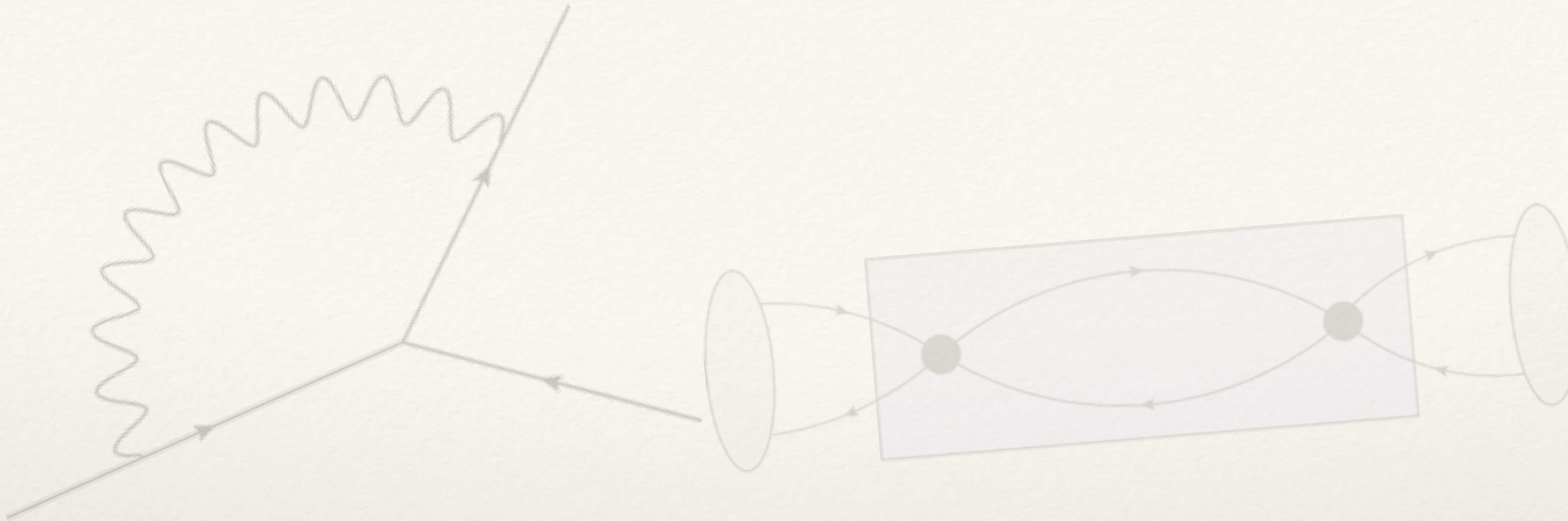


Current Trends In Flavour Physics
Institut Henri Poincaré, Paris, 29-31.03.2017

Novelties in kaon physics: a Lattice QCD perspective

Andreas Jüttner

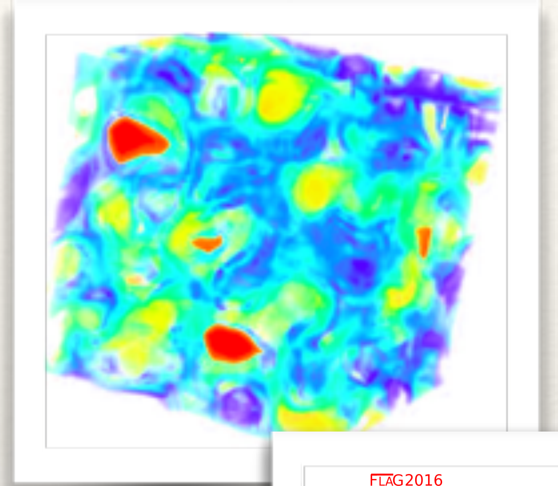
UNIVERSITY OF Southampton

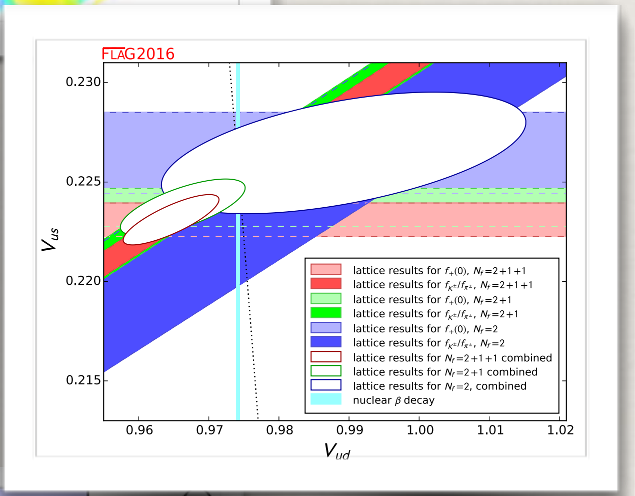
Outline

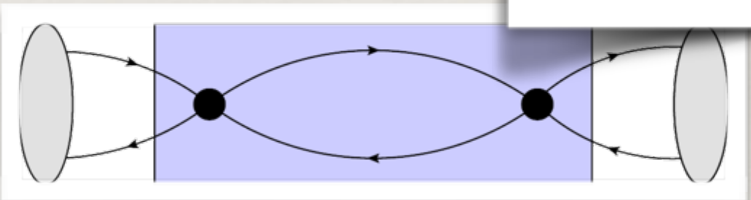
1. lattice QCD

2. precision kaon physics

3. novelties in kaon physics

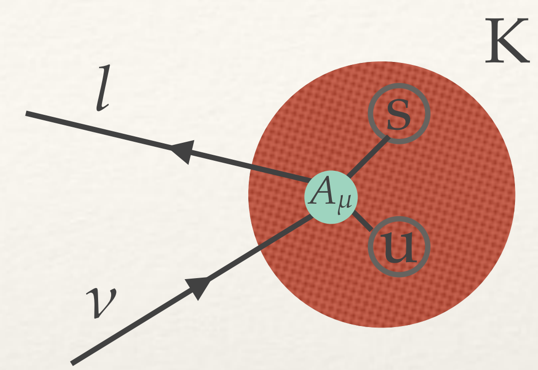






Motivation: Quark Flavour Physics

e.g tree level leptonic K decay:



Assumed factorisation: $\Gamma_{\text{exp.}} \stackrel{???}{=} V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$

$$\underbrace{\Gamma(K_{l2})}_{\text{experiment}} = \underbrace{|V_{us}|^2}_{\text{output}} \underbrace{\frac{G_F^2 M_K m_l^2}{4\pi}\left(1-\frac{m_l^2}{M_K^2}\right)^2 f_K}_{\text{theory prediction}} \quad \langle 0|A_\mu|K\rangle$$

Experimental measurement + theory prediction allows for extraction of CKM MEs

Lattice QCD

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_f \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \psi_f$$

Free parameters:

- gauge coupling $g \rightarrow \alpha_s = g^2/4\pi$
- quark masses $m_f = u,d,s,c,b,t$

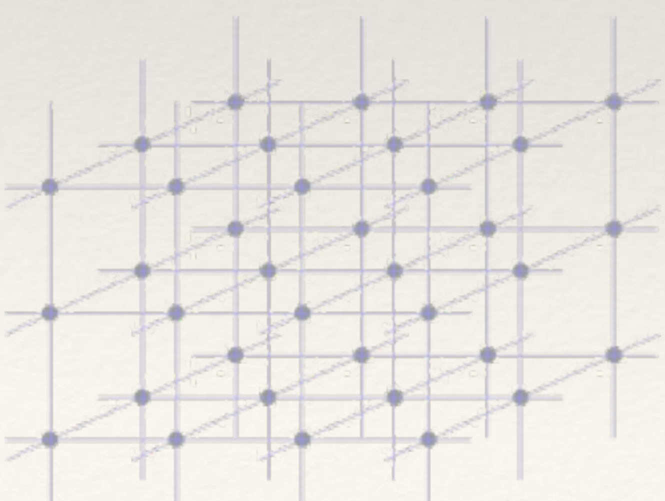
- Lagrangian of massless gluons and *almost massless quarks*
- what experiment sees are bound states, e.g. $m_\pi, m_P \gg m_{u,d}$
- underlying physics non-perturbative

