Gravitational waves

Theory, detection principles, VIRGO and some news…

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Graduate School in Particle and Astroparticle physics, Annecy, July 2017

- *Announcements :*

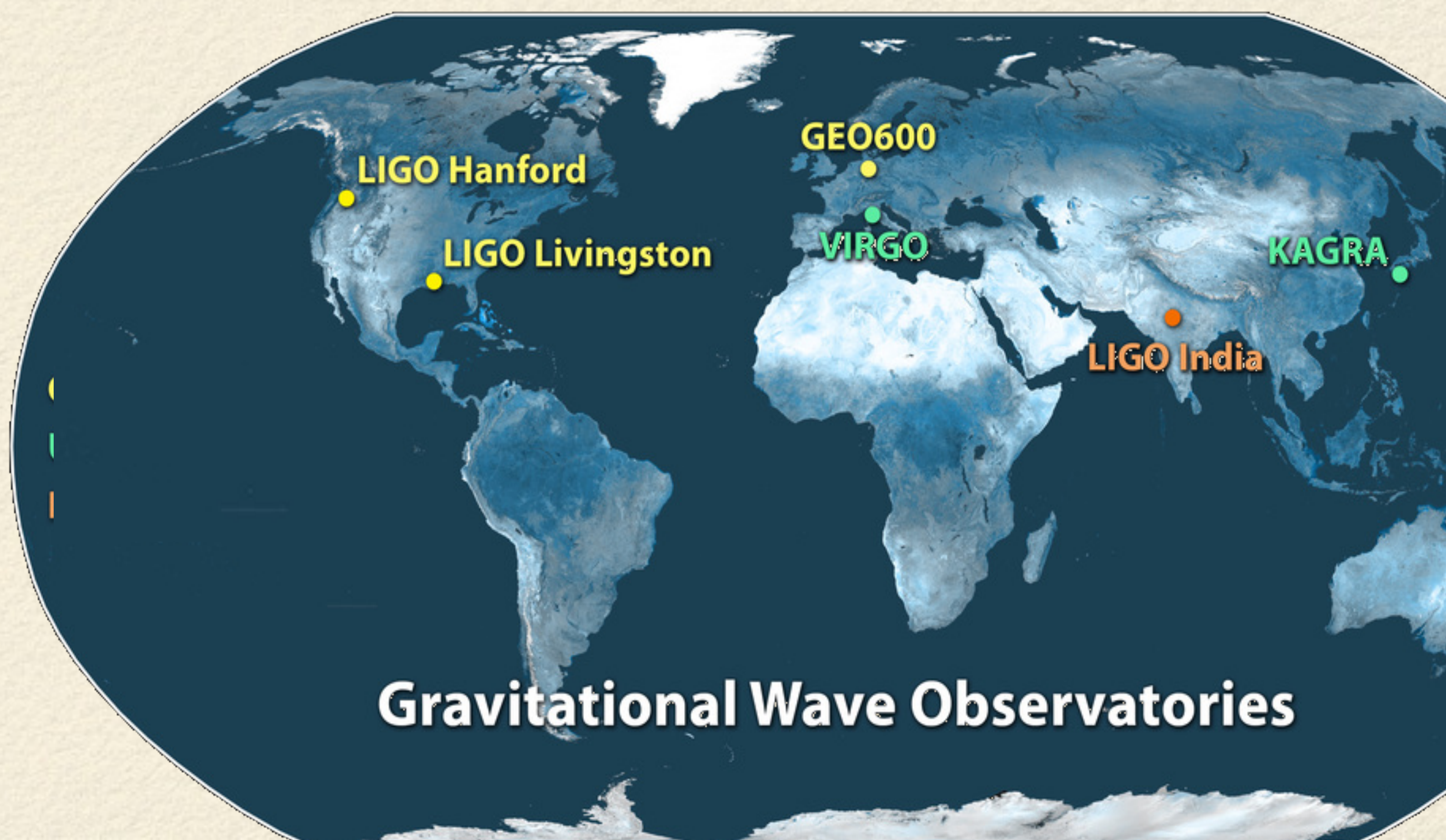
  « The LIGO and Virgo collaborations were very proud to announce that, on September 14th 2015 at 9:50 AM, the two LIGO interferometric detectors recorded an event, called GW150914, that was identified as the passage of a gravitational wave produced by the coalescence of two black holes of respectively 29 and 36 times the mass of the sun, located at a distance of 1.3 billion light-years. »

- « LIGO and Virgo did it again… twice, on December 26, 2015 and January 4, 2017 ! »

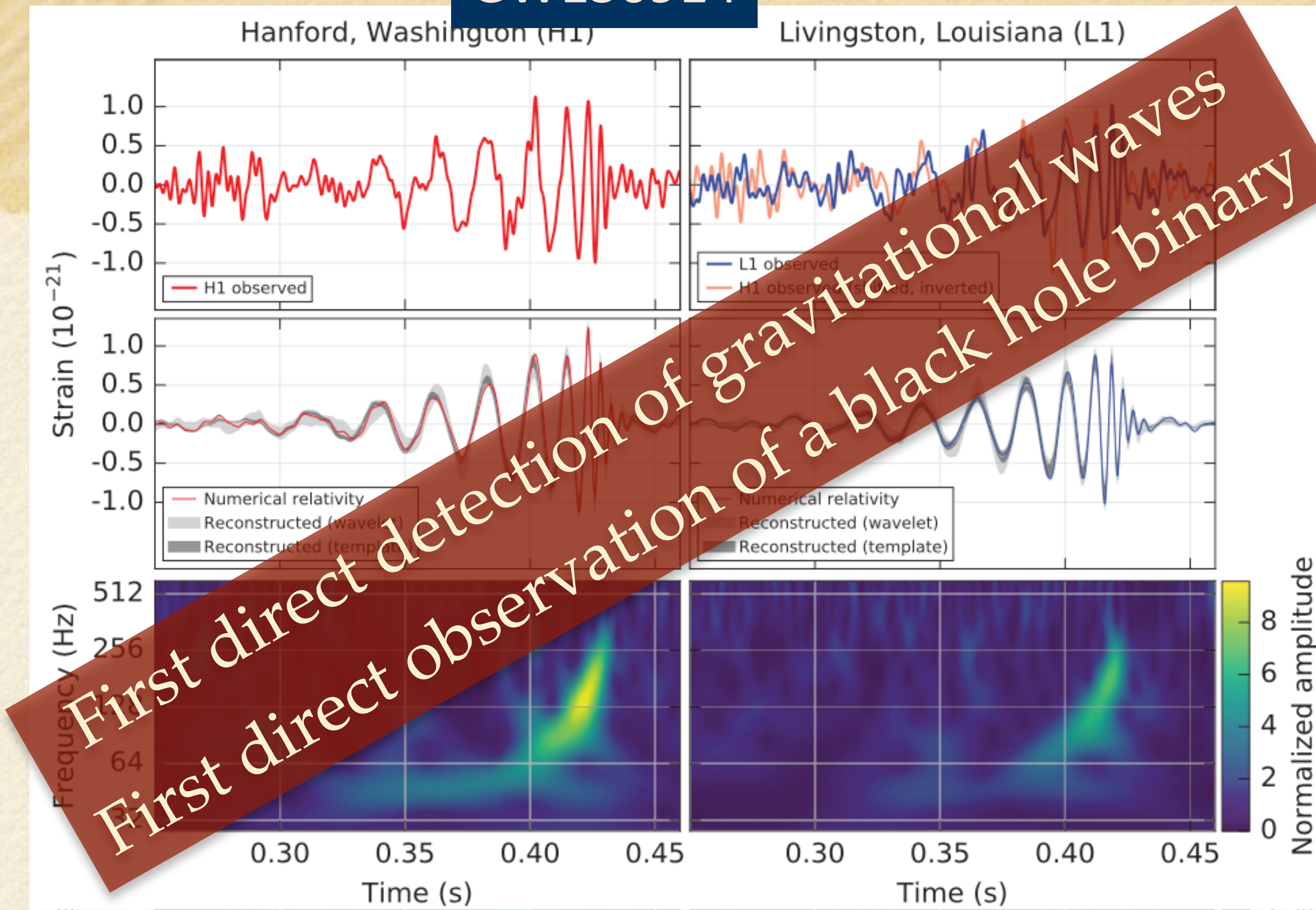
Who are "we" ?

- The collaborations
  - LSC (LIGO Scientific Collaboration)
  - Virgo
- Together they form the LVC (LSC-Virgo Collaboration) since 2007
- A network of interferometric observatories
  Advanced LIGO, Advanced Virgo and GEO

- Detectors that contributed :
  - LIGO Hanford
  - LIGO Livingston
- Data analysis :
  LIGO Scientific collaboration
  + Virgo



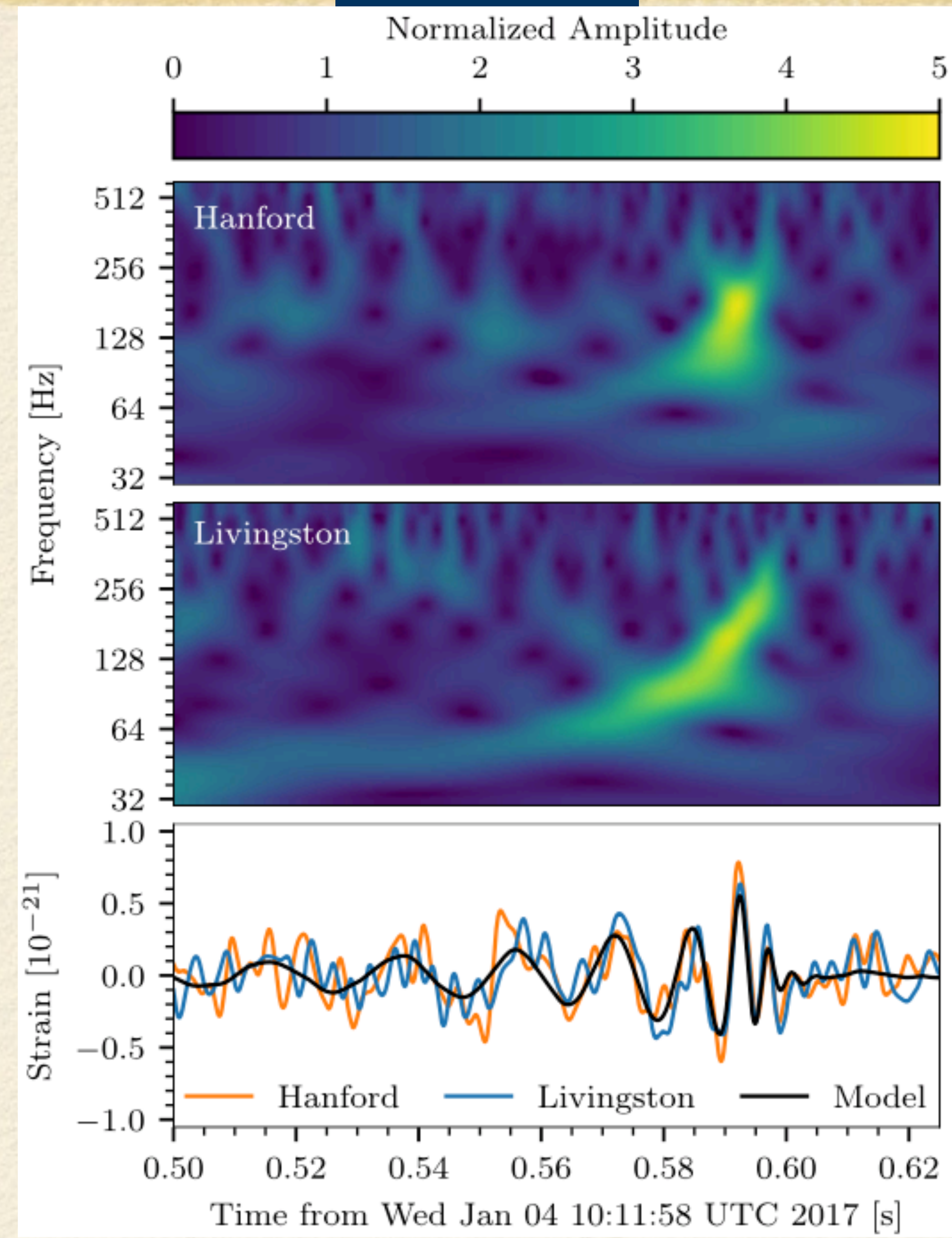
GW150914



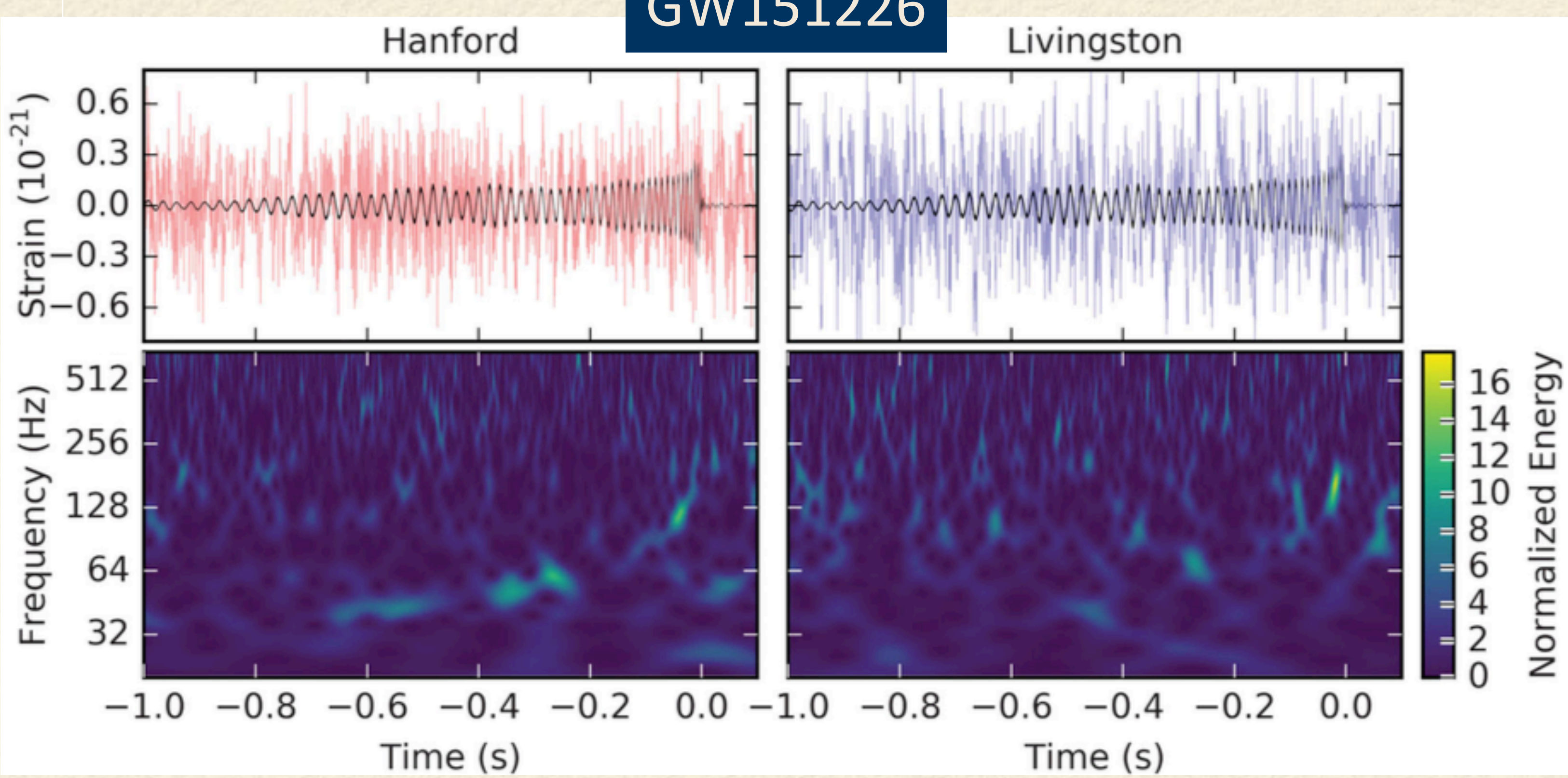
First direct detection of gravitational waves

First direct observation of a black hole binary

GW170104



GW151226



Context

- Gravity and General Relativity
- Linearized gravity
- Gravitational waves
- Generation of gravitational waves
- Scientific goals of a detection

The full calculations can be found, for example, in :

"General Relativity", M.P. Hobson, G. Efstathiou and N. Lasenby Cambridge University Press
"General Relativity", R.M. Wald, The University of Chicago Press

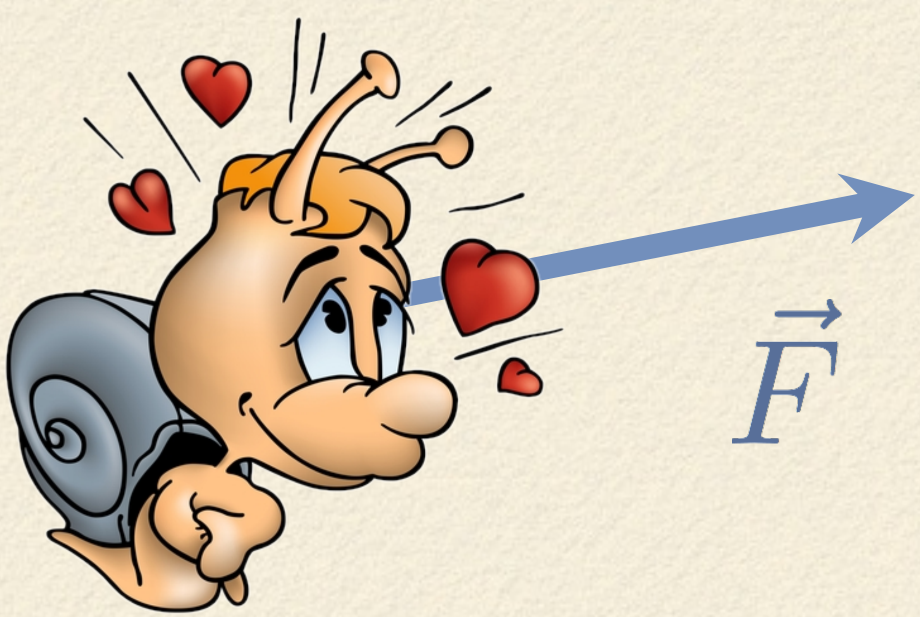
A very nice general public introductory book :

"A Journey into Gravity and Space-time", J. A. Wheeler, Scientific American Library

How does gravity work ?

