Bad Honnef, Jan. 19, 2015

Quantum trajectory of a field stored in a cavity: the past quantum state approach

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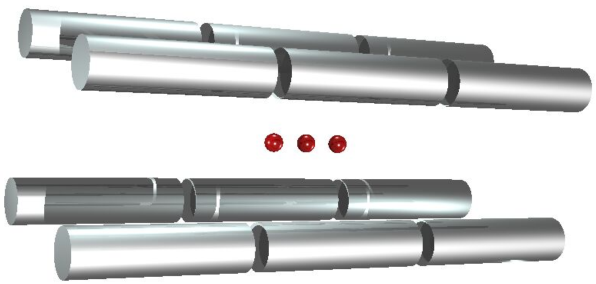
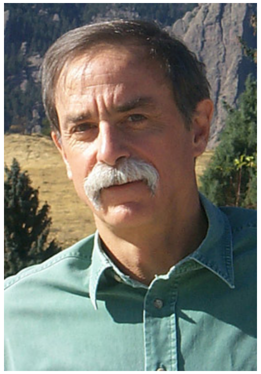
Trapped ions and photons

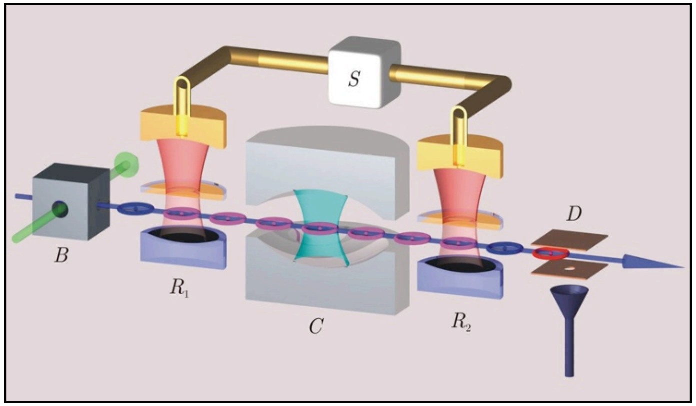
- Trapped ions

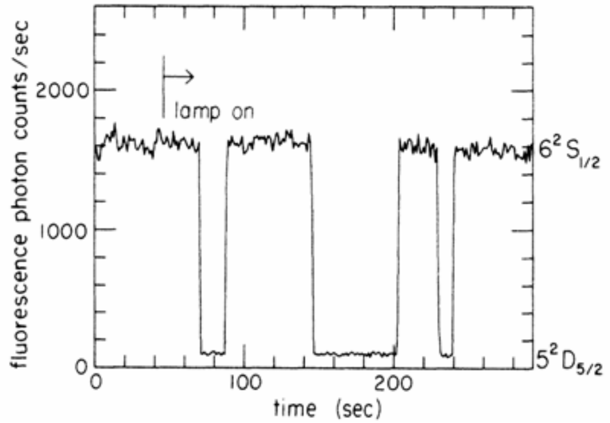
- Trapped photons



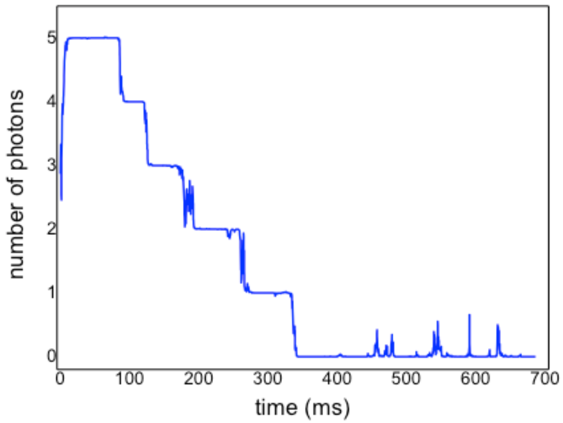


D.J. Wineland and S. Haroche





H. Dehmelt, JOSAB 1986



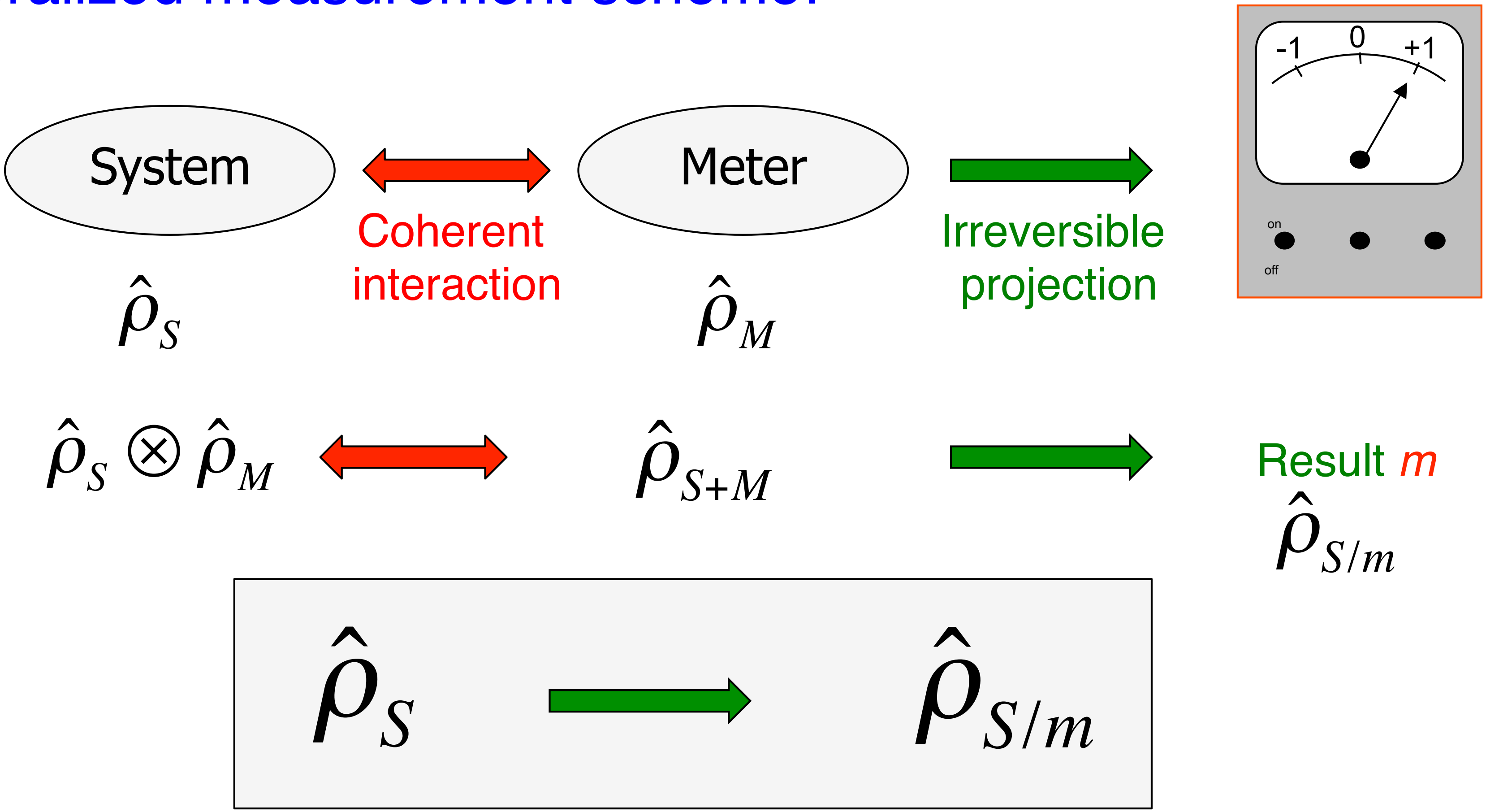
C. Gerlin et al. Nature 2007

→ Observation of quantum jumps in a single realization of an experiment

I. General state reconstruction methods

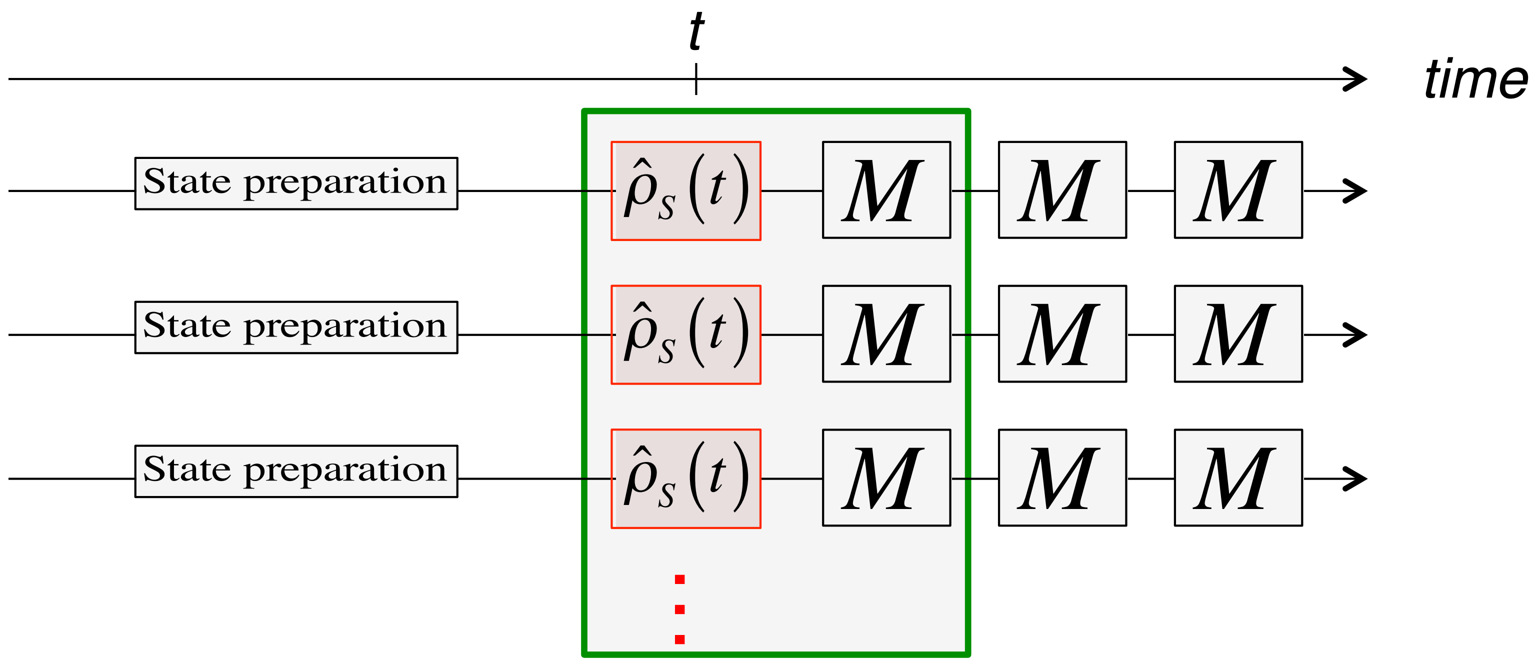
Quantum state reconstruction

- Generalized measurement scheme:



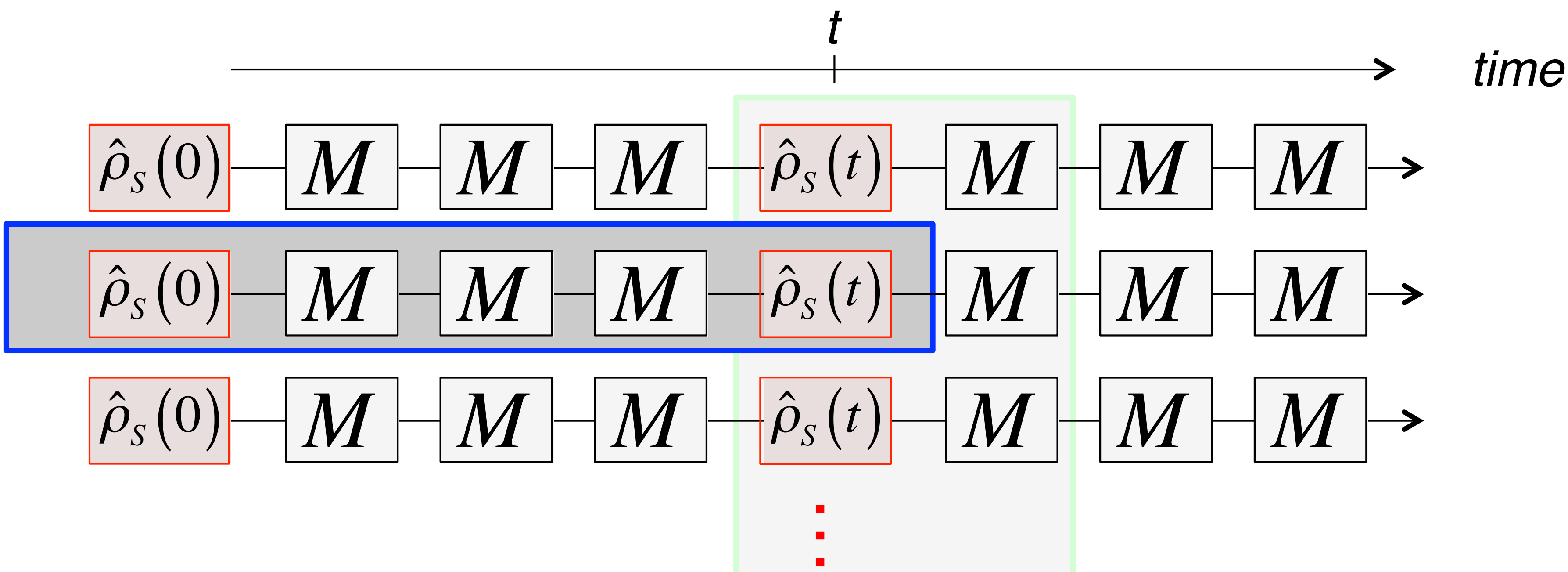
The measurement result provides (partial) information on S
General state reconstruction problem:
- optimize the amount of information extracted on S
- get the best estimate of the state after a measurement

