Hyperon transition form factors at low energies

Stefan Leupold

Uppsala University

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Historical remark

first hints for compositeness of proton came from

- non-trivial gyromagnetic ratio $\neq 2$

Historical remark

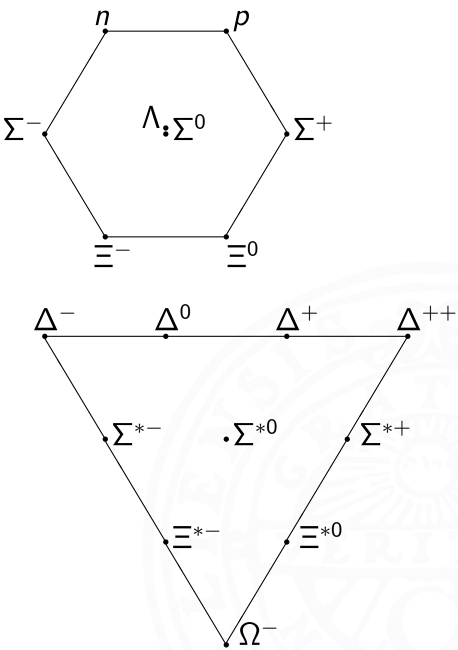
first hints for compositeness of proton came from

- non-trivial gyromagnetic ratio $\neq 2$

and from

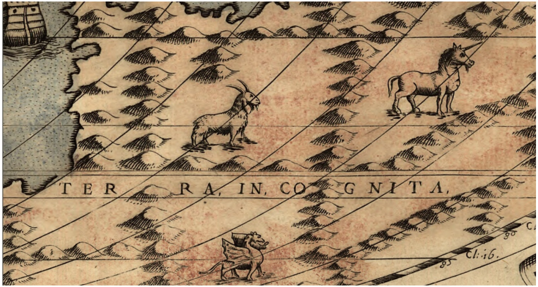
- Gell-Mann's multiplets containing strange hadrons

$\hookrightarrow$ expect valuable information from combining electromagnetism and strangeness



Electromagnetic form factors of hyperons

to large extent
terra incognita



- electron-hyperon scattering complicated

$\rightsquigarrow$ instead:

- reactions $e^+ e^- \longrightarrow$ hyperon anti-hyperon $(Y_1\ \bar{Y}_2) \rightsquigarrow$ BESIII

$\hookrightarrow$ form factors and transition form factors
for large time-like $q^2 > (m_{Y_1} + m_{Y_2})^2$
(time-like means $q^2 > 0$, i.e. energy transfer $>$ momentum transfer)

- decays $Y_1 \longrightarrow Y_2\ e^+\ e^- \rightsquigarrow$ HADES+PANDA

$\hookrightarrow$ transition form factors for small time-like $q^2 < (m_{Y_1} - m_{Y_2})^2$

Theory for low-energy form factors

- in general theorists aim for a good description/prediction of observables

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$\hookrightarrow$ but sometimes "good" is not good enough ...

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$\hookrightarrow$ but sometimes "good" is not good enough ...

... if one needs to know how good the theory is,
i.e. if one needs a reliable estimate of the theory uncertainty

- examples:
  - determination of standard-model parameters (e.g. quark masses)
  - hadronic contributions to high-precision standard model predictions (e.g. gyromagnetic ratio of muon*)

$\hookrightarrow$ develop/use effective field theories (EFTs)
and/or fundamental principles plus data (dispersion theory)

- systematically improvable, reliable uncertainty estimate
- cannot hurt to develop such a framework for hyperons

* see also Hoferichter/Kubis/SL/Niecknig/Schneider, Eur.Phys.J. C74, 3180 (2014)

Theory for baryon low-energy form factors

existing (in EFT spirit):

- for octet: chiral perturbation theory (EFT), Kubis/Meißner, Eur. Phys. J. C 18, 747 (2001)

$\hookrightarrow$ predictions for electric and magnetic radii (= slopes of form factors)

$\hookrightarrow$ shortcomings: no explicit decuplet, no explicit vector mesons

$\hookrightarrow$ for curvatures one already needs vector mesons

- for transitions decuplet-octet: chiral perturbation theory for Δ-N, Pascalutsa/Vanderhaeghen/Yang, Phys. Rept. 437, 125 (2007)

