Polarization and dilepton anisotropy in pion–nucleon collisions

Miklós Zétényi

with Enrico Speranza and Bengt Friman

Wigner RCP,
Budapest

ExtreMe Matter
Institute, Darmstadt

Space-like and time-like electromagnetic baryonic transitions
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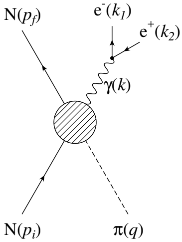
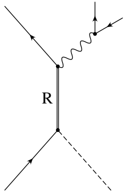
Introduction

$\pi N \rightarrow N e^+ e^-$ measured at HADES



resonance contributions:



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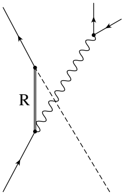

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Angular distributions of the dilepton

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Info on the polarization states of the virtual photon

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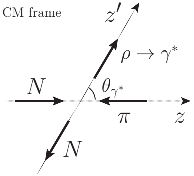
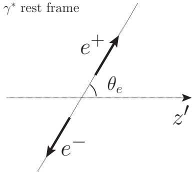
Info on the creation mechanism of the virtual photon
(e.g. on the s-channel resonance)

Anisotropy coefficient





- Triple differential cross section:

$$\frac{d\sigma}{dMd\cos\theta_{\gamma^*}\,d\cos_e} \propto \Sigma_\perp(1+\cos^2\theta_e) + \Sigma_\parallel(1-\cos^2\theta_e)$$

$$\propto A(1+\lambda_\theta(\theta_{\gamma^*}, M)\cos^2\theta_e)$$

- Anisotropy coefficient

$$\lambda_\theta(\theta_{\gamma^*}, M) = \frac{\Sigma_\perp - \Sigma_\parallel}{\Sigma_\perp + \Sigma_\parallel}$$

$\lambda_\theta(\theta_{\gamma^*}, M)$ contains information on the polarization of the virtual photon and hence on the quantum numbers of the baryon resonance

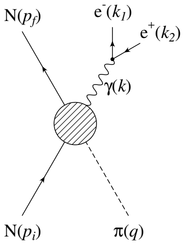
Emergence of anisotropy

- Initial state expanded in terms of orbital angular momentum states:

$$|\pi(\mathbf{p}); N(-\mathbf{p})\rangle \propto \sum_{lm} Y^*_{lm}(\theta,\phi)\,|lm\rangle \qquad (Y_{lm}(\theta=0,\phi)=0 \text{ for } m\neq 0)$$

- If the quantization axis is the z-axis
  $\Rightarrow$ orbital angular momentum does not contribute to total ang. mom.
  $\Rightarrow$ spin of the resonance in s-channel = incoming nucleon spin
- Unpolarized nucleon target $\Rightarrow$ $s_z = \frac{1}{2}$ and $-\frac{1}{2}$ are equally populated
- Resonance with $J_R = \frac{1}{2}$ $\Rightarrow$ all the states are equally populated
  $\Rightarrow$ Isotropic distribution in θ_{γ^*}
- Resonance with $J_R \geq \frac{3}{2}$ $\Rightarrow$ not all the states are populated
  $\Rightarrow$ Anisotropic distribution in θ_{γ^*}

Polarization density matrices



$$\sum_{\text{pol}} |\mathcal{M}|^2 = \frac{e^2}{k^4} H_{\mu\nu} L^{\mu\nu}$$

- Lepton tensor: $L^{\mu\nu} = 4\left(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - (k_1 \cdot k_2) g^{\mu\nu}\right)$
- Hadron tensor: $H_{\mu\nu} = \sum_{\text{pol}} \mathcal{M}_\mu^{\text{had}} \mathcal{M}_\nu^{\text{had}\,*}$

- Define the polarization density matrices:

$\rho^{\text{had}}_{\lambda,\lambda'} = \frac{e^2}{k^4} \epsilon^\mu(k,\lambda) H_{\mu\nu} \epsilon^\nu(k,\lambda')^*$, and $\rho^{\text{lep}}_{\lambda',\lambda} = \epsilon^\mu(k,\lambda') L_{\mu\nu} \epsilon^\nu(k,\lambda)^*$

$$\Rightarrow \qquad \sum_{\text{pol}} |\mathcal{M}|^2 = \sum_{\lambda,\lambda'} \rho^{\text{had}}_{\lambda,\lambda'} \rho^{\text{lep}}_{\lambda',\lambda}$$

Polarization density matrices

- The leptonic density matrix is explicitly known:

$$\rho^{\text{lep}}_{\lambda',\lambda} = 4|\mathbf{k}_1|^2 \begin{pmatrix} 1+\cos^2\theta_e+\alpha & \sqrt{2}\cos\theta_e\sin\theta_e e^{i\phi_e} & \sin^2\theta_e e^{2i\phi_e} \\ \sqrt{2}\cos\theta_e\sin\theta_e e^{-i\phi_e} & 2(1-\cos^2\theta_e)+\alpha & \sqrt{2}\cos\theta_e\sin\theta_e e^{i\phi_e} \\ \sin^2\theta_e e^{-2i\phi_e} & \sqrt{2}\cos\theta_e\sin\theta_e e^{-i\phi_e} & 1+\cos^2\theta_e+\alpha \end{pmatrix}$$

where $\alpha = 2m_e^2/|\mathbf{k}_1|^2$ (neglect in the following)

- This gives the angular distribution of e^+ and e^- in the virtual photon rest frame:

$$\sum_{\text{pol}} |\mathcal{M}|^2 \propto (1+\cos^2\theta_e)(\rho^{\text{had}}_{-1,-1}+\rho^{\text{had}}_{1,1}) + 2(1-\cos^2\theta_e)\rho^{\text{had}}_{0,0}$$
$$+ \sin^2\theta_e(e^{2i\phi_e}\rho^{\text{had}}_{-1,1} + e^{-2i\phi_e}\rho^{\text{had}}_{1,-1})$$
$$+ \sqrt{2}\cos\theta_e\sin\theta_e\left[e^{i\phi_e}(\rho^{\text{had}}_{-1,0}+\rho^{\text{had}}_{0,1}) + e^{-i\phi_e}(\rho^{\text{had}}_{1,0}+\rho^{\text{had}}_{0,-1})\right]$$

- c.f.:

$$\frac{d\sigma}{dMd\cos\theta_{\gamma^*}\,d\cos_e} \propto \Sigma_\perp(1+\cos^2\theta_e) + \Sigma_\parallel(1-\cos^2\theta_e)$$

$$\Rightarrow \quad \Sigma_\perp = \rho^{\text{had}}_{-1,-1}+\rho^{\text{had}}_{1,1}, \quad \text{and} \quad \Sigma_\parallel = 2\rho^{\text{had}}_{0,0}$$

Models

- Gauge invariant vector meson dominance for $\rho^0 - \gamma^*$ coupling (N. M. Kroll, T. D. Lee and B. Zumino, Phys. Rev. **157** (1967) 1376)

$$\mathcal{L}_{\rho\gamma} = -\frac{e}{2g_\rho} F^{\mu\nu} \rho^0_{\mu\nu}$$

- Interactions for spin-1/2 resonances with π and ρ

$$\begin{aligned} \mathcal{L}_{R_{1/2}N\pi} &= -\frac{g_{RN\pi}}{m_\pi} \bar{\psi}_R \Gamma \gamma^\mu \vec{\tau} \psi_N \cdot \partial_\mu \vec{\pi} + \text{h.c.} \\ \mathcal{L}_{R_{1/2}N\rho} &= \frac{g_{RN\rho}}{2m_\rho} \bar{\psi}_R \vec{\tau} \sigma^{\mu\nu} \tilde{\Gamma} \psi_N \cdot \vec{\rho}_{\mu\nu} + \text{h.c.} \end{aligned}$$