Quantum state reconstruction and time evolution



- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  → quantum state tomography

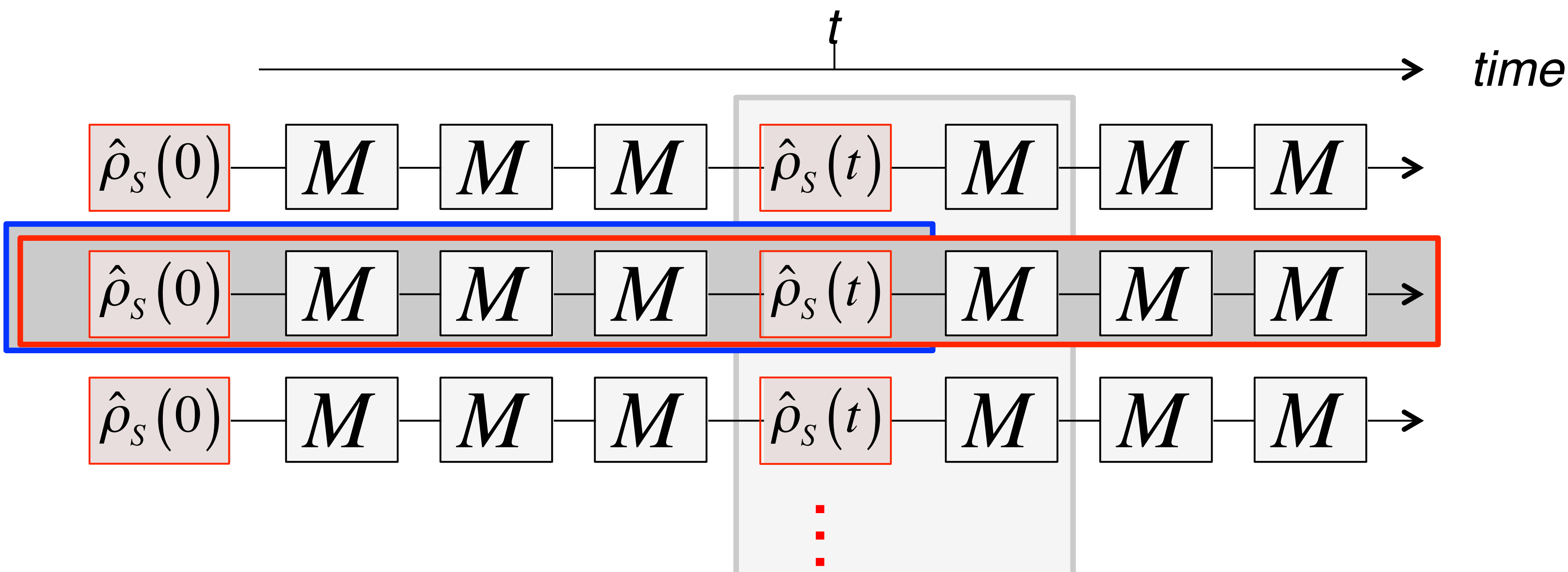
Quantum state reconstruction and time evolution



- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  → quantum state tomography

- Estimate $\hat{\rho}_S(t)$ in a given realization knowing measurement results before t_0 → quantum trajectory reconstruction
  "standard approach"

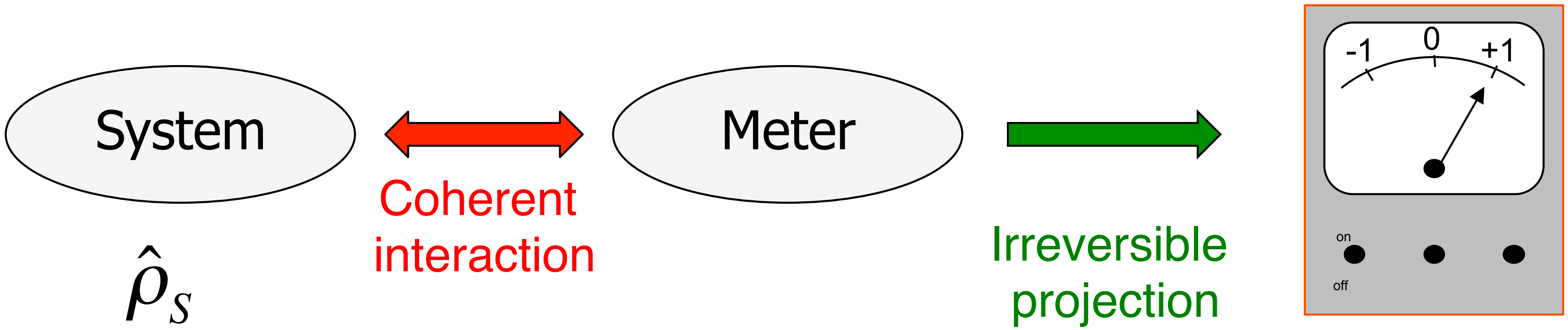
Optimal quantum state reconstruction and time evolution



- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  → quantum state tomography

- Estimate $\hat{\rho}_S(t)$ in a given realization knowing measurement results before t_0 → quantum trajectory reconstruction
  "standard approach"
- Estimate $\hat{\rho}_S(t)$ in a given realization knowing measurement results **before and after** t_0 → "Past quantum state" *(Mölmer PRL 2013)*

Generalized quantum measurement



$\hat{\rho}_S$

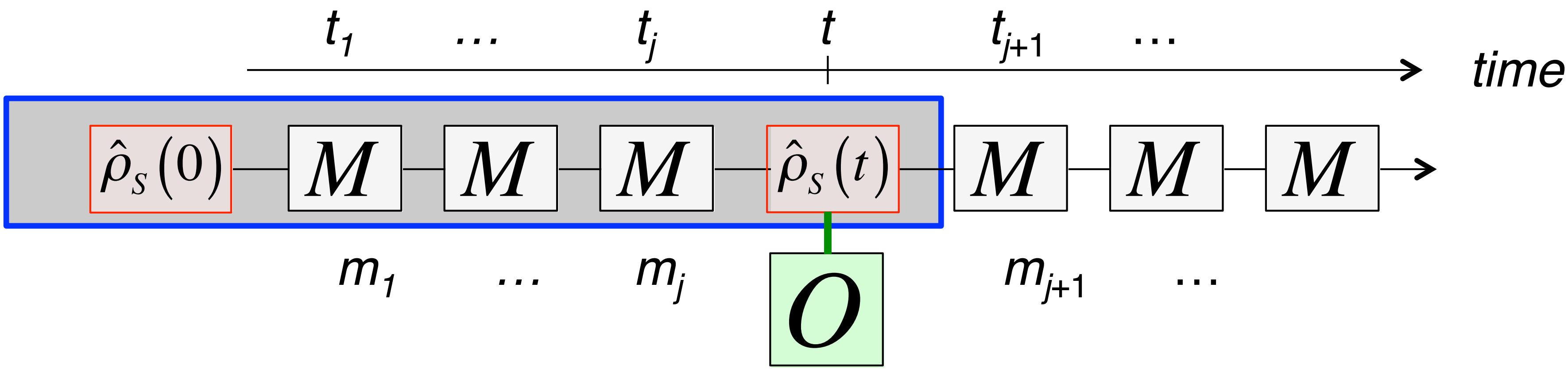
$$\hat{\rho}_S \longrightarrow \hat{\rho}_{S/m} = \frac{\hat{M}_m \hat{\rho}_S \hat{M}_m^+}{Norm}$$

- Operators $\left\{\hat{M}_m\right\}$: set of operators of S such that $\sum_m \hat{M}_m^+ \hat{M}_m = \hat{1}$.
- Proba of result m: $P(m) = tr\ \hat{M}_m \hat{\rho}_S \hat{M}_m^+$

→ **describes any evolution**:

- any measurement
- unitary: only one operator $\hat{M}_0 = \hat{U}(t_0, t)$
- relaxation can be seen as unread measurement in some environment

Quantum trajectory reconstruction: "standard approach"



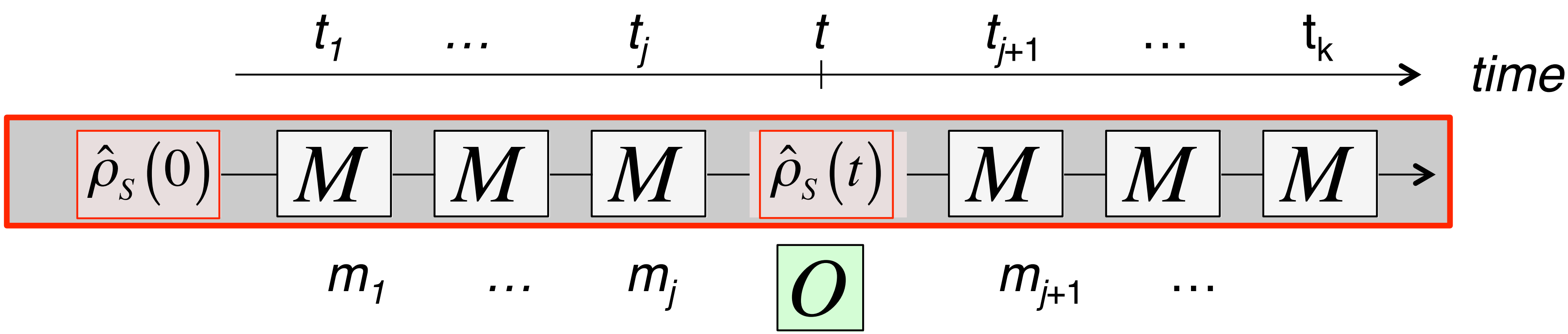
$$\hat{\rho}_S(t) = \hat{\rho}_{S/\{m_j\}} = \frac{\hat{M}_{m_j} \ldots \hat{M}_{m_0} \hat{\rho}_S(0) \hat{M}^+_{m_0} \ldots \hat{M}^+_{m_j}}{Norm}$$

With $\hat{\rho}_{S/\{m_j\}}$ one can describe the results of any measurement $\{\hat{O}_i\}$ performed at time *t*.

➞ one gets the probability of the measurement result o_i conditional to previous measurements

$$P\left(o_i, t / \{m_{1\ldots j}\}\right) = \frac{tr\, \hat{O}_i\, \hat{\rho}_S(t_j)\, \hat{O}^+_i}{Norm}$$

The "past quantum state approach"



We are now interested in another conditional probability: description of the measurement of $\{\hat{O}_i\}$ knowing the past and future measurement results.

$$P\left(o_i, t / \{m_{1\ldots k}\}\right) = \frac{tr\, \hat{O}_i\, \hat{\rho}_S(t)\, \hat{O}_i^+\, \hat{E}_S(t)}{Norm}$$

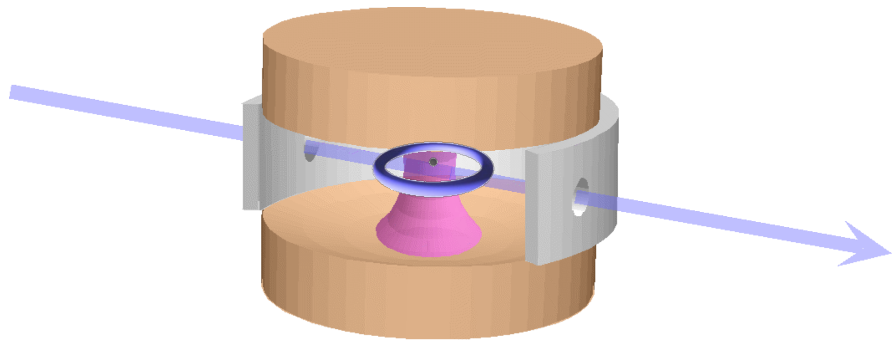
Mölmer PRL 2013

$$\hat{\rho}_S(t) = \hat{\rho}_{S/\{m_k\}} = \frac{\hat{M}_{m_j} \ldots \hat{M}_{m_0} \hat{\rho}_S(0) \hat{M}_{m_0}^+ \ldots \hat{M}_{m_j}^+}{Norm}$$

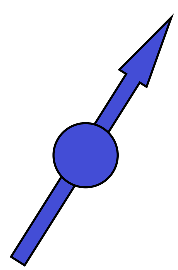
$$\hat{E}_S(t) = \frac{\hat{M}_{m_{j+1}}^+ \ldots \hat{M}_{m_k}^+\, \hat{1}\, \hat{M}_{m_k} \ldots \hat{M}_{m_{j+1}}}{Norm}$$

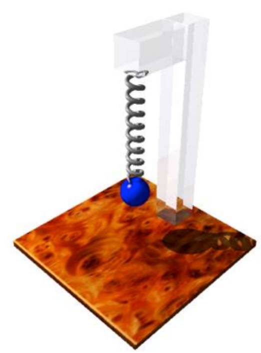
The "effect" matrix $\hat{E}_S(t)$ is similar to $\hat{\rho}_S(t)$, it involves the same measurement operators but in a different order.

