Towards generic adiabatic elimination for bipartite open quantum systems

22ème conférence Claude Itzykson
Manipulation of Simple Quantum Systems

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Joint work with R. Azouit, F. Chittaro and A. Sarlette (arXiv.1704.00785)

Slow/fast bipartite master equations

Model reduction and geometric singular perturbations

Geometric singular perturbations for bipartite quantum systems

Several contributions on adiabatic elimination (partial list)

- Lambda systems :
  E. Brion, L.H. Pedersen, K. Mølmer : Adiabatic elimination in a lambda system Journal of Physics A : Mathematical and Theoretical, 2007, 40, 1033.
  M. Mirrahimi, PR :. Singular perturbations and Lindblad-Kossakowski differential equations IEEE Trans. Automatic Control , 2009, 54, 1325-1329
  F. Reiter, A. Sørensen : Effective operator formalism for open quantum systems Phys. Rev. A, 2012, 85, 032111-
- Slow/fast Lindblad dynamics :
  E.M. Kessler : Generalized Schrieffer-Wolff formalism for dissipative systems. Phys. Rev. A, 2012, 86, 012126-
  D. Burgarth et al. : Non-Abelian Phases from a Quantum Zeno Dynamics. Phys. Rev. A 88, 042107 (2013)
  P. Zanardi, L. Campos Venuti : Coherent quantum dynamics in steady-state manifolds of strongly dissipative systems. Phys. Rev. Lett. 113, 240406 (2014)
  K. Macieszczak, M. Guta, I. Lesanovsky,J.P. Garrahan : Towards a Theory of Metastability in Open Quantum Dynamics. Phys. Rev. Lett. 116, 240404 (2016)
  L. Campos Venuti, P. Zanardi : Dynamical Response Theory for Driven-Dissipative Quantum Systems. Phys. Rev. A 93, 032101 (2016)
- Quantum stochastic models :
  J. Gough, R. van Handel : Singular perturbation of quantum stochastic differential equations with coupling through an oscillator model. J. Stat. Phys. 2007, 127 pp :575.
  L. Bouten, A. Silberfarb : Adiabatic elimination in quantum stochastic model, Commun. Math. Phys., 283, 491-505 (2008)
  L. Bouten, R. van Handel, A. Silberfarb : Approximation and limit theorems for quantum stochastic models with unbounded coefficients. Journal of Functional Analysis 254 (2008) 3123-3147.
  O. Cernotik, D. Vasilyev, K. Hammerer : Adiabatic elimination of Gaussian subsystems from quantum dynamics under continuous measurement Phys. Rev. A, , 92, 012124 (2015)

Bipartite slow/fast open quantum systems

Lindblad-Gorini-Kossakowsi-Sudarshan master equation [1] :

$$\frac{d}{dt}\rho = \mathcal{L}(\rho) = -i[\boldsymbol{H}, \rho] + \sum_{\mu} \left(\boldsymbol{L}_{\mu} \rho \boldsymbol{L}_{\mu}^{\dagger} - \tfrac{1}{2}(\boldsymbol{L}_{\mu}^{\dagger} \boldsymbol{L}_{\mu} \rho + \rho \boldsymbol{L}_{\mu}^{\dagger} \boldsymbol{L}_{\mu}) \right)$$

with ρ, $\boldsymbol{H}$ and $\boldsymbol{L}_{\mu}$ operators on the underlying Hilbert space $\mathcal{H}$.

- Sub-system A with Hilbert space $\mathcal{H}_A$ relaxing rapidly towards a unique equilibrium density operator $\overline{\rho}_A$ via the Lindbladian evolution :

$$\frac{d}{dt}\rho_A = \mathcal{L}_A(\rho_A) \text{ with } \lim_{t \mapsto +\infty} \rho_A(t) = \overline{\rho}_A;$$

- Sub-system B with Hilbert space $\mathcal{H}_B$ having a slow Lindbladian evolution

$$\frac{d}{dt}\rho_B = \epsilon \mathcal{L}_B(\rho_B) \text{ with } 0 \leq \epsilon \ll 1$$

- Weak (A, B) coupling via the Hamiltonian $\epsilon \sum_{k=1}^{m} \boldsymbol{A}_k \otimes \boldsymbol{B}_k^{\dagger}$