- Newton :

$$\vec{F} = G \cdot m_1 m_2 \cdot \frac{1}{r^2} \cdot \vec{u}$$

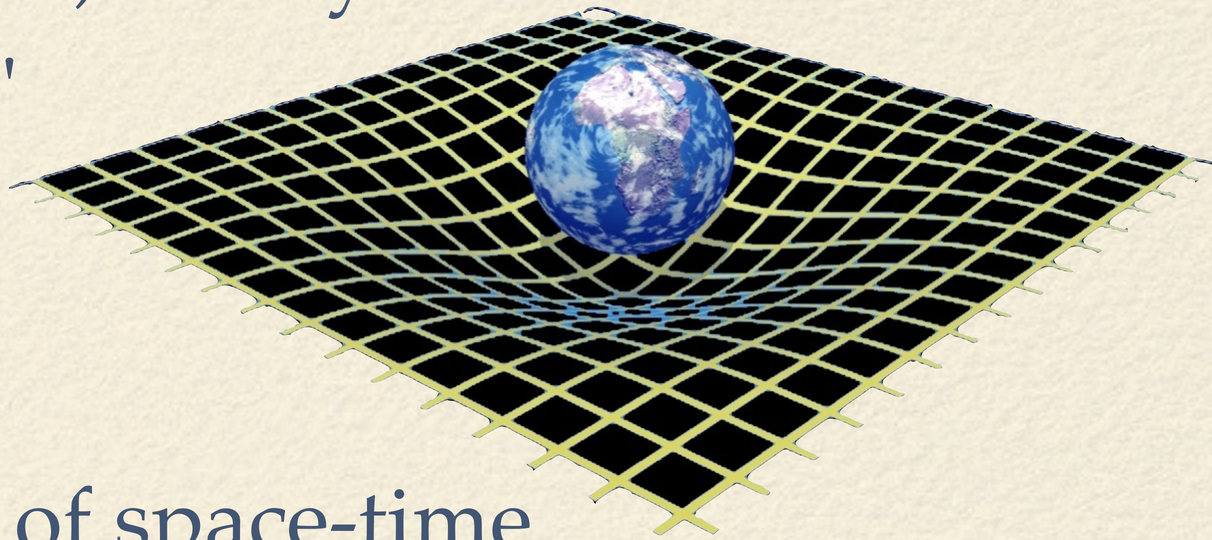


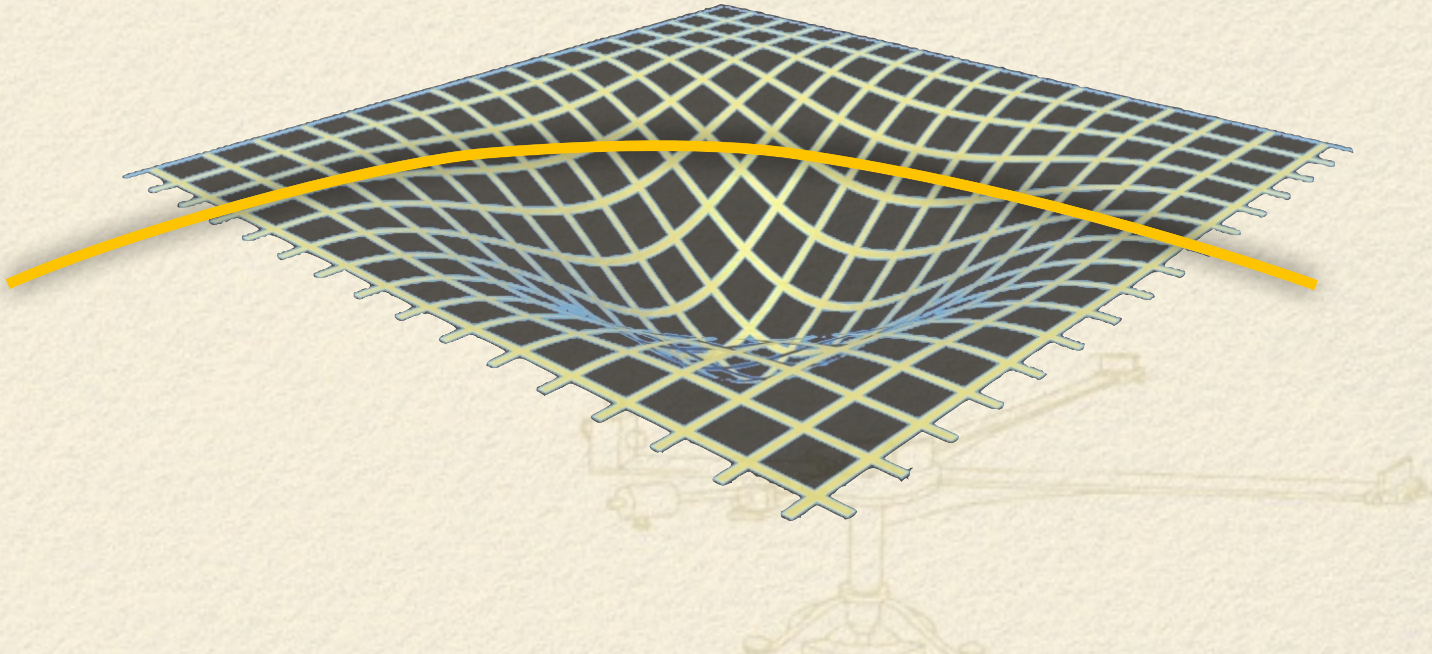
$\vec{F}$

$-\vec{F}$

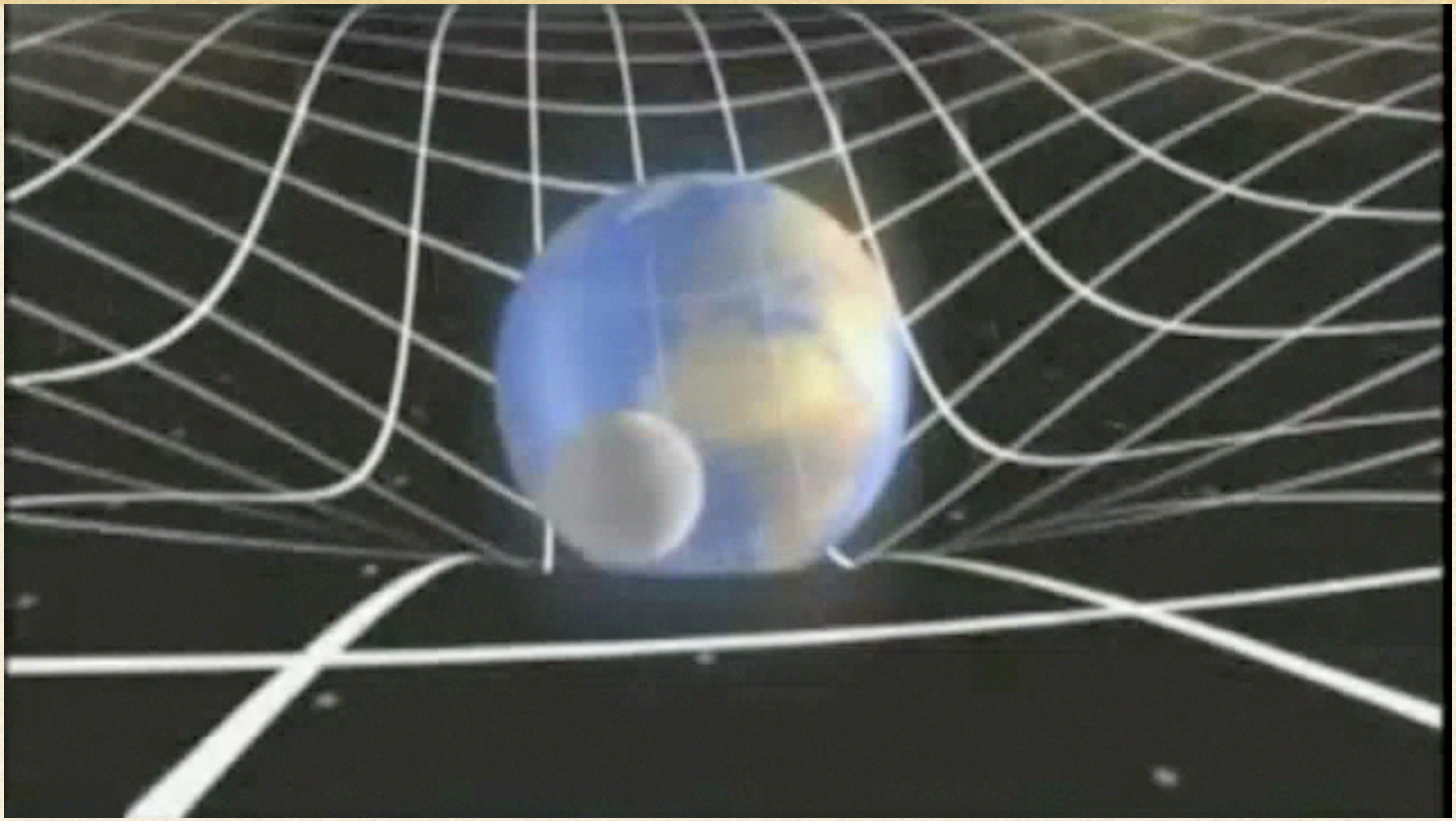
How does gravity work ?

- Experiences show that this is not a complete picture
- Einstein : «General Relativity» (GR) theory
  - A mass "bends" and "deforms" space-time
  - The trajectory of a mass is influenced by the curvature of space-time





How does gravity work ?

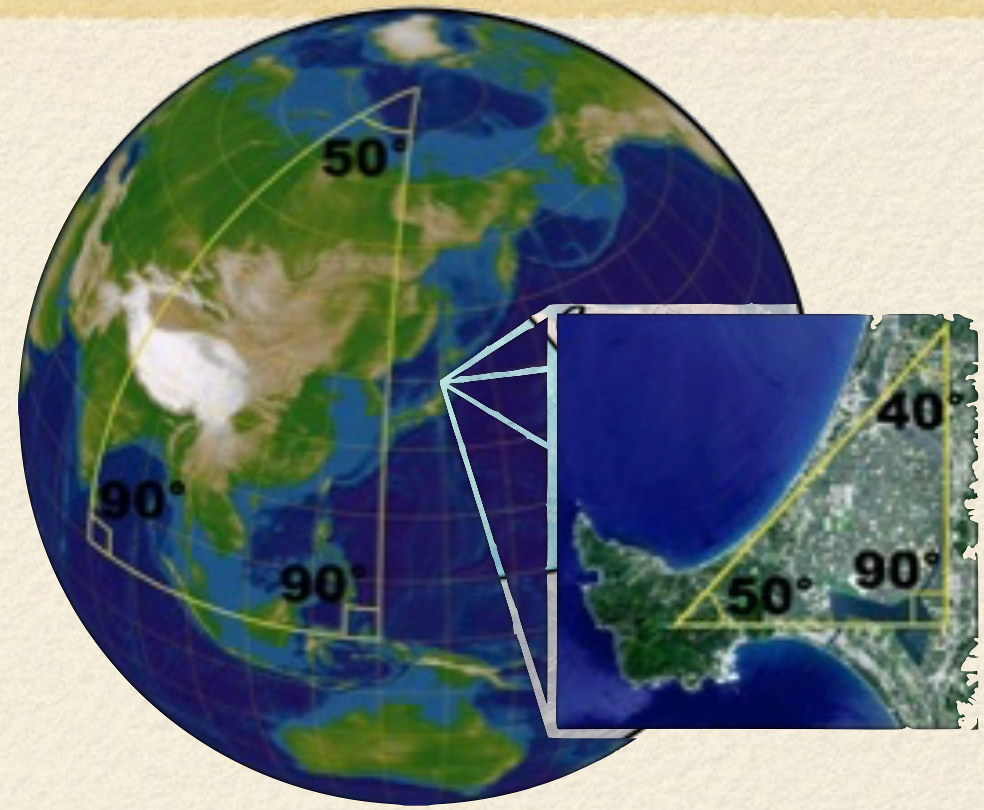


- But this is only a picture !
- Space-time is not an elastic surface in 2 dimensions !
- Very difficult to represent in 3 (rather 4) dimensions

"Curved" space-time

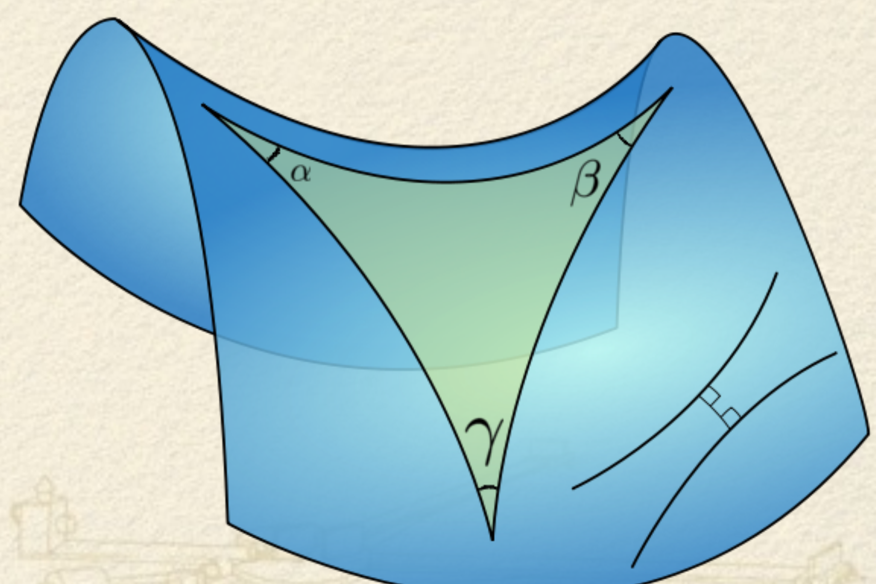
- What is a curved space ? (= "manifold")
  - examples : sphere, saddle
- Can we measure curvature ?
  - we cannot see our space from "outside"
  - but we can measure angles
  - the sum of the angles of a triangle is not always equal to π !
  - positive curvature

$$\sum \text{angles} = \alpha + \beta + \gamma > \pi$$



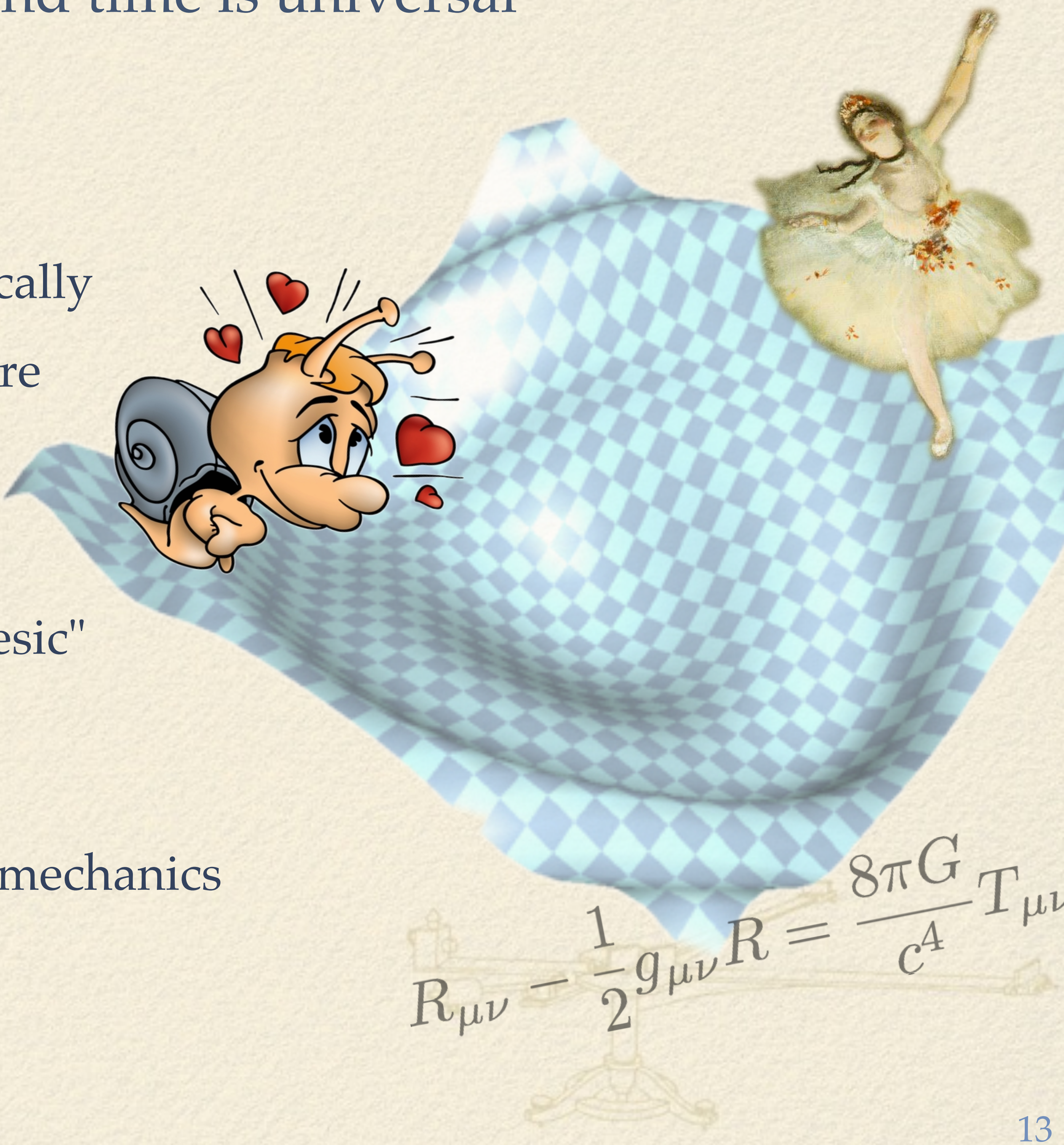
  - negative curvature

$$\sum \text{angles} = \alpha + \beta + \gamma < \pi$$



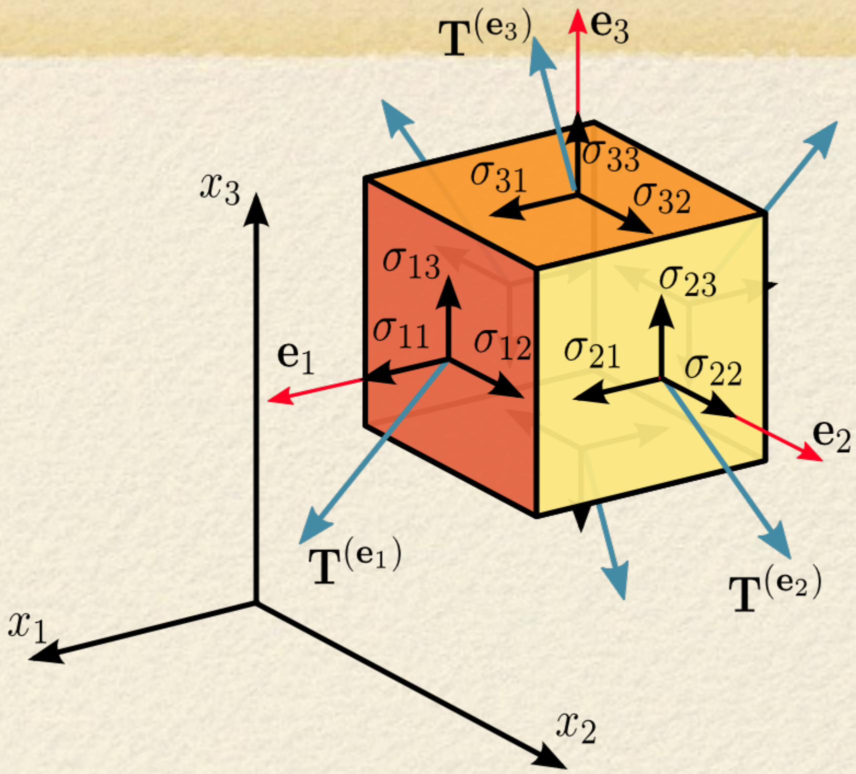
Curvature of space-time

- Newton : space is Euclidian (flat) and time is universal
  - flat space-time !
- General Relativity
  - space is curved and time is defined locally
  - one cannot go "out" to see the curvature
    - "intrinsically" curved space
    - intrinsic curvature
  - go straight (free fall) = follow a "geodesic"
  - note that the time is also curved !
  - as a first approximation, finds
    the results (trajectories) of newtonian mechanics



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

"Reminder" about tensors



- Tensor = mathematical object
- Does not depend on the coordinate system
- Extends the notion of vector
- In a specific coordinate system, multidimensional array
- Example : electrical conductivity of an anisotropic cristal

$$j^i = \sigma^i_j E^j$$

- Note : summation is implicit over repeated indices (Einstein convention)

$$\sigma^i_j E^j \equiv \sum_j \sigma^i_j E^j$$

The metric

- In space-time, measure
  - the distance between two points
  - the angle between two vectors
- Measure of the distance between two infinitesimally close events in spacetime
- Need a "metric", start from the "line element" seen in special relativity :

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

with c = 1 !