II. Cavity QED implementation: QND photon counting



The SPIN:
One atom, two levels



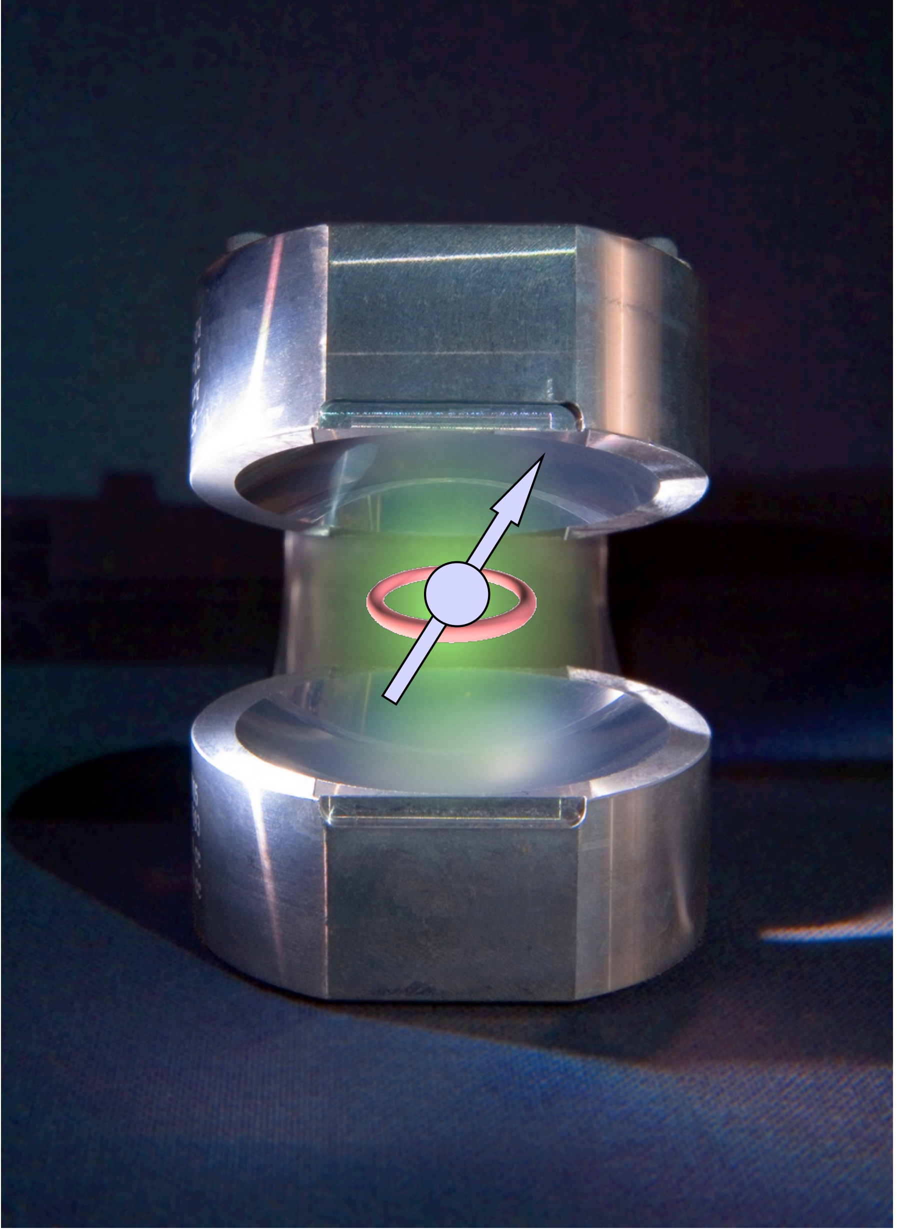


The SPRING:
One high Q cavity mode
as an harmonic oscillator

M. Brune and J.M. Raimond, EPL 110, 20001 (2015): "Trapped quantum light".

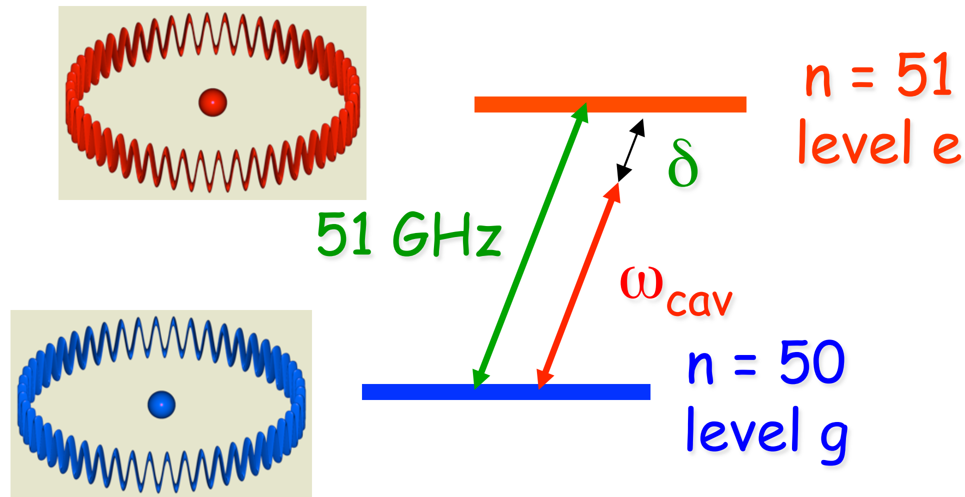
The "Spin"

- Photon box



- Photon probes

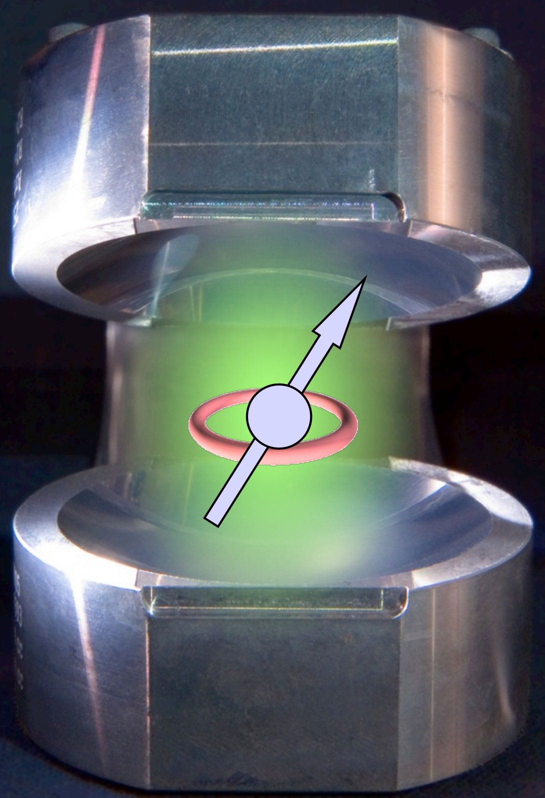
Circular Rydberg atoms



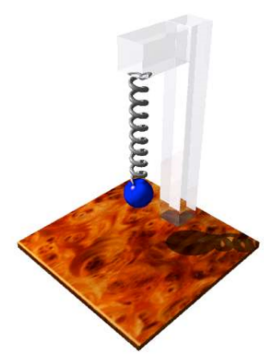
- Large dipole 1500 au
- Long lifetime: 30ms
- detected une by one

The "spring": a photon box

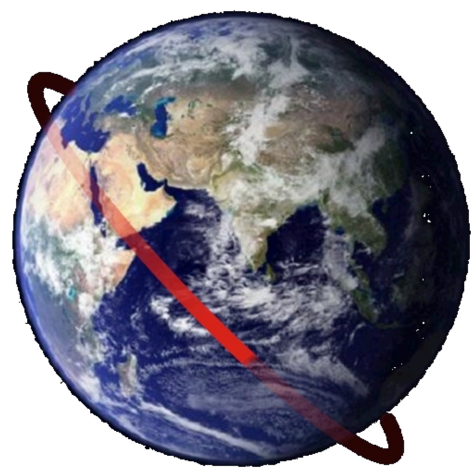
Superconducting cavity resonance: $\nu_{cav} = 51$ GHz



$T_{\rm cav} = 130\ {\rm ms}$

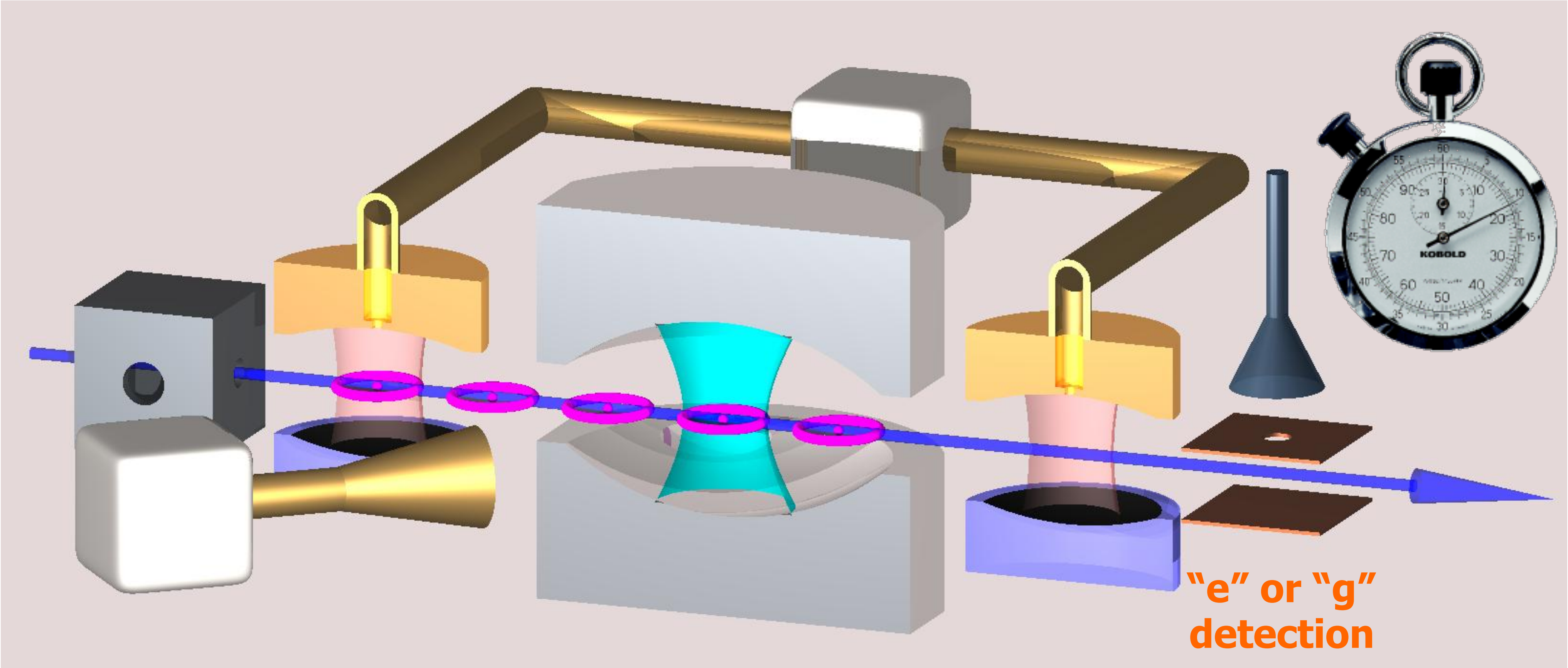


- Q factor = $4.2 \cdot 10^{10}$
- finesse= $4.\ 10^{9}$



Photons running for 39 000 km in the box before dying!

Experimental setup: an atomic clock



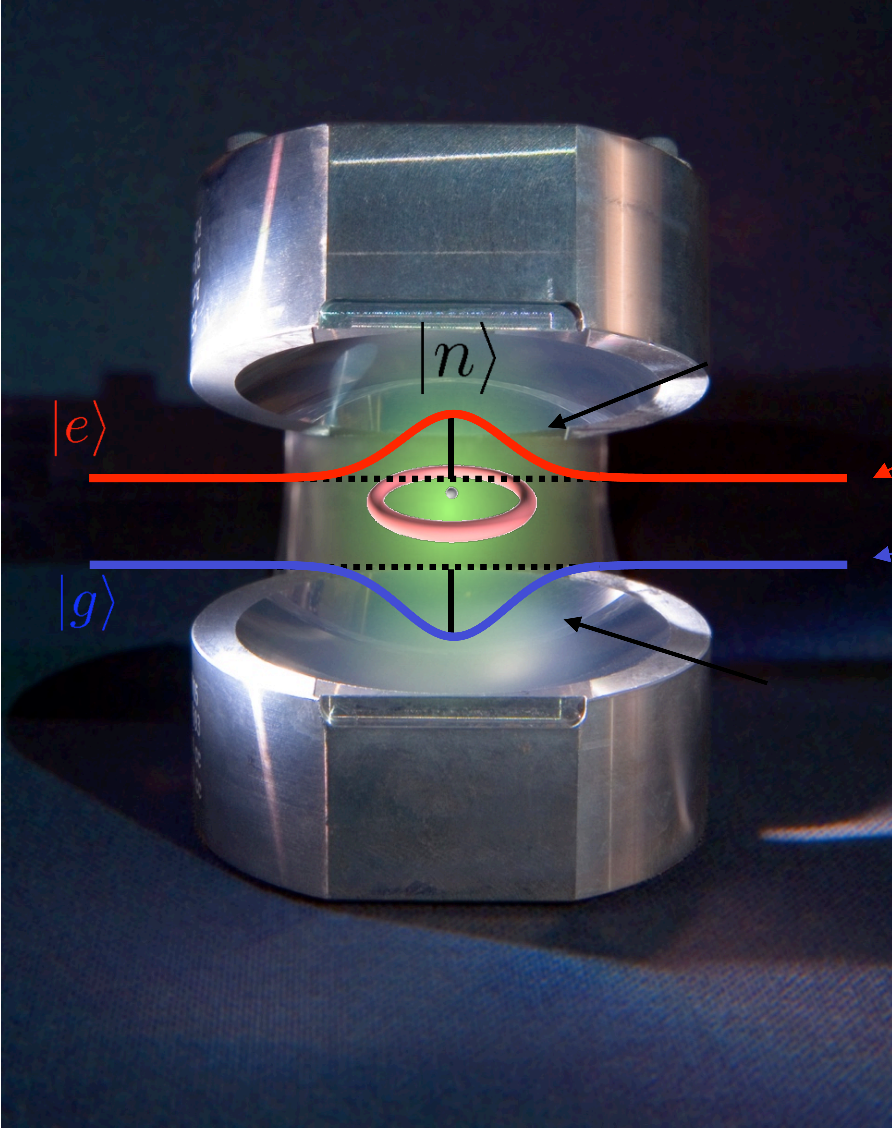
- An atomic clock (Ramsey setup) made of Rydberg for probing light-shifts induced by “trapped” photons

