

Direct CP -asymmetry in $D^0 \rightarrow \pi^+ \pi^-$, $K^+ K^-$: a QCD-based estimate

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(with Alexey Petrov, *work in progress*)



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Outline

- The direct CP asymmetry in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$
- The "penguin" hadronic matrix elements determining the magnitude of $a_{dir}^{CP}(\pi^+\pi^-)$ and $a_{dir}^{CP}(K^+K^-)$
- estimating the "pengiuns" with QCD light-cone sum rules \oplus duality; the method used before for $B \rightarrow \pi\pi$ decays
- some preliminary results

Direct CP asymmetry

- The width:

$$\Gamma(D \rightarrow P^+ P^-) = \frac{p_{D \rightarrow PP}^*}{8\pi m_D^2} |A(D \rightarrow P^+ P^-)|^2, \quad P = \pi, K.$$

$p_{D \rightarrow PP}^*$ - the decay 3-momentum in the D rest frame

- The asymmetry

$$a_{CP}^{dir}(P^+ P^-) = \frac{\Gamma(D^0 \rightarrow P^+ P^-) - \Gamma(\bar{D}^0 \rightarrow P^+ P^-)}{\Gamma(D^0 \rightarrow P^+ P^-) + \Gamma(\bar{D}^0 \rightarrow P^+ P^-)},$$

- The most recent LHCb update:

$$\Delta a_{CP}^{dir} = a_{CP}^{dir}(K^+ K^-) - a_{CP}^{dir}(\pi^+ \pi^-) = (-0.061 \pm 0.076)\%.$$

the indirect components to a large extent are independent of the final state and cancel in this difference

R. Aaij et al. [LHCb Collaboration], PRL 116, 191601 (2016)

- Can we make a quantitative estimate of this asymmetry in SM?
how small it should be in SM?

Single Cabibbo-suppressed (SCS) decays

- the effective Hamiltonian:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_d (c_1 O_1^d + c_2 O_2^d) + \lambda_s (c_1 O_1^s + c_2 O_2^s) - \lambda_b \sum_{i=3, \dots, 6, 8g} c_i O_i \right\},$$

$$O_1^d = (\bar{u} \Gamma_\mu d) (\bar{d} \Gamma^\mu c), \quad O_2^d = (\bar{d} \Gamma_\mu d) (\bar{u} \Gamma^\mu c) \xrightarrow[\bar{d} \rightarrow \bar{s}]{} O_1^s, \quad O_2^s$$

$$\lambda_D = V_{uD} V_{cD}^*, \quad (D = d, s, b), \quad \lambda_s \simeq -\lambda_d, \quad \lambda_b \ll \lambda_{s,d}$$

- the CKM unitarity in SM:

$$\sum_{D=d,s,b} \lambda_D = 0, \quad \text{or} \quad \lambda_d = -(\lambda_s + \lambda_b).$$

- we hereafter neglect $O_{i \leq 3}$ with $c_i \ll c_{1,2}$

Decomposition of decay amplitudes

- separating the contributions of $O_{1,2}^d$ and $O_{1,2}^s$ operators

$$A(D^0 \rightarrow \pi^+ \pi^-) = \lambda_d \langle \pi^+ \pi^- | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^+ \pi^- | \mathcal{O}^s | D^0 \rangle,$$
$$A(D^0 \rightarrow K^+ K^-) = \lambda_s \langle K^+ K^- | \mathcal{O}^s | D^0 \rangle + \lambda_d \langle K^+ K^- | \mathcal{O}^d | D^0 \rangle,$$

using a compact notation:

$$\mathcal{O}^D \equiv \frac{G_F}{\sqrt{2}} \sum_{i=1,2} c_i O_i^D, \quad (D = d, s).$$

- replacing $\lambda_d = -(\lambda_s + \lambda_b)$
- "penguin" type amplitudes - the central object of our interest

$$\mathcal{P}_{\pi\pi}^s = \langle \pi^+ \pi^- | \mathcal{O}^s | D^0 \rangle, \quad \mathcal{P}_{KK}^d = \langle K^+ K^- | \mathcal{O}^d | D^0 \rangle,$$

"penguin" indicates that the operator contains a quark-antiquark pair not belonging to the valence content of final state, otherwise no relation to "topological" diagrams

