

# The determination of the strong coupling $\alpha_s(m_Z)$

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Based on: M. Dalla Brida, P. Fritzsch, T. Korzec, A. R., S. Sint, R. Sommer,  
Phys.Rev.Lett. 117 (2016) no.18, 182001; Phys.Rev.D (2017) no.95, 014507.

M. Bruno, M. Dalla Brida, P. Fritzsch, T. Korzec, A. R.,  
S. Schaefer, S. Sint, H. Simma, R. Sommer,  
PoS LATTICE2016 (2016) 197

# OVERVIEW

Motivation

Lattice QCD

Finite size scaling

High energies

Low energies

Matching with QCD

Conclusions

# MOTIVATION

## Computing the strength of fundamental interactions

- ▶ Take some experimental observable  $O(\mu; p)$ .
- ▶ Work hard to get

$$O(\mu; p) = A(p)\alpha_{\overline{\text{MS}}}(\mu) + B(p)\alpha_{\overline{\text{MS}}}^2(\mu) + \dots$$

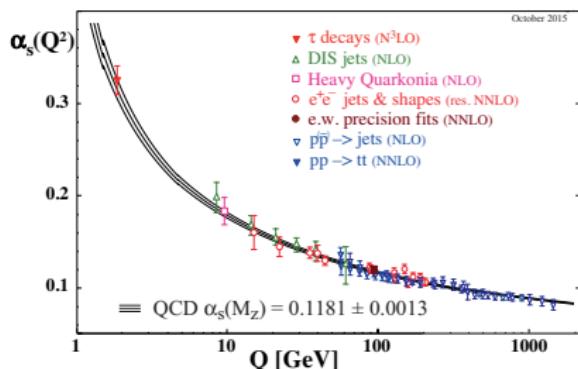
- ▶ Determine  $\alpha_{\overline{\text{MS}}}(\mu)$  by comparing experiment and theory computation

$g_e - 2 : \alpha_{em}$	$= 7.297\,352\,5698(24) \times 10^{-3}$	$\tau : \alpha_s(M_Z)$	$= 0.1198(15)$
$\text{recoil} : \alpha_{em}$	$= 7.297\,352\,585(48) \times 10^{-3}$	$e^+ e^- : \alpha_s(M_Z)$	$= 0.1172(37)$

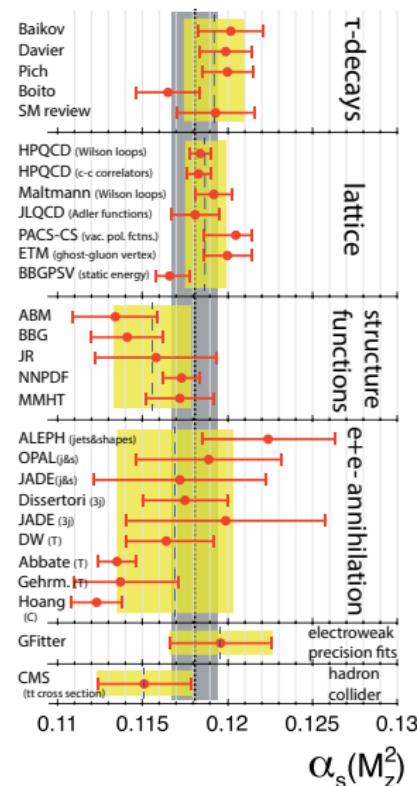
## Caveats

- ▶ In strong interactions asymptotic states are not quarks/gluons.
- ▶ What about higher orders in PT?. Resummation, Renormalons,  $\delta_{\text{NP}}$ , ...
- ▶ What about non-perturbative contributions?

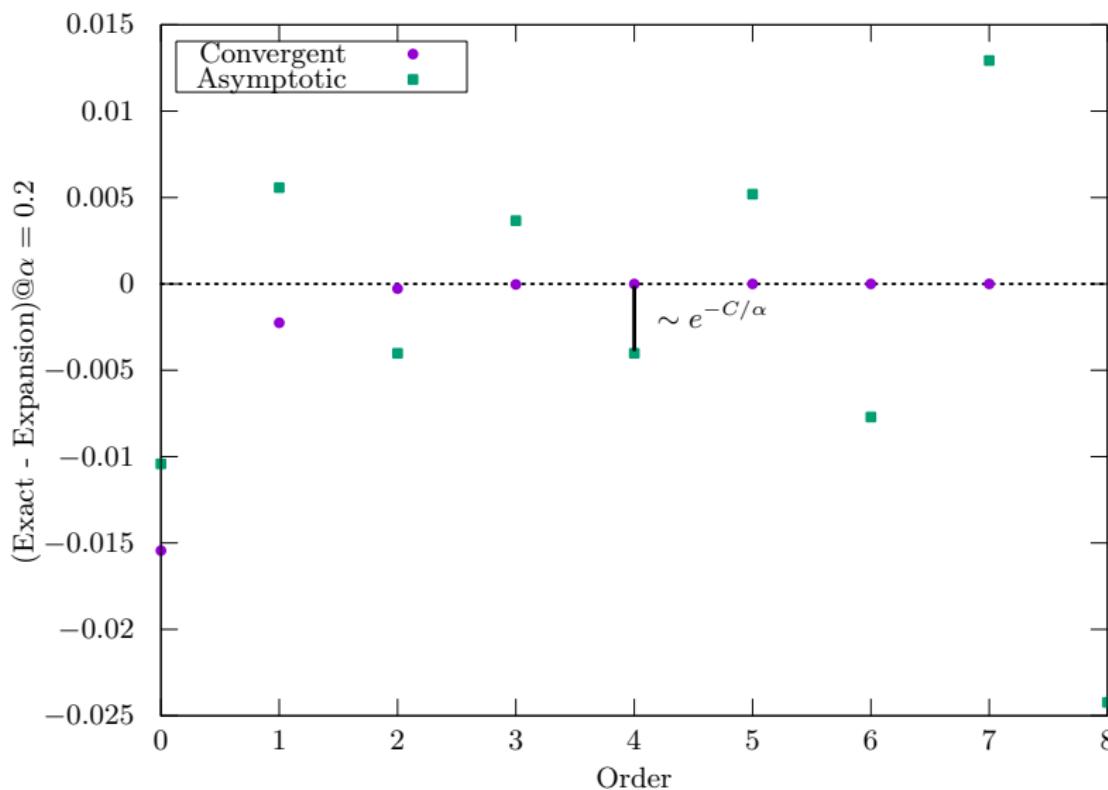
# DETERMINATIONS OF $\alpha_s(m_Z)$ (PDG '16)



