2. Introduction to Particle cosmology
Wimp paradigm, freeze out (and beyond)

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GRASPA School 24/07/2017

WHY ARE PARTICLE PHYSICISTS SO EXCITED ABOUT DM?

Dark Matter requires "new physics", beyond known theories, in order to be produced, and most likely is made of new degrees of freedom itself

Only a handful of similar indications for BSM: explains the interest of particle physicists!

Cosmology and astrophysics also give us some "particle physics" constraint

- *How much DM is out there*
- *DM is not "hot" (non-relativistic velocity distribution… as for the neutrinos)*
- *Must be stable or long-lived*
- *DM must be sufficiently heavy*
- *DM... is dark, and dissipationless*
- *DM is collisionless (or not very collisional)*
- *DM has small interactions with ordinary matter*

Won't review all of them, some detail on the first ones

NEUTRINOS AS DARK MATTER?

Condition 1. Must be massive (which is already a departure from SM...)

Fulfilled! Oscillations established, at least 2 massive states, measured splitting implies at least one state heavier than 0.05 eV

$$\Delta m^2_{\rm atm} \simeq 2.4 \times 10^{-3}\,{\rm eV}^2$$

More on that in the rest of the school!

Condition 2. Must match cosmological abundance

Failed! Direct mass limits combined with splittings from oscillation experiments impose upper limit of about 7 eV to the sum (After KATRIN, potentially improved to ~0.7 eV)

$$\Omega_\nu = \frac{\rho_\nu}{\rho_c} \simeq \frac{\sum_i m_i}{45\,{\rm eV}}$$

$\Omega_{DM} \approx 0.3$ (Planck) $\Rightarrow \sum m_i \approx 15$ eV

we will perform this computation

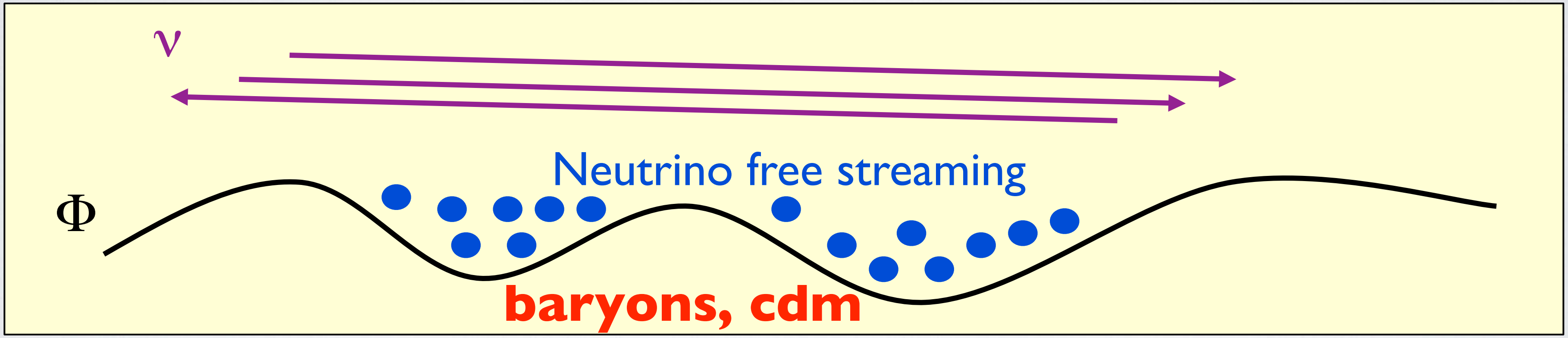
Condition 3. Must allow for structure formation (of the right kind)

Failed! This is a powerful argument excluding general classes of candidates (relativistic relics as DM, or so-called hot DM)

DM IS NOT "HOT" (IT IS NOT RELATIVISTIC)!

dark matter is not "hot": cannot have a relativistic velocity distribution (at least from matter-radiation equality for perturbation to grow)

This is the more profound reason why neutrinos would not work as DM, even if they had the correct mass: they were born with relativistic velocity distribution which prevents structures below O(100 Mpc) to grow till late!