Decomposition of decay amplitudes

- separating the $O(\lambda_b)$ contribution with CP-phase

$$A(D^0 \rightarrow \pi^+ \pi^-) = -\lambda_s \mathcal{A}_{\pi\pi} \left\{ 1 + \frac{\lambda_b}{\lambda_s} \left(1 + r_\pi \exp(i\delta_\pi) \right) \right\},$$

$$A(D^0 \rightarrow K^+ K^-) = \lambda_s \mathcal{A}_{KK} \left\{ 1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K) \right\},$$

the notation:

$$\mathcal{A}_{\pi\pi} = \langle \pi^+ \pi^- | \mathcal{O}^d | D^0 \rangle - \langle \pi^+ \pi^- | \mathcal{O}^s | D^0 \rangle,$$

$$\mathcal{A}_{KK} = \langle K^+ K^- | \mathcal{O}^s | D^0 \rangle - \langle K^+ K^- | \mathcal{O}^d | D^0 \rangle,$$

$$r_\pi = \left| \frac{\mathcal{P}_{\pi\pi}^s}{\mathcal{A}_{\pi\pi}} \right|, \quad r_K = \left| \frac{\mathcal{P}_{KK}^d}{\mathcal{A}_{KK}} \right|, \quad \delta_{\pi(K)} = \arg[\mathcal{P}_{\pi\pi(KK)}^{s(d)}] - \arg[\mathcal{A}_{\pi\pi(KK)}]$$

- to a good approximation

$$-\lambda_s \mathcal{A}_{\pi\pi} \simeq A(D^0 \rightarrow \pi^+ \pi^-), \quad \lambda_s \mathcal{A}_{KK} \simeq A(D^0 \rightarrow K^+ K^-)$$

The direct CP -asymmetry

- In terms of the parameters entering the decomposition:

$$a_{CP}^{dir}(K^+ K^-) = \frac{-2r_b r_K \sin \delta_K \sin \gamma}{1 - 2r_b r_K \cos \gamma \cos \delta_K + r_b^2 r_K^2},$$

$$a_{CP}^{dir}(\pi^+ \pi^-) = \frac{2r_b r_\pi \sin \delta_\pi \sin \gamma}{1 + 2r_b \cos \gamma (1 + r_\pi \cos \delta_\pi) + r_b^2 (1 + 2r_\pi \cos \delta_\pi + r_\pi^2)},$$

- the CKM elements involved

$$\frac{\lambda_b}{\lambda_s} \equiv r_b e^{-i\gamma}, \quad r_b = \left| \frac{V_{ub} V_{cb}^*}{V_{us} V_{cs}^*} \right|.$$

- the "clean" observable (after time-integration)

$$\begin{aligned} \Delta a_{CP}^{dir} &= a_{CP}^{dir}(K^+ K^-) - a_{CP}^{dir}(\pi^+ \pi^-) \\ &= -2r_b \sin \gamma (r_K \sin \delta_K + r_\pi \sin \delta_\pi) + O(r_b^2). \end{aligned}$$

- a QCD-based calculation of $\mathcal{P}_{\pi\pi}^s$ and \mathcal{P}_{KK}^d
- combined with $\mathcal{A}_{\pi\pi}$ and \mathcal{A}_{KK} extracted from experiment
⇒ an estimate of r_π and r_K

Calculation of the "penguin" hadronic matrix element

- The method employing QCD Light-Cone Sum Rules (LCSR) used earlier for the $B \rightarrow \pi\pi$ decays:

AK, Nucl. Phys. B 605 (2001) 558 [hep-ph/0012271];

AK, T. Mannel and B. Melic, Phys. Lett. B 571 (2003) 75 [hep-ph/0304179];

AK, T. Mannel, M. Melcher and B. Melic, Phys. Rev. D 72 (2005) 094012 [hep-ph/0509049].

- in gross features reproduce the QCD factorization results for $B \rightarrow \pi\pi$ adding nontrivial soft-gluon contributions
- reproduce the magnitudes of branching fractions, also for penguin dominated $B \rightarrow K\pi$ modes

M.Jung, AK, B.Melic, unpublished, work in progress

but I have the same problem as QCDF to reproduce the strong phases to fit in measured CP-violation

Some details of the calculation

- the correlation function for $D \rightarrow \pi^+ \pi^-$ case

$$F_\alpha(p, q, k) = i^2 \int d^4x e^{-i(p-q)x} \int d^4y e^{-i(p-k)y} \langle 0 | T\{j_{\alpha 5}^{(\pi)}(y) O_{1,2}^s(0) j_5^{(D)}(x)\} | \pi^+(q) \rangle$$