- Which can be written

$$ds^2 = \eta_{\alpha\beta} dx^\alpha dx^\beta$$

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and $dx^0 = dt, \quad dx^1 = dx,$
$dx^2 = dy, \quad dx^3 = dz$

- $\eta_{\mu\nu}$ is the metric of a flat spacetime, the Minkowski spacetime, used in special relativity

The metric

- But the space is not flat !
- The metric can be general : $g_{\mu\nu}$

- It contains all information about spacetime curvature
- It is a rank 2 tensor
- The curvature is also defined by another tensor, which depends on $g_{\mu\nu}$, the Ricci tensor $R_{\mu\nu}$
- But what generates curvature of spacetime ?

The Einstein Field Equations (EFE)

- Answer : the energy-momentum content of spacetime !
  - this includes mass
- Einstein Field Equations :

still c = 1 !
would be $\frac{8\pi G}{c^4}$

$$\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) = 8\pi G \left(T_{\mu\nu} \right)$$

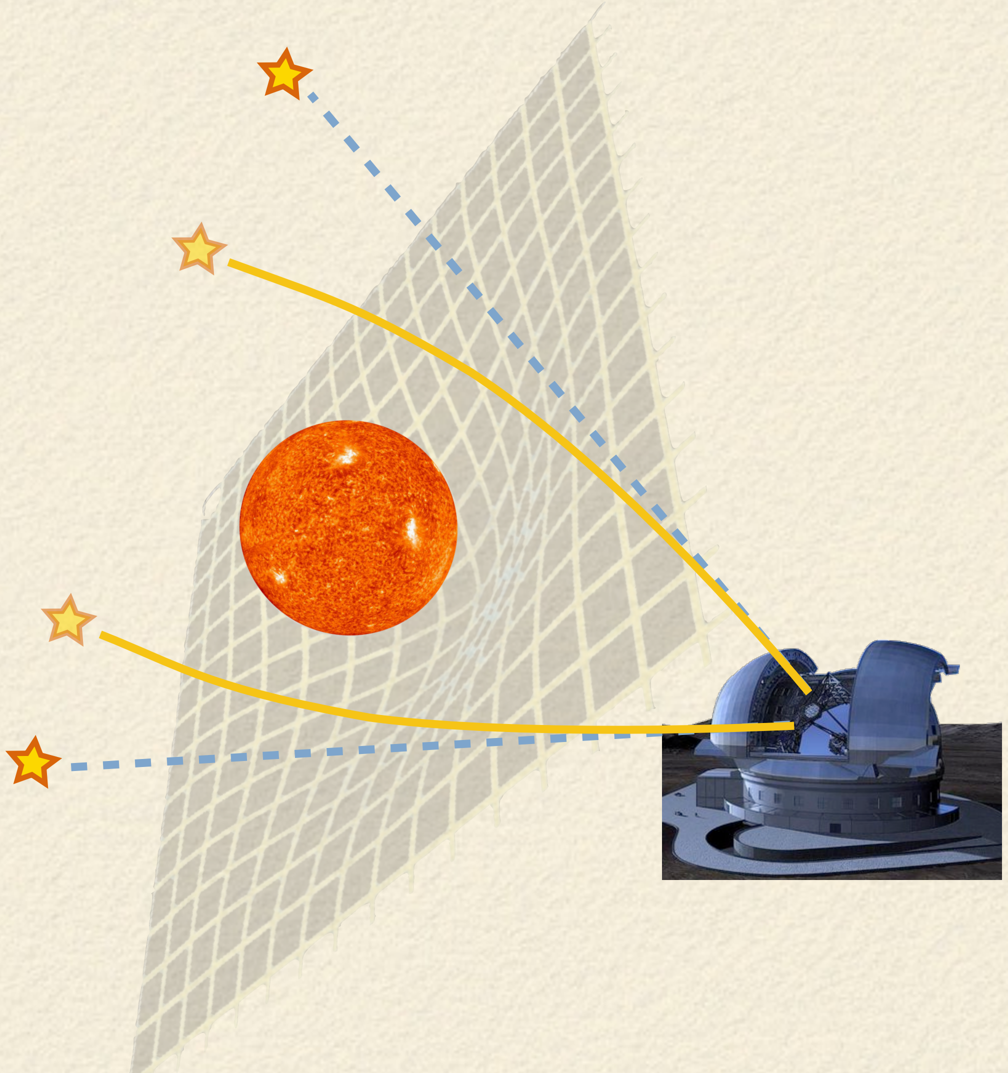
curvature term

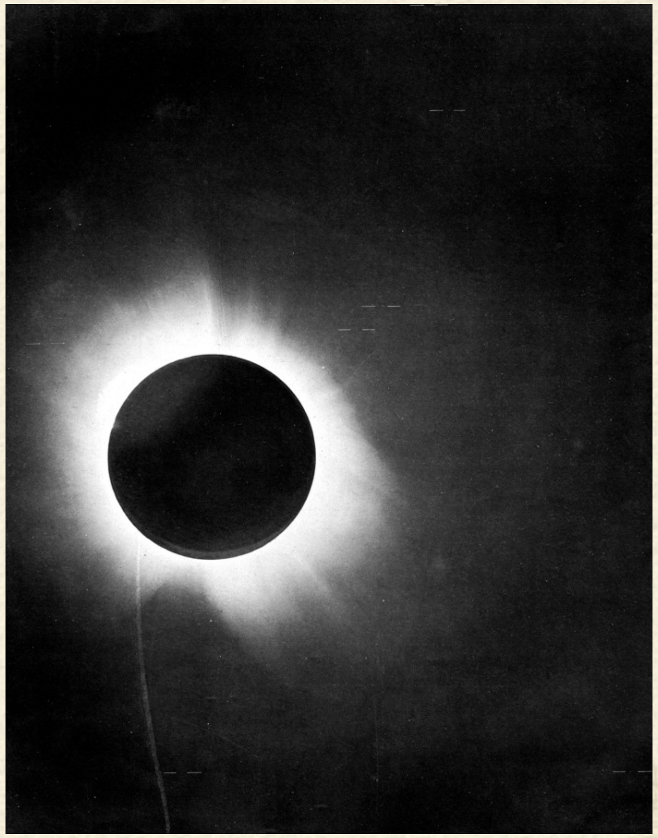
energy-momentum term

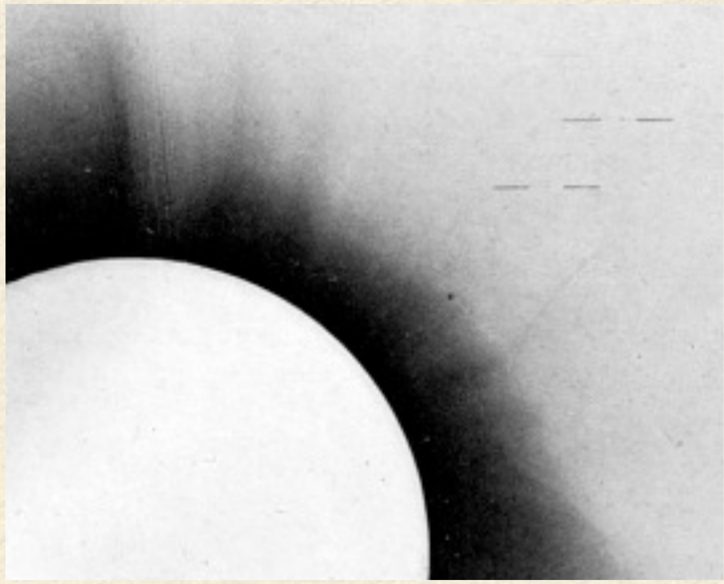
- Energy-momentum bends spacetime
  - being far from some energy density doesn't mean there is no bending !
- Spacetime tells mass (energy momentum) how to move
- These equations are non-linear

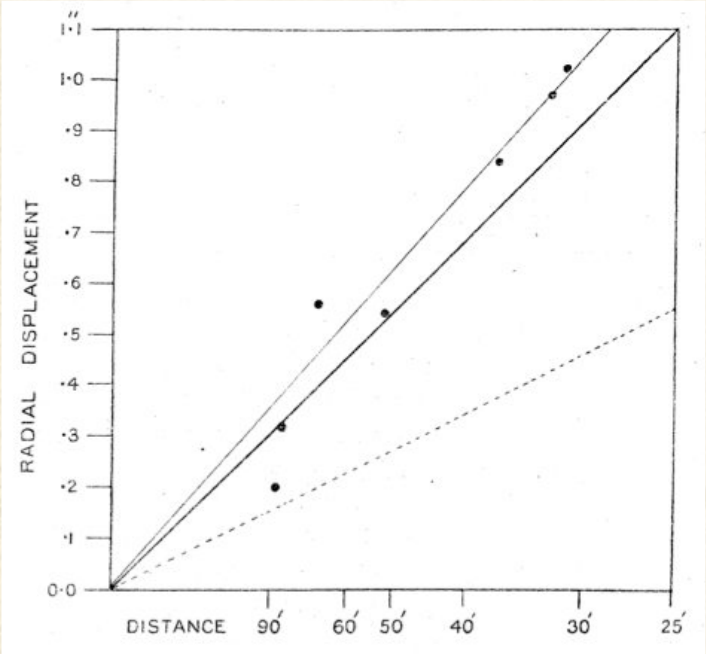
Novelties of the General Relativity

- New effects (w.r.t. Newtonian mechanics) but faint
  - the trajectory of some celestial bodies is modified (Mercury)
  - light follows the geodesics of space-time,
    its trajectory is curved nearby the sun
    (or any other body)









Eddington, 1919

Novelties of the General Relativity

Linearized gravity

- General Relativity (Einstein, 1916)
- Minkowski flat space-time with a small perturbation of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- where (Minkowski flat space-time metric) :

$$\eta_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- and $h_{\mu\nu} \ll 1$ is a perturbation of this metric

- then...

Linearized gravity

- start from the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G \left(T_{\mu\nu} \right)$$

- First step
  - linearization of all the constituents
    - replace $g_{\mu\nu}$ by $\eta_{\mu\nu} + h_{\mu\nu}$
    - remove the higher order terms in $h_{\mu\nu}$
- One obtains an equivalent equation which is still complicated
- Have to change variable $h_{\mu\nu} \rightarrow \bar{h}_{\mu\nu}$
- where the trace reverse is defined as : $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h$

Linearized gravity

- In General Relativity, the physics doesn't depend on the choice of coordinate system (gauge)
- Choose a particular set of coordinate systems where a certain condition is met, that simplifies the equations

$$\frac{\partial}{\partial x^\mu}\bar{h}^{\mu\nu} = \partial_\mu \bar{h}^{\mu\nu} = 0$$

Gauge condition

- The field equations may be written as :

$$\Box \bar{h}^{\mu\nu} = -2(8\pi G)T_{\mu\nu}$$

- Where $\Box$ is the d'Alembertian (or the wave operator) :

$$\Box \quad \Leftrightarrow \quad \{\nabla^2 - \frac{\partial^2}{\partial t^2}\} \quad \Leftrightarrow \quad \{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{\partial^2}{\partial t^2}\}$$

Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

definierten Größen $\gamma_{\mu\nu}$, welche linearen orthogonalen Transformationen gegenüber Tensorcharakter besitzen, gegen 1 als kleine Größen behandelt werden können, deren Quadrate und Produkte gegen die ersten Potenzen vernachlässigt werden dürfen. Dabei ist $\delta_{\mu\nu} = 1$ bzw. $\delta_{\mu\nu} = 0$, je nachdem $\mu = \nu$ oder $\mu \neq \nu$.

Wir werden zeigen, daß diese $\gamma_{\mu\nu}$ in analoger Weise berechnet werden können wie die retardierten Potentiale der Elektrodynamik. Daraus folgt dann zunächst, daß sich die Gravitationsfelder mit Lichtgeschwindigkeit ausbreiten. Wir werden im Anschluß an diese allgemeine Lösung die Gravitationswellen und deren Entstehungsweise untersuchen. Es hat sich gezeigt, daß die von mir vorgeschlagene Wahl des Bezugssystems gemäß der Bedingung $g = |g_{\mu\nu}| = -1$ für

Gravitational waves

- In vacuum ($T_{\mu\nu} = 0$), the Einstein field equations are equivalent to a wave equation :

$$\Box \, \bar{h}_{\mu\nu} = 0 \quad \Leftrightarrow \quad \{\nabla^2 - \frac{\partial^2}{\partial t^2}\}\bar{h}_{\mu\nu} = 0$$

with a gauge condition : $\partial_\mu \bar{h}^{\mu\nu} = 0$

- where c = 1 and a harmonic gauge choice

$$k^\rho = \begin{pmatrix} k_x \\ k_y \\ k_z \\ \frac{\omega}{c} \end{pmatrix}$$

- in the following, consider solutions

$$\bar{h}_{\mu\nu} = Re\,\{A_{\mu\nu} \exp\left(-ik_\rho x^\rho\right)\}$$

$$k_\rho x^\rho = k_0 x^0 + k_1 x^1 + k_2 x^2 + k_3 x^3 = -\omega t + \vec{k} \cdot \vec{x}$$

Gravitational waves

- Constraints
  - Satisfying the wave equation $\Box \bar{h}_{\mu\nu} = 0$:

$$k^\rho = \begin{pmatrix} k_x \\ k_y \\ k_z \\ \frac{\omega}{c} \end{pmatrix}$$

$$k_\rho k^\rho = 0$$

c = c for a moment

$$\Rightarrow \quad \omega^2 = c^2 |\vec{k}|^2$$ $\Longrightarrow$ the wave propagates at the speed of light c

  - Use the gauge conditions $\partial_\mu \bar{h}^{\mu\nu} = 0$:

$$k_\rho A^{\rho\sigma} = 0$$

$\Longrightarrow$ 6 remaining independent elements in the amplitude tensor

Gravitational waves

- Can still simplify the expression of the amplitude
- Among the set of coordinate systems, choose a particular one such that

$$A_{0\sigma} = 0 \quad \Rightarrow$$ number of independent elements further reduced to 2

- for a wave traveling along the z axis, the amplitude is then :

h = amplitude of the wave

$$A_{\mu\nu} = h.\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- choice $A_{11} = 1,\ A_{12} = 0$, polarization called "+"
- choice $A_{11} = 0,\ A_{12} = 1$, polarization called "x"
- This particular gauge (coordinate system) is called Transverse Traceless (TT)

Gravitational waves

- The complete metric **in this gauge** is now :

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

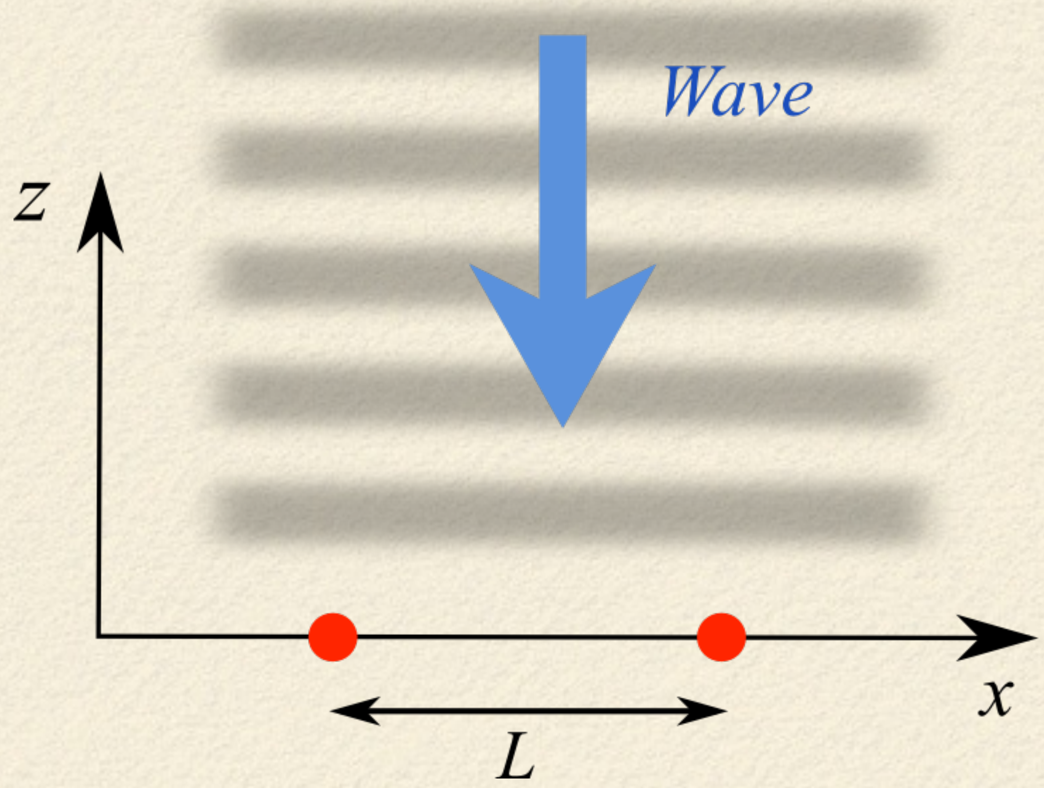
$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} + h.\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_{11} & A_{12} & 0 \\ 0 & A_{12} & -A_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}.\exp(-ik_\rho x^\rho)$$

- and a line element is

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$$

Gravitational waves

- Proper length between two test masses in free fall



$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

if dt=0 (proper length) and
look only along the "x" direction

$$\Rightarrow \quad ds^2 = g_{xx} dx^2$$

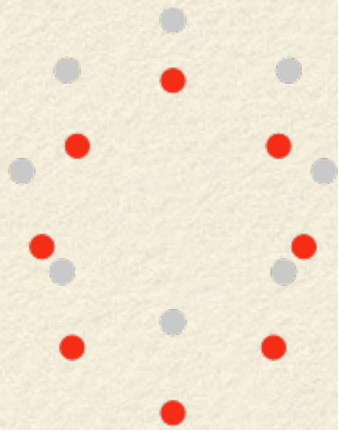
$$L = \int_0^{\Delta x} dx \sqrt{g_{xx}} = \int_0^{\Delta x} dx \sqrt{1 + h_{xx}^{TT}(t, z=0)}$$

