Gravitational Wave (GW) Standard Sirens



Samaya Nissanke (CITA) GW-HEN workshop, APC, 19th May 2009



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[Nissanke et al. (2009): arXiv 0904.1017; Nissanke et al. : in prep]

• (Very brief) Introduction to Standard Sirens:

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 - GW counterpart to Standard Candles.

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 - Methodology.
 - Results: (I). distance measures and (II). Hubble's parameter.
- Present and Future Work.

Introduction to Standard Sirens

Cosmology: expansion history of the universe

• Luminosity distance -- redshift relationship:

$$D_L(z_s) = rac{c(1+z)}{H_0\sqrt{\Omega_K}} \sinh\left[\sqrt{\Omega_K}\int_0^{z_s}rac{H_0}{H(z)}dz
ight]$$

where

$$rac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + \Omega_{
m de} (1+z)^{3(1+w)} + \Omega_K (1+z)^2}.$$

- Standard candles give a measure of the luminosity distance: cepheids, SNIa ...
- ...GW standard sirens: Inspiralling Compact Binaries (ICBs) -- an absolutely calibrated measure.

Space and worldwide network of gravitational wave (GW) interferometers





LIGO Livingston (4km) LIGO

LIGO Hanford (4km + 2km)

VIRGO (3km)

- Principal sources are inspiralling and merging compact binaries (ICBs/MCBs)
- Detection and measurement of GWs requires accurate source modelling in the form of GW templates.
- Matched filtering maximises SNR.

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Gravitational Waveforms for Standard Sirens

- Inspiral (analytic):
 - Post-Newtonian (PN) approximation

- Extraction of Source Parameters:

h (masses, spins, inclination angle, sky position......**luminosity distance**)

- Standard sirens: mapping out the expansion history of the universe

LUMINOSITY DISTANCE --REDSHIFT RELATIONSHIP



Gravitational Waveforms for Standard Sirens

Inspiral (analytic):
 Post-Newtonian (PN)
 approximation: expansion in

 $v/c \rightarrow 0$ as $c \rightarrow \infty$

Adiabatic: $T_{radn.reac.} \gg T_{orbital}$

- Extremely accurate and well modelled (tidal effects at 5PN).
- SNR dominated by inspiral for NS/NS and NS/BH binaries.



Standard sirens: measuring luminosity distance

 Absolute measure of luminosity distance at detector (ground/ space):

 $h_{\mathcal{M}}(t) = F_{+}(\theta,\phi,\psi)h_{+}(t) + F_{\times}(\theta,\phi,\psi)h_{\times}(t)$

where

$$\begin{split} h_+(t) &= 2 \frac{\mathcal{M}^{5/3} (\pi f(t))^{2/3}}{D_L} (1 + \cos^2(\hat{\mathbf{L}} \cdot \hat{\mathbf{n}})) \cos \Phi(t) \\ h_\times(t) &= -4 \frac{\mathcal{M}^{5/3} (\pi f(t))^{2/3}}{D_L} \cos(\hat{\mathbf{L}} \cdot \hat{\mathbf{n}}) \sin \Phi(t) \,. \end{split}$$

- Degeneracy of parameters in the amplitude.
- GW polarisation gives inclination angle.
- Network (timing of signals) or modulation of LISA gives source localisation and polarisation.

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EM counterpart for ICBs: Standard Sirens

• The ICB waveform (PN Inspiral):

Absolute measure of luminosity distance at detector (ground/space).

• Electromagnetic counterpart:

Schutz (1986); Chernoff and Finn (1993); Finn (1996).

Redshift: z!

Source Localisation: reduces parameter space and improves errors in luminosity distance.

Time of merger: reduces detection threshold SNR.

• LISA: electromagnetic counterpart? lensing?

e.g. Holz and Hughes (2005); Arun et al. (2006,2008).

• Advanced ground based detectors.

Nissanke et al. (2009): arXiv:0904.1017 Dalal, Holz, Hughes, Jain (2006).

Standard sirens for adv. LIGO/VIRGO type detectors

Enhanced and Advanced LIGO

www.ligo.caltech.edu

Initial LIGO: NS/NS - 15 Mpc; NS/10 M_{\odot} BH - 30 Mpc

Enhanced LIGO (2009--11): (x2 in sensitivity = x10 in volume)

Advanced LIGO (2015): (x10 in sensitivity = x1000 in volume) NS/NS - 300 Mpc; NS/BH - 650 Mpc



[see Marion and Van der Brand's talk y'day]

Electromagnetic counterpart?

www.ligo.caltech.edu

- LIGO today Enhanced LIGO (2009)[Dalal, Holz, Hughes, Jain (2006)] AdvLIGÖ [Kochanek and Piran (1993); Nakar (2007)] Advanced LIGO (2015
- Recent short hard gamma ray (SHB) observations.
- Advanced LIGO (2015): coincident SHB observations: NS/NS ~ 600 Mpc; NS/BH ~ 1300 Mpc.
- Leading progenitor model: NS/NS and NS/BH binaries.
- Lensing effects negligible.