- Low energy determinations are more precise (!!?)



# ASYMPTOTIC EXPANSIONS: HIGH ORDER $\neq$ HIGH ACCURACY



# ASYMPTOTIC EXPANSIONS: HIGH ORDER $\neq$ HIGH ACCURACY

## Asymptotic expansions

- ▶ Zero radius of convergence

$$O(\alpha) = \int_0^\infty \frac{e^{-t/\alpha}}{1+t} dt \sim \sum_n (-1)^n n! \alpha^{n+1}$$

- ▶ Useful ( $O(0.2) = 1.70\ 42\ 21\ 76\ 28 \times 10^{-1}$ )

$$O_2(0.2) = 1.60\ 00\ 00\ 00\ 00 \times 10^{-1} \quad O_5(0.2) = 1.66\ 40\ 00\ 00\ 00 \times 10^{-1}$$

- ▶ Limited by the size of  $e^{-C/\alpha}$  terms
- ▶  $e^{-C/\alpha}$  vanishes faster than any power  $\alpha^p \implies$  Negligible at “small”  $\alpha$

## Moral

With asymptotic series, smaller  $\alpha$  always better than higher order

# $\alpha_{\overline{\text{MS}}}(m_Z)$ FROM HADRONIC INPUT ( $M_p, M_\pi, \dots$ )

## This project

- ▶ Define a coupling (i.e. scheme) by using a “physical” observable

$$\alpha_s(\mu) = O(\mu) \sim \alpha_{\overline{\text{MS}}}(\mu) + c_1 \alpha_{\overline{\text{MS}}}^2(\mu) + \dots$$

- ▶ Determine non-perturbatively the  $\beta$ -function for  $\mu_{\text{had}} < \mu < \mu_{\text{EW}}$

$$\mu \frac{dg_s^2(\mu)}{d\mu} = \beta_s(x) \sim -b_0 x^3 - b_1 x^5 + \dots$$

- ▶ With hadronic input determine  $\Lambda_s$  (**PT only used at  $\mu_{\text{EW}}$ ,  $\alpha \sim 0.1$** )

$$\Lambda_s = \left[ b_0 g_s^2(\mu) \right]^{-b_1/2b_0^2} e^{-\frac{1}{2b_0 g_s^2(\mu)}} \exp \left\{ - \int_0^{g_s(\mu)} dx \left[ \frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0 x} \right] \right\}$$

- ▶ Determine  $\alpha_{\overline{\text{MS}}}(m_Z)$  from  $\Lambda_s$

## Key elements in this strategy

- ▶ Lattice QCD
- ▶ Finite size scaling

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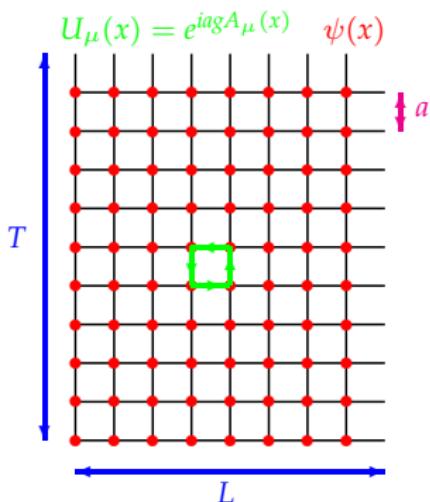
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# COMPUTING PATH INTEGRALS: LATTICE FIELD THEORY

Lattice field theory → Non Perturbative definition of QFT.



- Discretize space-time in an hyper-cubic lattice (spacing  $a$ )
- Path integral → multiple integral (one variable for each field at each point)
- Compute the integral numerically → Monte Carlo sampling.

$$\langle O \rangle = \frac{1}{N_{\text{conf}}} \sum_{i=1}^{N_{\text{conf}}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\text{conf}}})$$

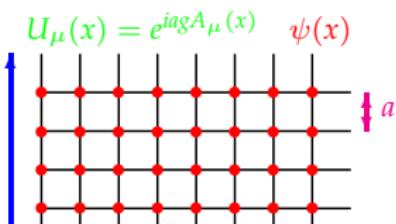
Observable computed averaging over samples

- This works both in the perturbative and non-perturbative regimes!

$$S_G[U] = \frac{\beta}{6} \sum_{p \in \text{Plaquettes}} \text{Tr}(1 - U_p - U_p^+) \xrightarrow{a \rightarrow 0} -\frac{1}{2} \int d^4x \text{Tr}(F_{\mu\nu} F_{\mu\nu})$$

# COMPUTING PATH INTEGRALS: LATTICE FIELD THEORY

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CLS ensembles ( $N_f = 3$  QCD) (Bruno et al. '15)

