Solving the Goldstone boson catastrophe in generic theories and two-loop Higgs masses in non-supersymmetric models

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in collaboration with Mark Goodsell and Florian Staub
based on *arXiv:1609.06977* and *arXiv:170x.xxxxx* (in progress)

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The context

Going Beyond the Standard Model

- 2012: discovery of a SM-Higgs-like particle by ATLAS and CMS
- No Physics beyond the SM found yet

$\Rightarrow$ properties of the Higgs as a probe for new Physics $\rightarrow$ Higgs mass m_h^2

State of the art

- **SM**: V_{eff} (relates $m_h^2 \leftrightarrow \lambda$) is known to full 2-loop (*Ford, Jack and Jones '92*) + leading – QCD – 3-loop and 4-loop (*Martin '13, Martin '15*)
- Some results for m_h^2 in specific SUSY theories: **MSSM** (leading – SQCD – 3-loop order); **NMSSM** (2-loop); **Dirac Gaugino models** (leading – SQCD – 2-loop: *J.B., Goodsell, Slavich '16*)
- **Generic theories**: V_{eff} computed to 2-loop (Martin '01), tadpoles and scalar masses (in gaugeless limit) implemented in SARAH (*Goodsell, Nickel, Staub '15*)

The Goldstone Boson Catastrophe

- Beyond one loop, $V_{\rm eff}$ only computed in Landau gauge $\Rightarrow$ Goldstones are treated as actual massless bosons *i.e.* $(m_G^2)^{\rm OS} = 0$

- By **choice** (simplicity) $V_{\rm eff}$ is computed with running masses:

$$(m_G^2)^{\rm run.} = (m_G^2)^{\rm OS} - \Pi_G((m_G^2)^{\rm OS}) = -\Pi_G(0),$$

  where Π_G is the Goldstone self-energy

- Under RG flow, $(m_G^2)^{\rm run.}$ may
  - $\rightarrow$ become 0 $\Rightarrow$ infrared divergence in $V_{\rm eff}$
  - $\rightarrow$ change sign $\Rightarrow$ imaginary part in $V_{\rm eff}$

$\equiv$ **Goldstone boson catastrophe**

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Remark:

$$\left.\begin{array}{l} \triangleright \text{ gauge symmetry } \rightarrow \text{ Goldstone } \notin \text{ physical spectrum} \\ \triangleright \text{ global symmetry } \rightarrow \text{ Goldstone } \in \text{ physical spectrum} \end{array}\right\} \begin{array}{c} \text{however,} \\ \text{Goldstone still} \\ \text{present in calculations!} \end{array}$$

First approaches to the GBC

By hand

- if $m_G^2 < 0$, drop the imaginary part of V_{eff}
- tune the renormalisation scale Q to ensure $m_G^2 > 0$ (and even m_G^2 not too small)

  $\Rightarrow$ may be impossible to achieve and is completely ad hoc

In automated codes (SARAH)

- For SUSY theories **only**: rely on the gauge-coupling dependent part of $V^{(0)}$
  - $\rightarrow$ minimize full $V_{\text{eff}} = V^{(0)} + \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)}|_{\text{gaugeless}}$
  - $\rightarrow$ compute tree-level masses with $V^{(0)}|_{\text{gaugeless}}$ (= turn off the D-term potential)
  - $\rightarrow$ yields a fake Goldstone mass of order $\mathcal{O}(m_{EW}^2) \Rightarrow$ no GBC

  $\rightarrow$ wrong mass for Goldstones hence wrong contribution to m_h

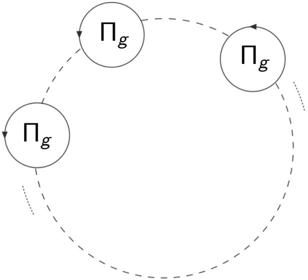
- Add a regulator mass $m_{\text{reg.}}^2 = RQ^2$ for massless particles $\rightarrow$ unwanted new dependence on R, changes the relative size of Goldstone contributions

\+ both methods spoil gauge invariance, etc.

$\Rightarrow$ especially wrong when the scalars – in particular the pseudo-scalars – give large contributions to the Higgs mass $\longrightarrow$ non-SUSY models

Resummation of the Goldstone contribution

SM: Martin 1406.2355; Ellias-Miro, Espinosa, Konstandin 1406.2652.
MSSM: Kumar, Martin 1605.02059.
Generic th.: JB, Goodsell 1609.06977.



