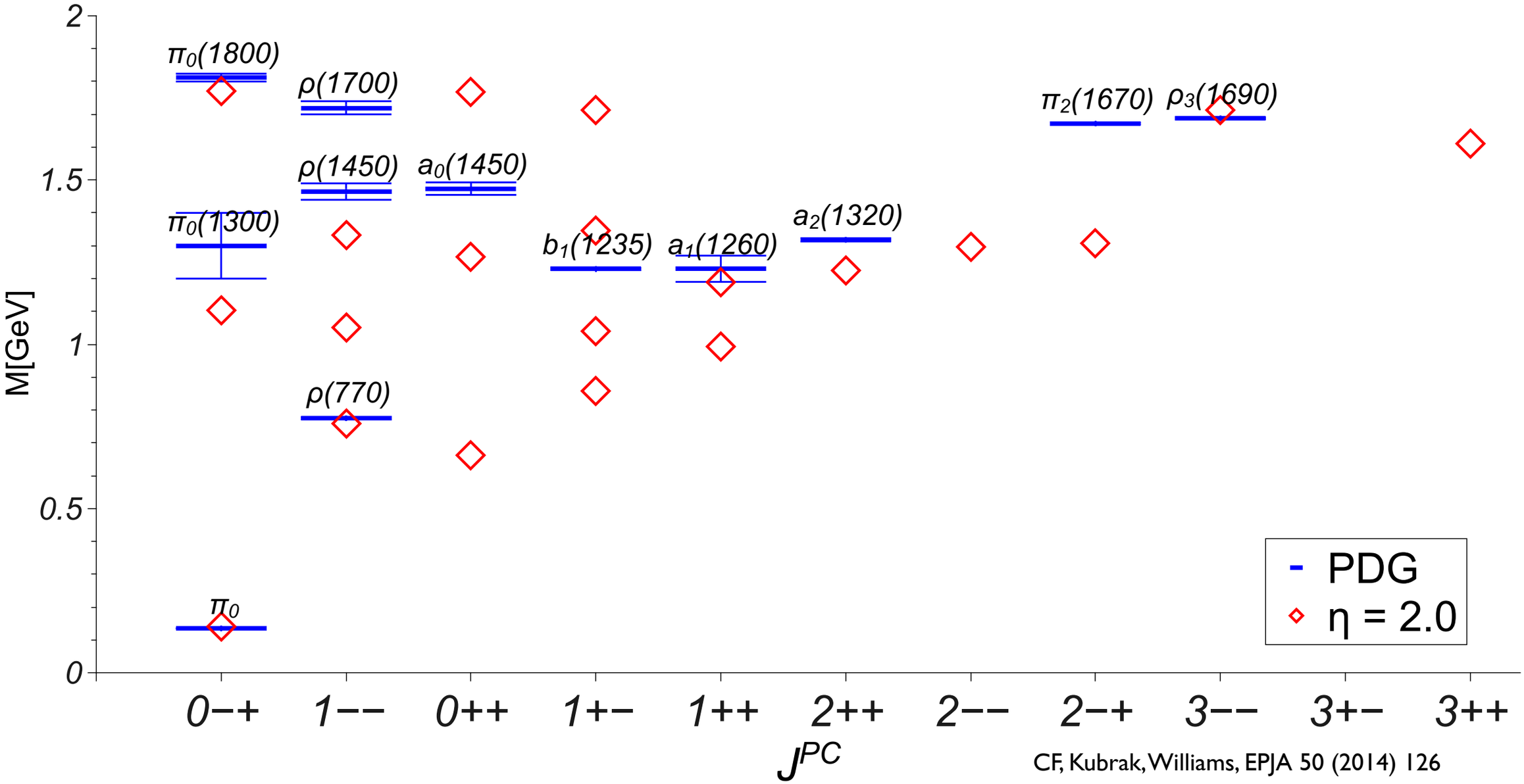

CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

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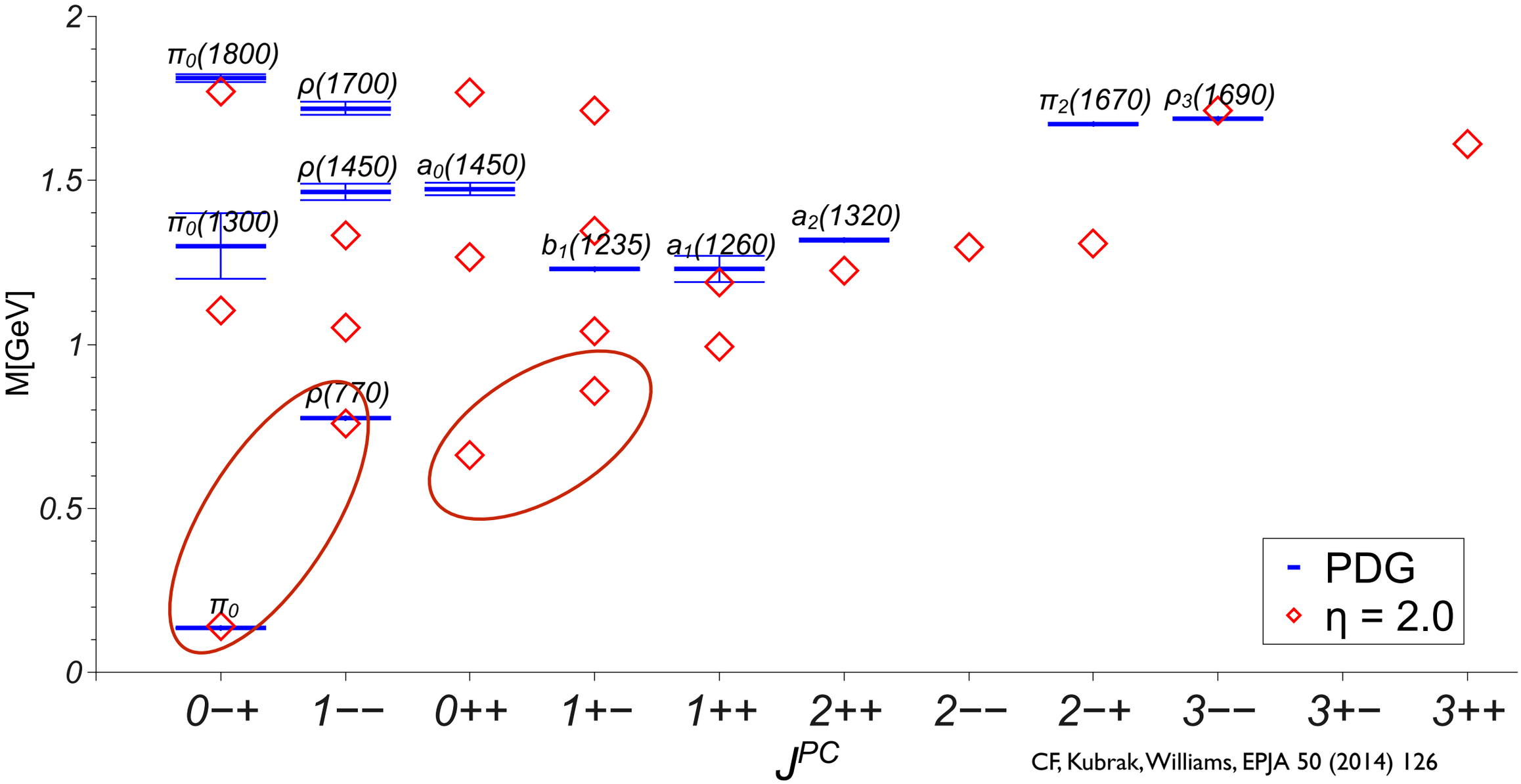

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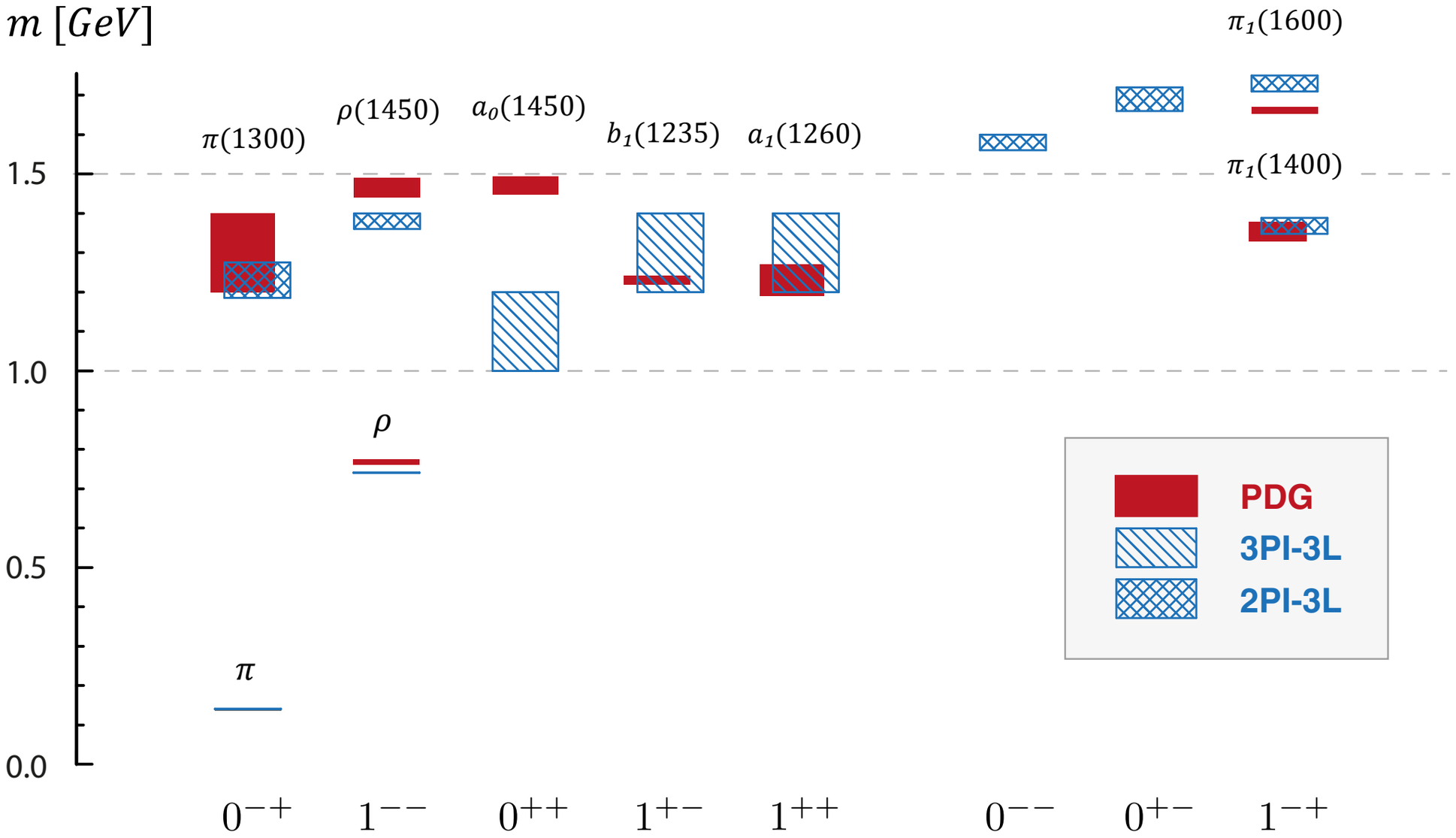

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- clear deficiencies in other channels and excited states
- drastic improvement beyond rainbow-ladder !

Overview

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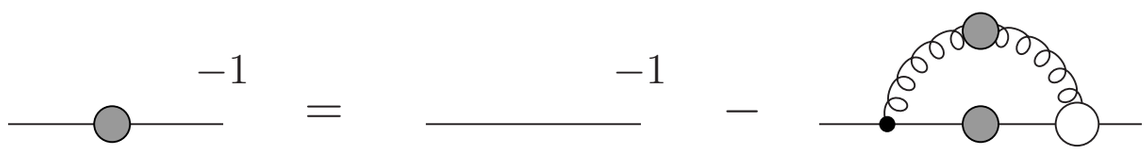

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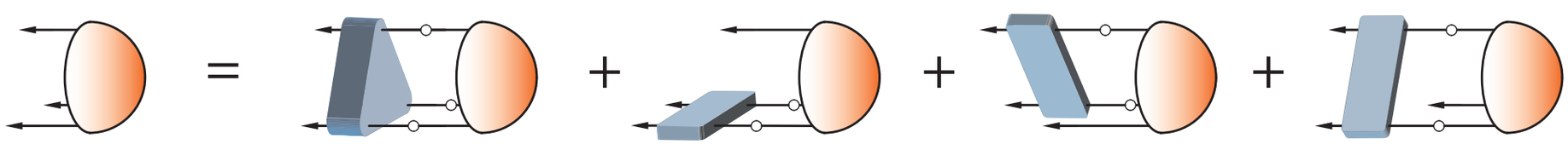
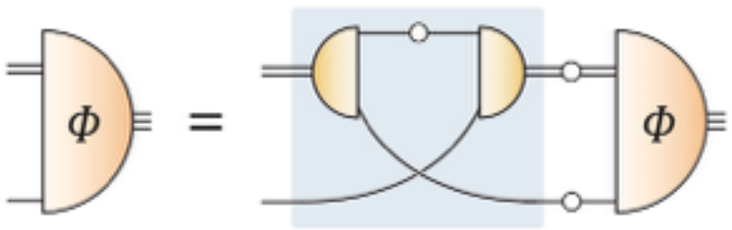
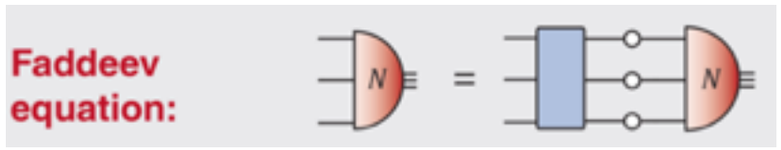
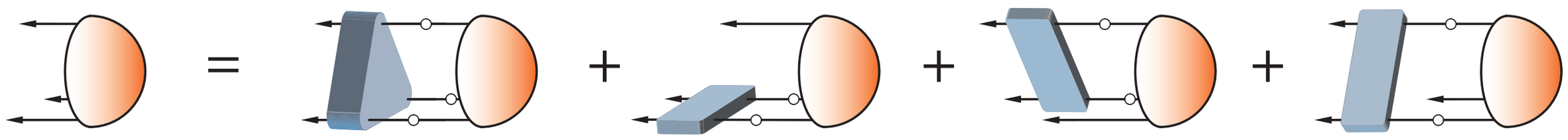




3. Pion TFF, baryon form factors and decays

Faddeev - equation



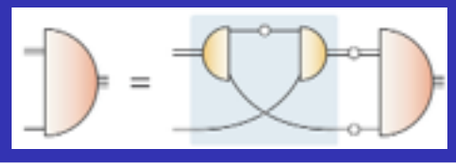


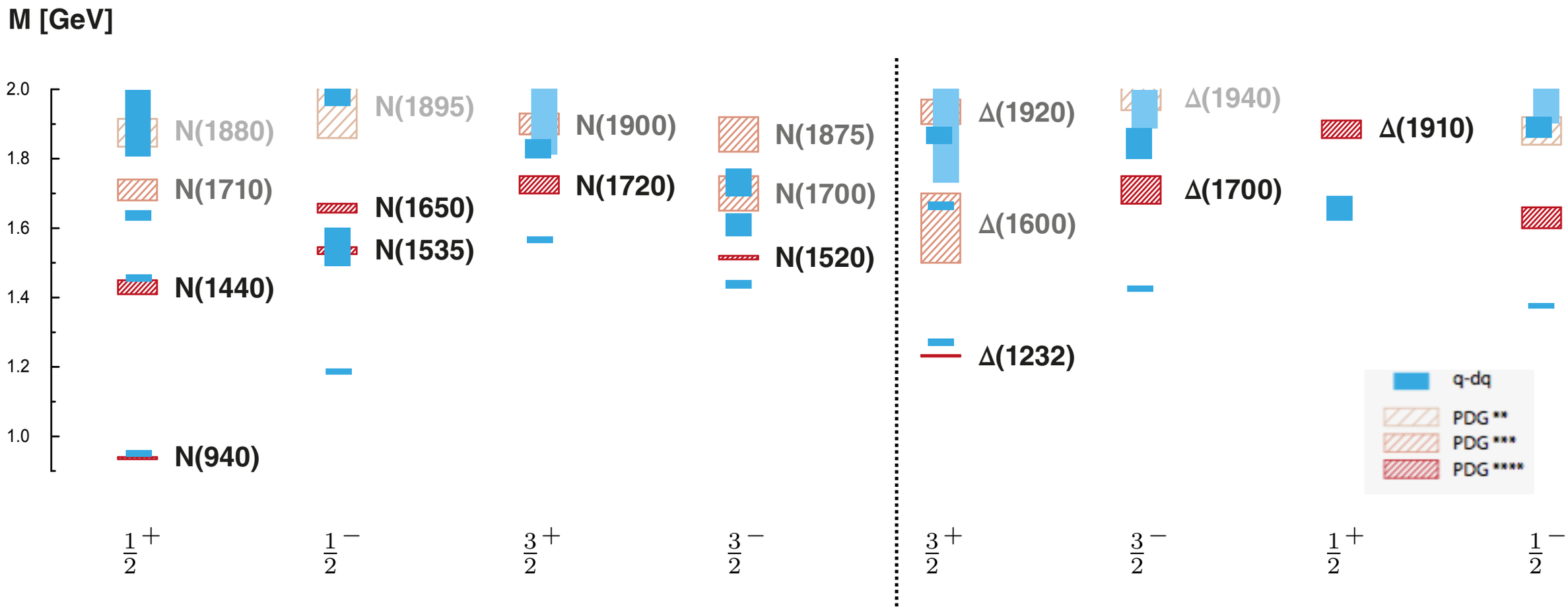
- relativistic bound state:
  - 64 tensor structures for octett: s, p, d - waves
  - 128 tensor structures for decuplett: s, p, d, f - waves

$$\Gamma(P,p,q) = \sum_l \tau_l F_l(P,p,q) \times (flavour \times colour)$$

$$D_i\,\gamma_5\,\mathcal{C} \otimes D_j\,\Lambda_+(P)\,, \qquad D_i = \left\{\mathbb{1},\ \not{p},\ \not{q},\ \not{P},\ [\not{p},\not{P}],\ [\not{q},\not{P}],\ [\not{p},\not{q}],\ [\not{p},\not{q},\not{P}]\right\},$$
$$\gamma_5\,D_i\,\gamma_5\,\mathcal{C} \otimes \gamma_5\,D_j\,\Lambda_+(P)\,, \qquad \Lambda_\pm(P) = \tfrac{1}{2}\left(\mathbb{1} \pm \hat{\not{P}}\right),$$