Bipartite Hilbert space (A, B) with Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ and density operator ρ governed by

$$\frac{d}{dt}\rho = \mathcal{L}_A(\rho) - i\epsilon \left[\sum_{k=1}^{m} \boldsymbol{A}_k \otimes \boldsymbol{B}_k^{\dagger} \,,\, \rho \right] + \epsilon \mathcal{L}_B(\rho)$$

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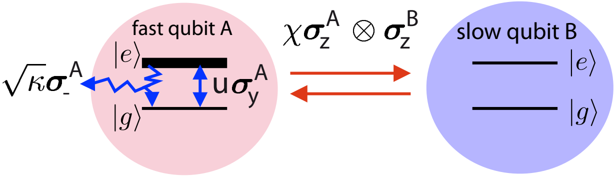
1. Lindblad G : On the generators of quantum dynamical semigroups. Commun. Math. Phys. 48 119-30 (1976)
Gorini V, Kossakowsi A and Sudarshan E C G : Completely positive dynamical semigroups of N-level systems. J. Math. Phys. 17 821-5 (1976)

Adiabatic elim. of fast qubit A dispersively coupled to slow qubit B[2]



The slow/fast dynamics

$$\frac{d\rho}{dt} = u\left[\boldsymbol{\sigma}_+^A - \boldsymbol{\sigma}_-^A\, ,\, \rho\right] + \kappa\left(\boldsymbol{\sigma}_-^A \rho \boldsymbol{\sigma}_+^A - \frac{\boldsymbol{\sigma}_+^A\boldsymbol{\sigma}_-^A\rho + \rho\boldsymbol{\sigma}_+^A\boldsymbol{\sigma}_-^A}{2}\right) - i\chi\left[\boldsymbol{\sigma}_z^A \otimes \boldsymbol{\sigma}_z^B, \rho\right]$$

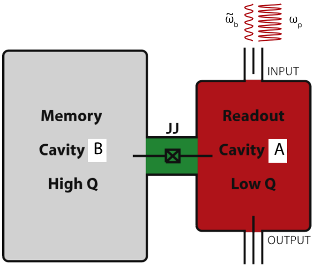
Slow dynamics (second order versus $\epsilon = \chi/\kappa$) :

$$\frac{d\rho_s}{dt} = i\frac{\chi\kappa^2}{\kappa^2+8u^2}[\boldsymbol{\sigma}_z, \rho_s] + \frac{(64\kappa\chi^2u^2)(\kappa^2+2u^2)}{(\kappa^2+8u^2)^3}\left(\boldsymbol{\sigma}_z\rho_s\boldsymbol{\sigma}_z - \rho_s\right)$$

Kraus (CPTP) map : $\rho = (\boldsymbol{I} - i\boldsymbol{Q}\otimes\boldsymbol{\sigma}_z)(\overline{\rho}_A \otimes \rho_s)(\boldsymbol{I} + i\boldsymbol{Q}^\dagger \otimes \boldsymbol{\sigma}_z)$ with
$\overline{\rho}_A = \frac{4\kappa u}{\kappa^2+8u^2}\boldsymbol{\sigma}_x - \frac{\kappa^2}{\kappa^2+8u^2}\boldsymbol{\sigma}_z + \frac{1}{2}\boldsymbol{I}$ and $\boldsymbol{Q} = \bullet\boldsymbol{\sigma}_x + \bullet\boldsymbol{\sigma}_y + \bullet\boldsymbol{\sigma}_z + \bullet\boldsymbol{I}$

2. R. Azouit, F. Chittaro, A. Sarlette, P.R., IFAC world congress 2017.

Two-photon pumping in super-conducting circuits [3]



$\frac{d}{dt}\rho = \mathcal{L}_A(\rho) - i[\boldsymbol{H}_{\text{int}}, \rho] + \mathcal{L}_B(\rho)$ where

$$\mathcal{L}_A(\rho) = [u\boldsymbol{a}^\dagger - u^*\boldsymbol{a}, \rho] + \kappa\mathcal{D}_{\boldsymbol{a}}(\rho)$$

$$\boldsymbol{H}_{\text{int}} = g[\boldsymbol{a}(\boldsymbol{b}^\dagger)^2 + \boldsymbol{a}^\dagger\boldsymbol{b}^2, \rho] + \chi(\boldsymbol{a}^\dagger\boldsymbol{a})(\boldsymbol{b}^\dagger\boldsymbol{b}) + \tfrac{\chi_a}{2}(\boldsymbol{a}^\dagger\boldsymbol{a})^2$$

$$\mathcal{L}_B(\rho) = -i\tfrac{\chi_b}{2}[(\boldsymbol{b}^\dagger\boldsymbol{b})^2, \rho]$$

with $\kappa \gg \max\left(|g|, |\chi|, |\chi_a|, |\chi_b|\right)$.

