Kaon Physics:
Theory Status

Based on work in collaboration with:

Andrzej Buras, Sebastian Jäger & Matthias Jamin [1507.06345]

Maria Cerda-Sevilla, Sebastian Jäger & Ahmet Kokulu [1611.08276]

[And based on older calculations with

Joachim Brod, Emanuel Stamou and Ulrich Haisch]

Current Trends in Flavour Physics
Institut Henri Poincare, Paris 30 March 2017

Martin Gorbahn

UNIVERSITY OF LIVERPOOL

LIVERPOOL THEORETICAL PHYSICS

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Kaon Physics:
Topics in Theory Calculations

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Current Trends in Flavour Physics
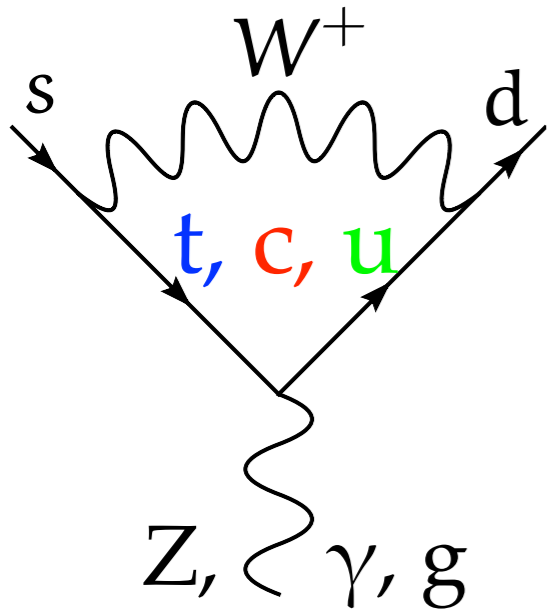## Institut Henri Poincare, Paris 30 March 2017

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CKM Factors in Kaon physics

Semi-leptonic decays (V_{us}): $\lambda = \mathcal{O}(0.2)$

$$V_{ij} = \mathcal{O} \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$
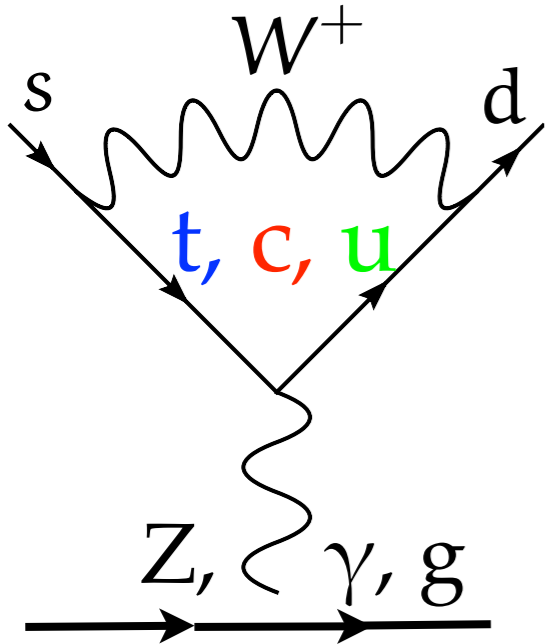
$$\mathrm{Im}V_{ts}^*V_{td} = -\mathrm{Im}V_{cs}^*V_{cd} = \mathcal{O}(\lambda^5) \qquad \mathrm{Im}V_{us}^*V_{ud} = 0$$

$$\mathrm{Re}V_{us}^*V_{ud} = -\mathrm{Re}V_{cs}^*V_{cd} = \mathcal{O}(\lambda^1) \qquad \mathrm{Re}V_{ts}^*V_{td} = \mathcal{O}(\lambda^5)$$

Kaon observables $\propto V_{ts}^*V_{td} \rightarrow$ suppressed in SM sensitive to flavour violating NP

Kaon observables $\propto V_{us}^*V_{ud}$ or $V_{cs}^*V_{cd} \rightarrow$ dominated by QCD, useful for extracting low energy constants

CKM Factors in Kaon physics

s W^+ d

$t,\ c,\ u$

$Z,\ \gamma,\ g$

Using the GIM mechanism,
we can eliminate either $V_{cs}^* V_{cd}$ or
$V_{us}^* V_{ud} \rightarrow -V_{cs}^* V_{cd} - V_{ts}^* V_{td}$

Z-Penguin and Boxes (high virtuality):
power expansion in: $A_c - A_u \propto 0 + O(m_c^2/M_W^2)$

γ/g-Penguin (momentum expansion + e.o.m.):
power expansion in: $A_c - A_u \propto O(\text{Log}(m_c^2/m_u^2))$

Content

Semileptonic decays: V_{us}, Lepton Flavour Universality, QCD

Leptonic decays: CP violation, Lepton Flavour Violation

Radiative decays: QCD
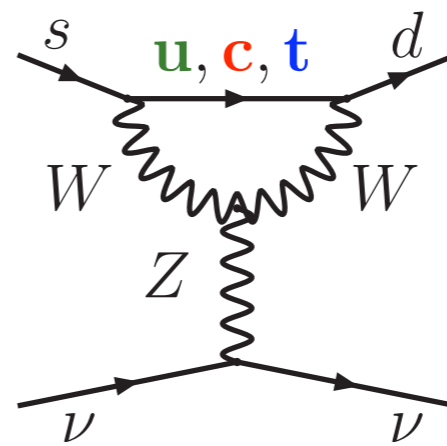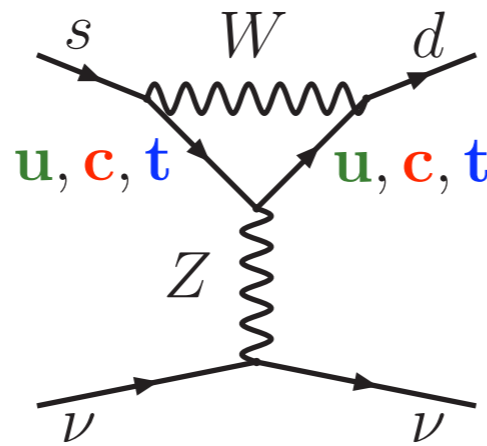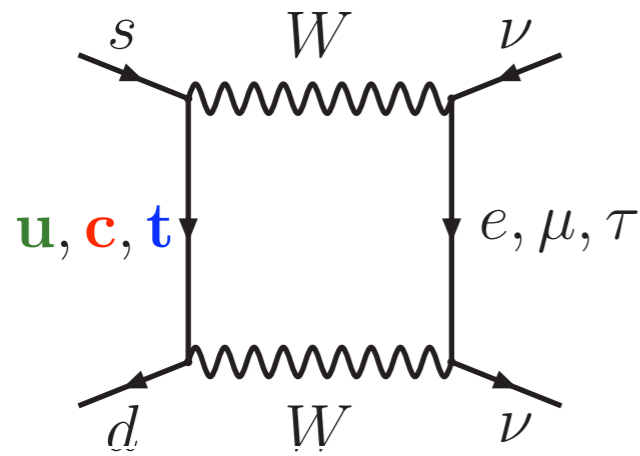
Rare decays: $K \to \pi\, l^+ l^-$ see talk by A. Jüttner

In this talk I will discuss:
1, $K \to \pi\, \bar{\nu}\, \nu$
2, ε_K
3, $\varepsilon'_K / \varepsilon_K$

$K \rightarrow \pi \, \bar{\upsilon} \, \upsilon$ at M_W
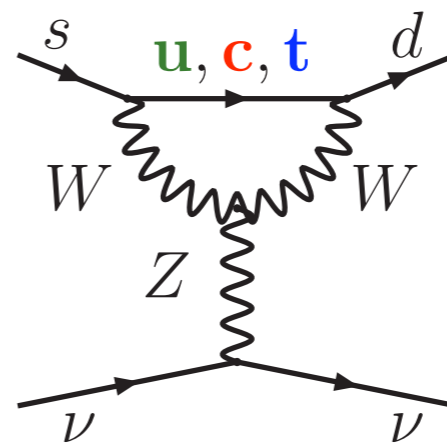

$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

$K \to \pi \, \bar{\upsilon} \, \upsilon$ at M_W
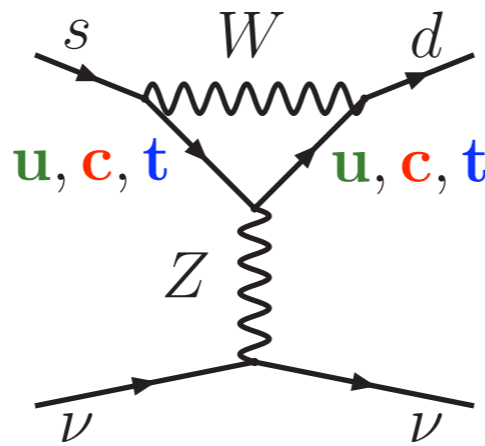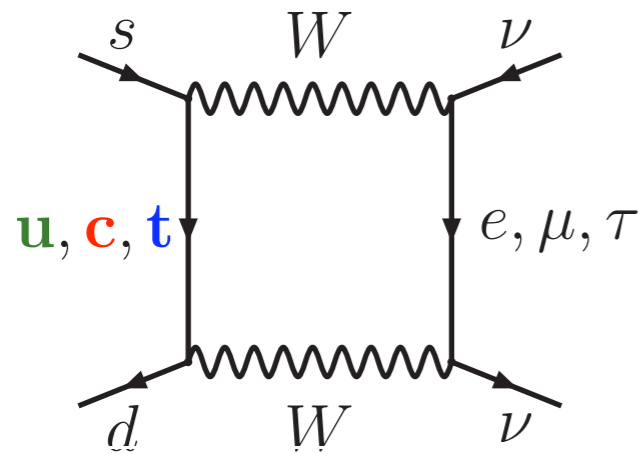
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Quadratic GIM: $\lambda^5 \dfrac{m_t^2}{M_W^2}$

Matching (NLO +EW):
[Misiak, Urban; Buras, Buchalla; Brod, MG, Stamou`11]