Lattice sp. $a$	UV cutoff $a^{-1}$	$L^{-1}$	$M_\pi$	$M_K$
0.086 fm	2.3 GeV	35 – 70 MeV	130 – 420 MeV	420 – 480 MeV
0.064 fm	3.1 GeV	50 – 64 MeV	200 – 420 MeV	420 – 480 MeV
0.05 fm	3.9 GeV	60 MeV	260 – 420 MeV	420 – 470 MeV

$$S_G[U] = \frac{\beta}{6} \sum_{p \in \text{Plaquettes}} \text{Tr}(1 - U_p - U_p^+) \xrightarrow[a \rightarrow 0]{} -\frac{1}{2} \int d^4x \text{Tr}(F_{\mu\nu} F_{\mu\nu})$$

s

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**Finite size scaling**

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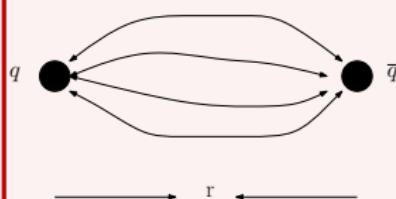
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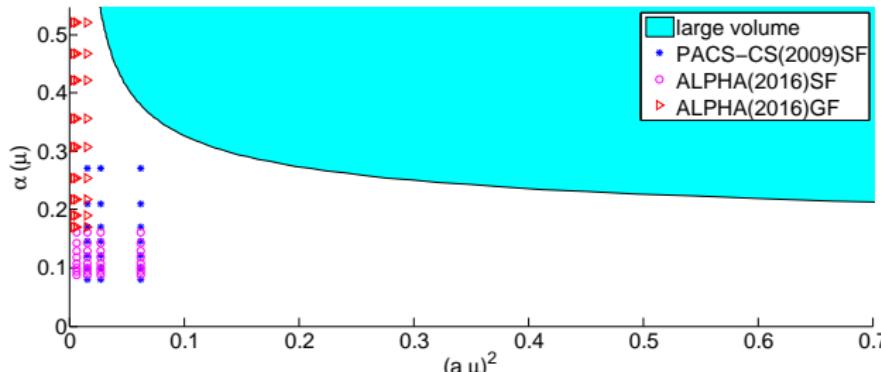
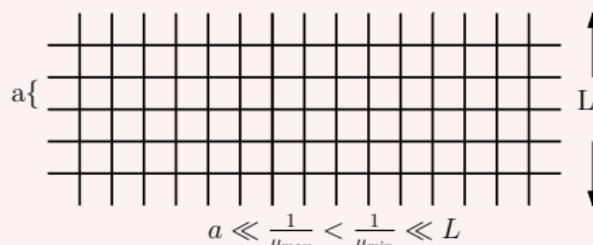
Conclusions

# THE WINDOW PROBLEM

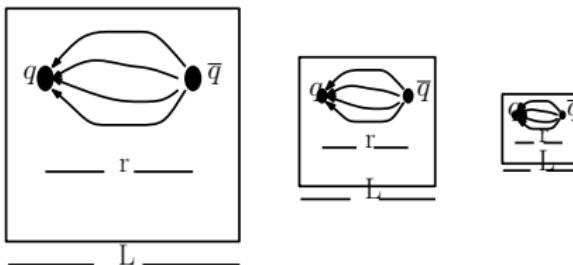
Brute force approach requires huge computer resources:  $L/a \gg 1000$ .



$$\alpha_{qq}(\mu) = \frac{3r^2}{4} F(r) \Big|_{\mu=1/r}$$



## THE SOUTION: FINITE SIZE SCALING (LÜSCHER, WEISZ, WOLFF '91)



Finite volume renormalization schemes: fix  $\mu L = \text{constant}$

- ▶ Coupling  $\alpha(\mu)$  depends on no other scale but  $L$  (Notation:  $\alpha(L), \alpha(1/L)$ ).
- ▶ Small  $L \implies$  small  $\alpha(L)$
- ▶  $a \ll 1/\mu$  easily achieved:  $L/a \sim 10 - 40$
- ▶ Step scaling function: How much changes the coupling when we change the renormalization scale:

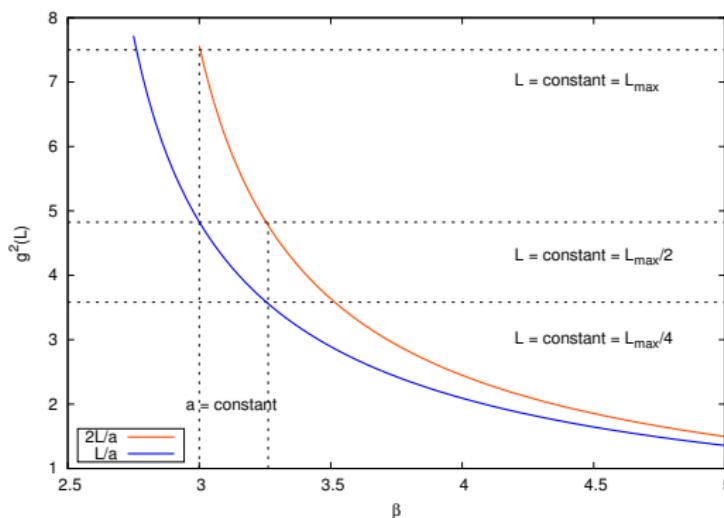
$$\sigma(u) = g^2(\mu/2) \Big|_{g^2(\mu)=u}$$

achieved by simple changing  $L/a \rightarrow 2L/a!$

- ▶  $1/L$  is a IR cutoff  $\Rightarrow$  simulate directly  $m_q = 0$
- ▶ We need dedicated simulations of the **femto-universe**