Path integral quantisation:

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-i S_{\text{lat}}[U,\psi,\bar{\psi}]}$$

$$\langle 0|O|0\rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O \boxed{e^{-S_{\text{lat}}[U,\psi,\bar{\psi}]}}$$

Euclidean space-time Boltzmann factor



finite volume, space-time grid (IR and UV regulators)

$\propto L^{-1}$ $\propto a^{-1}$

$\rightarrow$ well defined, finite dimensional Euclidean path integral

$\rightarrow$ from first principles

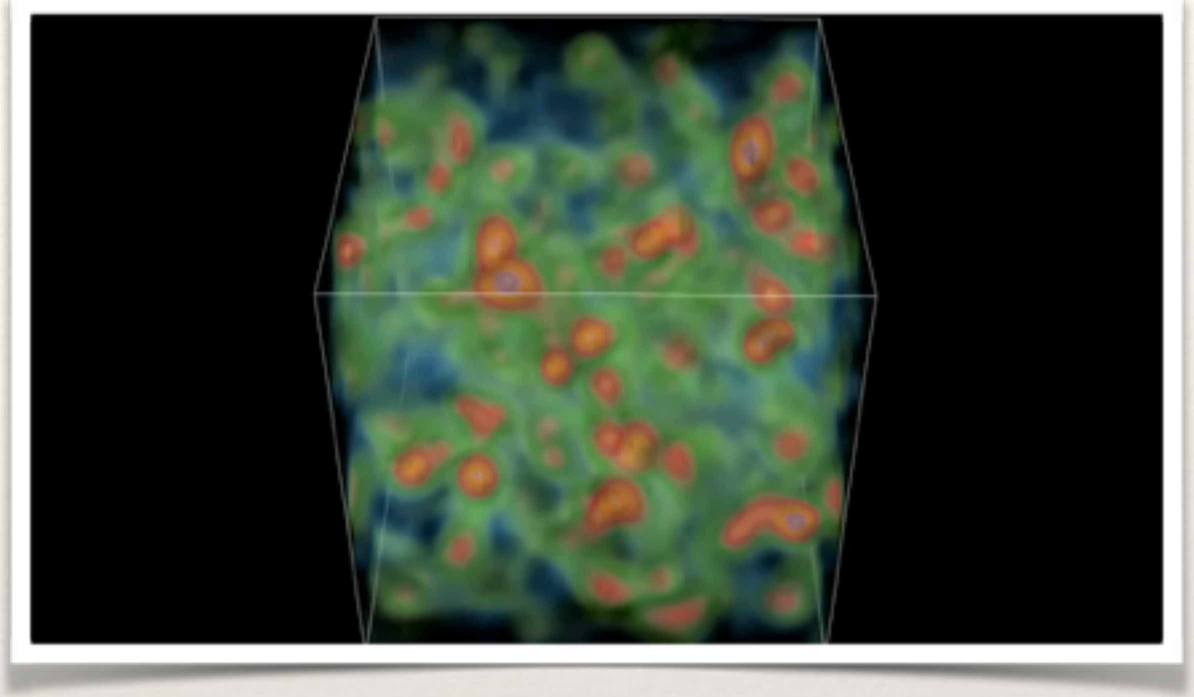
Lattice QCD

- gauge-invariant regularisation (Wilson 1974)
- finite volume lattice path integral still over large number of degrees of freedom > $O(10^{10})$
- Evaluate discretised path integral by means of Markov Chain Monte Carlo on state-of-the-art HPC installations

State of the art of lattice QCD simulations

What we can do

- simulations of QCD with dynamical (sea) *u,d,s,c* quarks with masses as found in nature ➔ $N_f = 2,\ 2+1,\ 2+1+1$
- bottom only as valence quark
- cut-off $a^{-1} \leq 4\text{GeV}$
- volume $L \leq 6fm$



action density of RBC/UKQCD physical point DWF ensemble

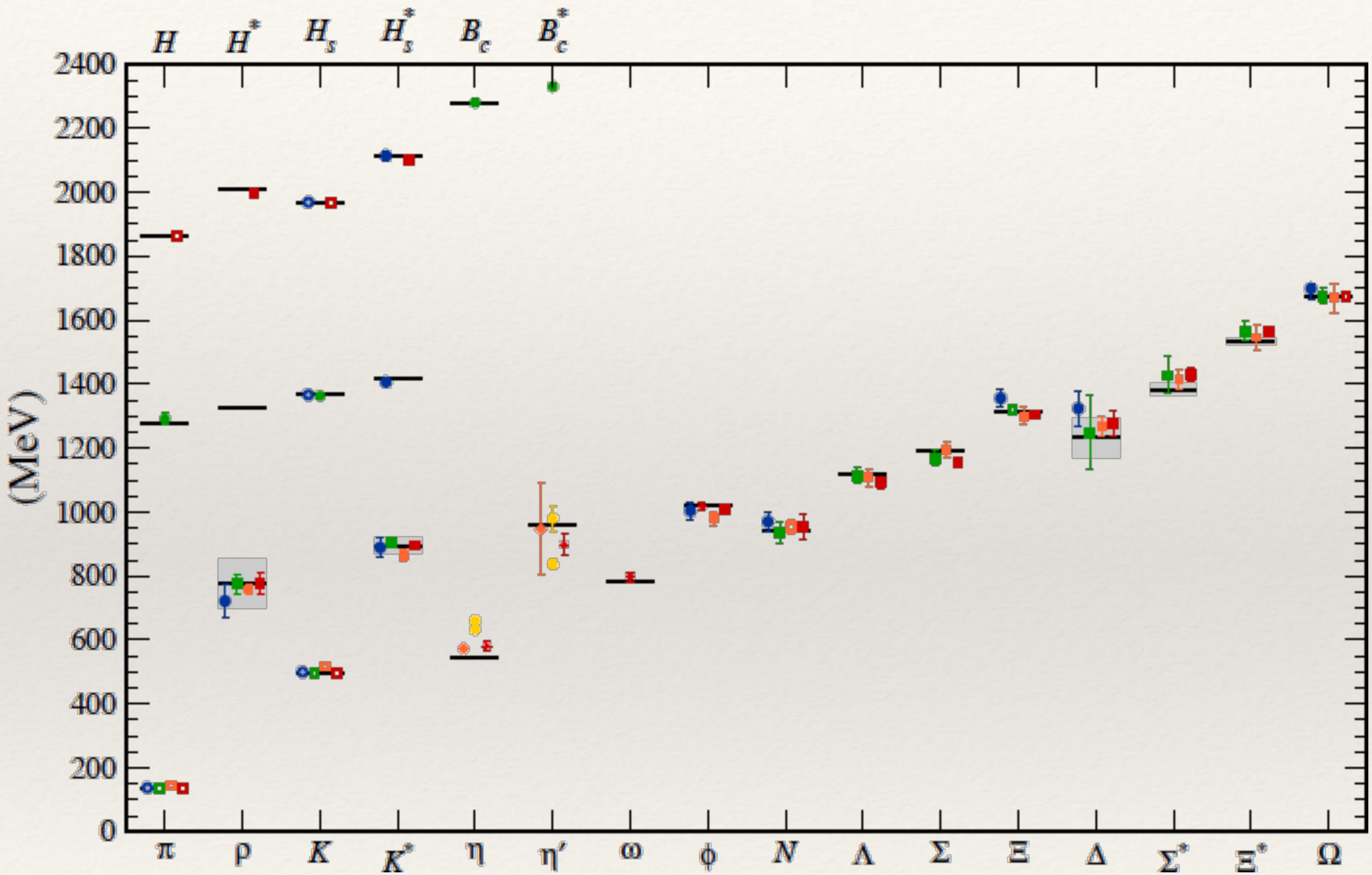
Parameter tuning

start from *educated guesses* and:

- tune light quark mass *am* such that $\frac{am_\pi}{am_P} = \frac{m_\pi^{PDG}}{m_P^{PDG}}$
- tune strange quark mass such that $\frac{am_\pi}{am_K} = \frac{m_\pi^{PDG}}{m_K^{PDG}}$
- determine physical lattice spacing $a = \frac{af_\pi}{f_\pi^{PDG}}$