$\Gamma = \gamma_5$ for $J^P = 1/2^+$, $\Gamma = 1$ for $J^P = 1/2^-$

Consistent interactions for higher spin resonances

- Lower spin components of the Rarita-Schwinger fields should not contribute
- Lagrangians must be invariant under the transformations:

$$\psi_\mu \to \psi_\mu + i\partial_\mu \chi \qquad (spin-3/2)$$

$$\psi_{\mu\nu} \to \psi_{\mu\nu} + \frac{i}{2}(\partial_\mu \chi_\nu - \partial_\nu \chi_\mu) \qquad (spin-5/2)$$

- Gauge invariant operators:

$$G_{\mu,\nu} = i(\partial_\mu \psi_\nu - \partial_\nu \psi_\mu)$$

$$G_{\mu\nu,\lambda\rho} = -\partial_\mu\partial_\nu\psi_{\lambda\rho} - \partial_\lambda\partial_\rho\psi_{\mu\nu} + \frac{1}{2}(\partial_\mu\partial_\lambda\psi_{\nu\rho} + \partial_\mu\partial_\rho\psi_{\nu\lambda} + \partial_\nu\partial_\lambda\psi_{\mu\rho} + \partial_\nu\partial_\rho\psi_{\mu\lambda})$$

$RN\pi$ vertex

$$\mathcal{L}_{R_{3/2}N\pi} = \frac{ig_{RN\pi}}{m_\pi^2}\bar{\Psi}_R^\mu \Gamma \vec{\tau}\psi_N \cdot \partial_\mu \vec{\pi}$$

$$\mathcal{L}_{R_{5/2}N\pi} = -\frac{g_{RN\pi}}{m_\pi^4}\bar{\Psi}_R^{\mu\nu} \Gamma \vec{\tau}\psi_N \cdot \partial_\mu\partial_\nu \vec{\pi}$$

$RN\rho$ vertex

$$\mathcal{L}_{R_{3/2}N\rho} = \frac{ig_{RN\rho}}{4m_N^2}\bar{\Psi}_R^\mu \vec{\tau}\gamma^\nu \tilde{\Gamma}\psi_N \cdot \vec{\rho}_{\nu\mu}$$

$$\mathcal{L}_{R_{5/2}N\rho} = -\frac{g_{RN\rho}}{(2m_N)^4}\bar{\Psi}_R^{\mu\nu} \vec{\tau}\tilde{\Gamma}\gamma^\rho(\partial_\mu\psi_N) \cdot \vec{\rho}_{\rho\nu}$$

$$\Psi_\mu = \gamma^\nu G_{\mu,\nu} \qquad \Psi_{\mu\nu} = \gamma^\lambda\gamma^\rho G_{\mu\nu,\lambda\rho}$$

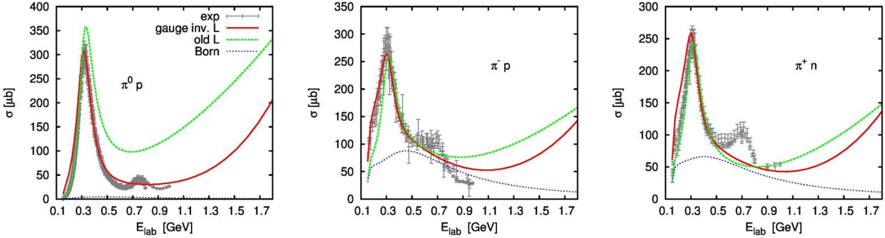
$\Gamma = \gamma_5$ for $J^P = 3/2^-, 5/2^+$ and $\Gamma = 1$ otherwise. $\tilde{\Gamma} = \gamma_5\Gamma$.