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

$K \to \pi \, \bar{\upsilon} \, \upsilon$ at M_W

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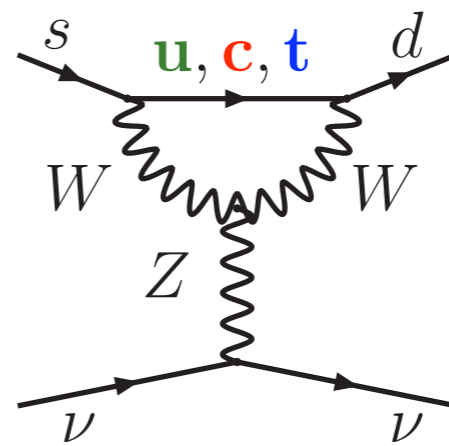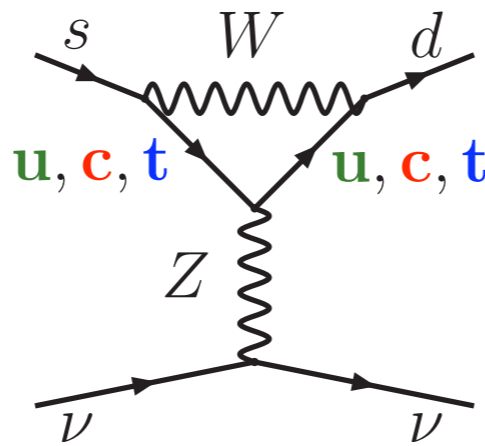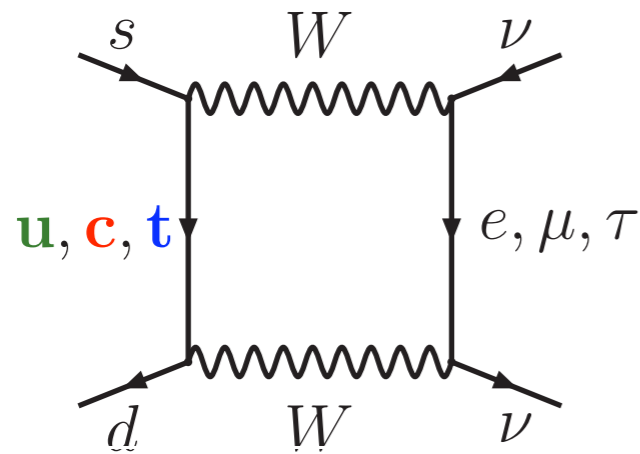
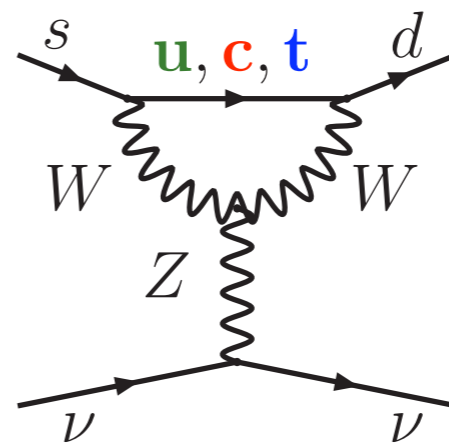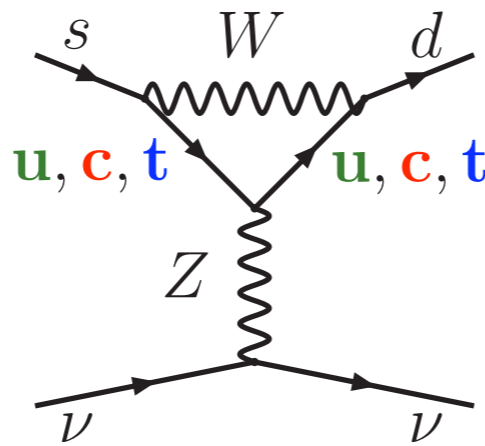Quadratic GIM: $\lambda^5 \dfrac{m_t^2}{M_W^2}$

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$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

Matrix element from K_{l3} decays
(Isospin symmetry: $K^+ \to \pi^0 \, e^+ \, \upsilon$)
[Mescia, Smith]

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K → π ῡ υ at M_W

$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

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After 2011 uncertainty at 1%

6

$K \rightarrow \pi \, \bar{\upsilon} \, \upsilon$ at M_W

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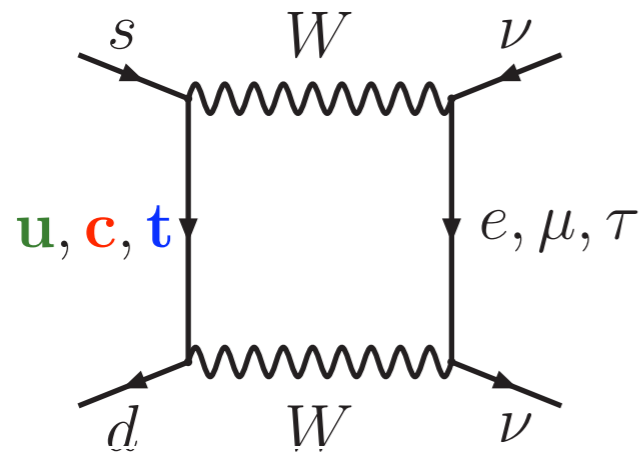Quadratic GIM: $\lambda^5 \dfrac{m_t^2}{M_W^2}$

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$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

For CP violating $K_L \rightarrow \pi^0 \, \bar{\upsilon} \, \upsilon$ only top contribution relevant. Clean theory and CKM suppression: NP sensitivity

$K^+ \rightarrow \pi^+ \bar{\nu} \nu$ at M_W

$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$
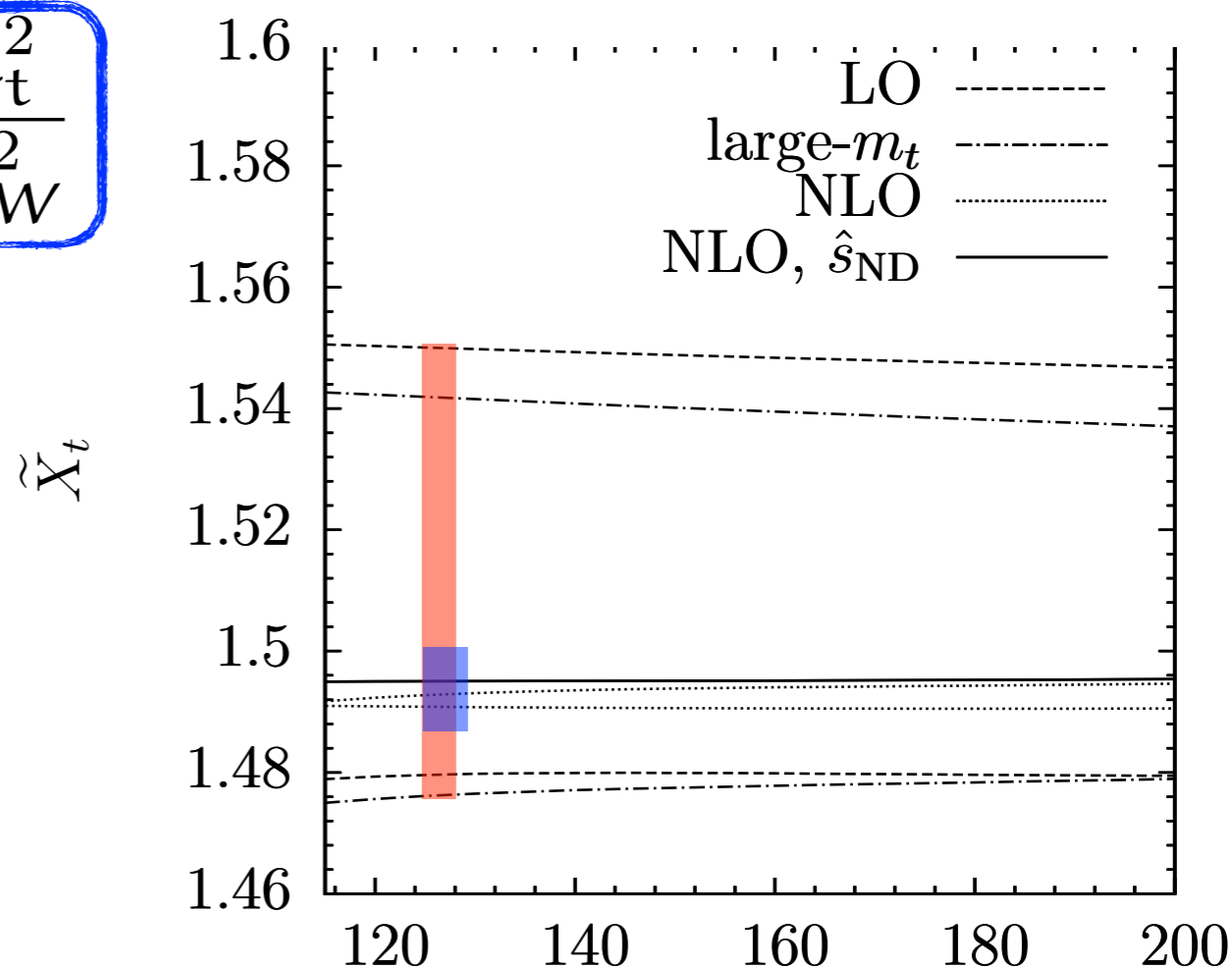
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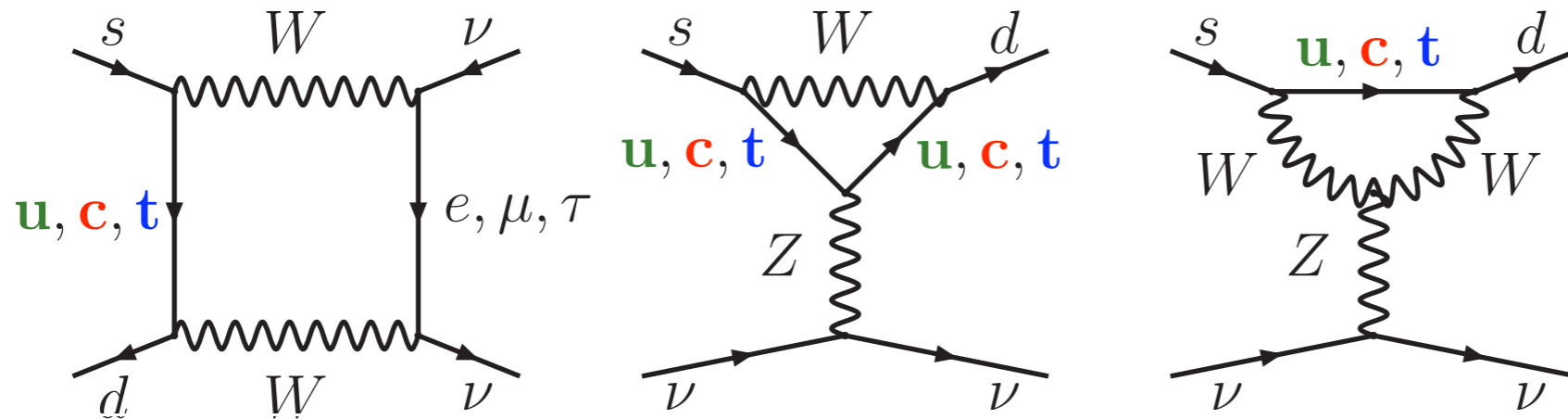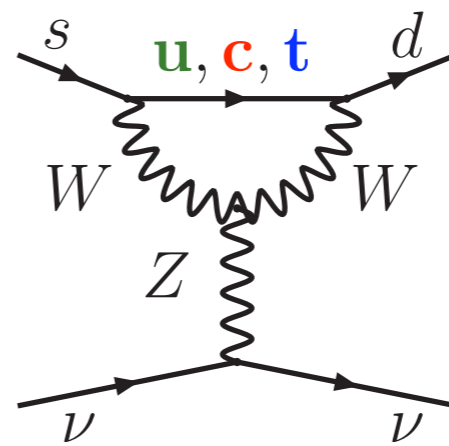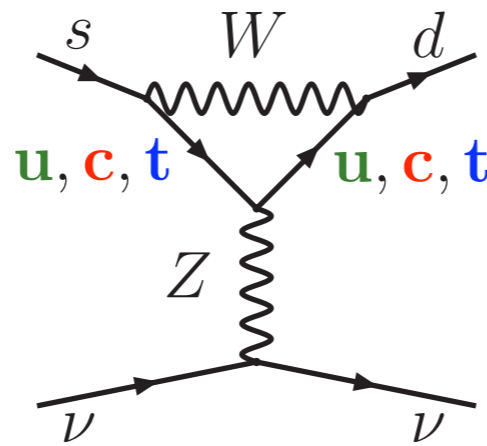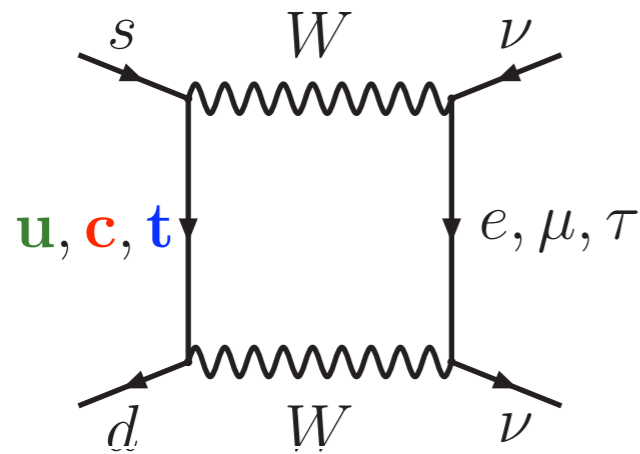[Misiak, Urban; Buras, Buchalla; Brod, MG, Stamou`11]