[Adapted from arXiv:1406.2652]

- Power counting $\to$ most divergent contribution to V_{eff} at ℓ-loop = ring of $\ell - 1$ Goldstone propagators and $\ell - 1$ insertions of 1PI subdiagrams Π_g involving **only** heavy particles
- Π_g obtained from Π_G, Goldstone self-energy, by removing "soft" Goldstone terms
- Resumming Goldstone rings $\Leftrightarrow$ shifting the Goldstone tree-level mass by Π_g in the 1-loop Goldstone term

$$\hat{V}_{\text{eff}} = V_{\text{eff}} + \frac{1}{16\pi^2}\left[f(m_G^2 + \Pi_g) - \sum_{n=0}^{\ell-1} \frac{(\Pi_g)^n}{n!} \left(\frac{d}{dm_G^2}\right)^n f(m_G^2) \right]$$

$\to$ ℓ-loop resummed V_{eff}, free of leading Goldstone boson catastrophe

A word on the extension of the resummation procedure for generic theories

arXiv:1609.06977

Additional difficulties

A priori: scalar mixing + several Goldstones!

→ Single out the Goldstones (index $G, G', \ldots$) and express their masses

$$m_G^2 = -\sum_i \frac{1}{v_i} (\tilde{R}_{iG})^2 \left. \frac{\partial (V_{\text{eff}} - V^{(0)})}{\partial \phi_i^0} \right|_{\phi_i^0 = 0} = \mathcal{O}(1\text{-loop})$$

($\tilde{R}_{ij}$: rotation matrices in tree-level minimum of V_{eff})

Issues with the resummation

- taking derivatives of $\hat{V}_{\text{eff}}$ can be very difficult (involves derivatives of the rotation matrices, etc.) → in practice resummation was **only** used to find the **tadpole equations**.
- the choice of "soft" Goldstone terms to remove from Π_G to find Π_g may be ambiguous and it is difficult to justify which terms to keep

Our solution: setting the Goldstone boson on-shell arXiv:1609.06977

Setting the Goldstone boson on-shell

- Adopt an on-shell scheme for the Goldstone(s): replace $(m_G^2)^{\text{run.}}$ by $(m_G^2)^{\text{OS}}(=0)$ and $\Pi_G(0)$

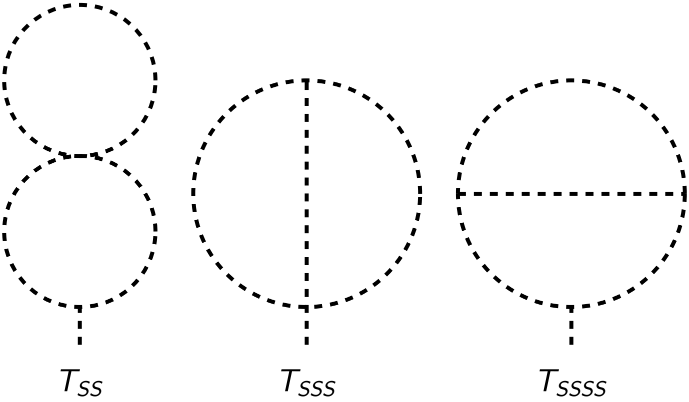
$$\underbrace{(m_G^2)^{\text{run.}}}_{\substack{\text{tree-level mass}\\ \text{in loop-corrected}\\ \text{minimum}}} = \underbrace{(m_G^2)^{\text{OS}}}_{\text{on-shell mass}} - \underbrace{\Pi_G((m_G^2)^{\text{OS}})}_{\text{self-energy}} = -\Pi_G(0)$$

- This can be done **directly** in the tadpole equations or mass diagrams!

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

2-loop tadpole diagrams involving scalars only:

The GBC also appears in diagrams with scalars and fermions or gauge bosons, and is cured with the same procedure $\rightarrow$ we present the purely scalar case.