IMPORTANT:
once the QCD-parameters are *tuned* no further parameters need to be fixed and we can make fully predictive simulations of QCD

benchmark - the hadron spectrum



Kronfeld, Ann. Rev. of Nucl. Part. Sci 2012 62

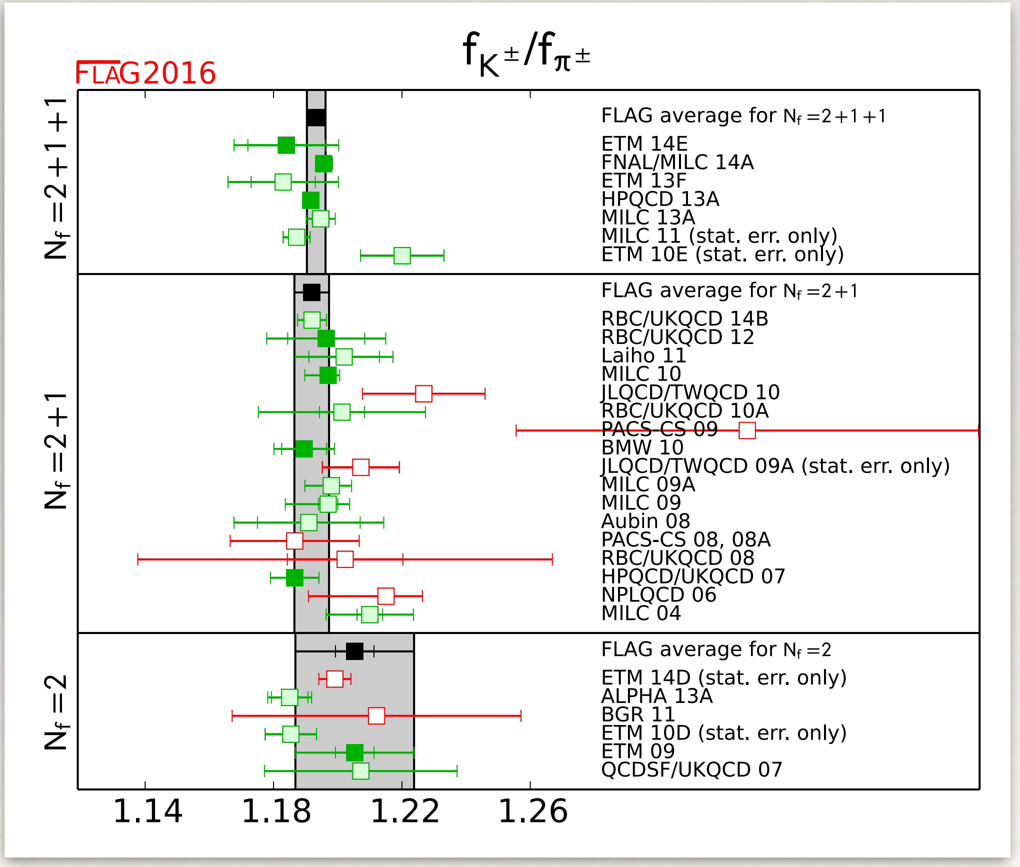
“tree” kaon/pion decays

$$\Gamma(K \to \mu \bar{\nu}_\mu) = \frac{G_F^2}{8\pi} f_K^2 m_\mu^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)^2 |V_{us}|^2$$

$$\langle 0 | \bar{s}/\bar{d} \gamma_\mu \gamma_5 u | K/\pi(p) \rangle = i f_{K/\pi} p_\mu$$

$$\frac{\Gamma(K \to \mu \bar{\nu}_\mu)}{\Gamma(\pi \to \mu \bar{\nu}_\mu)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K (1 - m_\mu^2/m_K^2)^2}{m_\pi (1 - m_\mu^2/m_\pi^2)^2} \times 0.9930(35)$$

Marciano, Phys.Rev.Lett. 2004



1

FLAG – Flavour Lattice Averaging Group

“What’s currently the best lattice value for a particular quantity?”

FLAG-1 **(Eur. Phys. J. C71 (2011) 1695)**

FLAG-2 **(Eur. Phys.J. C74 (2014) 2890)**

FLAG-3 **(Eur.Phys.J. C77 (2017) no.2, 112, http://itpwiki.unibe.ch/flag/)**

- quantities:

$$m_{u,d,s,c,b}$$

$$f_K/f_\pi,\, f_+^{K\pi}(0),\, B_K,\, SU(2)\text{ and }SU(3)\text{ LECs}$$

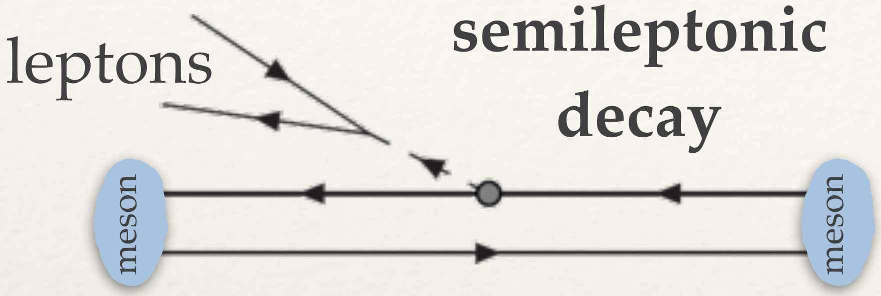
$$f_{D_{(s)}},\, f_{B_{(s)}},\, B_{B_{(s)}},\, B_{(s)}-\text{ and }\, D_{(s)}-\text{semileptonics}$$

$$\alpha_s$$

- summary of results
  - evaluation according to FLAG quality criteria (colour coding)
  - averages of best values where possible
  - detailed summary of properties of individual simulations

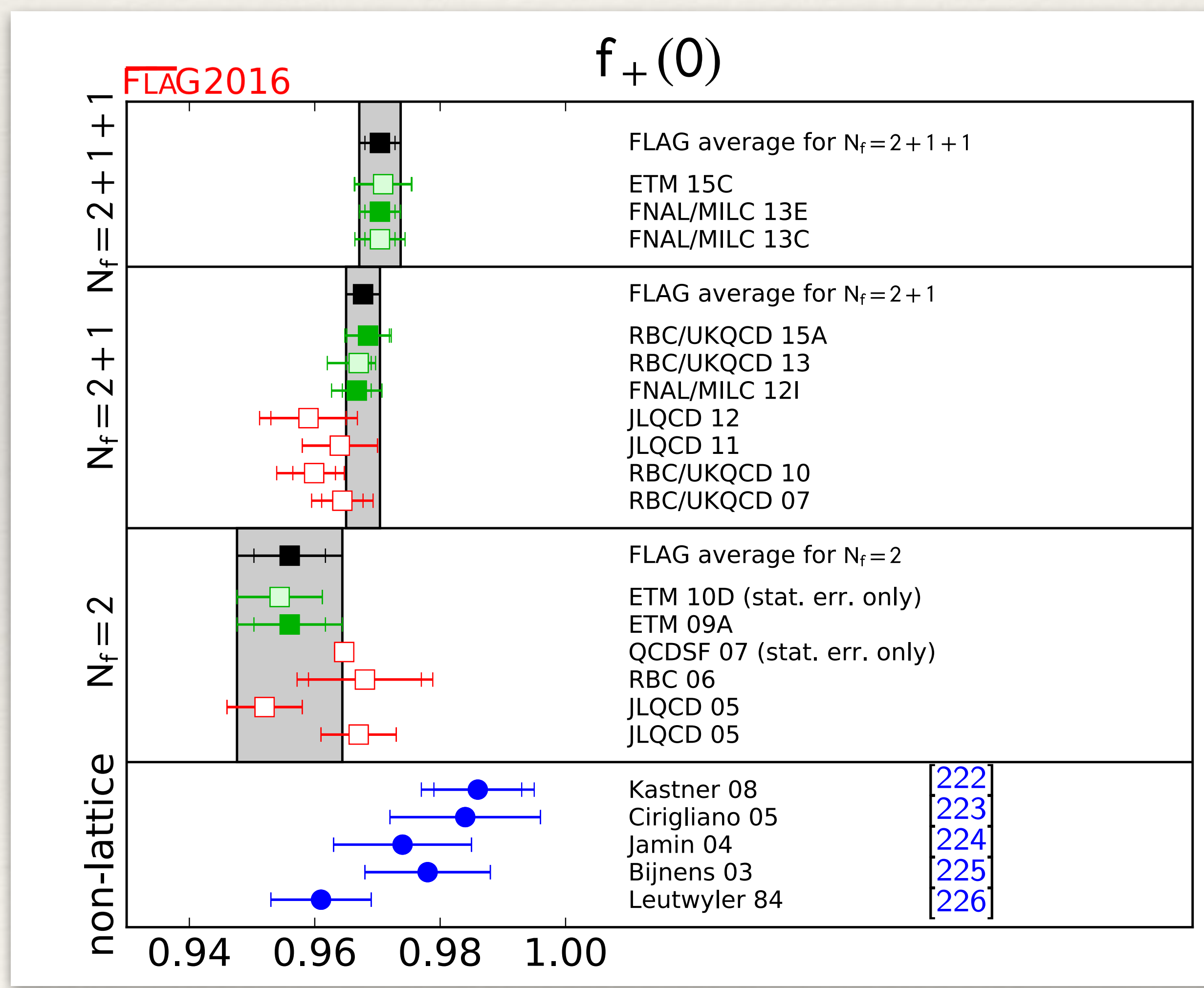
FLAG-4 kickoff meeting end of April at Higgs Centre for Theoretical Physics, Edinburgh