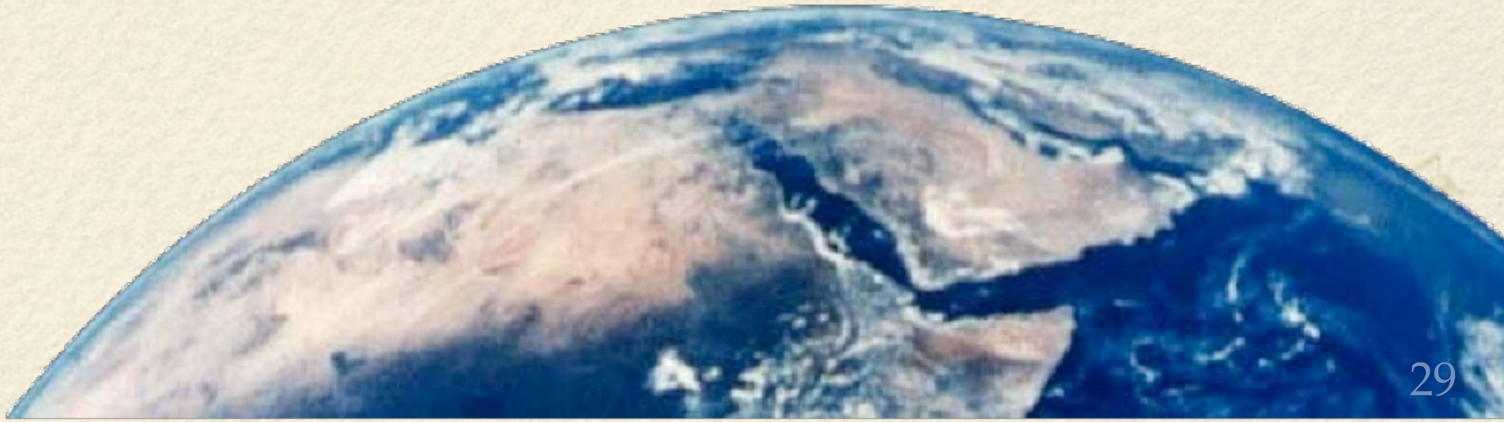
$$\approx \int_0^{\Delta x} dx \left[1 + \frac{1}{2} h_{xx}^{TT}(t, z=0)\right] = \Delta x \left[1 + \frac{1}{2} h_{xx}^{TT}(t, z=0)\right]$$

- h is the relative variation in proper length between the two test masses

Effect of spacetime curvature

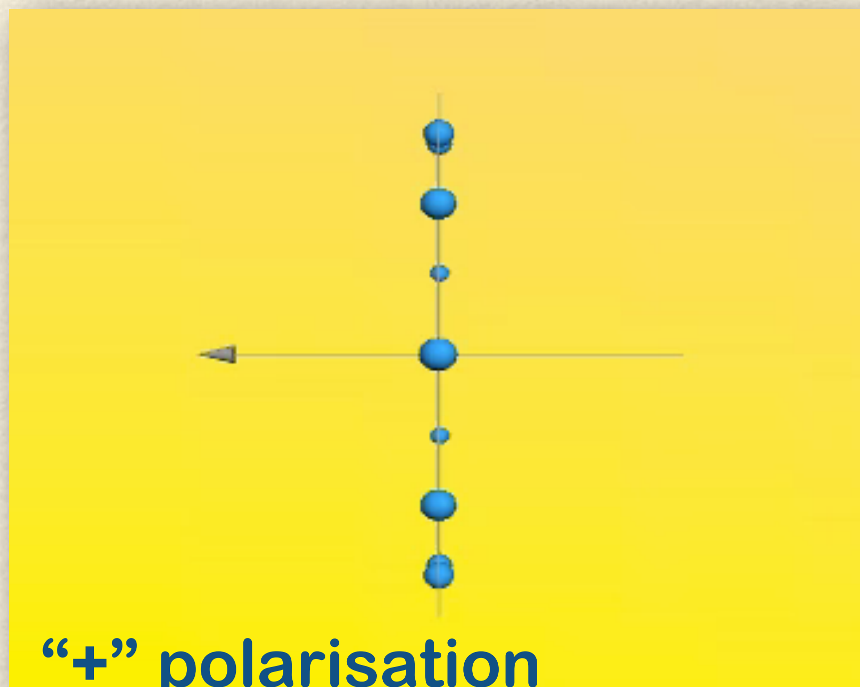
- Set of test masses
  - distributed on a circle
  - non interacting among themselves
  - freely floating above Earth's surface (static curvature)
- Effect of curvature on the set :
  - Lengthen in one dimension
  - Shrink in the perpendicular one

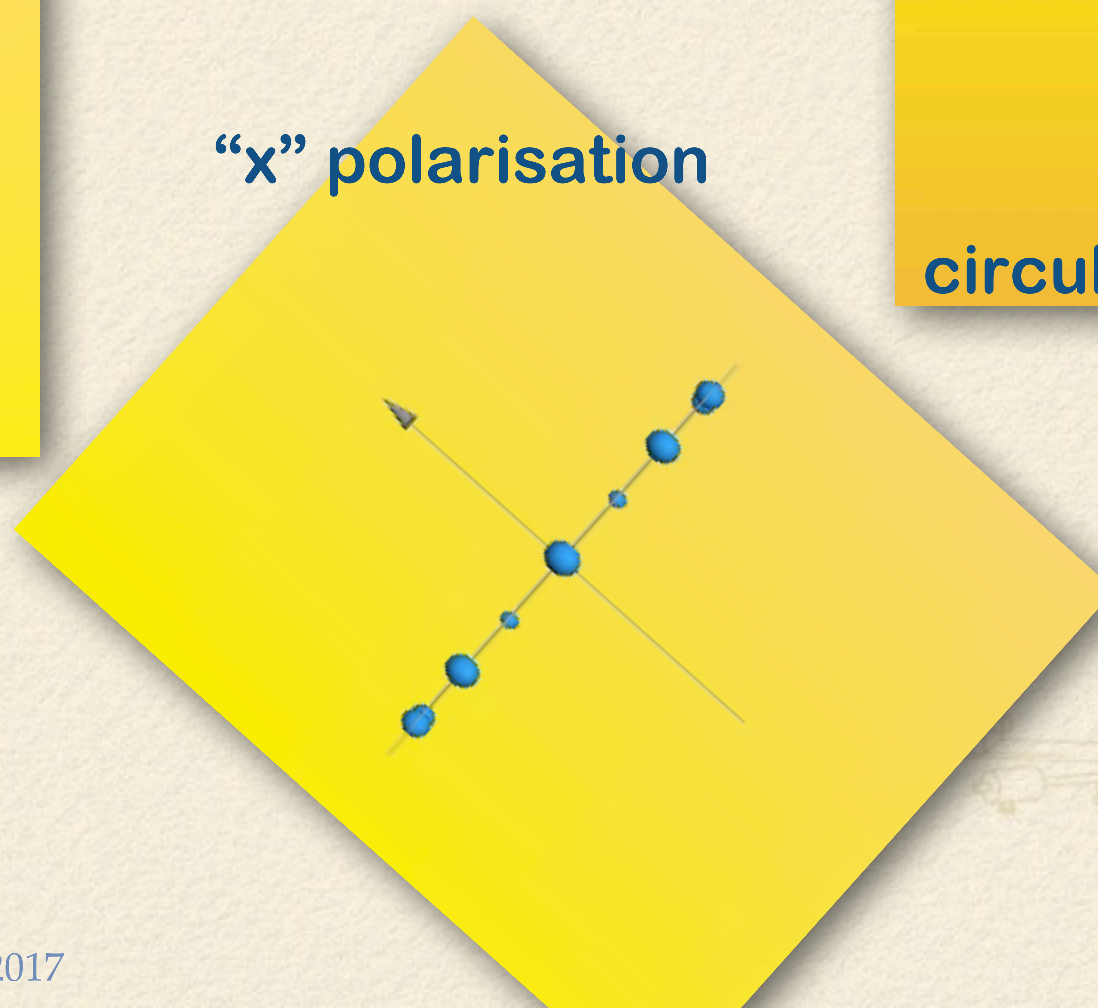


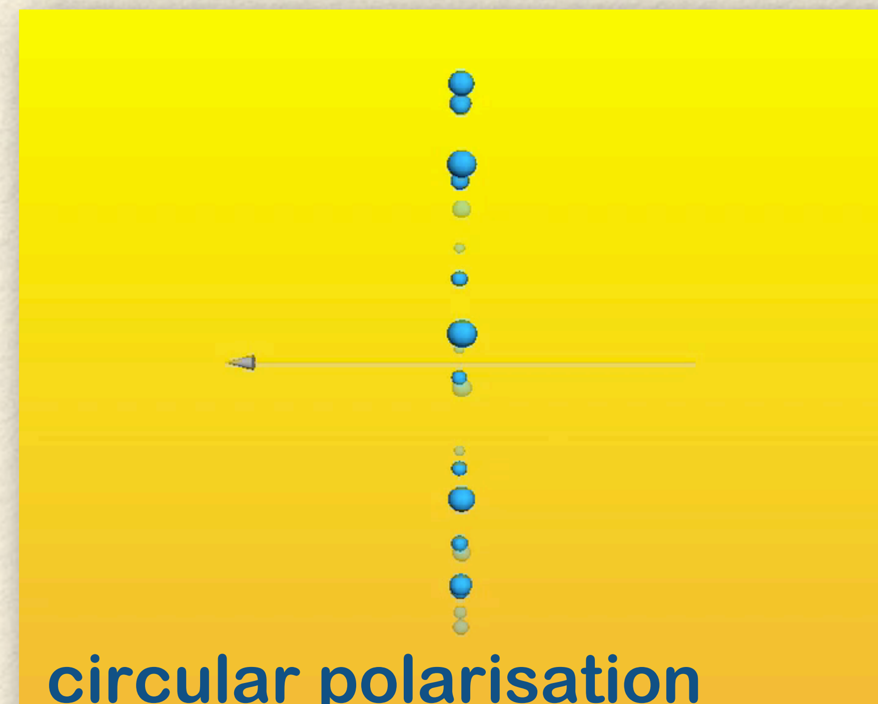


Effect of the gravitational waves

- Effect of GW on matter
- Same set of free falling test particles distributed on a circle
  illustration of the metric variation

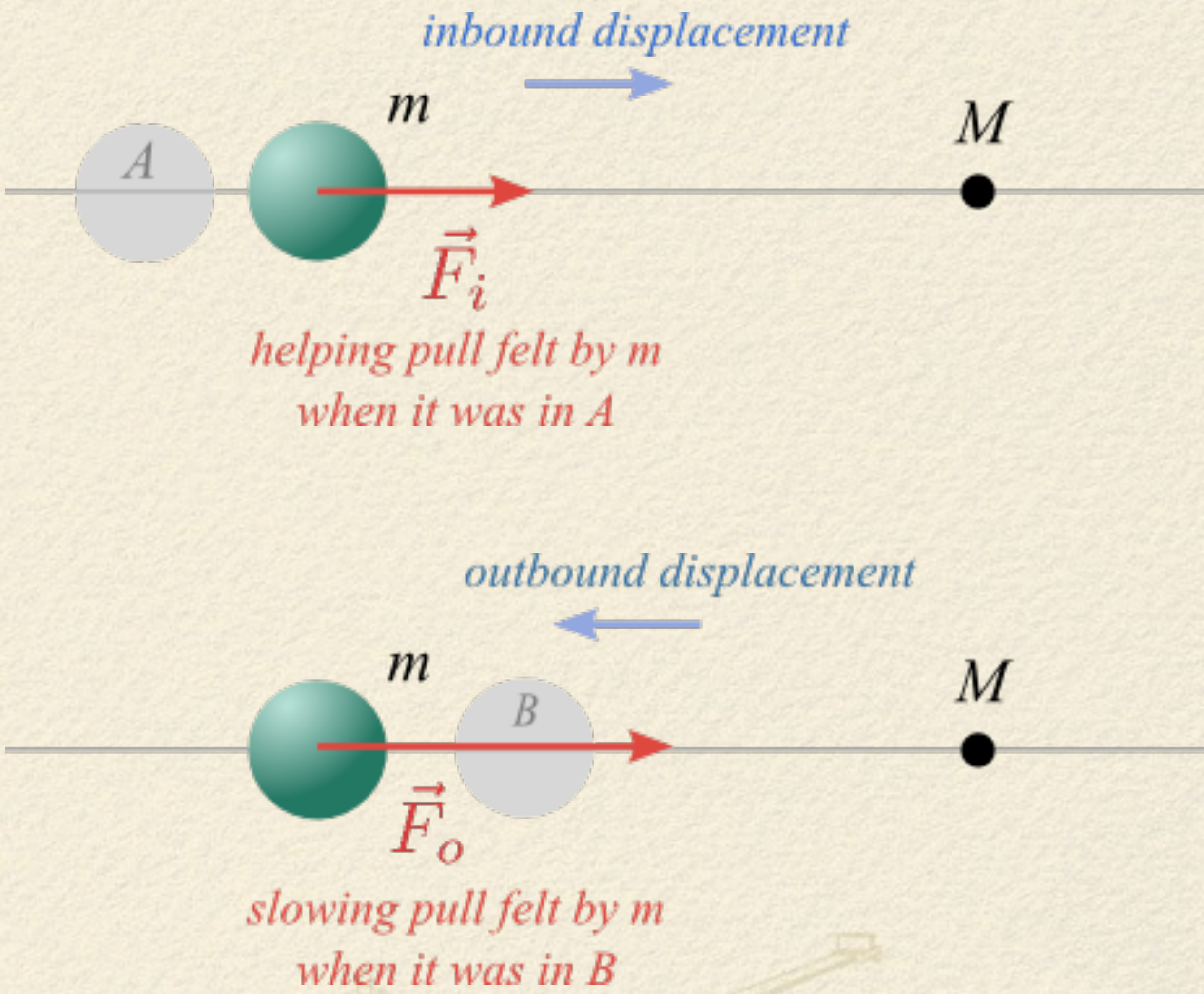






Why does a massive system lose energy ?

- Argument by Kalckar and Ulfbeck, central point : time delay
- System : a mass m oscillating around a fixed center of attraction of mass M
  - in 1 dimension
- Time needed for gravity to propagate
  - when traveling inbound (towards M) force $\vec{F}_i$ felt by m when it was farther
  - when traveling outbound (away from M) force $\vec{F}_o$ felt by m when it was closer
- $$\Rightarrow \quad \left|\vec{F}_o\right| > \left|\vec{F}_i\right|$$
- the oscillating motion of m is damped
- energy of the system reduces
- energy was taken away by gravity !



Generation of GW

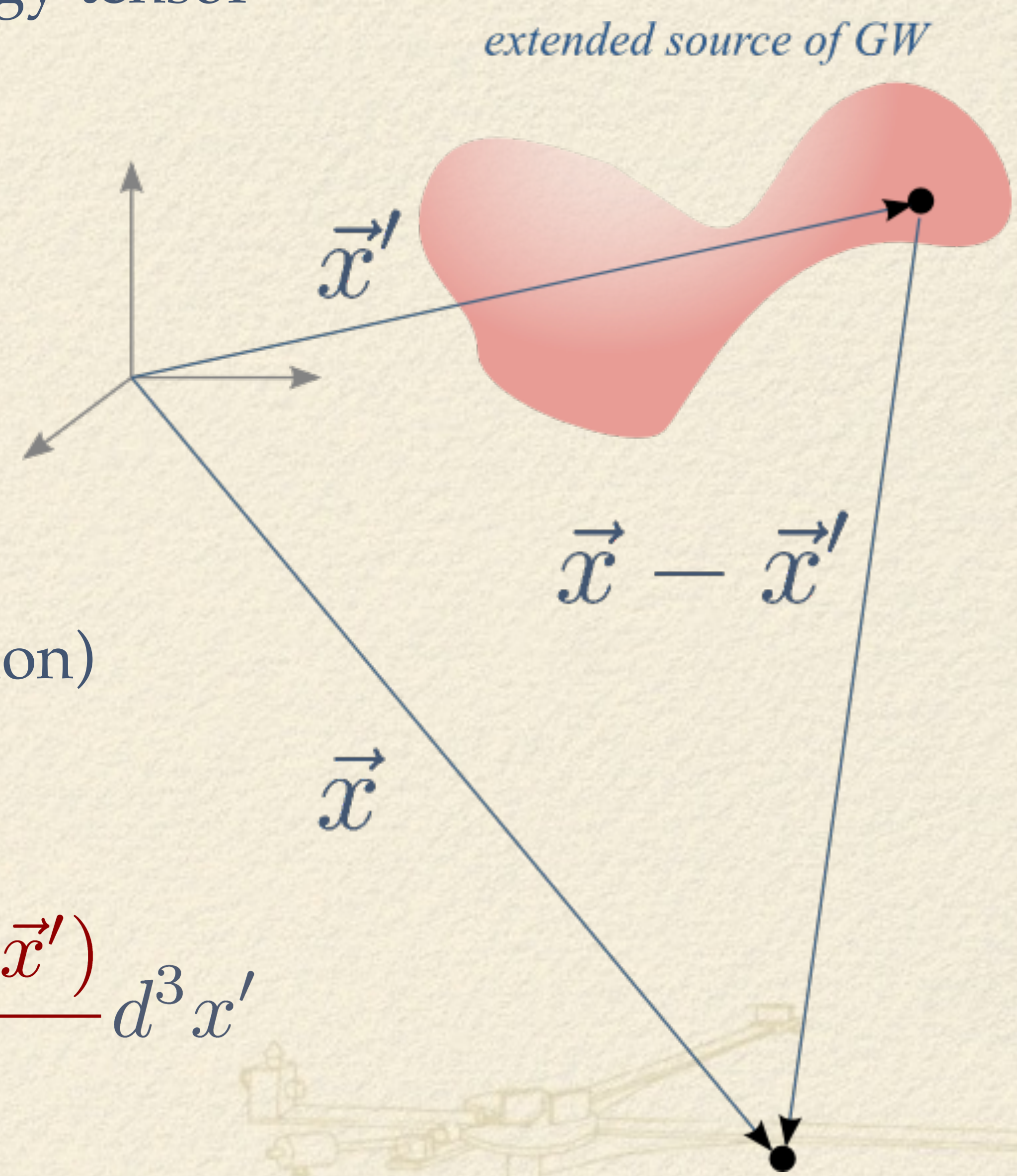
- Emission of the gravitational waves
  - Linearized Einstein equations with a stress-energy tensor

now c = c

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

  - Use Green functions
    - Solutions of the wave equation
      in the presence of a point source (delta function)
  - Retarded potential

$$\bar{h}_{\mu\nu}(t, \vec{x}) = -\frac{4G}{c^4} \int_{source} \frac{T_{\mu\nu}(t - \frac{|\vec{x}-\vec{x}'|}{c}, \vec{x}')}{|\vec{x} - \vec{x}'|} d^3x'$$



Generation of GW

- Approximations :
  - isolated source
  - compact source
  - observer far from the source ($R = |\vec{x} - \vec{x}'| >>$ typical size of the source)
- Amplitude of the wave written as a function of I_{ij}

$$\bar{h}_{ij}(t) = \frac{2G}{Rc^4} \frac{d^2 I_{ij}}{dt^2}\left(t - \frac{R}{c}\right)$$

I_{ij} = reduced quadrupolar moment of the source

$$= \int_{source} d\vec{x}\; x_i x_j\; T_{00}(t, \vec{x})$$

$$\frac{G}{c^4} \approx 8.24 \times 10^{-45}\ \mathrm{s}^2 \cdot \mathrm{m}^{-1} \cdot \mathrm{kg}^{-1}$$