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

$K^+ \rightarrow \pi^+ \bar{\nu} \nu$ at M_W
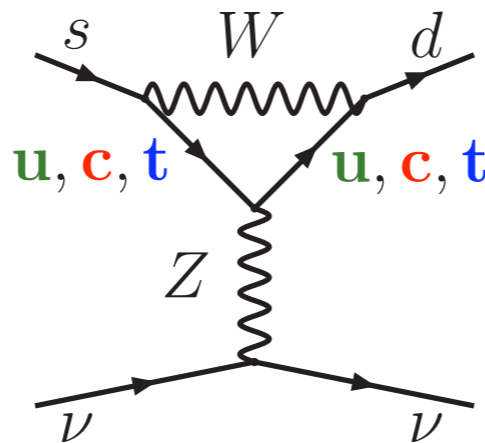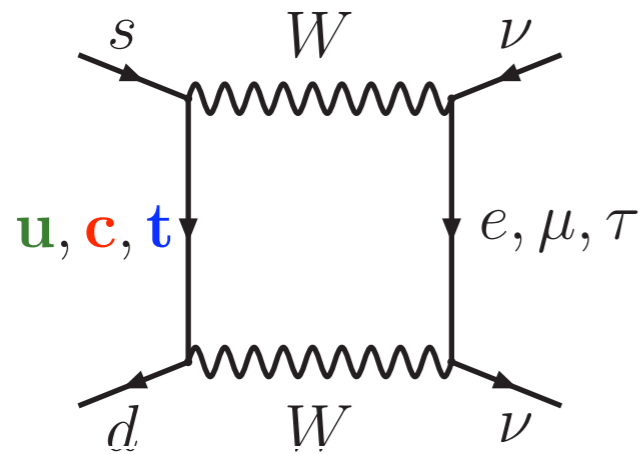
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Quadratic GIM: $\lambda^5 \dfrac{m_t^2}{M_W^2}$ $\lambda \dfrac{m_c^2}{M_W^2} \ln \dfrac{M_W}{m_c}$

Matching (NLO +EW):
[Misiak, Urban; Buras, Buchalla;
Brod, MG, Stamou`11]

Operator
Mixing (RGE)

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

$K^+ \to \pi^+ \bar{\nu} \nu$ at M_W
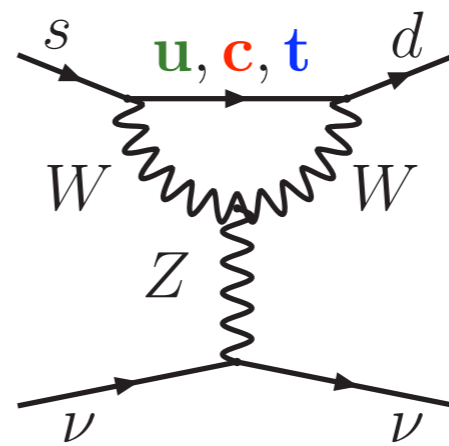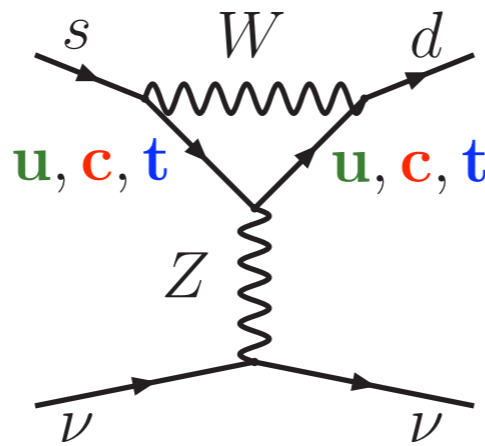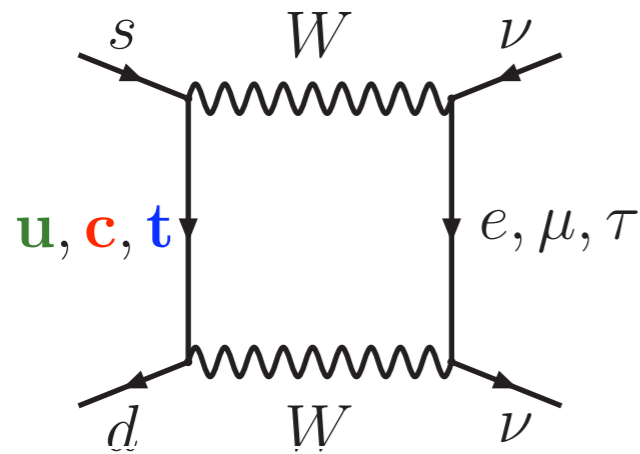
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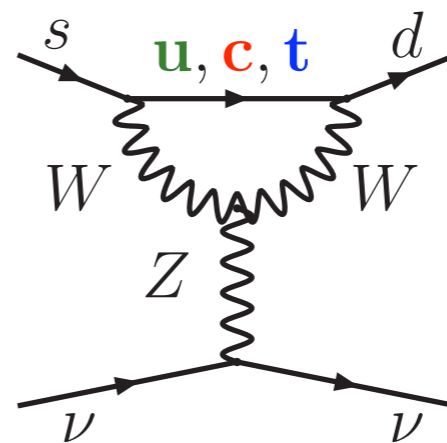
Quadratic GIM: $\lambda^5 \dfrac{m_t^2}{M_W^2}$ $\lambda \dfrac{m_c^2}{M_W^2} \ln \dfrac{M_W}{m_c}$ $\lambda \dfrac{\Lambda_{QCD}^2}{M_W^2}$

Matching (NLO +EW):
[Misiak, Urban; Buras, Buchalla;
Brod, MG, Stamou`11]

Operator
Mixing (RGE)

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$$

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$K^+ \rightarrow \pi^+ \bar{\upsilon} \upsilon$ from M_W to m_c

P_c: charm quark contribution
to $K^+ \rightarrow \pi^+ \bar{\upsilon} \upsilon$ (30% to BR)

Series converges very well
(NNLO:10%$\rightarrow$2.5% uncertainty)

NNLO+EW [Buras, MG, Haisch, Nierste; Brod MG]

K⁺ → π⁺ v̄ υ from M_W to m_c

$$K^+ \to \pi^+ \bar{\upsilon} \upsilon \text{ from } M_W \text{ to } m_c$$

P_c: charm quark contribution
to $K^+ \to \pi^+ \bar{\upsilon} \upsilon$ (30% to BR)

Series converges very well
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NNLO+EW [Buras, MG, Haisch, Nierste; Brod MG]

No GIM below the charm quark mass scale

higher dimensional operators UV scale dependent

One loop ChiPT calculation approximately cancels
this scale dependence $\delta P_{c,u} = 0.04 \pm 0.02$

[Isidori, Mescia, Smith `05]

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K$^+$ → π$^+$ ῡ υ from M$_W$ to m$_c$

P$_c$: charm quark contribution
to K$^+$ → π$^+$ ῡ υ (30% to BR)

Series converges very well
(NNLO:10%→2.5% uncertainty)

NNLO+EW [Buras, MG, Haisch, Nierste; Brod MG]

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One loop ChiPT calculation approximately cancels
this scale dependence $\delta P_{c,u} = 0.04 \pm 0.02$

[Isidori, Mescia, Smith `05]

Explorative (unphysical) Lattice calculation:
$\delta P_{c,u}$ =0.0040(±13)(±32)(-45) [Bai et.al. `17]