"tree" kaon/pion decays



$$\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW}(1+\Delta_{SU(2)}+\Delta_{\rm EM})^2\, I\, |f_+^{K\pi}(0)|^2 |V_{us}|^2$$

$$\langle\pi(p_\pi)|V_\mu(0)|K(p_K)\rangle = f_+^{K\pi}(q^2)(p_K+p_\pi)_\mu + f_-^{K\pi}(q^2)(p_K-p_\pi)_\mu$$



3‰!!!

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \approx |V_{ud}|^2 + |V_{us}|^2 \stackrel{?}{=} 1$$

$$|V_{us}| f_+^{K^0\pi^-}(0) = 0.2163(5) \qquad \frac{f_{K^+}}{f_{\pi^+}} \frac{|V_{us}|}{|V_{ud}|} = 0.2758(5)$$

FLAVIA Kaon WG EPJ C 69, 399-424 (2010)
KTeV, Istra, KLOE, NA48 [arXiv:1005.2323]

Summary I

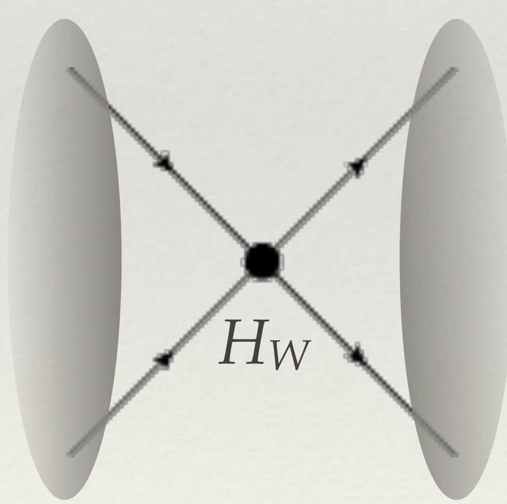
- (non-rare) Lattice QCD for Kaon Flavour Physics has *matured* (leptonic, semi-leptonic decays, kaon distribution amplitudes, hadronic kaon decay, …)
- independent groups with different approaches competing
- (sub-)percent precision for small set of quantities feasible in QCD
- FLAG summarises particularly mature quantities in a way accessible/usable to the wider community

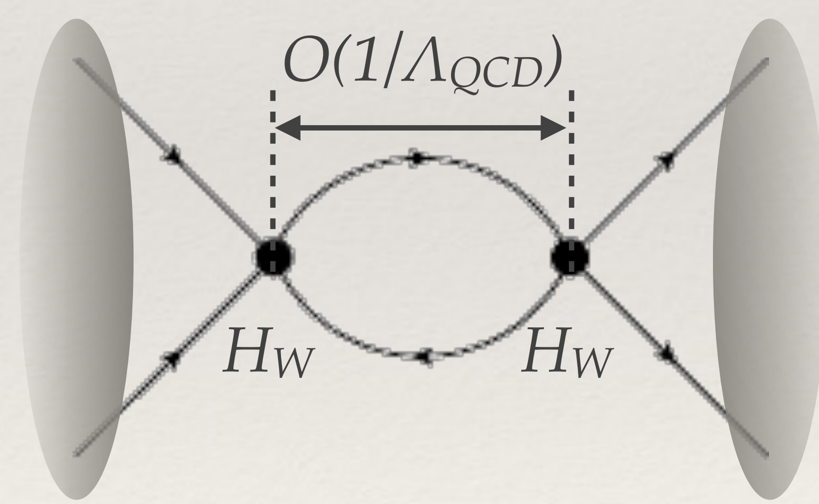
"Beyond Precision Lattice QCD"

Go beyond factorisation

$$\Gamma_{\mathrm{exp.}} \stackrel{???}{=} V_{\mathrm{CKM}}(\mathrm{WEAK})\underbrace{(\mathrm{EM})(\mathrm{STRONG})}_{\text{treat jointly in lattice QCD+QED}}$$

Go beyond short distance physics





Including QED in meson decay MEs

- most results in FLAG report based on QCD with $m_l = m_u = m_d$ and $\alpha_{EM} = 0$
- isospin breaking corrections computed in effective theory, e.g. ChPT based on factorisation of QCD and QED
- with 1% precision on QCD matrix elements isospin breaking needs to be taken into account properly

$$\alpha_{EM} \approx 1/137 \qquad (m_u - m_d)/\Lambda_{QCD} \approx O(1\%)$$

Isospin corrections are important

- e.g. $K \to \pi l \nu$: $\Gamma_{K\to\pi l\nu} = C_K^2 \frac{G_F^2 m_K^5}{192\pi^2} S_{\rm EW} (1 + \Delta_{SU(2)} + \Delta_{\rm EM})^2\, I\, |f_+^{K\pi}(0)|^2 |V_{us}|^2$

- precision now such that corrections need to be improved:

			Approx contrib to % err			
Mode	$V_{us}\, f_+(0)$	% err	BR	τ	Δ	I
$K_{L\,e3}$	0.2163(6)	0.26	0.09	0.20	0.11	0.05
$K_{L\,\mu3}$	0.2166(6)	0.28	0.15	0.18	0.11	0.06
$K_{S\,e3}$	0.2155(13)	0.61	0.60	0.02	0.11	0.05
$K^\pm_{e3}$	0.2172(8)	0.36	0.27	0.06	0.23	0.05
$K^\pm_{\mu3}$	0.2170(11)	0.51	0.45	0.06	0.23	0.06

Moulson@CKM 2014
arXiv:1411.5252

QCD+QED

QCD+QED Action:

$$S[U, A, \bar{\psi}, \psi] = S_g[U; g] + S_\gamma[A] + \sum_f \bar{\psi}_f D[U, A; e, q_f, m_f]\psi_f$$

$$S_\gamma^{naive} = -\frac{a^4}{4} \sum_{\mu,\nu,x} (\partial_\mu A_{\nu,x} - \partial_\nu A_{\mu,x})^2$$

- MC simulation of discretised theory

Very lively research topic — important questions:

- photon is massless - what to do with zero mode?
- finite volume effects more severe (QCD's mass gap ensures FVE$\propto e^{-m_\pi L}$) for simple MEs
- dealing with IR divergencies

QED zero mode

Feynman gauge
photon propagator (discretised)

$$\Delta_{\mu\nu}(x_1, x_2) = \delta_{\mu\nu} \frac{1}{L^4} \sum_{k=\frac{2\pi}{L}n} \frac{e^{ik\cdot(x_1-x_2)}}{4\sum_{\rho} \sin^2 \frac{k_\rho}{2}}$$