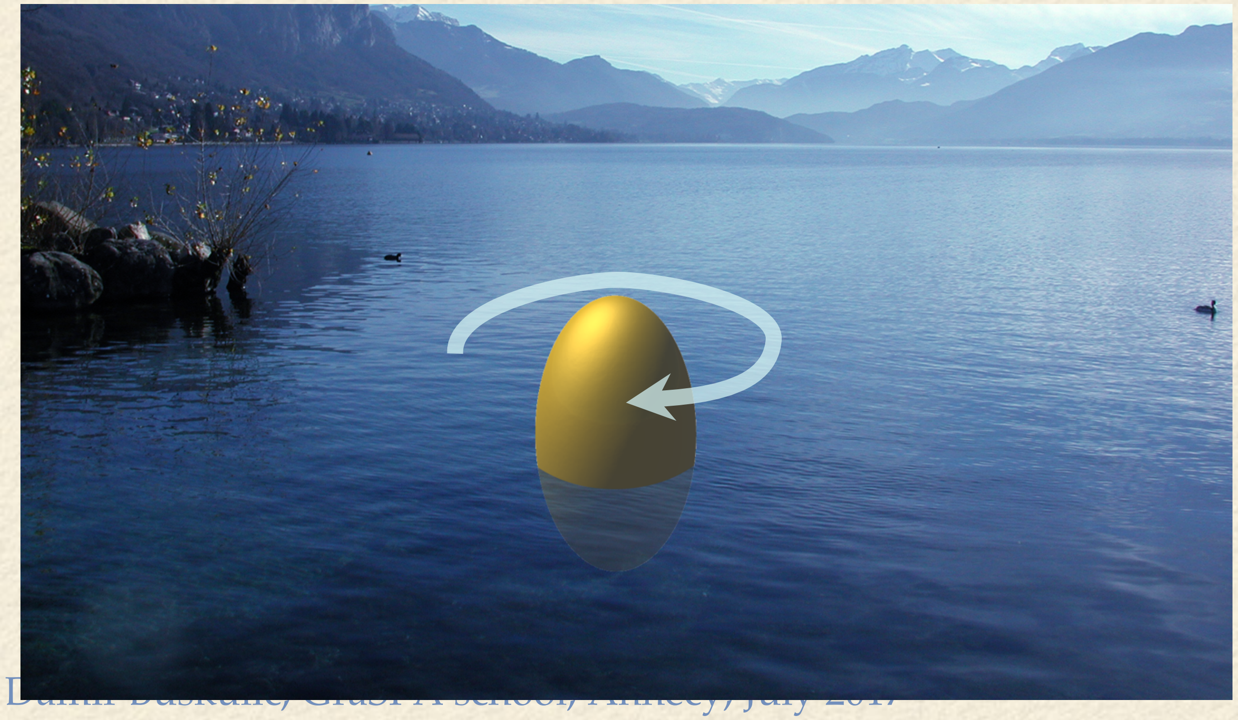
- Remark :

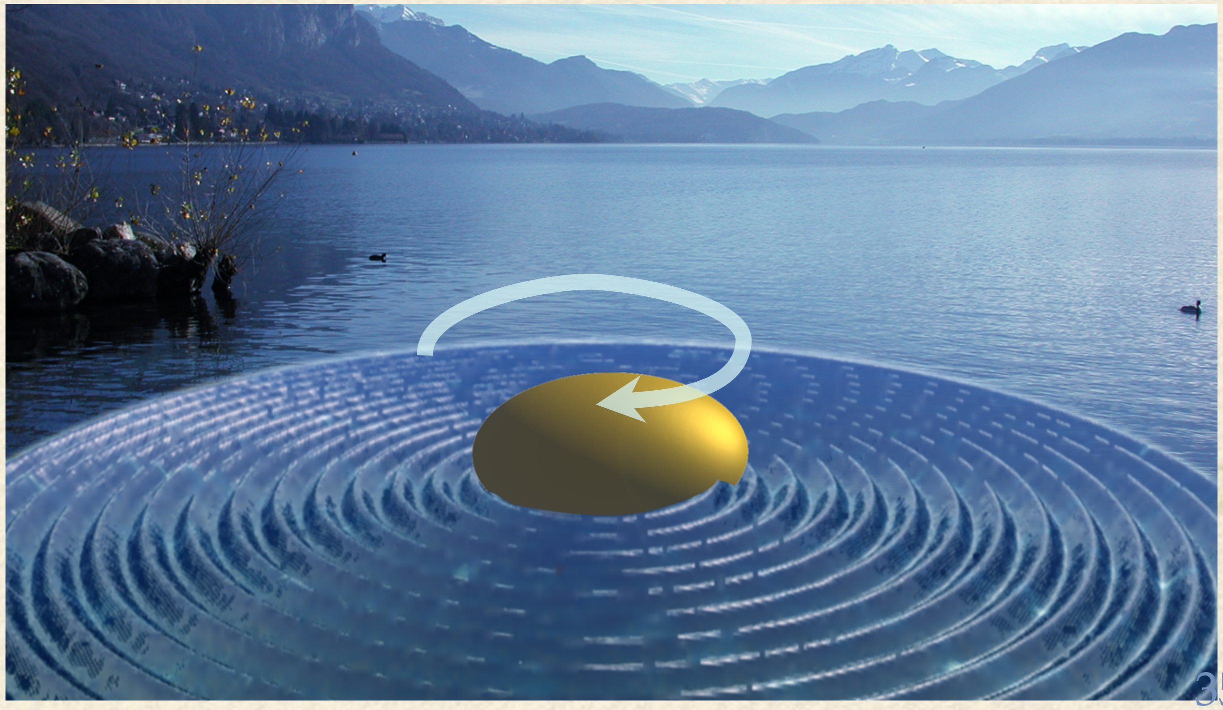
  Need a quadrupolar moment to generate a GW, the dipolar case is impossible (because of momentum conservation).

Generation of GW

- Example of a source : rotating neutron star
  - but not completely spherical (like a «rugby ball»)
- Or two neutron stars orbiting around each other
- Everything that rotates around an axis that is not a cylindrical symmetry axis

- ... raising your hand should generate gravitational waves !





Scientific goals

Done ! *Started !*

- Confirmation of GW — Detectors on earth, in space
- Study properties, test GR — Detectors on earth, in space
  - Speed = c ? Really quadrupolar ?
- Measure the Hubble constant
  - Coalescing binaries should be standard candles if the redshift and distance are known — Detectors on earth, in space

Impossible with only one detector for most of the sources → Build a worldwide observatory of gravitational waves

Scientific goals

Started !

- Study characteristics — Detectors on earth
  - of neutron stars
  - of solar mass black holes (BH)
    - Ellipticity, vibration modes, higher order moments, population…
- Study supermassive black holes — Detectors in space
  - cartography of space-time around a supermassive BH (Kerr).
  - study of their distribution, galactic evolution
- Stochastic background of GW : — Detectors on earth ?
  - first moments of the universe ? — Detectors in space ?
- …

Detection of gravitational waves by optical interferometry

- Historical introduction
- Principle of the interferometric detectors
- The noise makes the detector
- From Virgo to Advanced Virgo (AdV)
- A world wide detector network

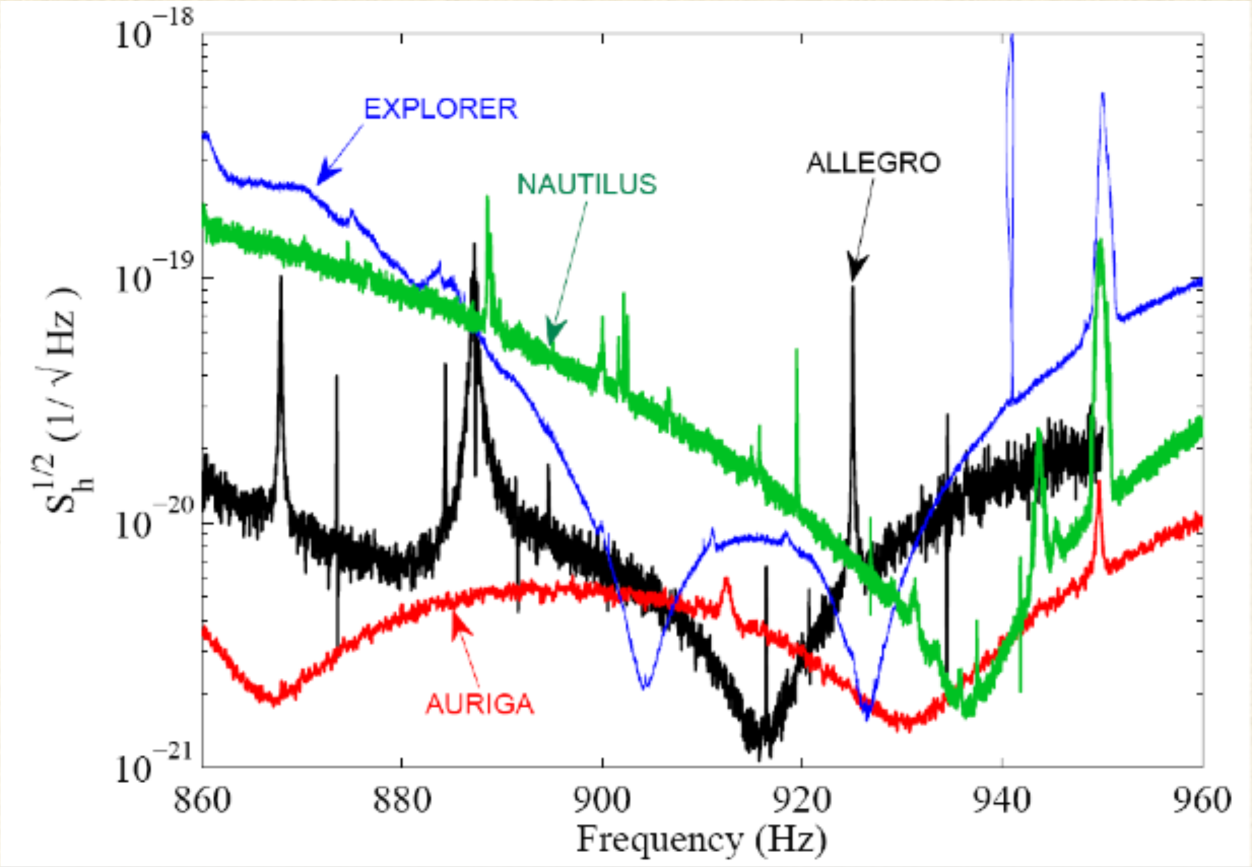
A rather complete and very pedagogical introduction :
"Fundamentals of Interferometric Gravitational Wave Detectors", P.R. Saulson, World Scientific, 1994

History

- First idea : resonant bars or spheres
  - J. Weber 1966 : the GW changes the resonance condition of a resonant bar of a few tons
  - Claims the detection of GW (1968-1969)
    - Various problems, other experiments have not confirmed the discovery
- Other idea : measure the time of flight of photons between two test masses, Michelson interferometer
  - Gertsenshtein & Pustovoit (1962)
  - First interferometer for GW :
  - R. L. Forward & al (1971)
  - Foundations for the modern interferometers : Rainer Weiss (1972)

Resonant detectors

- J. Weber 1966 : the GW changes the resonance condition of a resonant bar of a few tons
- Many other experiments until the mid-2000's
- $f_s = 700 - 1000$ Hz, $\Delta f = 50 - 200$ Hz
- sensitivity : $h \approx 10^{-19}$ à 10^{-21}



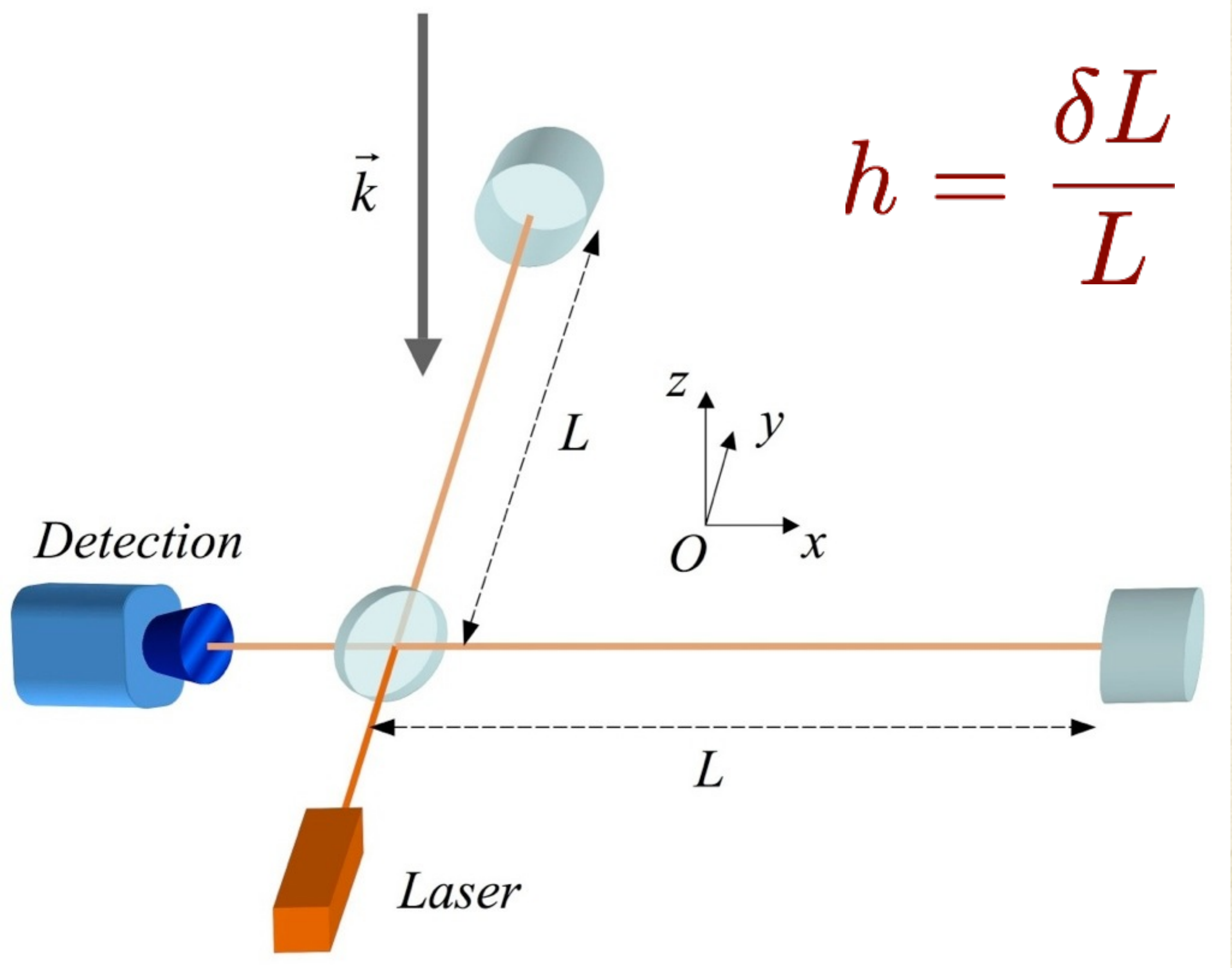
Interferometric detectors :
Principle of detection

Detection principle

- Michelson interferometer

The detector measures the optical path length difference between the two arms

most of the elements (mirrors, injection and detection systems) are suspended and behave like free falling masses in the interferometer plane (for $f \gg f_{pend}$)



Detection principle

- Photon in a field , general case

$$ds^2 = 0 = g_{\alpha\beta}dx^\alpha dx^\beta = \eta_{\alpha\beta}dx^\alpha dx^\beta + h_{\alpha\beta}dx^\alpha dx^\beta$$

- Particular case : wave along z, polarization "+" along one of the arms

$$ds^2 = 0 = -c^2dt^2 + (1 + h_+(t))dx^2 + (1 - h_+(t))dy^2 + dz^2$$

- Round trip time of the photons,
  integration on the path, for example for the arm along *x*

$$\frac{1}{c}\int_0^L dx = \int_0^{\tau_{aller}} \frac{1}{\sqrt{1 + h_+(t)}}\, dt \approx \int_0^{\tau_{aller}} \left(1 - \frac{1}{2}h_+(t)\right) dt$$

- Consider
  - round trip in one arm
  - wavelength of the GW >> length of one arm

    $\lambda_{OG} \gg L \quad \Rightarrow \quad h_+(t)$ independent of the position along the arm
  - period of the GW << round trip time of the light in one arm

    $\Rightarrow \quad h_+(t) = cte = h_+$

Detection principle

- For the arm along the "x" direction

$$\int_0^{\tau_{arx}} \left(1 - \frac{1}{2}h_+(t)\right) dt \approx \frac{1}{c}\left(\int_0^L dx - \int_L^0 dx\right) = \frac{2L_x}{c}$$

$$= \tau_{arx} - \frac{1}{2}\int_0^{\tau_{arx}} h_+(t)dt = \tau_{arx} - \frac{1}{2}\int_0^{\frac{2L_x}{c}} h_+(t)dt$$

$$\Rightarrow \quad \tau_{arx} = \frac{2L_x}{c} + \frac{1}{2}\int_0^{\frac{2L_x}{c}} h_+(t)dt$$

- arm along "y" : $\Rightarrow \quad \tau_{ary} = \frac{2L_y}{c} - \frac{1}{2}\int_0^{\frac{2L_y}{c}} h_+(t)dt$
- time difference (suppose h constant) if : $L_x = L_y = L$

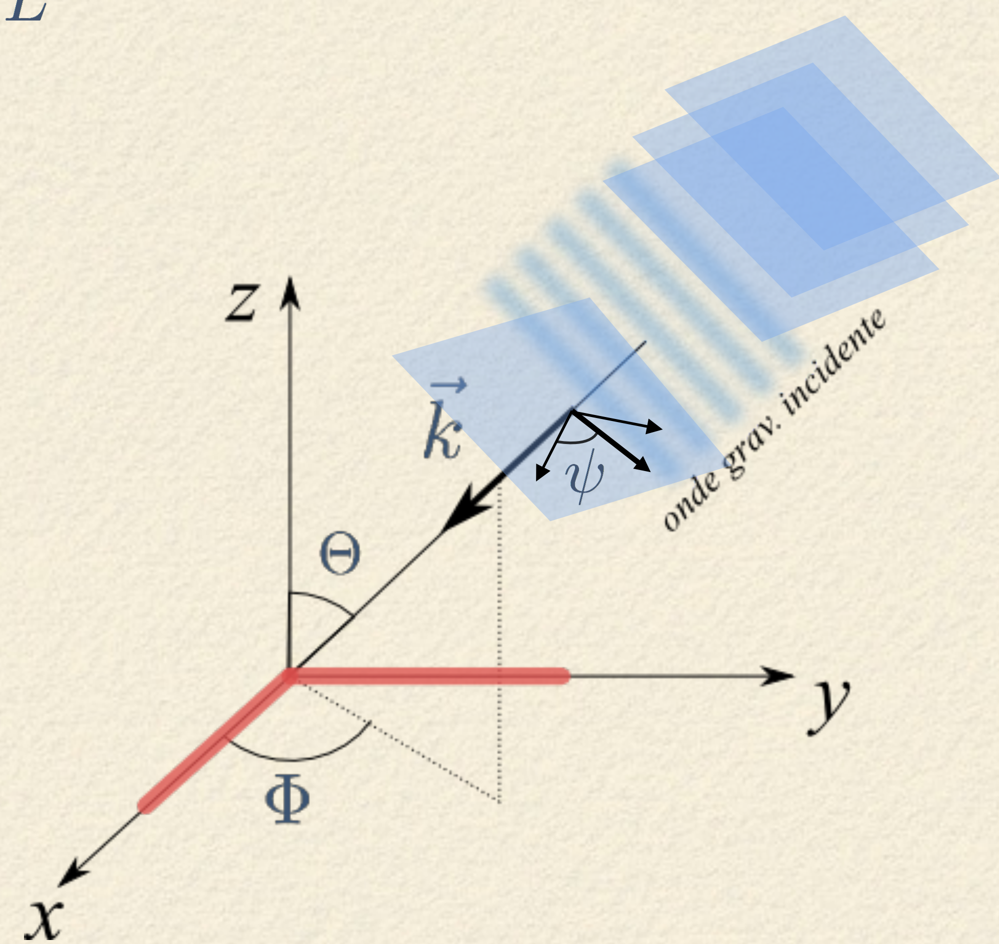
$$\delta\tau_{ar} = \frac{1}{2}h_+\left(\frac{2L_x}{c} + \frac{2L_y}{c}\right) = h_+\frac{2L}{c} \quad \Rightarrow \quad \frac{c \cdot \delta\tau_{ar}}{2} = \delta L = h_+ \cdot L \quad \Rightarrow \quad h_+ = \frac{\delta L}{L}$$