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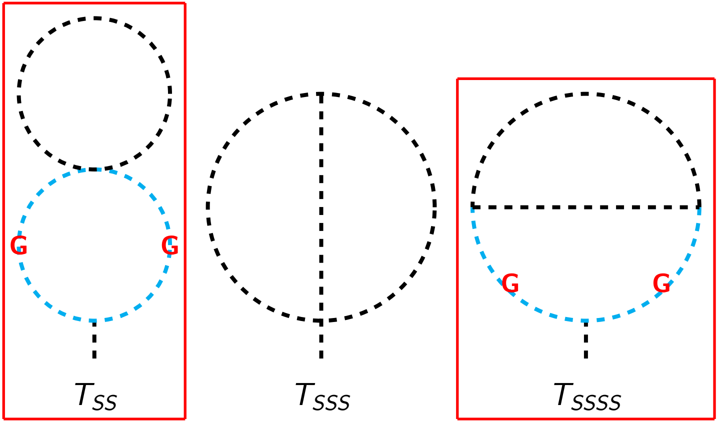
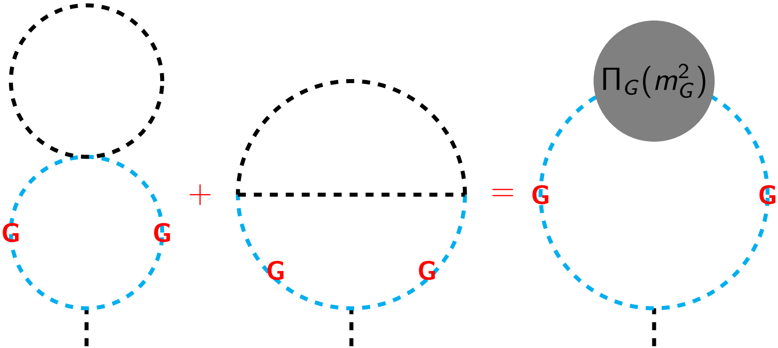


Some diagrams of T_{SS} and T_{SSSS} topologies diverge for $m_G^2 \to 0$

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

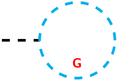
2-loop divergences in tadpole diagrams (involving scalars only) ...



... rewritten as a one-loop diagram with insertion of $\Pi_G(m_G^2)$

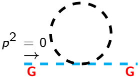
What happens when setting the Goldstone on-shell?

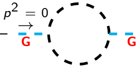
- Contribution of the Goldstone(s) to the 1-loop tadpole:

$T_S \supset$ 

$\propto A(m_G^2) = m_G^2\left(\log \frac{m_G^2}{Q^2} - 1\right)$

- At 1-loop order the scalar-only diagrams in $\Pi_G(0)$ are

$(m_G^2)^{\text{run.}} = \underbrace{(m_G^2)^{\text{OS}}}_{=0} -$ 

$-$ 

$+ \cdots$

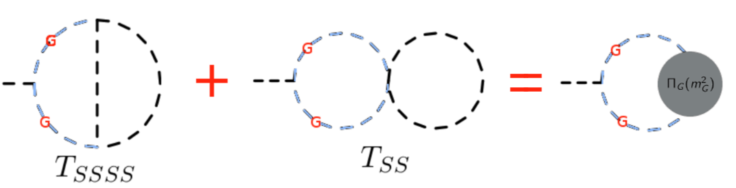
- Shifting m_G^2 by a 1-loop quantity, $\Pi_G(0)$, in the 1-loop tadpole

$$\Rightarrow \textbf{2-loop shift !}$$

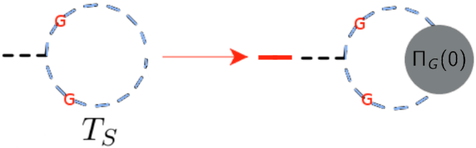
$$A((m_G^2)^{\text{run.}}) = \underbrace{A(0)}_{=0} - \underbrace{\log \frac{m_G^2}{Q^2}}_{\text{1-loop}} \underbrace{\Pi_G(0)}_{\text{1-loop}}$$

Canceling the IR divergences in the tadpole equations arXiv:1609.06977

- 2-loop divergent tadpole diagrams



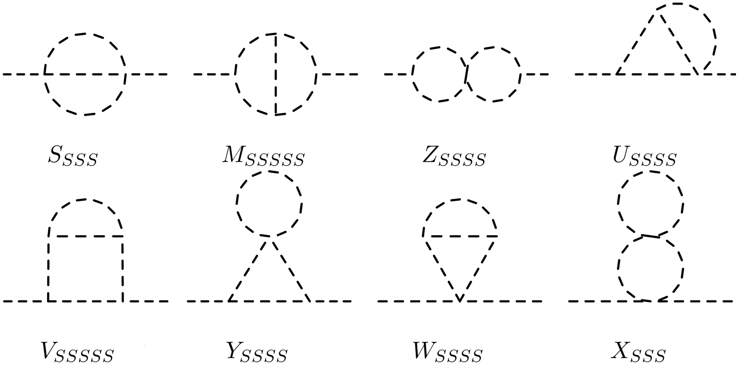
- shifting the Goldstone term in the 1-loop tadpole T_S



$\Rightarrow$ the divergent parts from the diagrams and the shift will cancel out!