Short Hard GRBs (SHBs)



GRB 050724 observed by Swift's X-ray telescope (red) and X-ray (Chandra) observations



GRB 050509b observed by Swift's γ-ray (blue) & X-ray (red) instruments

- Long (>2 s) and short GRBs ?
- Over past four years, γ-ray observatories have seen optical and X-ray afterglows for SHBs for the first time ...

z and (subarcsecond) **n** together.

- ~/> 30 SGRBs; small sample size.
- Diverse properties (host galaxies); large range of z (0.1--1.8).
- Two systems with tentative opening angles (~20°).

[see Fox and Guetta's talk]

Single GW channel

Science output	No EM counterpart	
NS/NS or NS/BH progenitor model	redshifted masses	
	redshifted spins	
collimation of jet: SHB engine	orientation	
host galaxy	source localisation	
	no redshift	
	luminosity distance	

Multi EM, GW (and HEv?) channels

e.g. HERMES (see Fox's talk)

Science output	No EM counterpart	γ-ray and afterglow
SNR threshold \downarrow		time of merger
NS/NS or NS/BH progenitor model	redshifted masses	masses
	redshifted spins	spins
collimation of jet: SHB engine	orientation	orientation?
host galaxy	source localisation	
Standard Sirens	no redshift	redshift
	luminosity distance	luminosity distance

Methodology

Parameter estimation?

• Central quantity: posterior PDF

$p(\theta|s) \propto p(\theta) \mathcal{L}_{ ext{total}}(s| heta)$

• Likelihood:

$$egin{split} \mathcal{L}_{ ext{det}}(s_{ ext{det}}|m{ heta}) \propto e^{-2\operatorname{Re}\int_0^\infty df rac{(ilde{h}(f;m{ heta})- ilde{s}(f))^*(ilde{h}(f;m{ heta})- ilde{s}(f))}{S_n(f)}} \ s_{ ext{det}}(t) &= h_{ ext{det}}(t;m{ heta}) + n_{ ext{det}}(t) \,. \end{split}$$

[Finn (1992); Cutler and Flanagan (1994)]

• Priors: more later - critical to selection criteria

MCMC exploration

- Metropolis-Hastings MCMC exploration of posterior PDF (using cosmoMC).
 [Bridle and Lewis (2002); http://cosmologist.info/cosmomc/]
- Restricted 2 PN waveform in the frequency domain: 7 parameter system.

(masses, D_L , cos(*inc*), ψ , t_c , ϕ_c)

- Gaussian and interferometric specific noise.
- Independent noise between interferometers: total likelihood of the network is product of individual likelihoods at each detector.

 Different network configurations: LIGO Hanford, LIGO Livingston, VIRGO, AIGO, LCGT.



Network I

 Different network configurations: LIGO Hanford, LIGO Livingston, VIRGO, AIGO, LCGT.



Network IIA

 Different network configurations: LIGO Hanford, LIGO Livingston, VIRGO, AIGO, LCGT.



Network IIB

 Different network configurations: LIGO Hanford, LIGO Livingston, VIRGO, AIGO, LCGT.



Network III

 Different network configurations: LIGO Hanford, LIGO Livingston, VIRGO, AIGO, LCGT.



• Two fiducial binaries: non-spinning NS-NS and NS-10 M_{\odot} BH systems.

 Different network configurations: LIGO Hanford, LIGO Livingston, VIRGO, AIGO, LCGT.



- Two fiducial binaries: non-spinning NS-NS and NS-10 M_{\odot} BH systems.
- Assume beaming of SHBs with a ~ 25° opening angle: how do errors in the luminosity distance decrease?

Selection criteria (NS/NS): detection rates

- Selection of a million binaries out to z = 1 with a constant comoving density.
- Detection threshold: for each binary and detector network, we ask whether its SNR > 7.5 ?
- Obtain detection rates for each network configuration. Red: not detected



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Blue: detected Red: not detected



Selection criteria (NS/NS): detection rates


Selection criteria : SNR vs. Luminosity distance



Selection criteria (NS/NS): prior densities

- **Detection prior** on selected sample with const. comoving volume.
- Uniform priors in all other parameters (including inclination angle).



Joint 2D prior density

Selection criteria: NS-NS beamed (prior densities)

 Beamed subsample selected from total sample assuming a gaussian inclination probability distribution with a ~ 25° opening angle.