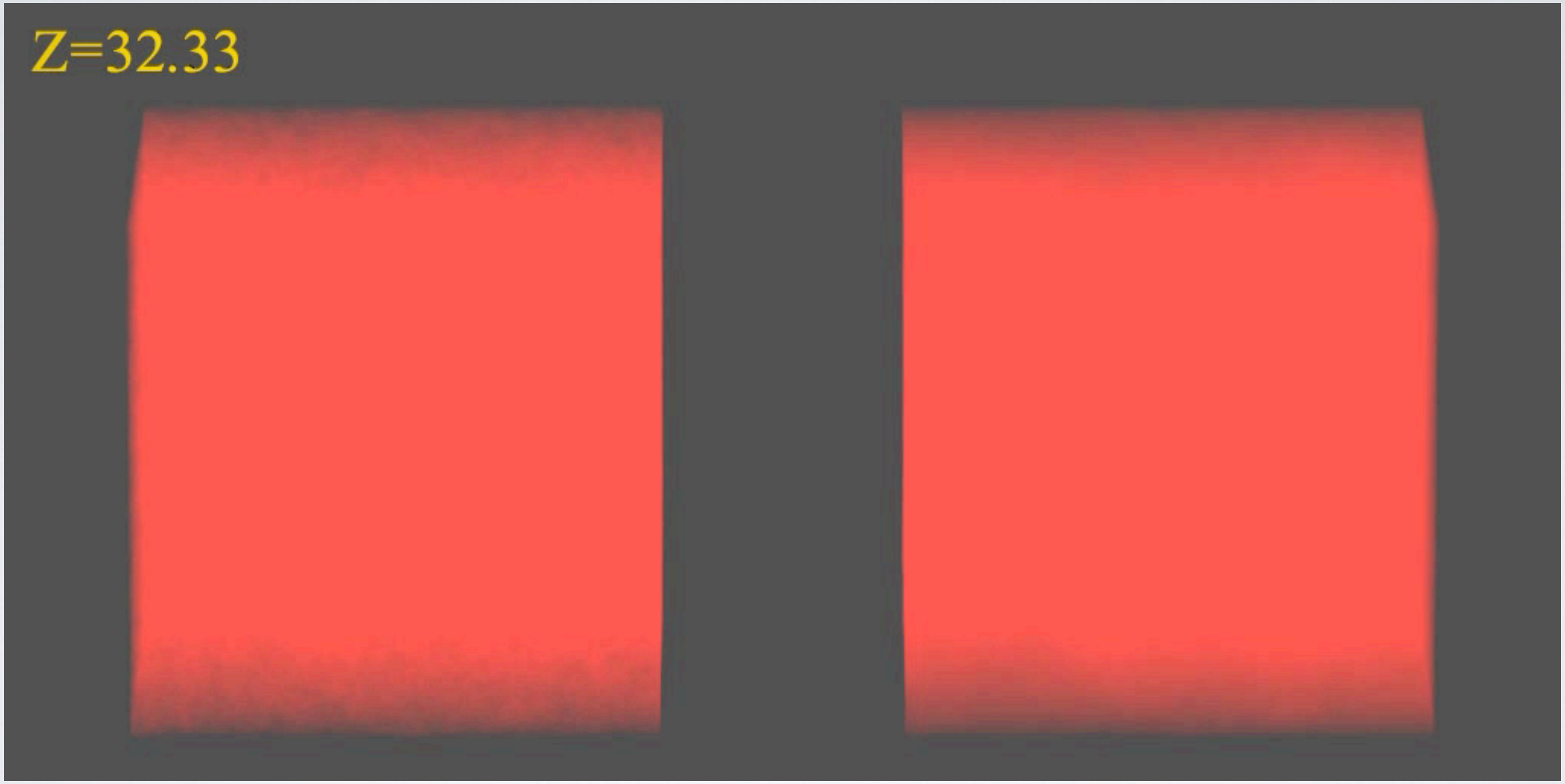


Cartoon Picture:

ν's "do not settle" in potential wells that they can overcome by their typical velocity: compared with CDM, they suppress power at small-scales

THE NUMERICAL PROOF

ΛCDM run vs. cosmology including neutrinos (total mass of 6.9 eV)



simulation by Troels Haugbølle

AN IMPORTANT NUMBER...

Recent determination (Planck 2015, 68% CL)

$\Omega_c h^2$=0.1188±0.0010, i.e. Ω_c~0.26

$$\rho_{X,0} = M_X\, n_{X,0} = M_X\, s_0\, Y_0$$

$$\rho_c = \frac{3H_0^2}{8\pi G_N} = 1.054 \times 10^{-5} h^2 \mathrm{GeV\,cm}^{-3}$$

$$s_0 = 2889 \left(\frac{T_{\gamma,0}}{2.725}\right)^3 \mathrm{cm}^{-3}$$

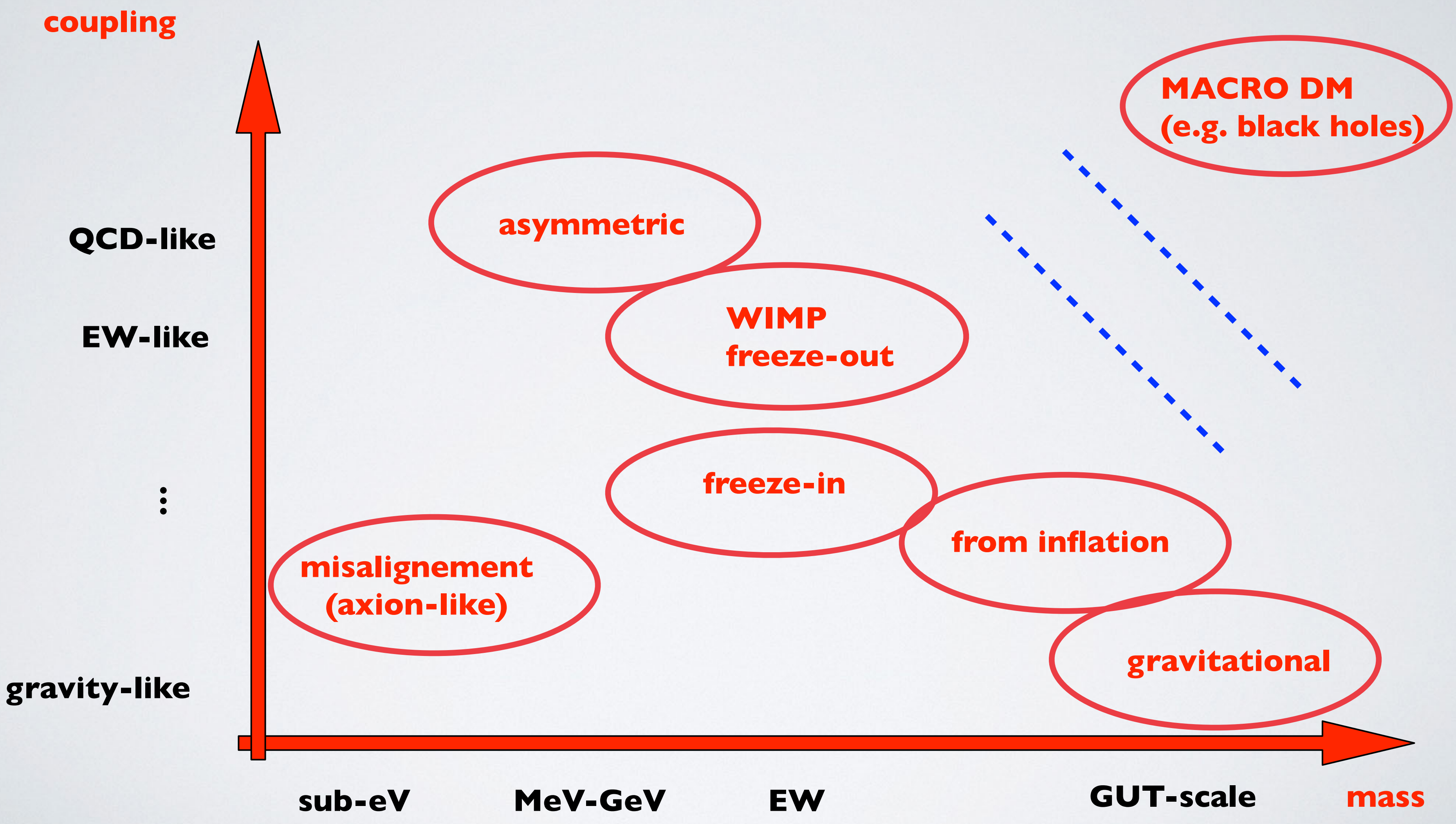
where $\mathbf{h_{eff}}$ ~ 2+3×2(4/11)×7/8~**3.91** comes from accounting for γ's & ν's

$$\Omega_X h^2 = 2.74 \times 10^8 \left(\frac{M_X}{\mathrm{GeV}}\right) Y_0$$

[Main] Goal *compute value of number to entropy density ratio, Y_0*

DM CLASSIFICATION / PARAMETER SPACE

Will discuss different classes based on production mechanisms. However, these are typically linked with masses and couplings as well!