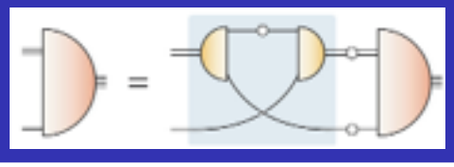
Light baryon spectrum: diquarks

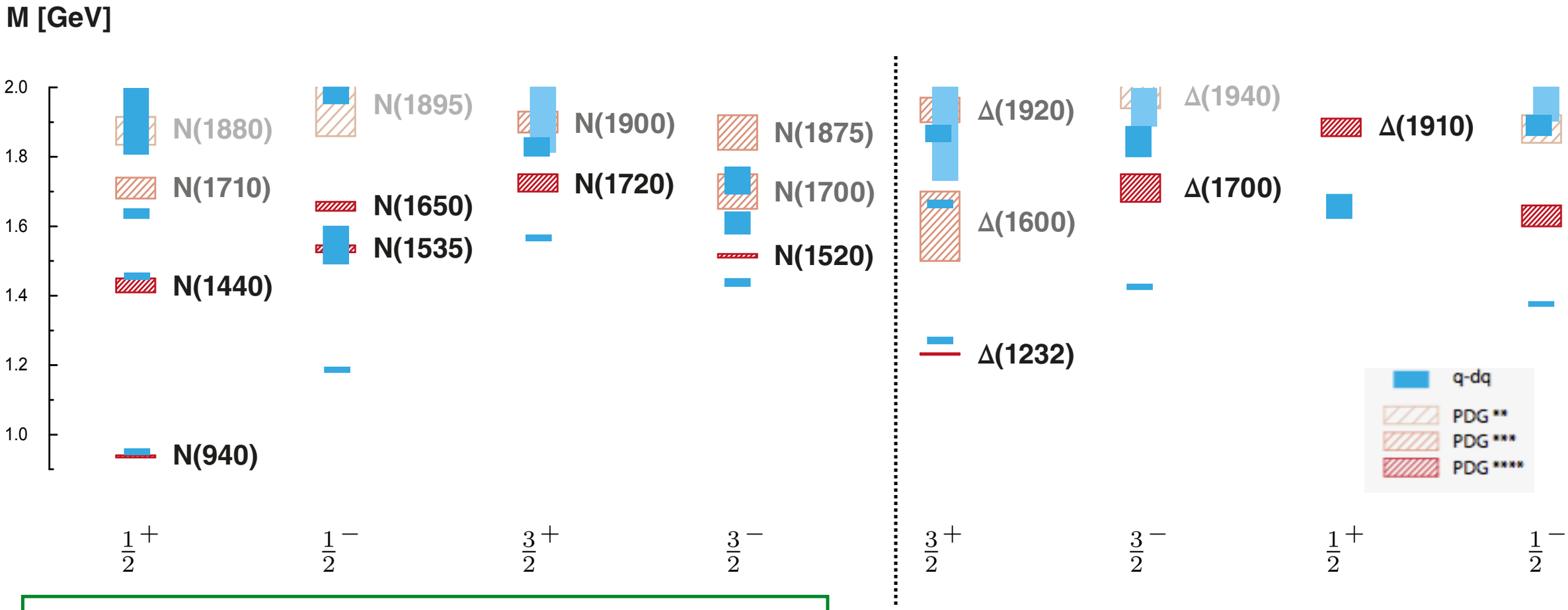




- nucleon and delta - channels: good results
- but: severe problems in all other channels

Light baryon spectrum: diquarks

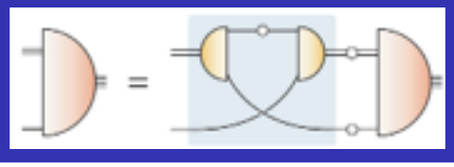


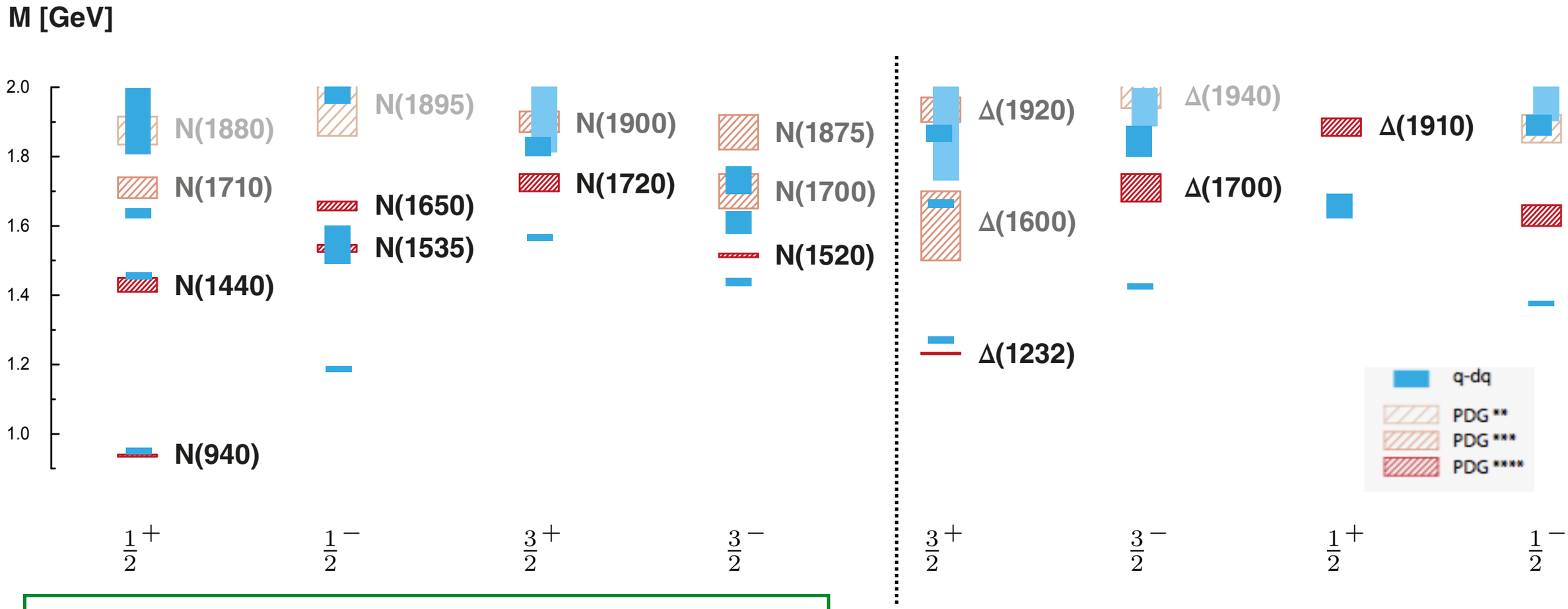


sc+av	ps+v	all	all

- nucleon and delta - channels: good results
- but: severe problems in all other channels

Light baryon spectrum: diquarks

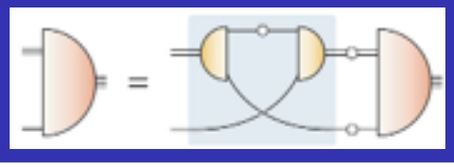


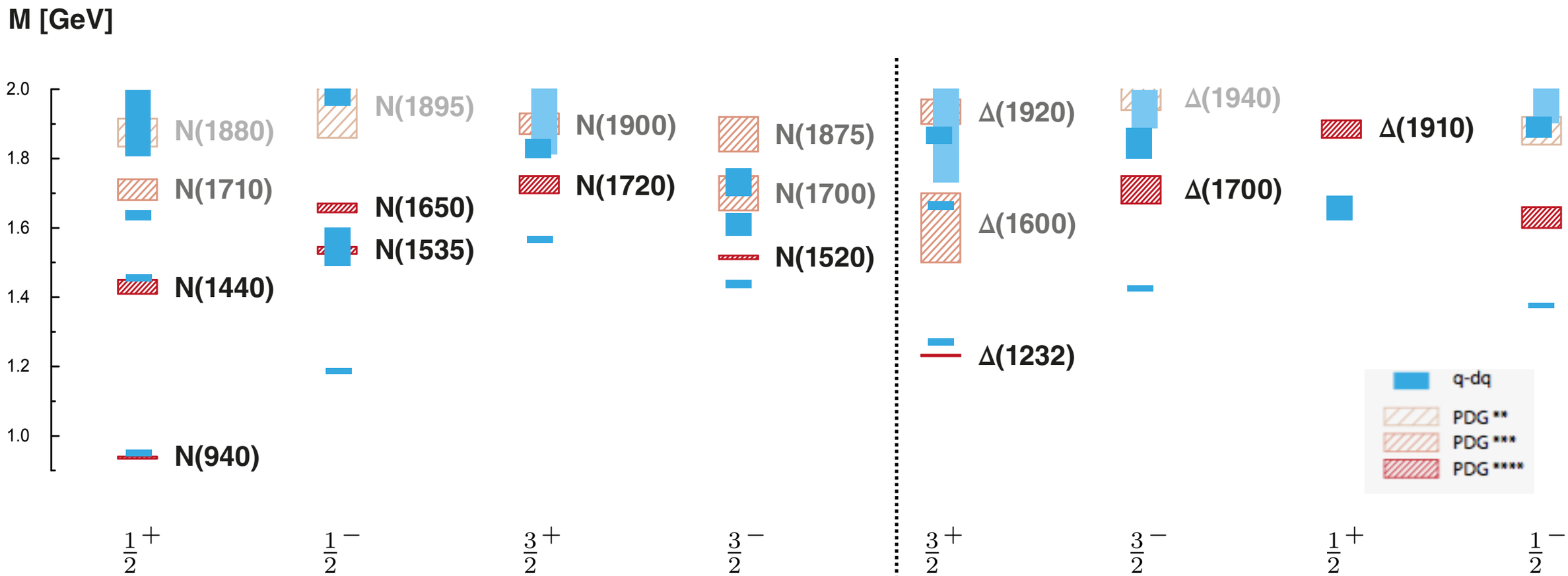


sc+av	ps+v	all	all

- nucleon and delta - channels: good results
- but: severe problems in all other channels
- artifact of rainbow-ladder: ps and v too strongly bound !

Light baryon spectrum: diquarks



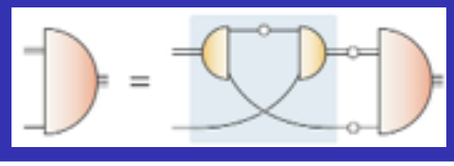


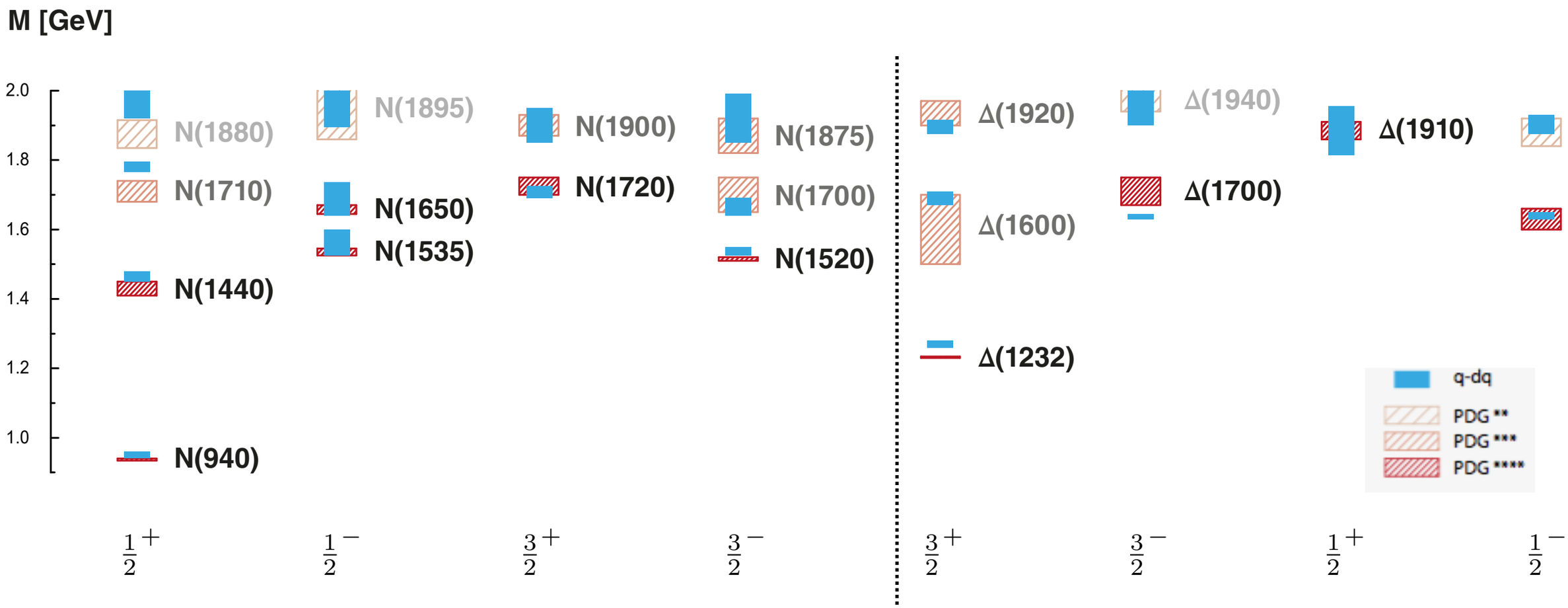
sc+av	ps+v	all	all

- nucleon and delta - channels: good results
- but: severe problems in all other channels
- artifact of rainbow-ladder: ps and v too strongly bound !

reduce binding in ps and v diquark channels, adjust to ρ-a_1-splitting

Light baryon spectrum: diquarks

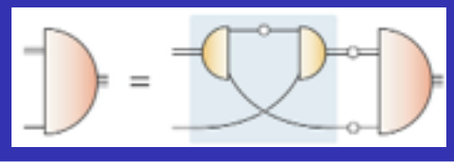


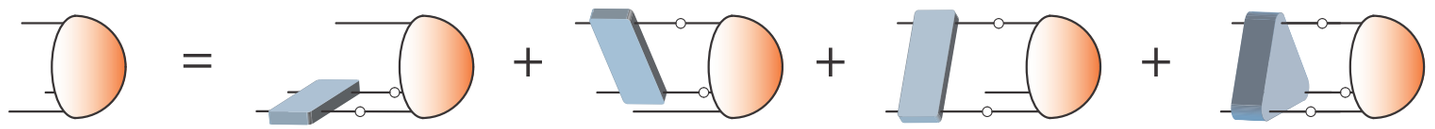


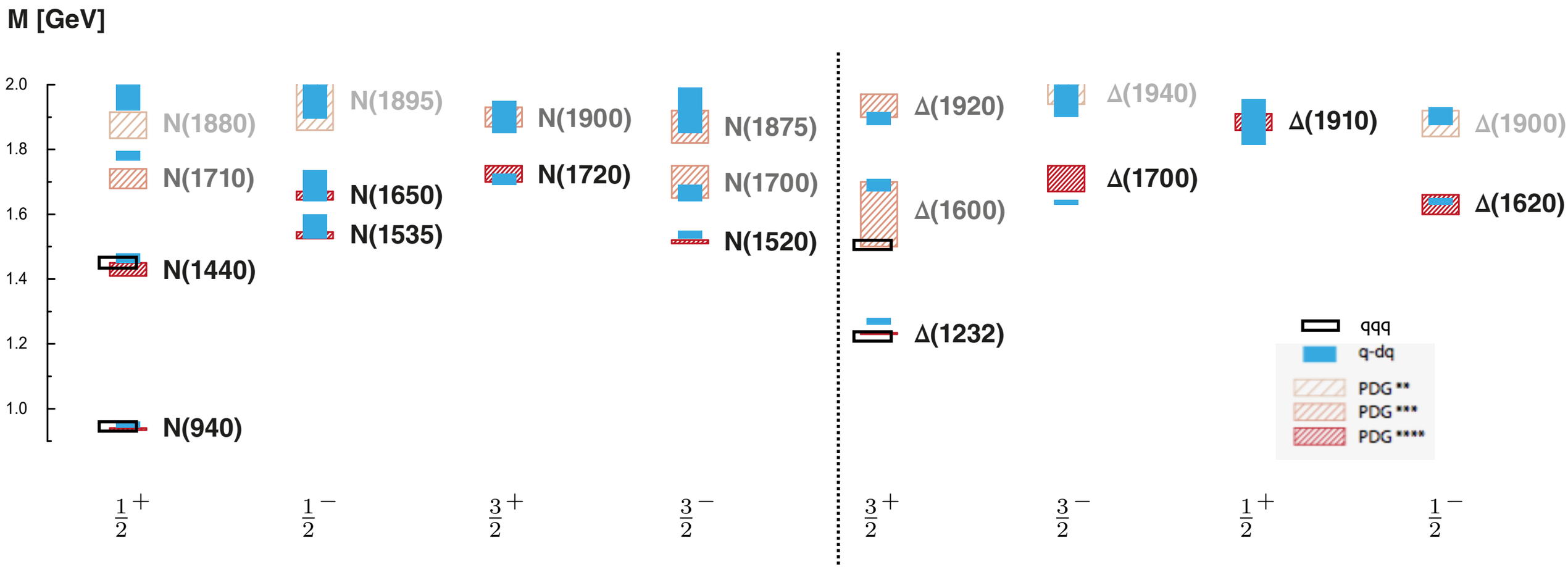
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects !)