The slow dynamics (second order approximation, $\alpha = 2u/\kappa$) :

$$\frac{d}{dt}\rho_s = -i\left[\alpha^2\chi\boldsymbol{b}^\dagger\boldsymbol{b} + \tfrac{\chi_b}{2}(\boldsymbol{b}^\dagger\boldsymbol{b})^2, \rho_s\right] - i\alpha g[\boldsymbol{b}^2 + (\boldsymbol{b}^\dagger)^2, \rho_s] + \left(\frac{4g^2}{\kappa}\right)\mathcal{D}_{\boldsymbol{L}_s}(\rho)$$

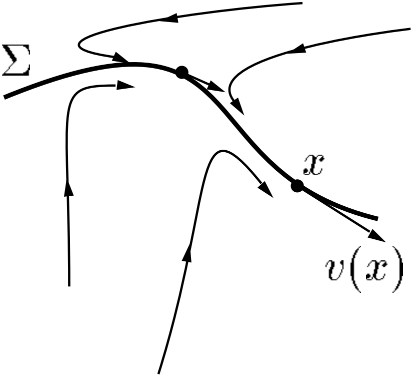
with $\boldsymbol{L}_s = \boldsymbol{b}^2 + \frac{\alpha}{g}\left(\chi\boldsymbol{b}^\dagger\boldsymbol{b} + \frac{\chi_a(1+2\alpha^2)}{2}\boldsymbol{I}\right)$.

Kraus (CPTP) map : $\rho = (\boldsymbol{I} - i\boldsymbol{M})\left(|\alpha\rangle\langle\alpha| \otimes \rho_s\right)(\boldsymbol{I} + i\boldsymbol{M}^\dagger)$ with

$$\boldsymbol{M} = (\boldsymbol{a}^\dagger - \alpha^*) \otimes \left(\frac{2g}{\kappa}\boldsymbol{b}^2 + \frac{2\alpha\chi}{\kappa}\boldsymbol{b}^\dagger\boldsymbol{b} + \frac{2\alpha(1+2\alpha^2)\chi_a}{\kappa}\boldsymbol{I}\right) + \frac{\alpha^2\chi_a}{\kappa}(\boldsymbol{a}^\dagger - \alpha^*)^2 \otimes \boldsymbol{I}.$$

3. M. Mirrahimi, Z. Leghtas, V.V. Albert, S. Touzard, R.J. Schoelkopf, L. Jiang, and M.H. Devoret. Dynamically protected cat-qubits : a new paradigm for universal quantum computation. New J. of Physics, 16 :045014, 2014.

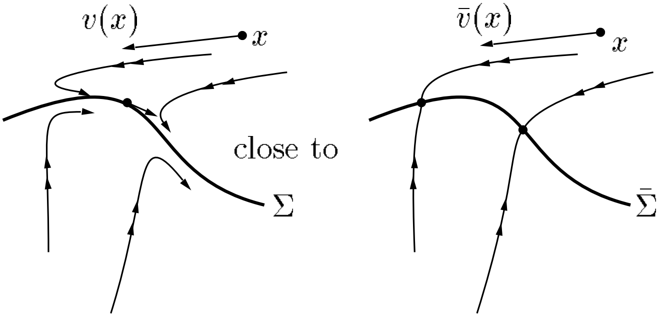
What is a dynamical reduced model for $\frac{d}{dt}x = v(x)$?



A possible answer :

restriction to an attractive invariant manifold Σ.