- State selective detection of atoms by field ionization: Atoms detected on “e” or “g” one by one

QND detection of photons: the tools

- Photon box

Superconducting cavity



- Photon probes

Circular Rydberg atoms

- Non-resonant interaction

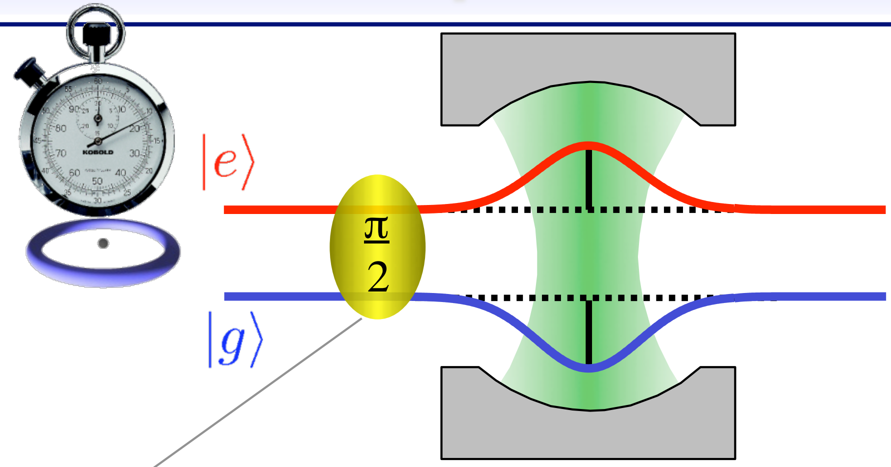
$\Rightarrow$ light shifts

$$\Delta E_e = \hbar \frac{\Omega_0^2}{4\delta}(n+1)$$

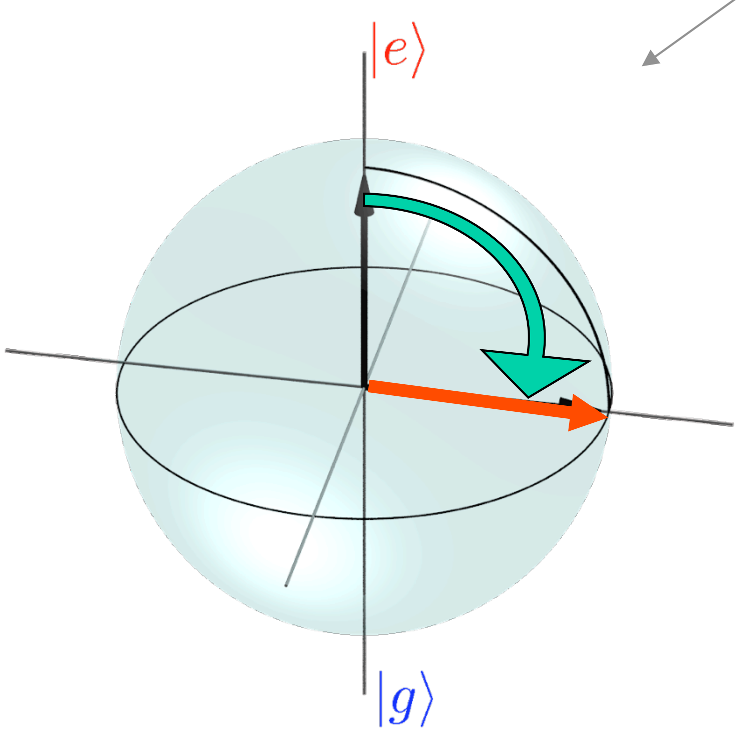
$$\Delta E_g = -\hbar \frac{\Omega_0^2}{4\delta} n$$

Atoms used as clock for counting n by measuring light shifts

QND measurement of photon number

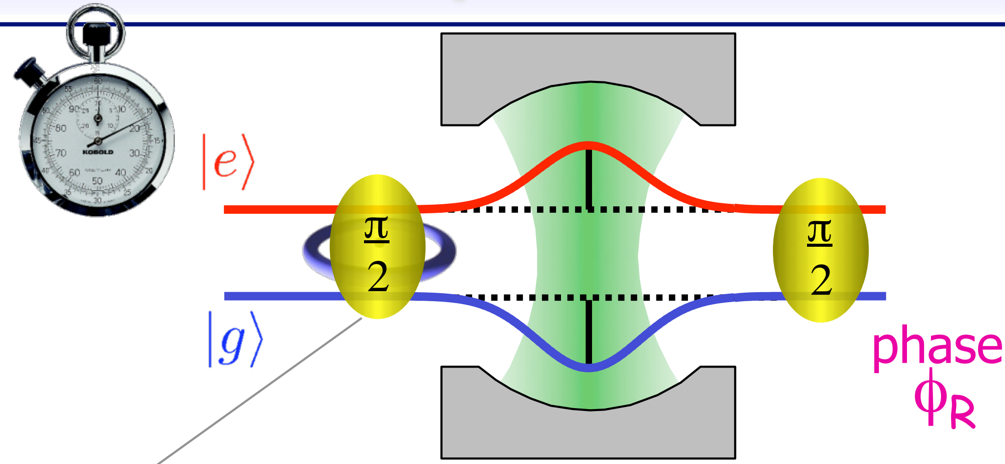


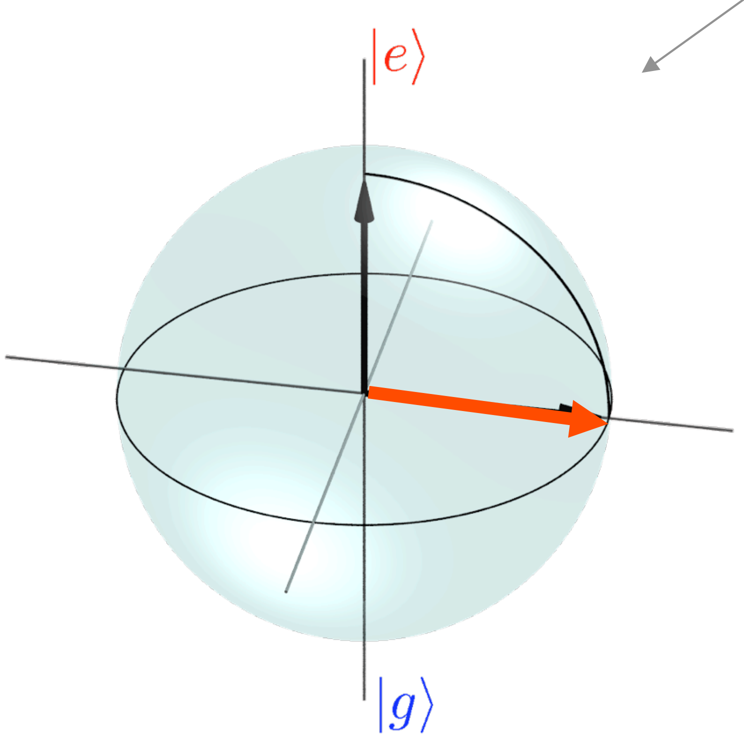
1. Trigger of the atom clock:
resonant π/2 pulse

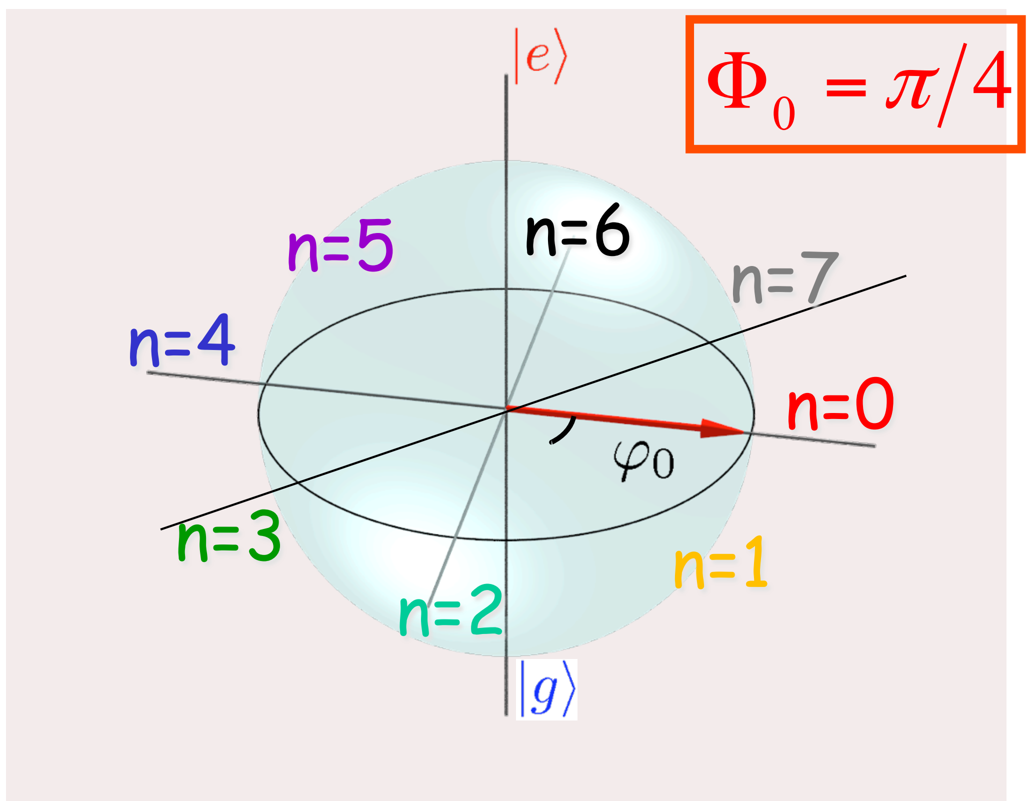


QND measurement of photon number

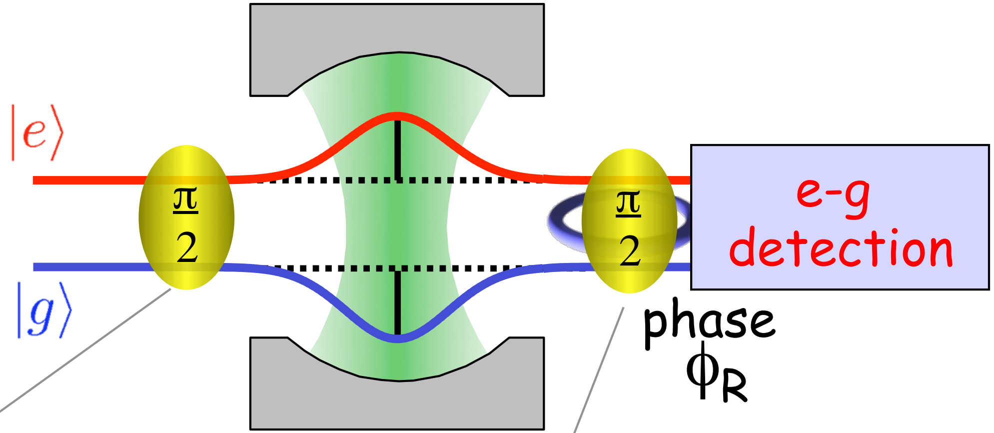
1. Trigger of the atom clock: resonant π/2 pulse
2. Dephasing of the clock: interaction with the cavity field