Light baryon spectrum: diquarks







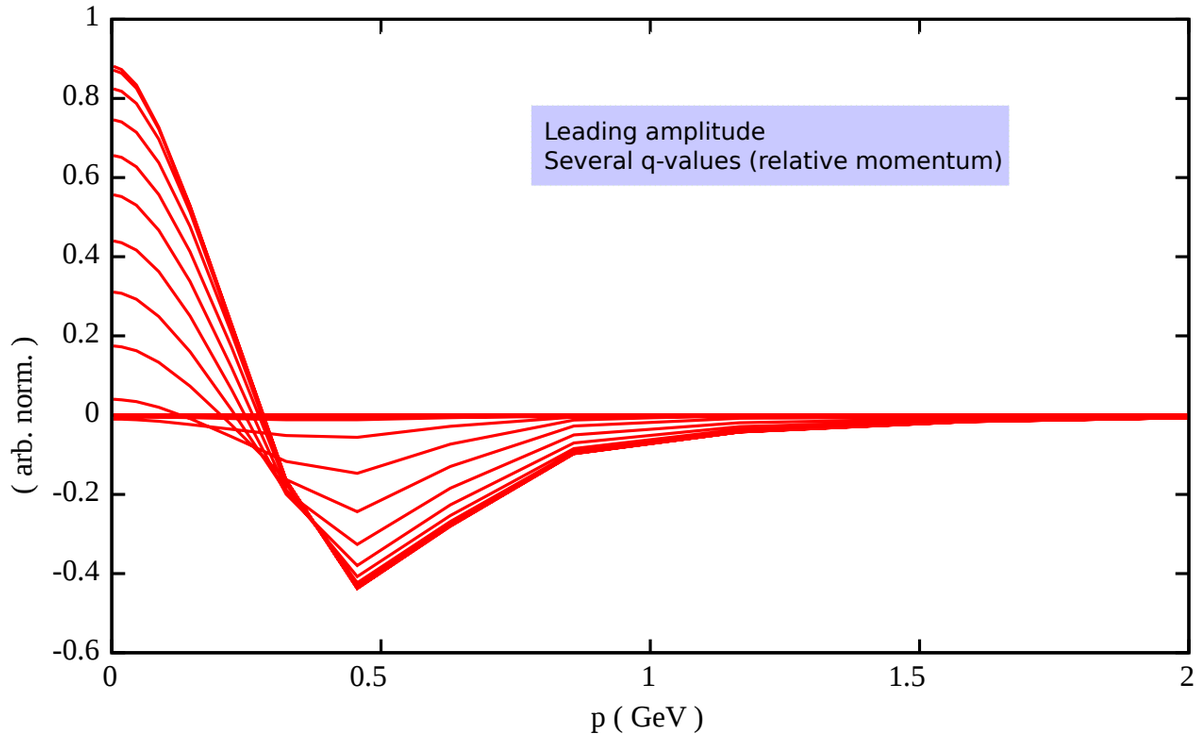
Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects !)
- three-body agrees with diquark-quark where applicable

Properties of the Roper

angular mom. decomposition

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

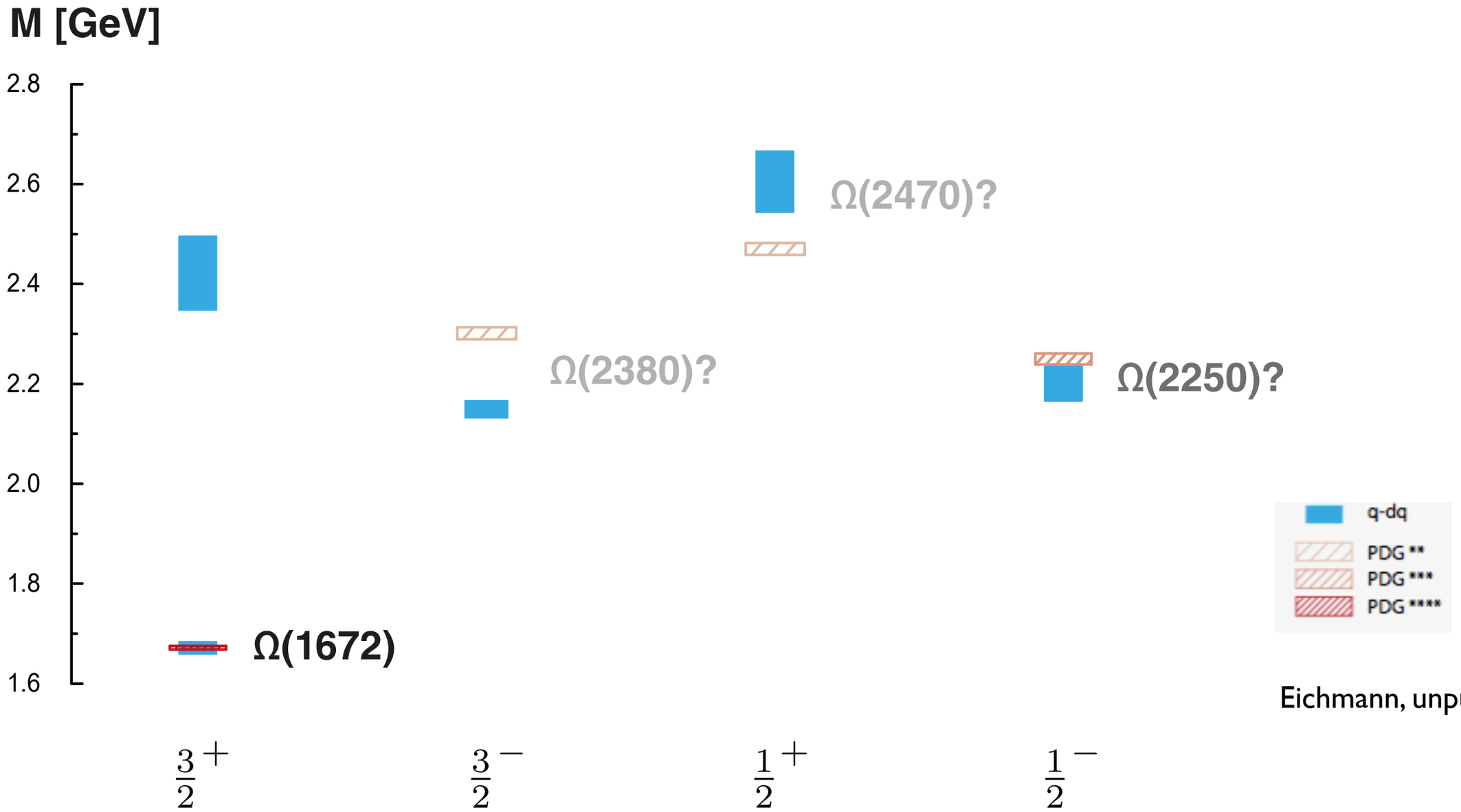


Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016)

- zero crossing of wave function: 2s-state
- every state is mixture of several partial waves !
- different internal structure of radial excitations

tension with simpler calculations ('contact interaction', 'QCD based model'):
Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17

Excited spectrum: Omega



Eichmann, unpublished

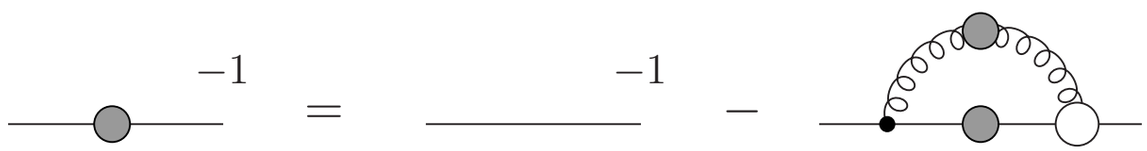
- same level ordering as quark model, but larger masses
- exp. quantum numbers ?

Ronniger, Metsch, EPJA 47 (2011) 162

assignment according to: Gamermann, Garcia-Recio, Nieves and Salcedo, PRD 84 (2011) 056017

Overview

1. Introduction - quarks, gluons and mesons



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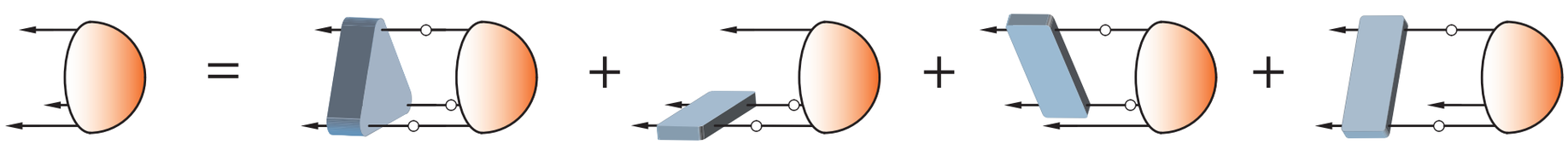
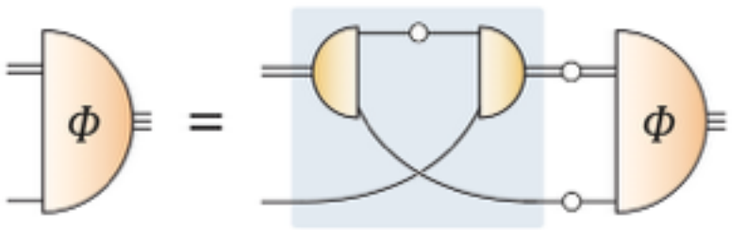
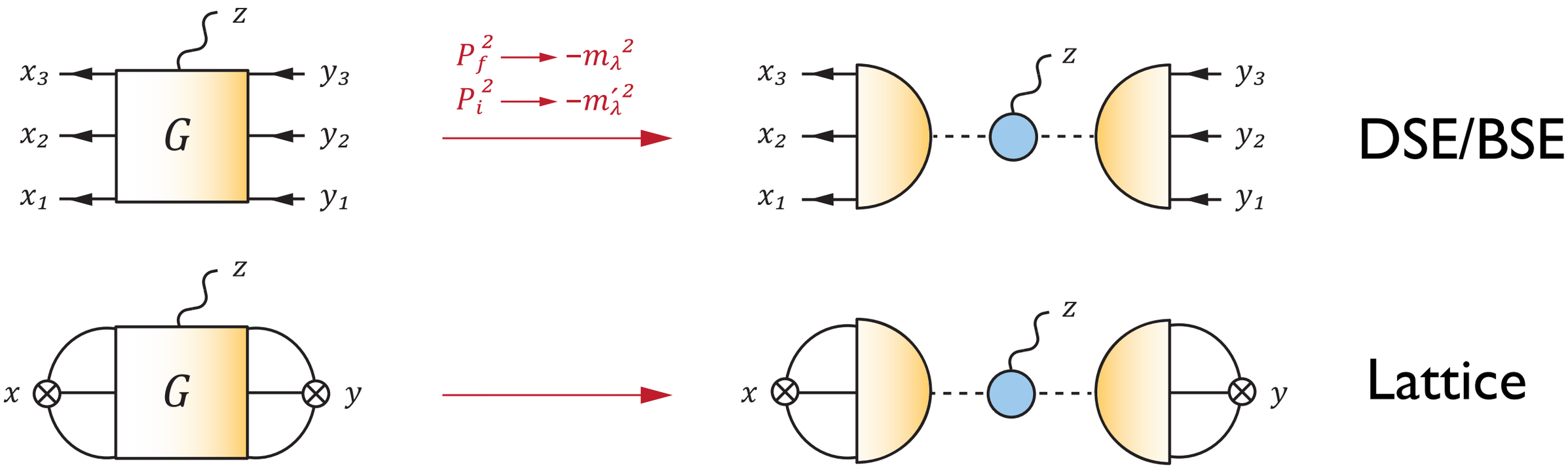


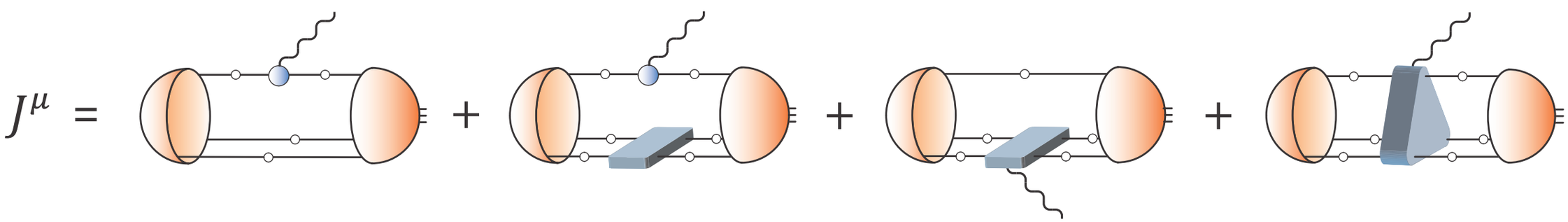


3. Pion TFF, baryon form factors and decays

Extracting form factors from correlators

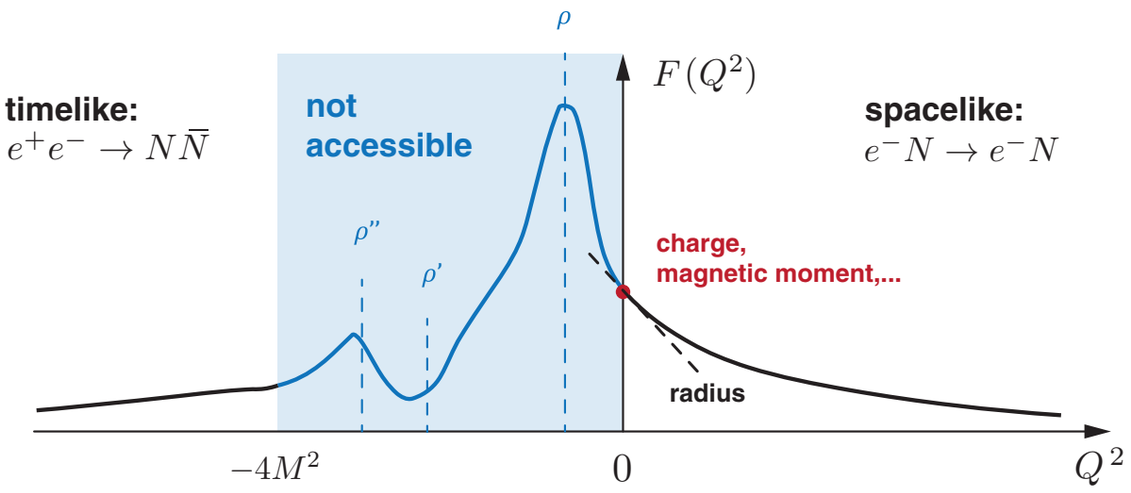


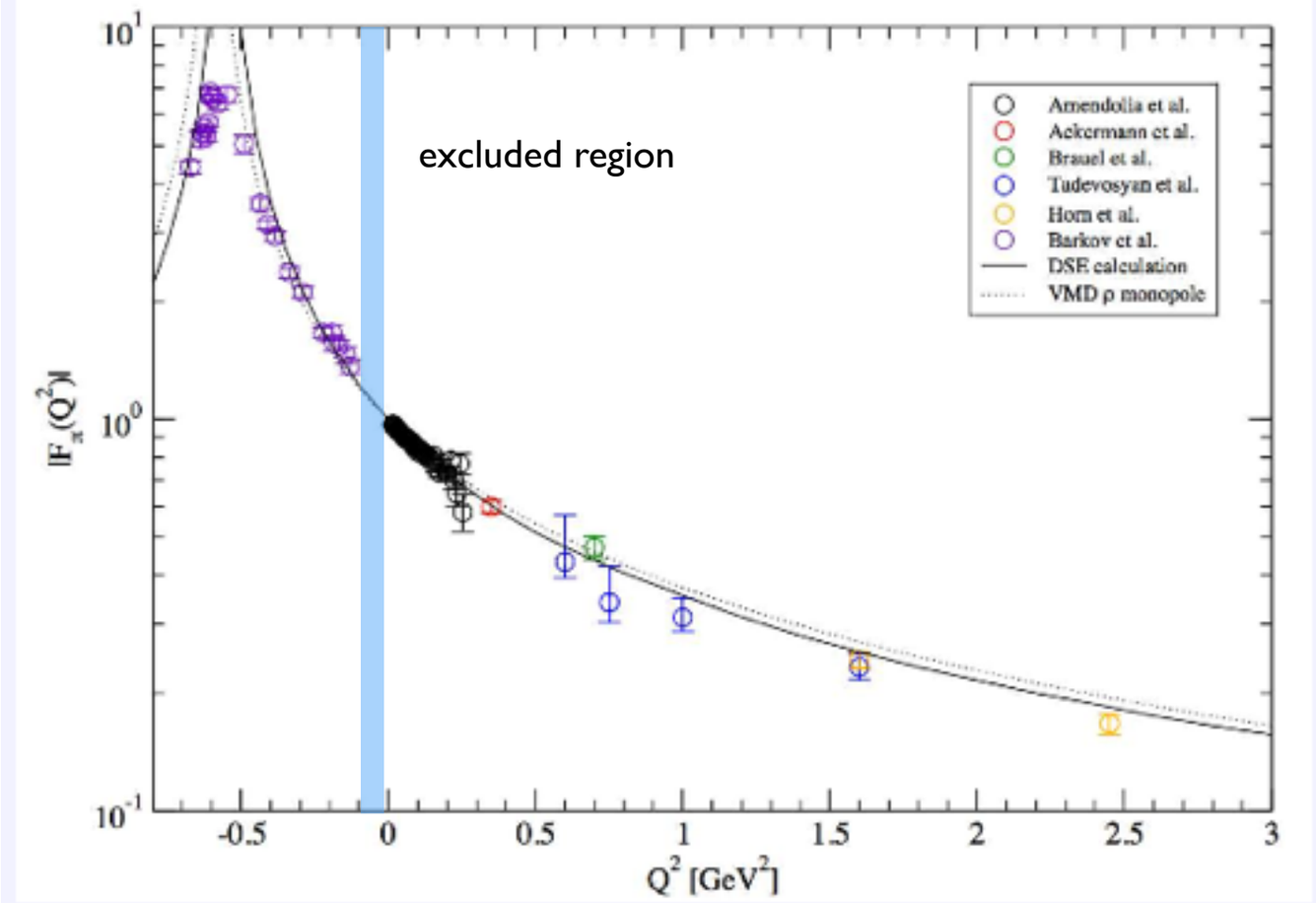
Form factor from BSEs (derived from equation of motion for G and 'gauging')