## THE SOUTION: FINITE SIZE SCALING (LÜSCHER, WEISZ, WOLFF '91)

$$\beta \iff a; \quad g^2(L) \iff L \iff \mu$$



Step scaling function

$$\Sigma^{-1}(u, a/L) = g^2(L/2) \Big|_{g^2(L)=u}$$

Continuum limit

$$\sigma^{-1}(u) = \lim_{a/L \rightarrow 0} \Sigma^{-1}(u, a/L)$$

Simulate several pair of lattices

# $\sigma(u) \implies \beta(g) \implies \text{WHATEVER!}$

(M. DALLA BRIDA, P. FRITZSCH, T. KORZEC, A. R., S. SINT, R. SOMMER)

- Once  $\sigma(u)$  is known, we can determine  $\beta(g) = \mu \frac{\partial g(\mu)}{\partial \mu}$

$$\log 2 = - \int_{g(\mu)}^{g(\mu/2)} \frac{dx}{\beta(x)} = - \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{dx}{\beta(x)}$$

- ... also the  $\Lambda$  parameter

$$\Lambda/\mu = \left[ b_0 g^2(\mu) \right]^{-b_1/2b_0^2} e^{-\frac{1}{2b_0 g^2(\mu)}} \exp \left\{ - \int_0^{g(\mu)} du \left[ \frac{1}{\beta(u)} + \frac{1}{b_0 u^3} - \frac{b_1}{b_0 u} \right] \right\}$$

(Equivalent to determining  $\alpha_s(M_Z)$ )

- ... and any ratio of scales

$$\log \frac{\mu_1}{\mu_2} = - \int_{g(\mu_1)}^{g(\mu_2)} \frac{dx}{\beta(x)}$$

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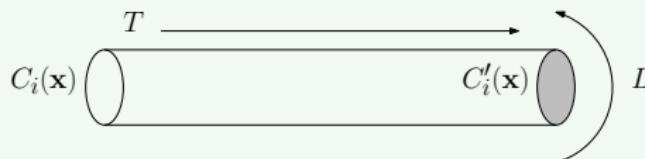
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## SF FAMILY OF COUPLINGS (SCHEMES)

Schrödinger Functional: Dirichlet bc at  $x_0 = 0, T$ , periodic in  $\mathbf{x}$



with the choice

$$C_i(\mathbf{x}) = \frac{t}{L} [\text{diag}(-\pi/3, 0, \pi/3) + \eta(\lambda_8 + \nu\lambda_3)]$$

$$C'_i(\mathbf{x}) = \frac{t}{L} [\text{diag}(-\pi, \pi/3, 2\pi/3) - \eta(\lambda_8 - \nu\lambda_3)]$$

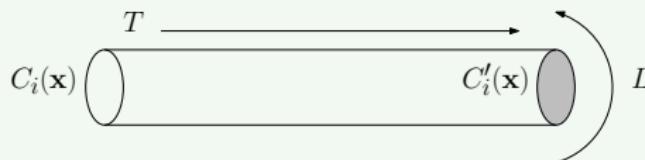
Coupling definition and properties

$$\frac{12\pi}{g_{\nu}^2(L)} = \left\langle \frac{\partial S}{\partial \eta} \Big|_{\eta=0} \right\rangle = \frac{12\pi}{g^2(L)} - 12\pi\nu\bar{v} \quad (1)$$

- ▶  $\delta_{\text{stat}} g^2 \sim \mathcal{O}(g^4) \implies$  high precision for small  $g^2$ .
- ▶ Finite volume  $\implies$  No IR renormalons. Known NP contribution  $\mathcal{O}(e^{-2.6/\alpha})$
- ▶ Known 3-loop  $\beta$ -function  $\implies \mathcal{O}(\alpha^2)$  corrections in the determination of  $\Lambda$

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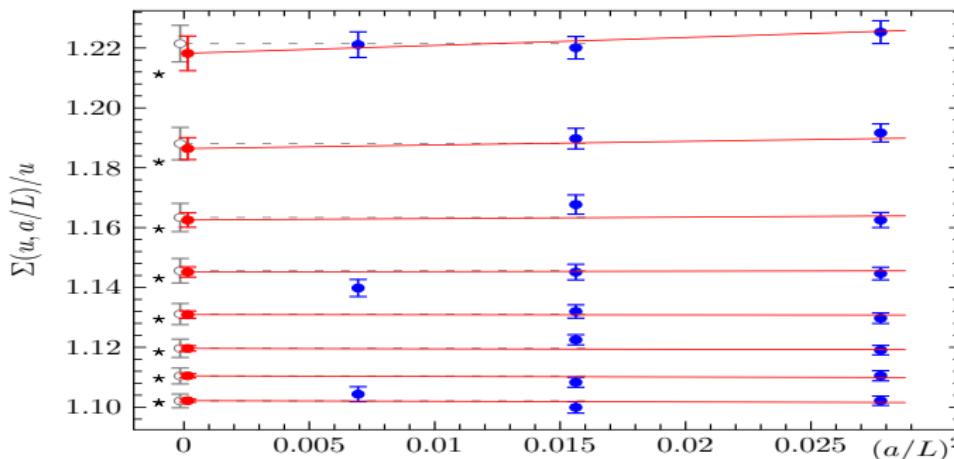
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- Each  $\nu$  a different scheme, different  $\Lambda_{\nu}$ , but exact relation known (S. Sint, R. Sommer '96)