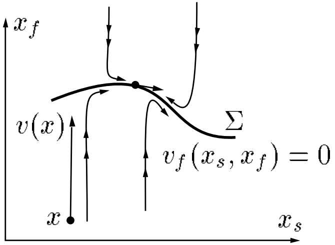
Slow/fast systems (coordinate free setting)



Geometric definition independent of coordinates due to Fenichel [4] :

- $x \mapsto v(x)$ close to $x \mapsto \overline{v}(x)$.
- $\overline{v}(x) = 0$ define a manifold $\overline{\Sigma}$ of dimension $n_s < n = \dim(x)$ of steady-states for $\overline{v}(x)$.
- $n_f = n - n_s$ eigenvalues of $\left.\frac{\partial \overline{v}}{\partial x}\right|_{\overline{\Sigma}}$ are stable (negative real parts).

4. N. Fenichel : Geometric singular perturbation theory for ordinary differential equations. J. Diff. Equations, 1979, 31, 53-98.



Any slow/fast system, can be put, after a suitable change of coordinates, in to a **quasi-vertical vector field** v :

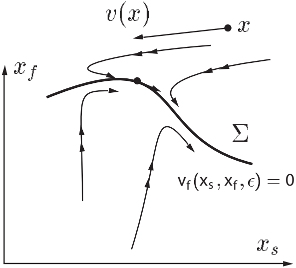
$$\frac{d}{dt}x_s = v_s(x_s, x_f) = \epsilon \tilde{v}_s(x_s, x_f, \epsilon)$$
$$\frac{d}{dt}x_f = v_f(x_s, x_f)$$

with $0 < \epsilon \ll 1$.

The reduced system $\frac{d}{dt}x_s = v_s(x_s, x_f)$ with $0 = v_f(x_s, x_f)$ is correct if $\frac{d}{dt}\xi_f = v_f(x_s, \xi_f)$ hyperbolically stable for any fixed x_s.

In general, modeling variables x are **not** Tikhonov variables.

5. See, e.g., F. Verhulst : Methods and Applications of Singular Perturbations : Boundary Layers and Multiple Timescale Dynamics. Springer, 2005



Example with the heuristic method :

$$\frac{d}{dt}x_s = 2(x_s - x_f) + \epsilon x_s \quad \frac{d}{dt}x_f = x_s - x_f$$

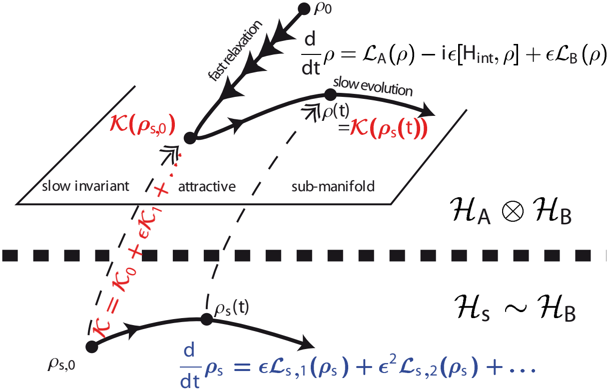
1- compute x_f versus x_s from $\frac{d}{dt}x_f = 0$;
2- plug $x_f = x_s$ into $\frac{d}{dt}x_s$ to obtain
$\boldsymbol{\frac{d}{dt}x_s = +\epsilon x_s}$ (wrong slow model !)

The reduced model of $\frac{d}{dt}x_s = v_s(x_s, x_f, \epsilon)$, $\frac{d}{dt}x_f = v_f(x_s, x_f, \epsilon)$ is [6]

$$\frac{d}{dt}x_s = \left(1 + \frac{\partial v_s}{\partial x_f}\left(\frac{\partial v_f}{\partial x_f}\right)^{-2}\frac{\partial v_f}{\partial x_s}\right)^{-1} v_s(x_s, x_f, \epsilon) + O(\epsilon^2), \quad v_f(x_s, x_f, \epsilon) = 0.$$

Same example with the correct method : with $\frac{\partial v_s}{\partial x_f} = -2$, $\frac{\partial v_f}{\partial x_s} = 1 = -\frac{\partial v_f}{\partial x_f}$, we get the correct slow model , $\boldsymbol{\frac{d}{dt}x_s = -\epsilon x_s}$.

6. J. Carr : Application of Center Manifold Theory. Springer, 1981. P. Duchêne, P.R. : Kinetic scheme reduction via geometric singular perturbation techniques. Chem. Eng. Science, 1996, 51, 4661-4672.



