SU(2)CS and SU(2NF) hidden symmetries

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What is origin of hadron mass?

Are chiral symmetry breaking and confinement uniquely connected?

Is it possible to separate confinement and chiral symmetry breaking physics?

What physics is responsible for confinement and for chiral symmetry breaking?

What is the underlying systematics that drives a genesis of hadrons and hadron spectra?

Low mode truncation

Banks-Casher:

$$< \bar{q}q >= -\pi\rho(0).$$

What we do:

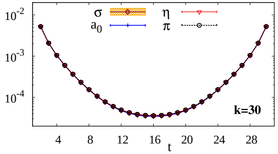
$$S = S_{Full} - \sum_{i=1}^{k} \frac{1}{\lambda_i} |\lambda_i\rangle\langle\lambda_i|.$$

What one expects for $J = 0$ correlators or states, if they survive:

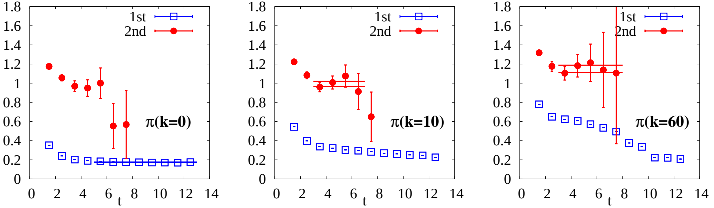
$$\begin{tikzcd}
(1/2,1/2)_a & \pi \arrow[r, leftrightarrow, "SU(2)_L * SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma \arrow[d, leftrightarrow, "U(1)_A"] \\
(1/2,1/2)_b & \mathbf{a}_0 \arrow[r, leftrightarrow, "SU(2)_L * SU(2)_R"'] & \eta
\end{tikzcd}$$

M.Denissenya, L.Ya.G., C.B.Lang, PRD 89(2014)077502; 91(2015)034505

We use JLQCD $N_f = 2$ overlap gauge configurations and quark propagators.



$SU(2)_L \times SU(2)_R \times U(1)_A$ is restored in correlators.



The ground states do not survive truncation: Without the near-zero modes π,

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What one expects for $J = 1$ mesons:

$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_\mathrm{F} \otimes \gamma^5\gamma^k)\Psi$		$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_\mathrm{F} \otimes \gamma^k)\Psi$	
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^0\gamma^k)\Psi$	$\xleftrightarrow{SU(2)_\mathrm{A}}$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_\mathrm{F} \otimes \gamma^0\gamma^k)\Psi$	
$U(1)_\mathrm{A}$ $\updownarrow$				$\updownarrow$ $U(1)_\mathrm{A}$
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^0\gamma^k)\Psi$	$\xleftrightarrow{SU(2)_\mathrm{A}}$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_\mathrm{F} \otimes \gamma^5\gamma^0\gamma^k)\Psi$	
$(1,0)\oplus(0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^k)\Psi$	$\xleftrightarrow{SU(2)_\mathrm{A}}$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^k)\Psi$	

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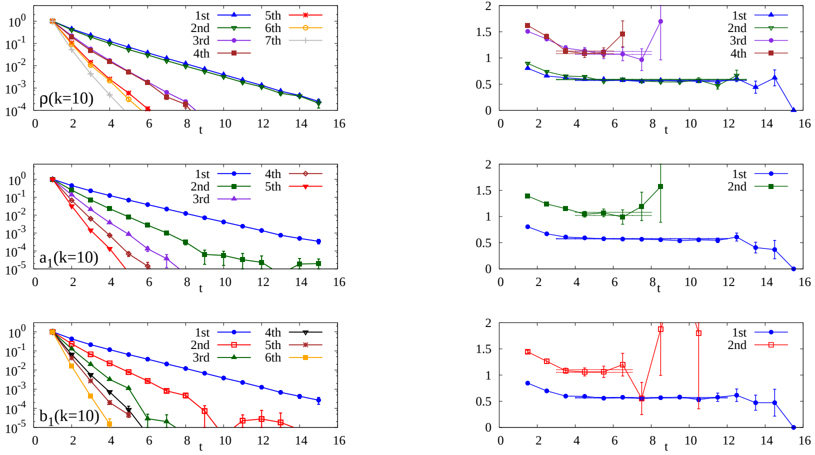
Isovector $J = 1$ states.



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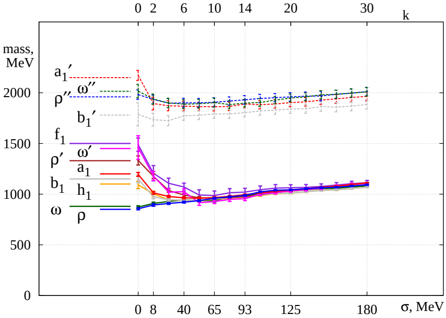
$$J = 1$$



We clearly see a larger degeneracy than the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry of the QCD Lagrangian. What does it mean !?