$$\frac{\Lambda}{\Lambda_{\nu}} = e^{-1.25516 \times \nu}$$

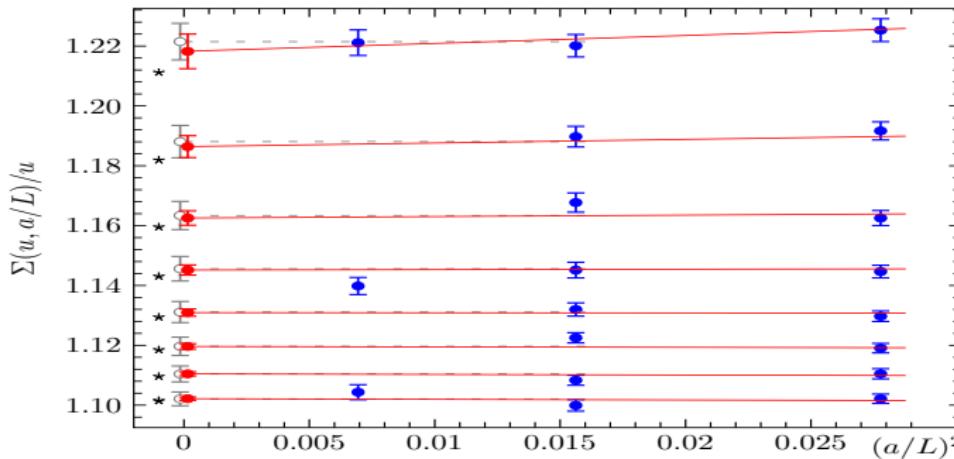
# STEP SCALING FUNCTION



- Perturbatively improve the lattice step scaling function ( $i = 1, 2$ )

$$\Sigma^{(i)}(u, a/L) = \frac{\Sigma(u, a/L)}{1 + \sum_k \delta_k(a/L) u^k}$$

# STEP SCALING FUNCTION



- ▶ Perform a global fit ( $n = 5$ ,  $s_0, s_1$  universal,  $s_2$  known).

$$\Sigma^{(i)}(u, a/L) = \sigma(u) + \left(\frac{a}{L}\right)^2 \rho^{(i)}(u)$$

$$\sigma(u) = \sum_{k=1}^3 s_k u^{k+1}; \quad \rho^{(i)}(u) = \sum_{k=1}^{n_\rho^{(i)}} \rho_k u^{k+i+1}$$

## STEP SCALING FUNCTION: EXCELLENT DESCRIPTION OF OUR DATA

fit	$u_n$	$i$	$\frac{L}{a} \Big _{\min}$	$n_\rho^{(i)}$	$n_c$	$L_0 \Lambda$ $\times 100$	$b_3^{\text{eff}}$ $\times (4\pi)^4$	$\chi^2$	d.o.f.
A	1.193(4)	0	6	2	1	3.04( 8)		14.7	16
B	1.194(4)	1	6	2	1	3.07( 8)		14.2	16
C	1.193(5)	2	6	2	1	3.03( 8)		14.5	16
D	1.192(7)	2	6	2	2	3.03(13)		14.5	15
E		2	6	2	1	3.00(11)	4(3)	14.6	16
F		2	8	1	1	3.01(11)	4(3)	12.7	9
G	1.191(11)	2	8	0	2	3.02(20)		13.0	9
H		1	6	2	1	3.04(10)	3(3)	14.1	16

fit	$\nu$	$i$	$\frac{L}{a} \Big _{\min}$	$n_\rho^{(i)}$	$n_c$	$L_0 \Lambda$ $\times 100$	$b_{3,\nu}^{\text{eff}}$ $\times (4\pi)^4$	$\chi^2$	d.o.f
H	-0.5	1	6	2	1	3.03(15)	11(5)	10.4	16
H	0.3	1	6	2	1	3.04(10)	0(3)	20.0	16

Choose a reference scale by the condition  $g_{\text{SF}}^2(L_0) = 2.012$

Excellent agreement in the determination of

$$L_0 \Lambda = 0.0303(8) \quad L_0 \Lambda_{\overline{\text{MS}}}^{(3)} = 0.0791(21) \quad (2.7\%)$$

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# CONNECTING WITH THE HADRONIC REGIME

$\delta g_{\text{SF}}^2 \sim g_{\text{SF}}^4$  is a killer for the precision at low energies

Welcome to the gradient flow...

## YANG-MILLS GRADIENT FLOW: BASICS (NARAYANAN, NEUBERGER '06; LÜSCHER '10)

- ▶ Add “extra” (flow) time coordinate  $t$  ( $\neq x_0$ ). Define gauge field  $B_\mu(x, t)$

$$\begin{aligned} G_{\nu\mu}(x, t) &= \partial_\nu B_\mu(x, t) - \partial_\mu B_\nu(x, t) + [B_\nu(x, t), B_\mu(x, t)] \\ \frac{dB_\mu(x, t)}{dt} &= D_\nu G_{\nu\mu}(x, t); \quad B_\mu(x, t=0) = A_\mu(x). \end{aligned}$$

- ▶ Since

$$\frac{dB_\mu(x, t)}{dt} = D_\nu G_{\nu\mu}(x, t) \quad \left( \sim -\frac{\delta S_{\text{YM}}[B]}{\delta B_\mu} \right)$$

$$\lim_{t \rightarrow \infty} B_\mu(t, x) = A_\mu^{\text{classical}}(x).$$

- ▶ Correlation functions of the “smooth” field  $B_\mu(x, t)$

$$G(x_1, x_2, \dots) = \langle B(x_1, t) B(x_2, t) \dots \rangle$$

are finite after the usual bare parameter renormalization (Lüscher, Weisz. '11).

- ▶ For example, in pure YM

$$E(x, t) = \frac{1}{4} \langle G_{\mu\nu}(x, t) G_{\mu\nu}(x, t) \rangle$$

is finite (for  $t > 0$ ) after the usual coupling renormalization.