$\hookrightarrow$ no vector mesons and not for hyperons

Theory for hyperon low-energy form factors

new approach:

- hadronic chiral perturbation theory plus dispersion theory

$\hookrightarrow$ easier to include decuplet

$\hookrightarrow$ dispersion theory includes ρ meson as measured in π-π

- Σ^0-Λ transition form factors:
  C. Granados, E. Perotti, SL, arXiv:1701.09130 [hep-ph]

$\hookrightarrow$ some results on next slides

- decuplet-octet transitions:
  E. Perotti, O. Junker, SL, work in progress

technically very similar: J.M. Alarcón, A.N. Hiller Blin, M.J. Vicente Vacas, C. Weiss, arXiv:1703.04534 [hep-ph]

Unitarity and analyticity

- constraints from local quantum field theory:
  partial-wave amplitudes for reactions/decays must be
  - unitary:

$$S\,S^\dagger = 1\,, \quad S = 1 + iT \quad \Rightarrow \quad 2\,\mathrm{Im}\,T = T\,T^\dagger$$

$\hookrightarrow$ note that this is a matrix equation:
$\mathrm{Im}\,T_{A\to B} = \sum_X T_{A\to X}\,T^\dagger_{X\to B}$

$\rightsquigarrow$ in practice: use most relevant intermediate states X

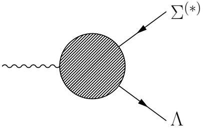
  - analytical (dispersion relations):

$$T(q^2) = T(0) + \frac{q^2}{\pi}\int\limits_{-\infty}^{\infty} ds\,\frac{\mathrm{Im}\,T(s)}{s\,(s-q^2-i\epsilon)}$$

$\rightsquigarrow$ can be used to calculate whole amplitude from imaginary part

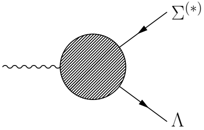
Hyperon transition form factors

- for $\Sigma/\Sigma^* \to \Lambda\, e^+ e^-$ need transition form factors



- dispersive framework: at low energies q^2 dependence is governed by lightest intermediate states

$\hookrightarrow$ obtain



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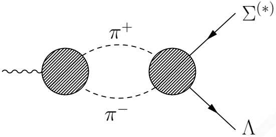
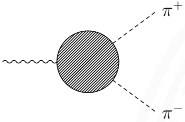
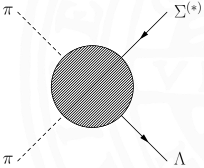


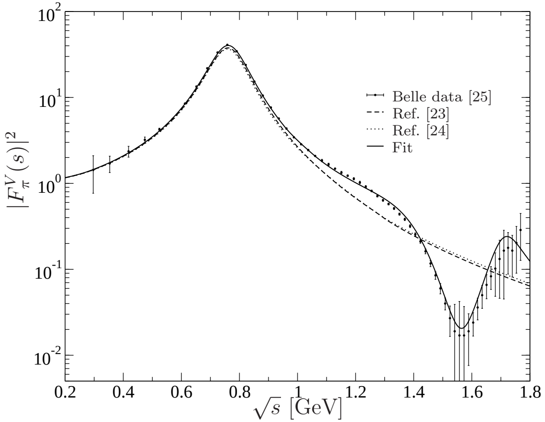
$\hookrightarrow$ need pion vector form factor

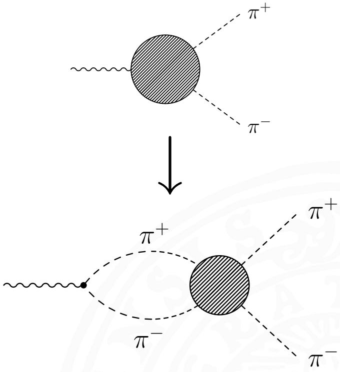


and hyperon-pion scattering amplitudes



Pion vector form factor

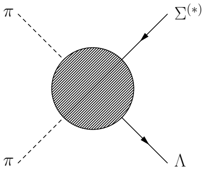




pion phase shift very well known; fits to pion vector form factor

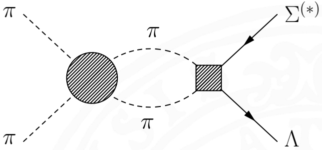
Sebastian P. Schneider, Bastian Kubis, Franz Niecknig, Phys.Rev.D86:054013,2012