Canceling the IR divergences in the mass diagrams arXiv:1609.06977

- Earlier literature: inclusion of momentum cures all the IR divergences
- We found
  ⇒ true at 1-loop order
  ⇒ at 2-loop, ∃ diagrams that still diverge for $m_G^2 \to 0$ **even with external momentum included**



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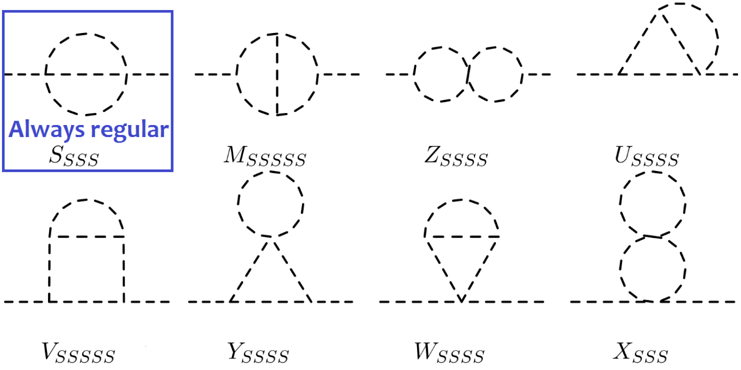

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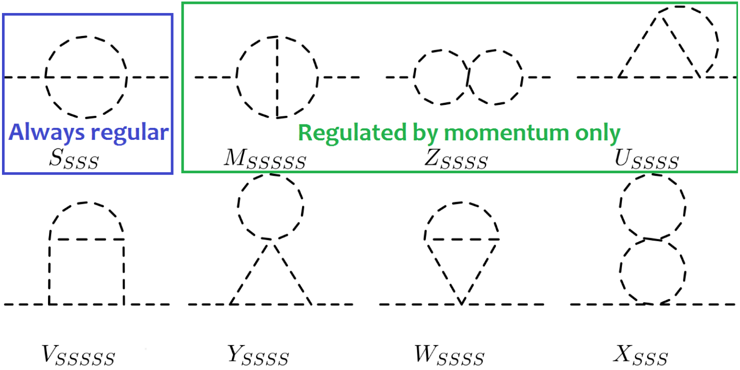

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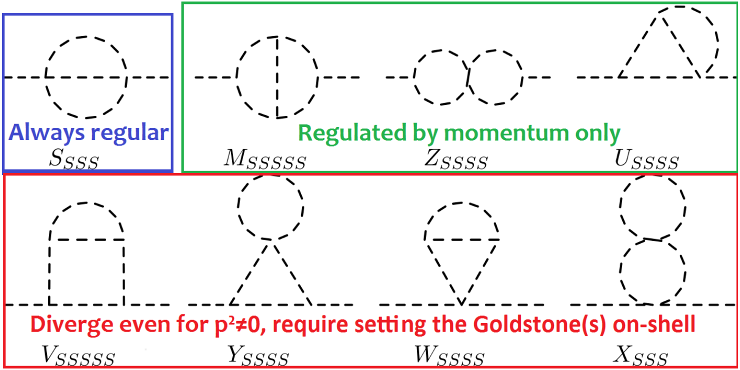

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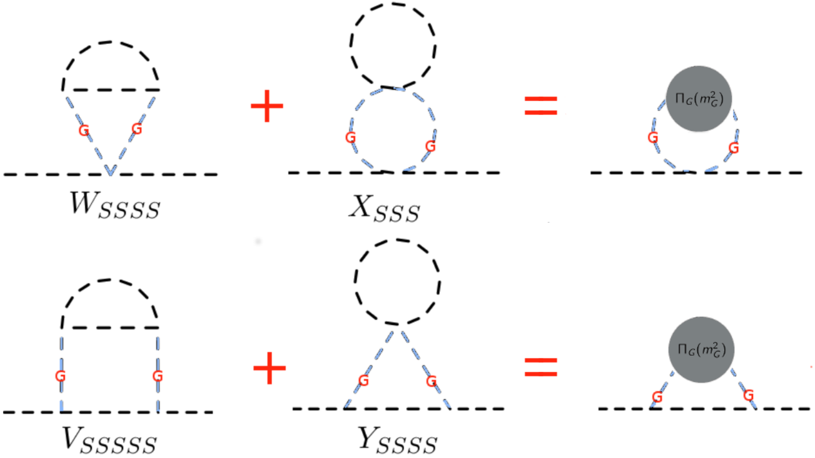
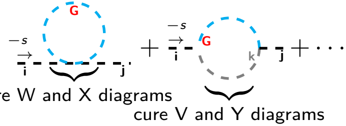


