The state-of-the-art of the Hadronic Uncertainties in $B \to K^* \mu\mu$

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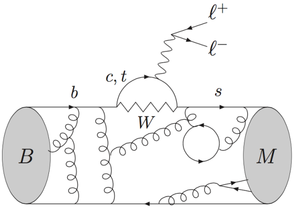
Based on **1701.08672 JHEP (2017)**

Outline

1. Introduction
2. Updated global fit results
3. Anatomy of power corrections
4. Long-distance $c\bar{c}$ loops from fits to $B \to K^* \mu\mu$ data
5. Conclusions

Theoretical framework



$\mathcal{A} \sim C_i$ (short dist.)

$\times$ Hadronic Matrix Elements (long dist.)

$b \to s\gamma^{(*)}$ Effective Hamiltonian

$$\mathcal{H}^{\text{SM}}_{\Delta F=1} \propto V^*_{ts} V_{tb} \sum_i C_i \mathcal{O}_i$$

- $\mathcal{O}_7 = \dfrac{\alpha}{4\pi} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu}$
- $\mathcal{O}_9 = \dfrac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$
- $\mathcal{O}_{10} = \dfrac{\alpha}{16\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$

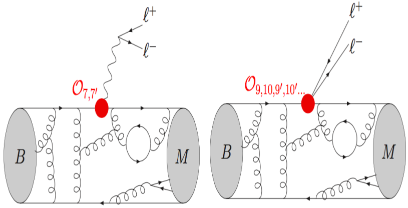
$C_7^{\text{SM}}(\mu_b) = -0.29 \qquad C_9^{\text{SM}}(\mu_b) = 4.1$

$C_{10}^{\text{SM}}(\mu_b) = -4.3 \qquad (\mu_b = m_b)$

$\Rightarrow$ In this picture, New Physics (NP) effects can enter through two mechanisms:

- Extra contributions to the WCs.
- Additional effective operators: $\mathcal{O}'_i$, $\mathcal{O}_S$, $\mathcal{O}_P$, $\mathcal{O}_T$,...

Theoretical framework



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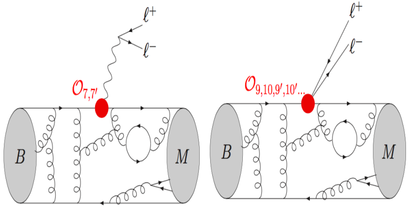

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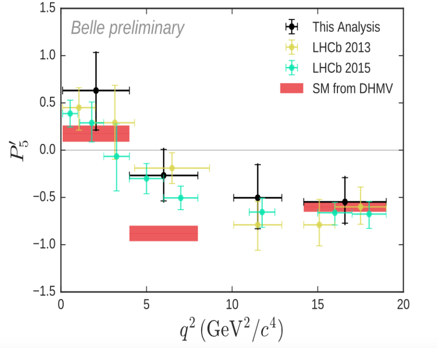
$\Rightarrow$ QCD factorisation (QCDF): Large recoil symmetries ($E_{K^*} \to \infty$ and HQL),

$$\{V, A_0, A_1, A_2, T_1, T_2, T_3\} \sim \{\xi_\perp, \xi_\|\}$$

up to $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\Lambda/m_B)$.

The P_5' anomaly

$b \to s\ell\ell$ driven processes have provided some interesting anomalies during the recent years.



P_5' was proposed in DMRV, JHEP 1301(2013)048

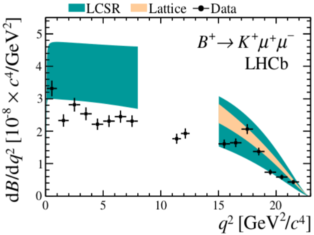
$$P_5' = \sqrt{2}\frac{\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*})}{\sqrt{|A_0|^2(|A_\perp|^2 + |A_\parallel|^2)}}$$

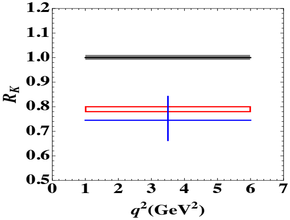
Optimized Obs.: Soft form factor ($\xi_\perp$) cancellation at LO.