BOLTZMANN EQ. FOR DM DENSITY CALCULATION

Assume that binary interactions of our particle X are present with species of the thermal bath

$$X\,X \leftrightarrow (\text{thermal bath particles})$$

If interaction rate **Γ=n σ v** very slow wrt Hubble rate **H**, # of particles conserved covariantly, i.e.

$$\frac{dn}{dt} + 3H\,n = 0 \Rightarrow n \propto a^{-3}$$

If interaction rate **Γ>> H**, # of particles follows equilibrium, e.g. for non-relativistic particles

$$n_{\text{eq}} = g \left(\frac{m\,T}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right)$$

The following equation has the right limiting behaviours

$$\frac{dn}{dt} + 3H\,n = -\langle\sigma v\rangle[n^2 - n_{\text{eq}}^2]$$

must be quadratic, for binary processes

for now, symbolic only

REWRITING IN TERMS OF Y AND x

$$\frac{dn}{dt} + 3H\,n = -\langle\sigma v\rangle[n^2 - n_{\rm eq}^2] \qquad \frac{dY}{dt} = -s\langle\sigma v\rangle[Y^2 - Y_{\rm eq}^2]$$

$$\frac{dY}{dt} = \frac{d}{dt}\left(\frac{n}{s}\right) = \frac{d}{dt}\left(\frac{n\,a^3}{s\,a^3}\right) = \frac{1}{s\,a^3}\frac{d}{dt}\left(n\,a^3\right) = \frac{1}{s\,a^3}\left(a^3\frac{dn}{dt} + 3a^2\dot{a}\,n\right) = \frac{1}{s}\left(\frac{dn}{dt} + 3H\,n\right)$$

Define *x=m/T* (*m arbitrary mass, either* M_X *or not*); for an iso-entropic expansion one has

$$\frac{d}{dt}(a^3 s) = 0 \Longrightarrow \frac{d}{dt}(a\,T) = 0 \Longrightarrow \frac{d}{dt}(a/x) = \frac{\dot{a}}{x} - \frac{a}{x^2}\dot{x} = 0 \Longrightarrow \frac{dx}{dt} = H\,x$$

$$\frac{dY}{dx} = -\frac{x\,s\langle\sigma v\rangle}{H(T=m)}[Y^2 - Y_{\rm eq}^2]$$

radiation-dominated period

More in general (arbitrary *s(t)* and *H(t)*):

$$\frac{dY}{dx} = -\sqrt{45\pi}M_{\rm Pl}\,m\frac{h_{\rm eff}(x)\langle\sigma v\rangle}{\sqrt{g_{\rm eff}(x)}\,x^2}\left(1 - \frac{1}{3}\frac{d\log h_{\rm eff}}{d\log x}\right)(Y^2 - Y_{\rm eq}^2)$$

M. Srednicki, R. Watkins and K.A. Olive,
"Calculations of Relic Densities in the Early Universe,"
Nucl. Phys. B 310, 693 (1988)

P. Gondolo and G. Gelmini,
"Cosmic abundances of stable particles: Improved analysis,"
Nucl. Phys. B 360, 145 (1991).

FREEZE-OUT CONDITION

The previous equation is a *Riccati equation*: no closed form solution exist!

Approximate analytical solutions exist for different hypotheses/regimes

(In the following, we shall assume the choice $m = M_X$)

For $h_{eff} \sim$ const., we can re-write

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm eq}}{H}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right] \quad \text{with} \quad \Gamma_{\rm eq} = \langle\sigma v\rangle n_{\rm eq}$$

If $\Gamma_{eq} >> H$ the particle starts from equilibrium condition at sufficiently small x (high-T), when relativistic. Crucial variable to determine the Y_{final} is the freeze-out epoch x_F from condition

$$\Gamma_{\rm eq}(x_F) = H(x_F)$$

RELATIVISTIC FREEZE-OUT

$$\Gamma_{\rm eq}(x_F) = H(x_F)$$

If the solution to this condition yields x_F<<*1*, then *(Lecture 1)*

$$n = g\frac{\zeta(3)}{\pi^2}T^3 \times \left\{1(\text{B}), \frac{3}{4}(\text{F})\right\}$$

comoving abundance stays constant, and independent of *x (if dof do not change)*

$$Y(x_F) = 0.28\frac{g \times \{1(\text{B}), 3/4(\text{F})\}}{h_{\rm eff}(x_F)}$$

Today's abundance of such a relativistic freeze-out relic is thus

$$\Omega_X h^2 = 0.0762 \times \left(\frac{M_X}{\text{eV}}\right)\frac{g \times \{1(\text{B}), 3/4(\text{F})\}}{h_{\rm eff}(x_F)}$$

For the neutrino case, $h_{\rm eff}$=10.75, g×{ }=3/2, thus

$$\Omega_\nu h^2 \simeq \frac{\sum m_\nu}{94\,\text{eV}}$$

Inconsistent with DM for current upper limits!