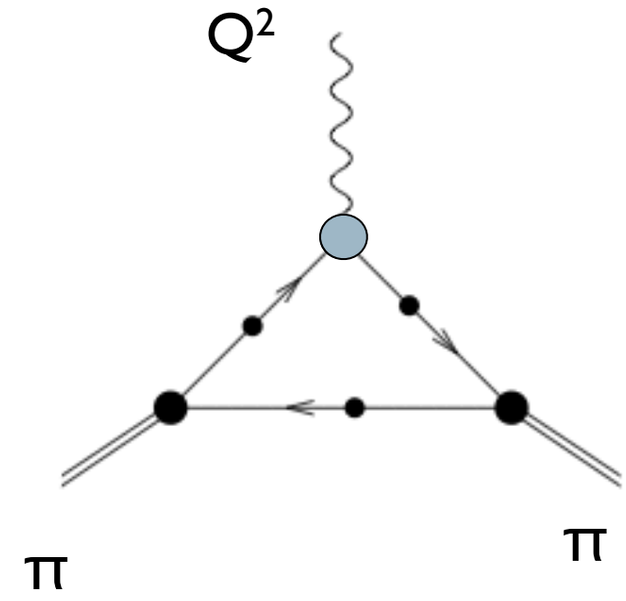
- exact equation for baryon form factors (similar for mesons)

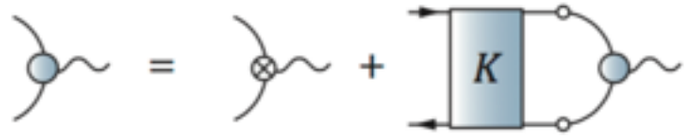
Quark-photon vertex and pion form factors





Pion form factor:

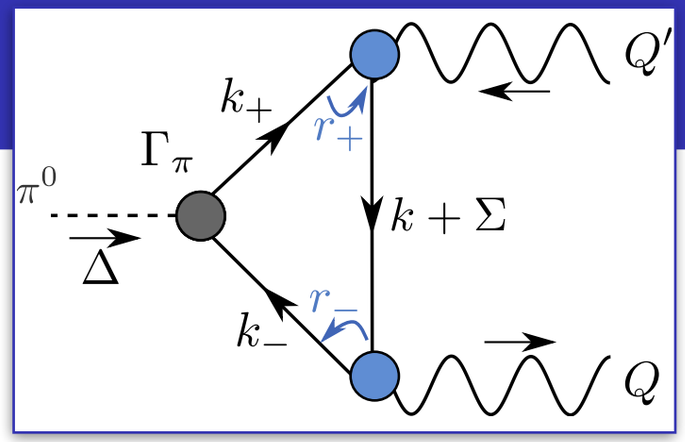




Maris, Tandy NPPS 161, 2006

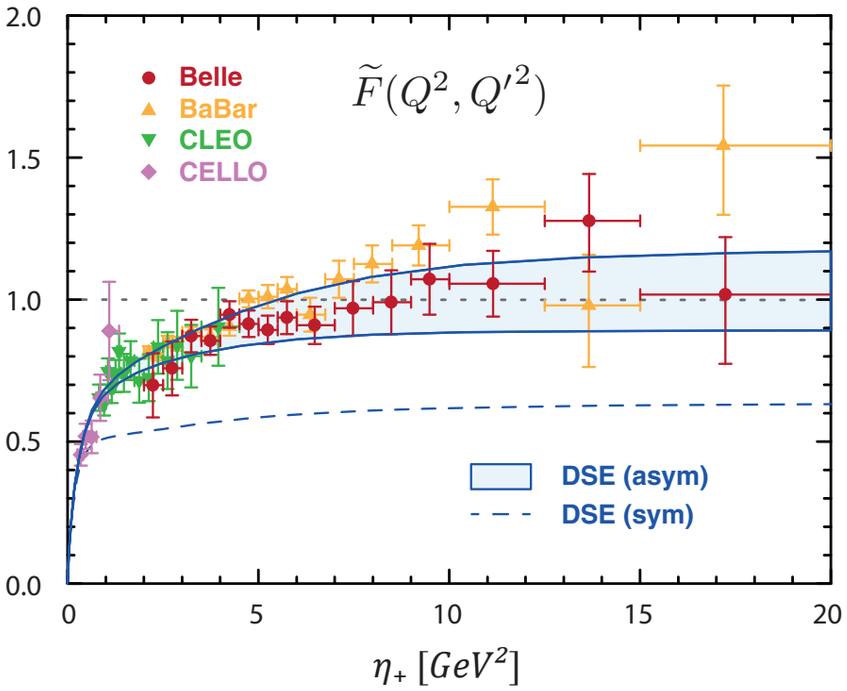
Vector meson poles dynamically generated!

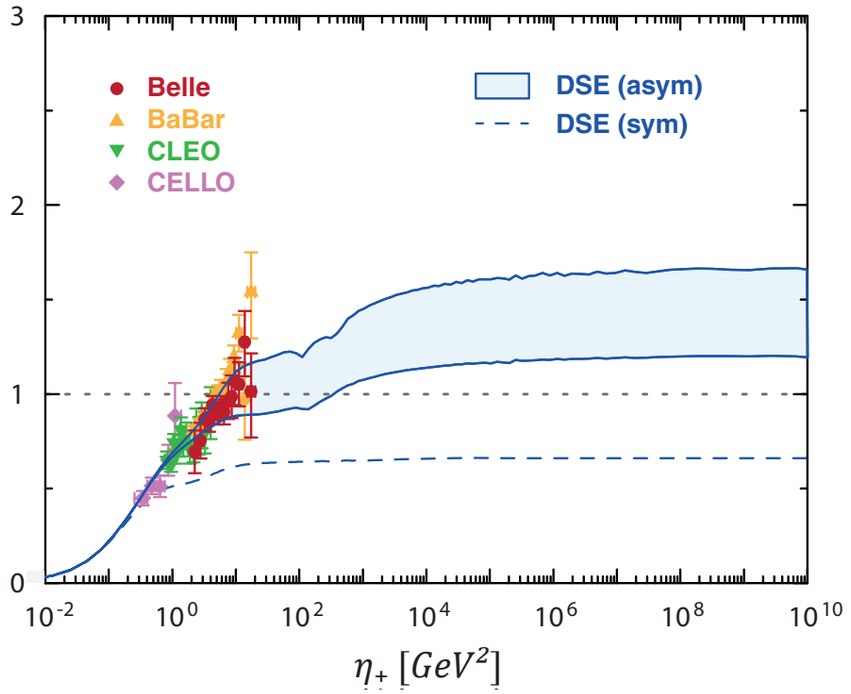
Pion transition form factor



- Large space-like momenta:

$$\eta^+ = \frac{Q^2 + Q'^2}{2}$$





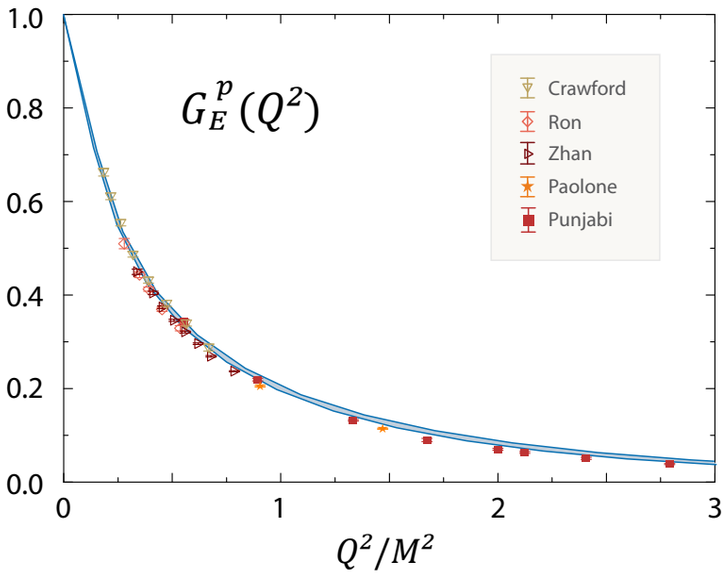
Weil, Eichmann, CF, Williams, arXiv:1704.05774

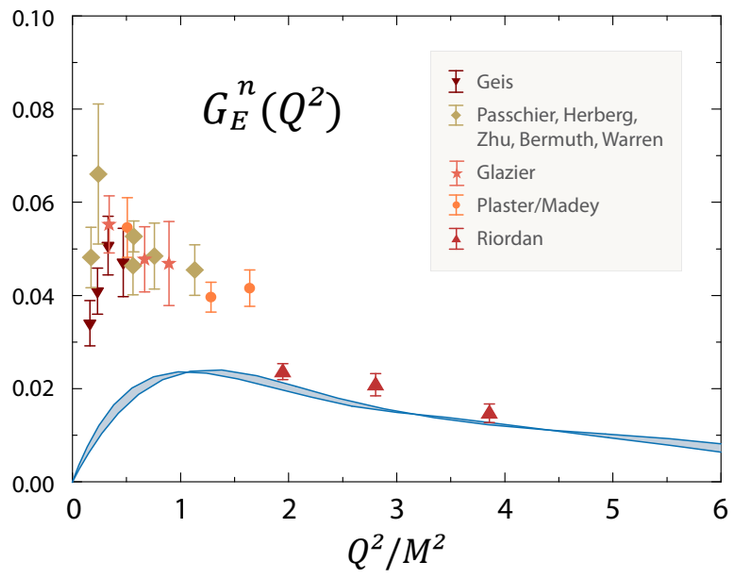
- Small time-like momenta:

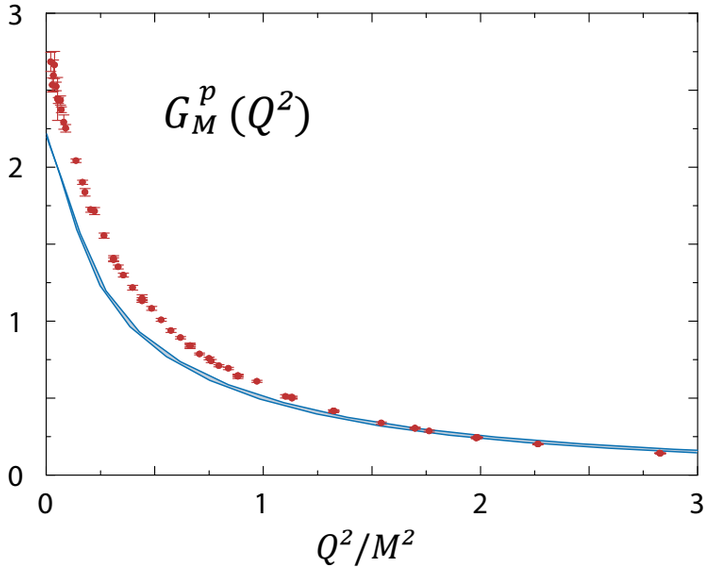
	our result	PDG
$\Gamma(\pi^0 \to e^+e^-\gamma)$	$9.11(4) \times 10^{-11}$ GeV	$9.06(18) \times 10^{-11}$ GeV
$\Gamma(\pi^0 \to e^+e^-e^+e^-)$	$2.63(1) \times 10^{-13}$ GeV	$2.58(12) \times 10^{-13}$ GeV
$\mathrm{B}(\pi^0 \to e^+e^-)$	$6.21(3) \times 10^{-8}$	$6.87(36) \times 10^{-8}$

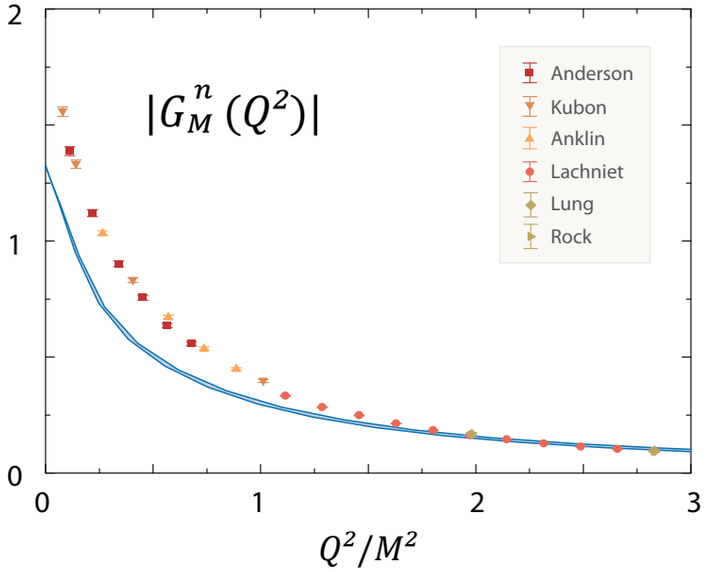
Eichmann, CF, Weil, Williams, arXiv:1704.06046

Nucleon form factors and magnetic moments

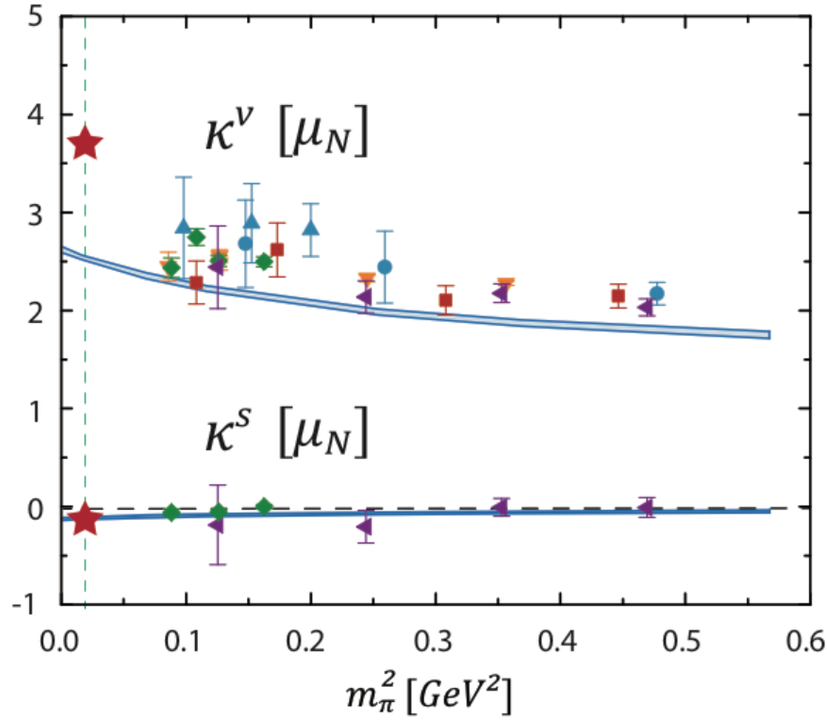






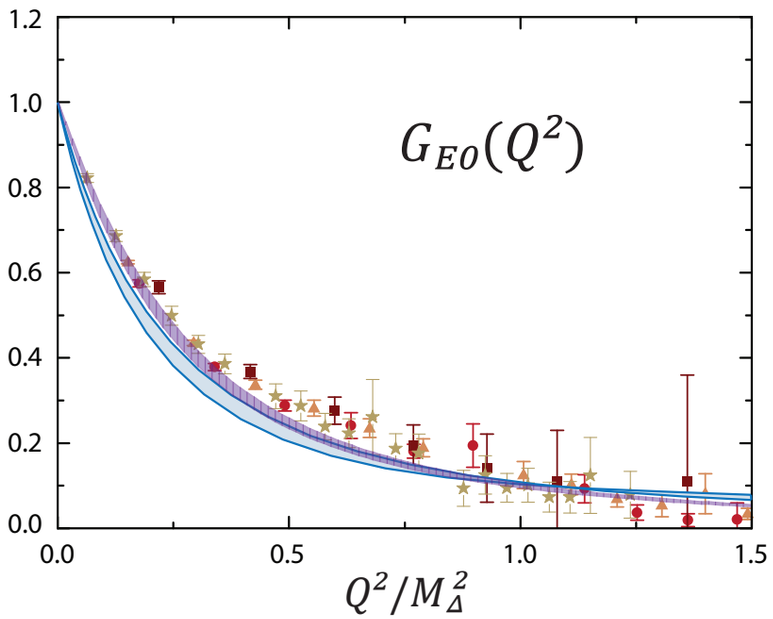


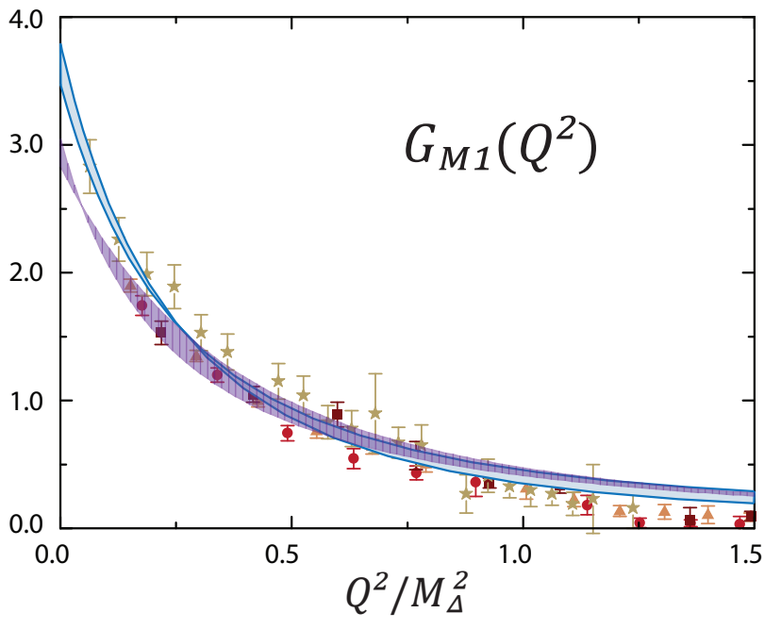
Isovector (p-n), isoscalar (p+n):

