Nucleon Axial Form Factor at Large Momentum Transfers

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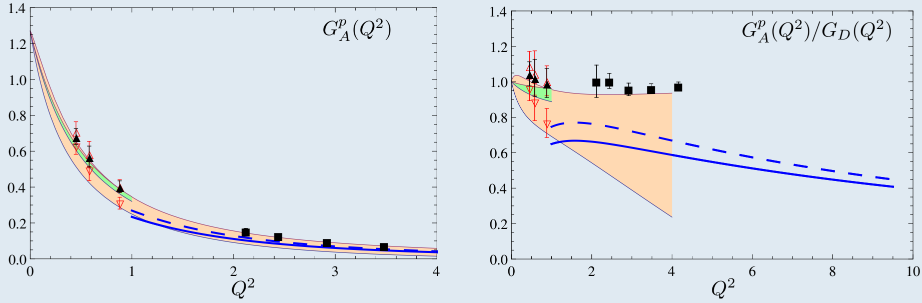
ECT*, Trento, 08.05.2017

Axial form factor of the nucleon

I.V. Anikin, V.M. Braun, N. Offen, Phys. Rev. D ***94*** *(2016) 034011*



- **curves: QCD (light-cone sum rules)**
- **shaded regions: neutrino scattering data with dipole vs. z-parametrization**
- **data points: pion electroproduction near threshold**

What are the options?

- **neutrino scattering**

— Dipole parametrization

$$G_A^p(Q^2) = \frac{g_A}{(1+Q^2/M_A^2)^2}\,, \qquad M_A = 1.026(21)\ \text{GeV} \quad \text{[Bernard:2001rs]}$$

— z-parametrization [Meyer:2016oeg]

$$G_A^p(Q^2) = a_0 + a_1 z + a_2 z^2 + \ldots\,, \qquad z = \frac{\sqrt{9m_\pi^2 - q^2} - \sqrt{9m_\pi^2 - t_0}}{\sqrt{9m_\pi^2 - q^2} - \sqrt{9m_\pi^2 - t_0}}$$

- **threshold pion production** [Park:2012yf]

$$\frac{1}{\sqrt{2}} G_A^p = \frac{Q^2}{m_N^2} G_1^{\pi^+ n} - \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2 + 2m_N^2)} G_M^n + \text{corrections}\,, \qquad E_{0+}^{\pi^+ n} \sim Q^2 G_1^{\pi^+ n}$$

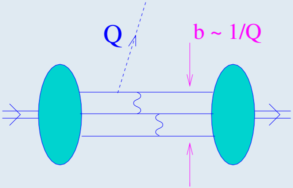
— Exact for $\forall Q^2$ in the chiral limit $m_\pi^2 = 0$

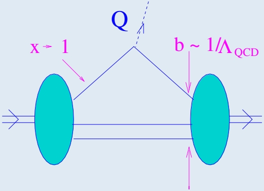
— Corrections $\sim m_\pi/M_N$ and $\sim m_\pi Q^2/m_N^3$

target accuracy?

Heuristic picture:

- quarks can acquire large transverse momenta when they exchange gluons
- "hard" gluon exchanges can be separated from "soft" nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations





Hard rescattering:

Small b
Average $0 < x < 1$

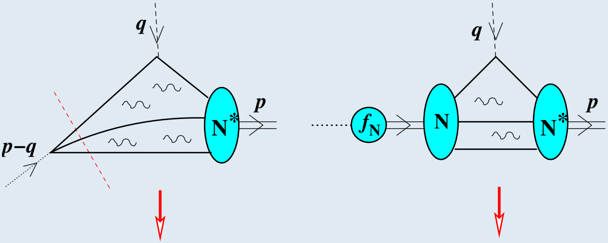
Soft (Feynman):

Average b
Large $x \to 1$

In practice three-quark states indeed seem to dominate, however

- "Squeesing" to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- $\Rightarrow$ More complicated nonperturbative input needed

- Duality + Dispersion Relations ⇒ Light-Cone Sum Rules:



$$\frac{1}{\pi}\int_0^{s_0} ds\, \text{Im}\, T(p,q) \stackrel{duality}{=} f_N\, F_{N\to N^*}(Q^2)$$

- $T(p,q)$ is calculated in terms of N^* distribution amplitudes
  Balitsky, Braun, Kolesnichenko, Nucl.Phys.B312:509-550,1989
  Braun, Halperin, Phys.Lett.B328:457-465,1994

- Leading term is a Feynman (soft) contribution; hard terms can be added systematically and without double counting