8
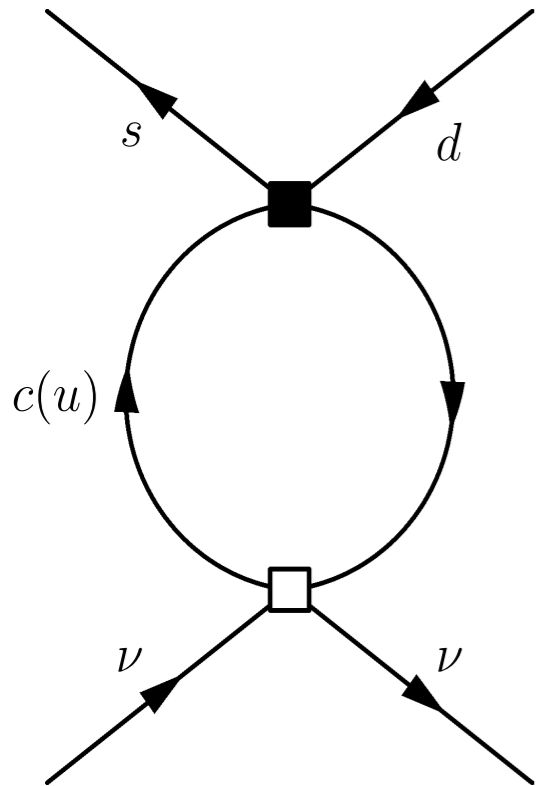
K → π ū υ: Error Budget

$BR^{th}(K^+ \to \pi^+ \bar{\nu}\nu) = 7.8(8)(3) \cdot 10^{-11}$ $BR^{th}(K_L \to \pi^0 \bar{\nu}\nu) = 2.43(39)(6) \cdot 10^{-11}$

$BR^{exp}(K^+ \to \pi^+ \bar{\nu}\nu) = 17(11) \cdot 10^{-11}$ $BR^{exp}(K_L \to \pi^0 \bar{\nu}\nu) < 6.7 \cdot 10^{-8}$

[E787, E949 ´08] NA62 → 10% accuracy   [E391a ´08]

[Brod, MG, Stamou `2011]
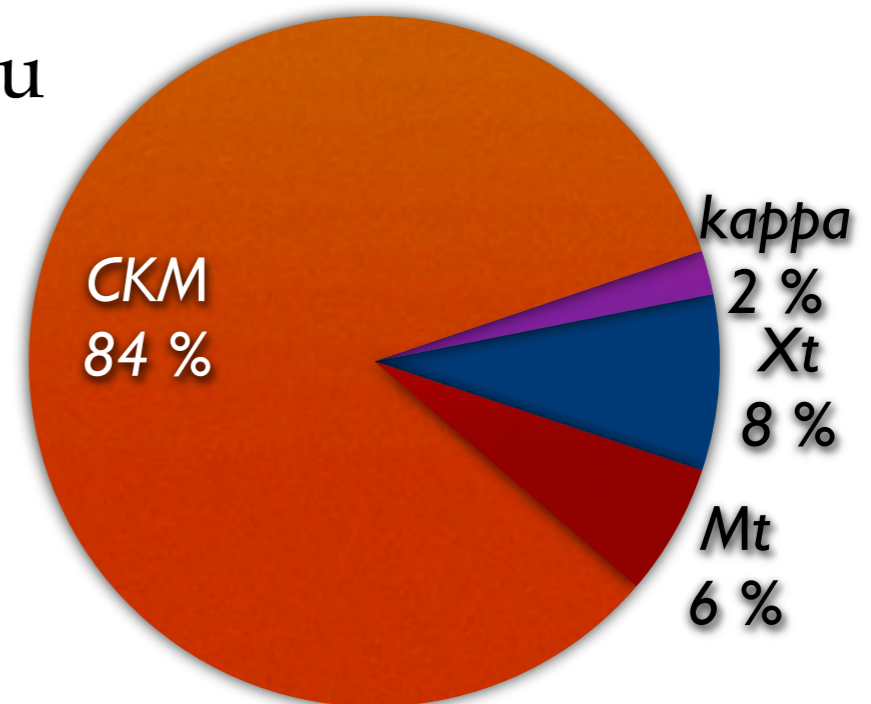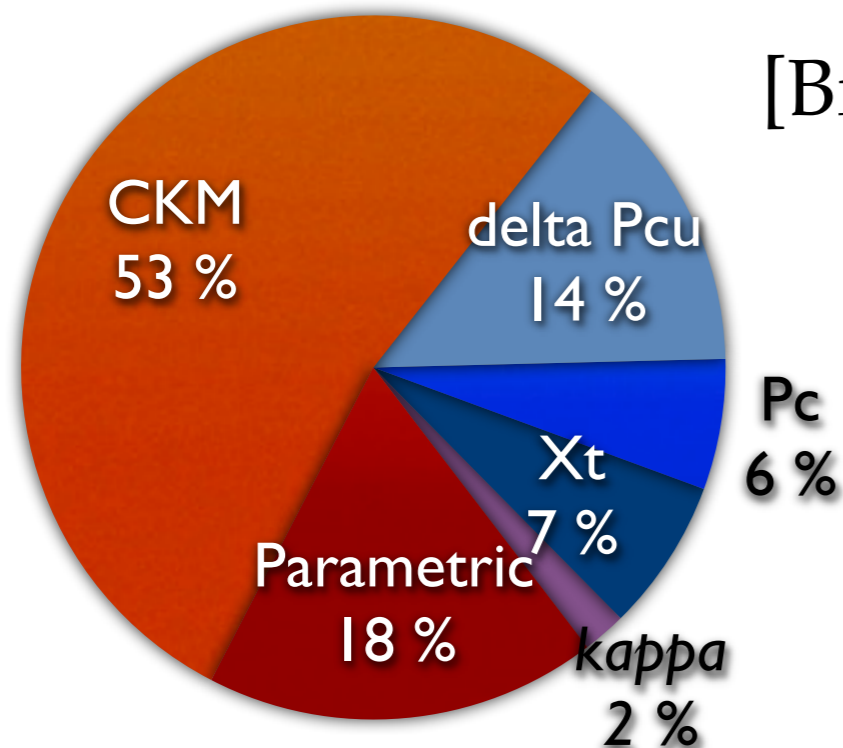
$BR^+ = 8.4(6) \cdot 10^{-11}$ (CKM tree) $BR_L = 3.4(6) \cdot 10^{-11}$ (CKM tree)

Using the same calculations: [Buras et.al. `15]

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K Meson Mixing

Schrödinger type equation for meson mixing

$$i\frac{d}{dt}\begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix} = \left[\begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix} \right] \begin{pmatrix} |K^0(t)\rangle \\ |\overline{K}^0(t)\rangle \end{pmatrix}$$

Diagonalise

$$|K_S\rangle = p|K^0\rangle + q|\overline{K}^0\rangle$$

$$|K_L\rangle = p|K^0\rangle - q|\overline{K}^0\rangle$$

M_{12} from $\Delta_s = 2$ Box $\longleftrightarrow$ Electroweak process

$\Gamma_{12} \longleftrightarrow \Delta\Gamma$ maximal and $\Delta I = 1/2$ saturates $\Gamma_{12} = A_0\,\overline{A}_0$

CP violation in Kaons

CP violation in mixing, interference & decay $\rightarrow$ non-zero

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | K_L^0 \rangle}{\langle \pi^+\pi^- | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | K_L^0 \rangle}{\langle \pi^0\pi^0 | K_S^0 \rangle}$$

Only CP violation in mixing (Re ε), interference of mixing and decay (Im ε, Im ε') and direct CP violation (Re ε')

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

$$\epsilon_K \simeq \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} \qquad \epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right)$$

from experiment, Lattice   small

$\xi = \text{Im } A_0 / \text{Re } A_0$ Individual: phase convention dependent

ε_K: CP violation in Kaon Mixing

$$2M_K M_{12} = \langle K^0| H^{|\Delta S|=2} |\bar{K}^0\rangle - \frac{i}{2} \int d^4x \, \langle K^0| H^{|\Delta S|=1}(x) \, H^{|\Delta S|=1}(0) |\bar{K}^0\rangle$$

dispersive part
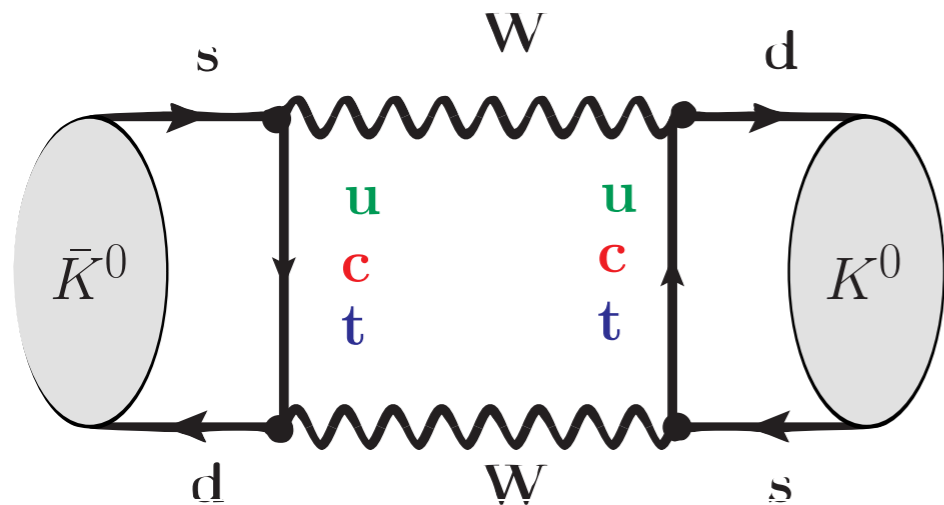
Local Interaction:
$$\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$$

Lattice: $\quad \langle K^0|\tilde{Q}|\bar{K}^0\rangle$

$(+75(1)\%)$: $\lambda_t \lambda_t \, m_t^2 / M_W^2 +$

Only known at NLO

$(+40(6)\%)$: $\lambda_c \lambda_t \, m_c^2 / M_W^2$
$\log(m_c^2 / M_W^2) +$

η_{ct}: 3-loop RGE,
2-loop Matching
[Brod, MG `10]