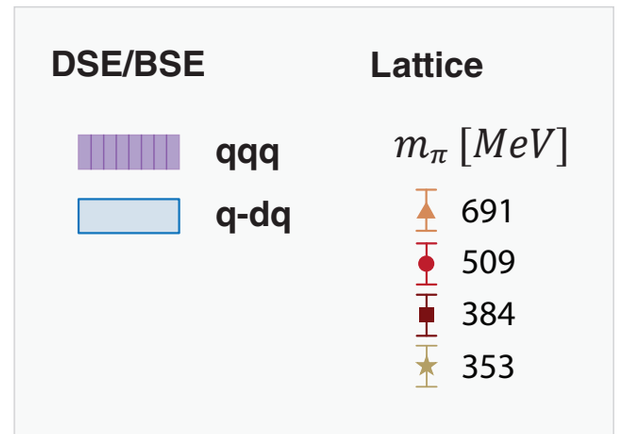


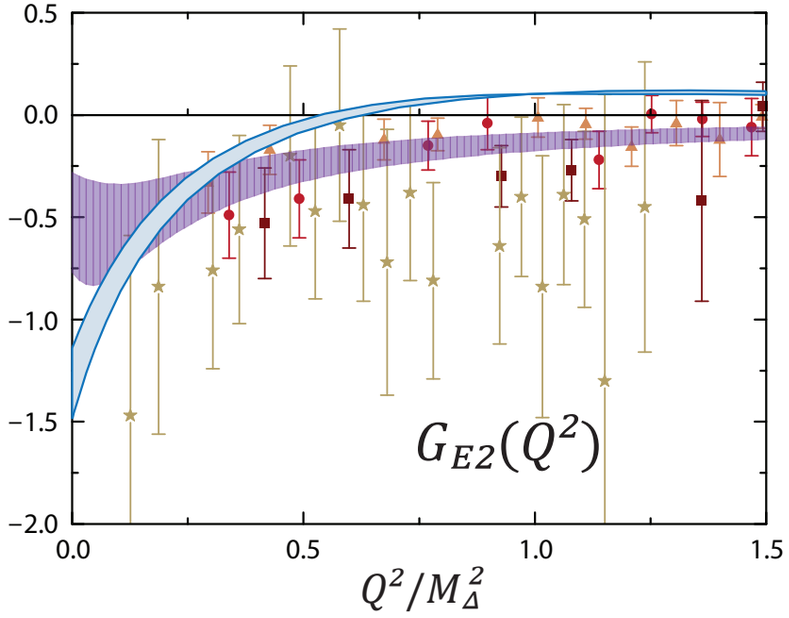
- missing pion cloud effects — Eichmann, PRD 84 (2011)
- similar for axial form factors — Eichmann and CF, EPJ A48 (2012) 9

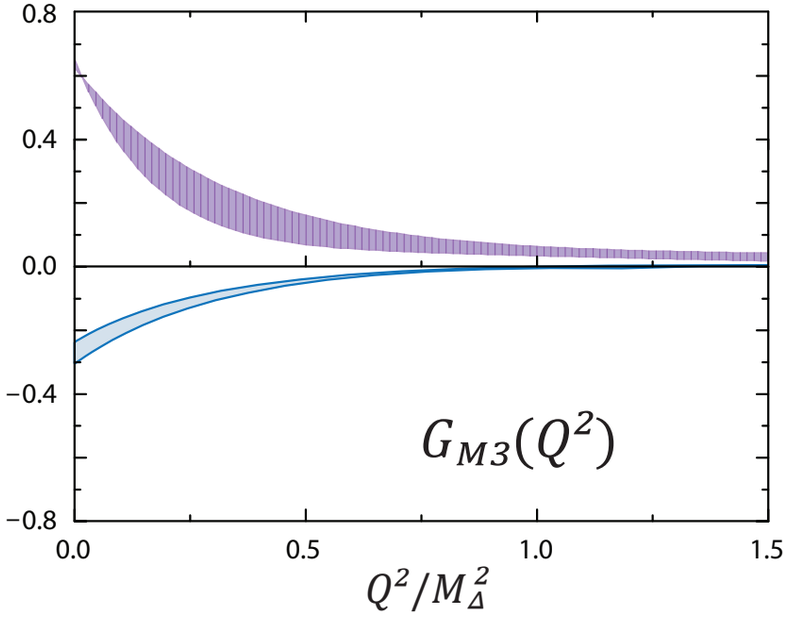
Δ-form factors

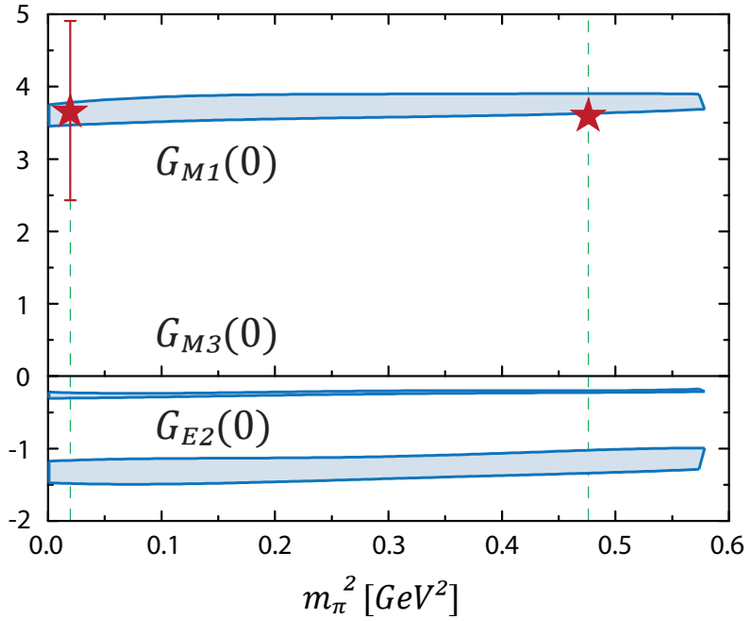








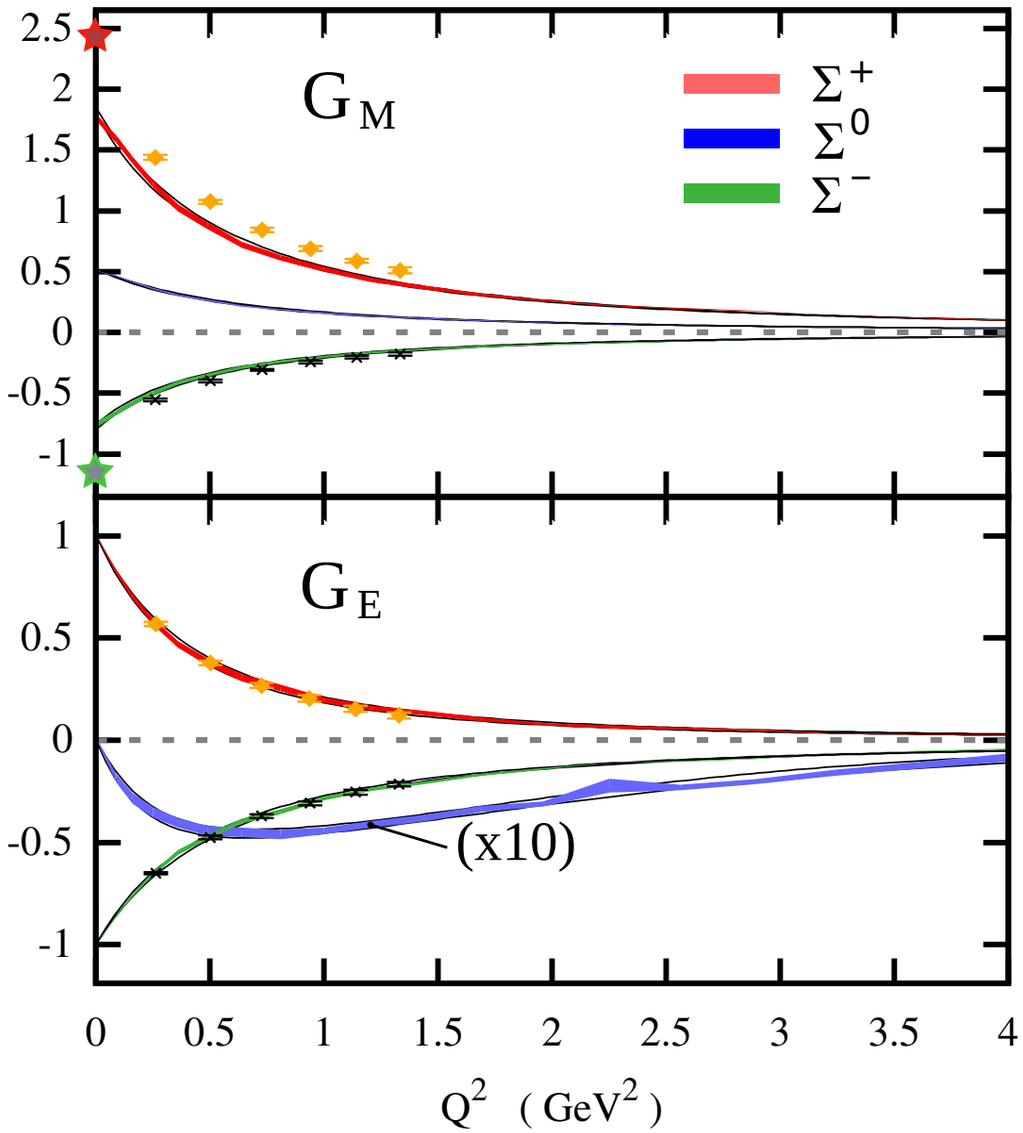


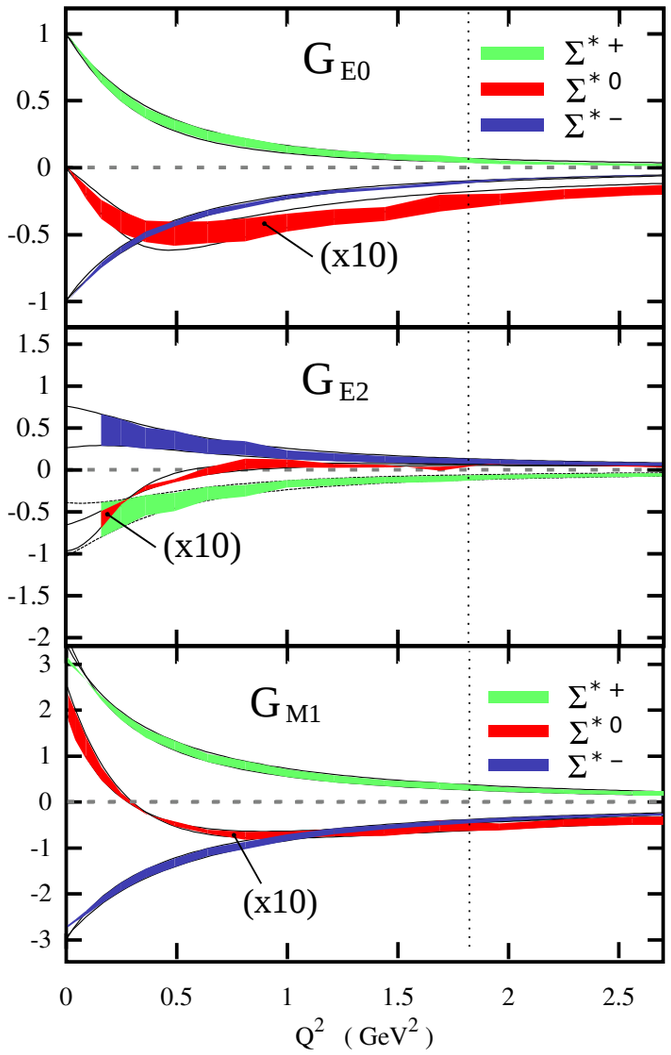


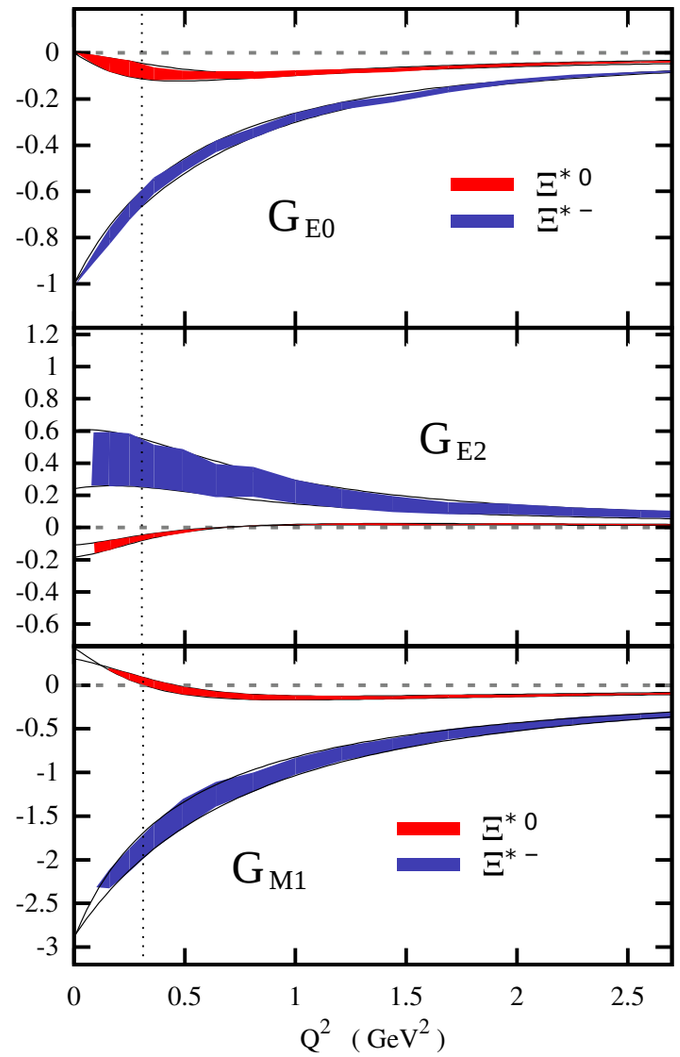
- may serve to distinguish between qqq and q-dq !

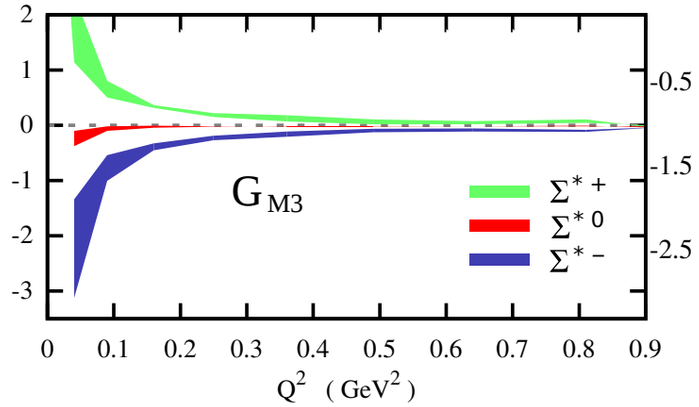
Sanchis-Alepuz, Williams, Alkofer, PRD87 (2013)
Nicmorus, Eichmann, Alkofer, PRD82 (2010)

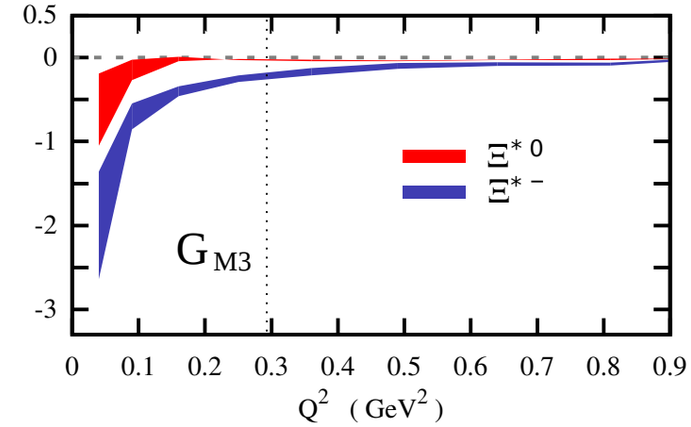
Strange form factors: octet and decuplet









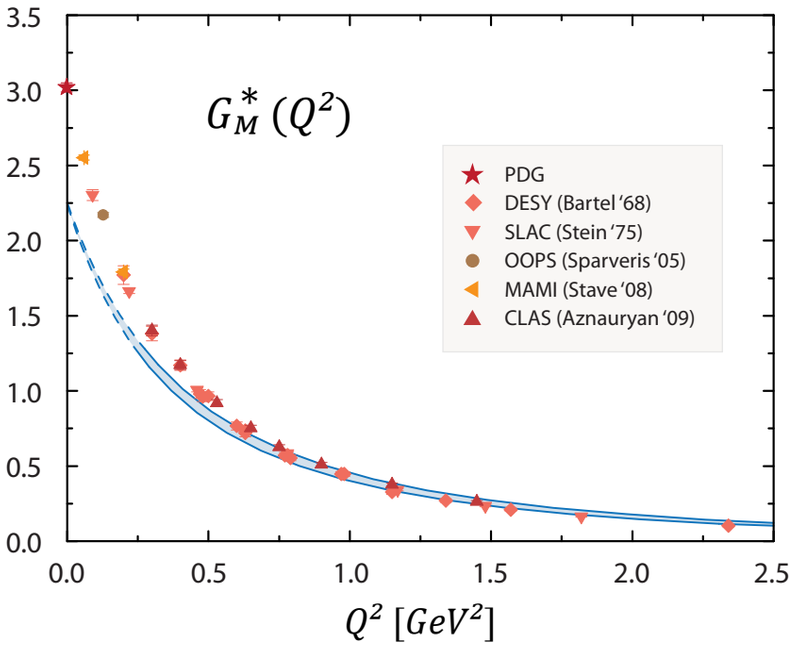


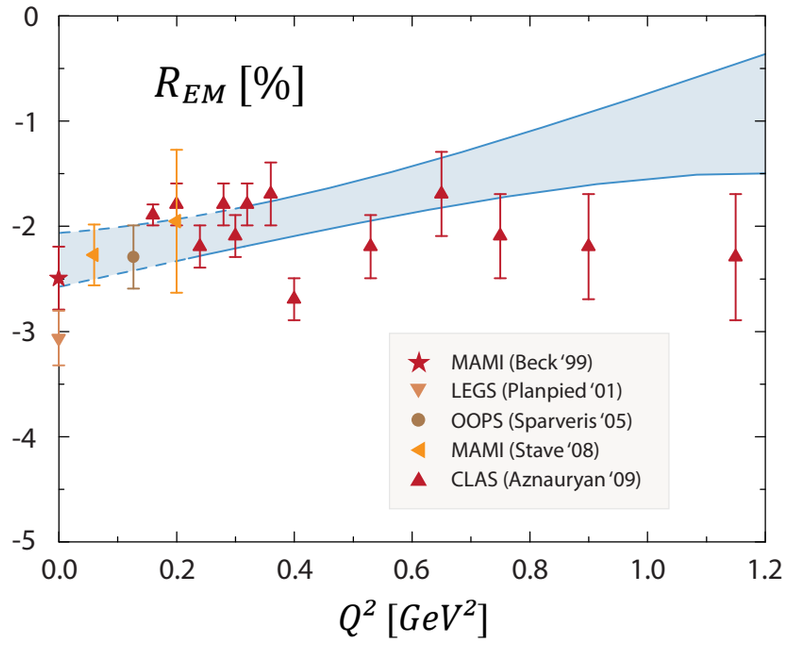
- Deculplet: prediction

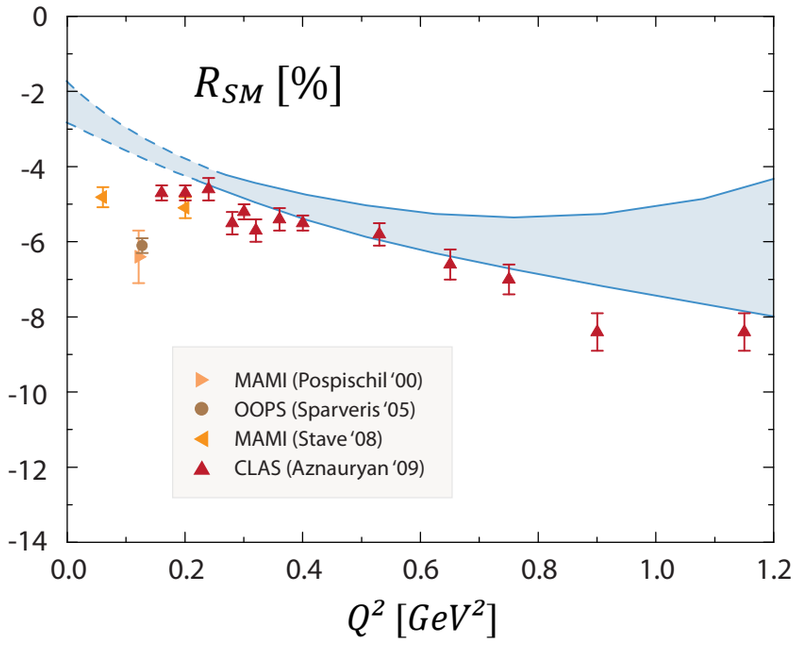
DSE: Sanchis-Alepuz, CF, EPJA 52 (2016)
Lattice: Shanahan et al, PRD 89 (2014), PRD 90 (2014)