- sorting out the operators:

$$c_1 O_1^s + c_2 O_2^s = 2c_1 \tilde{O}_2^s + \left(\frac{c_1}{3} + c_2 \right) O_2^s,$$

- the colour-octet operator provides dominant contribution

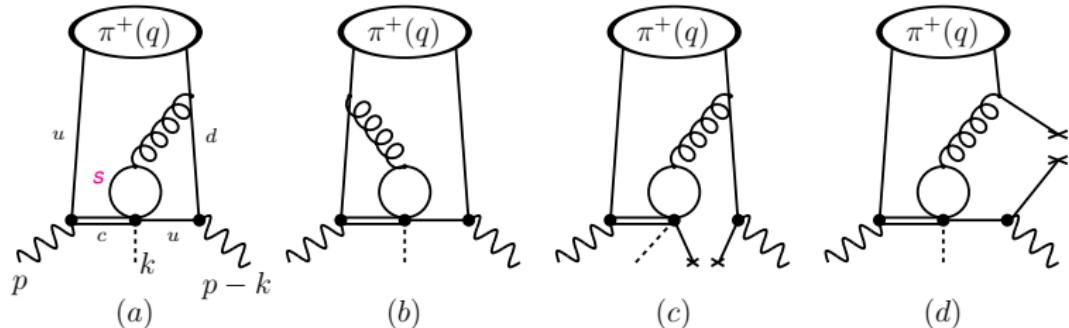
$$\tilde{O}_2^s = \left(\bar{s} \Gamma_\mu \frac{\lambda^a}{2} s \right) \left(\bar{u} \Gamma^\mu \frac{\lambda^a}{2} c \right),$$

- the hadronic matrix element entering r_π :

$$\mathcal{P}_{\pi\pi}^s \simeq 2c_1 \frac{G_F}{\sqrt{2}} \langle \pi^+ \pi^- | \tilde{O}_2^s | D^0 \rangle.$$

Some details of the calculation

- the dominant diagrams for $D \rightarrow \pi^+ \pi^-$ case:



- The method, adapted to $D \rightarrow PP$ ($D = \pi, K$):
 - calculate a series of OPE diagrams in terms of light-cone distribution amplitude of pion(kaon)
 - interpolate the second pion(kaon) and D -meson by quark currents.
 - switch from the spacelike region of P^2 (artificial 4-momentum $k \neq 0$) the final state invariant mass and analytically continue to $P^2 = m_D^2$, relying on the local quark-hadron duality

The light-cone sum rule

$$\begin{aligned}
\langle \pi^+ \pi^- | \tilde{O}_2^s | D^0 \rangle = & -i \frac{\alpha_s C_F m_c^2}{8\pi^3 m_D^2 f_D} \left[\int_0^{s_0^\pi} ds e^{-s/M_1^2} \int_{u_0^D}^1 \frac{du}{u} e^{\left(m_D^2 - \frac{m_c^2}{u} \right) / M_2^2} \right. \\
& \times \left\{ P^2 \int_0^1 dz I(z u P^2, m_s^2) \left(z(1-z) \varphi_\pi(u) \right. \right. \\
& + (1-z) \frac{\mu_\pi}{2m_c} \left[\left(2z + \frac{m_c^2}{u P^2} \right) u \varphi_p(u) + \frac{1}{3} \left(2z - \frac{m_c^2}{u P^2} \right) \left(\varphi_\sigma(u) - \frac{u \varphi'_\sigma(u)}{2} \right) \right] \left. \right] \\
& - \frac{\mu_\pi m_c}{4} \int_0^1 dz I(-z \bar{u} m_c^2/u, m_s^2) \frac{\bar{u}^2}{u} \left[\left(1 + \frac{3m_c^2}{u P^2} \right) \varphi_p(1) + \left(1 - \frac{5m_c^2}{u P^2} \right) \frac{\varphi'_\sigma(1)}{6} \right] \left. \right\} \\
& + \frac{2\pi^2}{3} m_c (-\langle \bar{q} q \rangle) \int_{u_0^D}^1 \frac{du}{u^2} e^{\left(m_D^2 - \frac{m_c^2}{u} \right) / M_2^2} \left\{ I(u P^2, m_s^2) \left(2\varphi_\pi(u) + \frac{\mu_\pi}{m_c} \left[3u \varphi_p(u) \right. \right. \right. \\
& \left. \left. \left. + \frac{\varphi_\sigma(u)}{3} - \frac{u \varphi'_\sigma(u)}{6} \right] \right) \right\} \Big]_{P^2 \rightarrow m_D^2} ,
\end{aligned}$$

the loop integral: $I(\ell^2, m_q^2) = \frac{1}{6} + \int_0^1 dx x(1-x) \ln \left[\frac{m_q^2 - x(1-x)\ell^2}{\mu^2} \right]$.