$(-15(6)\%)$: $\lambda_c \lambda_c \, m_c^2 / M_W^2$

η_{cc}: 3-loop RGE,
3-loop Matching
[Brod, MG `12]

NNLO

Long Distance contributions ε_K

Lattice + charm could reduce dominant error from η_{cc}
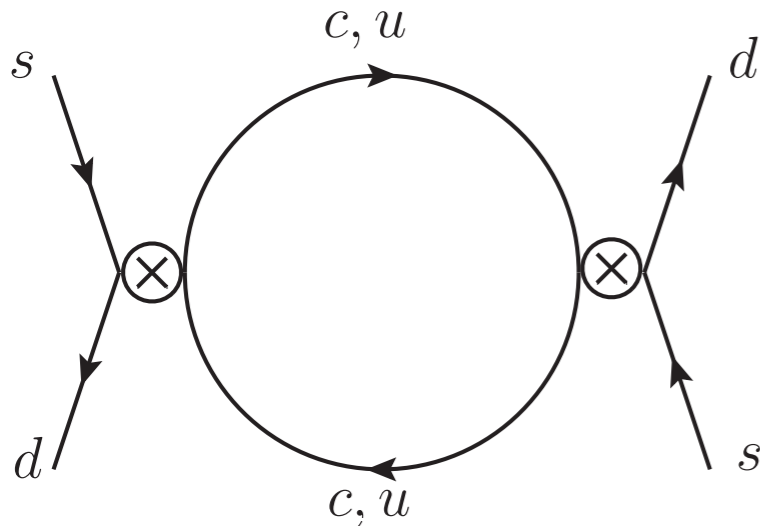
$$\int d^4x\, d^4y\, \langle K^0 | T\{H(x)\, H(y)\} | \bar{K}^0 \rangle$$

Integrate over $t_A < t_{x,y} < t_B$ on the Lattice, see talk by Jüttner

Comment on in my opinion not useful approach:

With a phase convention where $V_{cs}^* V_{cd}$ is real, η_{cc} vanishes

$\rightarrow$ new LD contributions for ε_K via modified ξ
(standard convention: 2π loop leading contribution to ξ,
$V_{cs}^* V_{cd}$ real conventions: ξ dominated by $\Delta M_K^{(LD)}$)

Effectively, one would estimate η_{cc} from ΔM_K^{exp} - ΔM_K^{SD}

Residual Theory Uncertainty

After Lattice QCD & NNLO progress: η_{cc} dominant uncertainty

ε_K is very important for phenomenology:
Future improvements are expected from Lattice QCD and interplay with perturbative QCD

[Brod, MG `12] V_{cb} dominates parametric uncertainty:

2012 $|\varepsilon_K| = 1.81(28)\ 10^{-3}$

CKMFitter 2016:

$|\varepsilon_K| = 2.27[+0.21\ -0.42]\ 10^{-3}$

Experimental:

$|\varepsilon_K| = 2.22(1)\ 10^{-3}$
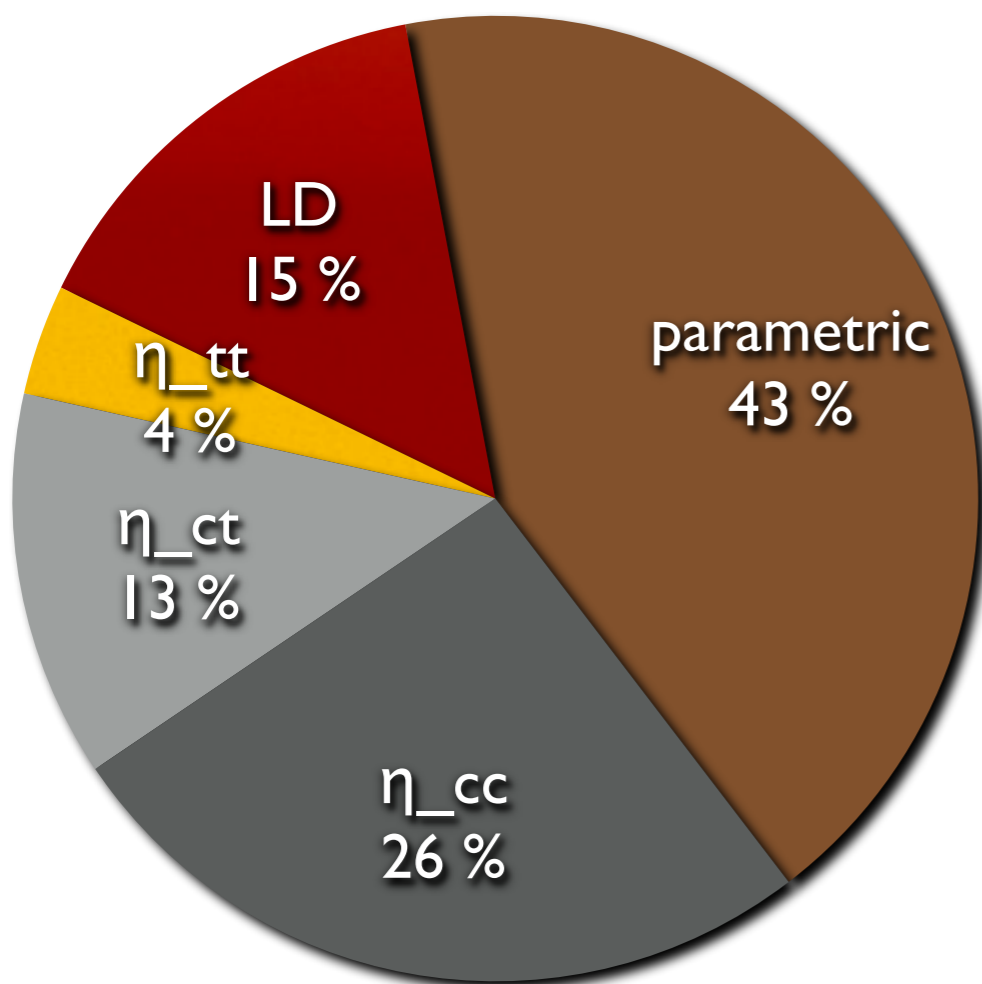
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CP violation in mixing, interference & decay → non-zero

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Only CP violation in mixing (Re ε), interference of mixing and decay (Im ε, Im ε') and direct CP violation (Re ε')

$$\epsilon_K = (\eta_{00} + 2\eta_{+-})/3 \qquad \epsilon' = (\eta_{+-} - \eta_{00})/3$$

Using: $\lambda_{ij} = \dfrac{q}{p} \dfrac{\langle \pi^i \pi^j | \bar{K}^0 \rangle}{\langle \pi^i \pi^j | K^0 \rangle}$ and $|1 - \lambda_{ij}| \ll 1$

$$\epsilon' \approx \frac{1}{6}(\lambda_{00} - \lambda_{+-}) + \frac{1}{12}(\lambda_{00} - \lambda_{+-})(2 - \lambda_{00} - \lambda_{+-}) + \ldots$$

Formula for ε' / ε

a₀, a₂ & a₂⁺ from experiment

[Cirigliano, et.al. `11]

a₀ & a₂: isospin amplitudes
for isospin conservation

$$\langle \pi^0 \pi^0 | K^0 \rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2} / \sqrt{2}$$

$$\langle \pi^+ \pi^- | K^0 \rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2} \sqrt{2}$$

$$\langle \pi^+ \pi^0 | K^+ \rangle = 3 a_2^+ e^{i\chi_2^+} / 2$$

Formula for ε′/ε

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$$\langle \pi^+ \pi^- | K^0 \rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2}\sqrt{2}$$

$$\langle \pi^+ \pi^0 | K^+ \rangle = 3a_2^+ e^{i\chi_2^+}/2$$

Current theory gives us only: $A_I = \langle (\pi\pi)_I | \mathcal{H}_{\text{eff}} | K \rangle$

Normalise to K⁺ decay (ω₊, a) and ε_K ,
expand in A₂/A₀ and CP violation:

Formula for ε'/ε

a_0, a_2 & a_2^+ from experiment

[Cirigliano, et.al. `11]

a_0 & a_2: isospin amplitudes for isospin conservation

$$\langle \pi^0 \pi^0 | K^0 \rangle = a_0 e^{i\chi_0} + a_2 e^{i\chi_2}/\sqrt{2}$$

$$\langle \pi^+ \pi^- | K^0 \rangle = a_0 e^{i\chi_0} - a_2 e^{i\chi_2}\sqrt{2}$$

$$\langle \pi^+ \pi^0 | K^+ \rangle = 3 a_2^+ e^{i\chi_2^+}/2$$

Current theory gives us only: $A_I = \langle (\pi\pi)_I | \mathcal{H}_{\text{eff}} | K \rangle$

Normalise to K^+ decay (ω_+, a) and ε_K, expand in A_2/A_0 and CP violation:

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{\epsilon'}{\epsilon} = -\frac{\omega_+}{\sqrt{2}\,|\epsilon_K|}\left[\frac{\text{Im}A_0}{\text{Re}A_0}(1 - \hat{\Omega}_{\text{eff}}) - \frac{1}{a}\frac{\text{Im}A_2}{\text{Re}A_2}\right]$$

[Buras, MG, Jäger, Jamin `15]

Adjusted to keep electroweak penguins in Im A_0 [Cirigliano, et.al. `11]

16

Current-Current & CKM

Study Unitarity & CKM Elements to get Im A_I & Re A_I

We use unitarity to eliminate $\qquad$ $V_{cs}^* V_{cd} = -V_{us}^* V_{ud} - V_{ts}^* V_{td} Q_2^c$