- 2013: 1fb^{-1} dataset LHCb found 3.7σ
- 2015: 3fb^{-1} dataset LHCb found 3σ in 2 bins.
- Belle confirmed it in a bin [4,8] few months ago.

Other tensions beyond P_5'

	bin	SM	EXP	Pull
$10^7 \times \mathrm{BR}(B_s \to \phi\mu^+\mu^-)$	[2,5]	1.55 ± 0.33	0.77 ± 0.14	**+2.2**
	[5,8]	1.89 ± 0.40	0.96 ± 0.15	**+2.2**
	[15,18.8]	2.20 ± 0.17	1.62 ± 0.20	**+2.2**
$10^7 \times \mathrm{BR}(B^0 \to K^0\mu^+\mu^-)$	[15,19]	0.91 ± 0.12	0.67 ± 0.12	+1.4
$10^7 \times \mathrm{BR}(B^0 \to K^{*0}\mu^+\mu^-)$	[16,19]	1.66 ± 0.15	1.23 ± 0.20	+1.7
$10^7 \times \mathrm{BR}(B^+ \to K^{*+}\mu^+\mu^-)$	[15,19]	2.59 ± 0.25	1.60 ± 0.32	**+2.5**





- Deviations in $BR(B_s \to \phi\mu\mu)$.
- Several systematic low-recoil small tensions in BR_μ.
- $BR(B \to K\mu\mu)$ small compared to SM predictions.
- $R_K = 0.745^{+0.090}_{-0.074} \pm 0.036$
  - $\Rightarrow$ SM Prediction: $R_K = 1$ (2.6σ).
  - $\Rightarrow$ Possible lepton-flavour universality violation.

Updated fits

Includes updated $BR(B \to K^* \mu^+ \mu^-)$ + corrected BSZ for $B_s \to \phi \mu^+ \mu^-$.
$P_5'^{\mu \text{BELLE}}$ adds from $+0.1$ to $+0.3\sigma$.

Coefficient	Best fit	1σ	Pull_{SM}	p-value (%)
$\mathcal{C}_7^{\text{NP}}$	-0.02	$[-0.04, -0.00]$	1.1	13.0
$\mathcal{C}_9^{\text{NP}}$	-1.05	$[-1.25, -0.85]$	4.7	61.0
$\mathcal{C}_{10}^{\text{NP}}$	0.55	$[0.34, 0.77]$	2.8	24.0
$\mathcal{C}_{7'}^{\text{NP}}$	0.02	$[-0.00, 0.04]$	0.9	13.0
$\mathcal{C}_{9'}^{\text{NP}}$	0.06	$[-0.18, 0.30]$	0.3	12.0
$\mathcal{C}_{10'}^{\text{NP}}$	-0.03	$[-0.20, 0.14]$	0.2	12.0
$\mathcal{C}_9^{\text{NP}} = \mathcal{C}_{10}^{\text{NP}}$	-0.18	$[-0.36, 0.02]$	0.9	13.0
$\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}}$	-0.59	$[-0.74, -0.44]$	4.3	51.0
$\mathcal{C}_{9'}^{\text{NP}} = -\mathcal{C}_{10'}^{\text{NP}}$	0.03	$[-0.08, 0.13]$	0.2	12.0
$\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}}$	-1.00	$[-1.20, -0.78]$	4.4	54.0
$\mathcal{C}_9^{\text{NP}} = -\mathcal{C}_{10}^{\text{NP}} = -\mathcal{C}_{9'}^{\text{NP}} = -\mathcal{C}_{10'}^{\text{NP}}$	-0.61	$[-0.45, -0.45]$	4.3	50.0

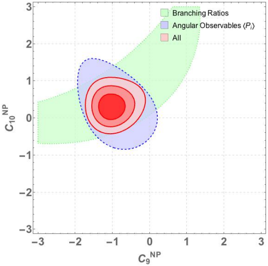
Global fit to ~ 100 obs. (radiative + $b \to s\ell^+\ell^-$, $\ell = e, \mu$)

All deviations add up constructively

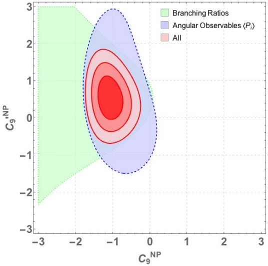
- New Physics contribution to $C_{9,\mu}^{\text{NP}}$=-1.1 alleviates all tensions.
- **At the moment**, NP contributions to the rest of Wilson coefficient are still not significantly different from zero.