NON-RELATIVISTIC FREEZE-OUT

to determine x_F

$$\Gamma_{\rm eq}(x_F) = H(x_F)$$

$$\frac{g\langle\sigma v\rangle}{(2\pi)^{3/2}} M_X^3\, x_F^{-3/2} e^{-x_F} = \sqrt{\frac{4\pi^3}{45} g_{\rm eff}}\, \frac{M_X^2}{x_F^2\, M_{\rm Pl}}$$

$$x_F^{1/2} e^{-x_F} = \sqrt{\frac{4\pi^3}{45} g_{\rm eff}}\, \frac{(2\pi)^{3/2}}{M_{\rm Pl} M_X\, g\, \langle\sigma v\rangle}$$

Thus one obtains

$$Y(x_F) = \frac{n(x_F)}{s(x_F)} = \frac{g}{h_{\rm eff}} \frac{45}{2\pi^2 (2\pi)^{3/2}} x_F^{3/2} e^{-x_F}$$

which also writes

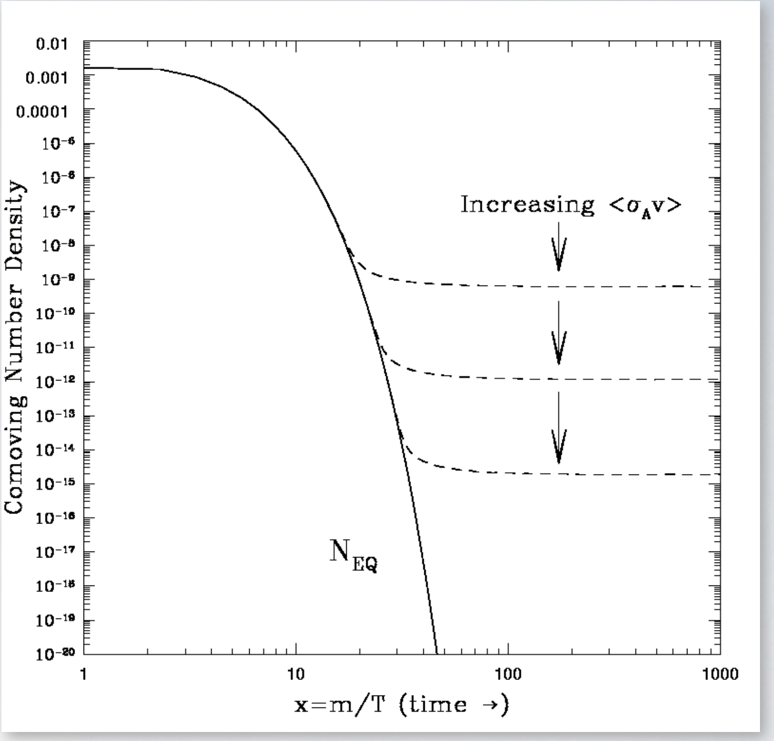
(Note the important result ***$Y(x_F)$~ 1/<σv>***)

$$Y(x_F) = \sqrt{\frac{45\, g_{\rm eff}}{\pi}}\, \frac{x_F}{h_{\rm eff} M_{\rm Pl}\, M_X\, \langle\sigma v\rangle} = \mathcal{O}(1)\, \frac{x_F}{M_{\rm Pl}\, M_X\, \langle\sigma v\rangle}$$

NON-RELATIVISTIC FREEZE-OUT: INTERPRETATION

$$Y(x_F) \simeq \mathcal{O}(1) \frac{x_F}{M_{\rm Pl}\, M_X\, \langle \sigma v \rangle}$$

makes sense, in the Boltzmann suppressed tail: The more it interacts, the later it decouples, the fewer particles around.



Also, plugging numbers (typically x_F~30), one has

$$\Longrightarrow \Omega_X h^2 \simeq \frac{0.1\,\mathrm{pb}}{\langle \sigma v \rangle}$$

dimensionally, for electroweak scale masses and couplings, one gets the right value!

$$\langle \sigma v \rangle \sim \frac{\alpha^2}{m^2} \simeq 1\,\mathrm{pb} \left(\frac{200\,\mathrm{GeV}}{m} \right)^2$$

But the pre-factor depends from widely different cosmological parameters (Hubble parameter, CMB temperature) and the Planck scale. Is this match simply a coincidence?

Dubbed sometimes "Weakly Interacting Massive Particle" (WIMP) Miracle

EXERCISE

By using any software of your choice (including symbolic ones like Mathematica©, etc.), write a simple code to solve for the relic abundance equation.

- Compare with the analytical approximations discussed during the lecture.
- Feel free to explore what happens under different conditions (e.g. different dependences for the cross section; epochs of entropy variations… for which the exercise assigned in Lec. 1 is needed!)

Have a look at 1204.3622 for comparison and for some "tricks" on how to make the computation more efficient (notably if you find, as you probably should, problems of numerical stiffness)

CARE SHOULD BE TAKEN WHEN DEALING WITH...

- cohannihilations with other particle(s) close in mass
- resonant annihilations*
- thresholds*

K. Griest and D. Seckel, "Three exceptions in the calculation of relic abundances," Phys. Rev. D 43, 3191 (1991).

* i.e., whenever **$\sigma(s)$** is a strongly varying function of the center-of-mass energy ***s***
(one recently popular example is the "Sommerfeld Enhancement")

For a pedagogical overview of generalization in presence of coannihilations (and decays), see

J. Edsjo and P. Gondolo, "Neutralino relic density including coannihilations," Phys. Rev. D 56, 1879 (1997) [hep-ph/9704361].

Nowadays, relic density calculations have reached a certain degree of sophistication and are automatized with publicly available software. But if you have a theory with "unusual" features... better to check!

MicrOMEGAs: a code for the calculation of Dark Matter Properties including the relic density, direct and indirect rates in a general supersymmetric model and other models of New Physics

Dark SUSY

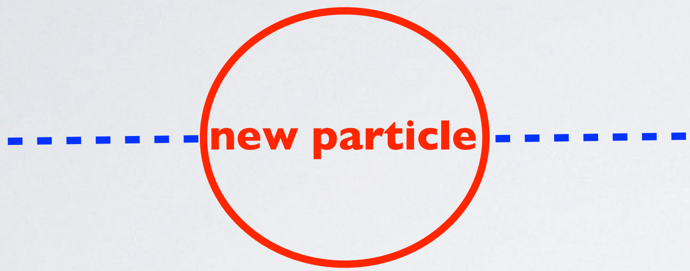
http://lapth.cnrs.fr/micromegas/

http://www.physto.se/~edsjo/darksusy/

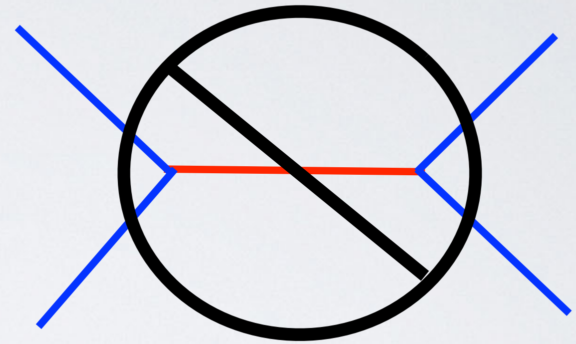
LINK WITH COLLIDERS

- If one has a strong prior for new TeV scale physics (~with ew. strength coupling) due to the hierarchy problem, precision ew data (e.g. from LEP) suggest that tree-level couplings SM-SM-BSM should be avoided!

we want it!



we want to avoid!