## GRADIENT FLOW: HOW IT WORKS

$$\frac{dB_\mu(x, t)}{dt} = D_\nu G_{\nu\mu}(x, t); \quad B_\mu(x, 0) = A_\mu(x)$$

Expand the flow field in powers of  $g_0$ .

$$B_\mu(x, t) = \sum_{n=1}^{\infty} B_{\mu,n}(x, t) g_0^n$$

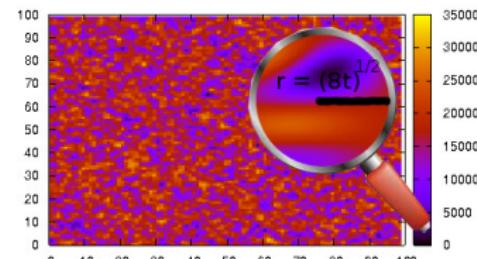
$GF \equiv$  Heat equation (+ gauge terms)

$$\frac{dB_{\mu,1}(x, t)}{dt} = \partial_\nu^2 B_{\mu,1}(x, t)$$

that has solution

$$B_{\mu,1}(x, t) = \sum_p e^{-p^2 t} e^{ipx} \tilde{A}_\mu(p)$$

$$B_{\mu,1}(x, t) = \frac{1}{4\pi t} \int d^4y e^{-\frac{(x-y)^2}{4t}} A_\mu(y)$$



We are “looking” at world with a resolution  $\sim \sqrt{8t}$ .

## GRADIENT FLOW: COUPLING (LÖSCHER '10)

Take the Energy density as a candidate observable

$$\langle E(t) \rangle = \frac{1}{4} \int \mathcal{D}A_\mu G_{\mu\nu}^a(x, t) G_{\mu\nu}^a(x, t) e^{-S[A]}$$

In perturbation theory we have:

$$\langle E(t) \rangle = \frac{3g_{\overline{MS}}^2}{16\pi^2 t^2} (1 + c_1 g_{\overline{MS}}^2 + \mathcal{O}(g_{\overline{MS}}^4))$$

and in terms of the running coupling  $\alpha(\mu)$  at scale  $\mu = 1/\sqrt{8t}$ .

$$t^2 \langle E(t) \rangle = \frac{3}{4\pi} \alpha_{\overline{MS}}(\mu) \left[ 1 + c'_1 \alpha_{\overline{MS}}(\mu) + \mathcal{O}(\alpha_{\overline{MS}}^2) \right]$$

Therefore one can define the strong coupling at a scale  $\mu = 1/\sqrt{8t} = 1/\textcolor{red}{c} L$

$$\alpha(\mu) = \# t^2 \langle E(t) \rangle = \alpha_{\overline{MS}}(\mu) + \dots$$

- ▶ Non-perturbative definition.
- ▶ Easy to evaluate on the lattice.
- ▶ precise (smooth observable).

# WHY IS A GOOD CHOICE? $N_f = 2$ AND $SU(3)$ SIMULATIONS (P. FRITZSCH, A.R. '13)

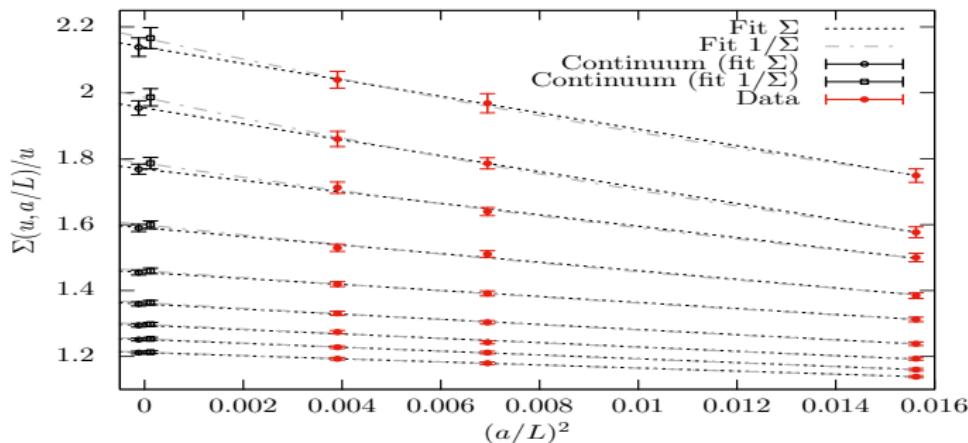
	$L/a$	6	8	10	12	16
$\beta$	5.2638	5.4689	5.6190	5.7580	5.9631	
$\kappa_{\text{sea}}$	0.135985	0.136700	0.136785	0.136623	0.136422	
$N_{\text{meas}}$	12160	8320	8192	8280	8460	
$\bar{g}_{\text{SF}}^2(L_1)$	4.423(75)	4.473(83)	4.49(10)	4.501(91)	4.40(10)	
$\bar{g}_{\text{GF}}^2(\mu)$ ( $c = 0.3$ )	4.8178(46)	4.7278(46)	4.6269(47)	4.5176(47)	4.4410(53)	
$\bar{g}_{\text{GF}}^2(\mu)$ ( $c = 0.4$ )	6.0090(86)	5.6985(86)	5.5976(97)	5.4837(97)	5.410(12)	
$\bar{g}_{\text{GF}}^2(\mu)$ ( $c = 0.5$ )	7.106(14)	6.817(15)	6.761(19)	6.658(19)	6.602(24)	

## Advantages of GF coupling definition

- $\mathcal{O}(10^3)$  less expensive at  $g^2 \sim 4$  (1 CPU day  $\rightarrow$  some CPU years).
- Finite variance when  $a \rightarrow 0$  (i.e.  $\mathcal{V} \sim \langle E^2(t) \rangle - \langle E(t) \rangle^2$ ).
- Statistical precision independent of coupling value  $\delta g^2/g^2 \sim \text{constant}$ .
- Smaller  $c \implies$  Larger cutoff effects, more precision. ( $c \in [0.3, 0.5]$ )