2D fits

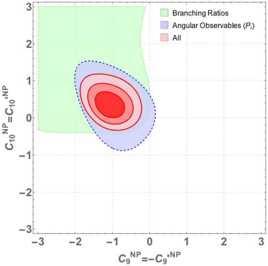
Fitting to data hypothesis where two Wilson coefficients are allowed to vary freely, we obtain several scenarios with pull-SM beyond 4σ:



$(C_9,\ C_{10})$



$(C_9,\ C_9')$



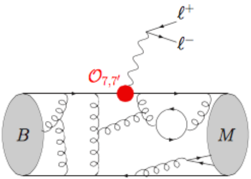
$(C_9 = -C_9',\ C_{10} = C_{10}')$

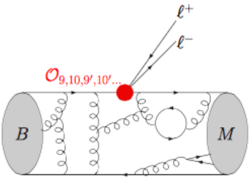
$\Rightarrow$ Both the confidence regions drawn from BR and $\langle P_i \rangle$ data, suggest $C_9^{\rm NP} \simeq -1$ in all the most significant scenarios.

Hadronic corrections: factorisable and non-factorisable

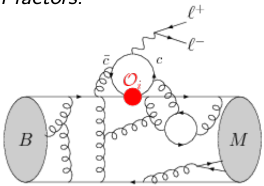
Theory predictions receive different types of QCD corrections.

- Factorisable Corrections: *corrections that **can** be absorbed into the definition of the (full) form factors.*





- Non-factorisable Corrections: *corrections that **cannot** be absorbed into the definition of the (full) form factors.*



CDHM17: *"We provide detailed arguments showing that factorisable power corrections cannot account for the observed anomalies and that an explanation through long-distance charm contributions is disfavoured."*

Improved QCDF

Improved QCDF (iQCDF) Approach: General decomposition of a full form factor (FF)

$$F^{\text{Full}}(q^2) = F^{\infty}(\xi_\perp(q^2), \xi_\parallel(q^2)) + \Delta F^{\alpha_s}(q^2) + \Delta F^{\Lambda}(q^2)$$

where F stands for any form factor, either from the helicity or transversity basis.

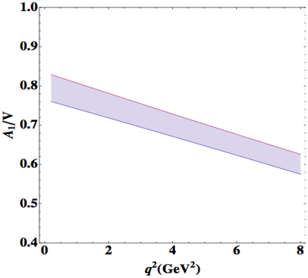
- Large recoil symmetries: low-q^2 and at LO in α_s and Λ/m_B
  $\Rightarrow$ Soft FF $\{\xi_\perp, \xi_\parallel\}$ (KMPW or any other FF parametrization)
  $\Rightarrow$ **Dominant correlations** automatically taken into account
- $\mathcal{O}(\alpha_s)$ corrections $\Rightarrow$ QCDF
- $\mathcal{O}(\Lambda/m_B)$ corrections $\Rightarrow$ **cannot** be explicitly computed within QCDF

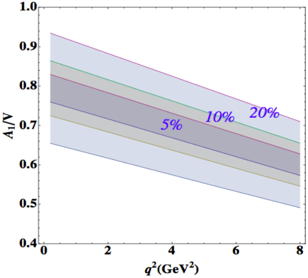
Parametrization of ΔF^{Λ} [JC12]

$$\Delta F^{\Lambda}(q^2) = a_F + b_F \frac{q^2}{m_B^2} + c_F \frac{q^4}{m_B^4} + \dots$$

Improved QCDF (vs full FF approach)

- How to estimate ΔF^{Λ}?
  $\Rightarrow$ Central values for a_F, b_F, c_F from **fit to full LCSR FF**.
  $\Rightarrow$ Error estimate: assign **uncorrelated** $\sim 100\%$ errors to $a_F, b_F, c_F = \mathcal{O}(\Lambda/m_B) \times F = 10\% \times F$
- Is our estimation of errors conservative?
  FF ratio A_1/V (that controls P_5'): BSZ (including correlations) vs iQCDF for different size of power corrections.