- QED_{TL} drop *4d* zero mode $A_\mu(k=0)=0$ Duncan, Eighteen PRL 76 3894 (1996)
- QED_L drop *T 3d* zero modes $A_\mu(k_0, \mathbf{k}=0)=0$ for all k_0 Hayakawa, Uno PRP 120 413 (2008)
- QED_γ introduce photon mass Endress et al. PRL 117 072002 (2016)
- C* boundary conditions Lucini et al. JHEP 02 (2016) 076
  (fields periodic in $\mu=i$ up to charge conjugation)

each one has its pros and cons …

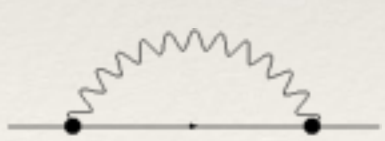
in the following QED_L

Finite volume effects for QCD+QED

- no mass gap in QED (as opposed to QCD)
- Infrared (finite volume effects) are universal
  $\rightarrow$ compute analytically in effective theory
- Leading finite volume effects

$$\xi' = \int \frac{dk_0}{(2\pi)} \left(\frac{1}{L^3} \sum_{\vec{k} \neq 0} - \int \frac{d^3k}{(2\pi)^3} \right) \frac{1}{(k^2)^{\frac{n}{2}}} = O\left(\frac{1}{L^{4-n}}\right)$$

- Example: FV correction to scalar QED self energy



$$\frac{(2p+k)^2}{(k^2+i\epsilon)((p+k)^2-m_P^2+i\epsilon)}$$

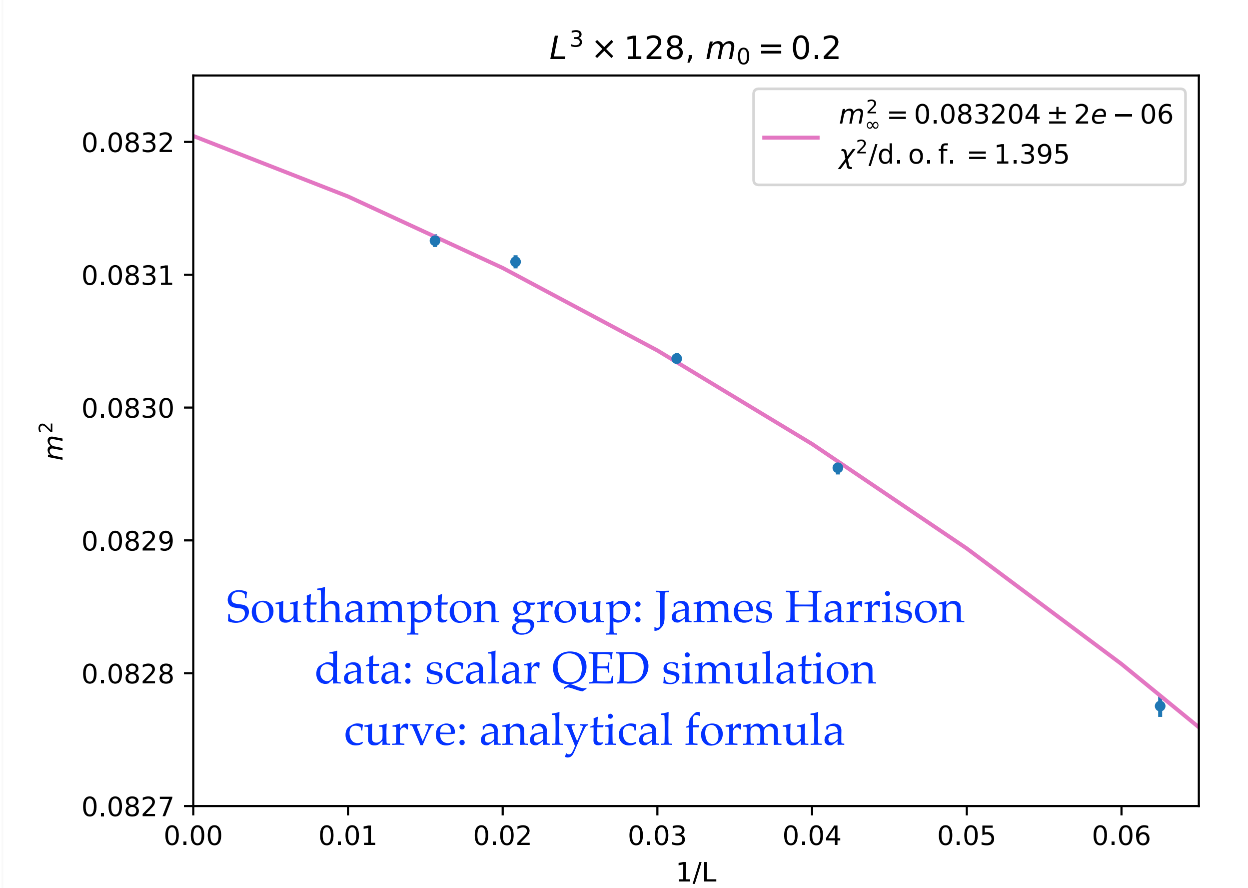
for small $k \rightarrow n=3$
$\rightarrow$ FVE ~ $O(1/L)$

Finite volume effects for QCD+QED

leading behaviour universal in κ (structure- and spin-independent)

$$m^2(L) = m_\infty^2 \left\{ 1 - q^2\alpha \left(\frac{\kappa}{m_\infty L} \left(1 + \frac{2}{m_\infty L} \right) \right) \right\}$$

BMW Collaboration
Science 347 (2015) 1452-1455
arXiv:1406.4088



Lot's of work going on to analytically predict finite volume effects in the presence of photons
(e.g. Lubicz et al. Phys.Rev. D95 (2017) no.3, 034504)

These predictions will be used to correct for QED-induced finite volume effects

Including QED in meson decay MEs

Beyond including QED effects in LQCD for spectra and a small number of hadronic matrix elements it gets quite tricky:

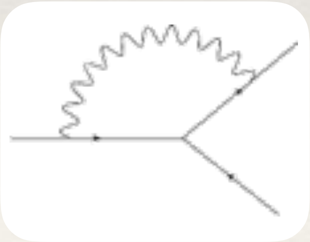
- leptonic decay at $O(\alpha^0)$:

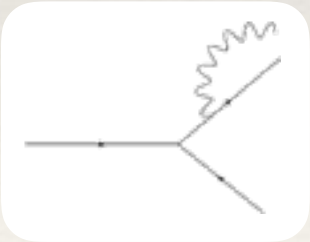
$$\Gamma(\pi^+ \to l^+\nu_l) = \frac{G_F^2 |V_{ud}|^2 f_\pi^2}{8\pi} m_\pi m_l^2 \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2$$

- including elm. effects @ $O(\alpha)$ - *we no longer speak in terms of the decay constant*

$$\Gamma(\pi^+ \to l^+\nu_l(\gamma)) = \Gamma(\pi^+ \to l^+\nu_l) + \Gamma(\pi^+ \to l^+\nu_l\gamma)$$

$$\equiv \quad \Gamma_0 \quad + \quad \Gamma_1$$





IR div. cancel between terms on r.h.s. between virtual and real photons (Bloch Nordsieck)

Including QED in meson decay MEs

Carrasco et al. PRD 91 074506 (2015) **arXiv:1502.00257**
Lubicz eta l. PRD 95 (2017) **arXiv:1611.08497**

- cut on small photon momentum < $\Delta E \rightarrow \gamma$ *sees* point-like π
  $\Delta E \approx 20$MeV experimentally accessible and π *"point like"*, i.e. no structure

point approximation

$$\Gamma(\Delta E) = \lim_{V\to\infty} \boxed{(\Gamma_0 - \Gamma_0^{\mathrm{pt}})} + \lim_{V\to\infty} \boxed{(\Gamma_0^{\mathrm{pt}} + \Gamma_1^{\mathrm{pt}}(\Delta E))}$$

$\Gamma(\pi^+ \to l^+ \nu_l)$
lattice and analytical
finite V

$\Gamma(\pi^+ \to l^+ \nu_l \gamma(\Delta E))$
analytically in $V \to \infty$

both terms separately IR finite, gauge invariant on its own

QCD+QED: first applications for ME

Carrasco et al. PRD 91 074506 (2015) arXiv:1502.00257
Lubicz eta l. PRD 95 (2017) arXiv:1611.08497

$$\Gamma(\Delta E) = \lim_{V\to\infty}(\Gamma_0 - \Gamma_0^{\rm pt}) + \lim_{V\to\infty}(\Gamma_0^{\rm pt} + \Gamma_1^{\rm pt}(\Delta E))$$