Ideal for matching with hadronic regime of QCD

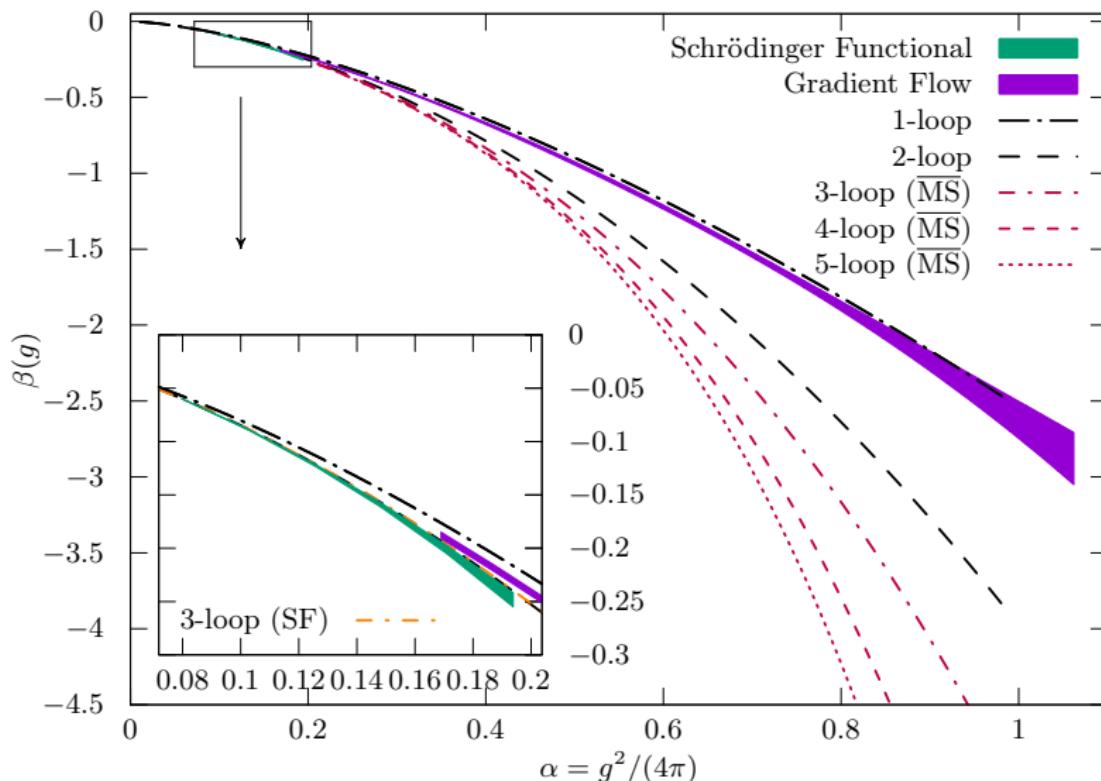
# PRECISE DETERMINATION OF $\sigma_{\text{GF}}(u)$ FOR $\alpha_{\text{GF}}(L) \in [0.2, 1]$ ( $\sim 4 - 0.2 \text{ GeV}$ )



- ▶ Cutoff effects larger than in the  $\alpha_{\text{SF}}$  scheme
- ▶ Detailed investigation in a EFT approach (A.R., S. Sint '16)
- ▶ +20 pages discussion in (Phys.Rev.D (2017) no.95, 014507)
- ▶ +4 variations to fit cutoff effects
- ▶ Accurate determination of  $\sigma_{\text{GF}}(u)$  with  $L/a = 8, 12, 16 \rightarrow 16, 24, 32$

# A PRECISE DETERMINATION OF $\alpha_s(m_Z)$ FROM THREE-FLAVOR QCD

( M. DALLA BRIDA ET AL. PHYS.REV.D (2017) NO.95, 014507 )



# A PRECISE DETERMINATION OF $\alpha_s(m_Z)$ FROM THREE-FLAVOR QCD

( M. DALLA BRIDA ET AL. PHYS.REV.D (2017) NO.95, 014507 )

Define hadronic scale by  $g_{GF}^2(L_{\text{had}}) = 11.31$

- ▶ Non-perturbative matching between SF and GF schemes at  $L_0$

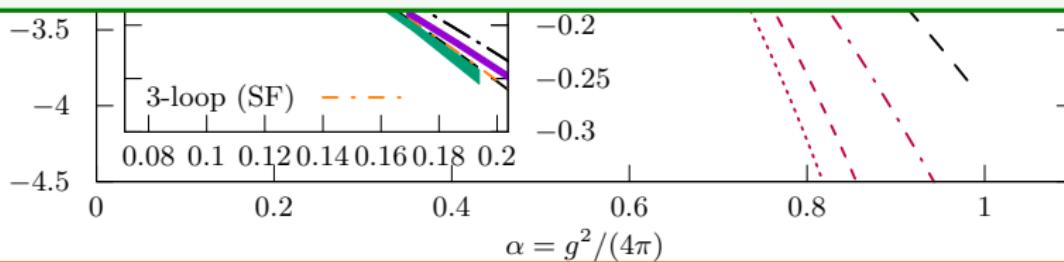
$$g_{GF}^2(2L_0) = 2.6723(64)$$

- ▶ Determination of  $\beta$ -function gives

$$\frac{L_{\text{had}}}{L_0} = 2 \exp \left\{ - \int_{\sqrt{2.6723(64)}}^{\sqrt{11.31}} \frac{dx}{\beta(x)} \right\} = 21.86(42)$$

- ▶ Using our previous matching with asymptotic behavior

$$L_{\text{had}} \Lambda_{\overline{\text{MS}}}^{(3)} = \frac{L_{\text{had}}}{L_0} \times L_0 \Lambda_{\overline{\text{MS}}}^{(3)} = 1.729(57) \quad (3.2\%)$$