- Straightforward solution (not unique!) is to impose a discrete "parity" symmetry e.g.: SUSY R-parity, K-parity in ED, T-parity in Little Higgs. New particles only appear in pairs!

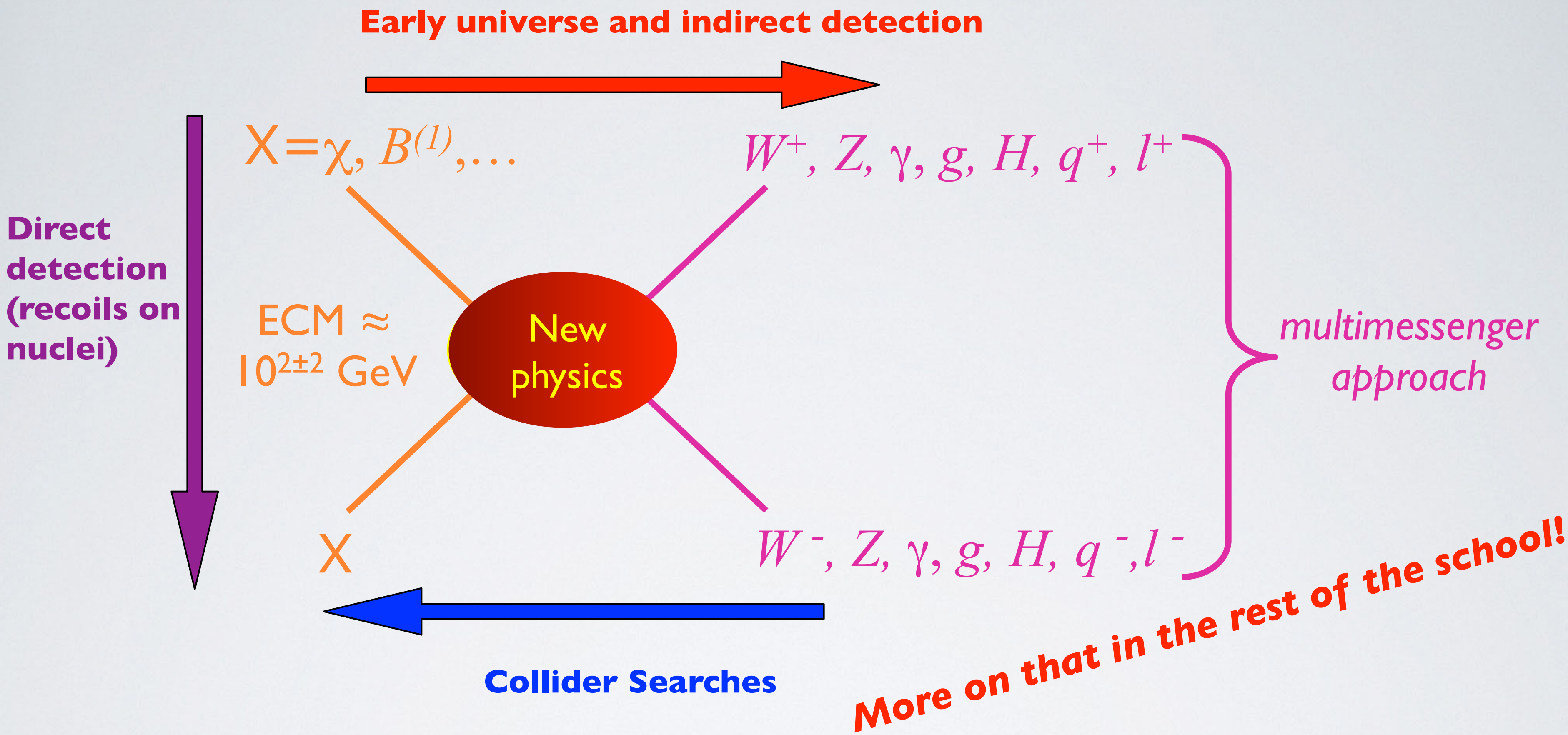
  ➡ Automatically makes lightest new particle stable!
  ➡ May have other benefits (e.g. respect proton stability bounds...)

In a sense, some WIMP DM (too few? too much?) is "naturally" expected for consistency of the currently favored framework for BSM physics at EW scale.

Beware of the reverse induction:

LHC is current our best tool to test this paradigm, but if no new physics is found at EW scale it is at best the WIMP scenario to be disfavored, not the "existence of DM"

WIMP (NOT GENERIC DM!) SEARCH PROGRAM



- ✓ demonstrate that astrophysical DM is made of particles (locally, via DD; remotely, via ID)
- ✓ Possibly, create DM candidates in the controlled environments of accelerators
- ✓ Find a consistency between properties of the two classes of particles. Ideally, we would like to calculate abundance and DD/ID signatures → link with cosmology/test of production

F("FEEBLY")IMPS… AND FREEZE-IN

We solved the evolution equation for Y under the *assumption* of initial equil. abundance, $Y(x<<1)=Y_{eq}$

$$\frac{dY}{dx} = -\frac{x\, s\langle\sigma v\rangle}{H(T=m)}\left[Y^2 - Y_{\rm eq}^2\right]$$