Canceling the IR divergences in the mass diagrams arXiv:1609.06977

Setting the Goldstone(s) on-shell in mass diagrams

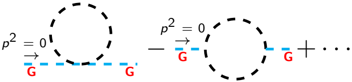
- Goldstone contributions to the 1-loop scalar self-energy

$$\Pi_{ij}^{(1)}(s = \text{-}p^2) =$$



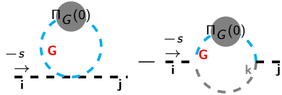
- Again, shifting the Goldstone mass to on-shell scheme gives

$$(m_G^2)^{\text{run.}} = -$$



$\rightarrow$ 2-loop shift to the mass diagrams

$$\delta\Pi_{ij}^{(1)}(s) = -$$



$\longrightarrow$ cancels the divergence in the V, X, Y, W diagrams !

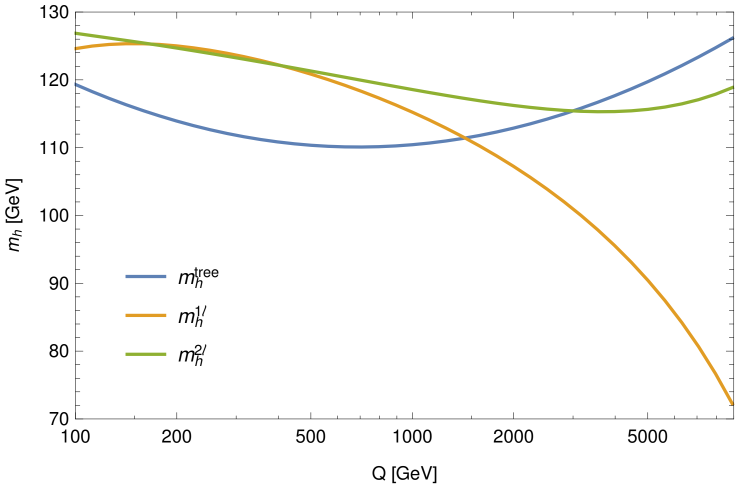
Automated two-loop mass computations free of the Goldstone boson catastrophe

- <u>New</u> routines, taking into account the on-shell Goldstones (*via regularised loop functions*), implemented in spectrum generator generator `SARAH` $\rightarrow$ generates `SPheno` code for the model to study.
- In particular useful for study of Higgs masses in non-SUSY theories where pseudo-scalar contributions are **large**.

I present in the following a few <u>preliminary</u> results for $m_h^{2\ell}$ in the Two-Higgs-Doublet Model (2HDM)

Two-loop Higgs masses in the 2HDM

Improved renormalisation scale dependence



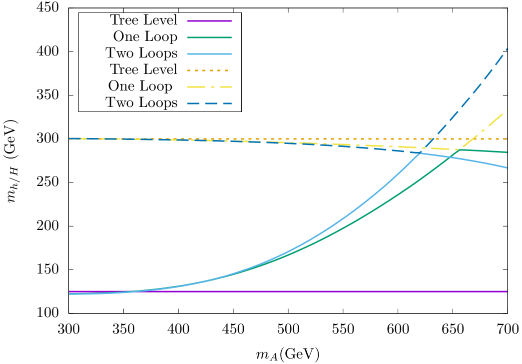
with $\tan\beta = 50$, $\lambda_1 = \lambda_2 = 0.114$, $\lambda_3 = 0.428$, $\lambda_4 = 0.8$,
$\lambda_5 = -1$, $m_{12}^2 = -5000\ \text{GeV}^2$ at scale $Q = 160$ GeV.

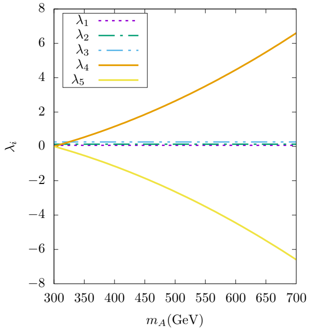
Two-loop Higgs masses in the 2HDM

Limits from perturbativity

Studies of 2HDM usually take tree-level Higgs masses as inputs instead of couplings from scalar potential, eg here inputs are

$$m_h, m_H, m_{H^\pm}, m_A, m_{12}^2, \tan\alpha, \tan\beta$$





→ size of two-loop corrections as a sign of the breakdown of perturbativity, instead of naive criterion $\lambda_i < 4\pi$

Our results

- Results for generic theories (scalars, fermions, gauge bosons), *avoiding the Goldstone boson catastrophe*
  - $\rightarrow$ full **two-loop tadpole equations**
  - $\rightarrow$ **two-loop mass diagrams** for neutral scalars in *gaugeless limit*, in a *generalised effective potential approach* (*i.e.* neglect terms of order $\mathcal{O}(s)$ and higher)
- Numerical implementation in `SARAH` (*soon made public*)
  - $\rightarrow$ no more numerical instability associated with the GBC
  - $\rightarrow$ **automated Higgs mass calculations in both SUSY and non-SUSY models**

$\rightsquigarrow$ SM, 2HDM, Georgi-Machacek model, etc.