Current-current interactions:
Two contributions if $\mu > m_c$.
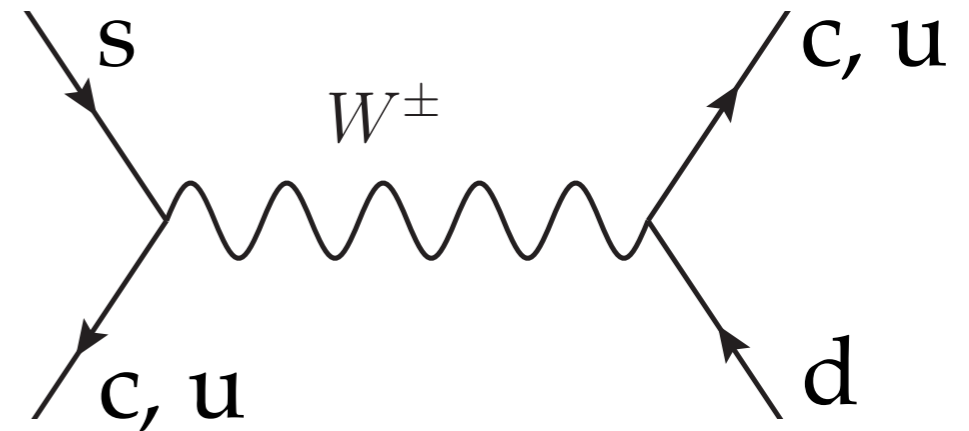
($\propto V_{ts}^* V_{td}$ and $\propto V_{us}^* V_{ud}$) $\quad V_{us}^* V_{ud} Q_{1/2}^u + V_{cs}^* V_{cd} Q_{1/2}^c \to$

$$V_{us}^* V_{ud} (Q_{1/2}^u - Q_{1/2}^c) - V_{ts}^* V_{td} Q_{1/2}^c$$

For $\mu < m_c$: $V_{ts}^* V_{td}$ is absent: $\quad V_{us}^* V_{ud} Q_{1/2}^u$

Penguin & CKM

Penguins: $f(m_u) - f(m_c) = 0$:

Only $V_{ts}^* V_{td}$ contribution
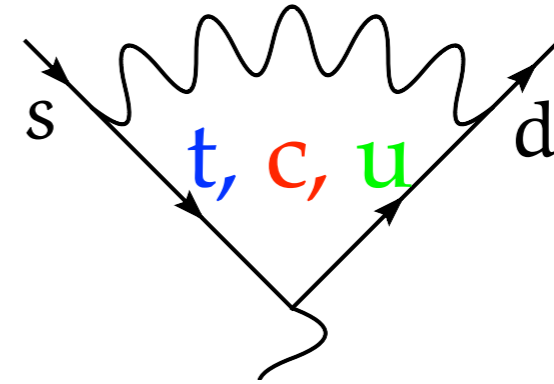
$$\{V_{us}^* V_{ud} f(m_u) + V_{cs}^* V_{cd} f(m_c) + V_{ts}^* V_{td} f(m_t)\} Q_{\text{Penguin}} \rightarrow$$

$$\{V_{us}^* V_{ud} [f(m_u) - f(m_c)] + V_{ts}^* V_{td} [f(m_t) - f(m_c)]\} Q_{\text{Penguin}}$$

Penguin & CKM

Penguins: f(m$_u$) - f(m$_c$) = 0:
Only V$_{ts}$* V$_{td}$ contribution
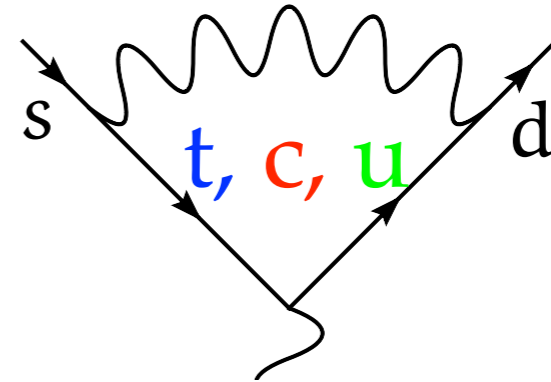
$$\{V_{us}^* V_{ud} f(m_u) + V_{cs}^* V_{cd} f(m_c) + V_{ts}^* V_{td} f(m_t)\} Q_{\text{Penguin}} \rightarrow$$

$$\{V_{us}^* V_{ud} [f(m_u) - f(m_c)] + V_{ts}^* V_{td} [f(m_t) - f(m_c)]\} Q_{\text{Penguin}}$$

$\mu >$ m$_c$: V$_{ts}$* V$_{td}$ Q$^c_{1/2}$ mixes into V$_{ts}$* V$_{td}$ Q$_{\text{Penguin}}$ (like usual).

$\mu >$ m$_c$: V$_{us}$* V$_{ud}$ (Q$^u_{1/2}$ – Q$^c_{1/2}$) does not mix into Q$_{\text{Penguin}}$.

$\mu <$ m$_c$: Match V$_{ts}$* V$_{td}$ Q$^c_{1/2}$ onto V$_{ts}$* V$_{td}$ Q$_{\text{Penguin}}$
  → CP violation from Q$_{\text{Penguin}}$
  → CP conserving from Q$^u_{1/2}$ (plus small Q$_{\text{Penguin}}$)

Effective Hamiltonian

Currently we use the effective Hamiltonian below the charm:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \, V_{ud} V_{us}^* \sum_{i=1}^{10} \left(z_i(\mu) + \tau \, y_i(\mu) \right) Q_i(\mu) \,, \quad \tau \equiv - \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$

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current-current

$$Q_{1,2/\pm} = (\bar{s}_i u_j)_{V-A} \, (\bar{u}_k d_l)_{V-A}$$

QCD &
electroweak
penguins

$$Q_{3,\dots,6} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} (\bar{q}_k q_l)_{V \pm A}$$

$$Q_{7,\dots,10} = (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_k q_l)_{V \pm A}$$

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We have z_i & y_i at NLO [Buras et.al., Ciuchini et. al. `92 `93]

And now also a Lattice QCD calculation of: $\langle (\pi\pi)_I | Q_i | K \rangle = \langle Q_i \rangle_I$

by RBC-UKQCD [Blum et. al., Bai et. al. `15]

Im A₂/Re A₂ – (V-A)x(V-A)

A_2 only contributes in the ratio Im A_2/Re A_2

Let us first consider only (V-A)x(V-A) operators:

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} \, (\bar{u}_\beta d_\alpha)_{V-A} \qquad Q_2 = (\bar{s}u)_{V-A} \, (\bar{u}d)_{V-A}$$

$$Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q \, (\bar{q}q)_{V-A} \qquad Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q \, (\bar{q}_\beta q_\alpha)_{V-A}$$

Isospin limit: $2 <Q_9>_2 = 2 <Q_{10}>_2 = 3 <Q_1>_2 = 3 <Q_2>_2$

Re A_2: $(z_1+z_2)<Q_1+Q_2>_2 = z_+<Q_+>_2$ Im A_2: $y_9<Q_9>_2 + y_{10}<Q_{10}>_2$

$$\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right)_{V-A} = \mathrm{Im}\tau \, \frac{3(y_9 + y_{10})}{2z_+} \,, \qquad \tau = \frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}}$$

Im A_0/Re A_0 – (V-A)x(V-A)

More operators contribute to Im A_0/Re A_0

$$\mathrm{Re}A_0 = \frac{G_F}{\sqrt{2}} V_{ud}V_{us}^* \left(z_+ \langle Q_+ \rangle_0 + z_- \langle Q_- \rangle_0 \right)$$

Fierz relations for (V-A)x(V-A) give, e.g.: $\langle Q_4 \rangle_0 = \langle Q_3 \rangle_0 + 2\langle Q_- \rangle_0$

$$\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right)_{V-A} = \mathrm{Im}\tau \, \frac{2y_4}{(1+q)z_-} + \mathcal{O}(p_3)$$

Is only a function of Wilson coefficients and of the ratio

$$q = (z_+(\mu)\langle Q_+(\mu)\rangle_0)/(z_-(\mu)\langle Q_-(\mu)\rangle_0)$$

Expression with $p_3 = \langle Q_3 \rangle_0 / \langle Q_4 \rangle_0$ and EW penguins given in
[Buras, MG, Jäger & Jamin `15]

(V-A)x(V+A) Contributions

Q_6 & Q_8 give the leading contribution to
$\text{Im}A_0$ & $\text{Im}A_2$ respectively

$$\left(\frac{\text{Im}A_0}{\text{Re}A_0}\right)_6 = -\frac{G_F}{\sqrt{2}}\,\text{Im}\lambda_t\,y_6\,\frac{\langle Q_6\rangle_0}{\text{Re}A_0}$$

$$\left(\frac{\text{Im}A_2}{\text{Re}A_2}\right)_8 = -\frac{G_F}{\sqrt{2}}\,\text{Im}\lambda_t\,y_8^{\text{eff}}\,\frac{\langle Q_8\rangle_2}{\text{Re}A_2}$$

Here: Take Re A_0 from data

One can re-express $<Q_6>_0$ & $<Q_8>_2$ in terms of B_6 & B_8

Prediction for ε'/ε

I=2 Similarly for (V−A)x(V−A):

$$\text{I=0 (V-A)x(V-A)} \qquad \text{I=2 (V-A)x(V-A)}$$

$$\frac{\varepsilon'}{\varepsilon} = 10^{-4} \left[\frac{\text{Im}\lambda_t}{1.4 \cdot 10^{-4}} \right] \left[a \left(1 - \hat{\Omega}_{\text{eff}}\right)\left(-4.1(8) + 24.7 B_6^{(1/2)}\right) + 1.2(1) - 10.4 B_8^{(3/2)} \right]$$

(V−A)x(V+A) Matrix elements $B_6 = 0.57(19)$ and $B_8 = 0.76(5)$

from Lattice QCD [Blum et. al., Bai et. al. `15]

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\text{SM}} = 1.9(4.5) \times 10^{-4}$$

2.9 σ difference

$$\left(\frac{\epsilon'}{\epsilon}\right)_{\text{exp}} = 16.6(2.3) \times 10^{-4}$$

Similar findings by Kitahara et.al. 16

quantity	error on ε'/ε
$B_6^{(1/2)}$	4.1
NNLO	1.6
$\hat{\Omega}_{\text{eff}}$	0.7
p_3	0.6
$B_8^{(3/2)}$	0.5
p_5	0.4
$m_s(m_c)$	0.3
$m_t(m_t)$	0.3

NLO vs NNLO

Theory prediction only at NLO at the moment

Convergence at m_c is not clear – should calculate next order

Long term use Lattice QCD

Also the error estimate does not include $O(p^2/m_c^2)$ corrections which for $K \rightarrow \pi\pi$ are expected to be small

Status of ε'/ε NNLO

Energy	Fields	Order
μ_W	g,γ,W,Z,h, u,d,s,c,b,t	NNLO Q_1-Q_6 & Q_{8g} i) NNLO EW Penguins (traditional Basis) ii)
RGE	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_b	γ,g,u,d,s,c,b	NNLO Q_1-Q_6 iv)
RGE	γ,g,u,d,s,c	NNLO Q_1-Q_6 & Q_{8g} iii)
μ_c	γ,g,u,d,s,c	NLO Q_1-Q_{10} v)
RGE	γ,g,u,d,s	NNLO Q_1-Q_6 & Q8g iii)
$M_{Lattice}$	g,u,d,s	NLO Q_1-Q_{10} (traditional Basis) vi)

i) [Misiak, Bobeth, Urban]
ii) [Gambino,Buras, Haisch]
iii)[Gorbahn, Haisch]

iv)[Gorbahn, Brod]
v) [Buras, Jamin, Lautenbacher]
vi)[Blum et. al., Bai et. al. '15]