Already a 5% power corrections (right) reproduces the BSZ full FF approach errors (left).

Scheme dependence

- Different possibilities for what to take as input for the two independent soft FFs $\{\xi_\perp, \xi_\|\}$

  e.g. scheme 1 [DHMV] $\{V, a_1 A_1 + a_2 A_2\}$ or scheme 2 [JC] $\{T_1, A_0\}$ or...

  $\Rightarrow$ choice defines **input scheme**.
- Observables are scheme independent **if and only if** all the correlations among FF are included.

  $\Rightarrow$ also correlations among $\Delta a_F, \Delta b_F, \ldots$!

  $\Rightarrow$ Uncorrelated errors in $\Delta F^\Lambda \Rightarrow$ scheme dependence at $\mathcal{O}(\Lambda/m_B)$.
- Input FF **do not receive** power corrections

  $\Rightarrow$ Appropiate scheme choices reduce the impact of ΔF^Λ.

  $\Rightarrow$ Non-optimal schemes **can artificially inflate** the errors due to ΔF^Λ.

An illustrative example: $BR(B \to K^*\gamma)$

- How a non-optimal scheme can artificially inflate the errors?

  $\Rightarrow$ Take $BR(B \to K^*\gamma)$ as an example:

  $$BR(B \to K^*\gamma) \propto T_1(0)$$

  $\Rightarrow$ **Natural choice**: Choose an scheme where T_1 is used as input,

  $$T_1(0) = T_1^{\text{LCSR}}(0) \pm \Delta T_1^{\text{LCSR}}(0) \;\Rightarrow\; \Delta BR(B \to K^*\gamma) \propto \Delta T_1^{\text{LCSR}}(0)$$

  $\Rightarrow$ **"Wrong" choice**: Use any other FF related to T_1 as input (e.g. T_2),

  $$T_1(0) = \left(T_2^{\text{LCSR}}(0) + a_{T_1}\right) \pm \left(\Delta T_1^{\text{LCSR}}(0) + \Delta a_{T_1}\right)$$
  $$\Rightarrow\; \Delta BR(B \to K^*\gamma) \propto \Delta T_1^{\text{LCSR}}(0) + \Delta a_{T_1}$$

- **Unnatural scheme choices** generate **extra** contributions in error computations.

Scheme dependence of P_5'

Explicit analytic formulae for the power corrections to P_5' [CDHM]:

- Helicity basis,

$$P_5' = P_5'|_\infty \left(1 + \frac{2a_{V_-} - 2a_{T_-}}{\xi_\perp} \frac{C_7^{\rm eff}(C_{9,\perp}C_{9,\parallel} - C_{10}^2)}{(C_{9,\perp} + C_{9,\parallel})(C_{9,\perp}^2 + C_{10}^2)} \frac{m_b m_B}{q^2} \right.$$
$$\left. + \frac{2a_{V_0} - 2a_{T_0}}{\tilde{\xi}_\parallel} \frac{C_7^{\rm eff}(C_{9,\perp}C_{9,\parallel} - C_{10}^2)}{(C_{9,\perp} + C_{9,\parallel})(C_{9,\parallel}^2 + C_{10}^2)} \frac{m_b}{m_B} - \frac{2a_{V_+}}{\xi_\perp} \frac{C_{9,\parallel}}{C_{9,\perp} + C_{9,\parallel}} + \ldots \right)$$