L.Ya.G., EPJA 51(2015)27

(i) (0,0):

$$|(0,0);\pm;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}R \pm \bar{L}L\rangle_J.$$

(ii) $(1/2,1/2)_a$ **and** $(1/2,1/2)_b$**:**

$$|(1/2,1/2)_a;+;I=0;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L + \bar{L}R\rangle_J,$$

$$|(1/2,1/2)_a;-;I=1;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L - \bar{L}\boldsymbol{\tau}R\rangle_J,$$

$$|(1/2,1/2)_b;-;I=0;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}L - \bar{L}R\rangle_J,$$

$$|(1/2,1/2)_b;+;I=1;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}L + \bar{L}\boldsymbol{\tau}R\rangle_J.$$

(iii) (0,1)⊕(1,0):

$$|(0,1)+(1,0);\pm;J\rangle = \frac{1}{\sqrt{2}}|\bar{R}\boldsymbol{\tau}R \pm \bar{L}\boldsymbol{\tau}L\rangle_J,$$

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Consider rotations in an imaginary 3-dim space of doublets constructed from the Weyl spinors

$$\mathrm{U} = \begin{pmatrix} u_L \\ u_R \end{pmatrix} \qquad \mathrm{D} = \begin{pmatrix} d_L \\ d_R \end{pmatrix}$$

$$\mathrm{U} \to \mathrm{U}' = e^{i\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}}{2}}\mathrm{U}\;, \qquad \mathrm{D} \to \mathrm{D}' = e^{i\frac{\boldsymbol{\varepsilon}\cdot\boldsymbol{\sigma}}{2}}\mathrm{D}\;,$$

where $\boldsymbol{\sigma}$ are the standard Pauli matrices: $[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k$.
We refer to this imaginary three-dimensional space as the chiralspin space and denote this symmetry group as $SU(2)_{\rm CS}$

A group that contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS}$ is $SU(4)$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\rm L} \\ u_{\rm R} \\ d_{\rm L} \\ d_{\rm R} \end{pmatrix}$$

L.Ya.G., M. Pak, PRD 92(2015)016001

Instead of the states constructed with Weyl spinors we can consider the left- and right-handed Dirac bispinors and bilinear operators.
Then the $SU(2)_{\rm CS}$ chiralspin rotations are generated through

$$\boldsymbol{\Sigma} = \{\gamma^0, i\gamma^5\gamma^0, -\gamma^5\}\,, \qquad [\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk}\,\Sigma^k\,.$$

The $SU(4)$ group contains at the same time $SU(2)_L \times SU(2)_R$ and $SU(2)_{CS} \supset U(1)_A$ with the fundamental vector

$$\Psi = \begin{pmatrix} u_{\rm L} \\ u_{\rm R} \\ d_{\rm L} \\ d_{\rm R} \end{pmatrix}$$

and has the following set of generators:

$$\{(\tau^a \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i)\}$$

L.Ya.G., M. Pak, PRD 92(2015)016001

$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(1\!\!1_{\mathrm{F}} \otimes \gamma^5\gamma^k)\Psi$	$\omega(0,1^{--})$ $\overline{\Psi}(1\!\!1_{\mathrm{F}} \otimes \gamma^k)\Psi$
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^0\gamma^k)\Psi$	$\omega(0,1^{--})$ $\overline{\Psi}(1\!\!1_{\mathrm{F}} \otimes \gamma^0\gamma^k)\Psi$
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^0\gamma^k)\Psi$	$h_1(0,1^{+-})$ $\overline{\Psi}(1\!\!1_{\mathrm{F}} \otimes \gamma^5\gamma^0\gamma^k)\Psi$
$(1,0)\oplus(0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^k)\Psi$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^k)\Psi$

$SU(4)$

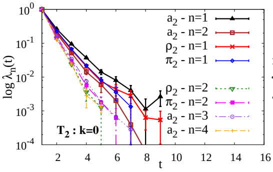
$SU(2)_{\mathrm{CS}}$

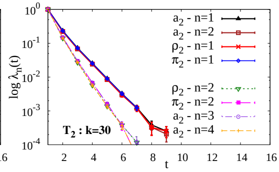
$SU(2)_{\mathrm{CS}}$

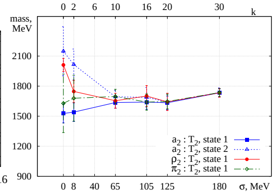
M. Denissenya, L.Ya.G, M.Pak, PRD 91(2015)114512

J=2 mesons.