Results I: Distance measures

Nissanke et al. (2009): arXiv:0904.1017

Review of distance measurement accuracies

• Fisher matrix method breaks down at relatively high SNR: the probability density function (D_L) can be significantly non-Gaussian:

10-30 % errors in D_L out to 600 Mpc in NS/NS and to 1400 Mpc in NS/BH.

- Non-negligible and important degeneracies between luminosity distance and inclination angle of binary.
 e.g. Cutler and Flanagan (1994)
- Face-on or edge-on nature of binary introduces systematic bias...
- ... but taking sky averages for many differently orientated standard sirens removes this bias cf. GW astronomy!
- Strong dependence on different instances of noise realisations when signal is simulated.

Distance Measurement Accuracies: NS-NS binaries

- Keep a unique instance of noise realisation constant at each detector.
- 200 unbeamed and 200 beamed NS/NS binaries randomly selected from our total detected sample.

$$\frac{\Delta D_L}{\tilde{D}_L}$$

Distance Measurement Accuracies: NS-NS binaries



Distance Measurement Accuracies: NS-NS binaries

- Two distinct distributions: beamed and unbeamed.
- Beaming prior improves distance measurements by a factor of ~2 by breaking cos (inc) - D_L degeneracy.
- Large scatter, in particular, for isotropic case.

Fisher matrix breaks down even at relatively low D_L: Fundamental cos(inc) - D_L degeneracy.



 Addition of detectors increases number of detected binaries (detectable volume and increase in sky coverage)

Future work for distance measures

 Understanding impact of priors and systematic errors arising from limitation in the theoretical source modelling.

• Will the addition of spin precession and higher harmonics in PN waveform break important degeneracies in parameters such as the luminosity distance -- inclination angle to the binary?

Results II: constraining Hubble's parameter

Nissanke et al. (2009): in prep.

Ensemble vs. individual standard siren measurements?

- At first blush, results do not seem encouraging....
- ...however, we are interested in ensemble and not individual standard siren measurements.
- The joint post. PDF in H₀ (or any cosmological parameter) for n events becomes increasingly well constrained as n increases.
- The detected number of binaries is critical (number of detectors and SHB progenitor model).

Results: NS/NS binary (5 yrs.)



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- Five year observation period assuming 10 GW-SHB events per year per Gpc⁻³.
- Increase in number of detectors substantially helps in pinning down H₀ up to a factor ~1.5:

measurement errors in luminosity distance decrease and detected number of binaries increase.



Results: NS/NS binary (5 yrs.)

- Five year observation time assuming 10 GW-SHB events per year per Gpc⁻³.
- Errors in H₀ improve by at least a factor of > two when we assume collimation of sources.



Results: NS/BH binary (1 yr.)

- One year observation time assuming 10 GW-SHB events per year per Gpc⁻³.
- Total number of detected binaries significantly increases ~ 55 NS-BH GW-SHBs per year cf. ~ 6 in the NS-NS case.
- Increase in number of detectors substantially helps in pinning down H₀.



Results: NS/BH binary (1 yr.)

- One year observation time assuming 10 GW-SHB events per year per Gpc⁻³.
 Total number of detected binaries significantly increases ~ 55 NS-BH per year cf. ~ 6 in the NS-NS case.
 - Errors in H₀ improve by at least a factor of > two when we assume collimation of sources.



Summary of H_0 results

	NS-NS ~>	4 NS-BH
H ₀ : Observation time (nos. of binaries)	5.5% ~ 5 years (15)	3.5% ~1 year (26)
beamed SHB	improves by > ×2	
3 →5 detector network	improves by ~ ×1.5	

• Expect errors to decrease with $\sqrt{(Observation Time)}$.

Summary for standard sirens

 GW standard sirens offer a complementary technique to constrain cosmological parameters.
 Independent of the cosmological distance ladder.

 Clean, well modelled systems: no need to treat systematic problems as with SNIa (evolutionary aspects, dust extinction etc...).

Beyond Standard Sirens

 SHB observations (redshift, source localisation, beaming) will help GW astronomy and cosmology...

• ... but converse case is true too!

Information about the extent of collimation of SHBs and source localisation (host environment) will constrain progenitor models.

Single GW channel: constraints on SHB progenitor model

 Assume no electromagnetic counterpart -- how well can one localise the ICBs.?

~ few to 10s of square degrees with advanced 3 det. network.

[Blair et al. (2006); Cavalier et al.(2008); Nissanke et al. (2009): in prep.]

• What can be said about the extent of gamma-ray collimation?

~ |cos (inc)| < 0.3 (predominantly face-on binaries).

Thank you.

Test case: Introducing the CF binary

SNR: 12.4 (5.8) Best fit Distance: 432 Mpc cos(inc) = 0.31 NS/NS system ADV. LIGO H, LIGO L, VIRGO



Fisher information matrix?