- Schematic structure of a LCSR for baryon form factors

Braun, Lenz, Mahnke, Stein, Phys. Rev. D **65**, *074011 (2002)*
Braun, Lenz, Wittmann, Phys. Rev. D **73**, *094019 (2006)*

$$F(Q^2) \simeq \underbrace{\frac{1}{Q^6}\left\{F_{tw=3} + F_{tw=4} + \frac{\Lambda^2}{s_0}F_{tw=5} + \ldots\right\}}_{\text{soft+hard}} + \underbrace{\frac{1}{Q^4}\left(\frac{\alpha_s(Q)}{\pi}\right)^2 F^{tw=3}_{pQCD}}_{\text{hard (pQCD)}}$$

where

$$F_{tw=3} = f_{tw=3}(Q^2, s_0) \otimes \Phi_3(\mu_F = s_0)\,, \qquad F_{tw=4} = f_{tw=4}(Q^2, s_0) \otimes \Phi_4(\mu_F = s_0)\,,$$

- for each twist obtain an expansion

$$f_{tw=n} = f^{(0)}_{tw=n} + \left(\frac{\alpha_s(s_0)}{\pi}\right) f^{(1)}_{tw=n} + \left(\frac{\alpha_s(s_0)}{\pi}\right)^2 f^{(2)}_{tw=n} + \ldots$$

$$f^{(1)}_{tw=n} = c_2 \ln^2(Q^2/s_0) + c_1 \ln(Q^2/s_0) + c_0\,, \qquad \text{etc.}$$

- hierarchy of twists based on $s_0 \gg \Lambda_{QCD}$, hence $\alpha_s(s_0) \ll 1$

NLO LCSRs for nucleon EM form factors

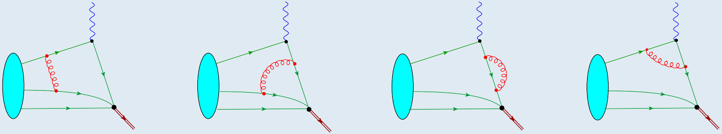
- **A large project completed:**

  *Anikin, Braun, Offen; PRD **88**, 114021 (2013) [arXiv:1310.1375]*

  - A consistent renormalization scheme for three-quark operators

    Krankl, Manashov; PLB703, 519 (2011)

  - Light-cone expansion for three-quark operators for generic coordinates
  - NLO coefficient functions including twist-three and twist-four DAs



  - Nucleon mass corrections for all twists

    *Anikin, Manashov; PRD **89** (2014) 014011*

  - A new calculation of twist-five off-light cone contributions
  - A more general model for the DAs, including second-order polynomials

Examples of coefficient functions

APPENDIX E: SUMMARY OF NLO COEFFICIENT FUNCTIONS

The NLO corrections (39) to the correlation functions $\mathcal{A}(Q^2, P'^2)$ and $\mathcal{B}(Q^2, P'^2)$ can be written as a sum of contributions of a given quark flavor $q = u, d$ and expanded in contributions of nucleon DAs as shown in Eqs. (46) and (47). Our results for the coefficient functions $C_q^F(x_i, W)$ are collected below. We use a shorthand notation $L = \ln Q^2/\mu^2$, where μ^2 is the factorization scale. The dependence on $W = 1 + P'^2/Q^2$ is not shown for brevity.

$$\begin{aligned} x_2 C_d^{\mathbb{V}_1}(x_i) = {} & 2x_2x_3[3(L-2)g_1(x_3) + 2(L-1)g_{11}(x_3,x_3) + g_{21}(x_3,x_3)] + [2x_2 + (4L-3)x_3]h_{11}(x_3) \\ & + (3-4L)\bar{x}_1 h_{11}(\bar{x}_1) + 2x_3 h_{21}(x_3) - 2\bar{x}_1 h_{21}(\bar{x}_1) - 2[3(x_2/x_3)(2L-3) + 5L - 7]h_{12}(x_3) \\ & + 2(5L-7)h_{12}(\bar{x}_1) - [6(x_2/x_3) + 5]h_{22}(x_3) + 5h_{22}(\bar{x}_1) + (6/x_3)(L-2)h_{13}(x_3) \\ & - (6/\bar{x}_1)(L-2)h_{13}(\bar{x}_1) + (3/x_3)h_{23}(x_3) - (3/\bar{x}_1)h_{23}(\bar{x}_1), \end{aligned} \tag{E1}$$