$(0,0)$	$\omega_2(0,2^{--})$		$f_2(0,2^{++})$	
		$SU(4)$		
$(1/2,1/2)_a$	$\pi_2(1,2^{-+})$		$f_2(0,2^{++})$	$SU(2)_{\mathrm{CS}}$
$SU(2)_{\mathrm{CS}}$				
$(1/2,1/2)_b$	$a_2(1,2^{++})$		$\eta_2(0,2^{-+})$	
$(1,0)\oplus(0,1)$	$a_2(1,2^{++})$		$\rho_2(1,2^{--})$	





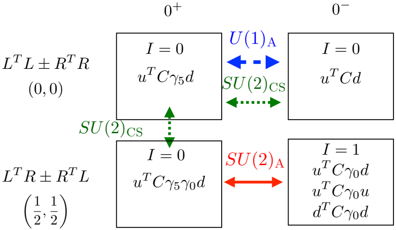


$J = 1/2$ baryons: M. Denissenya, L.Ya.G, M.Pak, arXiv:1508.01413

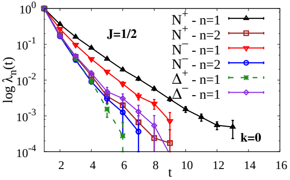
$$(1/2,0)+(0,1/2): \quad \mathcal{O}_{N\pm} = \varepsilon^{abc}\mathcal{P}_{\pm}u^{a}\left[u^{bT}C\gamma_5 d^{c}\right] \quad \leftarrow \underline{20} \text{ of } SU(4)$$

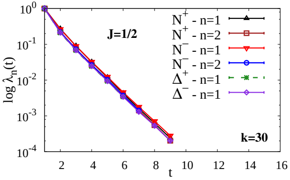
$$(1,1/2)+(1/2,1): \quad \mathcal{O}_{N\pm} = i\varepsilon^{abc}\mathcal{P}_{\pm}u^{a}\left[u^{bT}C\gamma_5\gamma_0 d^{c}\right] \quad \leftarrow \underline{20} \text{ of } SU(4)$$

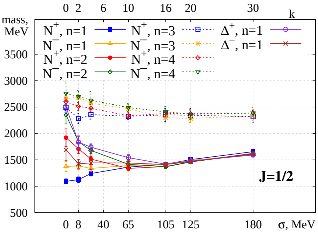
$$(1,1/2)+(1/2,1): \quad \mathcal{O}_{\Delta\pm} = i\varepsilon^{abc}\mathcal{P}_{\pm}\gamma_i\gamma_5 u^{a}\left[u^{bT}C\gamma_i u^{c}\right] \quad \leftarrow \underline{20'} \text{ of } SU(4)$$



$J = 1/2$ baryons: M. Denissenya, L.Ya.G, M.Pak, arXiv:1508.01413







L.Ya.G. EPJA 51(2015)27

What does this symmetry mean ?

A unitary transformation (L.Ya.G., A. Nefediev, PRD 76 (2007) 096004):

$$
\begin{aligned}
|(0,1)+(1,0);1\ 1^{--}\rangle &= \sqrt{\frac{2}{3}}\,|1;{}^3S_1\rangle + \sqrt{\frac{1}{3}}\,|1;{}^3D_1\rangle, \\
|(1/2,1/2)_b;1\ 1^{--}\rangle &= \sqrt{\frac{1}{3}}\,|1;{}^3S_1\rangle - \sqrt{\frac{2}{3}}\,|1;{}^3D_1\rangle, \\
|(0,0);0\ 1^{--}\rangle &= \sqrt{\frac{2}{3}}\,|0;{}^3S_1\rangle + \sqrt{\frac{1}{3}}\,|0;{}^3D_1\rangle, \\
|(1/2,1/2)_a;0\ 1^{--}\rangle &= \sqrt{\frac{1}{3}}\,|0;{}^3S_1\rangle - \sqrt{\frac{2}{3}}\,|0;{}^3D_1\rangle, \\
|(0,1)+(1,0);1\ 1^{++}\rangle &= |1;{}^3P_1\rangle, \\
|(0,0);0\ 1^{++}\rangle &= |0;{}^3P_1\rangle, \\
|(1/2,1/2)_a;1\ 1^{+-}\rangle &= |1;{}^1P_1\rangle, \\
|(1/2,1/2)_b;0\ 1^{+-}\rangle &= |0;{}^1P_1\rangle.
\end{aligned}
$$

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We can invert this unitary transformation and obtain a chiral decomposition of vectors

$$|0;{}^3S_1\rangle, |1;{}^3S_1\rangle, |0;{}^3D_1\rangle, |1;{}^3D_1\rangle, |0;{}^1P_1\rangle, |1;{}^1P_1\rangle, |0;{}^3P_1\rangle, |1;{}^3P_1\rangle.$$

A degeneracy of all these states requires that there are no spin-spin, spin-orbit and tensor forces in the system. These forces are mediated by the magnetic field.