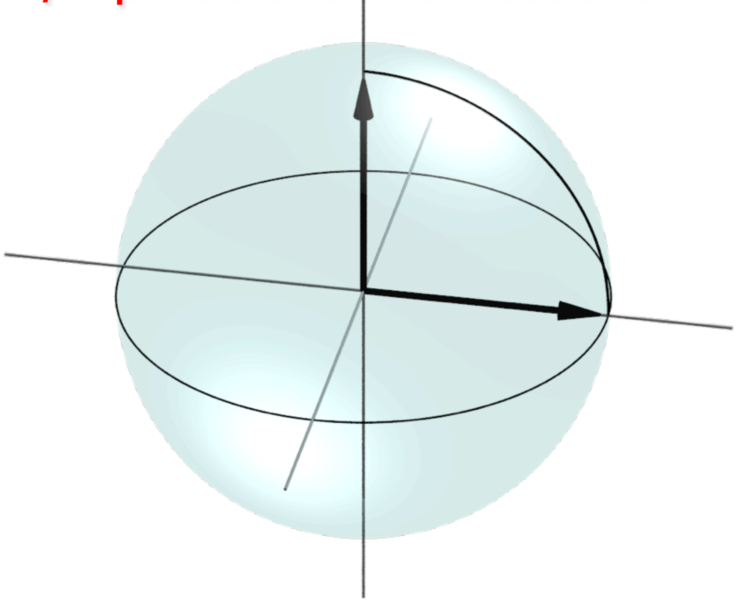


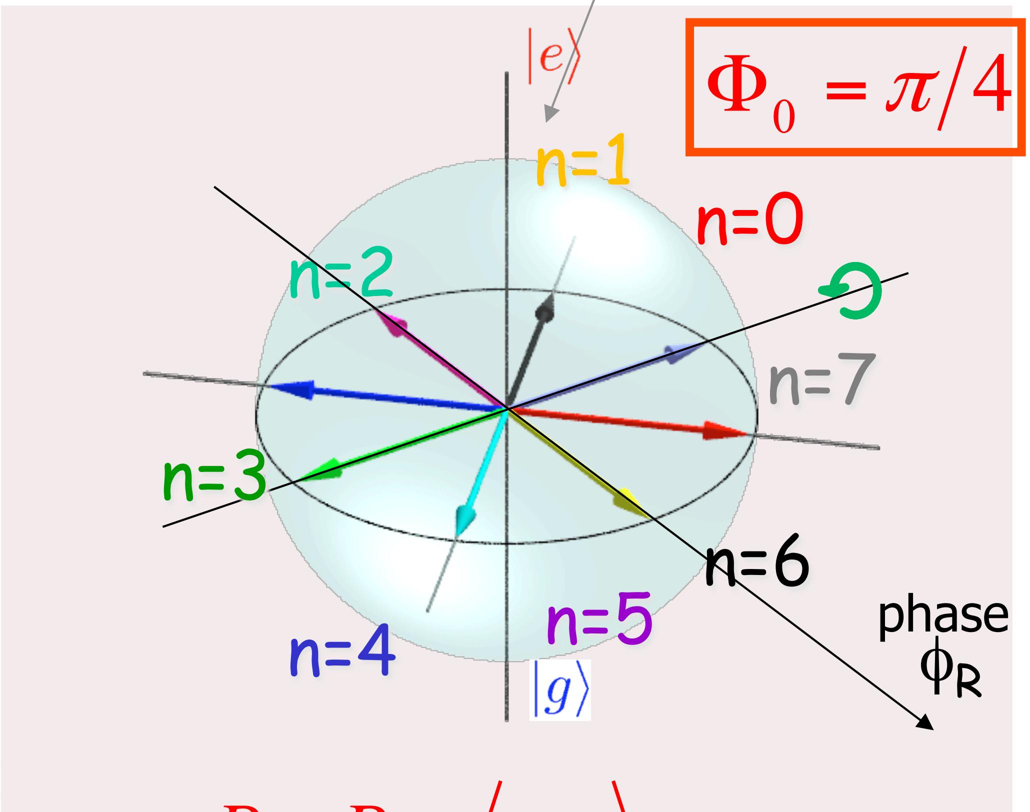


QND measurement of photon number



1. Trigger of the atom clock: resonant π/2 pulse
2. Dephasing of the clock: interaction with the cavity field
3. Measurement of the clock: second π/2 pulse & state detection





Pseudo-spin measurement in arbitrary direction:

$$P_e - P_g = \langle \sigma_{\phi_R} \rangle$$

Field measurement operators

- Measurement result: *j = e,g*
- Transformation of the field density matrix ρ_S after a measurement with result *j*:

$$\hat{\rho}_{S/j} = \frac{\hat{M}_j \hat{\rho}_S \hat{M}_j^+}{tr\ \hat{M}_j \hat{\rho}_S \hat{M}_j^+}$$

$$\hat{M}_g = \sin\left(\frac{\phi_R + \phi_0 \hat{N}}{2}\right)$$

$$\hat{M}_e = \cos\left(\frac{\phi_R + \phi_0 \hat{N}}{2}\right)$$

ϕ_r : variable Ramsey interferometer phase

ϕ_0 : phase shift per photon

- Assume ρ_S initially diagonal

→ simplified measurement description in term of $P(n) = \hat{\rho}_{S\,n,n}$

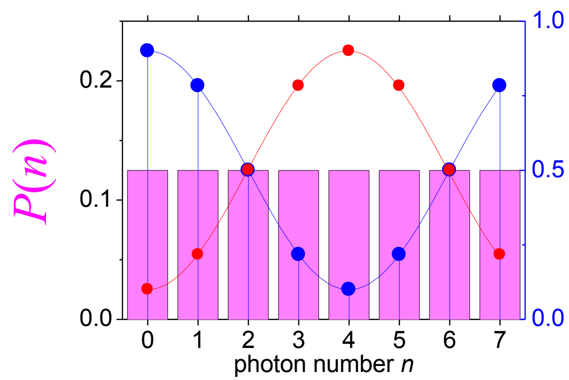
Information acquisition by detecting 1 atom

Bayes law:

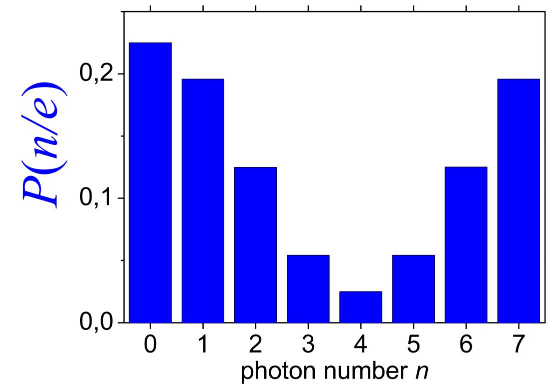
$$P_{after}(n) = P(n / j, \phi_R) = P_{before}(n).\frac{P(j,/n,\phi_R)}{P(j/\phi_R)}$$

Measurement result:
$j = e$ or g

$\varphi = 0$



e



Probability of n that are incompatible with the measurement result j are cancelled.

Information acquisition by detecting 1 atom

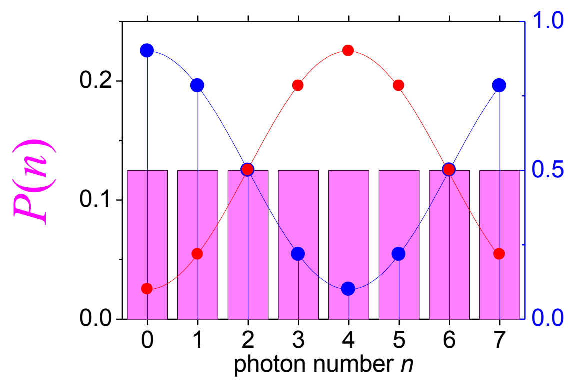
Bayes law:

$$P_{after}(n) = P(n/j,\phi_R) = P_{before}(n).\frac{P(j,/n,\phi_R)}{P(j/\phi_R)}$$

Measurement result:

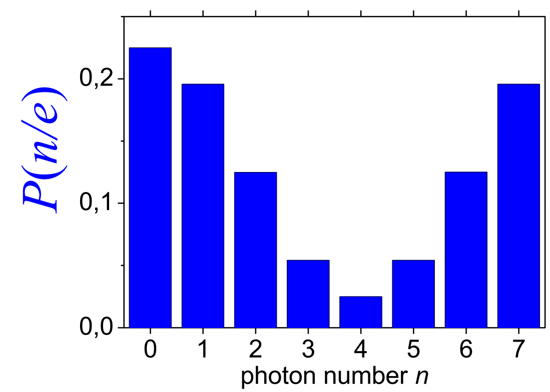
$j = e$ or g

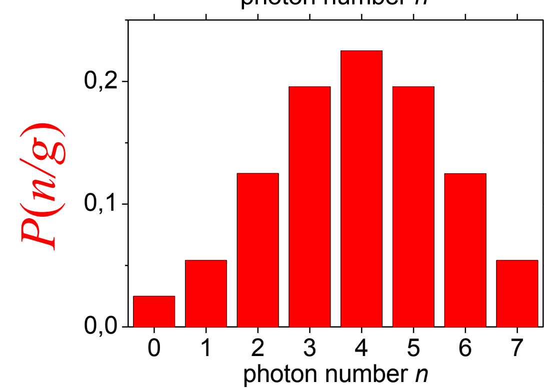
$\phi_R = 0$



e

g

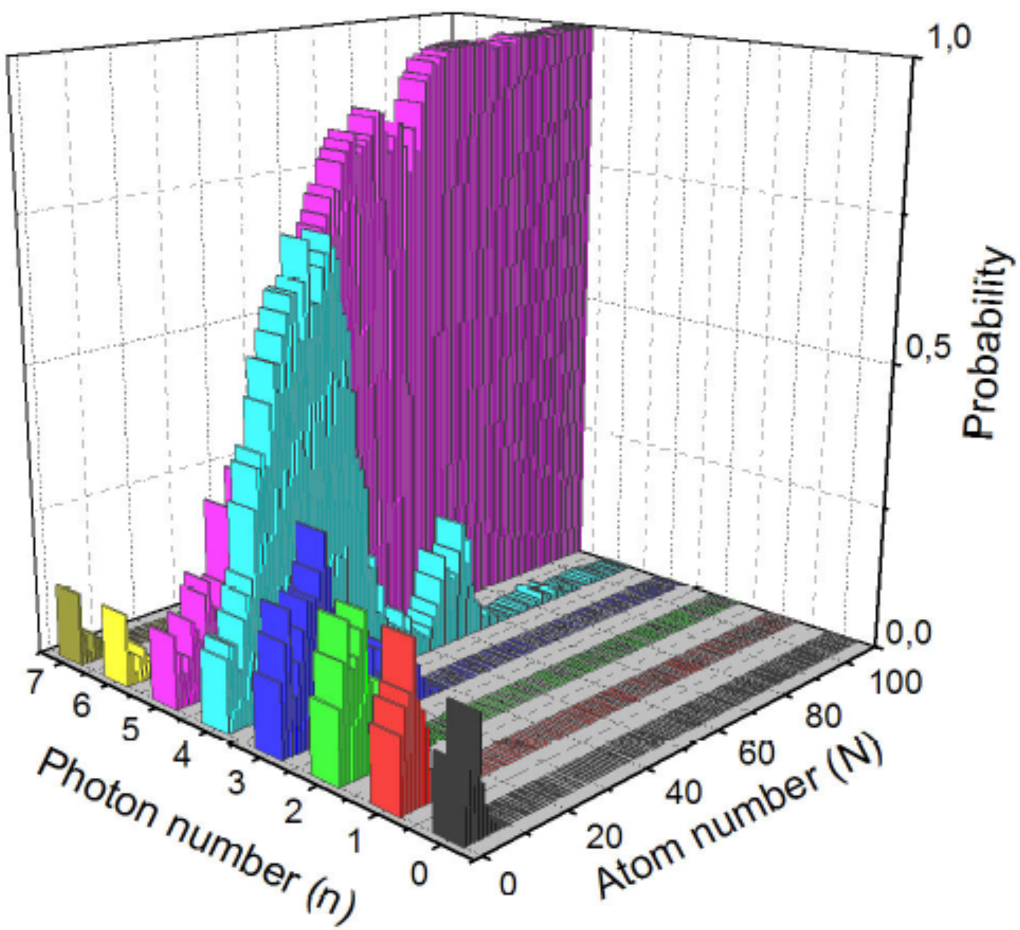




Repeating the measurement with other values of the measurement phase φ decimates other photon numbers

Atom by atom analysis of the measurement process

For each detected atom, one projects the field state according to the measurement result e or g



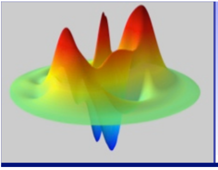
Progressive collapse of the field state on $n=5$

Measurement of a coherent field $\langle n \rangle = 3.7\ (\pm 0.008)$

Initial knowledge of the photon number distribution is not needed

*C. Guerlin, J. Bernu, S. Deléglise, C. Sayrin, S. Gleyzes, S. Kuhr, M. Brune, J.M. R., S. H. Nature **448**, 889 (07)*

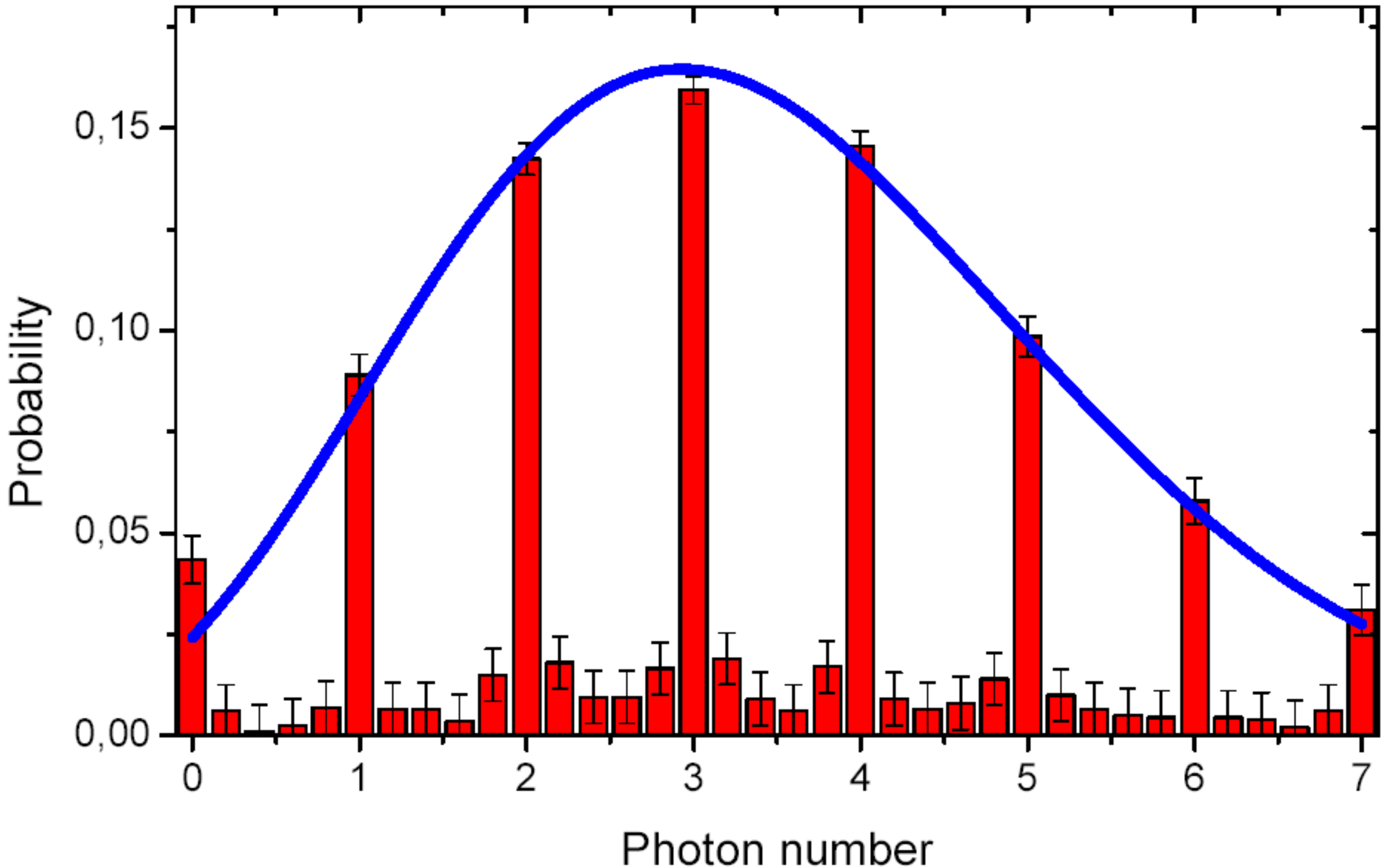
Convergence analysis: M. Bauer, D. Bernard. PRA 84, 044103 (2011)