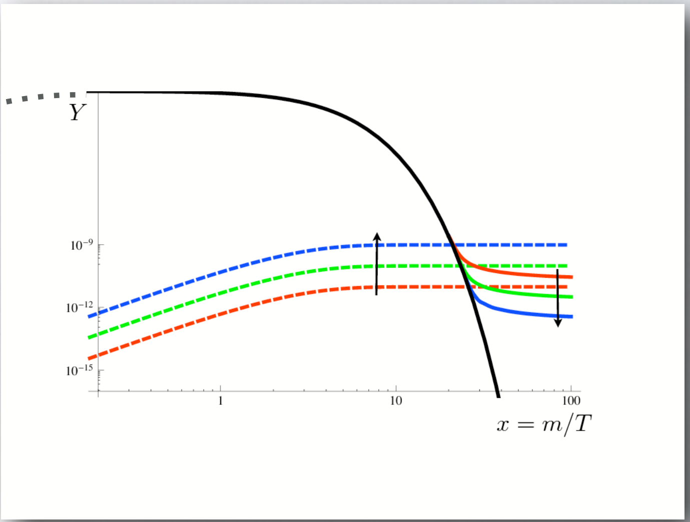
This is unnecessary: had we started with $Y(x_0<<1)=0$, provided that $\Gamma_{eq} / H = K>>1$ the equation

$$\frac{x}{Y_{\rm eq}}\frac{dY}{dx} = -\frac{\Gamma_{\rm eq}}{H}\left[\left(\frac{Y}{Y_{\rm eq}}\right)^2 - 1\right]$$ with $$\Gamma_{\rm eq} = \langle\sigma v\rangle n_{\rm eq}$$

admits the solution $Y \sim Y_{eq}\, K \ln(x/x_0)$ *[assuming K constant…which is not!]* so equilibrium is attained when $x \sim x_0 \exp(1/K)$, i.e. only a 10% increase wrt x_0 for K=10!

However, if $\Gamma_{eq}/H = K<<1$ (i.e., ***feeble*** coupling!) it never attains equilibrium: yet it can match the required DM value via the residual production from the plasma

That's called "***Freeze In***", since it's the reverse of Freeze out



L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, "Freeze-In Production of FIMP Dark Matter," JHEP 1003, 080 (2010) [0911.1120]

SOME PROPERTIES OF FREEZE-IN

In the eq., we can then neglect Y wrt Y_{eq}

$$\frac{dY}{dx} \simeq \frac{x\, s \langle \sigma v \rangle}{H(m)} Y_{eq}^2$$

Assuming negligible initial abundance (otherwise it's not produced via freeze-in!)

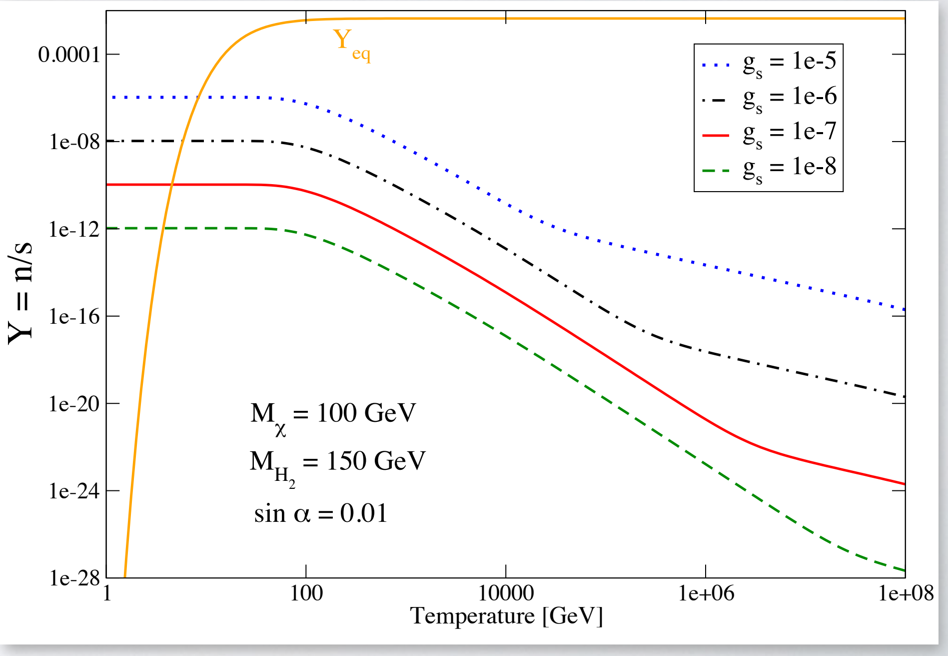
$$Y_\infty \simeq \int_{x_0}^{\infty} dx' \frac{x'\, s\, \langle \sigma v \rangle}{H(m)} Y_{eq}^2$$

- Note that now $Y_\infty \propto \langle \sigma v \rangle$ **inverse dependence** wrt WIMP freeze-out

- Can also check that ***Y* saturates at smaller *x*** (order 1) wrt x_{fo}~20-30 (early universe history more important)

- Can be generalized to other production mechanisms, e.g. via decays in the plasma (similarly, Y_∞ proportional to decay rate…)

- Since it typically requires small couplings, it is ***harder to test*** (possible ***signatures more model dependent***)



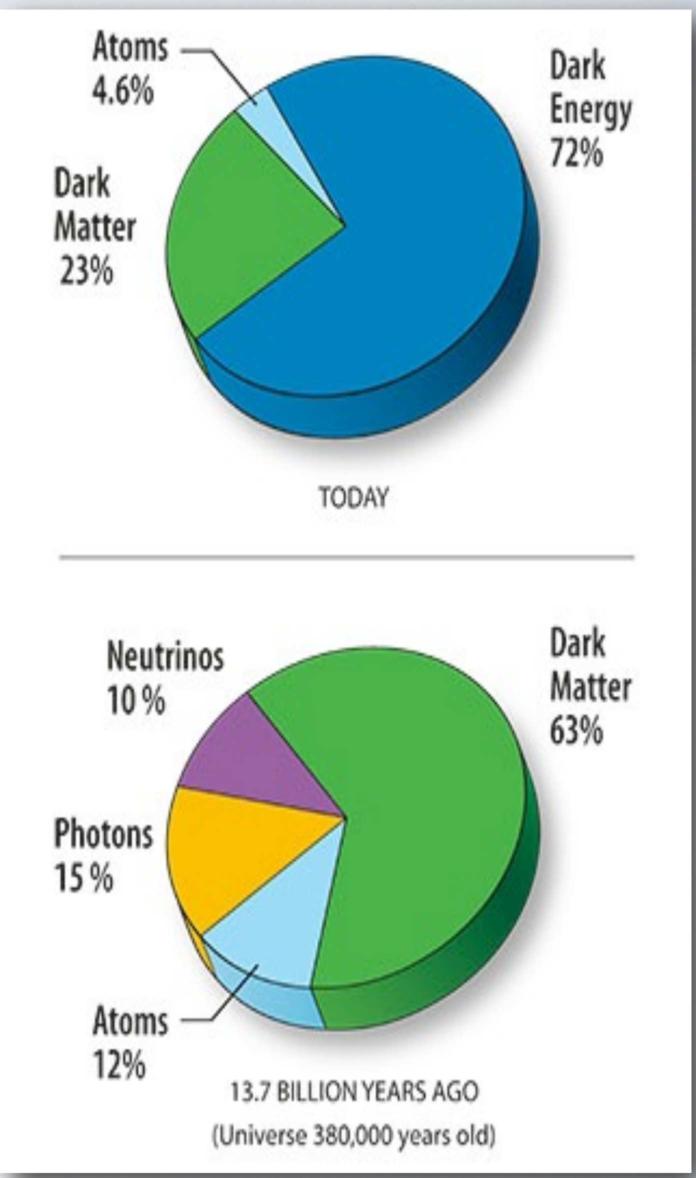
M. Klasen and C. E. Yaguna, "Warm and cold fermionic dark matter via freeze-in," JCAP 1311, 039 (2013)