Outlook

- Further work on the GBC
  - investigate further the link between resummation and on-shell method
  - extend the solution of GBC to higher loop order
    - $\rightarrow$ on-shell method still working?
    - $\rightarrow$ how to formalise/prove the resummation prescription? ($i.e.$ how to find Π_g)
  - extend mass-diagram calculations to quartic order in the gauge couplings (go beyond the gaugeless limit)
  - Apply similar techniques to address other IR divergences ?

- A great wealth of models to study with increased precision using `SARAH`!

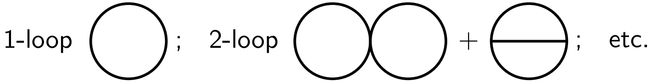
Thank you for your attention !

Backup

The effective potential

$$V_{\text{eff}} = V^{(0)} + \text{quantum corrections}$$

- Potential for scalars, including quantum corrections = 1PI vacuum graphs computed loop by loop



- Expressed as a function of **running tree-level masses** of particles, in some **minimal substraction scheme** ($\overline{\mathrm{MS}}$, $\overline{\mathrm{DR}}'$, etc.)
- First derivative of V_{eff}: **tadpole equation** ($\leftrightarrow$ minimum condition), relates vev and mass-squared parameters
- Second derivative: same as self-energy diagrams, but with **zero external momentum** $\rightarrow$ **approximate scalar masses**

Illustration: the abelian Goldstone model

- 1 complex scalar $\phi = \frac{1}{\sqrt{2}}(v + h + iG)$, no gauge group and only a potential

$$V^{(0)} = \mu^2|\phi|^2 + \lambda|\phi|^4$$

  v: true vev, to all orders in perturbation theory (PT)

- SM: G^+, G^0 Goldstones do not mix, and can be treated separetely
  $\rightarrow$ this model captures the behaviour of the GBC in the SM

- V_{eff} at 2-loop order:

$$V_{\text{eff}} = V^{(0)} + \underbrace{\frac{1}{16\pi^2}\Big[f(m_h^2) + f(m_G^2)\Big]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{1}{(16^2)^2}\left[\lambda\left(\frac{3}{4}A(m_G^2)^2 + \frac{1}{2}A(m_G^2)A(m_h^2)\right) - \lambda^2 v^2 I(m_h^2, m_G^2, m_G^2) + \overbrace{\cdots}^{\text{no Goldstone}}\right]}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

  where $f(x) = \frac{x^2}{4}(\log x/Q^2 - 3/2)$, $A(x) = x(\log x/Q^2 - 1)$ and $I \propto$ 

- Tree-level masses: $m_h^2 = \mu^2 + 3\lambda v^2$, $m_G^2 = \mu^2 + \lambda v^2$

Illustration: the abelian Goldstone model

Tree-level tadpole

$$\left.\frac{\partial V^{(0)}}{\partial h}\right|_{h=0,G=0} = 0 = \mu^2 v + \lambda v^3 = m_G^2 v$$

Loop-corrected tadpole

$$\left.\frac{\partial V_{\text{eff}}}{\partial h}\right|_{h=0,G=0} = 0 = m_G^2 v + \underbrace{\frac{\lambda v}{16\pi^2}\left[3A(m_h^2) + A(m_G^2)\right]}_{\text{1-loop}}$$

$$+ \underbrace{\frac{\log\frac{m_G^2}{Q^2}}{(16^2)^2}\left[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\right] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

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Tree-level tadpole equation

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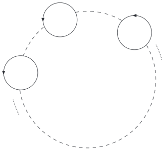
$$+ \underbrace{\frac{\overbrace{\log\frac{m_G^2}{Q^2}}^{\textbf{GBC!}}}{(16^2)^2}\Big[3\lambda^2 v\, A(m_G^2) + \frac{4\lambda^3 v^3}{m_h^2} A(m_h^2)\Big] + \overbrace{\cdots}^{\text{regular for } m_G^2 \to 0}}_{\text{2-loop}} + \mathcal{O}(\text{3-loop})$$