Reconstructing the photon number statistics

Coherent field at measurement time

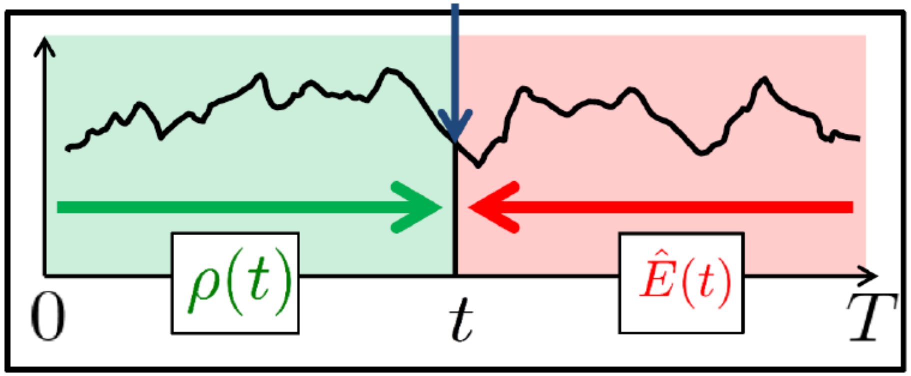
$$\langle n \rangle = 3.4 \pm 0.008$$



III. Following a quantum trajectory

Following a quantum trajectory

S. Gammelmak et al. PRL 111, 160401(2013)



$$P\left(o_i, t / \{m_k\}\right) = \frac{tr\ \hat{O}_i\ \hat{\rho}_S(t)\ \hat{O}_i^{+}\ \hat{E}_S(t)}{Norm}$$

Apply to photon number operator $\hat{O} = \hat{N}$: $\hat{O}_n = |n\rangle\langle n|$

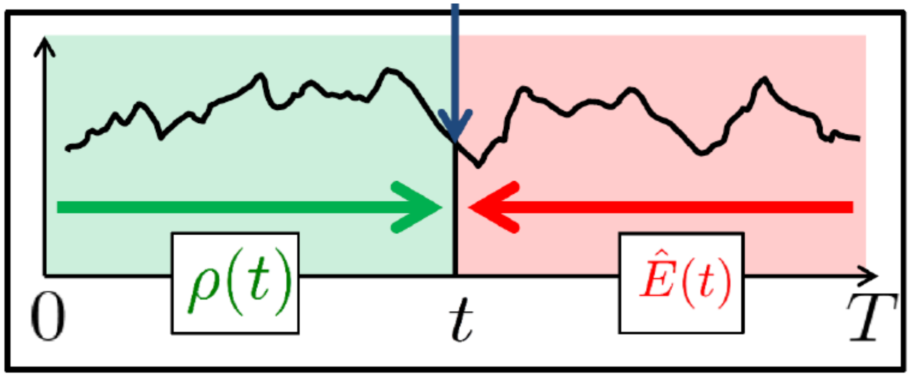
$$P\left(n, t / \{m_k\}\right) = \frac{tr\ |n\rangle\langle n|\ \hat{\rho}_S(t)\ |n\rangle\langle n|\ \hat{E}_S(t)}{Norm}$$

$\Longrightarrow$

$$P\left(n, t / \{m_k\}\right) = \frac{\hat{\rho}^{S}_{n,n}(t)\ \hat{E}^{S}_{n,n}(t)}{Norm}$$

→ only diagonal matrix elements

Following a quantum trajectory



$$P\left(n,t/\{m_k\}\right) = \frac{tr\ \hat{\rho}^S_{n,n}(t)\ \hat{E}^S_{n,n}(t)}{Norm}$$

→ Photon number distributions:

- Forward estimation: $P^f(n,t) = \hat{\rho}^S_{n,n}(t)$

standard calculation of the density matrix $\rho(t)$ taking into account

  - projection at measurement
  - relaxation between measurements

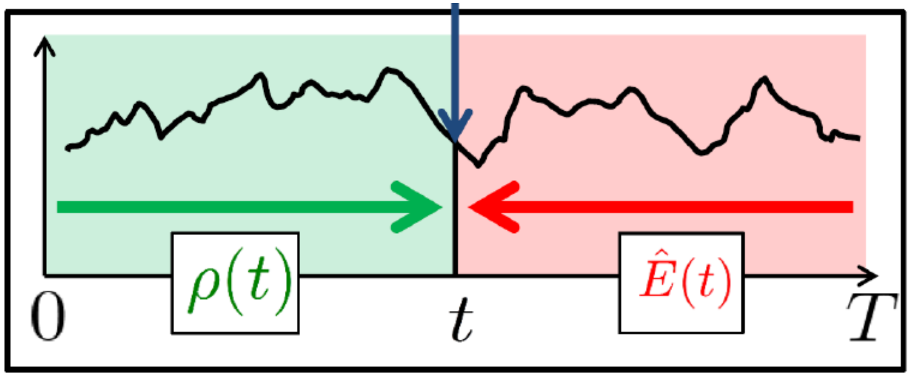
- Backward estimation: $P^b(n,t) = \hat{E}^S_{n,n}(t)$

calculation effect matrix $E(t)$:

- Flat distribution at final time T: describes an unknown final state
- Same measurement operators as forward
- 'inverse' relaxation (annihilation and creation operators exchanged)

  → Exponential growth of the photon number in "backward time"

Following a quantum trajectory



$$P\left(n,t/\{m_k\}\right)=\frac{tr\ \hat{\rho}^S_{n,n}(t)\ \hat{E}^S_{n,n}(t)}{Norm}$$

→ Photon number distributions:

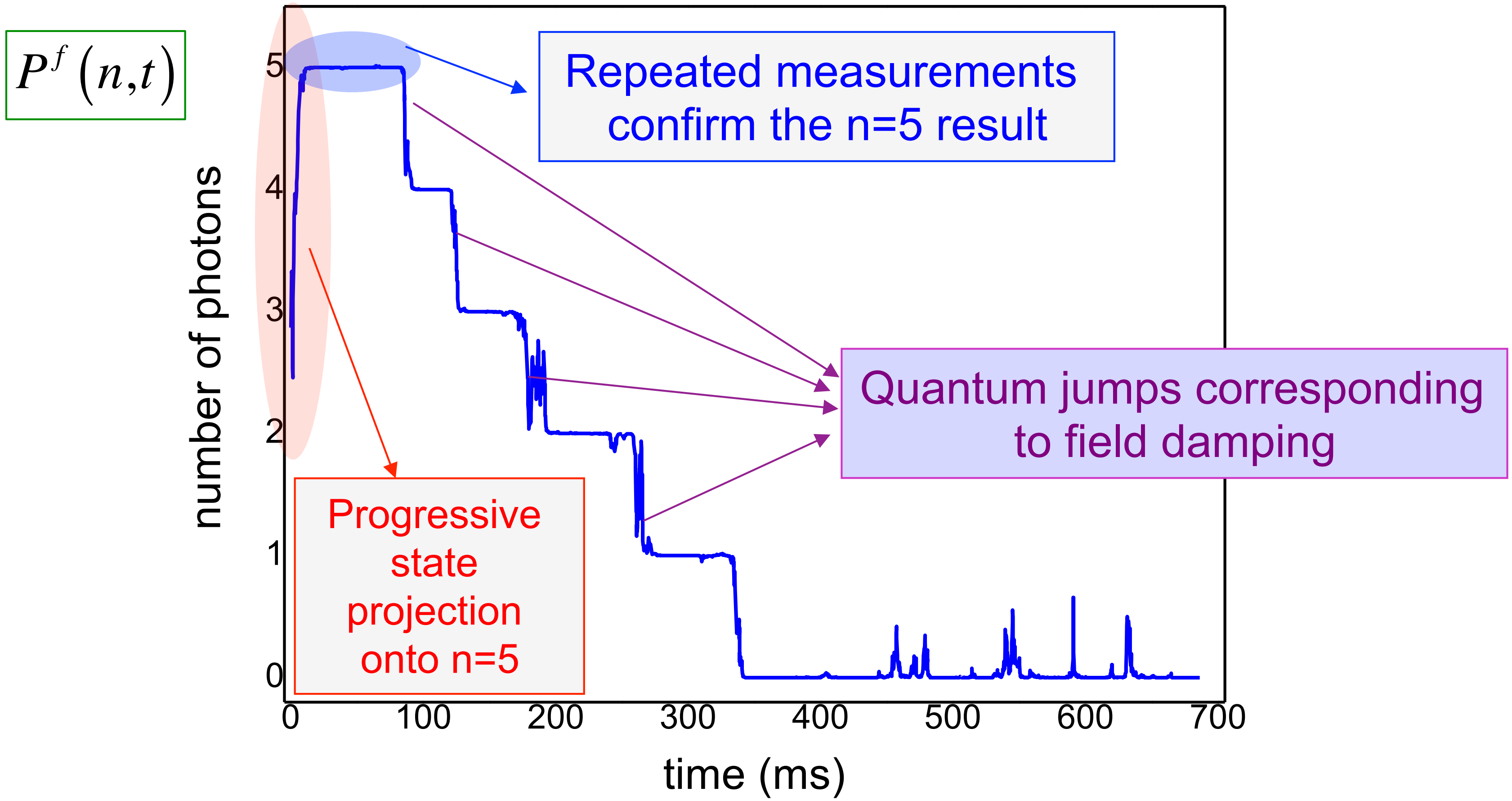
- Forward estimation: → $P^f(n,t)=\hat{\rho}^S_{n,n}(t)$
- Backward estimation: ← $P^b(n,t)=\hat{E}^S_{n,n}(t)$
- Past quantum state / forward-backward estimation

$$P^{fb}(n,t)=\frac{P^f(n,t).P^b(n,t)}{Norm}$$

→ P(n) is the product of two photon number distributions computed forward and backward in time.

In our case PQS reduces to the "forward/backward smoothing algorithm", which can be safely used in this quantum context

Repeated measurements: Forward photon number distribution

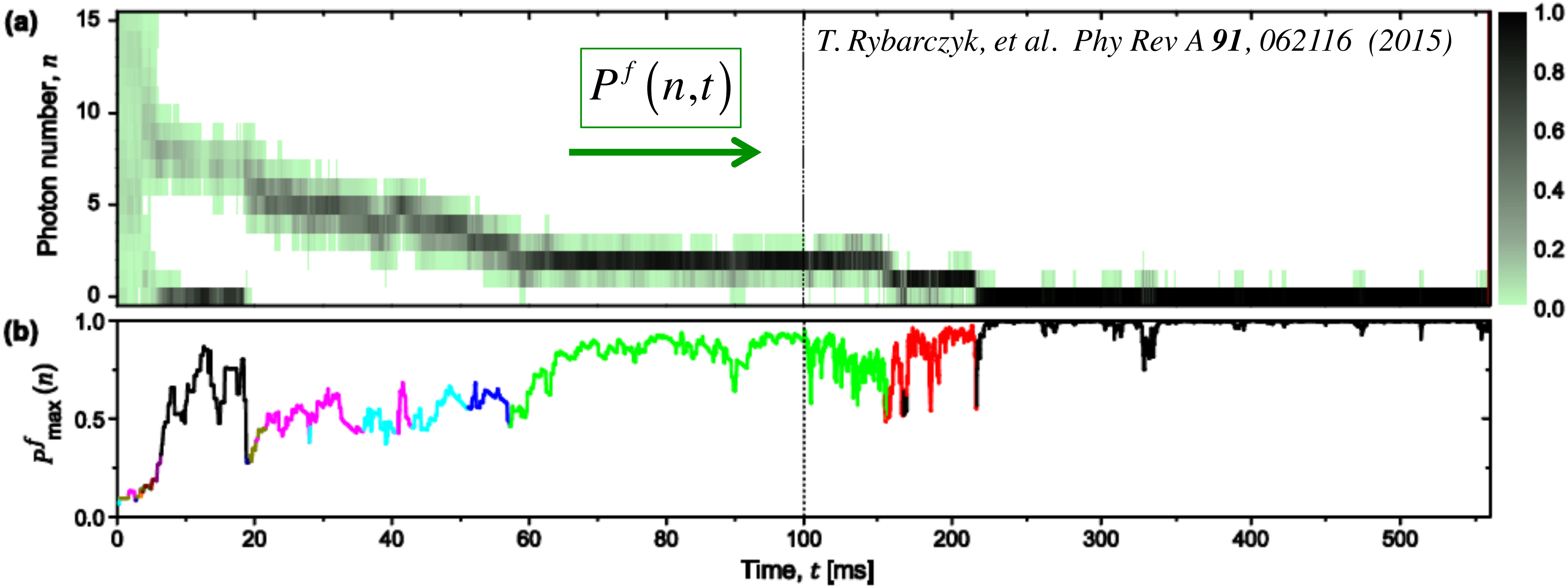


Field evolution due to cavity damping: not to QND measurement

- Exhibits all features of quantum theory of measurement:
  - State collapse / Random result / repeatability