$\Gamma(\pi^+ \to l^+ \nu_l)$ $\quad$ $\Gamma(\pi^+ \to l^+ \nu_l \gamma(\Delta E))$

- Finite volume effects computed on the lattice:

$$\Gamma_0^{\rm pt}(L) = C_0(r_l) + \tilde{C}_0(r_l)\log(m_\pi L) + \frac{C_1(r_l)}{m_\pi L} \qquad (r_l = m_l/m_\pi)$$

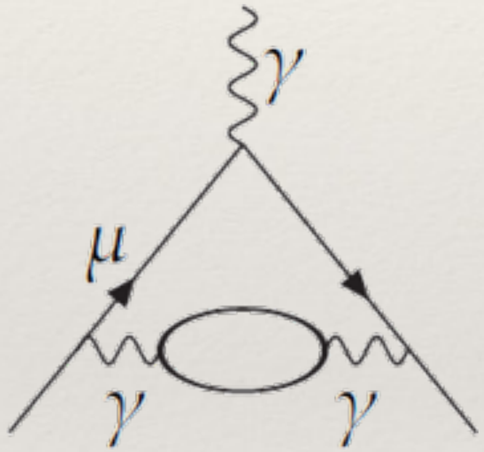
- these finite volume effects are universal, i.e. structure independent

  they should therefore cancel in $\Gamma_0 - \Gamma_0^{\rm pt}$

- $O(1/L^2)$ corrections are structure dependent

QCD+QED: first applications for ME

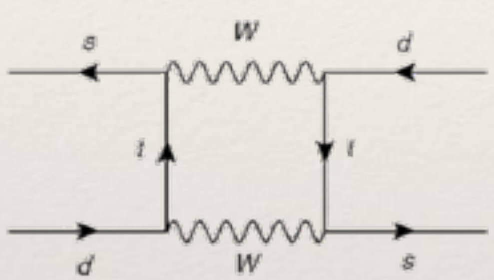
- light flavour matrix elements f_π, f_K, $f_+(0)$, …
  the way we do exp+th. analyses will change - reference to *decay constants* and *form factors* may disappear
- lattice predictions of leading hadronic contribution to muon g-2

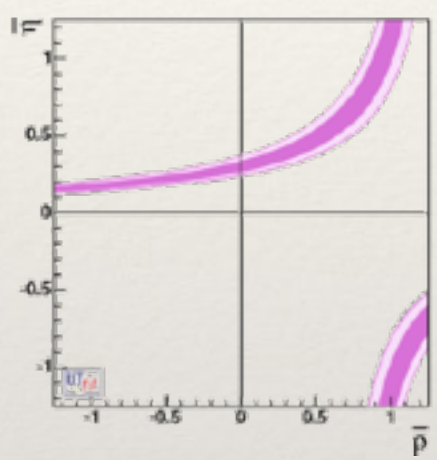


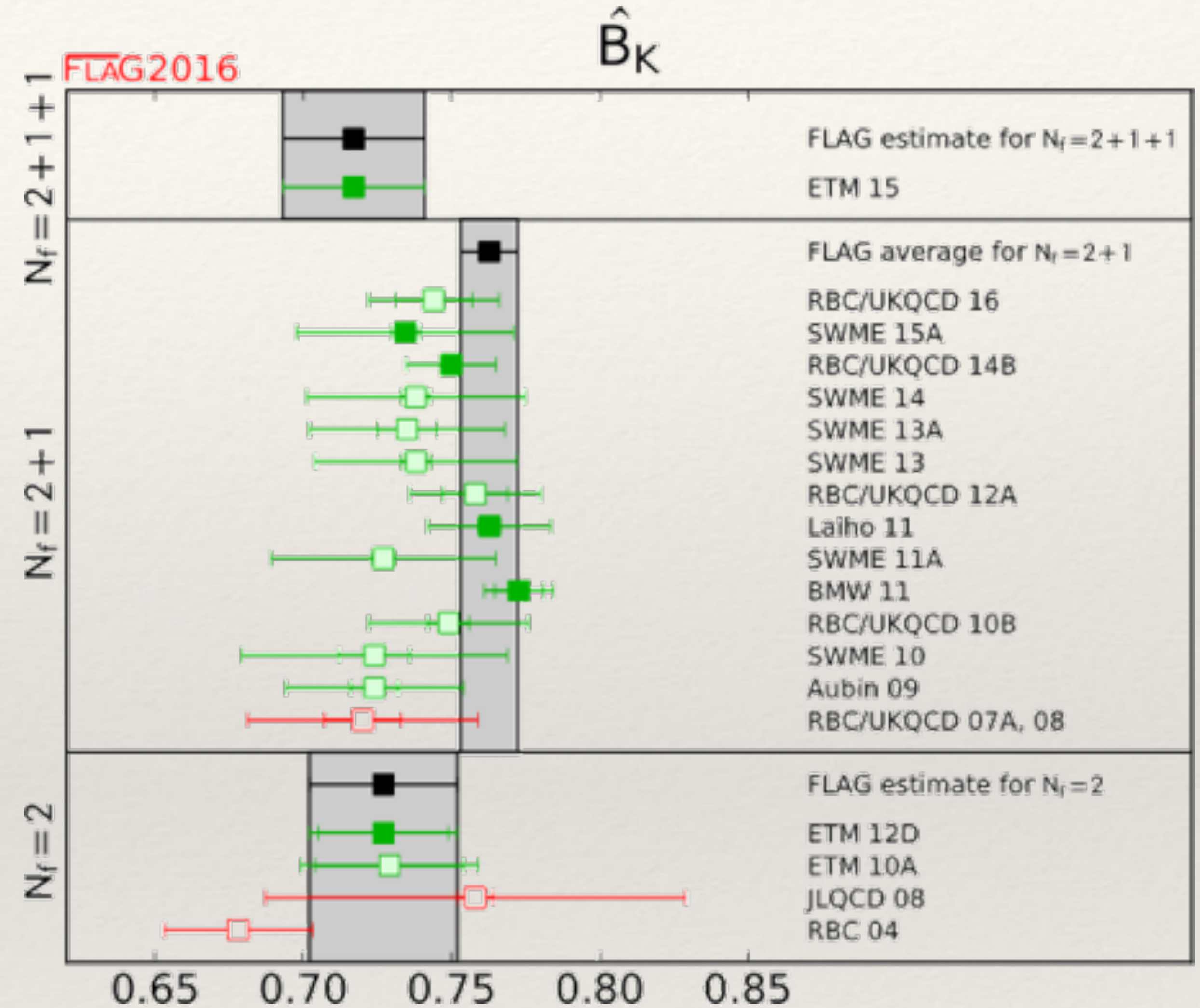
- lattice (m_u-$m_d = 0$, $\alpha_{EM} = 0$) is getting competitive with experimental determination ($e^+e^-\rightarrow hadrons$))
- next step would be inclusion of isospin breaking effects

inclusion of QED effects will be one of the big challenges in Lattice phenomenology over the next years

Neutral kaon mixing - short distance







- SM kaon bag parameter here excellent agreement at the 1.3%-level
- results for BSM bag parameters B_2, B_3, B_4, B_5 also available → FLAG report

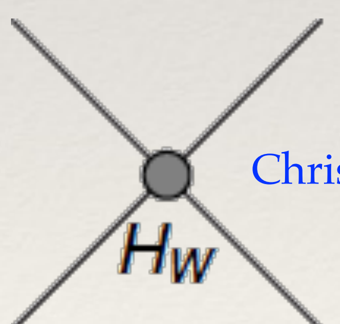
Long distance effects in neutral kaon mixing

$$\epsilon_K = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = e^{i\Phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}\langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle}{\Delta M_K} + \text{L.D. effects} \right)$$

L.D. effects
Buras, Guadagnoli PRD 78 (2008)
Buras, Guadagnoli, Isidori, PLB 688 (2010)

Long Distance effects amount to O(5%), so certainly worth considering on the lattice