Two other possibilities for the $RN\rho$ vertices

V. Pascalutsa, Phys. Rev. D **58** (1998) 096002
T. Vrancx *et al.*, Phys. Rev. C **84** (2011) 045201

Consistent interactions for higher spin resonances

Illustration – pion-photoproduction:

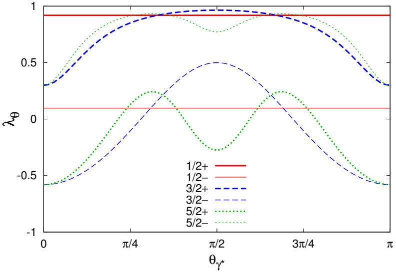


- Born + s- and u-channel Δ contribution
- u-channel violates unitarity $\rightarrow$ unphysical rise at high E_{lab}
- "gauge invariant" Lagrangian is better

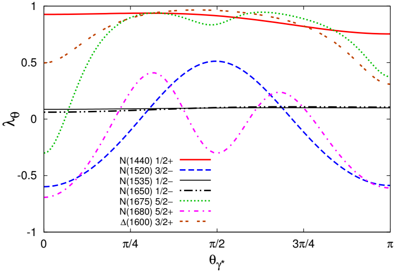
Anistropy coefficients

$$\sqrt{s} = 1.49\,\text{GeV} \qquad M = 0.5\,\text{GeV}$$

$|A_s|^2$



$|A_s^i + A_u^i|^2$

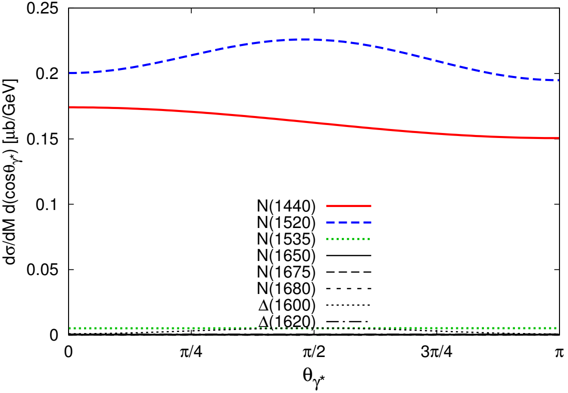


- Spin and parity of the intermediate resonance is reflected in a characteristic angular dependence of λ_θ
- The *u*-channel is negligible on-shell

Angular distributions

$$\sqrt{s} = 1.49\,\text{GeV} \qquad M = 0.5\,\text{GeV}$$

$$|A_s^i + A_u^i|^2$$

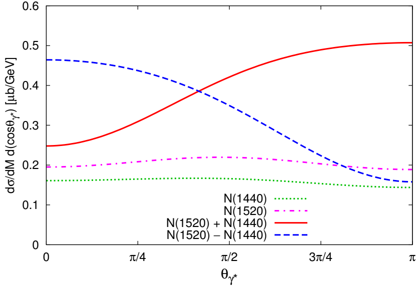


- N(1440) and N(1520) are dominant
- Coupling constants are determined from decay rates (PDG)

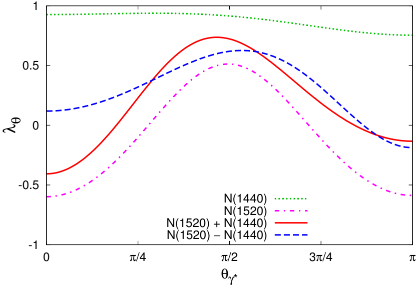
N(1440) and N(1520)

$$\sqrt{s} = 1.49\,\text{GeV} \qquad M = 0.5\,\text{GeV}$$

$|A_s^i + A_u^i|^2$



$|A_s^i + A_u^i|^2$

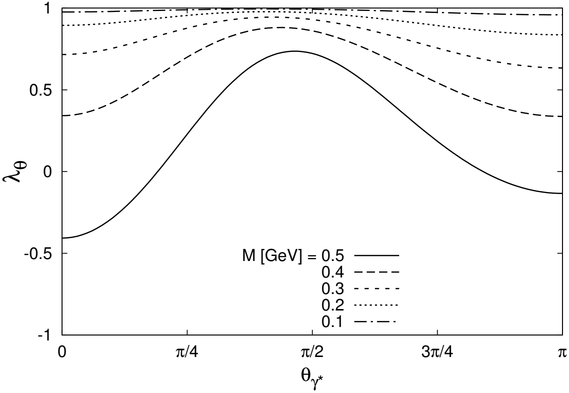


- Unknown relative phase between contributions of the two resonances
- λ_θ does not depend strongly on the relative phase