THE MISLEADING PIE CHART PROBLEM

Perhaps you heard that we do not know what 95% of the Universe is made of (because of DM and DE)

The **situation is worse!** We do not know where baryons come from either! (and actually for neutrinos we think we know how they were produced, but not the origin & value of their mass, hence their contribution to pie)

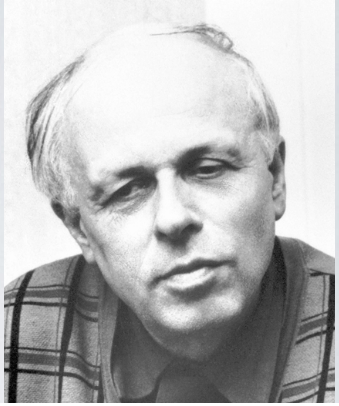
The (yet unknown) physical mechanism behind the observed matter-antimatter asymmetry is called **Baryogenesis**



ANDREI SAKHAROV CONDITIONS

One can **generate baryon asymmetry dynamically** in QFT provided that

- **B is violated** *(obvious, otherwise B does not change with t)*
- **violation of C** *(to avoid extra B-production reactions balanced by extra anti-B production reactions)* **& CP** *(to avoid excess of left-handed B compensated by right-handed anti-B)*
- **departure from thermal equilibrium takes place** *(otherwise processes increasing & decreasing B @ equilibrium by CPT-symmetry, i.e. equil. distributions only depend on mass, equal for part/antipart. due to CPT)*



Nobel Peace Prize 1975

Remarkably, these **conditions are met in the SM as well**... but (to cut a long story short) "**too weakly**" to be useful (2nd order EW phase transition, small CP-violation…)

The two main (not unique!) classes of BSM model trying to explain that are:

- **EW baryogenesis:** in extended models of TeV scale physics (SUSY or not), the problem mentioned with the SM could be overcome.

- **Leptogenesis:** initial L production, then reshuffled via sphalerons (violating B+L but conserving B-L in the SM at T~100 GeV). L asymmetry linked to neutrino mass generation, but typically (although not necessarily) high mass scale phenomenon

ASYMMETRIC DM?

$$\frac{\Omega_{dm}}{\Omega_b} \simeq 5$$

Is this relation suggestive of a common origin (co-genesis)?

Not in WIMP paradigm! DM result of thermal freeze-out, baryons from some unknown **baryogenesis** mechanism. A nice avenue would be some similar process happening in the dark sector, too!

Theoretically, not hard: all classes of models considered for baryogenesis can be considered (actually more, since phenomenologically more freedom in dark sector...)

K. Zurek "Asymmetric Dark Matter: Theories, Signatures, and Constraints," Phys. Rept. 537 91 (2014) 1308.0338

- Introduce a DM candidate which is not self-conjugated, allowing for asymmetry in number density $$n_{dm} - \bar{n}_{dm} \neq 0$$
- Use dynamics to relate it to the baryon asymmetry $$n_{dm} - \bar{n}_{dm} \propto n_b - \bar{n}_b$$
- Generically one has (κ model dependent!) $$\frac{\Omega_{dm}}{\Omega_b} = \frac{|n_{dm} - \bar{n}_{dm}| m_{dm}}{n_b\, m_b} \simeq \kappa \frac{m_{dm}}{m_N}$$

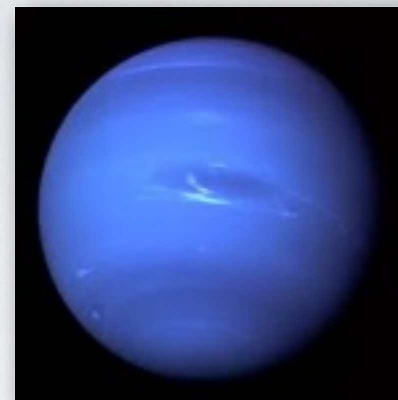
SUMMARY OF WHAT WE LEARNED

- We described heuristically how to derive the relic abundance via freeze-out mechanism
- We saw why non-relativistic relics seem to work... WIMP cold DM paradigm.
- WIMPs rich in collider, direct and indirect signatures, hence well studied.
- We described the "freeze-in" alternative (harder to detect!)
- Asymmetric DM alternative. Introduction of Baryogenesis, and links with it

Hopefully, you have an idea of key concepts in cosmology, of key evidences for puzzling physics (notably DM, but also baryon asymmetry) and of some ideas we have on what that could be, and how to search for it, at the interface of particle physics, astrophysics and cosmology