Hyperon-pion scattering amplitudes

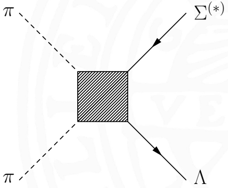


contain

- "right-hand cuts" (pion rescattering)
  - $\hookrightarrow$ straightforward from unitarity and analyticity (and experimental pion phase shift)



- and rest: left-hand cuts, polynomial terms
  - $\hookrightarrow$ not straightforward
  - $\hookrightarrow$ use three-flavor baryon chiral perturbation theory



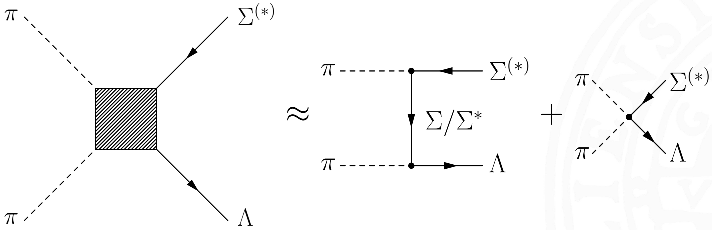
$= \ ?$

Input for hyperon-pion scattering amplitudes

- ideally use data

$\hookrightarrow$ available for pion-nucleon, but not for pion-hyperon

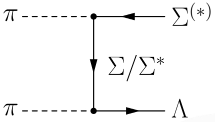
$\hookrightarrow$ instead: three-flavor baryon chiral perturbation theory (χPT) at leading and next-to-leading order (NLO) including decuplet states (optional for $\Sigma^0 \to \Lambda$ transition)



χPT input for hyperon-pion scattering amplitudes

how to determine three-point coupling constants:

- Σ-Λ-π and Σ-Σ-π related to weak octet decays (F and D parameter)
- Σ^*-Λ-π and Σ^*-Σ-π from Σ^* decays (h_A)



interesting observations:

- pole of Σ-exchange contribution is close to 2π threshold

$\hookrightarrow$ creates structure, i.e. energy dependence, that is not covered by simple vector-meson dominance models

see also Frazer/Fulco, Phys.Rev. 117, 1609 (1960)

- in general (away from threshold): large cancelation between Σ- and Σ^*-exchange

$\hookrightarrow$ inclusion of decuplet is not an option but a necessity

χPT input for hyperon-pion scattering amplitudes

how to determine NLO four-point coupling constants:

- only one parameter (b_{10}) for Σ-Λ transition

$\hookrightarrow$ but not very well known

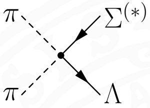
- "resonance saturation" estimates

  Meißner/Steininger/Kubis, Nucl.Phys. B499, 349 (1997);

  Eur.Phys.J. C18, 747 (2001)

or from fit to πN and KN scattering data with coupled-channel Bethe-Salpeter approach

  Lutz/Kolomeitsev, Nucl.Phys. A700, 193 (2002)



- maybe in the future:
  cross-check from lattice QCD

parameter is directly related to magnetic transition radius of Σ-Λ

Uncertainties of input

- parameter variations:
  - $2.2 < h_A < 2.4$
  - $0.85 < b_{10} < 1.35$ (in inverse GeV)
    (Lutz/Kolomeitsev value is at lower edge)
- cut off formal $\Sigma\bar{\Lambda} \to \pi^+\pi^-$ amplitude when other channels except for 2π become important

$\hookrightarrow$ physically at $K\bar{K}$ threshold, but at the latest at $\Sigma\bar{\Lambda}$ threshold

$\hookrightarrow$ vary cutoff in range 1-2 GeV (mild effect)