RG-invariant factorisation

Traditional the contribution of running ($U(\mu, \mu_0)$) and matching ($M(\mu)$) are combined as:

$$\langle \vec{Q} \rangle^{(3)}(\mu_L) \vec{C}^{(3)}(\mu_L) = \langle \vec{Q} \rangle(\mu_L) U^{(3)}(\mu_L, {\color{red}\mu_c}) {\color{red}M^{(34)}(\mu_c)} {\color{red}U^{(4)}(\mu_c,} {\color{blue}\mu_b)}$$

$$\quad {\color{blue}M^{(45)}(\mu_b) U^{(5)}(\mu_b,} \mu_W) \vec{C}^{(5)}(\mu_W)$$

Alternatively we can also factorise as

$$\langle \vec{Q} \rangle^{(3)}(\mu_L) \vec{C}^{(3)}(\mu) = \langle \vec{Q} \rangle(\mu_L)^{(3)} u^{(3)}(\mu_L)$$

$$\quad {\color{red}u^{(3)^{-1}}(\mu_c) M^{(34)}(\mu_c) u^{(4)}(\mu_c)}$$

$$\quad {\color{blue}u^{(4)^{-1}}(\mu_b) M^{(45)}(\mu_b) u^{(5)}(\mu_b)}$$

$$\quad u^{(5)^{-1}}(\mu_W) \vec{C}^{(5)}(\mu_W)$$

or write in terms of scheme and scale independent quantities:

$$\langle \vec{Q} \rangle^{(3)}(\mu_L) \vec{C}^{(3)}(\mu) = \langle \hat{\vec{Q}} \rangle^{(3)} {\color{red}\hat{M}^{(34)}} \hat{M}^{(45)} \hat{\vec{C}}^{(5)}$$

RG-invariant factorisation

All hatted quantities $\langle \hat{\vec{Q}} \rangle^{(3)}, \hat{M}^{(34)}, \hat{M}^{(45)}$ and $\hat{\vec{C}}^{(5)}$ and also their products

$$\hat{\vec{C}}^{(3)} = \hat{M}^{(34)} \hat{M}^{(45)} \hat{\vec{C}}^{(5)}$$
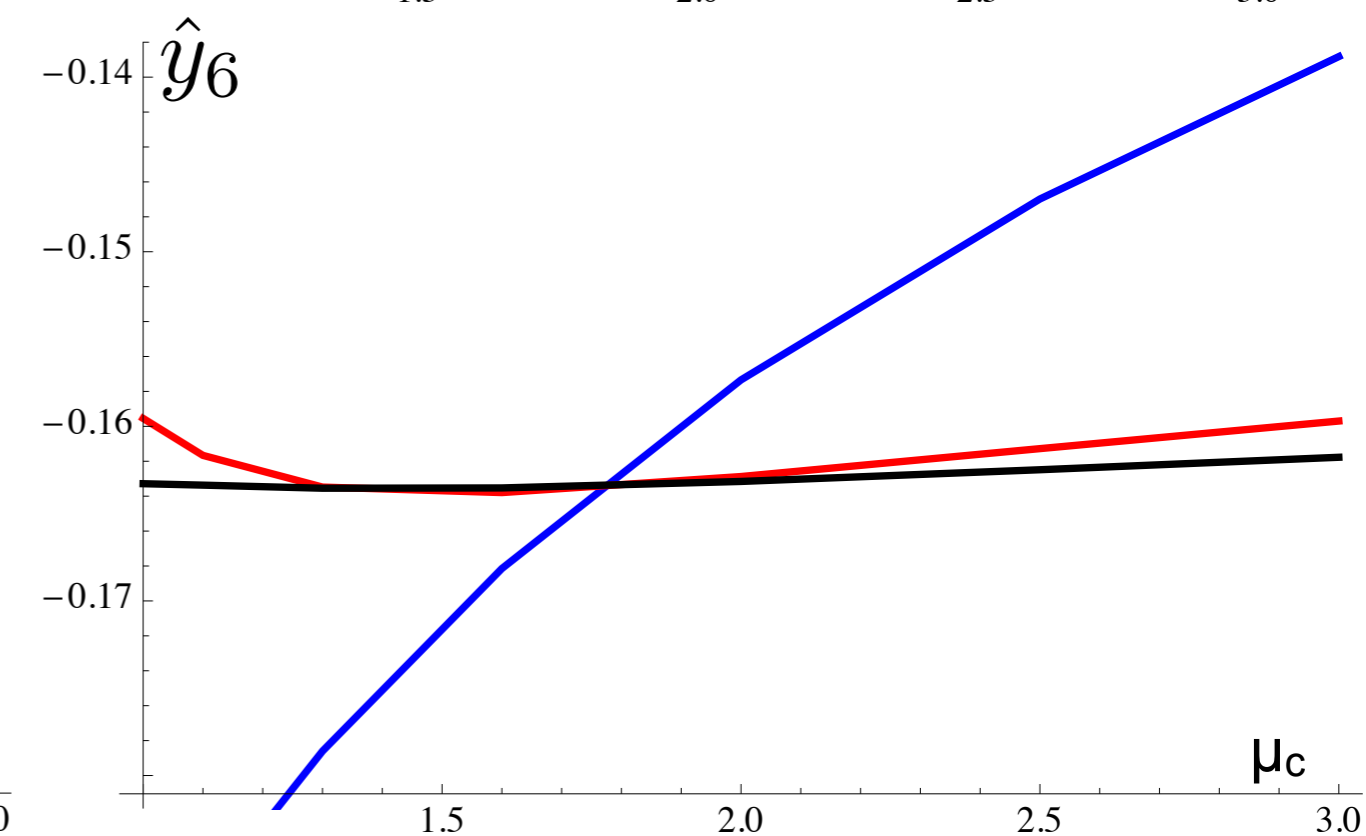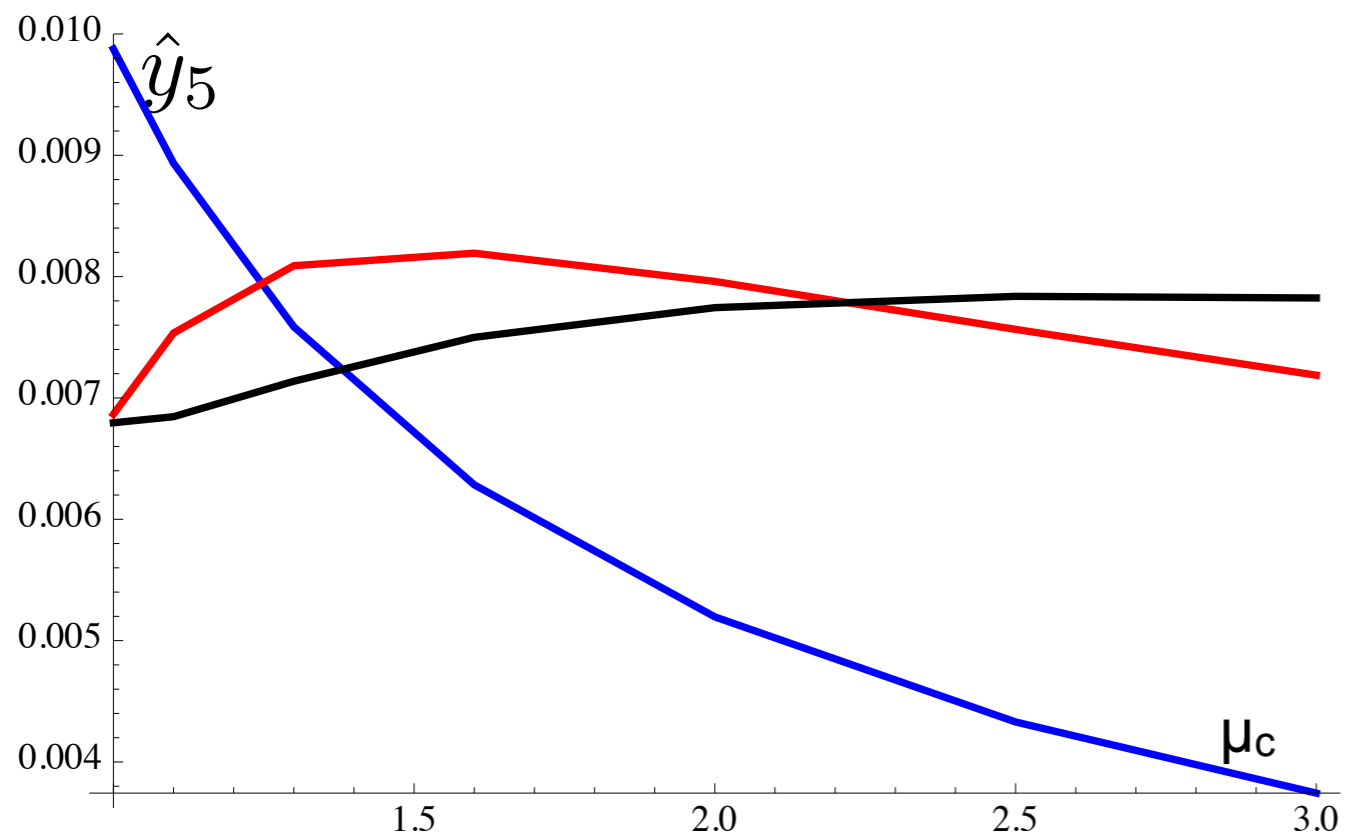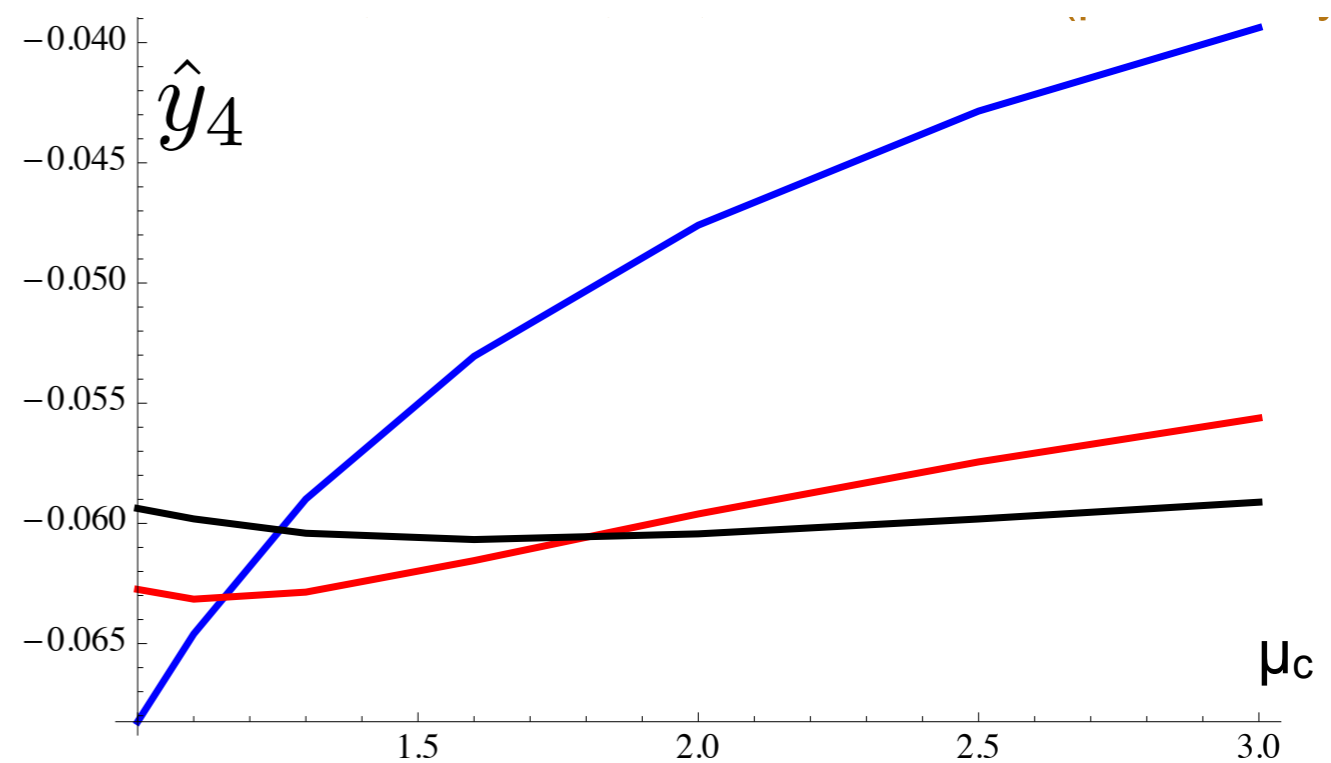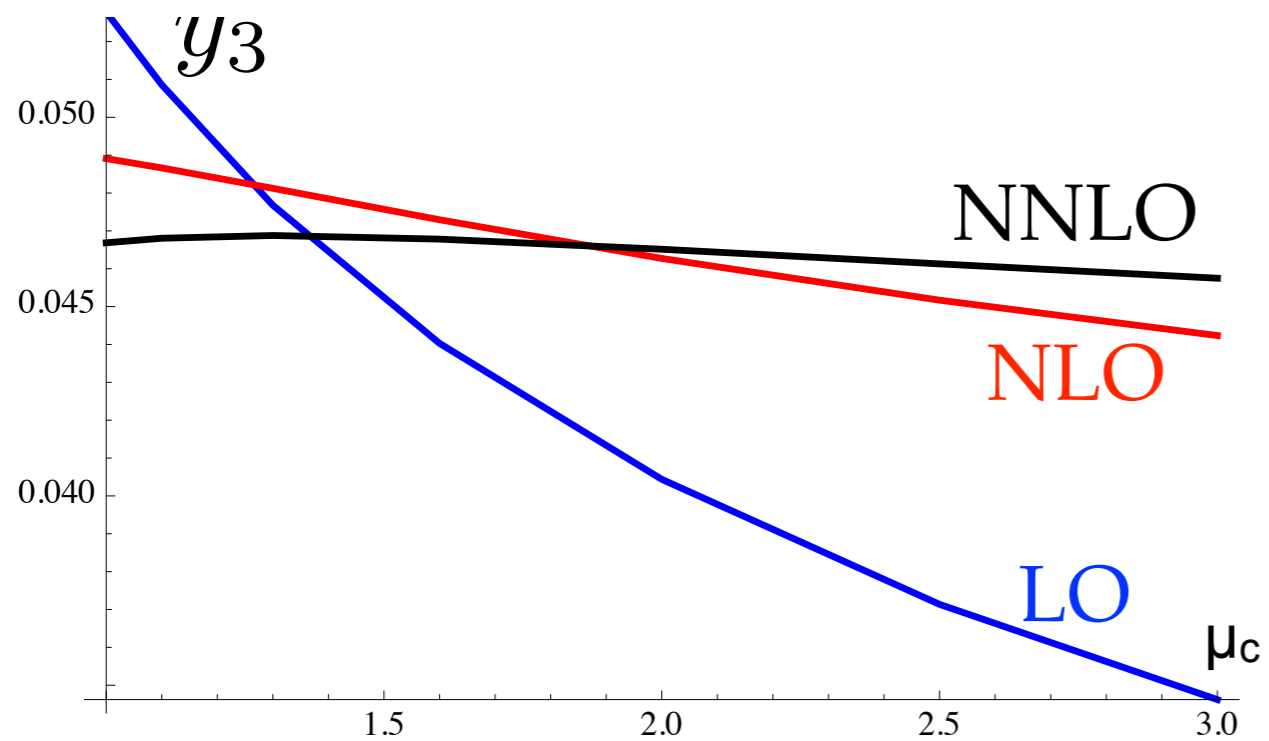
are formally scheme and scale independent.