Lindbladian slow dynamics in a copy $\mathcal{H}_s$ of $\mathcal{H}_B$

$$\frac{d}{dt}\rho_s = \mathcal{L}_s(\rho_s) = \epsilon\mathcal{L}_{s,1}(\rho_s) + \epsilon^2\mathcal{L}_{s,2}(\rho_s) + \dots$$

with **Kraus map** to recover the physical density operator ρ from ρ_s :

$$\rho = \mathcal{K}(\rho_s) = \mathcal{K}_0(\rho_s) + \epsilon\mathcal{K}_1(\rho_s) + \dots$$

7. R. Azouit et al. IEEE CDC 2016.

An iterative procedure based on center manifold approximation

Plug $\rho = \mathcal{K}(\rho_s) = \overline{\rho}_A \otimes \rho_s + \epsilon \mathcal{K}_1(\rho_s) + \ldots$ and $\frac{d}{dt}\rho_s = \mathcal{L}_s(\rho_s) = \epsilon \mathcal{L}_{s,1}(\rho_s) + \epsilon^2 \mathcal{L}_{s,2}(\rho_s) + \ldots$ into invariance condition

$$\mathcal{L}_A(\mathcal{K}(\rho_s)) - \epsilon i \left[\boldsymbol{H}_{\text{int}}, \mathcal{K}(\rho_s) \right] + \epsilon \mathcal{L}_B(\mathcal{K}(\rho_s)) = \frac{d}{dt}\rho = \mathcal{K}(\mathcal{L}_s(\rho_s))$$

and identify terms of same orders :

order 1 : $\mathcal{L}_A(\mathcal{K}_1(\rho_s)) - i \left[\boldsymbol{H}_{\text{int}}, \mathcal{K}_0(\rho_s) \right] + \mathcal{L}_B(\mathcal{K}_0(\rho_s)) = \mathcal{K}_0(\mathcal{L}_{s,1}(\rho_s))$

order 2 : $\mathcal{L}_A(\mathcal{K}_2(\rho_s)) - i \left[\boldsymbol{H}_{\text{int}}, \mathcal{K}_1(\rho_s) \right] + \mathcal{L}_B(\mathcal{K}_1(\rho_s)) = \mathcal{K}_0(\mathcal{L}_{s,2}(\rho_s)) + \mathcal{K}_1(\mathcal{L}_{s,1}(\rho_s))$

$\ldots$

At each order

1. take the trace versus A to get the correction to $\mathcal{L}_s$
2. compute the correction to $\mathcal{K}$ via $-\mathcal{L}_A^{-1}$, a super operator for zero-trace operators $\boldsymbol{W}$ on $\mathcal{H}_A$

$$-\mathcal{L}_A^{-1}(\boldsymbol{W}) = \int_0^{+\infty} e^{t\mathcal{L}_A}(\boldsymbol{W}) dt$$

that coincides with the restriction to zero-trace operators of a completely positive (CP) map.

First order expansion : Zeno dynamics and **"Zeno map"** [8]

The full dynamics

$$\frac{d}{dt}\rho = \mathcal{L}_A(\rho) - i\epsilon \left[\sum_{k=1}^{m} \boldsymbol{A}_k \otimes \boldsymbol{B}_k^\dagger \,,\ \rho \right] + \epsilon \mathcal{L}_B(\rho)$$

can be approximated by

$$\frac{d}{dt}\rho_s = \underbrace{-i\epsilon \left[\sum_{k=1}^{m} \mathrm{tr}(\boldsymbol{A}_k \overline{\rho}_A) \boldsymbol{B}_k^\dagger \,,\ \rho_s \right] + \epsilon \mathcal{L}_B(\rho_s)}_{\text{Zeno dynamics}} + O(\epsilon^2)$$

$$\rho = \underbrace{\left(\boldsymbol{I} - i\epsilon \boldsymbol{M}\right) \left(\overline{\rho}_A \otimes \rho_s\right) \left(\boldsymbol{I} + i\epsilon \boldsymbol{M}^\dagger\right)}_{\text{completely positive map} \triangleq \text{"Zeno map"}} + O(\epsilon^2)$$

where $\boldsymbol{M} = \sum_{k=1}^{m} \boldsymbol{F}_k \otimes \boldsymbol{B}_k^\dagger$ with $\boldsymbol{F}_k$ given by

$$\boldsymbol{F}_k \overline{\rho}_A = -\mathcal{L}_A^{-1}\left(\boldsymbol{A}_k\, \overline{\rho}_A - \mathrm{tr}(\boldsymbol{A}_k\, \overline{\rho}_A) \overline{\rho}_A \right).$$