- NNLO corrections not yet calculated

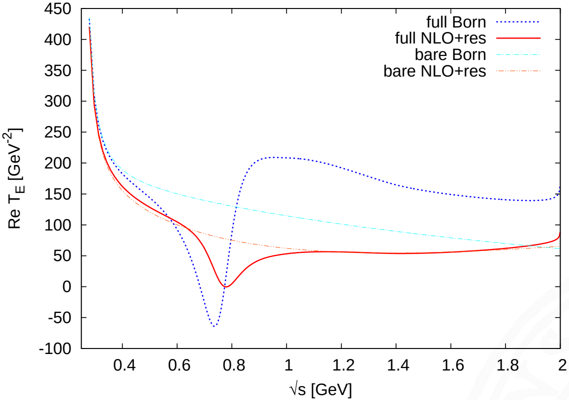
$\hookrightarrow$ no reliable uncertainty estimates yet

- variations in input for pion phase shift not explored yet, but expected to be small

Colangelo/Gasser/Leutwyler, Nucl.Phys. B603, 125 (2001)

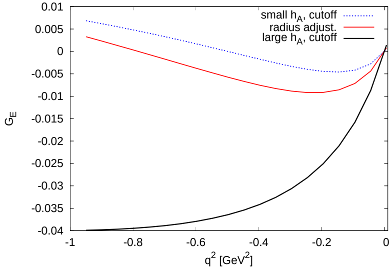
Garcia-Martin/Kaminski/Pelaez/Ruiz de Elvira/Yndurain, Phys.Rev. D83, 074004 (2011)

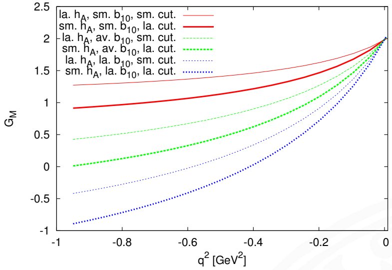
First results



- shows electric (helicity non-flip) part of formal $\Sigma\bar{\Lambda} \to \pi^+\pi^-$ p-wave amplitude (real part, sub threshold)
- impact of ρ meson visible when comparing "full" (dispersive) vs. "bare" (χPT input)
- inclusion of decuplet exchange ("res") important

Transition form factors Σ-Λ





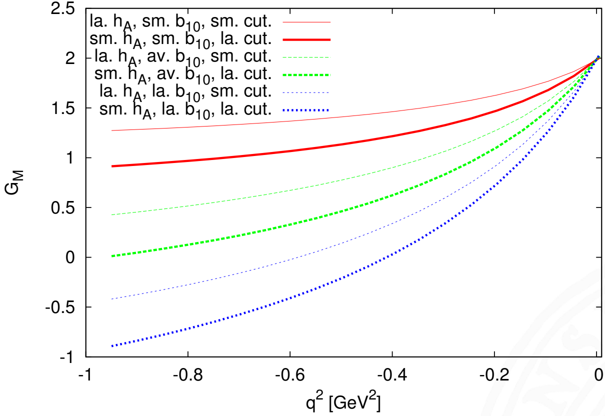
- electric transition form factor very small over large range

$\hookrightarrow$ what one might measure at low energies is magnetic transition form factor

$\hookrightarrow$ data integrated over Λ-e^- angle, but differential in q^2 might be sufficient

- note: Dalitz decay region $4m_e^2 < q^2 < (m_\Sigma - m_\Lambda)^2$ hardly visible here

Magnetic transition form factor Σ-Λ



- large uncertainty

↪ directly related to uncertainty in NLO low-energy constant b_{10}

↪ can be determined from measuring magnetic transition radius

Summary

structure of hadrons

- learned a lot about hadrons from electromagnetic probes
- . . . from strangeness

$\hookrightarrow$ high time to combine these lines of research

$\rightsquigarrow$ electromagnetic (transition) form factors of hyperons

- at low energies: decays $Y_A \to Y_B\, e^+ e^-$
- not even all decays $Y_A \to Y_B\, \gamma$ are measured for initial decuplet states
- complementary theory program: combining dispersion theory with baryon octet+decuplet chiral perturbation theory

$\hookrightarrow$ first results: electric part of Σ-Λ transition very small; magnetic part can be predicted if radius is measured (slope at photon point)