Transition form factor: NΔγ

$$R_{EM} = -\frac{G_E^*}{G_M^*}, \qquad R_{SM} = -\frac{|\vec{Q}|}{2M_\Delta}\frac{G_C^*}{G_M^*}$$



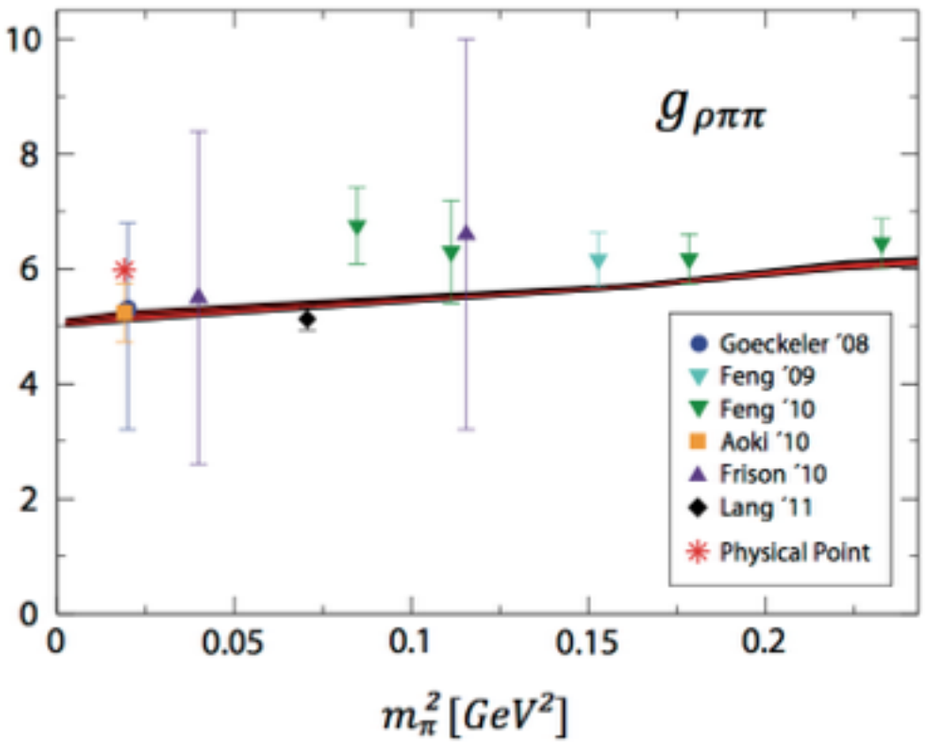


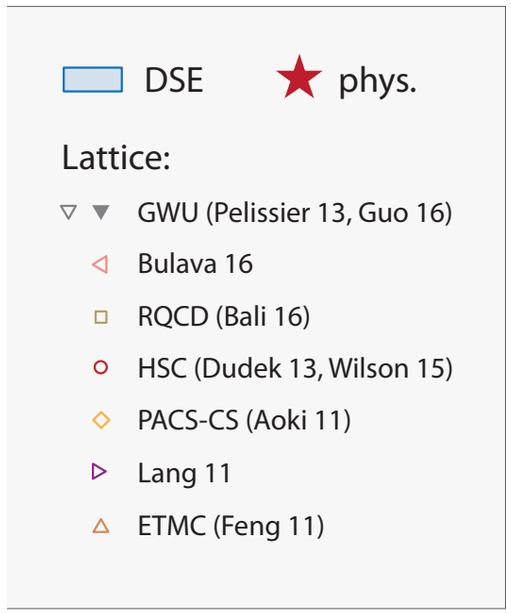


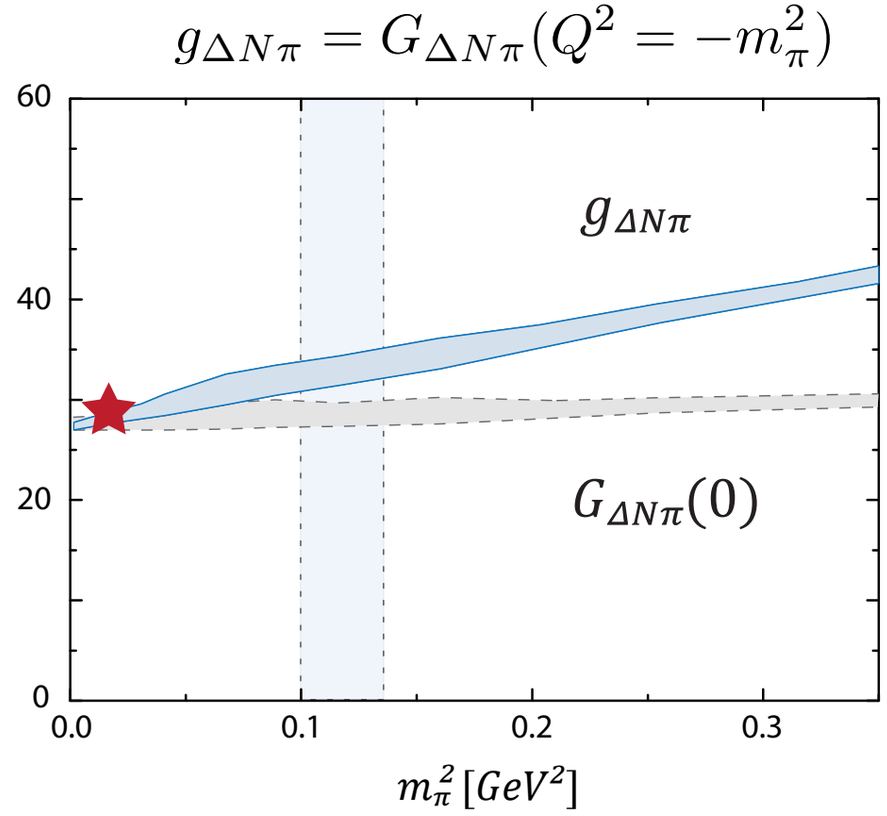
- R_{EM} highly dominated by p-waves !

Eichmann, Nicmorus, PRD 87 (2012)

Decays: ρππ and ΔNπ

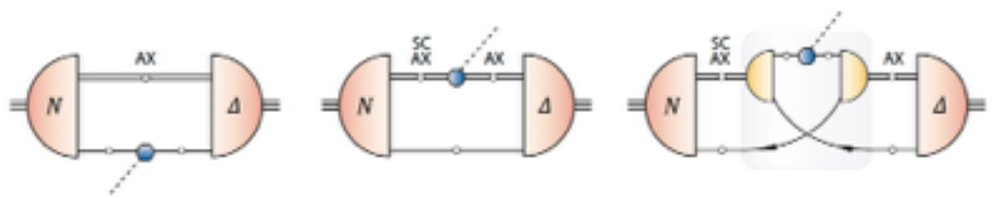




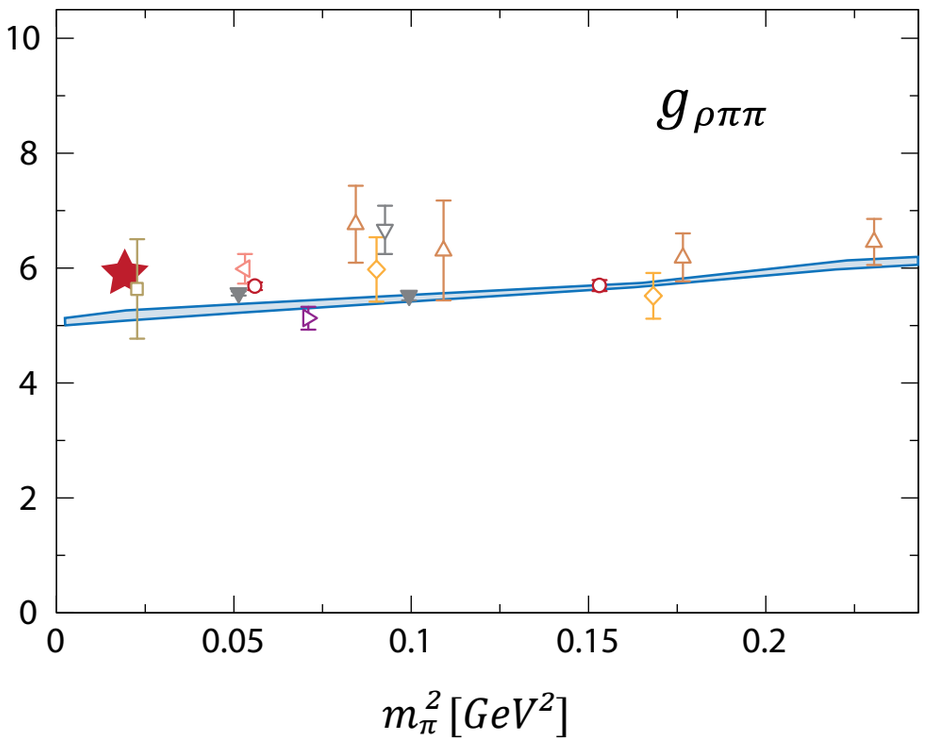


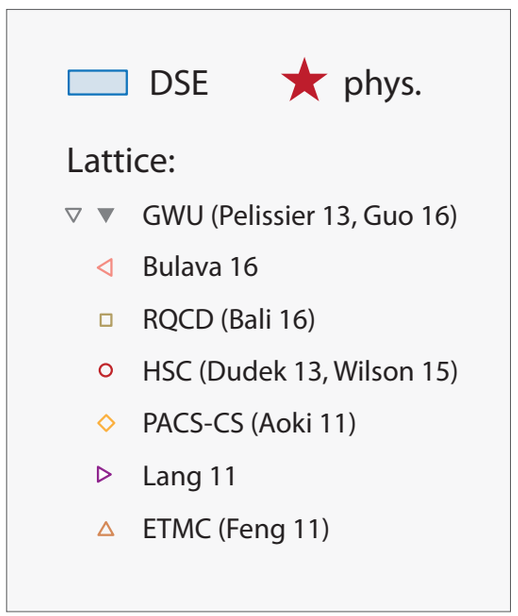
Mader, Eichmann, Blank, Krassnigg PRD84 (2011)

- Decay constants can be calculated in rainbow-ladder
  (although bound states have no width)
- Good agreement with lattice and experiment

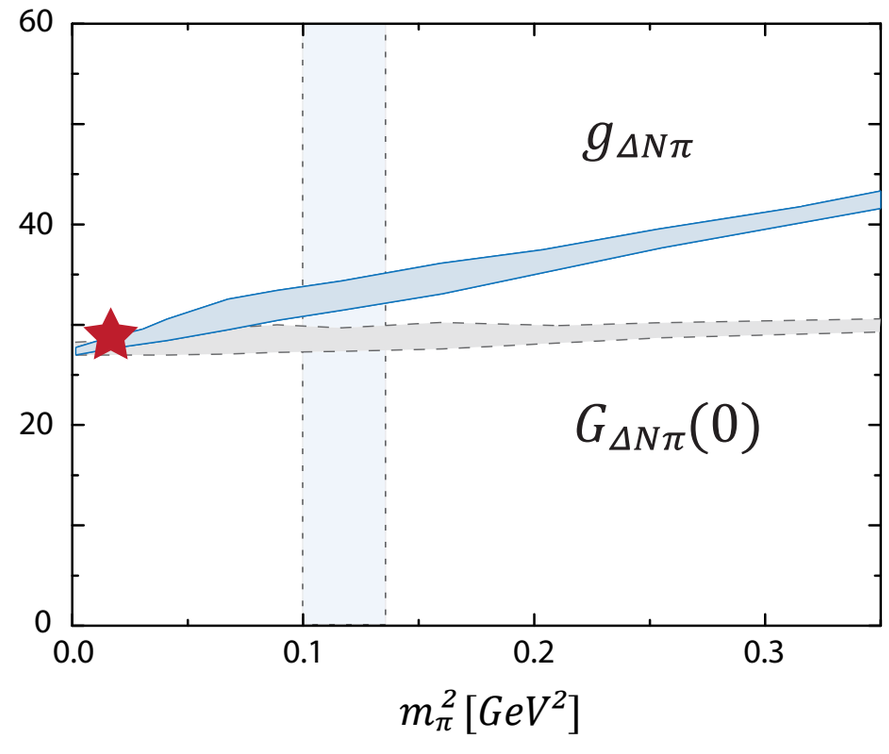


Decays: ρππ and ΔNπ





$g_{\Delta N\pi} = G_{\Delta N\pi}(Q^2 = -m_\pi^2)$



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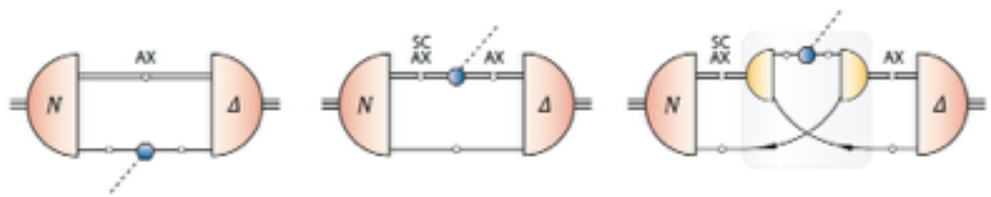


Summary and outlook

- Goal: get control over microscopic QCD forces
- Light baryon spectrum in good agreement with experiment
  - No tightly bound diquarks, correct level ordering
- Baryon form factors determined in rainbow-ladder
  - missing pion cloud effects, good results at intermediate Q^2
- pion-TFF: connecting space- and time-like FF

Outlook

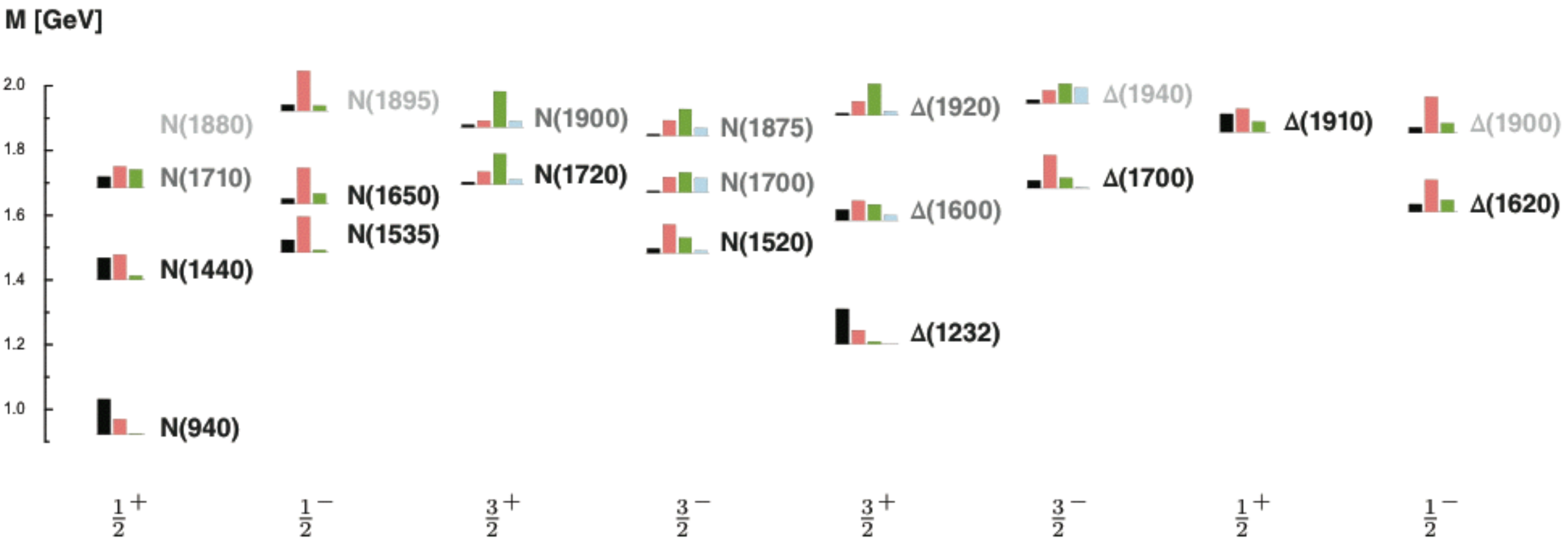
- QCD forces beyond rainbow ladder

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

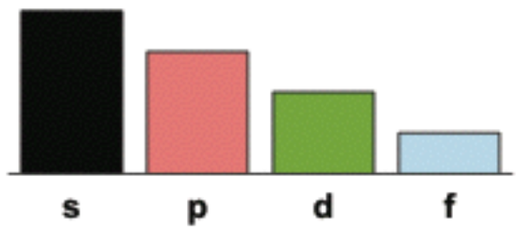
- Details of strange spectrum
- Transition form factors of strange baryons