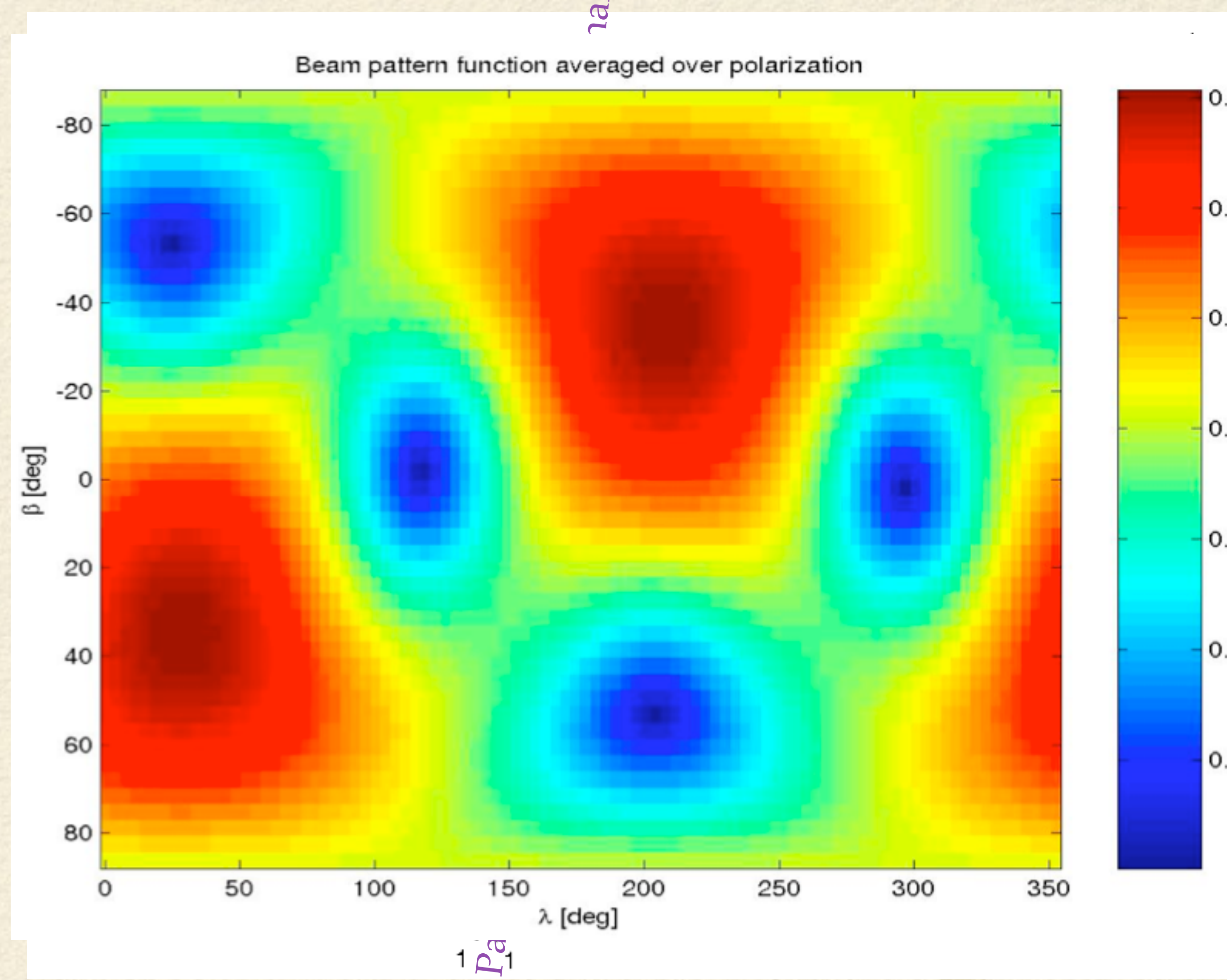
- accumulated phase difference : $\delta\phi = \omega_{\text{laser}}\,\delta\tau_{ar} = \frac{4\pi}{\lambda_{\text{laser}}} L h_+$
- proportional to h and L

Angular response

- Interferometer angular response
  - Average on the polarization of the incident wave

$$\frac{\Delta L}{L} = F_+(\Theta, \Phi, \psi)\, h_+ + F_\times(\Theta, \Phi, \psi)\, h_\times$$





$$F_+ = -\frac{1}{2}(1+\cos^2\Theta)\cos 2\Phi \cos 2\psi - \cos\Theta \sin 2\Phi \sin 2\psi$$

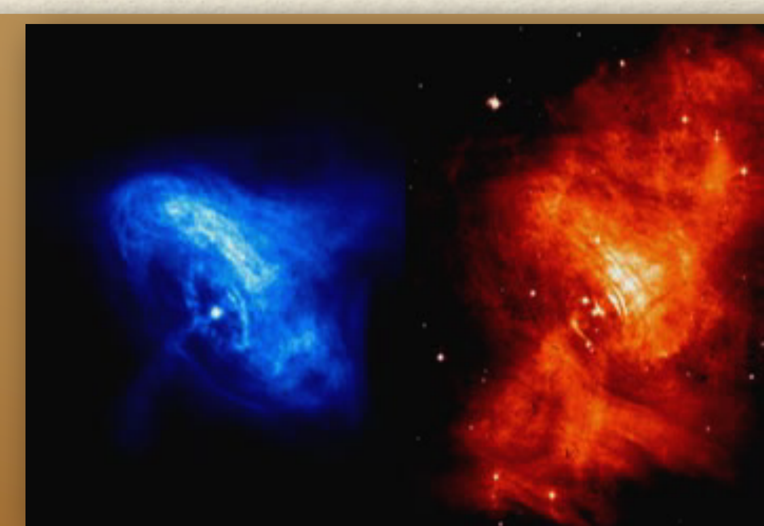
$$F_\times = \frac{1}{2}(1+\cos^2\Theta)\cos 2\Phi \sin 2\psi - \cos\Theta \sin 2\Phi \cos 2\psi$$

"quasi" omni-directional detector

Sources of gravitational waves

Main sources





- Impulsive (or burst) sources
  - Supernovae $T \sim$ ms, $\nu \sim$ kHz, $h \sim 10^{-21} - 10^{-24}$ à 15 Mpc
  - Compact binary system coalescences (neutron stars or black holes) $T \sim$ mn, $\nu \sim$ 10 Hz - 1 kHz, $h \sim 10^{-23}$ à 10 Mpc
- Periodic sources
  - rotating neutrons stars $\nu \sim$ 1 Hz - 1 kHz, $h \sim 10^{-25}$ à 3 kpc
- What about the Gamma Ray Bursts ?
  - short GRBs may be coalescences

D Eichler, M Livio, T Piran, and D Schramm. Nature, 340:126, 1989.
R Narayan, Paczynski, and T Piran. Astroph. J., 395:L83, 1992.

Sources : Coalescences

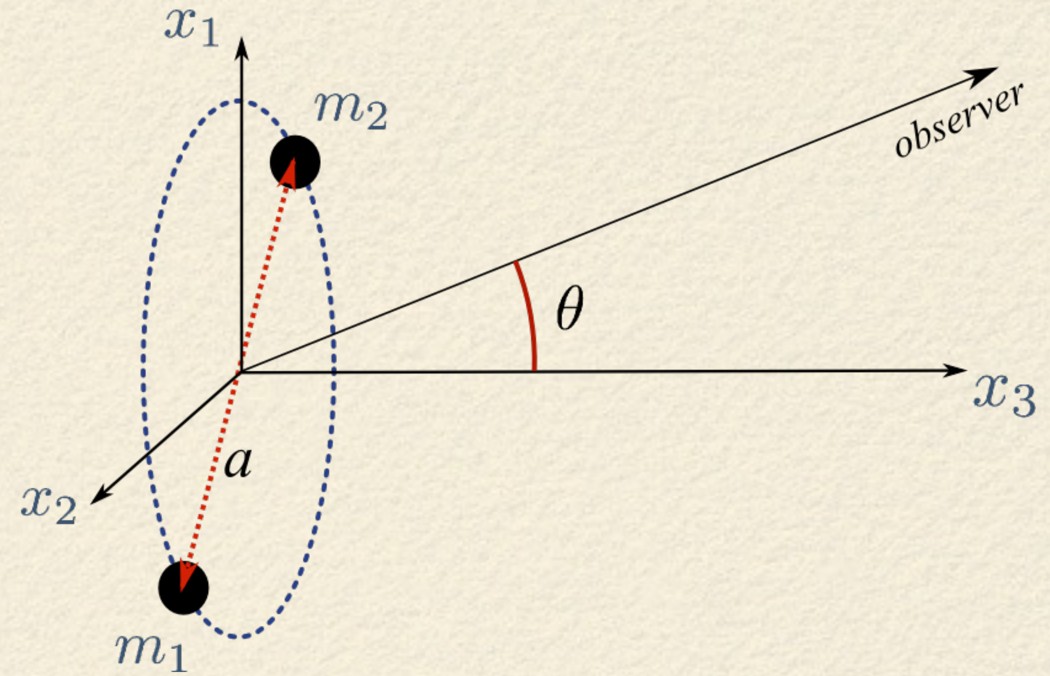
- Binary system of compact objects
- One of the most promising sources for detection :

- Black hole - black hole (BHBH)
- BH - Neutron star (NS)
- NS - NS

$T \sim$ mn, $\nu \sim$ 10 Hz - 1 kHz,
$h \sim 10^{-23}$ à 10 Mpc

Generation : binary system

- Example : binary system of two compact objects
  - Masses m_1 and m_2
  - Distance between the objects : a
  - Total mass : $M = m_1 + m_2$
  - Reduced mass : $\mu = \dfrac{m_1 m_2}{M}$
- Newtonian approximation
  - 3[d] Kepler law : $\omega = \sqrt{\dfrac{GM}{a^3}}$
- Take circular orbits
- Compute h_+ and $h_\times$, the amplitude of the two modes of the emitted wave seen by an observer situated at a distance $R \gg a$
- Relative coordinates : $x_1(t) = \dfrac{a}{2}\cos\omega t, \;\; x_2(t) = \dfrac{a}{2}\sin\omega t, \;\; x_3(t) = 0$

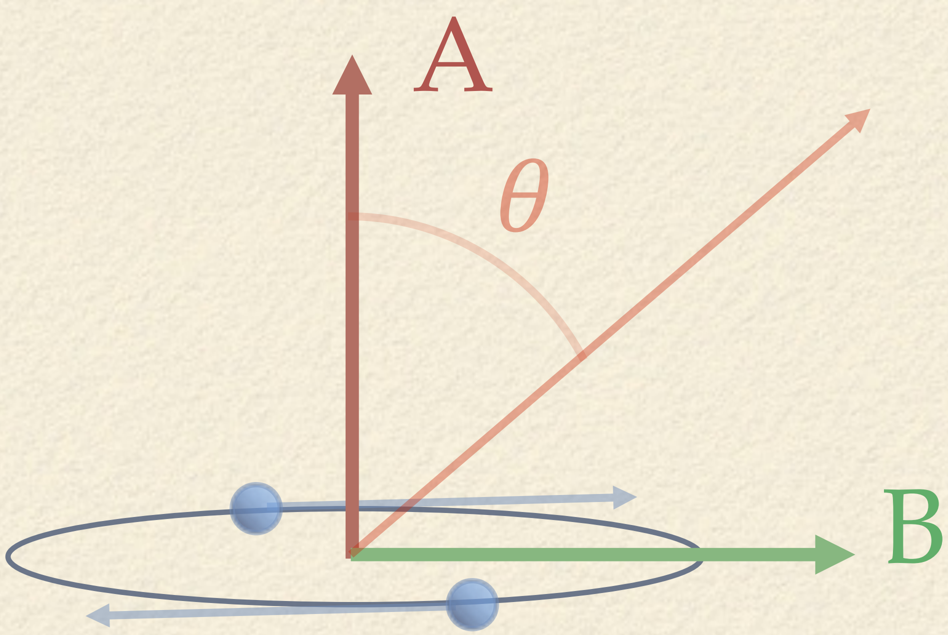


Generation : binary system

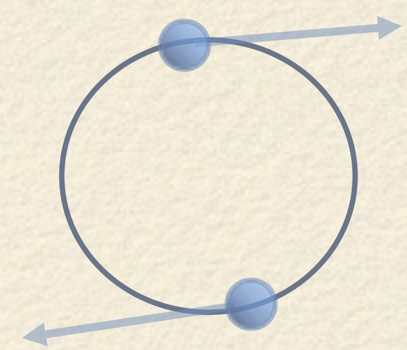
- One obtains

$$h_+(t) = \frac{4G\mu a^2\omega^2}{Rc^4}\frac{1+\cos^2\theta}{2}\cos 2\omega t$$

$$h_\times(t) = \frac{4G\mu a^2\omega^2}{Rc^4}\cos\theta \sin 2\omega t$$



Observer A : $\cos\theta = 1$
sees the two polarizations



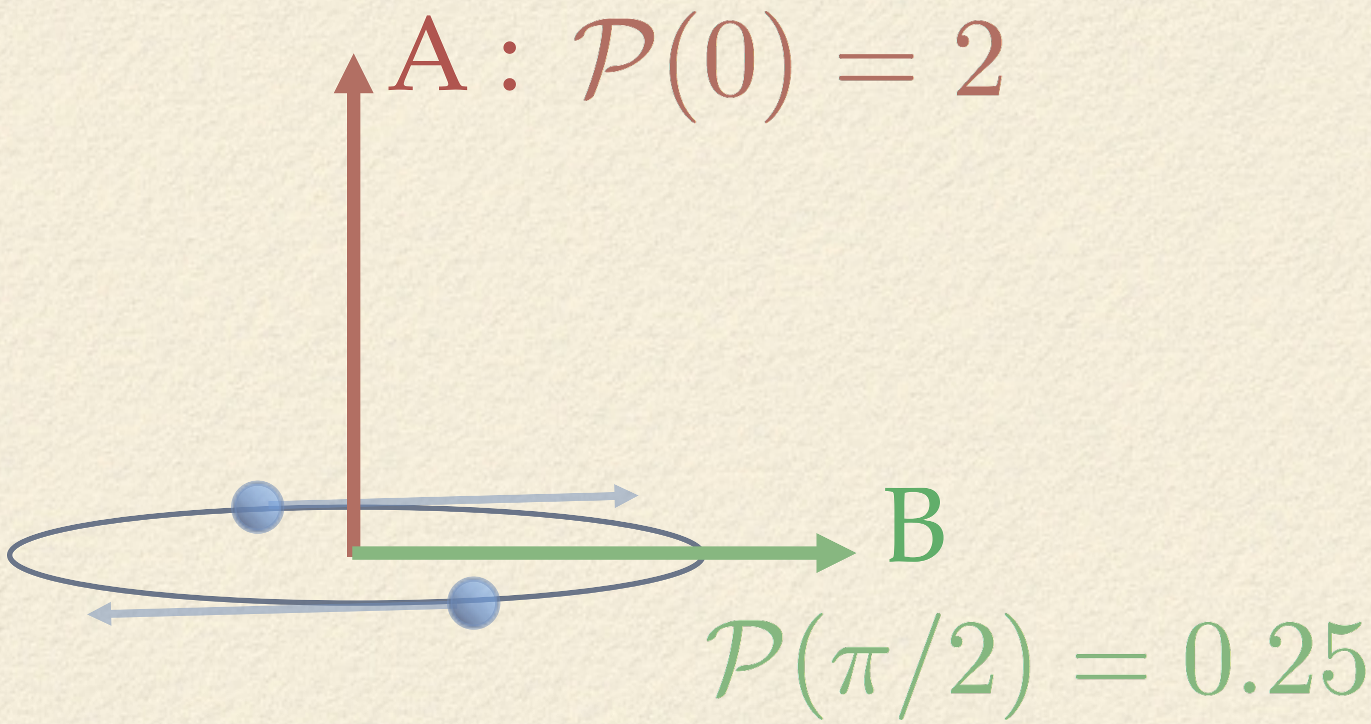
Observer B : $\cos\theta = 0$
sees a linear polarization

Generation : binary system

- Radiated power per unit solid angle $$\frac{dP}{d\Omega} = \frac{2G\mu^2 a^4 \omega^6}{\pi c^5}\mathcal{P}(\theta)$$

$$\mathcal{P}(\theta) = \frac{1}{4}(1 + 6\cos^2\theta + \cos^4\theta)$$

- Radiated power non zero whatever the direction of emission



- Total radiated power

$$P = \frac{32G\mu^2 a^4 \omega^6}{5c^5}$$

Generation : binary system

- Some examples
- Sun-Jupiter system

$$m_J = 1.9 \times 10^{27}\text{kg}, \quad a = 7.8 \times 10^{11}\text{m}, \quad \omega = 1.68 \times 10^{-7}\text{s}^{-1}$$

$$\Rightarrow \quad P = 5 \times 10^3 \quad \text{J/s}$$

  - Very small, compared to the light power emitted by the sun :

$$L_\odot \approx 3.8 \times 10^{26}\ \text{J/s}$$

- Binary pulsar PSR1913+16 (Hulse and Taylor) : $P = 7.35 \times 10^{24}\ \text{J/s}$

Generation : binary system

- Consider simplified Newtonian case (so called "order 0").
- Radiated energy taken to the gravitational energy of the system
  - Grav. energy of the system decreases, radius of the orbits decreases
  - Frequency of the GW increases
  - Conservation of energy : $\frac{dE}{dt} = -P$ (E total energy of the system)
- Newtonian $E = -G\frac{m_1 m_2}{2a}, \quad \omega^2 = \frac{GM}{a^3} \qquad (M = m_1 + m_2)$
- For $m_1 = m_2$

$$E = \frac{1}{2}m\left(\omega\frac{a}{2}\right)^2 + \frac{1}{2}m(\omega\frac{a}{2})^2 - G\frac{m^2}{a} = -G\frac{m^2}{2a} = -\frac{G^{\frac{2}{3}}M^{\frac{5}{3}}}{8}\omega^{\frac{2}{3}}$$

Generation : binary system

- Goal : calculate the evolution of the frequency of the wave

$$E = -G\frac{m_1 m_2}{2a}, \quad \omega^2 = \frac{GM}{a^3} \quad \Rightarrow \quad \dot{E} = -G^{2/3}\frac{m_1 m_2}{2M^{1/3}}\frac{2}{3}\,\dot{\omega}\,\omega^{-1/3}$$

$$\boxed{\dot{E} = -P} \quad \Rightarrow \quad G^{2/3}\frac{m_1 m_2}{2M^{1/3}}\frac{2}{3}\,\dot{\omega}\,\omega^{-1/3} = \frac{32G\mu^2 a^4 \omega^6}{5c^5}$$

- Replace *a* by its expression as a function of ω : $\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\frac{G^{5/3}}{c^5}\frac{\mu}{M}(M\omega)^{5/3}$
- Frequency : $\dot{f}_{OG} = \frac{96}{5}\frac{G^{5/3}}{c^5}\pi^{8/3}\mathcal{M}^{5/3} f_{OG}^{11/3}$ (since $2\pi f_{OG} = 2\omega$)