$$\begin{aligned} x_1x_3 C_u^{\mathbb{V}_1}(x_i) = {} & x_1x_2x_3[(17-7L)g_1(x_2) + (1+2L)g_{11}(\bar{x}_1,x_2) + 2(2L-3)g_{11}(\bar{x}_3,x_2) + 2(5-7L)g_{11}(x_2,x_2) \\ & + g_{21}(\bar{x}_1,x_2) + 2g_{21}(\bar{x}_3,x_2) - 7g_{21}(x_2,x_2)] - x_1x_3[(1+2L)h_{11}(\bar{x}_1) + 2(2L-3)h_{11}(\bar{x}_3) \\ & + 2(5-7L)h_{11}(x_2)] + (1+2L)x_1h_{12}(\bar{x}_1) + 4x_3h_{12}(\bar{x}_3) - [4x_3 + (x_1/x_2)[x_2(1+2L) \\ & + 4x_3(4-L)]]h_{12}(x_2) - 2(L-2)(x_1/\bar{x}_1)h_{13}(\bar{x}_1) + 2(2L-7)(x_3/\bar{x}_3)h_{13}(\bar{x}_3) + (1/x_2)[2(L-2)x_1 \\ & + 2(7-2L)x_3]h_{13}(x_2) - x_1x_3[h_{21}(\bar{x}_1) + 2h_{21}(\bar{x}_3) - 7h_{21}(x_2)] + x_1h_{22}(\bar{x}_1) \\ & + x_1[2(x_3/x_2) - 1]h_{22}(x_2) - (x_1/\bar{x}_1)h_{23}(\bar{x}_1) + 2(x_3/\bar{x}_3)h_{23}(\bar{x}_3) + [(x_1 - 2x_3)/x_2]h_{23}(x_2), \end{aligned} \tag{E2}$$

$$\begin{aligned} x_2 C_d^{\mathbb{V}_3}(x_i) = {} & 2x_2x_3[(5-3L)g_1(x_3) + (3-4L)g_{11}(\bar{x}_1,x_3) + 2(2L-1)g_{11}(x_3,x_3) - 2g_{21}(\bar{x}_1,x_3) + 2g_{21}(x_3,x_3)] \\ & + 2(4L-3)(2x_2+x_3)h_{11}(\bar{x}_1) + [8(1-2L)x_2 + 2(3-4L)x_3]h_{11}(x_3) + 4(2x_2+x_3)[h_{21}(\bar{x}_1) - h_{21}(x_3)] \\ & + 6(3-4L)h_{12}(\bar{x}_1) + 6[4L - 3 + 4(x_2/x_3)(L-1)]h_{12}(x_3) - 12h_{22}(\bar{x}_1) + 12(\bar{x}_1/x_3)h_{22}(x_3) \\ & + (4/\bar{x}_1)(2L-1)h_{13}(\bar{x}_1) - (4/x_3)(2L-1)h_{13}(x_3) + (4/\bar{x}_1)h_{23}(\bar{x}_1) - (4/x_3)h_{23}(x_3), \end{aligned} \tag{E3}$$

$$\begin{aligned} x_1x_3 C_u^{\mathbb{V}_3}(x_i) = {} & 2x_1x_2x_3[5(L-3)g_1(x_2) + 2(1-2L)g_{11}(\bar{x}_1,x_2) + (5-4L)g_{11}(\bar{x}_3,x_2) + 2(8L-5)g_{11}(x_2,x_2) \\ & - 2g_{21}(\bar{x}_1,x_2) - 2g_{21}(\bar{x}_3,x_2) + 8g_{21}(x_2,x_2)] + 2x_3[(6L-8)x_1 + (2L-3)x_2]h_{11}(\bar{x}_3) \\ & + 4x_1[Lx_2 + (3L-1)x_3]h_{11}(\bar{x}_1) - 2[4x_1x_3(5L-3) + x_2x_3(2L-3) + 2x_1x_2L]h_{11}(x_2) \\ & + 2x_1(x_2+3x_3)h_{21}(\bar{x}_1) + 2x_3(3x_1+x_2)h_{21}(\bar{x}_3) - 2[10x_1x_3 + x_2\bar{x}_2]h_{21}(x_2) - 4(3+2L)x_1h_{12}(\bar{x}_1) \\ & + 2x_3(15-8L)h_{12}(\bar{x}_3) + 2(x_3/x_2)[4(L-1)x_1 + (8L-15)x_2]h_{12}(x_2) + 4(x_1/x_2)[(3+2L)x_2 + 6x_3]h_{12}(x_2) \\ & - 4x_1h_{22}(\bar{x}_1) - 8x_3h_{22}(\bar{x}_3) + (4/x_2)[2x_2x_3 + x_1\bar{x}_1]h_{22}(x_2) + 12(x_1/\bar{x}_1)h_{13}(\bar{x}_1) + 8(x_3/\bar{x}_3)(L-2)h_{13}(\bar{x}_3) \\ & - (4/x_2)[3x_1 + 2x_3(L-2)]h_{13}(x_2) + 4(x_3/\bar{x}_3)h_{23}(\bar{x}_3) - 4(x_3/x_2)h_{23}(x_2), \end{aligned} \tag{E4}$$