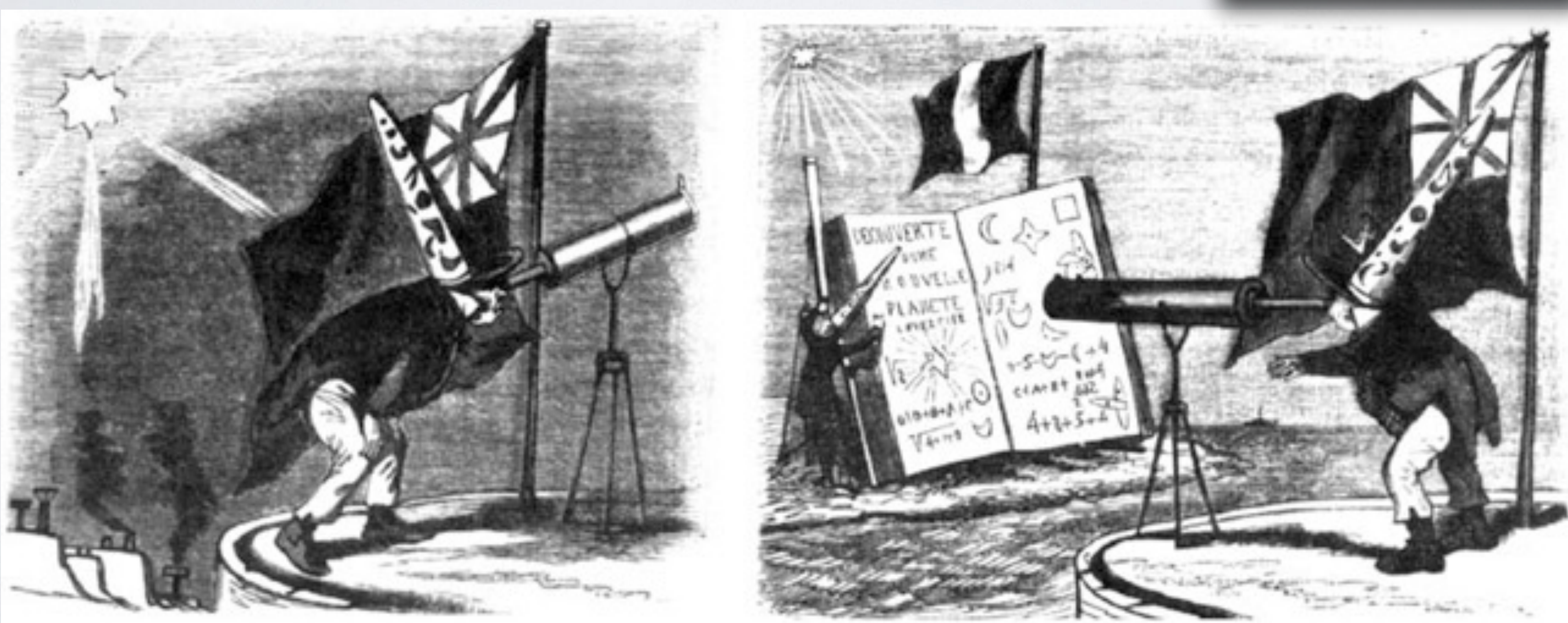
IF YOU'RE PESSIMIST, REMEMBER

An additional "species" inferred from gravitational effects has been already identified (*electromagnetically detected*) once!



Adams (1844-45) and independently Le Verrier (1845-46) interpreted irregularities in Uranus's orbit as due to perturbation by a yet unknown planet, calculating its orbital elements "by inversion"

On September 24, 1846 Galle found that "the planet whose place you [Le Verrier] have [computed] *really exists*"

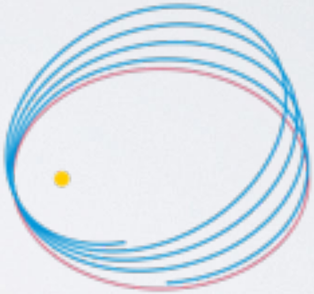


A cartoon published in France at the time of the controversy over the discovery of Neptune
Adams is shown looking for it in vain and then finding it in the pages of Leverrier's book.

BUT... SOMETIMES SURPRISES SHOW UP!

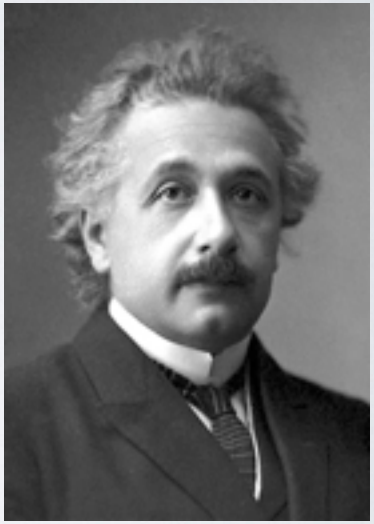
In 1859, Le Verrier analyzed the effect of gravitational perturbations of other planets on the perihelion shift of Mercury, finding a residual "anomalous" shift of 38 arcsec/century.

He re-used his "old" trick, hypothesizing that this was the result of another planet, which he named *Vulcan* whose orbital elements he inferred.



This planet was claimed to be found *several times*...

... but its existence was eventually disproved and Mercury's anomaly (re-evaluated in 43 arcsec/century) was finally explained thanks to GR effects (first major postdiction that convinced A. Einstein that GR was right)



hence, "Dark Matter" (just like "Modified Gravity") has already been discovered... but only after several trials & errors, hard work, and fake claims: Be patient, and be ready for the unexpected, too!

BY THE WAY, STILL FRUITFUL STRATEGY…

THE ASTRONOMICAL JOURNAL, 151:22 (12pp), 2016 February
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EVIDENCE FOR A DISTANT GIANT PLANET IN THE SOLAR SYSTEM

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ABSTRACT

Recent analyses have shown that distant orbits within the scattered disk population of the Kuiper Belt exhibit an unexpected clustering in their respective arguments of perihelion. While several hypotheses have been put forward to explain this alignment, to date, a theoretical model that can successfully account for the observations remains elusive. In this work we show that the orbits of distant Kuiper Belt objects (KBOs) cluster not only in argument of perihelion, but also in physical space. We demonstrate that the perihelion positions and orbital planes of the objects are tightly confined and that such a clustering has only a probability of 0.007% to be due to chance, thus requiring a dynamical origin. We find that the observed orbital alignment can be maintained by a distant eccentric planet with mass $\gtrsim 10\, m_{\oplus}$ whose orbit lies in approximately the same plane as those of the distant KBOs, but whose perihelion is 180° away from the perihelia of the minor bodies. In addition to accounting for the observed orbital alignment, the existence of such a planet naturally explains the presence of high-perihelion Sedna-like objects, as well as the known collection of high semimajor axis objects with inclinations between 60° and 150° whose origin was previously unclear. Continued analysis of both distant and highly inclined outer solar system objects provides the opportunity for testing our hypothesis as well as further constraining the orbital elements and mass of the distant planet.

Key words: Kuiper Belt: general – planets and satellites: dynamical evolution and stability