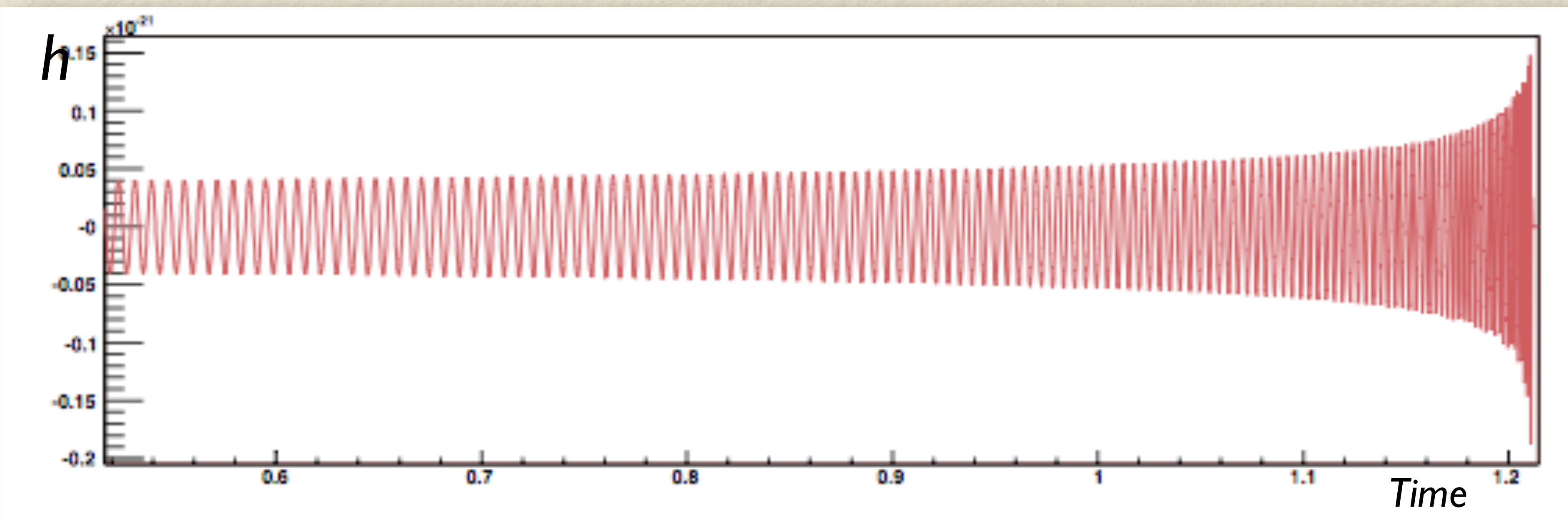
Mass drives the evolution of the waveform

- where we define the "chirp mass" :

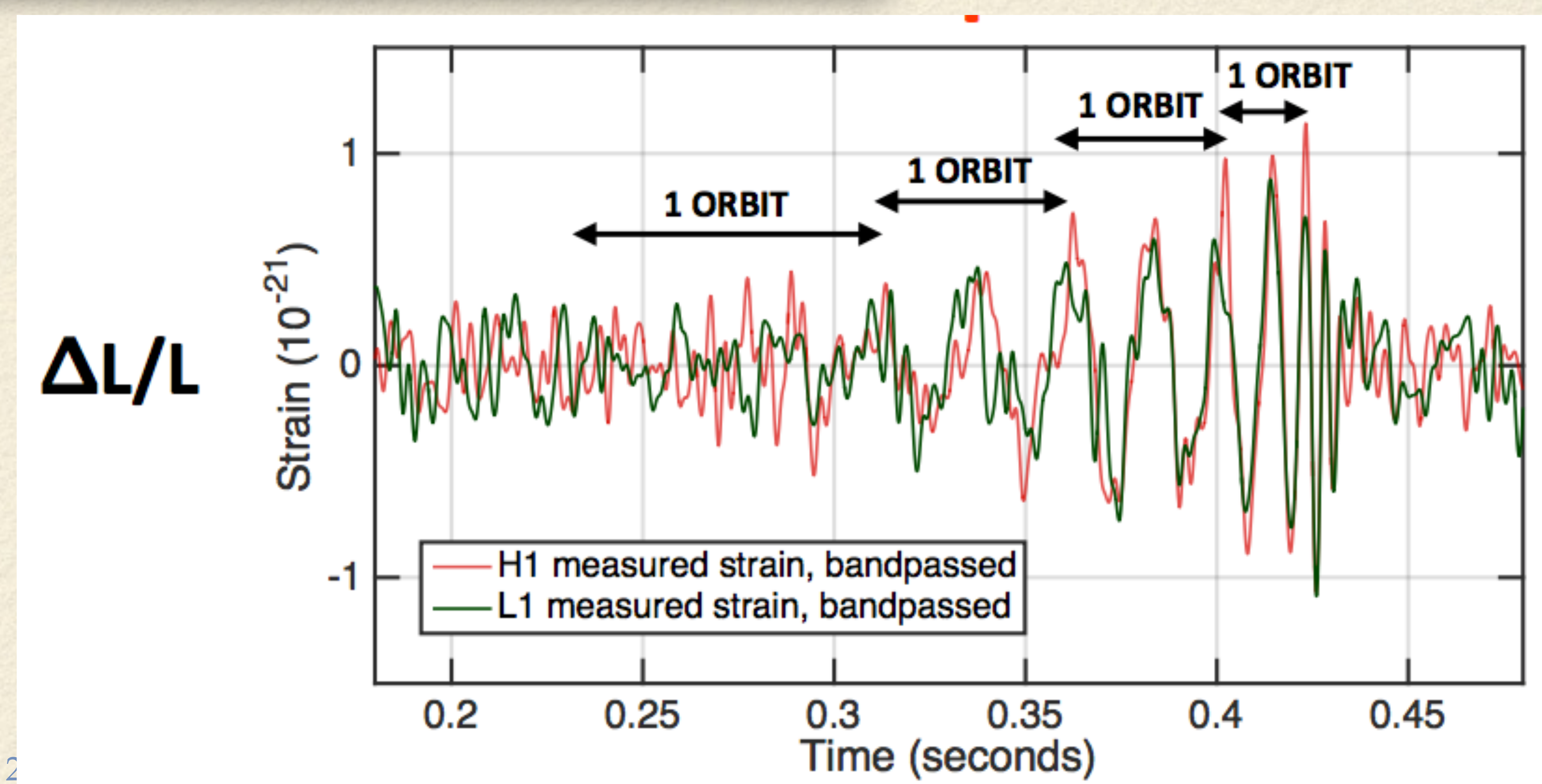
$$\mathcal{M} = \mu^{3/5} M^{2/5}$$

Generation : binary system

- Computed waveform before the "plunge" :



one orbit
=
two GW cycles



Generation : binary system

- GR → post-newtonian corrections → more complex !
- Development around the newtonian limit in $\epsilon = \left(\frac{v}{c}\right)^2$
  - v = relative speed of the two stars (dimensionless) $v = (M\omega)^{1/3}$
- For example, development of the orbital phase

$$\phi(t) = \phi_{ref} + \phi_N \sum_{k=0}^{n} \phi_{\frac{k}{2}PN} v^k$$

- The successive terms become more and more complex
  - higher order effect, spin-spin interaction, spin-orbit, radiation.

k	N	2	3	4	5
$\mathcal{F}_k$	$\frac{32\eta^2 v^{10}}{5}$	$-\frac{1247}{336} - \frac{35\eta}{12}$	4π	$-\frac{44711}{9072} + \frac{9271\eta}{504} + \frac{65\eta^2}{18}$	$-\left(\frac{8191}{672} + \frac{535\eta}{24}\right)\pi$
t_k^v	$-\frac{5m}{256\eta v^8}$	$\frac{743}{252} + \frac{11\eta}{3}$	$-\frac{32\pi}{5}$	$\frac{3058673}{508032} + \frac{5429\eta}{504} + \frac{617\eta^2}{72}$	$-\left(\frac{7729}{252} + \eta\right)\pi$
ϕ_k^v	$-\frac{1}{16\eta v^5}$	$\frac{3715}{1008} + \frac{55\eta}{12}$	-10π	$\frac{15293365}{1016064} + \frac{27145\eta}{1008} + \frac{3085\eta^2}{144}$	$\left(\frac{38645}{672} + \frac{15\eta}{8}\right)\pi \ln\left(\frac{v}{v_{\rm lso}}\right)$
ϕ_k^t	$-\frac{2}{\eta\theta^5}$	$\frac{3715}{8064} + \frac{55\eta}{96}$	$-\frac{3\pi}{4}$	$\frac{9275495}{14450688} + \frac{284875\eta}{258048} + \frac{1855\eta^2}{2048}$	$\left(\frac{38645}{21504} + \frac{15\eta}{256}\right)\pi \ln\left(\frac{\theta}{\theta_{\rm lso}}\right)$
F_k^t	$\frac{\theta^3}{8\pi m}$	$\frac{743}{2688} + \frac{11\eta}{32}$	$-\frac{3\pi}{10}$	$\frac{1855099}{14450688} + \frac{56975\eta}{258048} + \frac{371\eta^2}{2048}$	$-\left(\frac{7729}{21504} + \frac{3}{256}\eta\right)\pi$
τ_k	$\frac{3}{128\eta}$	$\frac{5}{9}\left(\frac{743}{84} + 11\eta\right)$	-16π	$2\phi_4^v$	$\frac{1}{3}\left(8\phi_5^v - 5t_5^v\right)$

Generation : binary system

- PSR 1913+16
  (Hulse et Taylor)
- points = observations,
  line = GR prediction

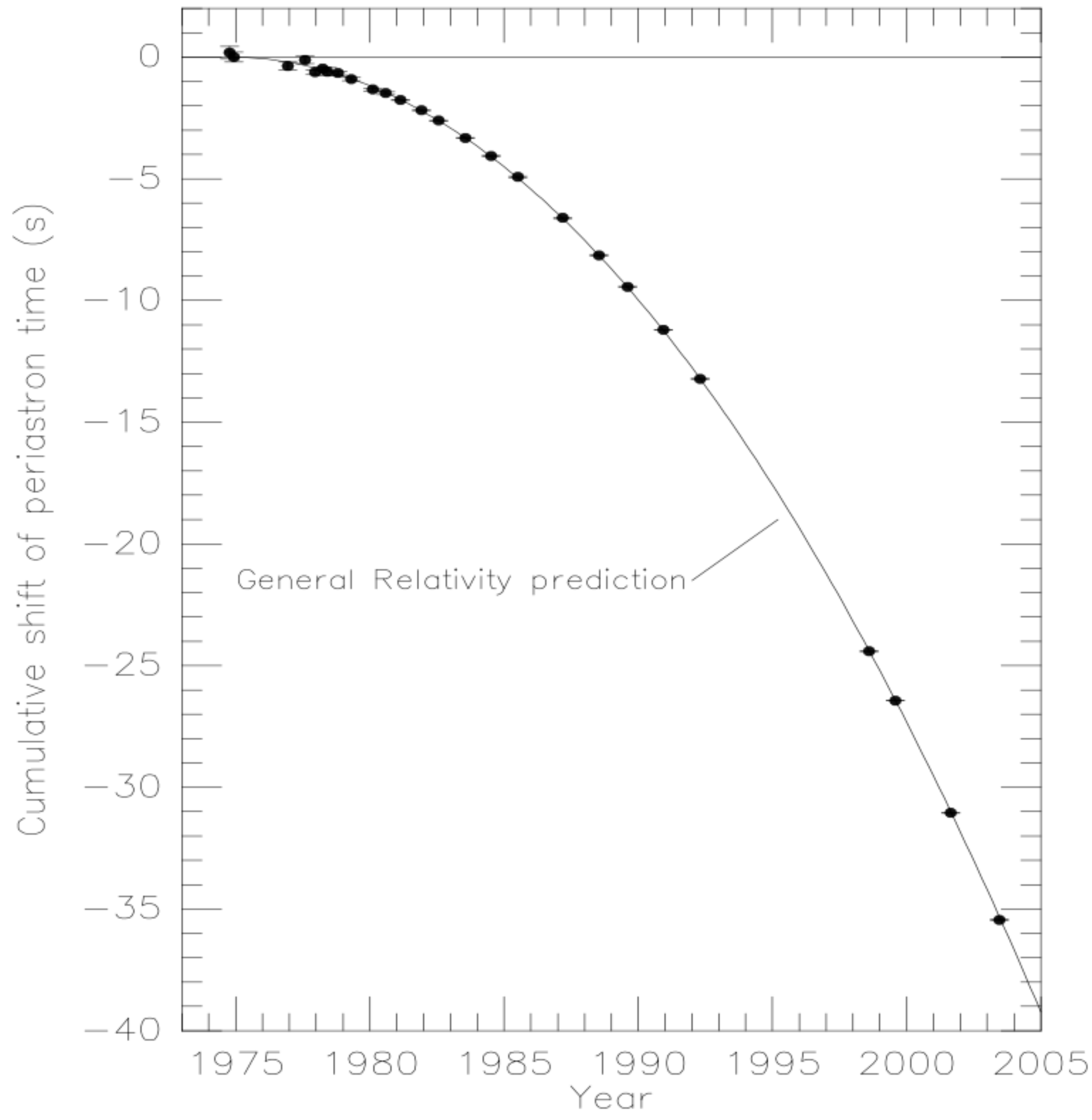
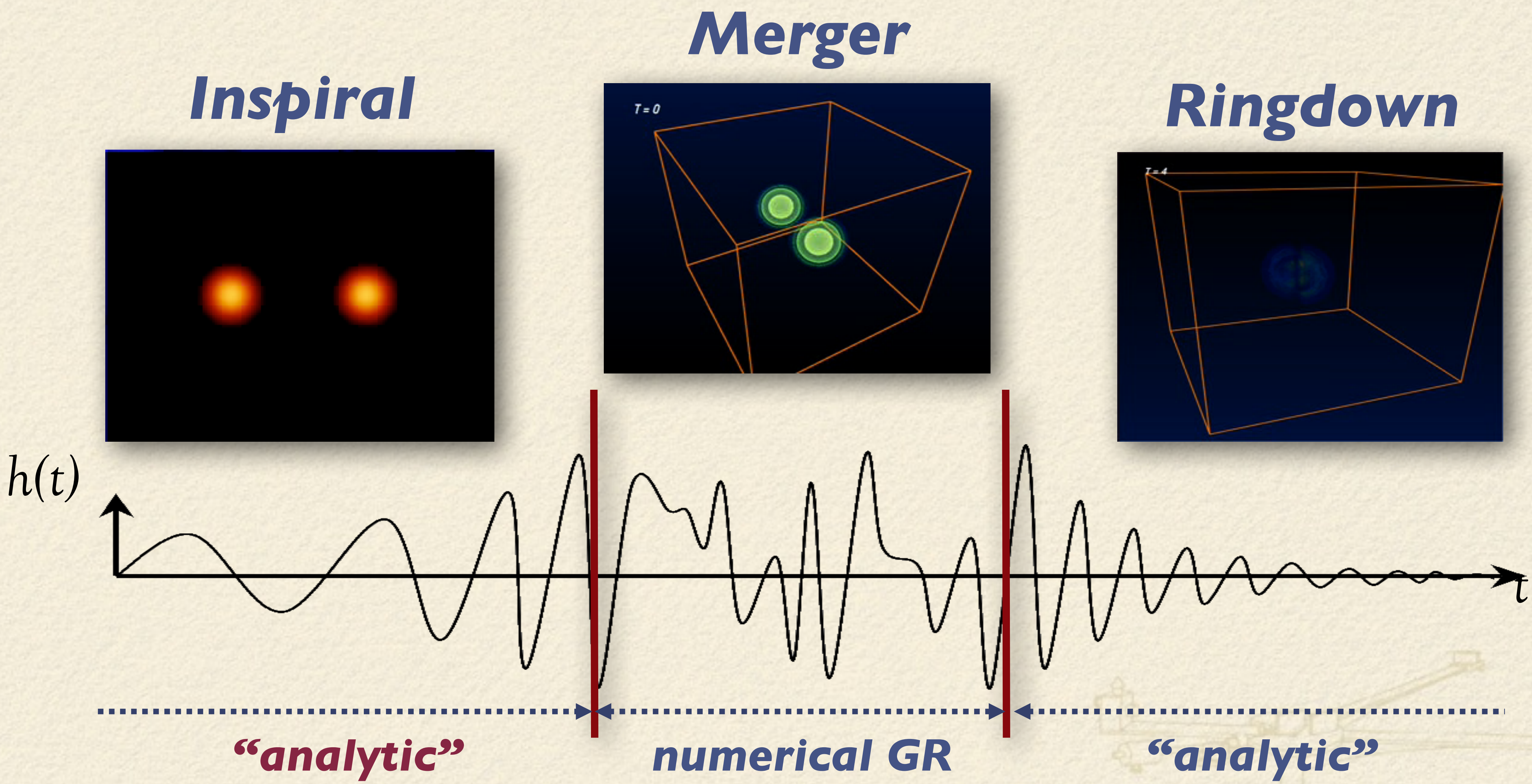


Figure 1. Orbital decay of PSR B1913+16. The data points indicate the observed change in the epoch of periastron with date while the parabola illustrates the theoretically expected change in epoch for a system emitting gravitational radiation, according to general relativity.