Invariant mass dependence for λ_θ

$$\sqrt{s} = 1.49\,\text{GeV}$$

$$\sum_i |A_s^i + A_u^i|^2 \quad N(1440),\ N(1520)$$

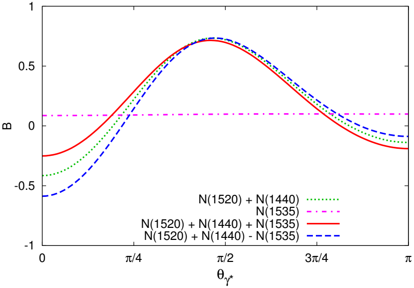


- $\lambda_\theta = \frac{\Sigma_\perp - \Sigma_\parallel}{\Sigma_\perp + \Sigma_\parallel} \to 1$ as $M \to 0$ (real photon limit $\Sigma_\parallel \to 0$)
- Rough binning both in M and θ_{γ^*} would be sufficient

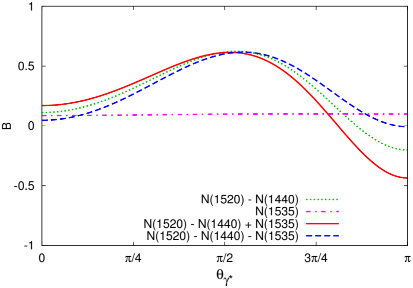
Adding N(1535)

$\sqrt{s} = 1.49\,\text{GeV} \qquad M = 0.5\,\text{GeV}$

$|A_s^i + A_u^i|^2$



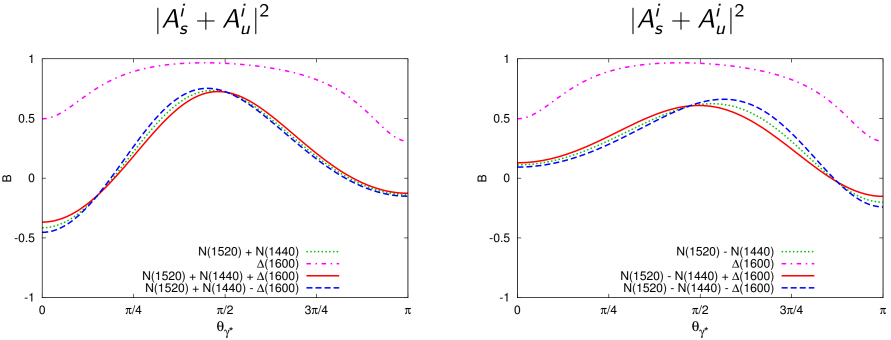
$|A_s^i + A_u^i|^2$



- λ_θ does not depend strongly on the relative sign of the couplings

Adding $\Delta(1600)$

$$\sqrt{s} = 1.49\,\text{GeV} \qquad M = 0.5\,\text{GeV}$$



- λ_θ does not depend strongly on the relative phase of the couplings

Conclusions

Summary

- The anisotropy coefficient λ_θ reflects the polarization state of the virtual photon
- The shape of $\lambda_\theta(\theta_{\gamma^*})$ is characteristic of the spin parity of the resonances
- N(1440) and N(1520) are dominant at $\sqrt{s} = 1.49\,\text{GeV}$. The interference of resonances has a weak effect on the anisotropy coefficient, but a strong effect on the angular distributions
- Adding N(1535) or Δ(1600) does not influence strongly λ_θ

Outlook

- Calculations with the other two kinds of $RN\rho$ vertices needed
- Non-resonant contributions must be added
- Study the two pion final state
- Rough binning both in M and θ_{γ^*} would be sufficient for extraction information on polarization

E. Speranza, M. Z., B. Friman, PLB **764** (2017) 282.