Quantum trajectory for a larger initial field

- Forward estimation of the field at time *t*



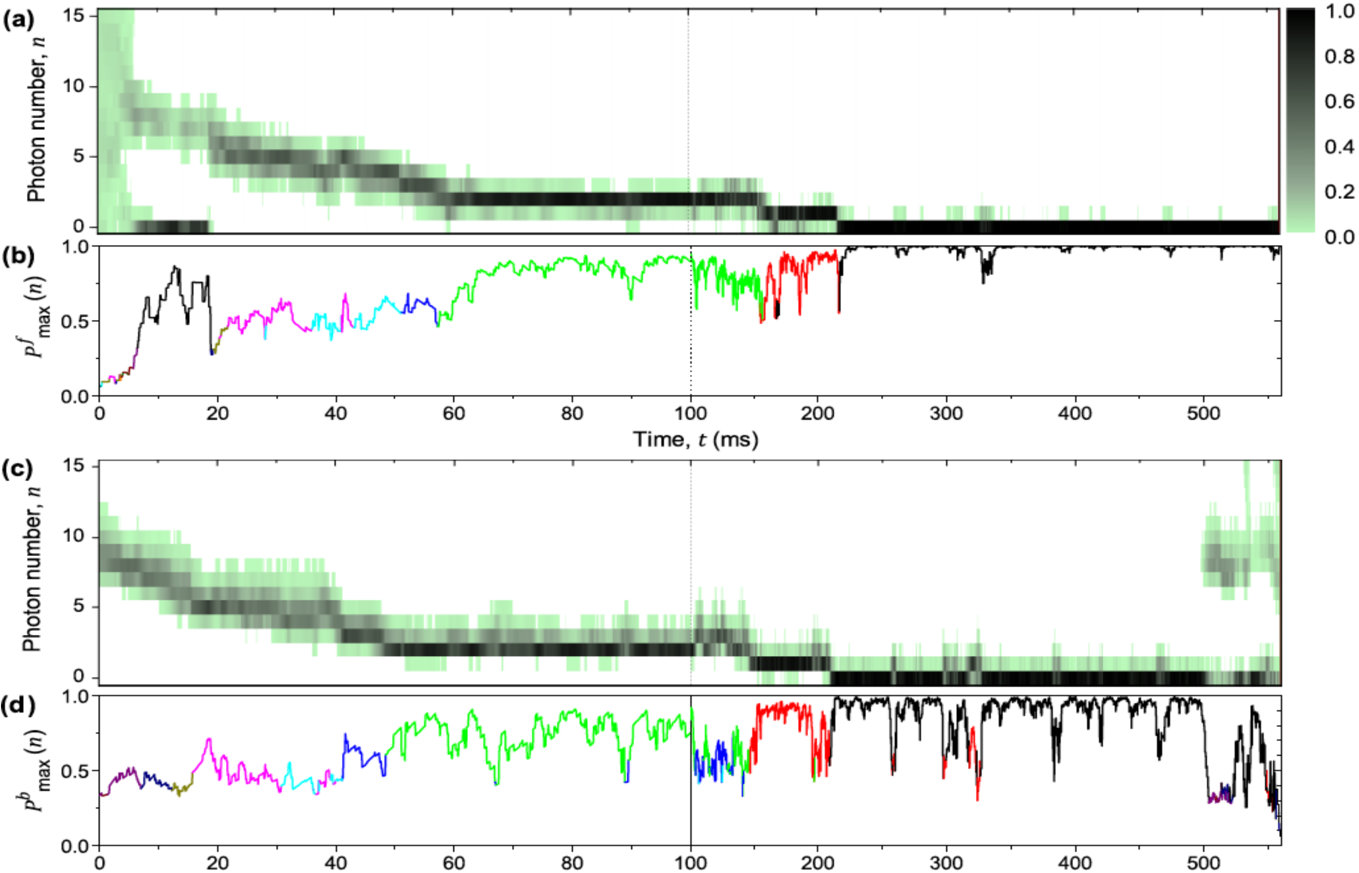
Obvious limitations

→ Noise due to statistical fluctuations of atomic detections

→ Initial ambiguity in the photon number due to the periodicity of the measurement operators

– Absurd photon number jumps (from 0 to 7)

Forward and backward estimations



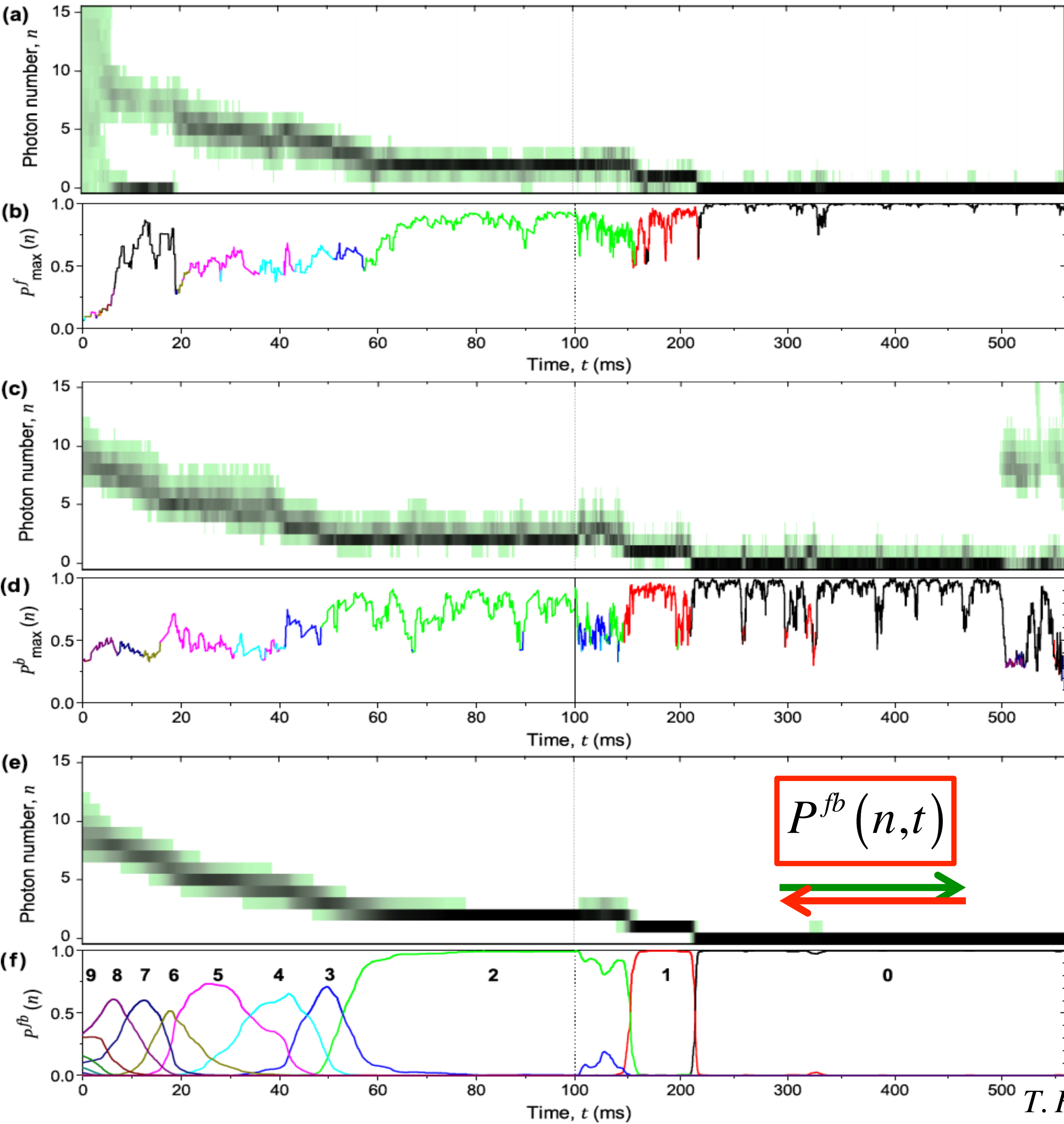
$P^f(n,t)$ →

$P^b(n,t)$ ←

- → Noise due to statistical fluctuations of atomic detections
- → Final ambiguity in the photon number due flat distribution at T and to the periodicity of the measurement operators
- → "Reverse" relaxation makes a good job!

T. Rybarczyk, et al. Phy Rev A ***91****, 062116 (2015)*

Forward and backward estimations



$P^f(n,t)$

$P^b(n,t)$

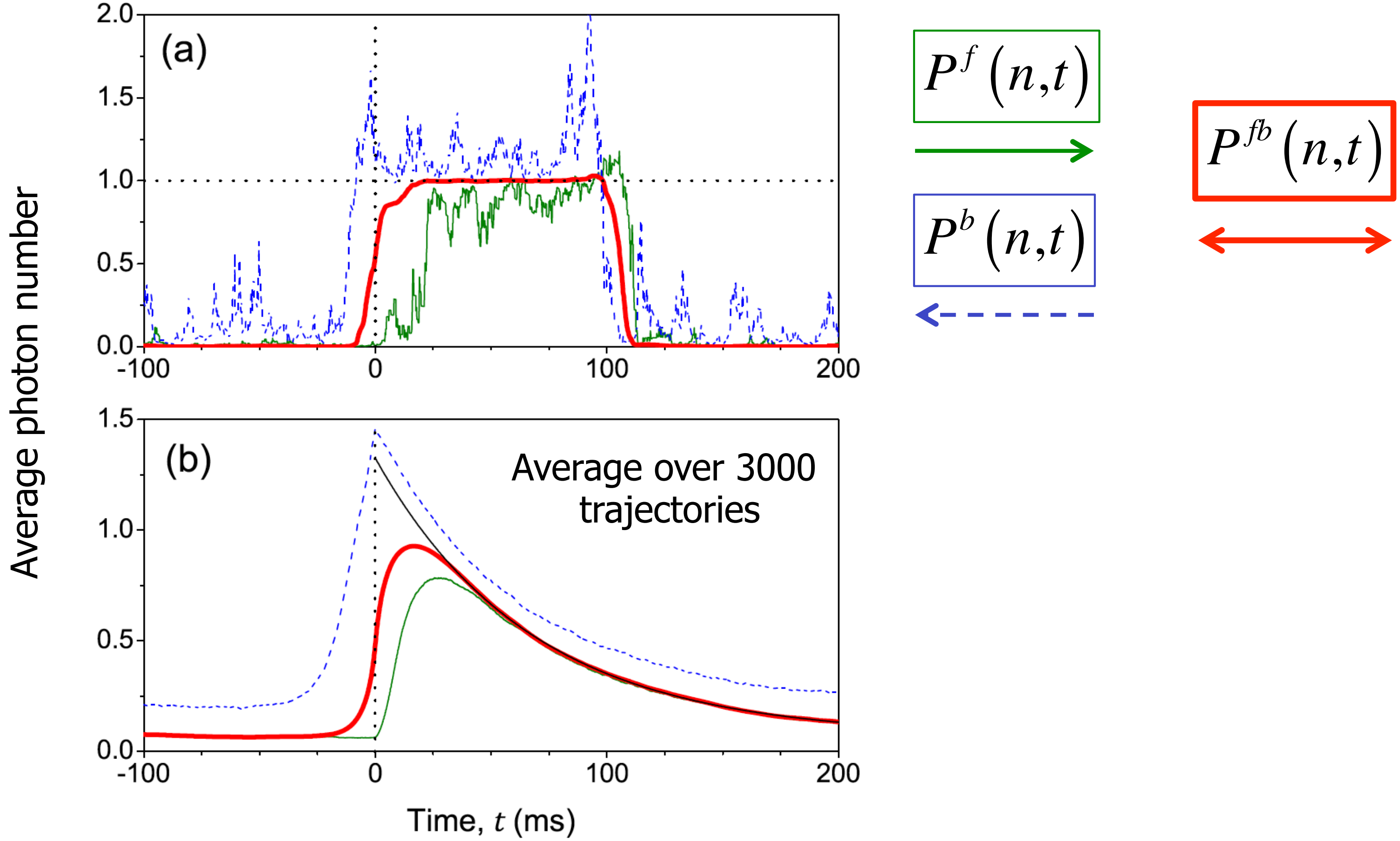
- Measurement ambiguities lifted
- Considerable noise reduction:

All estimations take into account **ALL** available information

T. Rybarczyk, et al. Phy Rev A ***91****, 062116 (2015)*

PQS estimation of a single-photon quantum jump

- A single photon is emitted by a resonant atom at t=0
- The estimator only knows QND measurement results