$$\begin{aligned} x_1x_2 C_u^{\mathbb{V}_2^{(1)}}(x_i) = {} & -8\bar{x}_3x_2g_1(\bar{x}_3) + 2x_2^2[4(L-2) + (2L-5)x_1]g_1(x_2) - 4x_2^2[(L-3) + (2L-3)x_1]g_{11}(\bar{x}_3,x_2) \\ & + 2x_2^2(1+2x_1)[2(L-1)g_{11}(x_2,x_2) - g_{21}(\bar{x}_3,x_2) + g_{21}(x_2,x_2)] + 4x_2[(L-3) + (2L-3)x_1]h_{11}(\bar{x}_3) \\ & - 2x_2(1+2x_1)[2(L-1)h_{11}(x_2) - h_{21}(\bar{x}_3) + 2h_{21}(x_2)] + 2x_2[(4L-12)/\bar{x}_3 + (4L-13)/x_1 - 4]h_{12}(\bar{x}_3) \\ & + 2[4(1+x_2-L) + (x_2/x_1)(13-4L)]h_{12}(x_2) + 4[1 - (x_1/\bar{x}_3) + (x_2/x_1)]h_{22}(\bar{x}_3) - 4[1 + (x_2/x_1)]h_{22}(x_2) \\ & + 2[(x_1/\bar{x}_3^2)(8L-25) - 2(x_2/\bar{x}_3)(2L-7) - (1/x_1)(8L-25)]h_{13}(\bar{x}_3) + 2[2(2L-7) \\ & + (1/x_1)(8L-25)]h_{13}(x_2) + 4[2(x_1/\bar{x}_3^2) - (x_2/\bar{x}_3) - (2/x_1)]h_{23}(\bar{x}_3) + 4[1 + (2/x_1)]h_{23}(x_2), \end{aligned} \tag{E5}$$

$$\begin{aligned} x_2 C_d^{\mathbb{V}_2^{(2)}}(x_i) = {} & 4\bar{x}_1g_1(\bar{x}_1) + 2x_3(3-4L)g_1(x_3) - 8x_3g_{11}(\bar{x}_1,x_3) + 2[4 + (4L-3)\bar{x}_1]h_{11}(\bar{x}_1) - 2[4(L-1)\bar{x}_1 + x_3]h_{11}(x_3) \\ & + 4\bar{x}_1[h_{21}(\bar{x}_1) - h_{21}(x_3)] + 2[2(7-5L) + (11-2L)/x_2 + 2(5-2L)/\bar{x}_1]h_{12}(\bar{x}_1) - 2[2(7-5L) \\ & + (11-2L)/x_2 + 4(1-L)(1+2x_2)/x_3]h_{12}(x_3) - [10 + (4/\bar{x}_1) + (2/x_2)]h_{22}(\bar{x}_1) + 2[5 + (1/x_2) \\ & + 2(1+2x_2)/x_3]h_{22}(x_3) + (4/\bar{x}_1)[(3L-8)(1/\bar{x}_1 + 1/x_2) + 3(L-2)]h_{13}(\bar{x}_1) - (4/x_3)[(3L-8)/x_2 \\ & + 3(L-2)]h_{13}(x_3) + (6/\bar{x}_1)[1/\bar{x}_1 + 1/x_2 + 1]h_{23}(\bar{x}_1) - (6/x_3)[1/x_2 + 1]h_{23}(x_3), \end{aligned} \tag{E6}$$