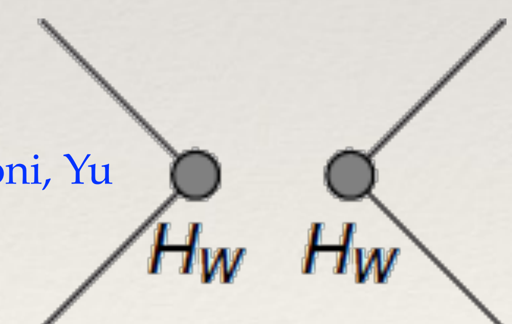
1st order Weak



single 4-quark OP,
length scale 10^{-18}m

Christ, Izubuchi, Sachrajda, Soni, Yu
arXiv:1212.5931

2nd order Weak

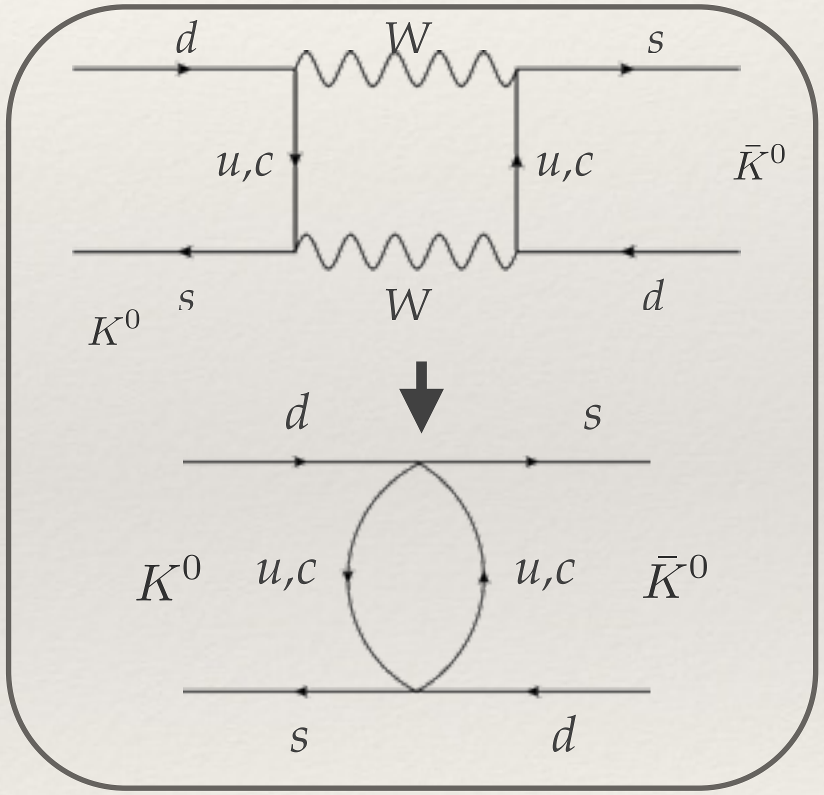


two 4-quark OP
length scale $1/\Lambda_{QCD}$

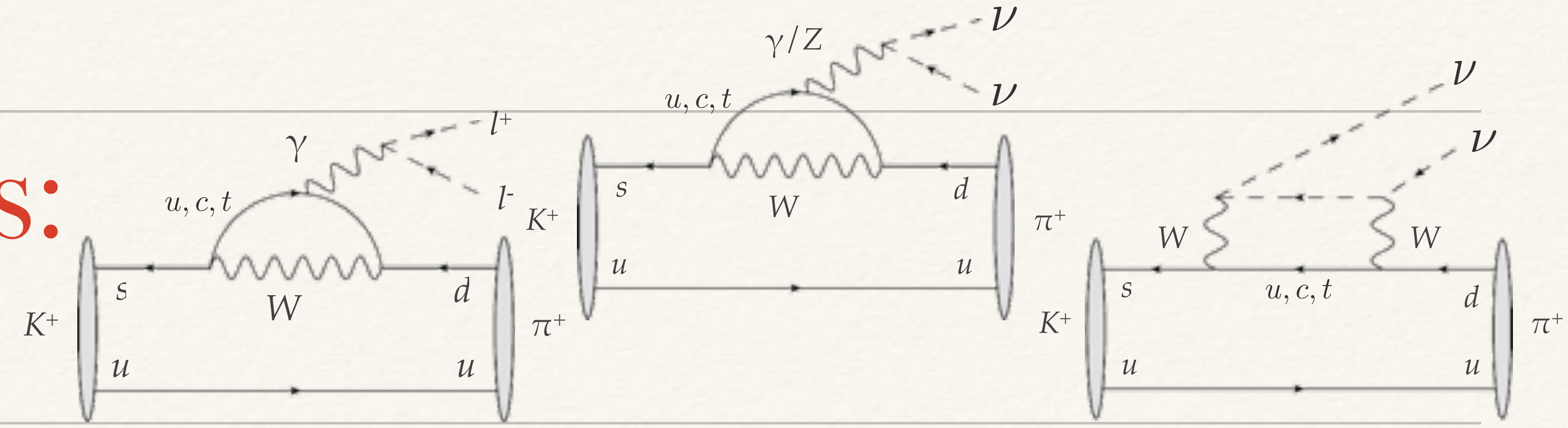
Long distance effects in kaon mixing: ΔM_K

$$\boxed{\Delta M_K = m_{K_S} - m_{K_L} = \boxed{2\mathrm{Re}M_{\bar{0}0}}} \qquad M_{\bar{0}0} = \mathcal{P}\sum_\lambda \frac{\langle \overline{K^0}|H_W|\lambda\rangle\langle\lambda|H_W|K^0\rangle}{m_K - E_\lambda}$$

- experimentally ΔM_K=3.483(6)×10^{-12}MeV (PDG)
- 2nd order EW, suppressed by 14 orders of magnitude with respect to QCD $\rightarrow$ poses strong BSM constraints (e.g. $(1/\Lambda)^2$ $\bar{s}d\bar{s}d$ BSM contribution) knowing ΔM_K at 10%-level → **Λ≥10⁴TeV**
- SD about 70% of experimental value - rest LD?
- PT large contributions at $\mu \sim m_c$ where PT turns out to converge badly (NLO->NNLO constitutes 36% correction) Brod, Gorbahn PRL 108 121801 (2012) **arXiv:1108.2036**



long distance effects: Rare kaon decays



Two new experiments dedicated to rare kaon decays NA62 (CERN) and KOTO (J-PARC) are running

- FCNC (W-W or γ/Z-exchange diagrams)
- deep probe into flavour mixing and SM/BSM due to suppression in the SM
- can determine V_{td}, V_{ts} and test SM

$K_L \to \pi^0 \nu\bar{\nu}$

- KOTO (J-PARC)
- direct CP violation
- exp. BR $\leq 2.6 \times 10^{-8}$
  theory BR $3.0(3) \times 10^{-11}$
- GIM → top dominated and charm suppressed, purely SD

$K^+ \to \pi^+ l^+ l^-$ $\quad$ $K_S \to \pi^0 l^+ l^-$

- 1-photon exchange LD dominated
- indirect contribution to CP-violating rare K_L decay
- SM prediction mainly ChPT
- lattice can predict ME and LECs
- experimenters will be able to look at these channels as well

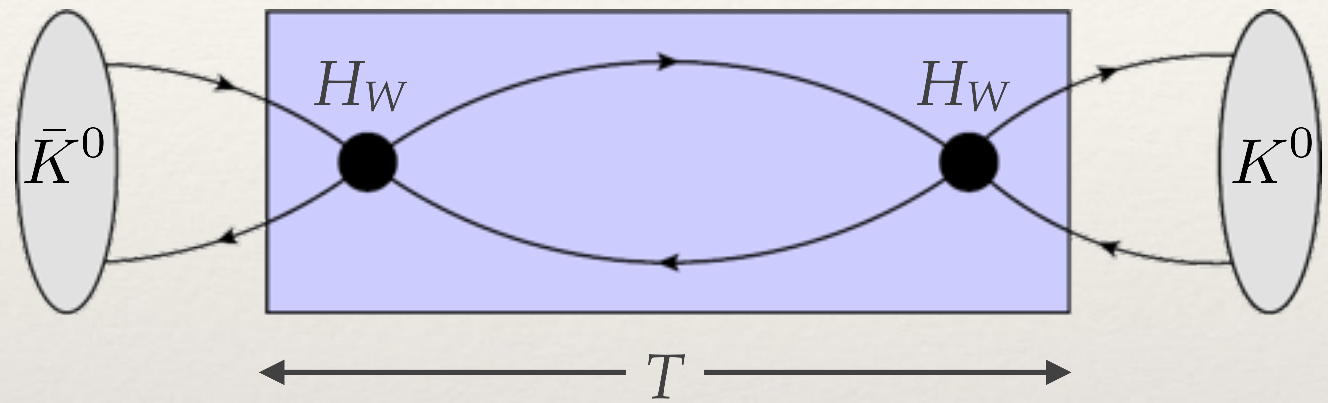
$K^+ \to \pi^+ \nu\bar{\nu}$

- NA62 (CERN)
- CP conserving
- exp. BR $1.73(^{+1.15}_{-1.05}) \times 10^{-10}$
  theory BR $0.911(72) \times 10^{-10}$
- small LD contribution, candidate for lattice

compute in lattice QCD

long distance effects

In previous examples generic situation is two Weak Hamiltonians separated by hadronic length scales