8. A. Azouit et al. arXiv.1704.00785

Second order dynamics [9]

The full dynamics

$$\frac{d}{dt}\rho = \mathcal{L}_A(\rho) - i\epsilon\left[\sum_{k=1}^{m} \boldsymbol{A}_k \otimes \boldsymbol{B}_k^\dagger \ , \ \rho\right] + \epsilon\mathcal{L}_B(\rho)$$

can be approximated by

$$\frac{d}{dt}\rho_s = -i\left[\epsilon\sum_k \mathrm{tr}(\boldsymbol{A}_k\overline{\rho}_A)\boldsymbol{B}_k + \epsilon^2\sum_{k,j} y_{k,j}\,\boldsymbol{B}_k\boldsymbol{B}_j^\dagger \ , \ \rho_s\right]$$

$$+\epsilon\mathcal{L}_B(\rho_s) + \epsilon^2\sum_{k=1}^{m}\mathcal{D}_{\boldsymbol{L}_k}(\rho_s) + O(\epsilon^3)$$

$$\rho = \left(\boldsymbol{I} - i\epsilon\boldsymbol{M}\right)\left(\overline{\rho}_A \otimes \rho_s\right)\left(\boldsymbol{I} + i\epsilon\boldsymbol{M}^\dagger\right) \ + O(\epsilon^2)$$

where $\boldsymbol{M} = \sum_{k=1}^{m} \boldsymbol{F}_k \otimes \boldsymbol{B}_k^\dagger$ with $\boldsymbol{F}_k\overline{\rho}_A = -\mathcal{L}_A^{-1}\left(\boldsymbol{A}_k\,\overline{\rho}_A - \mathrm{tr}(\boldsymbol{A}_k\,\overline{\rho}_A)\overline{\rho}_A\right)$
where $y_{k,j} = \frac{1}{2i}\mathrm{tr}\left(\boldsymbol{F}_j\overline{\rho}_A\boldsymbol{A}_k^\dagger - \boldsymbol{A}_j\overline{\rho}_A\boldsymbol{F}_k^\dagger\right)$ and $\boldsymbol{L}_k = \sum_{j=1}^{m}\lambda_{j,k}\boldsymbol{B}_j$ with matrix λ given by $\lambda\lambda^\dagger = x$ and $x_{k,j} = \mathrm{tr}\left(\boldsymbol{F}_j\overline{\rho}_A\boldsymbol{A}_k^\dagger + \boldsymbol{A}_j\overline{\rho}_A\boldsymbol{F}_k^\dagger\right)$

9. A. Azouit et al. arXiv.1704.00785

Key formula of $\mathcal{L}_A^{-1}$ for A being a driven low-Q cavity.

- $\boldsymbol{N} = \boldsymbol{a}^\dagger \boldsymbol{a}$, u drive amplitude, Δ detuning, $1/\kappa$ damping time :

$$\mathcal{L}_A(\rho) = [u\boldsymbol{a}^\dagger - u^*\boldsymbol{a}, \rho] - i\Delta[\boldsymbol{N}, \rho] + \kappa\mathcal{D}_{\boldsymbol{a}}(\rho)$$

Steady state $\overline{\rho}_A = |\alpha\rangle\langle\alpha|$ with $\alpha = u/(\kappa/2 + i\Delta)$.

- For any zero-trace operator $\boldsymbol{W}$, zero-trace solution $\boldsymbol{X}$ of $-\mathcal{L}_A(\boldsymbol{X}) = \boldsymbol{W}$ is given by $\int_0^{+\infty} e^{t\mathcal{L}_A}(\boldsymbol{W})dt$.