$$\begin{aligned} x_1x_3 C_u^{\mathbb{V}_2^{(2)}}(x_i) = {} & 4\bar{x}_1x_1g_1(\bar{x}_1) - 8\bar{x}_3x_3g_1(\bar{x}_3) + 2[4x_2x_3 - 2x_1x_2 + x_1x_3[13L - 25 + (7L-17)x_2]]g_1(x_2) \\ & - 2x_1[2x_3L + x_2x_3(1+2L) - 2x_2]]g_{11}(\bar{x}_1,x_2) - 2x_3[x_1(1+2x_2)(2L-3) - 2x_2]g_{11}(\bar{x}_3,x_2) \\ & + 4[3x_1x_3(2L-1) + x_1x_2x_3(7L-5) - x_2\bar{x}_2]g_{11}(x_2,x_2) - 2x_1x_3[(1+x_2)g_{21}(\bar{x}_1,x_2) \\ & + (1+2x_2)g_{21}(\bar{x}_3,x_2) - (6+7x_2)g_{21}(x_2,x_2) + (1+5L)g_2(x_2) + 2Lg_{12}(\bar{x}_1,x_2) - (3-2L)g_{12}(\bar{x}_3,x_2) \\ & - 2(4L-7)g_{12}(x_2,x_2) + g_{22}(\bar{x}_1,x_2) + g_{22}(\bar{x}_3,x_2) - 4g_{22}(x_2,x_2)] + 2x_1[x_3(1+2L) \\ & - 2(1+L)]h_{11}(\bar{x}_1) + 2x_3[1 - 2L + 2x_1(2L-3)]h_{11}(\bar{x}_3) - (2/x_2)[x_3(x_2 - 4x_1)(2L-1) \\ & + 2x_1x_2[1 + L + x_3(5-7L)]]h_{11}(x_2) - 2x_1\bar{x}_3h_{21}(\bar{x}_1) - 2(1-2x_1)x_3h_{21}(\bar{x}_3) \\ & + (2/x_2)[x_2\bar{x}_2 - x_1x_3(4+7x_2)]h_{21}(x_2) - 2x_1(1 + 2L - 2/\bar{x}_1)h_{12}(\bar{x}_1) - 8x_3[(3-L)/\bar{x}_3 + 1]h_{12}(\bar{x}_3) \\ & + (2/x_2^2)[4x_2x_3[(3-L) + x_2 + x_1(4-L)] + x_1x_2[x_2(1+2L) - 2] + 2x_1x_3(13-6L)]h_{12}(x_2) \\ & - 2x_1h_{22}(\bar{x}_1) + 4(x_3/\bar{x}_3)h_{22}(\bar{x}_3) + (2/x_2)[x_1x_2 - 2x_3[1 + x_1 + 3(x_1/x_2)]]h_{22}(x_2) \\ & - 2(x_1/\bar{x}_1^2)[9 - 2L + 2\bar{x}_1(2-L)]h_{13}(\bar{x}_1) + 2(x_3/\bar{x}_3^2)[25 - 8L + 2\bar{x}_3(7-2L)]h_{13}(\bar{x}_3) \\ & + (2/x_2^2)[x_1[9 - 2L + 2x_2(2-L)] - x_3[25 - 8L + 2x_2(7-2L)]]h_{13}(x_2) + 2(x_1/\bar{x}_1^2)(1+\bar{x}_1)h_{23}(\bar{x}_1) \\ & - 4(x_3/\bar{x}_3^2)(2+\bar{x}_3)h_{23}(\bar{x}_3) - (2/x_2^2)[x_1(1+x_2) - 2x_3(2+x_2)]h_{23}(x_2), \end{aligned} \tag{E7}$$

$$\begin{aligned} x_2 C_d^{\mathbb{V}_2^{(3)}}(x_i) = {} & 4\bar{x}_1g_1(\bar{x}_1) + 2x_2[25 - 13L + 6x_3(2-L) - 2(x_3/x_2)]g_1(x_3) + [2x_2(4L-3) - 8x_3]g_{11}(\bar{x}_1,x_3) \\ & + 2x_2[[6(1-2L) + 4x_3(1-L) + 4(x_3/x_2)]g_{11}(x_3,x_3) + 2g_{21}(\bar{x}_1,x_3) - 2(3+x_3)g_{21}(x_3,x_3) \\ & + (1+5L)g_2(x_3) - (3-4L)g_{12}(\bar{x}_1,x_3) + 2(7-4L)g_{12}(x_3,x_3) + 2g_{22}(\bar{x}_1,x_3) - 4g_{22}(x_3,x_3)] \\ & + 2[(1+4L) - \bar{x}_1(3-4L)\bar{x}_1]h_{11}(\bar{x}_1) - 2[4(x_2/x_3)(1-2L) + 2x_2 + 1 + 4L - x_3(3-4L)]h_{11}(x_3) \\ & + 4(1+\bar{x}_1)h_{21}(\bar{x}_1) + 4[2x_2/x_3 - (1+x_3)]h_{21}(x_3) + 4[7 - 5L + (5-2L)/\bar{x}_1]h_{12}(\bar{x}_1) + (4/x_3^2)[x_3(2L-5) \\ & + x_3^2(5L-7) + x_2(6L-13) + 3x_2x_3(2L-3)]h_{12}(x_3) - 2[5 + 2/\bar{x}_1]h_{22}(\bar{x}_1) + (2/x_3^2)[6x_2(1+x_3) \\ & + x_3(2+5x_3)]h_{22}(x_3) + (4/\bar{x}_1^2)[3L - 8 + 3\bar{x}_1(L-2)]h_{13}(\bar{x}_1) - (4/x_3^2)[3L - 8 + 3(L-2)x_3]h_{13}(x_3) \\ & + (6/\bar{x}_1^2)(1+\bar{x}_1)h_{23}(\bar{x}_1) - (6/x_3^2)(1+x_3)h_{23}(x_3), \end{aligned} \tag{E8}$$