Integrate operators (here H_W) over time interval where initial and final kaon dominate

$$\mathcal{A} = \langle 0| T \left\{ K^0(t_f) \frac{1}{2} \int_{t_A}^{t_B} dt_2 \int_{t_A}^{t_B} dt_1 H_W(t_2) H_W(t_1) {K^0}^\dagger(t_i) \right\} |0\rangle$$

long distance effects – ΔM_K

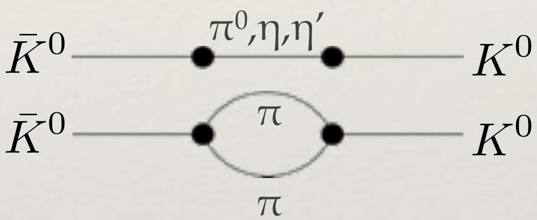
N. Christ et al. PRD 88 (2013) 014508 **arXiv:1212.5931**
Bai et al. PRL 113 (2014) 112003 **arXiv:1406.0916**

$$\mathcal{A} = N_K^2 e^{-M_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(-T - \frac{1}{M_K - E_n} + \frac{e^{(M_K - E_n)T}}{M_K - E_n} \right)$$

amplitude Δm_K^{FV}

irrelevant constant

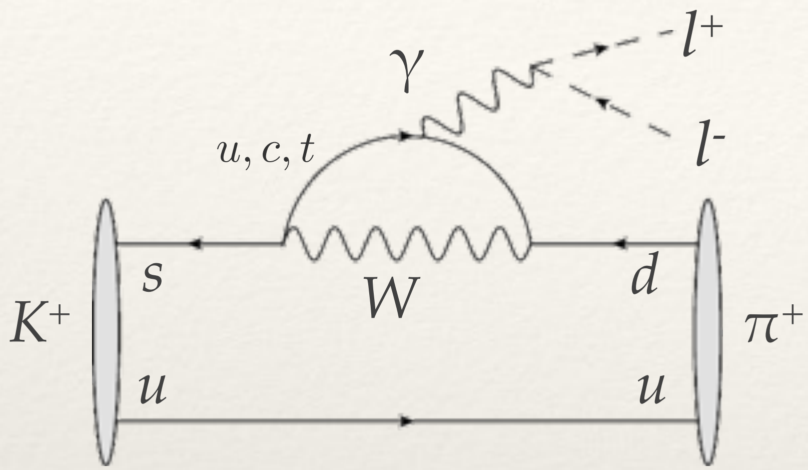
exponential term needs to be subtracted



- multiple hadrons in intermediate states causing difficulties and need to be subtracted
- finite volume corrections from two-particle intermediate state can be sizeable
  N. Christ et al. PRD91 (2015) 114510 **arXiv:1504.01170** also: Briceno, Hansen **arXiv:1502.04314**
  extension of Lellouch-Lüscher correction to 2nd order weak MEs
  $\Delta^{\mathrm{FV}}(\Delta M_K) = -\cot(\phi(M_K) + \delta_0(M_K)) \frac{d(\phi(E)+\delta_0(E))}{dE}\Big|_{E=M_K} |\langle \bar{K}^0 | H_W | \pi\pi, M_K \rangle^{\mathrm{V}'}|^2$
- what happens when the two H_W approach each other (GIM in action)?

Rare kaon decays $K^+ \to \pi^+ l^+ l^-$

N. Christ et al. arXiv:1507.03094
arXiv:1602.01374



LD contribution given through $K \to \gamma^*$ contribution which is computed as

$$\mathcal{A}_\mu = (q^2) \int d^4x \langle \pi(p) | T\left[J_\mu(0) H_W(x)\right] | K(k) \rangle$$

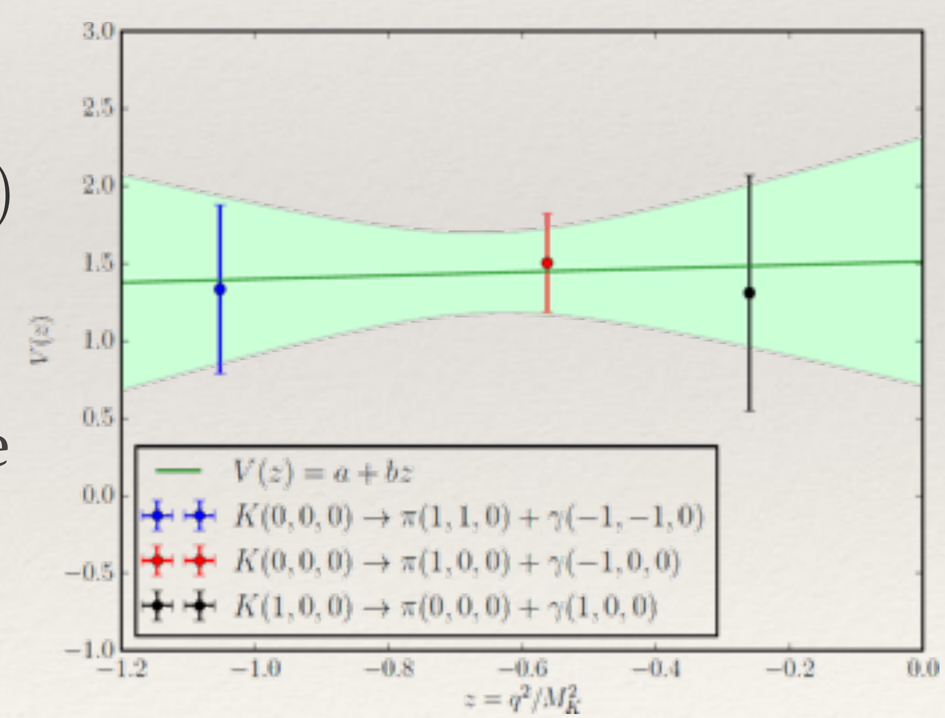
dominant operators: $Q_1^q = \left(\bar{s}_i \gamma_\mu^L d_i\right)\left(\bar{q}_j \gamma_\mu^L q_j\right)\,, \qquad Q_2^q = \left(\bar{s}_i \gamma_\mu^L q_i\right)\left(\bar{q}_j \gamma_\mu^L d_j\right)$

Decay amplitude in terms of elm. transition form factor

$$A_i = -\frac{G_F \alpha}{4\pi} V_i(z)(k+p)^\mu \bar{u}_l(p_-)\gamma_\mu \nu_l(p_+) \qquad (i = +, S)$$

$$V_i(z) = a_i + b_i z + V_i^{\pi\pi}(z)$$

- the a_S and a_+ can be extracted from experiment or lattice
- a_S parameterises also the CP-violating contribution to the K_L decay



Summary II

- considerable set of SM parameters, spectra and matrix elements now reliably and precisely predicted in full lattice QCD — "bread and butter"
- results with good control over systematics summarised by Flavour Lattice Averaging Group (FLAG) (3rd edition is out)
- **New challenges in Flavour physics:**
  - precision on "bread and butter" such that isospin breaking in matrix elements and spectra needs to be taken into account
  - long distance effects (neutral main mixing, rare kaon decays, …)

loads of new questions and theoretical problems and potential impact on SM and BSM phenomenology