The matrix elements $\langle \hat{\vec{Q}} \rangle$ satisfy $d = 4$ Fierz identities.

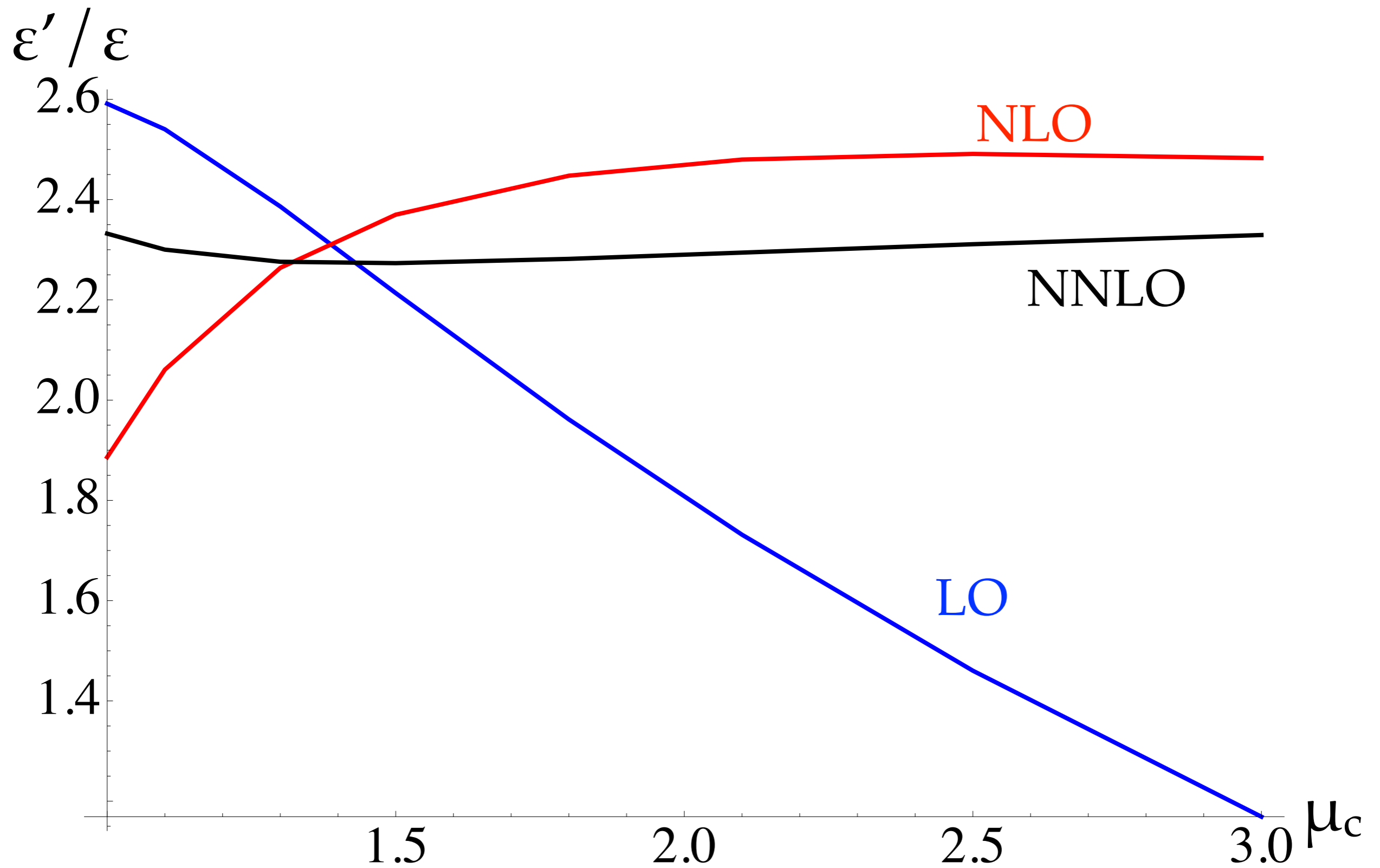$\hat{\vec{C}}^{(3)}$ is μ *independent*, but shows *residual* μ *dependence*.

Plot this for the $\hat{y}(\mu_c)$ (the ones $\propto \text{Im}(V_{ts}^* V_{td})$):

Residual μ_c dependence

Residual μ_c dependence

Conclusion

Perturbative calculations for $K \to \pi \bar{\nu} \nu$ under very good control, with only sub-leading non-perturbative effects.

Ongoing Lattice efforts improve the estimate of non-perturbative effects for $K \to \pi \bar{\nu} \nu$ and ε_K.

New perturbative NNLO calculation removes large part of the perturbative uncertainty in ε'_K.

Interesting tension with experiment.