Outlook — speculation

beyond exploring structure of hadrons $\rightsquigarrow$ baryonic CP violation

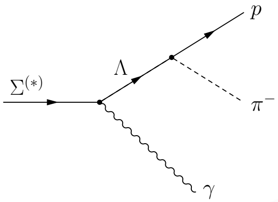
- important for baryon asymmetry of universe (if C is violated) and for strong CP problem
- never observed so far (recently 3σ evidence from LHCb)
- standard observables: angular distributions in weak decays of hyperons vs. antihyperons
- maybe worth to explore: angular distribution in $Y_A \to \gamma\, Y_B \to \gamma\, \pi h$ (related to electric dipole moments)

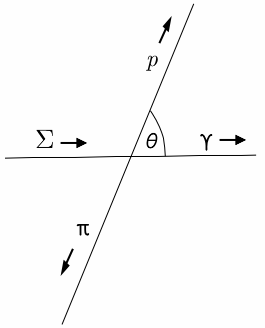
$\hookrightarrow$ S. Nair, SL, work in progress

terra incognita . . . . . . hic sunt dracones

$\Sigma^0 \to \Lambda\,\gamma$

- branching ratio is $\approx 100\%$
- basic: differential distribution for $\Sigma^0 \to \Lambda\,\gamma \to p\pi^-\,\gamma$





- why could this be interesting?

$\hookrightarrow$ parity symmetry of first decay demands isotropic distribution in $\cos\theta$ (as measured in Λ rest frame)

$\hookrightarrow$ advanced:
check for deviation from isotropy as sign for baryonic P and CP violation

backup slides

$\Sigma^0 \to \Lambda\, e^+ e^-$

- basic: branching ratio (QED prediction $5 \cdot 10^{-3}$)
- advanced: differential distribution;
  resolve effect from non-trivial transition form factors G_E, G_M

$$\frac{d^2\Gamma}{dq^2\, dz} \sim \left\{ |G_E(q^2)|^2\, (m_\Lambda + m_\Sigma)^2\, (1 - z^2) + |G_M(q^2)|^2\, q^2\, (1 + z^2) \right\}$$

as compared to (leading-order) QED prediction

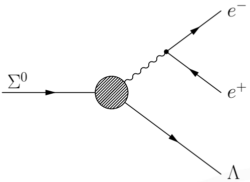
$$\frac{d^2\Gamma_{\rm QED}}{dq^2\, dz} \sim \mu_{\Sigma\Lambda}^2\, q^2\, (1 + z^2)$$

with transition magnetic moment $\mu_{\Sigma\Lambda}$ (known from $\Sigma^0 \to \Lambda\, \gamma$)

- note: proportionality factor is function of $q := p_{e^+} + p_{e^-}$,
  but not of $z :=$cos(angle(e^-, Λ) in dilepton rest frame)

Challenge to extract form factors

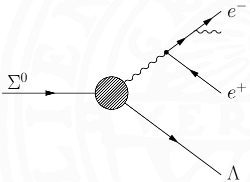
- $\Sigma^0 \to \Lambda\, e^+ e^-$ does not produce large invariant masses for the lepton pair, $q^2 < (77\,\mathrm{MeV})^2$



$\hookrightarrow$ transition form factor is close to unity (normalization at photon point)

$\hookrightarrow$ need high precision in experiment and theory to deduce transition form factor

$\hookrightarrow$ expect effects on 1-2% level



$\hookrightarrow$ effects from form factor compete with QED corrections

$\hookrightarrow$ tedious but calculable

(T. Husek and SL, in preparation)

Full formulae for differential distribution

double differential decay width $\Sigma^0 \to \Lambda\, e^+ e^-$:

$$\frac{d^2\Gamma}{ds\, dz} = \frac{1}{(2\pi)^3\, 64 m_\Sigma^3}\, \lambda^{1/2}(m_\Sigma^2, s, m_\Lambda^2) \left(1 - \frac{4m_e^2}{s}\right)^{1/2} \overline{|\mathcal{M}|^2}$$

$$\begin{aligned} \overline{|\mathcal{M}|^2} \; = \; & \frac{e^4}{s^2}\, 2((m_\Sigma - m_\Lambda)^2 - s) \\ & \left\{ |G_E(s)|^2\, (m_\Lambda + m_\Sigma)^2 \left(1 - \left(1 - \frac{4m_e^2}{s}\right) z^2\right) \right. \\ & \left. \quad + |G_M(s)|^2\, (s\,(1 + z^2) + 4m_e^2\,(1 - z^2)) \right\} . \end{aligned}$$

note: electron mass neglected in main presentation

independent kinematical variables defined on next slide

Full formulae for differential distribution, cont.