Sources : Coalescences

Phases of the coalescence of a binary system of compact objects

(neutron stars or black holes)



Sources : "Pulsars"

- rotating neutron stars

$\nu \sim$ 1 Hz - 1 kHz, $h \sim 10^{-25}$ à 3 kpc

Amplitude of the wave :

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz}\, \varepsilon\, f_{gw}^2}{d}$$

I_{zz} : moment of inertia along the axis of rotation

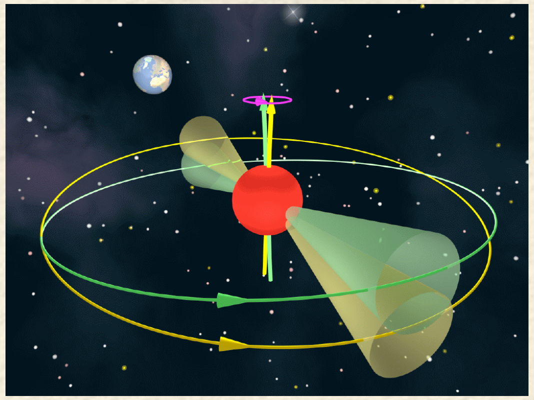
$$\varepsilon = \frac{I_{xx} - I_{yy}}{I_{zz}}$$

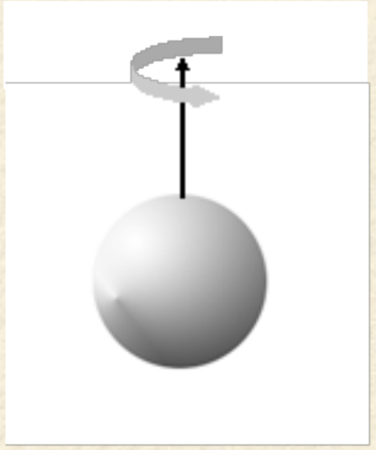
: ellipticity in the equatorial plane

very poorly known or estimated

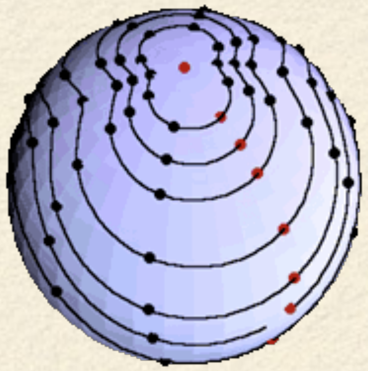
modulated (Doppler effect) by the motion and orientation of the detector around the sun

precession





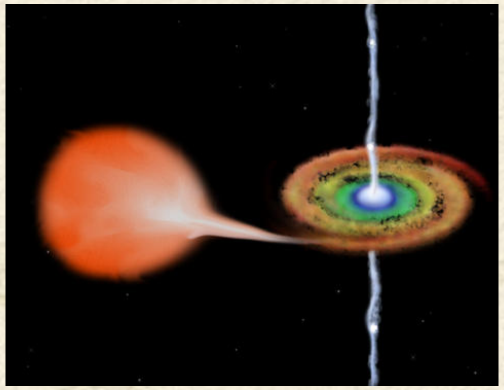
"mountains"



Oscillating modes

LMXB

(Low Mass X-ray Binaries)



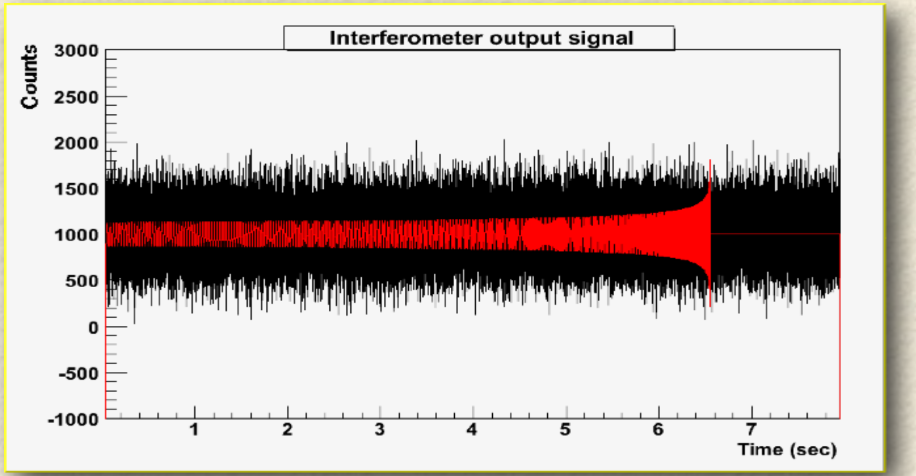
A glimpse of data analysis

The problem

- Signal buried into the noise

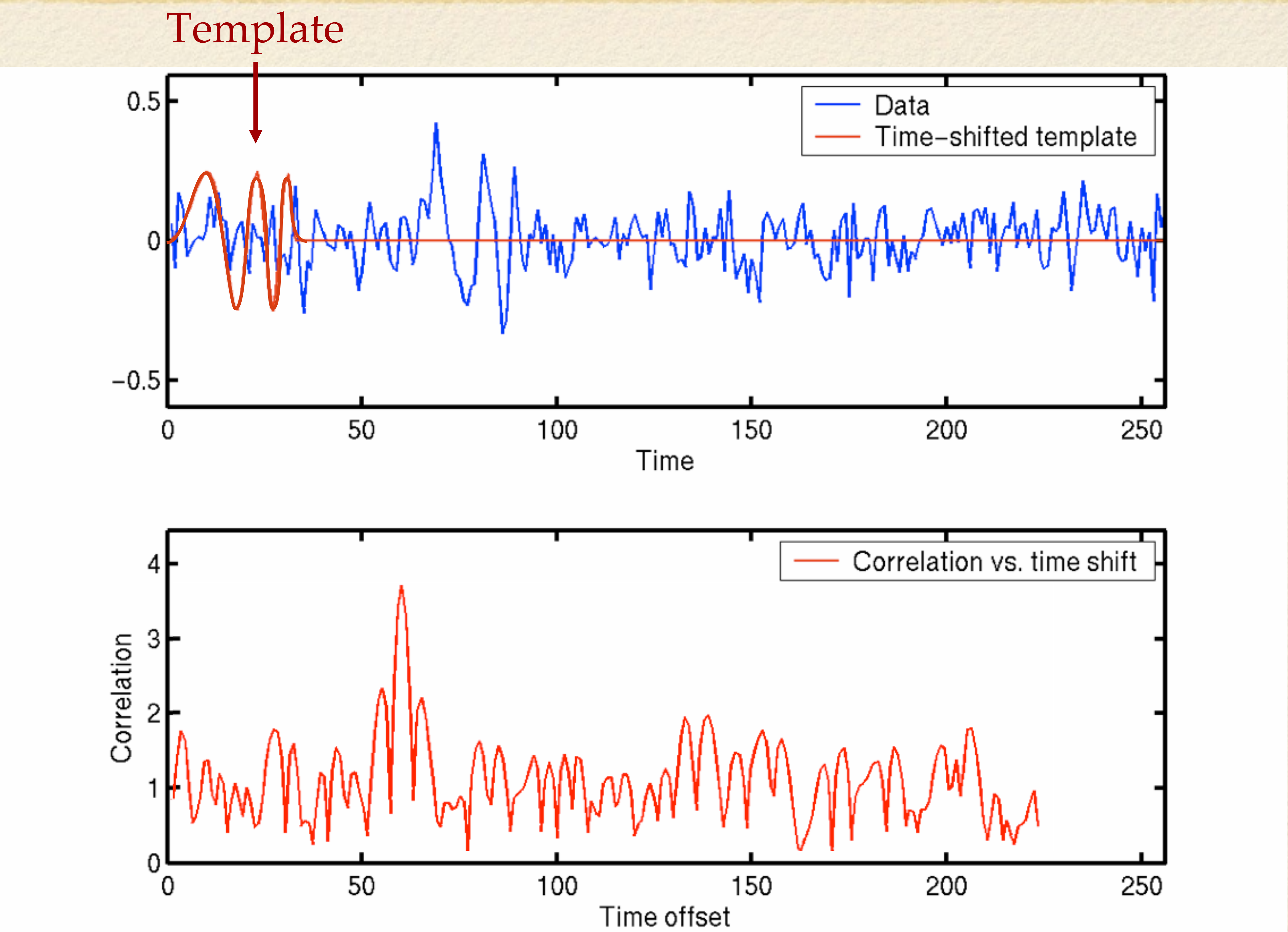
$$a(t) = n(t) + s(t)$$

Detector output = Noise + Signal



- Signal is deterministic $\Rightarrow$ expressed (in frequency domain) in $1/\text{Hz}$
- Noise is stochastic $\Rightarrow$ expressed (in frequency domain) in $1/\sqrt{\text{Hz}}$
- ***Not of the same nature !***
- How to recognize that a waveform is hidden in the noise ?

Intercorrelation

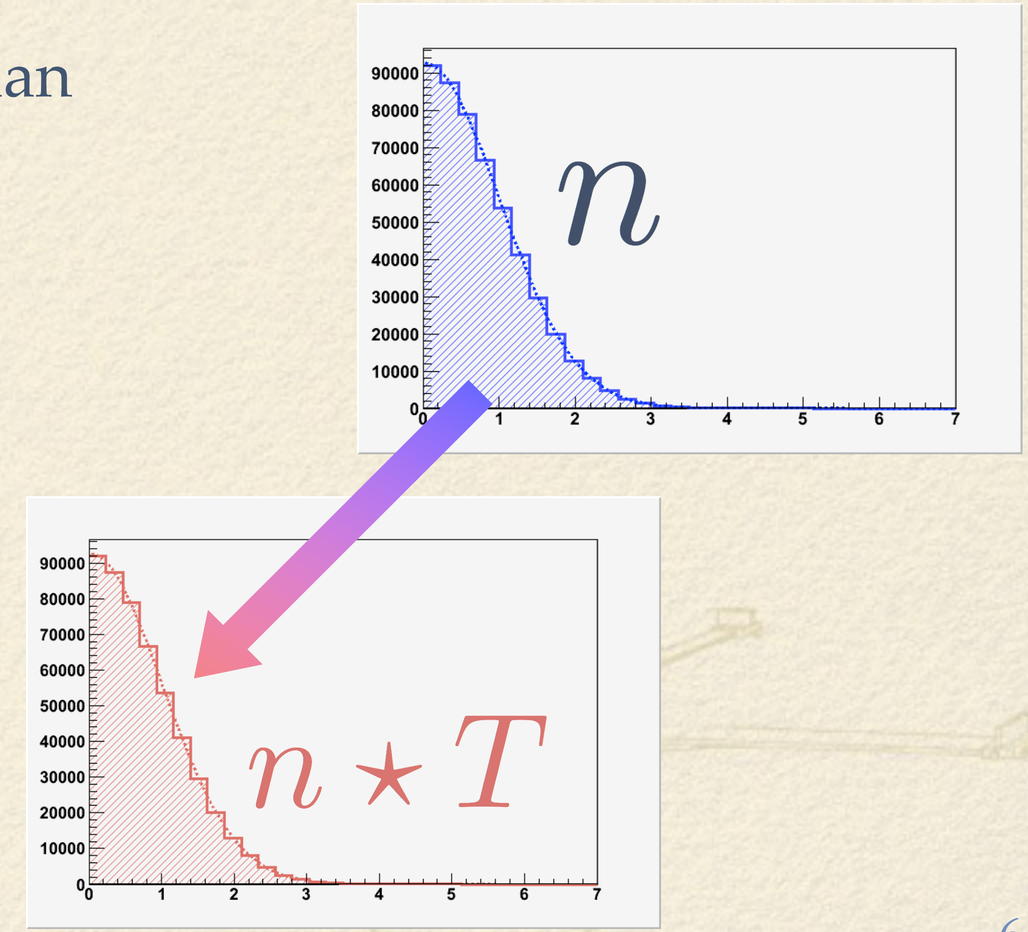


Intercorrelation

- Resemblance of two waveforms : intercorrelation

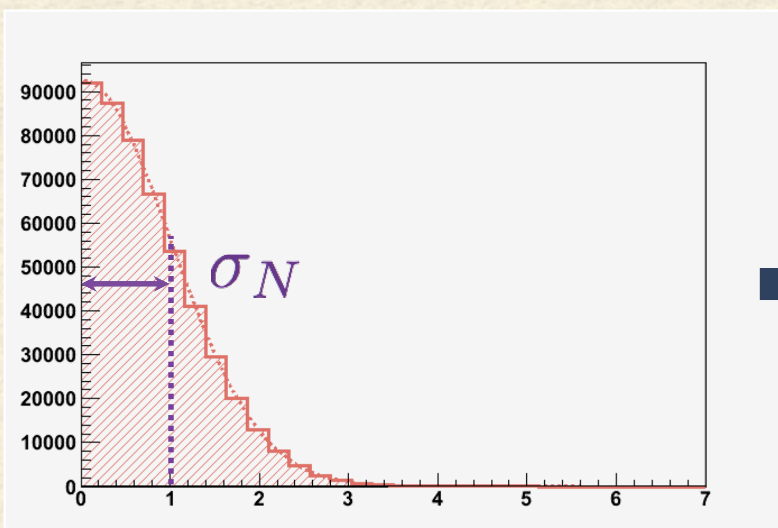
$$a_1 \star a_2(\tau) = \int_{-\infty}^{+\infty} a_1(t)\, a_2(t+\tau)\, dt$$

- If
  - The distribution of the noise values is gaussian
  - There is no signal present in the data
- Then
  - The distribution of the values of intercorrelation between the data and and a test signal (template) *T* is also gaussian

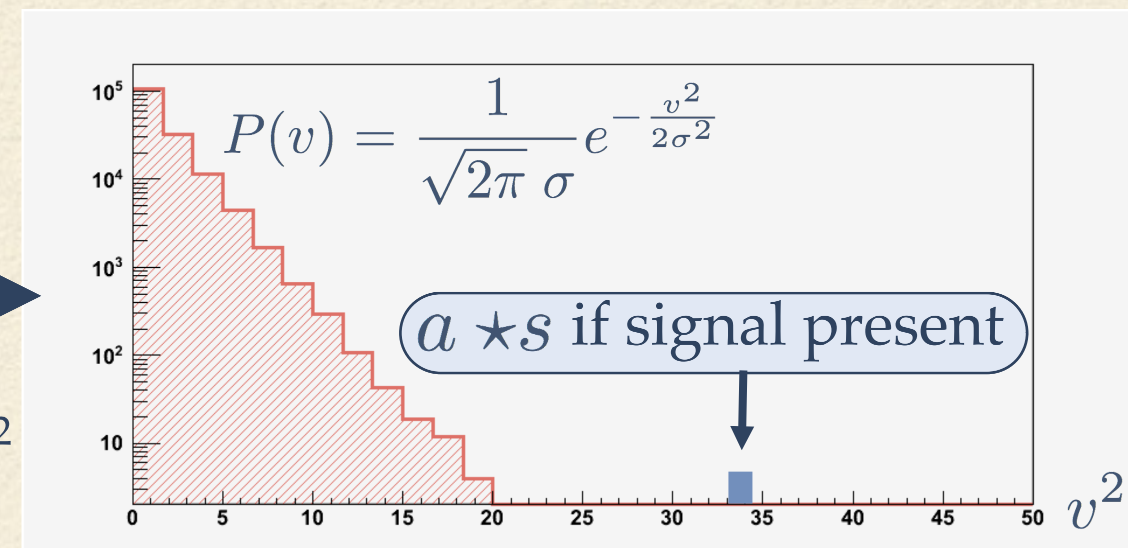


Signal over noise ratio

- If a signal s_0 is present in the noise
  - value of $a \star s$ high for s of the same shape as the signal s_0



$\ln(P)$
as a fonction of v^2



  - width of the noise distribution :

$$\sigma_N = \sqrt{\langle n \star s\,(\tau)^2 \rangle - \underbrace{\langle n \star s(\tau) \rangle}_{=0}{}^2}$$

  - signal :

$$S \equiv |a \star s\,(\tau)|$$

  - Signal over Noise Ratio (SNR)

$$SNR = \frac{S}{\sigma_N}$$

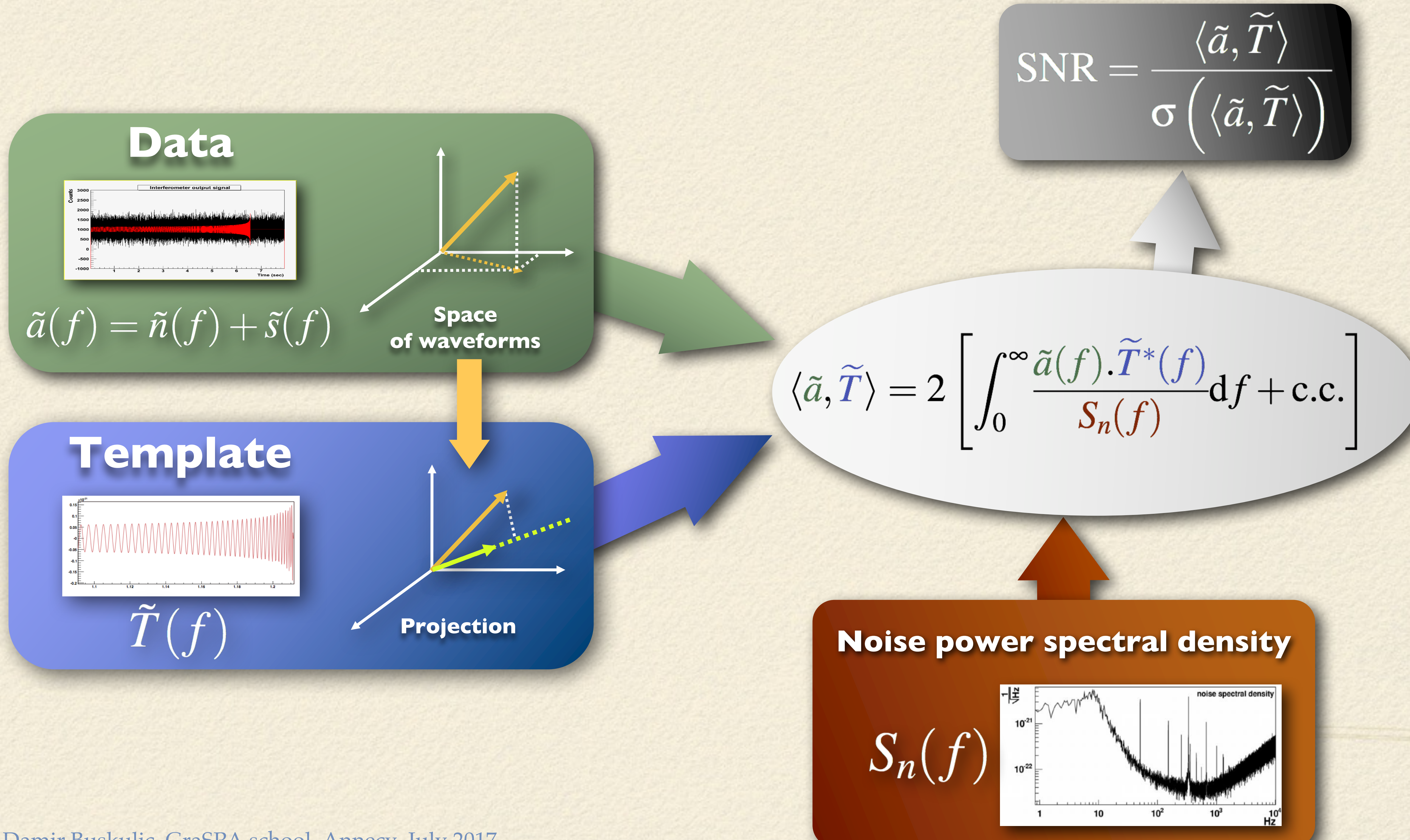
Optimal filtering

- In our case...
- Search for a waveform *s* buried in a noise *n* (radars in the 40')
- Intercorrelation gives an optimal SNR if *n* is a white noise (power spectral density $P_a(f) = cte$)
  - But in practice the noise is not white !
- One can show that an optimal filter can be found in the frequency domain

$$\langle \tilde{a}, \widetilde{T} \rangle = 2 \left[\int_0^\infty \frac{\tilde{a}(f).\widetilde{T}^*(f)}{S_n(f)} \mathrm{d}f + \mathrm{c.c.} \right]$$

  - c.c. = complex conjugate
  - can be interpreted
    - as a "weighted intercorrelation", where the weight is the noise power spectral density or
    - as a scalar product in the space of signals

Optimal filtering



Detection rate

$$\mathcal{R}_{det} \propto R.\Gamma.T$$

- R: Astrophysical rate (per galaxy and per unit time)
- Γ: Number of reachable galaxies
- T: Observation time

- Example of NS-NS estimated rates for current and future detectors
  (takes into account the sensitivity and galaxy distribution)

Epoch		2015 – 2016	2016 – 2017	2017 – 2018	2019+	2022+ (India)
Estimated run duration		4 months	6 months	9 months	(per year)	(per year)
Burst range/Mpc	LIGO	40 – 60	60 – 75	75 – 90	105	105
	Virgo	—	20 – 40	40 – 50	40 – 80	80
BNS range/Mpc	LIGO	40 – 80	80 – 120	120 – 170	200	200
	Virgo	—	20 – 60	60 – 85	65 – 115	130
Estimated BNS detections		0.0005 – 4	0.006 – 20	0.04 – 100	0.2 – 200	0.4 – 400
90% CR	% within 5 deg^2	< 1	2	$> 1 - 2$	$> 3 - 8$	> 20
	% within 20 deg^2	< 1	14	> 10	$> 8 - 30$	> 50
	median/deg^2	480	230	—	—	—
searched area	% within 5 deg^2	6	20	—	—	—
	% within 20 deg^2	16	44	—	—	—
	median/deg^2	88	29	—	—	—

Ze End

of ze feurst part

(as we say in french)