$$\begin{aligned} x_3 C_u^{\mathbb{V}_2^{(3)}}(x_i) = {} & 4\bar{x}_1g_1(\bar{x}_1) + 2x_2[5 + x_3(8-3L)]g_1(x_2) - 2x_2[2(1-L) + x_3(1+2L)]g_{11}(\bar{x}_1,x_2) \\ & - 4x_2\bar{x}_3(L-1)g_{11}(x_2,x_2) + 2x_2\bar{x}_3[g_{21}(\bar{x}_1,x_2) - g_{21}(x_2,x_2)] + 2[2(1-L) + x_3(1+2L)]h_{11}(\bar{x}_1) \\ & - 2\bar{x}_3[2(1-L)h_{11}(x_2) + h_{21}(\bar{x}_1) - h_{21}(x_2)] - 2[1 + 2L - 2/\bar{x}_1 + 2(L-1)/x_3]h_{12}(\bar{x}_1) \\ & + 2[1 + 2L + 2(L-1)(1/x_3 + 2x_3/x_2)]h_{12}(x_2) - 2[1 + 1/x_3]h_{22}(\bar{x}_1) + 2[1 + 1/x_3 + 2x_3/x_2]h_{22}(x_2) \\ & + (2/\bar{x}_1)[(1/\bar{x}_1 + 1/x_3)(2L-9) + 2(L-2)]h_{13}(\bar{x}_1) + (2/x_2)[(9-2L)/x_3 - 2(L-2)]h_{13}(x_2) \\ & + (2/\bar{x}_1)[1 + 1/\bar{x}_1 + 1/x_3]h_{23}(\bar{x}_1) - (2/x_2)[1 + 1/x_3]h_{23}(x_2), \end{aligned} \tag{E9}$$

$$\begin{aligned} x_2 C_d^{\mathbb{A}_1}(x_i) = {} & 3\bar{x}_1h_{11}(\bar{x}_1) - 3x_3h_{11}(x_3) + 2(3L-10)h_{12}(\bar{x}_1) - 2(3L-10)h_{12}(x_3) + 3h_{22}(\bar{x}_1) - 3h_{22}(x_3) \\ & - (6/\bar{x}_1)(L-3)h_{13}(\bar{x}_1) + (6/x_3)(L-3)h_{13}(x_3) - (3/\bar{x}_1)h_{23}(\bar{x}_1) + (3/x_3)h_{23}(x_3), \end{aligned} \tag{E10}$$

Nonperturbative input

- **from lattice QCD** [Braun:2014wpa]

— Normalization constants (wave functions at the origin) for S- and P-wave DAs (twist-3,4)

$$
\begin{aligned}
f_N &= 2.84(1)(33)\ 10^{-3}\ \text{GeV}^2 \\
\lambda_1 &= -4.13(2)(20)\ 10^{-2}\ \text{GeV}^2 \\
\lambda_2 &= 8.19(5)(39)\ 10^{-2}\ \text{GeV}^2
\end{aligned}
$$

— Quark "momentum fractions" in the S-wave DA (twist-3)

$$
\begin{aligned}
\langle x_1 \rangle &= 0.372(7)(?) \\
\langle x_2 \rangle &= 0.314(3)(?) \\
\langle x_3 \rangle &= 0.314(7)(?)
\end{aligned}
$$

- **Fit parameters** [Anikin:2013aka]

— Quark "momentum fractions" in the P-wave DAs (twist-4)

NLO LCSRs for nucleon EM form factors (II)

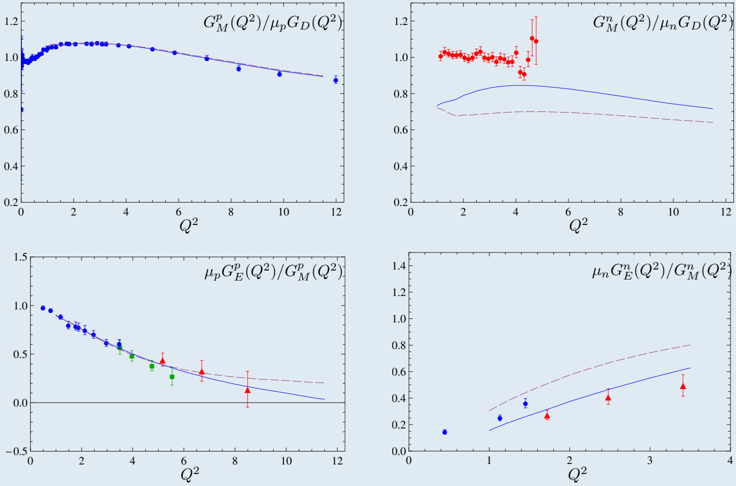


Figure: Nucleon electromagnetic form factors from LCSRs compared to the experimental data

NLO LCSRs for nucleon EM form factors (III)

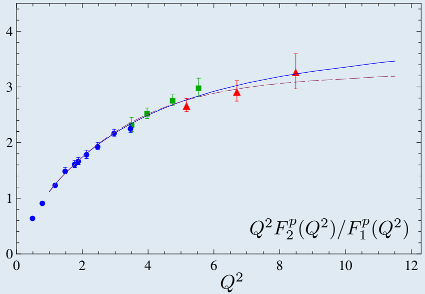


Figure: The ratio of Pauli and Dirac electromagnetic proton form factors vs. data