Backup Slides

Partial wave content

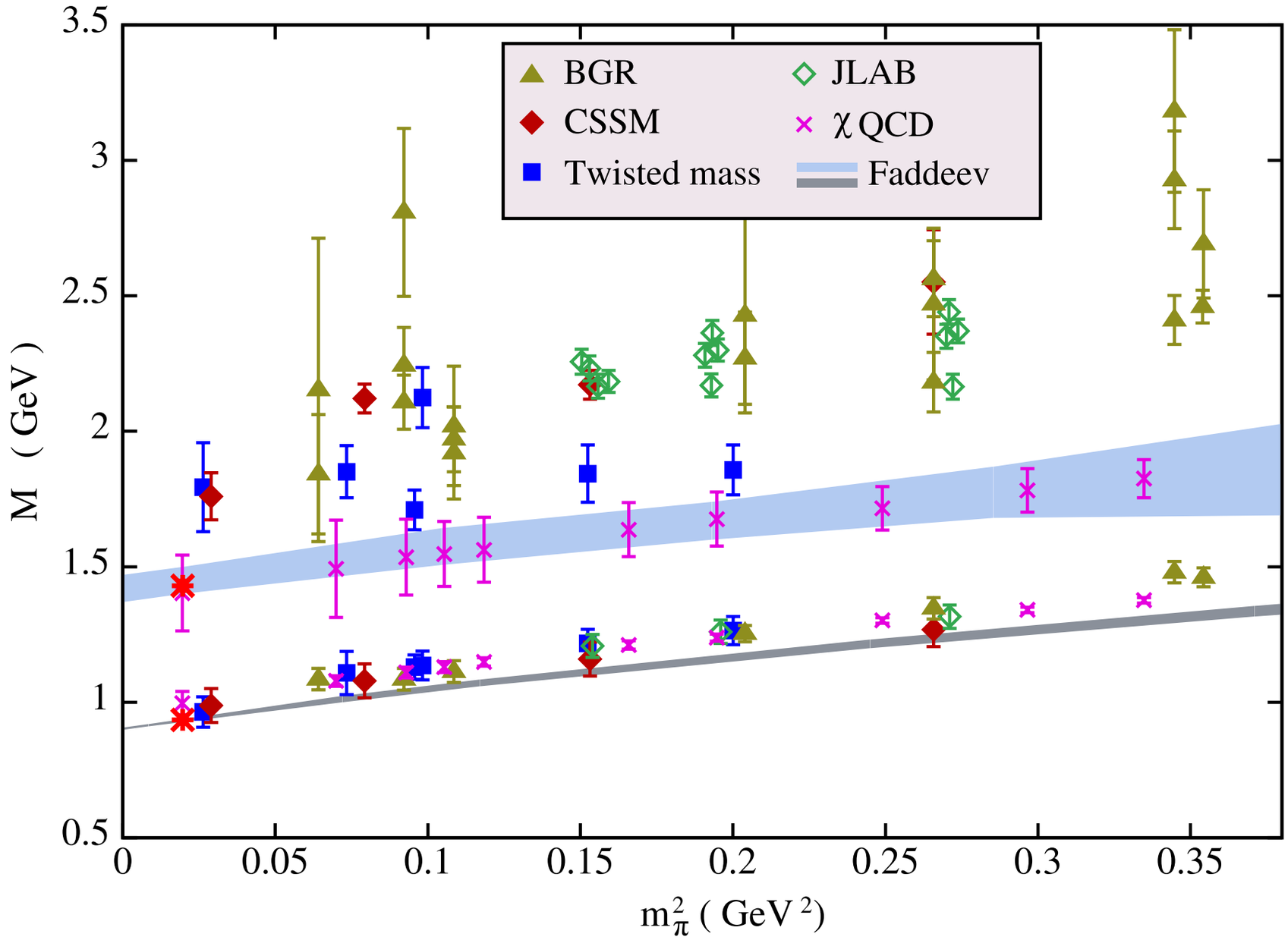


Partial-wave content:



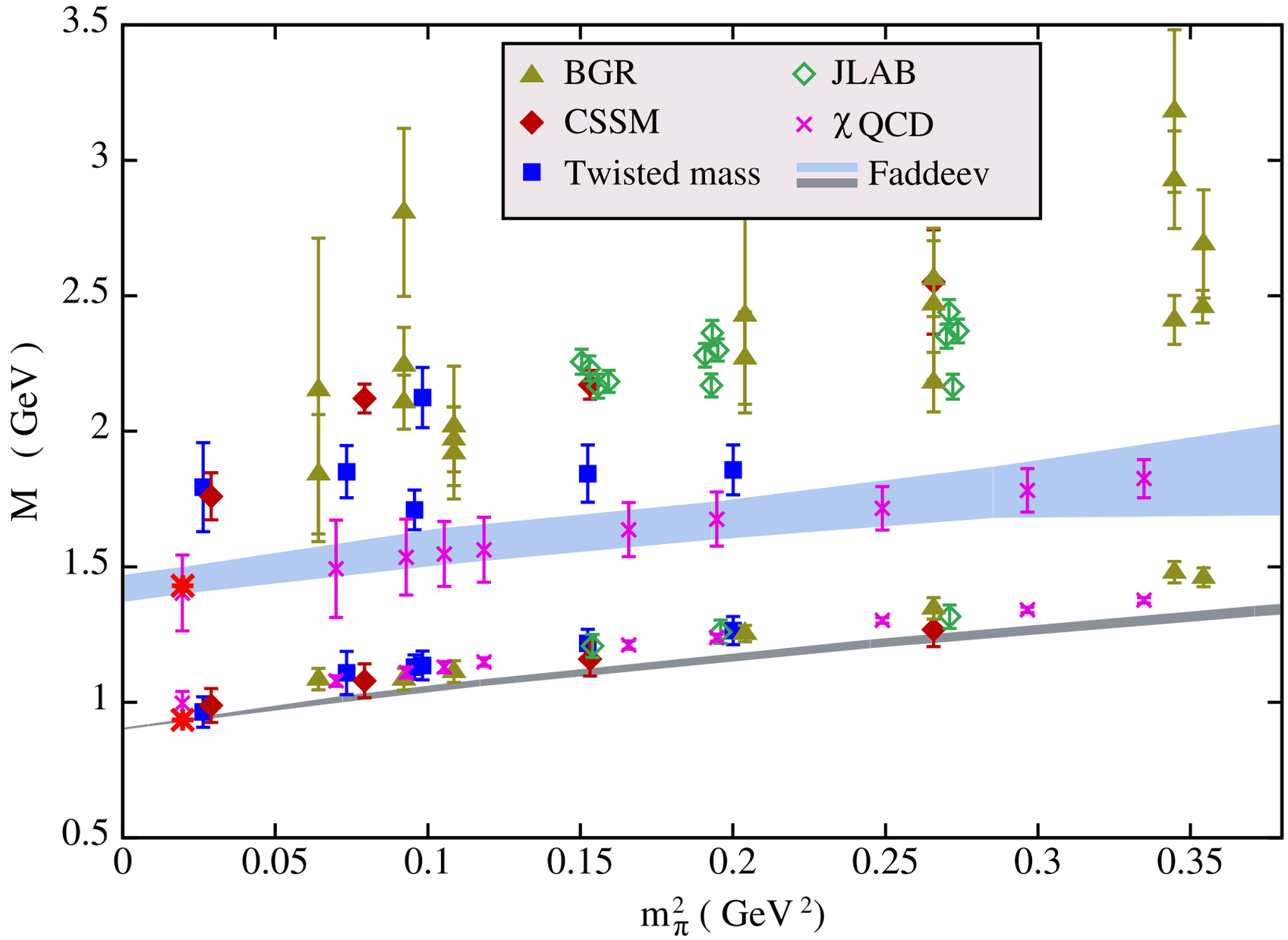
- N and Δ ground states dominated by **s waves,** negative-parity states typically by **p waves** (as expected)
- But 'quark-model forbidden' contributions are always present, e.g. **Roper: dominated by p waves ⇒ relativity is important!**

Mass evolution



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016)

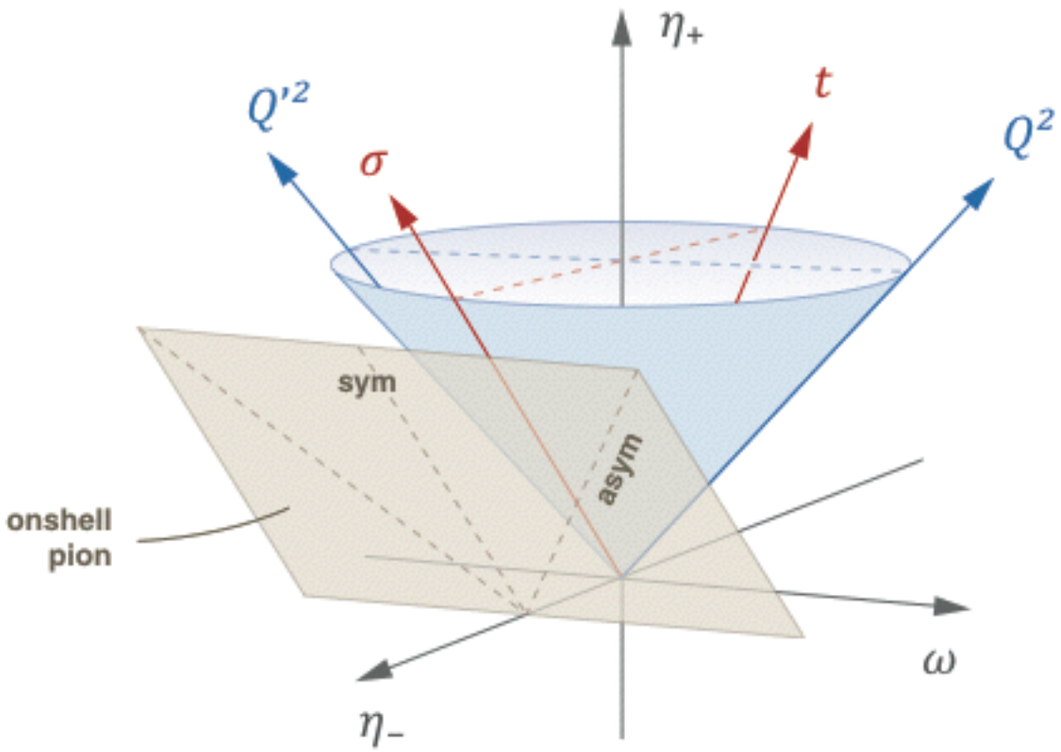
Mass evolution



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)
Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91 (2016)

- Mass evolution as expected for three-body state…

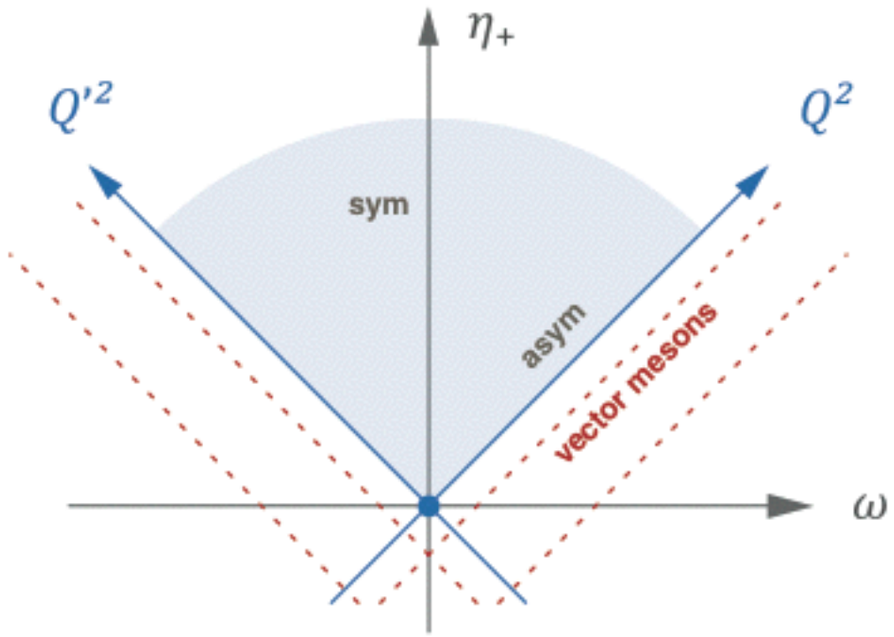
pion TFF - general idea

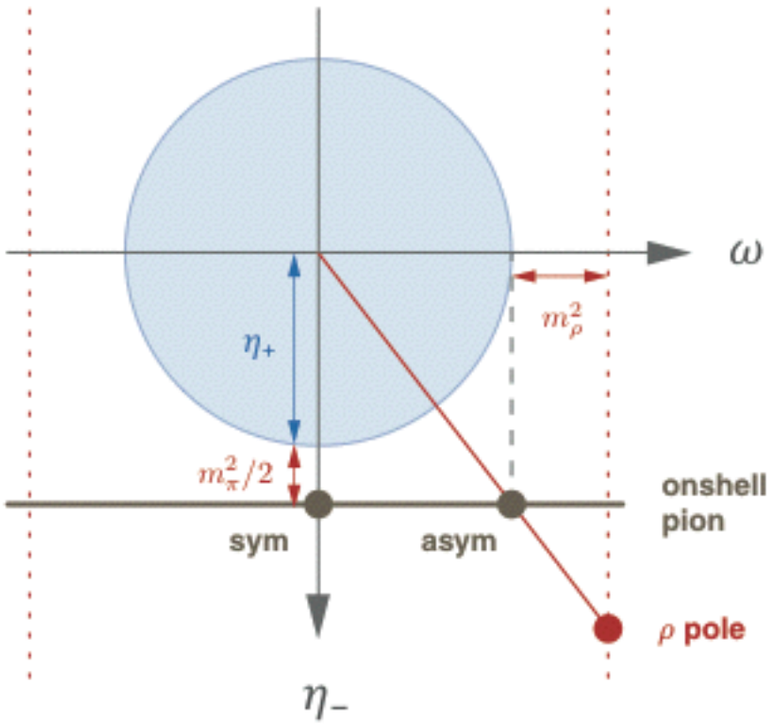


$$\eta_+ = \frac{Q^2+Q'^2}{2}$$

$$\omega = \frac{Q^2-Q'^2}{2}$$

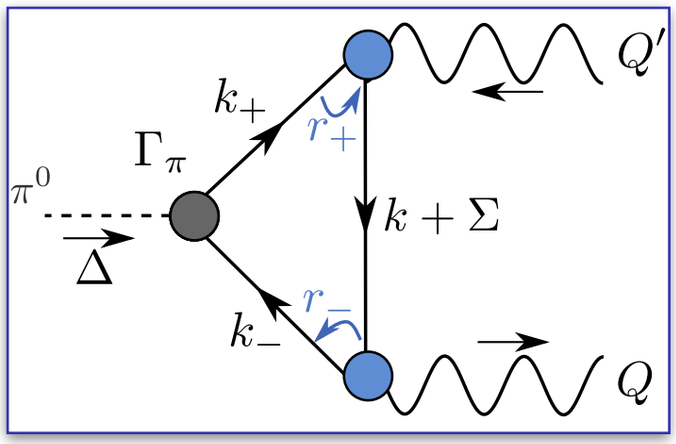
$$\eta_- = Q \cdot Q'$$



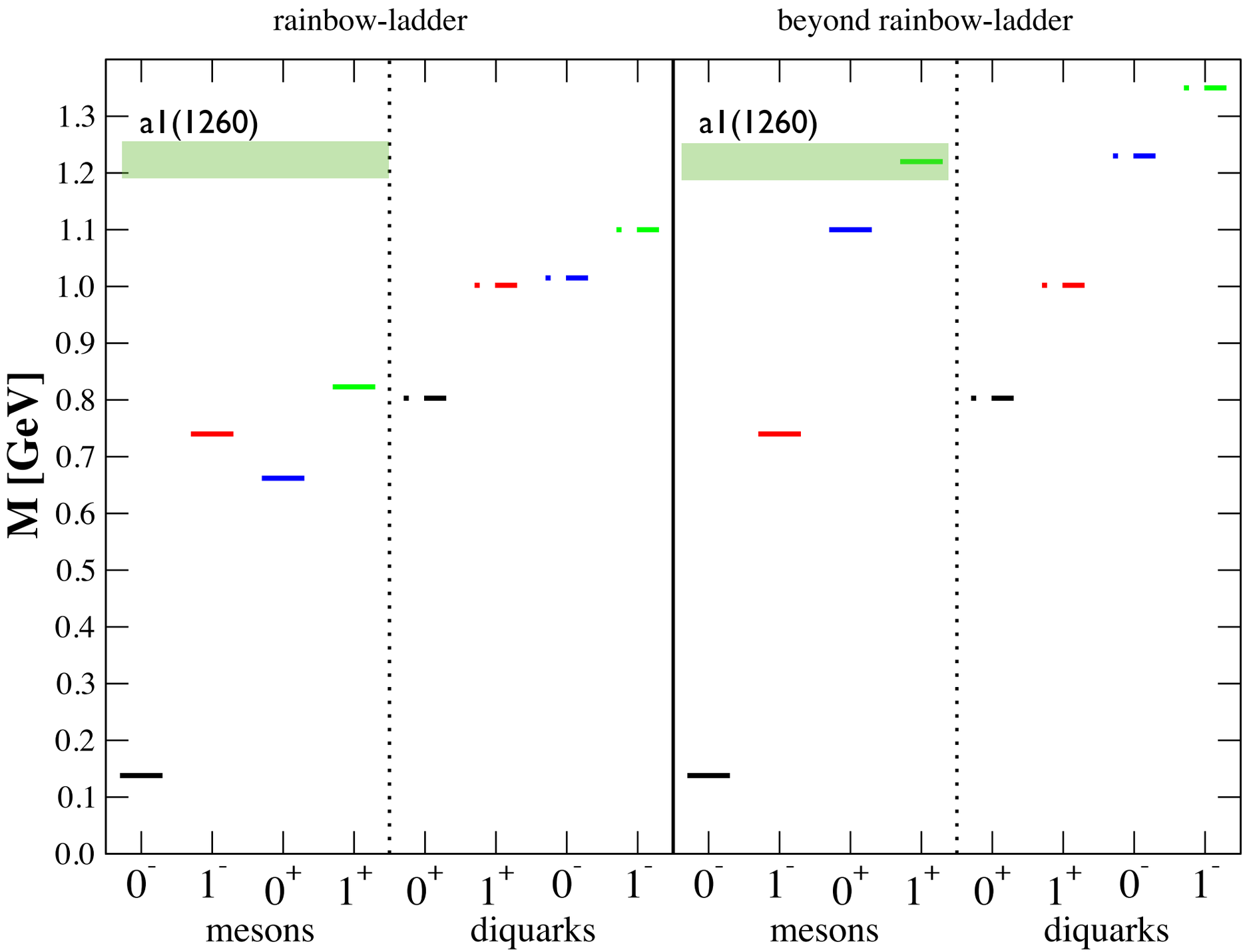


Idea:

- calculate FF inside cone
- interpolate to physical plane using VM pole as constraint
- can be done for arbitrary Q^2



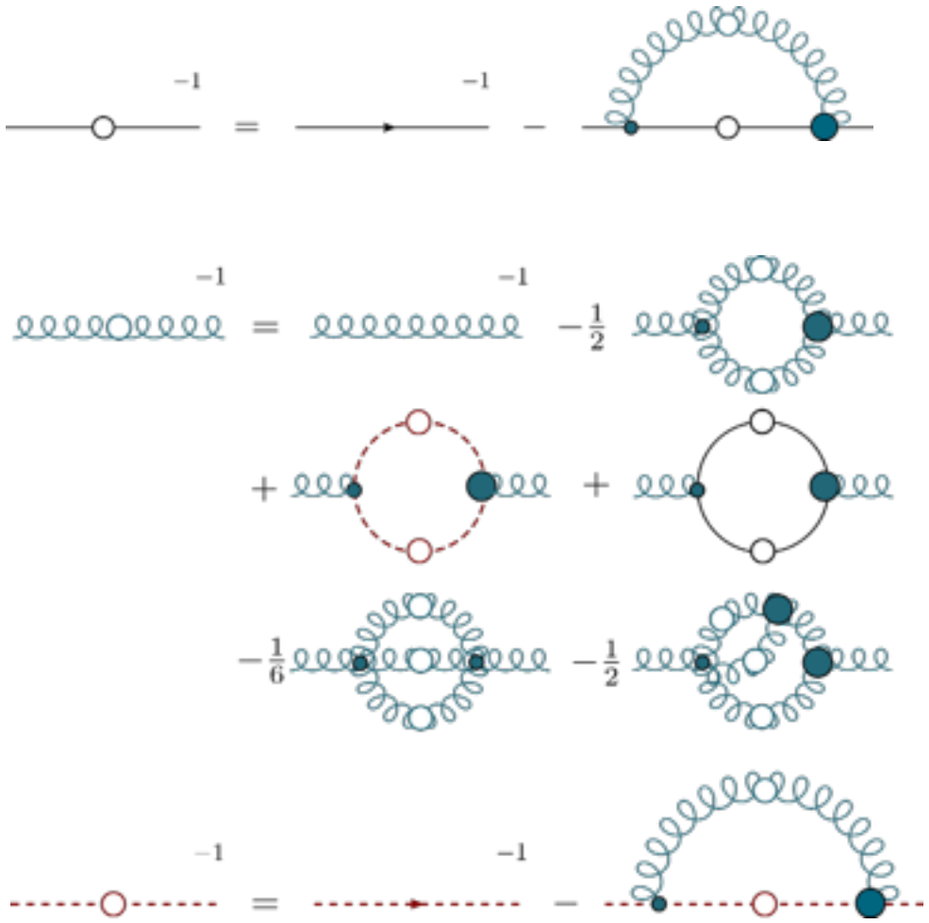
Diquarks with modified rainbow-ladder



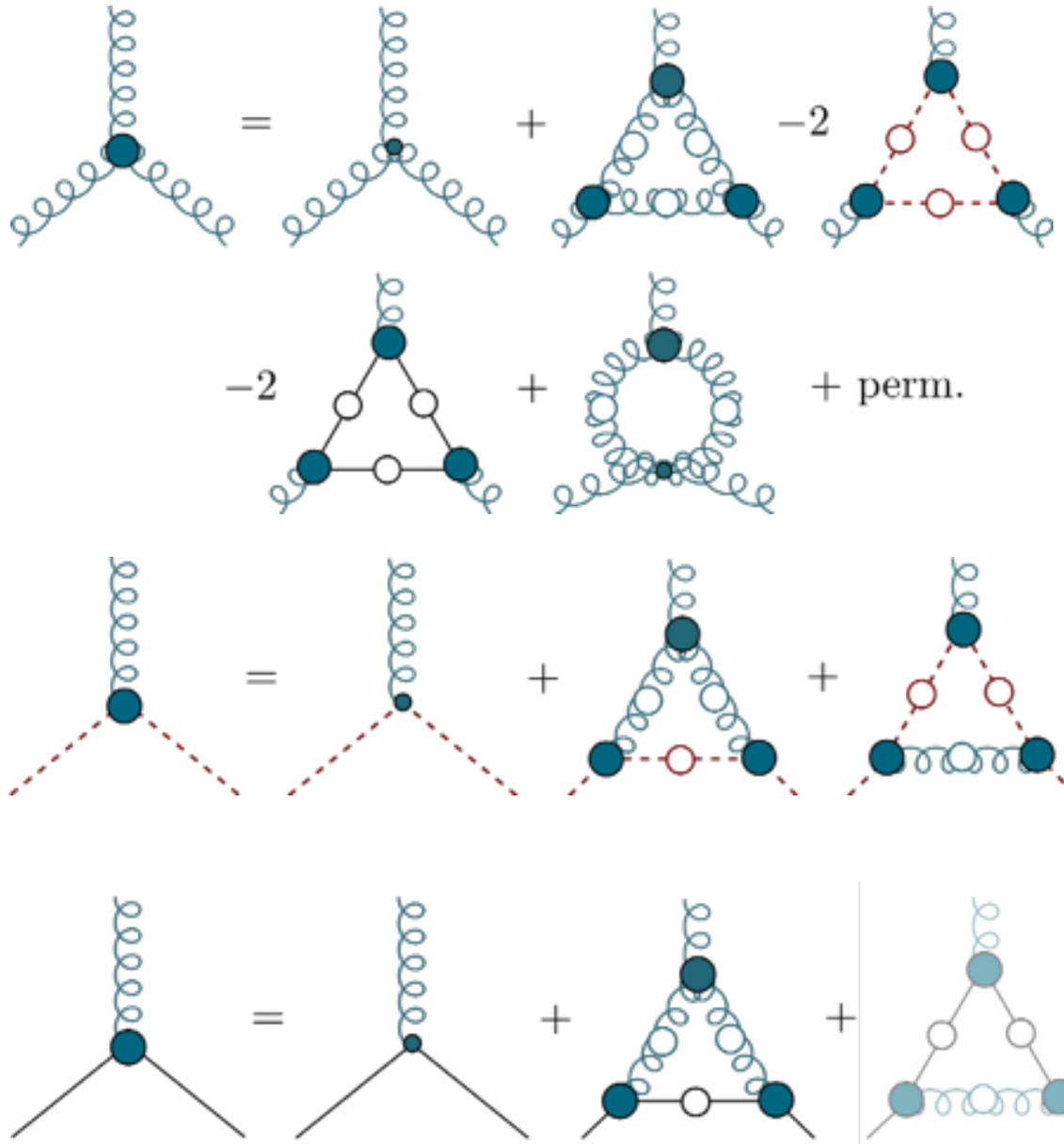
- α multiplied with 0.35 in 'bad' channels

3PI-truncation

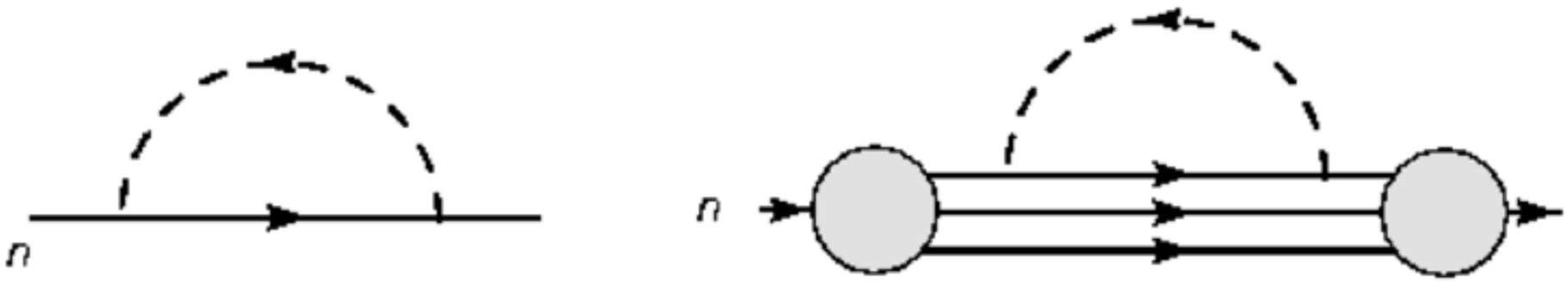
propagators



vertices

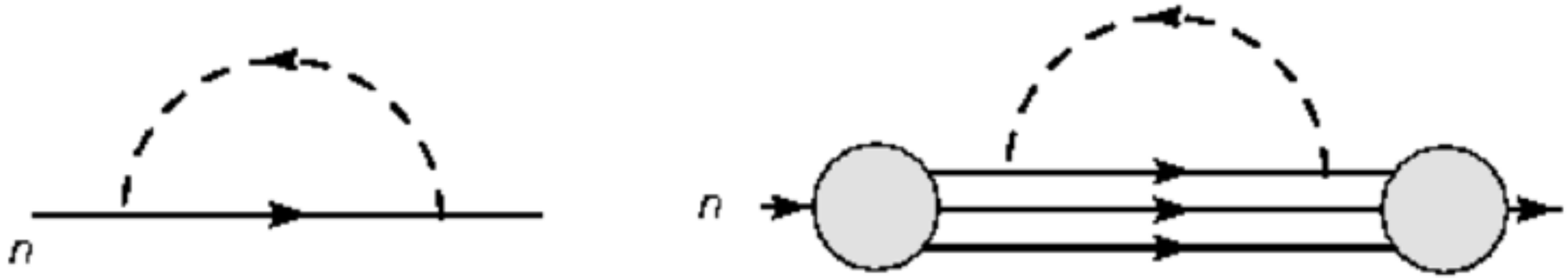


Pion cloud effects



- Hadron level: πN-contributions to nucleon self-energy
- Quark-level: π-contributions to quark self-energy and interactions

Pion cloud effects



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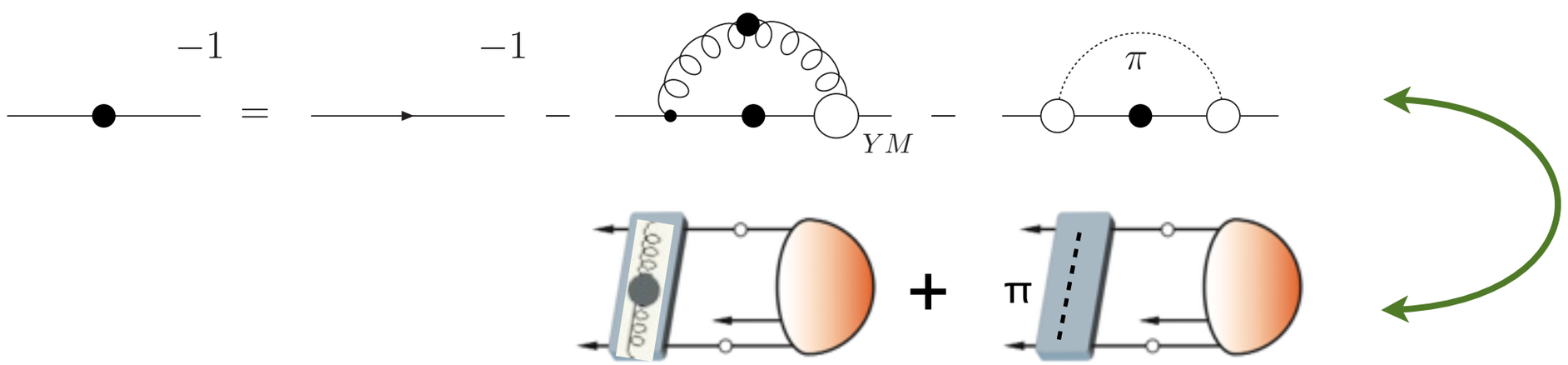
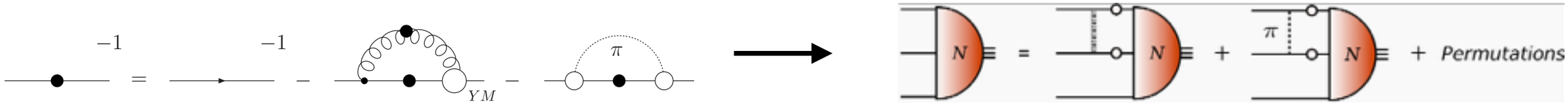
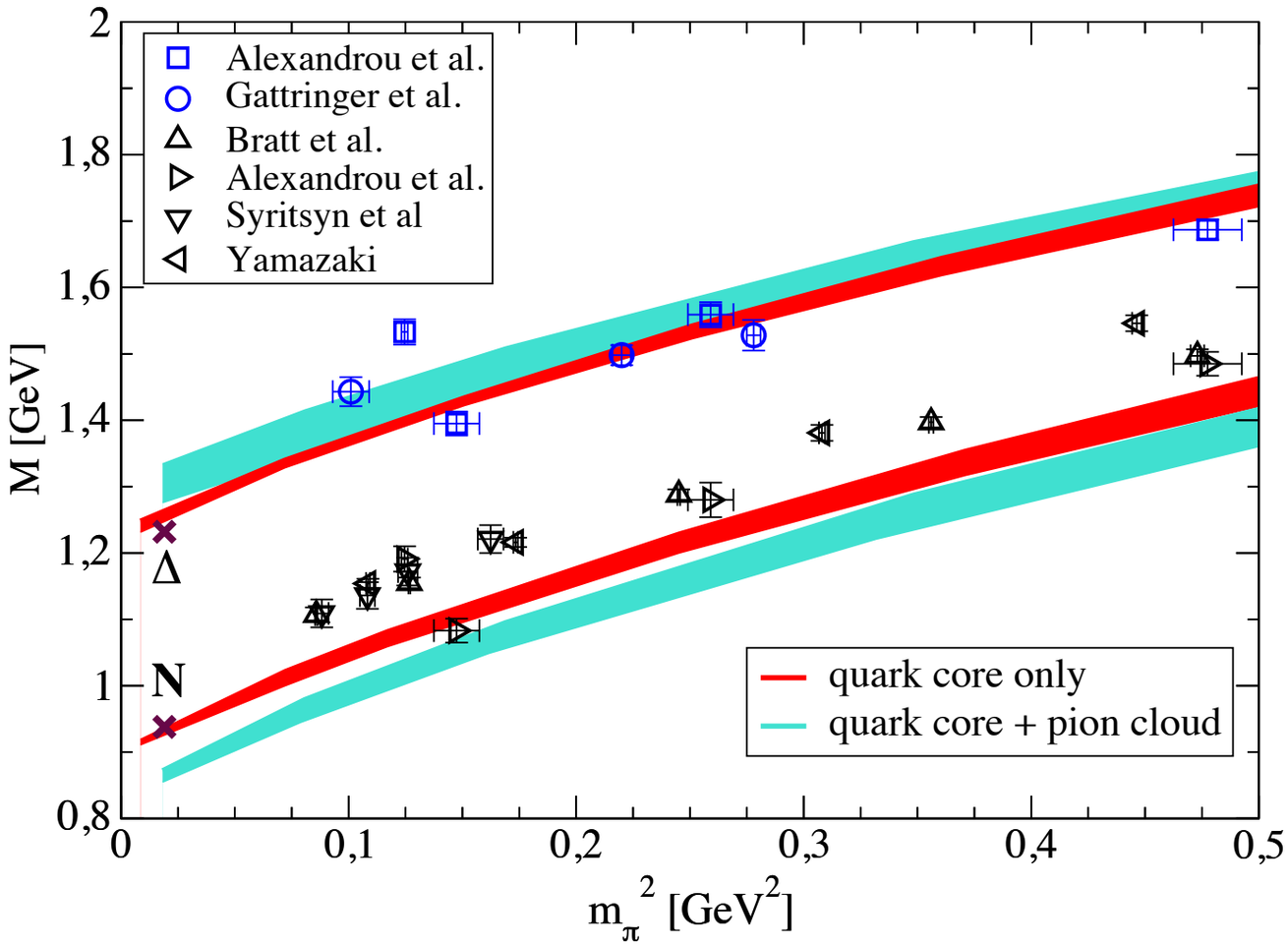


Pion not an elementary field!
Derived from DSE for quark-gluon interaction!

CF, Nickel and Wambach, PRD 76 (2007) 094009

Baryon masses- including pion cloud





Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014) [1401.3183]

- fix Λ by f_π, vary η s.t. f_π still ok
- effects of the order of 50-100 MeV
- missing: gluon self-interaction effects

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

Pion cloud effects in baryons: structure

	Nucleon			Delta			
	s-wave	p-wave	d-wave	s-wave	p-wave	d-wave	f-wave
quark core	75	24	1	61	31	7	0.2
quark core plus pion cloud	75	24	1	60	31	8	0.2

$$\sigma_{\pi N} = 30(3)\,\mathrm{MeV} \quad (\text{quark core only})$$
$$\sigma_{\pi N} = 31(3)\,\mathrm{MeV} \quad (\text{quark core + pion cloud})$$

Sanchis-Alepuz, CF, Kubrak, PLB 733 (2014) [1401.3183]

- pion cloud does not change shape of nucleon: uniform skin
- sigma-term small...