[Dalal et al. (2006)]

'Gaussian approximation'(Fisher matrix method) in high SNR limit:

$$\exp\left[-\left(h(\boldsymbol{\theta}) - s|h(\boldsymbol{\theta}) - s\right)/2\right] \simeq \exp\left[-\frac{1}{2}\left(\frac{\partial h}{\partial \theta^a} \left|\frac{\partial h}{\partial \theta^b}\right) \delta \theta^a \delta \theta^b\right]$$

where $\delta \theta^a = \theta^a - ilde{ heta}^a$

• Fisher and covariance matrix:

$$\Gamma_{ab} \equiv \left(\frac{\partial h}{\partial \theta^a} \bigg| \frac{\partial h}{\partial \theta^b} \right) \quad ; \quad \Sigma^{ab} = (\Gamma^{ab})^{-1}$$

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Single GW channel

• Parameters of interest:

$$\boldsymbol{\theta} = \{ \boldsymbol{\mathcal{M}}, \boldsymbol{\mu}, \boldsymbol{\mathsf{D}}_{\mathsf{L}}, \cos(\textit{inc}) \equiv \mathbf{\hat{L}} \cdot \mathbf{\hat{n}}, \boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\psi}, t_{c}, \boldsymbol{\phi}_{c} \}$$

NS/NS or NS/BH: **Progenitor model**



Single GW channel

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$$\boldsymbol{\theta} = \{\mathcal{M}, \mu, \mathbf{D}_{\mathbf{L}}, \mathbf{cos}(inc) \equiv \mathbf{\hat{L}} \cdot \mathbf{\hat{n}}, \theta, \phi, \psi, t_{c}, \phi_{c}\}$$

extent of collimation: SHB engine

where

$$\mu = \frac{m_1 m_2}{m_1 + m_2} (1 + z)$$
$$\mathcal{M} = \mu^{3/5} (m_1 + m_2)^{2/5} (1 + z)$$

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source position:
host galaxy
$$\mu = \frac{m_1 m_2}{m_1 + m_2} (1 + z)$$
$$\mathcal{M} = \mu^{3/5} (m_1 + m_2)^{2/5} (1 + z)$$
Single GW channel

• Parameters of interest:

$$(\boldsymbol{\theta}) = \{\mathcal{M}, \boldsymbol{\mu}, \mathbf{D}_{\mathsf{L}}, \cos(\textit{inc}) \equiv \mathbf{\hat{L}} \cdot \mathbf{\hat{n}}, \boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\psi}, \boldsymbol{t_{c}}, \boldsymbol{\phi_{c}}\}$$

frequency chirp: dynamical flow time dependence of source



Multi- EM and GW channels

• Parameters of interest:

$$\boldsymbol{\theta} = \{\mathcal{M}, \boldsymbol{\mu}, \mathbf{D}_{\mathbf{L}}, \cos(inc) \equiv \hat{\mathbf{L}} \cdot \hat{\mathbf{n}}, \boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\psi}, t_{c}, \phi_{c}\}$$

$$M_{1}$$

$$M_{2}$$

$$N_{1}$$

$$M_{2}$$

$$N_{1}$$

$$M_{2}$$

$$N_{1}$$

$$N_{2}$$

$$N_{1}$$

$$N_{2}$$

$$N_{1}$$

$$N_{2}$$

$$N_{1}$$

$$N_{2}$$

$$N_{2}$$

$$N_{1}$$

$$N_{2}$$

Standard sirens for LISA

2000

1500

500

0

0

Z 1000

• Without counterpart,

sky positions < 1 deg; luminosity distance < 10%

- With optical counterpart,
 luminosity distance < 0.1%
- Possible electromagnetic counterparts?

gas within binary is driven onto larger BH: super-Eddington accretion, outflows/ jets.

delayed afterglows: inspiral hollows out circumbinary gas, which subsequently infalls after merger.

[e.g. Milosavljevic, Phinney, etc.]

No em counterpart 1200 1000 800 600 400 200 10⁻³ $\Delta D_L^{10^{-2}} D_L$ 10⁻⁴ 10^{-1} 10 Lang and Hughes (2006,2007) = 10⁵ M_{\odot} , m_2 = 6×10⁵ M_{\odot} at z = Assuming counterpart 0.005 0.01 0.015 0.02

 $\delta D_L / D_L$

Holz and Hughes (2005)

Standard sirens for LISA



Holz and Hughes (2005)

- Main drawback ... universe in non homogeneous...gravitational lensing...
- For LISA standard sirens to be useful, must have ~100 (sufficient statistics) to average out lensing.
 - Merger rates, EM counterparts still uncertain.

Breakdown of Gaussian approx.: variation of D_L



Breakdown of Gaussian approx.: variation of D_L