- For $\boldsymbol{W} = \boldsymbol{A}\overline{\rho}_A - \mathrm{tr}(\boldsymbol{A}\overline{\rho}_A)\overline{\rho}_A$ and with

$$e^{t\mathcal{L}_A}(\boldsymbol{W}) = \sum_{n=0}^{+\infty} \left(\frac{(1-e^{-\kappa t})^n}{n!}\right) \boldsymbol{D}_\alpha \left(e^{-\left(\frac{\kappa}{2}+i\Delta\right)t\boldsymbol{N}}\boldsymbol{a}^n\right) \boldsymbol{D}_{-\alpha}\, \boldsymbol{W}\, \boldsymbol{D}_\alpha \left(\left(\boldsymbol{a}^\dagger\right)^n e^{-\left(\frac{\kappa}{2}-i\Delta\right)t\boldsymbol{N}}\right) \boldsymbol{D}_{-\alpha},$$

we get

$$-\mathcal{L}_A^{-1}\left(\boldsymbol{A}\overline{\rho}_A - \mathrm{tr}(\boldsymbol{A}\overline{\rho}_A)\overline{\rho}_A\right) = \int_0^{+\infty} \left(\boldsymbol{D}_\alpha e^{-\left(\frac{\kappa}{2}+i\Delta\right)t\boldsymbol{N}}\boldsymbol{D}_{-\alpha}\boldsymbol{A}\overline{\rho}_A - \mathrm{tr}(\boldsymbol{A}\overline{\rho}_A)\overline{\rho}_A\right) dt.$$

Conclusion

Interest of such geometric adiabatic elimination preserving the quantum structure (Lindblad master equation, CPTP maps) :

Some non Markovian dynamics can be modeled via a Lindbladian dynamics on a small Hilbert space combined with a CPTP map towards the physical Hilbert space of large dimension.

Coherent feedback where the quantum controller admits a fast relaxation compared to the quantum system to be controlled (elimination of rapidly relaxing sub-system in quantum feedback networks described by (S, L, H) formalism of Gough/James).

Extension when $\mathcal{H} = \mathcal{H}_0 \oplus \mathcal{H}_\infty$ with $\mathcal{H}_\infty = \bigoplus_k \mathcal{H}_{A_k} \otimes \mathcal{H}_{B_k}$ and the slow manifold is parameterized via

$$\rho_s = \sum_k \overline{\rho}_{A_k} \otimes \rho_{s,k} \text{ with } \rho_{s,k} \geq 0 \text{ and } \mathrm{tr}(\rho_{s,k}) \in [0, 1]$$

Conjecture : at any order it is always possible to obtain, up-to higher order terms, Lindbladian dynamics for ρ_s and CPTP maps relating ρ to ρ_s.

April 16th to July 13th, 2018

Organized by:

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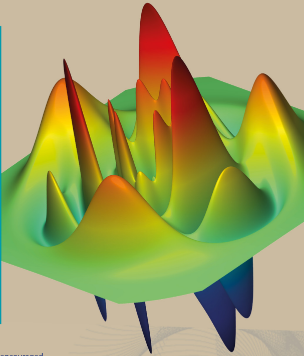

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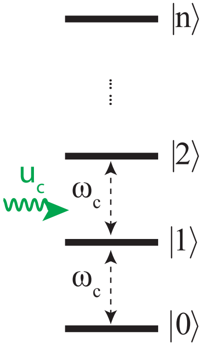
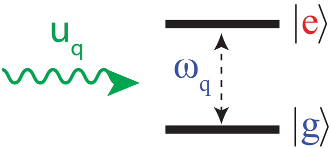
Quantum harmonic oscillator (spring system)

- Hilbert space :
  $\mathcal{H}_S = \left\{ \sum_{n\geq 0} \psi_n \left|n\right\rangle, \; (\psi_n)_{n\geq 0} \in l^2(\mathbb{C}) \right\} \equiv L^2(\mathbb{R},\mathbb{C})$
- Quantum state space :
  $\mathfrak{D} = \{\rho \in \mathcal{L}(\mathcal{H}_S), \rho^\dagger = \rho, \mathrm{tr}(\rho) = 1, \rho \geq 0\}$.
- Operators and commutations :
  $\boldsymbol{a}\left|n\right\rangle = \sqrt{n}\left|\boldsymbol{n-1}\right\rangle$, $\boldsymbol{a}^\dagger\left|n\right\rangle = \sqrt{n+1}\left|n+1\right\rangle$;
  $\boldsymbol{N} = \boldsymbol{a}^\dagger\boldsymbol{a}$, $\boldsymbol{N}\left|n\right\rangle = n\left|n\right\rangle$;
  $[\boldsymbol{a}, \boldsymbol{a}^\dagger] = \boldsymbol{I}$, $\boldsymbol{a}f(\boldsymbol{N}) = f(\boldsymbol{N}+\boldsymbol{I})\boldsymbol{a}$;
  $\boldsymbol{D}_\alpha = e^{\alpha\boldsymbol{a}^\dagger - \alpha^\dagger\boldsymbol{a}}$.
  $\boldsymbol{a} = \boldsymbol{X} + i\boldsymbol{P} = \frac{1}{\sqrt{2}}\left(x + \frac{\partial}{\partial x}\right)$, $[\boldsymbol{X}, \boldsymbol{P}] = \imath\boldsymbol{I}/2$.