independent kinematical variables:
$s := (p_{e^+} + p_{e^-})^2 = q^2$
z is cos of angle between e^- and Λ in rest frame of dilepton

$$z := \frac{\Delta m^2}{(\Delta m^2)_{\max}}$$

with $\Delta m^2 := (p_{e^+} + p_\Lambda)^2 - (p_{e^-} + p_\Lambda)^2$ and

$$\left(\Delta m^2\right)_{\max} := \lambda^{1/2}(m_\Sigma^2, s, m_\Lambda^2) \sqrt{1 - \frac{4m_e^2}{s}} \,.$$

kinematical variables cover the ranges
$z \in [-1, 1]$ and $4m_e^2 \leq s \leq (m_\Sigma - m_\Lambda)^2$

Källén function $\lambda(a, b, c) := a^2 + b^2 + c^2 - 2(ab + bc + ac)$

Theory for low-energy form factors

Confessions of a theorist:

- every theorist has favorite toys
- mine are at the moment
  effective field theories (EFT) and dispersion theory

$\hookrightarrow$ some arguments in favor of this choice:

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$\hookrightarrow$ some arguments in favor of this choice:

- effective theories are systematic $\leftrightarrow$ phenom. models are not
  "systematic" means that one can estimate the theory uncertainty/precision
- dispersion theory uses data instead of phenomenological models

$\hookrightarrow$ (improvable) data uncertainties instead of
(not improvable) model uncertainties

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$\hookrightarrow$ (improvable) data uncertainties instead of
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but: not for every problem there exists an effective theory;
dispersion theory not of practical use if one has to deal with
too many channels, too many particles

Example: effective theory $\leftrightarrow$ phenom. model

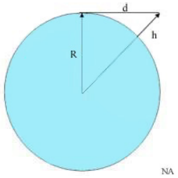


figure from wikipedia

- determine potential energy of object with mass m and height h above ground

$\hookrightarrow$ develop phenomenological model:

$$V_{\text{pheno}}(h) = m\,g\,h \tag{1}$$

- perform measurements for some h (and m) to determine g

$\hookrightarrow$ obtain predictive power for any h

Example: effective theory ↔ phenom. model

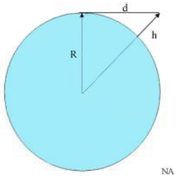


figure from wikipedia

- determine potential energy of object with mass m and height h above ground

↪ develop phenomenological model:

$$V_{\text{pheno}}(h) = m\,g\,h \tag{1}$$

- perform measurements for some h (and m) to determine g

↪ obtain predictive power for any h?

↪ if (1) is not completely correct, then how accurate is it?

↪ phenomenological model cannot answer this,
effective theory can

credits for this example: Emil Ryberg, Gothenburg

Example: effective theory $\leftrightarrow$ phenom. model

- determine potential energy of object with mass m and height h above ground

$\hookrightarrow$ effective theory (systematic):

$$V_{\text{eff}}(h) = m\,(g\,h + g_2\,h^2 + g_3\,h^3 + \dots) \qquad (2)$$

- how to use it:
  - truncate (2) e.g. after $\mathcal{O}(h^2)$ and perform measurements to determine g and g_2
  - theory uncertainty/accuracy $\Delta V \approx |g_2\,h^2|$
  - if unsatisfied with accuracy

  $\hookrightarrow$ truncate (2) only after $\mathcal{O}(h^3)$ and perform (more!) measurements to determine g, g_2 and g_3
  - ...