$\Rightarrow$ We recovered the expression in JC12 + an additional term

- Transversity basis,

$$P_5' = P_5'|_\infty \left(1 + \frac{a_{A_1} + a_V - 2a_{T_1}}{\xi_\perp} \frac{C_7^{\rm eff}(C_{9,\perp}C_{9,\parallel} - C_{10}^2)}{(C_{9,\perp} + C_{9,\parallel})(C_{9,\perp}^2 + C_{10}^2)} \frac{m_b m_B}{q^2} \right.$$
$$\left. - \frac{a_{A_1} - a_V}{\xi_\perp} \frac{C_{9,\parallel}}{C_{9,\perp} + C_{9,\parallel}} - \frac{a_{T_1} - a_{T_3}}{\tilde{\xi}_\parallel} \frac{C_7^{\rm eff}(C_{9,\perp}C_{9,\parallel} - C_{10}^2)}{(C_{9,\perp} + C_{9,\parallel})(C_{9,\parallel}^2 + C_{10}^2)} \frac{m_b}{m_{K^*}} + \ldots \right)$$

with $C_{9,\perp} = C_9^{\rm eff} + \frac{2m_b m_B}{q^2} C_7^{\rm eff}$ and $C_{9,\parallel} = C_9^{\rm eff} + \frac{2m_b}{m_B} C_7^{\rm eff}$

Scheme dependence of P_5'

- The FF ratio A_1/V dominates P_5',
  $\Rightarrow$ **Convenient**: scheme 1 [DHMV] $\{V, a_1 A_1 + a_2 A_2\}$
  $\Rightarrow$ **Inconvenient**: scheme 2 [JC] $\{T_1, A_0\}$
- Evaluating the expression for the power corrections to P_5' at $q^2 = 6\ \text{GeV}^2$ (around the anomaly),

$$P_5'(6\,\text{GeV}^2) = P_5'|_\infty(6\,\text{GeV}^2)\left(1+0.18\frac{a_{A_1}+a_V-2a_{T_1}}{\xi_\perp}-0.14\frac{a_{T_1}-a_{T_3}}{\tilde{\xi}_\parallel}-0.73\frac{a_{A_1}-a_V}{\xi_\perp}\right)$$

$\Rightarrow$ Scheme 1: $P_5'(6\,\text{GeV}^2) \simeq P_5'|_\infty(6\,\text{GeV}^2)\left(1-0.73\frac{a_{A_1}}{\xi_\perp}\right) \Rightarrow$ reduced errors.

$\Rightarrow$ Scheme 2: $P_5'(6\,\text{GeV}^2) \simeq P_5'|_\infty(6\,\text{GeV}^2)\left(1-0.73\frac{a_{A_1}-a_V}{\xi_\perp}\right) \Rightarrow$ increased errors.

Correlations and scheme dependence of P_5'

Assessing the impact of the correlations among power corrections (PC) + scheme dependence,

$\boxed{1}$ PC Analysis

- $\Delta F^\Lambda = F \times \mathcal{O}(\Lambda/m_B) \sim 10\% \times F$
- **correlations** from large-recoil sym. $\Rightarrow \xi_{\perp,\parallel}$, ΔF^Λ uncorr.

$\boxed{2}$ PC Analysis

- ΔF^Λ from fit to LCSR [BSZ].
- **correlations** from large-recoil sym. $\Rightarrow \xi_{\perp,\parallel}$, ΔF^Λ uncorr.

$\boxed{3}$ PC Analysis

- ΔF^Λ from fit to LCSR [BSZ].
- **correlations** from LCSR [BSZ] $\Rightarrow \xi_{\perp,\parallel}$, ΔF^Λ corr.

$P_5'[4.0, 6.0]$	scheme 1 [CDHM]	scheme 2 [JC]
$\boxed{1}$	$-0.72 \pm \mathbf{0.05}$	$-0.72 \pm \mathbf{0.12}$
$\boxed{2}$	-0.72 ± 0.03	-0.72 ± 0.03
$\boxed{3}$	-0.72 ± 0.03	-0.72 ± 0.03
full BSZ	$-0.72 \pm \mathbf{0.03}$	

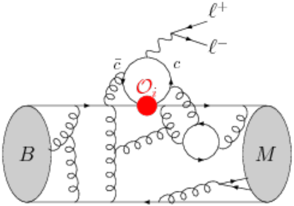
errors only from pc with BSZ form factors