More details on the resummation of Goldstone contributions

$R_\ell \equiv$ 

$$
\begin{aligned}
&\propto \int \frac{d^d k}{i(2\pi)^d} \left(\frac{\Pi_g}{k^2 - m_G^2} \right)^{\ell - 1} \\
&\propto \frac{(\Pi_g)^{\ell-1}}{(\ell-1)!} \left(\frac{d}{dm_G^2} \right)^{\ell-1} \int \frac{d^d k}{i(2\pi)^d} \log(k^2 - m_G^2) \\
&= \frac{1}{16\pi^2} \frac{(\Pi_g)^{\ell-1}}{(\ell-1)!} \left(\frac{d}{dm_G^2} \right)^{\ell-1} f(m_G^2)
\end{aligned}
$$

so $\displaystyle\sum_\ell R_\ell = \frac{1}{16\pi^2} f(m_G^2 + \Pi_g)$

where $f(x) = \frac{x^2}{4}(\overline{\log}\, x - \frac{3}{2})$

Extending the resummation to generic theories arXiv:1609.06977

Generic theories: J.B., Goodsell arXiv:1609.06977

Real scalar fields $\varphi_i^0 = v_i + \phi_i^0$, where v_i are the vevs **to all order in PT**

$$V^{(0)}(\{\varphi_i^0\}) = V^{(0)}(v_i) + \frac{1}{2} m^2_{0,ij}\phi_i^0\phi_j^0 + \frac{1}{6}\hat{\lambda}_0^{ijk}\phi_i^0\phi_j^0\phi_k^0 + \frac{1}{24}\hat{\lambda}_0^{ijkl}\phi_i^0\phi_j^0\phi_k^0\phi_l^0$$

$m^2_{0,ij}$ solution of the tree-level tadpole equation

To work in minimum of loop-corrected $V_{\text{eff}} \to$ new couplings m^2_{ij}

$\Downarrow$

Diagonalise to work with mass eigenstates in both bases

$$(\phi_i^0, m^2_{0,ij}) \xrightarrow{\phi_i^0 = \tilde{R}_{ij}\tilde{\phi}_j} (\tilde{\phi}_i, \tilde{m}_i) \ \textit{(no loop corrections)}$$

$$(\phi_i^0, m^2_{ij}) \xrightarrow{\phi_i^0 = R_{ij}\phi_j} (\phi_i, m_i) \ \textit{(with loop corrections)}$$

$\Downarrow$

Single out the Goldstone boson(s), index $G, G', \ldots$ and its/their mass(es)

$$m_G^2 = -\sum_i \frac{1}{v_i}(\tilde{R}_{iG})^2 \left.\frac{\partial(V_{\text{eff}} - V^{(0)})}{\partial \phi_i^0}\right|_{\phi_i^0=0} = \mathcal{O}(\text{1-loop})$$

More details about the calculations for the scalar-only tadpole

Divergent terms

- From T_{SS}:

$$\left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} \supset \frac{1}{4} R_{rp} \sum_{l \neq G} \lambda^{GGll} \lambda^{GGp} \overline{\log}\, m_G^2 A(m_l^2)$$

- From T_{SSSS}:

$$\left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} \supset \frac{1}{4} R_{rp} \lambda^{pGG} \lambda^{Gkl} \lambda^{Gkl} \overline{\log}\, m_G^2 P_{SS}(m_k^2, m_l^2)$$

Setting the Goldstone mass on-shell

$$\Pi_{GG}^{(1),S}\left(p^2\right) = \frac{1}{2} \lambda^{GGjj} A(m_j^2) - \frac{1}{2} (\lambda^{Gjk})^2 B(p^2, m_j^2, m_k^2)$$

- Hence a 2-loop shift:

$$\frac{\partial V_S^{(2)}}{\partial \phi_r^0}((m_G^2)^{\text{OS}}) = \left.\frac{\partial V_S^{(2)}}{\partial \phi_r^0}\right|_{m_G^2 \to (m_G^2)^{\text{OS}}} - \frac{1}{4} R_{rp} \lambda^{GGp} \overline{\log}(m_G^2)^{\text{OS}} \left(\lambda^{GGjj} A(m_j^2) - (\lambda^{Gjk})^2 B(0, m_j^2, m_k^2)\right).$$

The full 2-loop tadpole equation free of GBC

$$\left.\frac{\partial \hat{V}^{(2)}}{\partial \phi_r^0}\right|_{\varphi=v} = R_{rp}\Big[\overline{T}^p_{SS} + \overline{T}^p_{SSS} + \overline{T}^p_{SSSS} + \overline{T}^p_{SSFF} + \overline{T}^p_{FFFS} + \overline{T}^p_{SSV} + \overline{T}^p_{VS} + \overline{T}^p_{VVS} + \overline{T}^p_{FFV} + \overline{T}^p_{\overline{FF}V} + \overline{T}^p_{\mathrm{gauge}}\Big].$$