Axial form factor does not involve new parameters

— see slide 2

Threshold pion production

Generalized Form Factors $\sim$ S-wave Multipoles at Threshold

$$\langle \pi N | j_\mu^{\rm em} | p \rangle = -\frac{i}{f_\pi} \bar{N}(P_2)\gamma_5 \left\{ \left(\gamma_\mu q^2 - q_\mu \not{q}\right) \frac{1}{m_N^2} G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu} q^\nu}{2m_N} G_2^{\pi N}(Q^2) \right\} N(P_2)$$

related to S-wave multipoles in the PWA

$$E_{0+}^{\pi N}(Q^2, W_{\rm th}) = \frac{\sqrt{4\pi\alpha_{\rm em}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_1^{\pi N}$$

$$L_{0+}^{\pi N}(Q^2, W_{\rm th}) = \frac{\sqrt{4\pi\alpha_{\rm em}}}{32\pi} \frac{Q^2 \sqrt{Q^2 + 4m_N^2}}{m_N^3 f_\pi} G_2^{\pi N}$$

e.g. the differential cross section at threshold is given by

$$\left.\frac{d\sigma_{\gamma^*}}{d\Omega_\pi}\right|_{\rm th} = \frac{2|\vec{k}_f|\, W}{W^2 - m_N^2} \left[(E_{0+}^{\pi N})^2 + \epsilon \frac{Q^2}{(\omega_\gamma^{\rm th})^2} (L_{0+}^{\pi N})^2 \right]$$

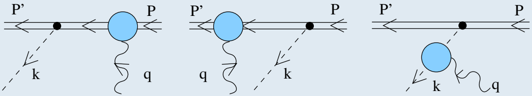
Low-Energy Theorems

- *Current algebra: predate ChPT and QCD*

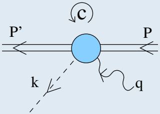
Chiral symmetry:

1. pion mass $m_\pi \to 0$
2. pion coupling $\sim |k| \to 0$

- **Pion emission from external legs**



- **Chiral Rotation**



$$\langle \pi^a N | j_\mu^{\rm em} | N \rangle \sim \frac{i}{f_\pi} \langle N | [j_\mu^{\rm em}, Q_5^a] | N \rangle$$

Kroll, Ruderman '54

Low-Energy Theorems – *continued*

PCAC + current algebra:

Vainshtein, Zakharov, NPB36(1972)589
Scherer, Koch, NPA534(1991)461

$$\frac{Q^2}{m_N^2} G_1^{\pi^0 p} = \frac{g_A}{2} \frac{Q^2}{(Q^2+2m_N^2)} G_M^p\,, \qquad G_2^{\pi^0 p} = \frac{2g_A m_N^2}{(Q^2+2m_N^2)} G_E^p\,,$$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{(Q^2+2m_N^2)} G_M^n + \frac{1}{\sqrt{2}} G_A\,, \qquad G_2^{\pi^+ n} = \frac{2\sqrt{2} g_A m_N^2}{(Q^2+2m_N^2)} G_E^n\,,$$

Derivation does not imply $Q^2 \sim m_\pi^2$!

- **Threshold photoproduction of π^0 is suppressed compared to π^+**
- **The π^0/π^+–ratio is rapidly increasing with Q^2**

- ◇ **The $\mathcal{O}(m_\pi)$ corrections can be added**
- ◇ **ChPT allows to treat $\mathcal{O}(m_\pi^2)$ terms if $Q \sim m_\pi$**

Chiral Perturbation Theory

Bernard, Kaiser, Meissner; PRL69 (1992)1877

Nambu, Lurié, Shrauner

$$E_{0+}^{(-)}(m_\pi = 0, q^2) = \frac{eg_A}{8\pi f_\pi}\left\{1 + \frac{q^2}{6}r_A^2 + \frac{q^2}{4m_N^2}\left(\kappa_v + \frac{1}{2}\right) + \frac{q^2}{128 f_\pi^2}\left(1 - \frac{12}{\pi^2}\right)\right\}$$

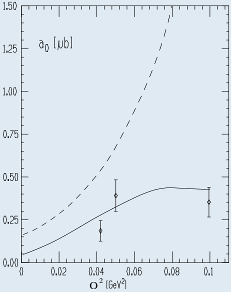
$$G_A(q^2) = g_A\left(1 + \frac{q^2}{6}r_A^2 + \dots\right)$$

Experiment: $r_A = 0.65 \pm 0.03$ (elastic ep); $r_A = 0.59 + 0.04 \pm 0.05$ (pion el.prod)