Non-factorisable hadronic corrections

There are two different types of non-factorisable hadronic corrections

- α_s-corrections from hard gluon exchange ($\mathcal{O}_{1-6}$, $\mathcal{O}_8$ topologies) $\Rightarrow$ QCDF
- $\mathcal{O}(\Lambda/m_B)$ corrections involving $c\bar{c}$ loops
  - $\Rightarrow$ [KMPW] LCSR + dispersion relations (only th. calculation)
  - $\Rightarrow$ Non-factorisable $\mathcal{O}(\Lambda/m_B)$ power corrections (charm loops) yield q^2- and hadronic-dependent contributions with $\mathcal{O}_{7,9}$ structures that may mimic NP:

$$C_9^{\text{eff}\,i}(q^2) = C_{9\,\text{pert}}^{\text{eff SM}}(q^2) + C_9^{\text{NP}} + C_9^{c\bar{c}\,i}(q^2) \quad i = \perp, \|, 0$$



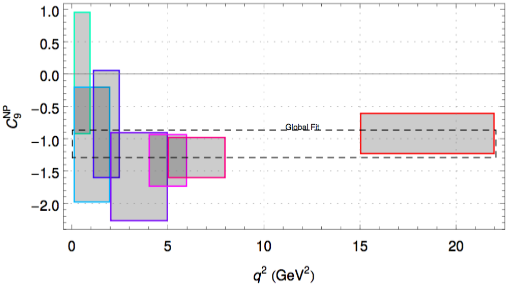
Disentangling $c\bar{c}$ loops from New Physics

NP and hadronic effects have different signatures on C_9:

- NP effects: universal and q^2-independent.
- Hadronic effects: transversity dependent and (most likely) q^2-depedent.

Testing the q^2 dependence of the contributions to C_9 by means of data,

- $C_9^{\rm NP}$ bin-by-bin fit to $b \to s\ell\ell$ data (assuming KMPW-like $C_9^{c\bar{c}\,i}(q^2)$):



$\Rightarrow$ Excellent agreement with a q^2-independent $C_9 \simeq -1$.

Fitting a charm-loop parametrization to data

Following *Ciuchini et al.*, we performed a fit of the charm loop contributions to data using a polinomial paramatrization,

$$A^0_{L,R} = A^0_{L,R}(Y(q^2)) + \frac{N}{q^2}\left(h_0^{(0)} + \frac{q^2}{1GeV^2}h_0^{(1)} + \frac{q^4}{1GeV^4}h_0^{(2)} + \frac{q^6}{1GeV^6}h_0^{(3)}\right)$$

- Non-zero $h_\lambda^{(2),(3)}$ $(\lambda = +,-,0)$ introduce q^2-dependent terms in C_9.
  $\Rightarrow$ Disclaimer: $C_{7,9}^{\rm NP}$ contribute to $h_i^{(2),(3),\dots} \Rightarrow C_i^{\rm NP} \times F(q^2)$.
- Frequentist fit of $h_\lambda^{(i)}$ to $B \to K^* \mu\mu$ data: using KMPW FF and without including any charm-loop estimate to $C_{7,9}$.
- Comparing hypothesis with increasing orders of the h_λ polinomials $(n = 0, 1, 2, 3)$, we conclude [CDHM]:
  $\Rightarrow$ Hypotheses with linear h_λ polinomials are the ones with better improvement of the fit.
  $\Rightarrow$ Setting $C_9^{\rm NP} = -1.1$ significantly improves the fit (already with $h_\lambda = 0$).

Conclusions

- Global fits: $C_{9,\mu}^{\text{NP}} = -1.1$ with a pull-SM of 4.7σ alleviates all the tensions.
- Analysis of the two main sources of hadronic uncertainties:
  $\Rightarrow$ Factorisable power corrections can artificially inflate the errors when considered as uncorrelated by means of non-optimal scheme choices.
  $\Rightarrow$ Non-factorisable $c\bar{c}$-loops: our tests do not show signs of large q^2 contributions to C_9.