$\hookrightarrow$ systematically improvable

(but requires more and more measurements to gain predictive power)

Example: effective theory and fundamental theory

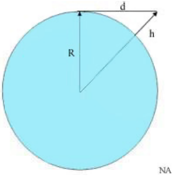


figure from wikipedia

effective theory

$$V_{\rm eff}(h) = m\,(g\,h + g_2\,h^2 + g_3\,h^3 + \ldots)$$

- for this physics problem (potential energy ...) Newton provided the fundamental theory:

$$V_{\rm fund}(h) = -\frac{G\,M\,m}{h+R} + \frac{G\,M\,m}{R}$$

$\hookrightarrow$ parameters g, g_2 , ... can be calculated instead of measured

$\hookrightarrow$ just Taylor expand in h/R

$\hookrightarrow$ range of applicability of effective theory is $h \ll R$

- effective theories always have a limited range of applicability

Back to hadrons

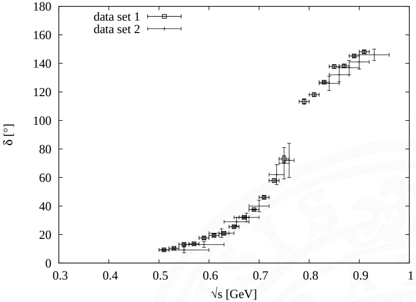
- fundamental theory: QCD
- effective field theory at low energies (below resonances): chiral perturbation theory
- there exist plenty of phenomenological models
  (and some colleagues call them "effective theories" :-(
- for region of hadronic resonances there is no established effective field theory (yet)
  (active field of research: Lutz, Kolomeitsev, SL, Scherer, Meißner, . . .)

$\hookrightarrow$ is there a way to get controlled theory uncertainties in the resonance region?

$\hookrightarrow$ dispersion theory! (sometimes)

Dispersion theory

- if a resonance
  - is important
    (e.g. vector mesons for
    electromagnetic reactions)
  - is known from rather well
    measured phase shifts
  - does not have too many
    decay channels



$\hookrightarrow$ use phase shifts instead of modeling

$\hookrightarrow$ dispersion theory

- based on fundamental principles of local quantum field theory

Right- and left-hand cuts

$$\mathrm{Im}\, T_{A\to B} = \sum_X T_{A\to X}\, T^\dagger_{X\to B}$$

$$T(q^2) = T(0) + \frac{q^2}{\pi} \int\limits_{-\infty}^{\infty} ds\, \frac{\mathrm{Im}\, T(s)}{s\,(s - q^2 - i\epsilon)}$$

- can be used to calculate whole amplitude from imaginary part

$\hookrightarrow$ but need to know imaginary part for all values of s, not only for physical ones restricted by thresholds $s_{\rm thr}$ of A, B, X

- for instance, if $X = 2\pi$ then $s \geq 4m_\pi^2 = s_{\rm thr}$ is physical range

$$\hookrightarrow \quad \int\limits_{s_{\rm thr}}^{\infty} ds \ldots \quad \rightsquigarrow \quad \text{“right-hand cut”}$$

Right- and left-hand cuts

$$T(q^2) = T(0) + \frac{q^2}{\pi} \int\limits_{-\infty}^{\infty} ds \, \frac{\mathrm{Im}\, T(s)}{s\,(s - q^2 - i\epsilon)}$$

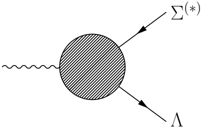
- crossing symmetry: imaginary part in $s \geq s_{\mathrm{thr}}$ leads in crossed channel to imaginary part in Mandelstam variable t (or u)
- but condition $t \geq s_{\mathrm{thr}}$ is in crossed channel related to $s \leq \tilde{s}_{\mathrm{thr}}$

$$\hookrightarrow \quad \int\limits_{-\infty}^{\tilde{s}_{\mathrm{thr}}} ds \, \ldots \quad \rightsquigarrow \quad \text{“left-hand cut”}$$

- note: name “cut” is related to fact that amplitude has logarithmic structure

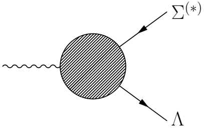
$\hookrightarrow$ Riemann sheets and cuts

Hyperon transition form factors

- for $\Sigma/\Sigma^* \to \Lambda\, e^+ e^-$ need transition form factors 

$\hookrightarrow$ separate long- from short-range physics, universal from quark-structure specific features

$\hookrightarrow$ use dispersion theory and encode short-range physics in subtraction constants

$\hookrightarrow$ obtain 

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