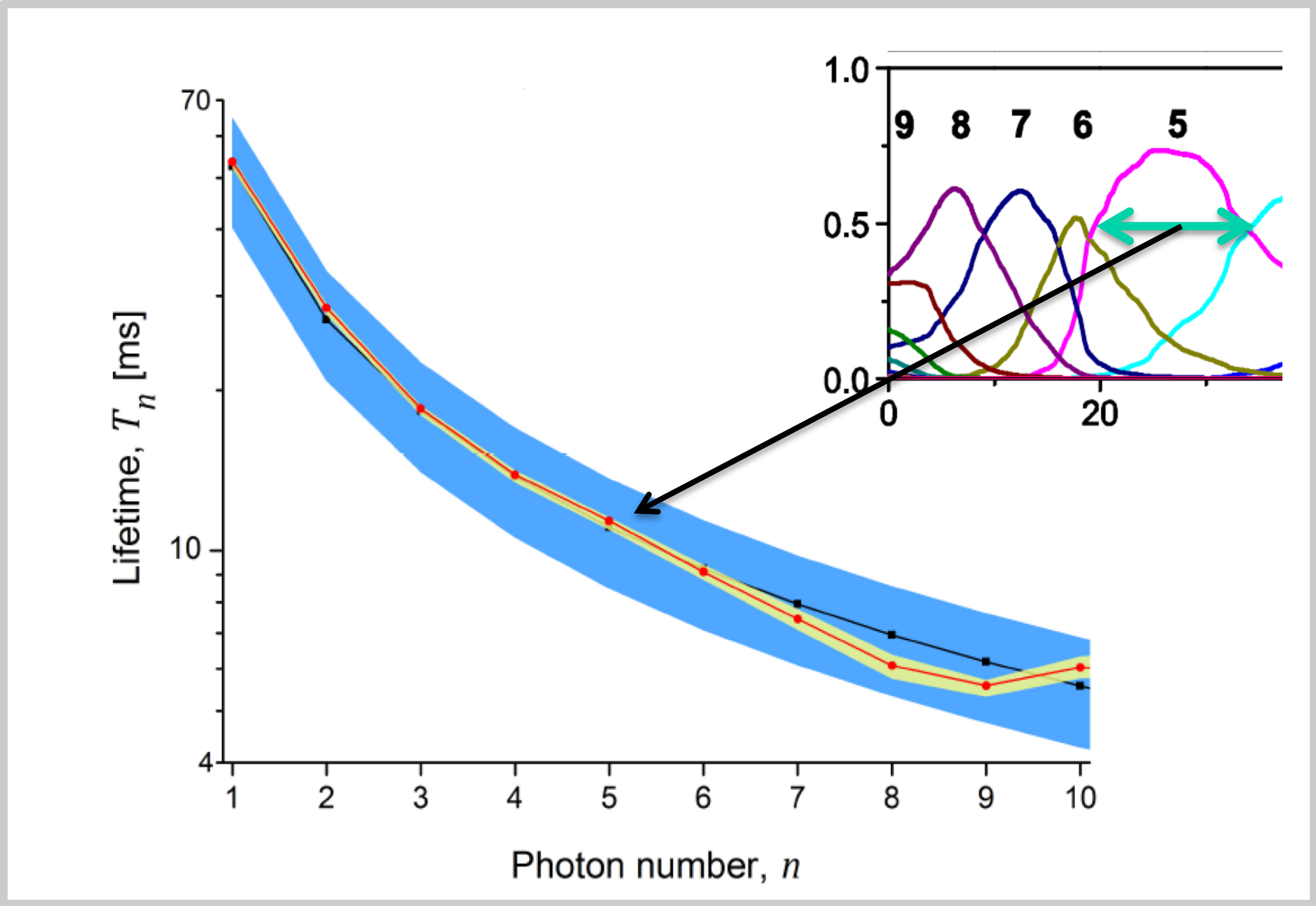


T. Rybarczyk, et al. Phy Rev A ***91****, 062116 (2015)*

Application: lifetime measurement of photon number states

- Analysis of average time between jumps

T. Rybarczyk, et al. Phy Rev A ***91****, 062116 (2015)*



→ An impossible feat with forward estimation only due to spurious noise-induced jumps *(Brune et al. PRL 101 240402)*

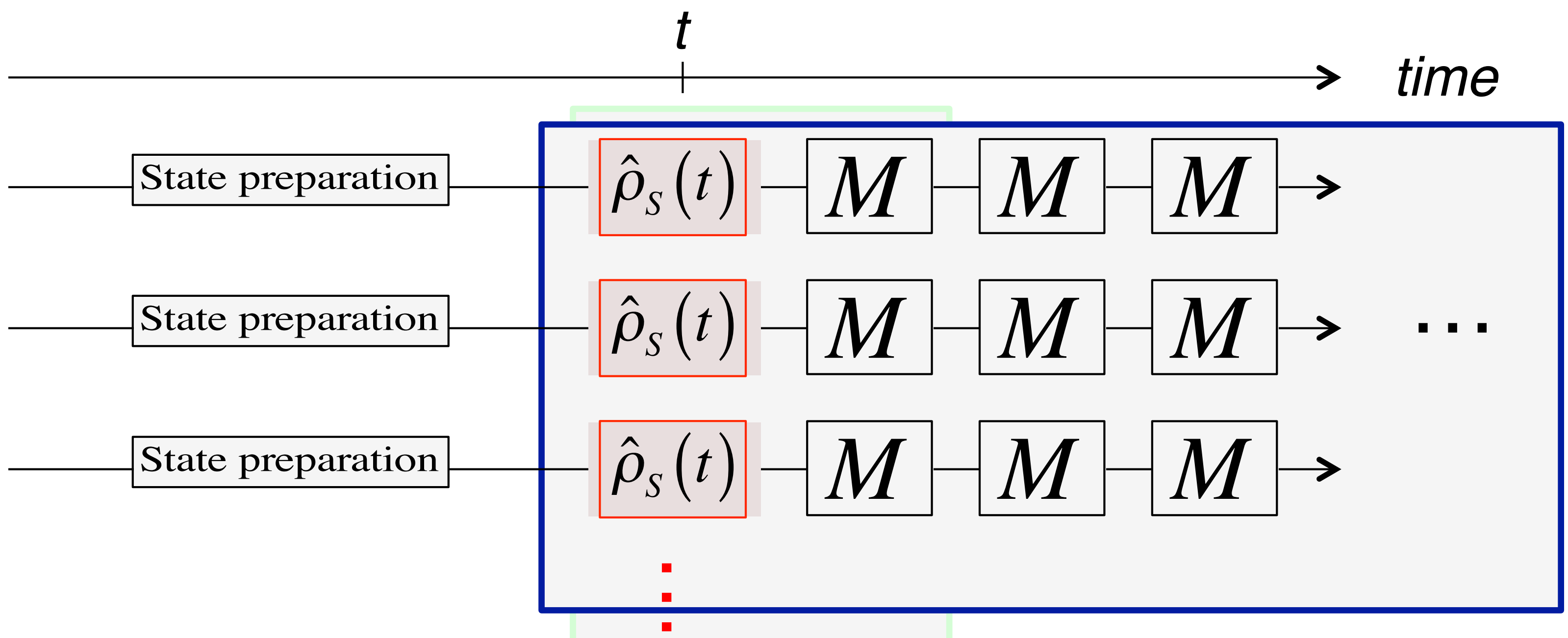
Past Quantum State: take home message

- PQS analysis is a fruitful tool for quantum state estimation
  - Also, for spin 1/2-like systems
    - → Gammelmark et al., PRA **89**, 043839
    - → Armen et al., PRL **103**, 173601
    - → Kerkhoff et al. Opt. Expr. **19**, 6478
    - → Tan et al., PRL **114**, 040903

- Cannot be used for real-time estimations
  - For instance in quantum feedback processes

(C. Sayrin et al. Nature **477**, 73; X.X. Zhou et al., PRL **108**, 243602)

The future: combining PQS and state tomography

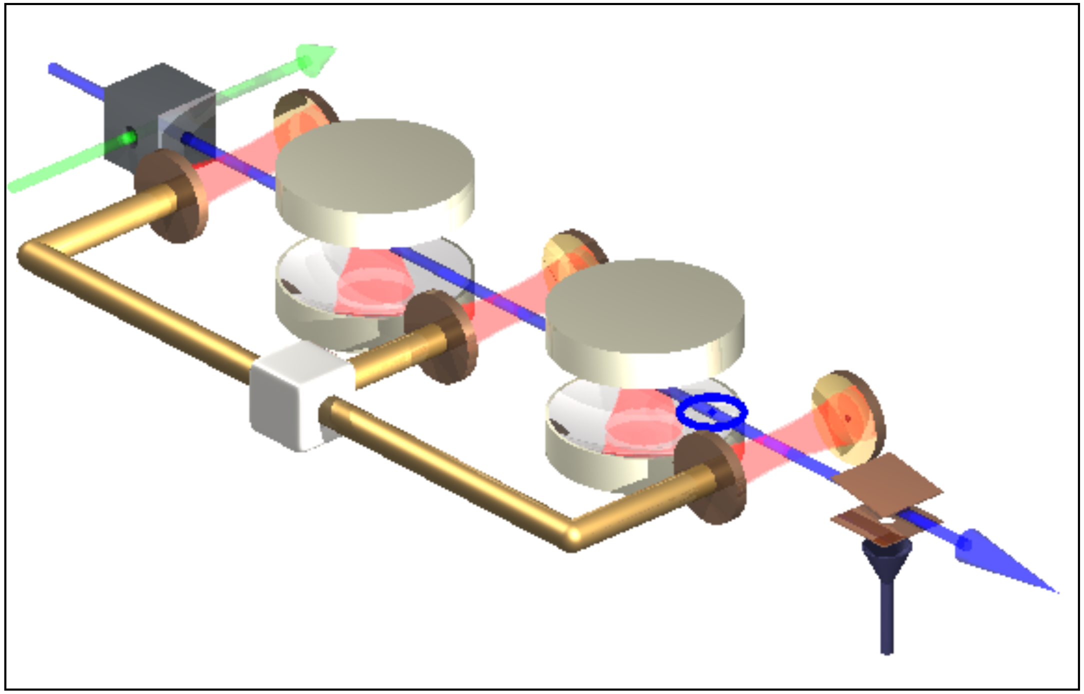


- Reconstruct $\hat{\rho}_S(t)$ given a large number of identical preparation
  → quantum state tomography

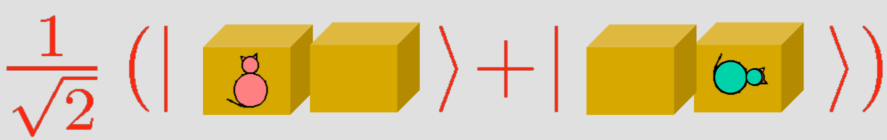
- Generalization:
  Reconstruct $\hat{\rho}_S(t)$ by using all measurements performed after state preparation

Six, P …. P. Rouchon,
PHYSICAL REVIEW A ***93*** *012109 (2016)*

CQED with two cavities



$$\frac{1}{\sqrt{2}}\left(|\alpha\rangle|0\rangle+|0\rangle|-\alpha\rangle\right)$$



➜ alive-here-and-dead-there state

Slow atoms cavity QED set-up

- Limitation of present experiments:

  Atom-cavity interaction time

  100 μs << 30 ms, 0.13 s

- Achieving long interaction times:

  A set-up with a nearly stationary Rydberg

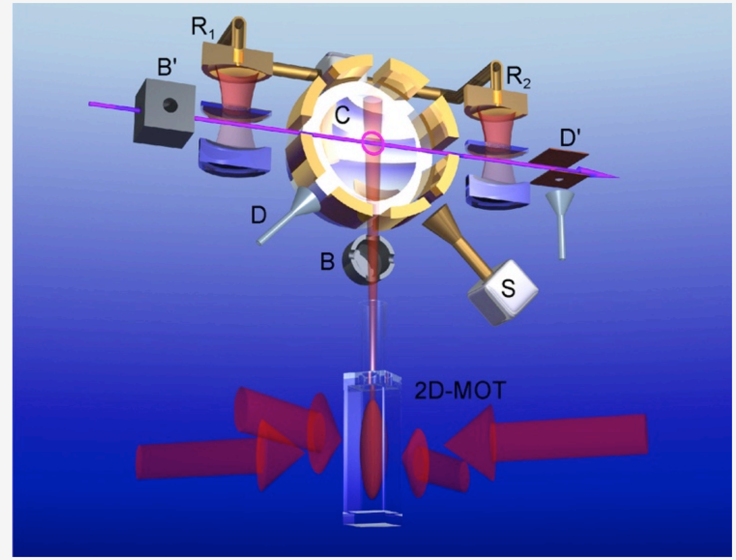
  atom in a cavity

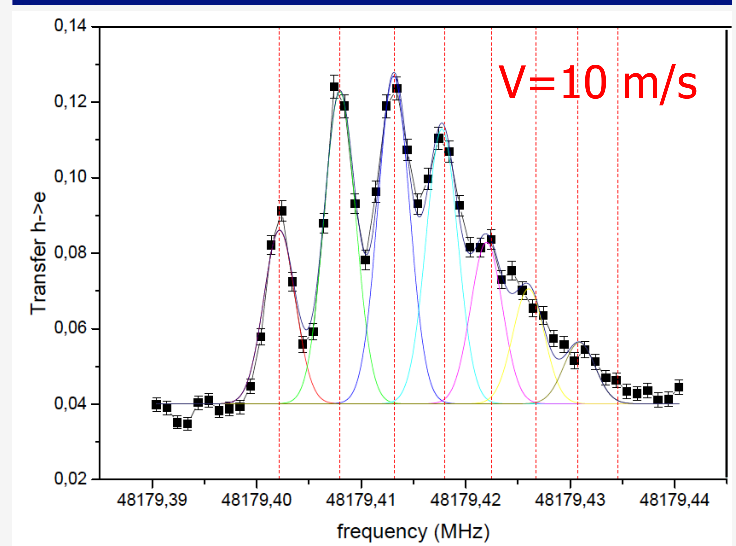
  - Interaction time: 10 ms range
  - Large cats, metrology of decoherence
  - Quantum Zeno dynamics

  J.M. Raimond et al PRL **105**, 213601 (2010)

  - Reservoir engineering

  A. Sarlette, A. et al. PRL **107**, 010402 (2011)





Dressed states spectroscopy

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