S-wave cross section

$\gamma^* p \to \pi^0 p$

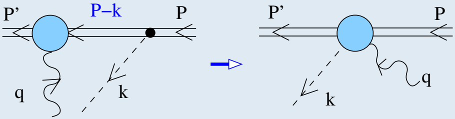
$W = 1074, \epsilon = 0.58$



Low-Energy Theorems – *continued (II)*

- expected to fail for $Q^2 \sim \frac{m_N^3}{m_\pi}$

since π cannot have small momentum w.r.t. the initial and final state protons simultaneously



at threshold

$$m_N^2 - (P-k)^2 \quad = \quad \frac{m_\pi}{m_N}\left[Q^2 + 2m_N^2\right]$$

⇒ **phenomenological Lagrangians to take into account nucleon resonances**

⇒ **or go over to quark-gluon description**

QCD Factorization

basic idea: decouple "hard" and "soft" momenta

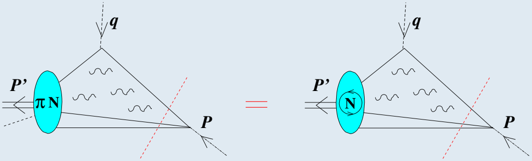
$$M(Q_1^2, Q_2^2, \ldots; k_1^2, k_2^2, \ldots) \quad = \quad H(Q_1^2, Q_2^2, \ldots; \mu^2) \otimes S(\mu^2; k_1^2, k_2^2, \ldots)$$

$\Uparrow$ $\Uparrow$

factorization scale

- **H: hard function can be calculated in pQCD**
- **S: soft function can be simplified using LET (ChPT)**

$Q^2 \sim m_N^3/m_\pi$: Light-Cone Sum Rules



[Braun:2006td], [Braun:2007pz]

$$|p\uparrow\rangle = \frac{\phi_s(x)}{\sqrt{6}}|2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle + \frac{\phi_a(x)}{\sqrt{2}}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

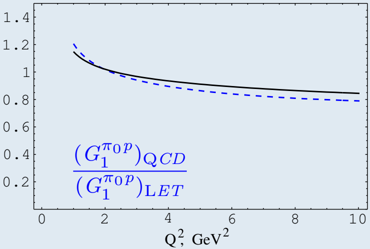
$$|p\uparrow\pi^0\rangle = \frac{\phi_s(x)}{2\sqrt{6}f_\pi}|6u_\uparrow d_\downarrow u_\uparrow + u_\uparrow u_\downarrow d_\uparrow + d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2\sqrt{2}f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

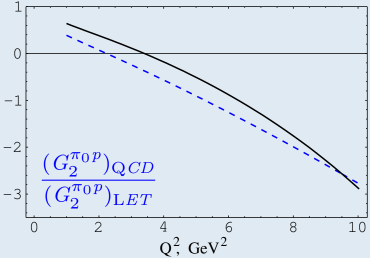
$$|n\uparrow\pi^+\rangle = \frac{\phi_s(x)}{\sqrt{12}f_\pi}|2u_\uparrow d_\downarrow u_\uparrow - 3u_\uparrow u_\downarrow d_\uparrow - 3d_\uparrow u_\downarrow u_\uparrow\rangle - \frac{\phi_a(x)}{2f_\pi}|u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow\rangle$$

Pobylitsa, Polyakov, Strikman, PRL87(2001)022001:

◇ **Chirally rotated nucleon DAs**

Deviation from LET (LO): [Braun:2007pz]

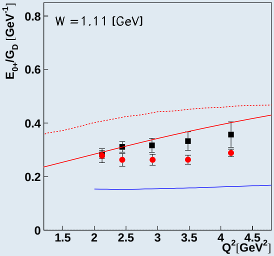




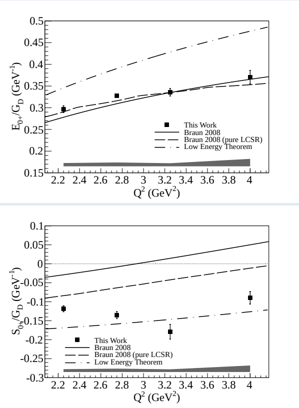
CLAS

$\gamma^* p \to \pi^+ n$

$\gamma^* p \to \pi^0 p$



[Park:2012yf]



[Khetarpal:2012vs]

Summary and Outlook

- Axial FF in a few GeV range can be accessed through threshold pion production

  — Need PWA with emphasize on the threshold region

  — Need theory for $\mathcal{O}(m_\pi Q/m)$ corrections to LET

    — expect 30% corrections

    — LCSRs, hadron models, $\pi^+ n$ vs. $\pi^- p$, etc.

- A new chapter in hard exclusive reactions: use a pion to “rotate” the nucleon WF