We conclude: After reduction of the near-zero modes there are no magnetic interactions and only electric interactions are left in the system.

Dynamical string?

L.Ya.G., M. Pak, PRD 92(2015)016001

$SU(2)_{CS}$ and $SU(4)$ are hidden in the QCD Lagrangian.
The interaction part of the Lagrangian consists of electric and magnetic interactions:

$$\overline{\Psi}\gamma^{\mu} D_{\mu}\Psi = \overline{\Psi}\gamma^{0} D_{0}\Psi - \overline{\Psi}\boldsymbol{\gamma}\cdot\mathbf{D}\Psi \ .$$

We apply $SU(2)_{CS}$ and $SU(4)$ transformations on the interaction part of the QCD Lagrangian,

$$\overline{\Psi}'\gamma^{0} D_{0}\Psi' = \overline{\Psi}\gamma^{0} D_{0}\Psi.$$

The γ^0-part is invariant (singlet) under these transformations. It encodes the electric (Coulombic) $Q\phi$ interactions in QCD.

The spatial (magnetic) part, $\sim \mathbf{j}\cdot\mathbf{A}$, is not invariant - it is a triplet under $SU(2)_{CS}$ and a 15-plet under $SU(4)$.

L.Ya.G., M. Pak, PRD 92(2015)016001

QCD Hamiltonian in Coulomb gauge (Christ and Lee):

$$H_{QCD} = H_E + H_B + \int d^3x \Psi^\dagger(\mathbf{x})[-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta m]\Psi(\mathbf{x}) + H_T + H_C,$$

$$H_T = -g \int d^3x \, \Psi^\dagger(\mathbf{x}) \boldsymbol{\alpha} \cdot \mathbf{A}(\mathbf{x}) \, \Psi(\mathbf{x}) \ ,$$

$$H_C = \frac{g^2}{2} \int d^3x \, d^3y \, J^{-1} \ \rho^a(\mathbf{x}) F^{ab}(\mathbf{x}, \mathbf{y}) \, J \, \rho^b(\mathbf{y}) \ .$$

The Coulombic H_C part is a $SU(2)_{CS}$- and $SU(4)$- singlet. It is a confining part of the QCD Hamiltonian. This part generates a $SU(4)$-symmetric spectrum.

The transverse (magnetic) part H_T is not $SU(2)_{CS}$- and $SU(4)$-symmetric and therefore its expectation value vanishes in the $SU(4)$-symmetric hadron wave function.

The quasi-zero modes are due to the magnetic part of QCD. The magnetic interactions break $SU(4)$ and $SU(2)_{CS}$ symmetries of confinement explicitly and $SU(2)_L \times SU(2)_R \times U(1)_A$ - dynamically. Instanton fluctuations?

(Near) zero modes of Euclidean QCD and $SU(2)_{CS}$-$SU(4)$ breaking.

The Euclidean $SU(2)_{CS}$ generators:

$$\mathbf{\Sigma} = \{\gamma^4, i\gamma^5\gamma^4, -\gamma^5\} \ . \qquad (2)$$

The $SU(2)_{CS}$ generators do not commute with the Dirac operator.

These symmetries are missing in the Lagrangian.
The intrinsic dynamical reason: the zero modes of the Dirac operator

$$\gamma_\mu D_\mu \Psi_0(x) = 0. \qquad (3)$$

The zero mode is chiral, L or R, depending on the topological charge $Q \neq 0$.
Atiyah-Singer:

$$Q = n_L - n_R$$

.
At $Q \neq 0$, there is an asymmetry between L and R.
Conclusion: The zero modes break explicitly $SU(2)_{CS}$ and $SU(2N_F)$.

Conclusions

Observed on the lattice $SU(4)$ symmetry of hadrons upon elimination of the near-zero modes is a symmetry of confinement in QCD that is due to color-electric charge-charge interaction.

The magnetic interactions in QCD are responsible for generation of the quasi-zero modes. They break explicitly the $SU(4)$ symmetry of confinement and dynamically the $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetry. Instantons?

The hadron spectra observed in real world can be viewed as a result of splitting of the primary energy levels of the dynamical QCD string with the $SU(4)$ symmetry by means of dynamics associated with the quasi-zero modes.