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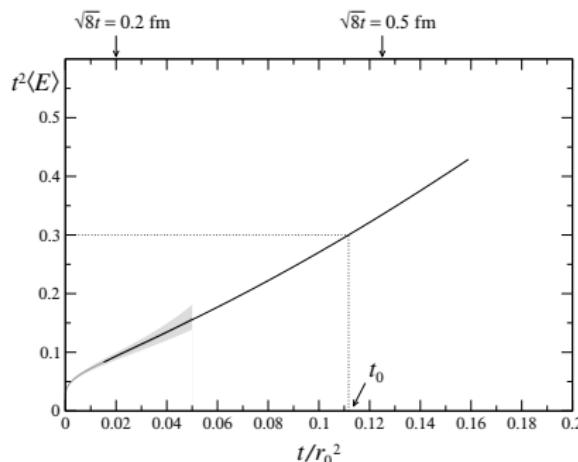
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# MATCHING WITH A QCD EXPERIMENTAL QUANTITY

The computation of  $\frac{L_{\text{had}}}{f_{\pi K}}$

Welcome to CLS

## $t_0^*$ AS AN INTERMEDIATE REFERENCE SCALE



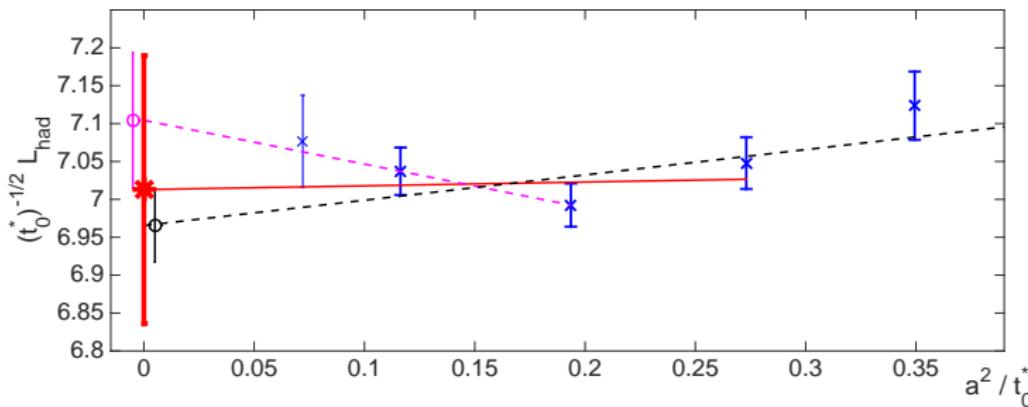
- ▶  $t^2 \langle E(t) \rangle$  is dimensionless.
- ▶ Depends on scale  $\mu = 1/\sqrt{8t}$
- ▶ Ideal candidate for scale setting:  $t_0$  (M. Lüscher JHEP 1008 '10).
- ▶  $t^2 \langle E(t) \rangle|_{t=t_0^*} = 0.3$  at  $m_\pi = m_K$

### Scale setting on CLS ensembles (Bruno et al. '16)

- ▶ Use PDG data for  $f_{\pi K} = (2f_K + f_\pi)/3 = 147.6 \text{ MeV}$
- ▶ Compute the dimensionless quantity  $\sqrt{8t_0^*} f_{\pi K}$
- ▶ Main result

$$\sqrt{8t_0^*} = 0.413(5)(2) \text{ fm}$$

# PRELIMINARY DETERMINATION OF $\Lambda_{\overline{\text{MS}}}^{(3)}$



## Preliminary determination

- Compute the dimensionless ratio (preliminary)  $L_{\text{had}} / \sqrt{t_0^*} = 7.01(18)$
- Combine with previous results

$$L_{\text{had}} = 1.03(3) \text{ fm}$$

$$\Lambda_{\overline{\text{MS}}}^{(3)} = L_{\text{had}} \times \frac{\Lambda_{\overline{\text{MS}}}^{(3)}}{L_{\text{had}}} = 332(14) \text{ MeV} \quad (4.2\%)$$

# PRELIMINARY DETERMINATION OF $\alpha_{\overline{\text{MS}}}(m_Z)$

Connecting to the 5-flavor theory

- ▶ Use 4-loop PT to connect the 3-flavor with the 5-flavor theory

$$\Lambda_{\overline{\text{MS}}}^{(3)} = 332(14) \text{ MeV} \rightarrow \Lambda_{\overline{\text{MS}}}^{(4)} = 289(14) \text{ MeV} \rightarrow \Lambda_{\overline{\text{MS}}}^{(5)} = 207(11) \text{ MeV}$$

- ▶ Use  $\Lambda_{\overline{\text{MS}}}^{(5)} = 207(11) \text{ MeV}$

$$\alpha_{\overline{\text{MS}}}(m_Z) = 0.1179(10)(2)$$

# OVERVIEW

Motivation

Lattice QCD

Finite size scaling

High energies

Low energies

Matching with QCD

Conclusions

## CONCLUSIONS

- ▶ Lattice QCD and Finite size scaling provide a sound theoretical approach to the computation of the fundamental parameters of the SM.
- ▶ Together with the new schemes based on the gradient flow, it has allowed us to determine  $\alpha_{\overline{\text{MS}}}(m_Z) = 0.1179(10)(2)$  (sub-percent precision)
- ▶ High energy determination of  $\Lambda_{\overline{\text{MS}}}^{(3)}$  from hadronic input
  - ▶ Non-perturbative running from 200 MeV to 100 GeV
  - ▶ PT only used at 100 GeV

### Perspectives

- ▶ Perturbative crossing of quark thresholds can be avoided ( $m_c, m_b$ )
- ▶ Better matching with asymptotic PT regime
- ▶ Reduction of uncertainty is definitively possible