- Hamiltonian : $\boldsymbol{H}_S/\hbar = \omega_c\boldsymbol{a}^\dagger\boldsymbol{a} + \boldsymbol{u_c}(\boldsymbol{a} + \boldsymbol{a}^\dagger)$.
  (associated classical dynamics :
  $\frac{dx}{dt} = \omega_c p, \; \frac{dp}{dt} = -\omega_c x - \sqrt{2}u_c$).
- Classical pure state $\equiv$ coherent state $\left|\alpha\right\rangle$
  $\alpha \in \mathbb{C}: \; \left|\alpha\right\rangle = \sum_{n\geq 0}\left(e^{-|\alpha|^2/2}\frac{\alpha^n}{\sqrt{n!}}\right)\left|n\right\rangle$; $\left|\alpha\right\rangle \equiv \frac{1}{\pi^{1/4}} e^{\imath\sqrt{2}x\Im\alpha} e^{-\frac{(x-\sqrt{2}\Re\alpha)^2}{2}}$
  $\boldsymbol{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle$, $\boldsymbol{D}_\alpha\left|0\right\rangle = \left|\alpha\right\rangle$.

- Hilbert space :
  $\mathcal{H}_M = \mathbb{C}^2 = \left\{ c_g \left| g \right\rangle + c_e \left| e \right\rangle, \; c_g, c_e \in \mathbb{C} \right\}.$
- Quantum state space :
  $\mathfrak{D} = \{ \rho \in \mathcal{L}(\mathcal{H}_M), \rho^\dagger = \rho, \mathrm{tr}(\rho) = 1, \rho \geq 0 \}$.
- Operators and commutations :
  $\boldsymbol{\sigma_-} = \left| g \right\rangle \left\langle e \right|, \; \boldsymbol{\sigma_+} = \boldsymbol{\sigma_-}^\dagger = \left| e \right\rangle \left\langle g \right|$
  $\boldsymbol{\sigma_x} = \boldsymbol{\sigma_-} + \boldsymbol{\sigma_+} = \left| g \right\rangle \left\langle e \right| + \left| e \right\rangle \left\langle g \right|$;
  $\boldsymbol{\sigma_y} = i\boldsymbol{\sigma_-} - i\boldsymbol{\sigma_+} = i \left| g \right\rangle \left\langle e \right| - i \left| e \right\rangle \left\langle g \right|$;
  $\boldsymbol{\sigma_z} = \boldsymbol{\sigma_+}\boldsymbol{\sigma_-} - \boldsymbol{\sigma_-}\boldsymbol{\sigma_+} = \boldsymbol{P}_e - \boldsymbol{P}_g$;
  ${\boldsymbol{\sigma_x}}^2 = \boldsymbol{I}, \; \boldsymbol{\sigma_x}\boldsymbol{\sigma_y} = i\boldsymbol{\sigma_z}, \; [\boldsymbol{\sigma_x}, \boldsymbol{\sigma_y}] = 2i\boldsymbol{\sigma_z}, \ldots$



- Hamiltonian : $\boldsymbol{H}_M/\hbar = \omega_q \boldsymbol{\sigma_z}/2 + \boldsymbol{u_q}\boldsymbol{\sigma_x}$.
- Bloch sphere representation :
  $\mathfrak{D} = \left\{ \frac{1}{2}\left(\boldsymbol{I} + x\boldsymbol{\sigma_x} + y\boldsymbol{\sigma_y} + z\boldsymbol{\sigma_z}\right) \;\middle|\; (x, y, z) \in \mathbb{R}^3, \; x^2 + y^2 + z^2 \leq 1 \right\}$