Preliminary numerical estimates

- only for the $D \rightarrow \pi^+ \pi^-$ mode, $D \rightarrow K^+ K^-$ in progress
- LCSR input: quark masses, pion, kaon DAs, Borel scales, effective thresholds from the LCSR calculation of $D \rightarrow \pi$, $D \rightarrow K$ and pion form factor

- the hadronic matrix element calculated from the sum rule

$$\langle \pi^+ \pi^- | \tilde{O}_2^s | D^0 \rangle = (3.2 \pm 0.3 \pm ..) \times 10^{-2} \exp[i(98 \pm 2 \pm ..)^o] \text{GeV}^3$$

not all parametric uncertainties analysed yet

- converting to the estimate of the penguin amplitude:
 $c_1(\mu_{\text{def}}) = 1.25$

$$|\mathcal{P}_{\pi\pi}^s| = (6.5 \pm 0.5) \times 10^{-7}$$

- extracting the absolute value of $\mathcal{A}_{\pi\pi}$ from exp. :

$$BR(D^0 \rightarrow \pi^+ \pi^-) = (1.42 \pm 0.025) \times 10^{-3} \text{ [PDG]}$$

$$|\mathcal{A}_{\pi\pi}| \simeq \frac{1}{\lambda_s} |\mathcal{A}(D^0 \rightarrow \pi^+ \pi^-)| = \frac{4.65 \times 10^{-7}}{0.219098} = 21.22 \times 10^{-7}$$

Comparing with experiment

- the CKM averages [PDG, global CKM fit] :
 $|V_{ub}| = 0.00357, |V_{cb}| = 0.0411, |V_{us}| = 0.22506, |V_{cs}| = 0.97351, \gamma = 73.2^\circ$

$$r_b \sin \gamma = 0.64 \times 10^{-3}.$$

- from the measured difference of CP asymmetries

$$[\Delta a_{CP}^{dir}]_{LHCb} = (-0.061 \pm 0.076)\% = -2r_b \sin \gamma (r_K \sin \delta_K + r_\pi \sin \delta_\pi)$$

$$\Rightarrow r_K \sin \delta_K + r_\pi \sin \delta_\pi = (0.48 \pm 0.59)$$

- we predict (*preliminary!*)

$$r_\pi = \frac{|\mathcal{P}_{\pi\pi}^s|}{|\mathcal{A}_{\pi\pi}|} = 0.309 \pm 0.03 \pm \dots \quad r_K = \text{(to be obtained soon)}$$

- assuming (temporarily !) the $SU(3)_{fl}$ limit:

$$|(r_\pi \sin \delta_\pi + r_K \sin \delta_K)| \sim 2|r_\pi \sin \delta_\pi| \leq 0.62 \pm 10\% \pm \dots \% \pm \Delta_{SU(3)}\%$$

- this limit consistent with the current LHCb result

Summary and outlook

- the magnitude of direct CP-violation in $D \rightarrow \pi^+ \pi^-$ and $D \rightarrow K^+ K^-$ can be predicted and constrained calculating the relevant hadronic matrix elements from LCSR
can lattice QCD do that ?
- no topological amplitude decomposition is used,
but ! the OPE hierarchy sorts out the leading penguin-loop diagrams
- the strong phase difference is not yet reliably accessible
need to calculate the full \mathcal{A}_{PP}
- the issue of duality: scalar resonances influencing hadronic matrix elements ?
one may argue that the ratio $\mathcal{P}_{\pi\pi}^S$ and $\mathcal{A}_{\pi\pi}$ is not influenced
- a complete analysis for $D \rightarrow \pi^+ \pi^-$ and $D \rightarrow K^+ K^-$, including SU(3)-violating effects and parametrical error analysis is in progress
- our preliminary results indicate that the current LHCb central value for the difference of CP-asymmetries is at the expected level in SM.