Notations: see 1609.06977, 1503.03098

The full 2-loop tadpole equation free of GBC

The all-scalar diagrams are

$$
\begin{aligned}
\overline{T}^p_{SS} =& \frac{1}{4} \sum_{j,k,l \neq G} \lambda^{jkll} \lambda^{jkp} P_{SS}(m_j^2, m_k^2) A(m_l^2) \\
&+ \frac{1}{2} \sum_{k,l \neq G} \lambda^{Gkll} \lambda^{Gkp} P_{SS}(0, m_k^2) A(m_l^2), \\
\overline{T}^p_{SSS} =& \frac{1}{6} \lambda^{pjkl} \lambda^{jkl} f_{SSS}(m_j^2, m_k^2, m_l^2)\big|_{m_G^2 \to 0}, \\
\overline{T}^p_{SSSS} =& \frac{1}{4} \sum_{(j,j') \neq (G,G')} \lambda^{pjj'} \lambda^{jkl} \lambda^{j'kl} U_0(m_j^2, m_{j'}^2, m_k^2, m_l^2) \\
&+ \frac{1}{4} \sum_{(k,l) \neq (G,G')} \lambda^{pGG'} \lambda^{Gkl} \lambda^{G'kl} R_{SS}(m_k^2, m_l^2),
\end{aligned}
$$

where by $(j, j') \neq (G, G')$ we mean that j, j' are not both Goldstone indices.

The full 2-loop tadpole equation free of GBC

The fermion-scalar diagrams are

$$\begin{aligned}
\overline{T}^{p}_{SSFF} = \sum_{(k,l)\neq(G,G')} & \left\{ \frac{1}{2} y^{IJk} y_{IJl} \lambda^{klp} f^{(0,0,1)}_{FFS}(m_I^2, m_J^2; m_k^2, m_l^2) \right. \\
& \left. -\mathrm{Re}\left[y^{IJk} y^{I'J'k} M^*_{II'} M^*_{JJ'} \right] \lambda^{klp} U_0(m_k^2, m_l^2, m_I^2, m_J^2) \right\} \\
& + \frac{1}{2} \lambda^{GG'p} y^{IJG} y_{IJG'} \left(-I(m_I^2, m_J^2, 0) - (m_I^2 + m_J^2) R_{SS}(m_I^2, m_J^2) \right) \\
& - \lambda^{GG'p} \mathrm{Re}\left[y^{IJG} y^{I'J'G'} M^*_{II'} M^*_{JJ'} \right] R_{SS}(m_I^2, m_J^2),
\end{aligned}$$

$$\overline{T}^{p}_{FFFS} = T^{p}_{FFFS}\Big|_{m_G^2 \to 0},$$

The full 2-loop tadpole equation free of GBC

The gauge boson-scalar tadpoles are

$$
\begin{aligned}
\overline{T}^{p}_{SSV} &= T^{p}_{SSV}\Big|_{m_G^2 \to 0}, \\
\overline{T}^{p}_{VS} &= \frac{1}{4} g^{abii} g^{abp} f^{(1,0)}_{VS}(m_a^2, m_b^2; m_i^2)\Big|_{m_G^2 \to 0} \\
&\quad + \sum_{(i,k)\neq(G,G')} \frac{1}{4} g^{aaik} \lambda^{ikp} f^{(0,1)}_{VS}(m_a^2; m_i^2, m_k^2), \\
\overline{T}^{p}_{VVS} &= \frac{1}{2} g^{abi} g^{cbi} g^{acp} f^{(1,0,0)}_{VVS}(m_a^2, m_c^2; m_b^2, m_i^2)\Big|_{m_G^2 \to 0} \\
&\quad + \sum_{(i,j)\neq(G,G')} \frac{1}{4} g^{abi} g^{abj} \lambda^{ijp} f^{(0,0,1)}_{VVS}(m_a^2, m_b^2; m_i^2, m_j^2) \\
&\quad - \frac{1}{4} g^{abG} g^{abG'} \lambda^{GG'p} R_{VV}(m_a^2, m_b^2).
\end{aligned}
$$

The full 2-loop tadpole equation free of GBC

The gauge boson-fermion and gauge diagrams are not affected by the Goldstone boson catastrophe

$$
\begin{aligned}
\overline{T}^{p}_{FFV} =& 2 g^{aJ}_{I} \overline{g}^{K}_{bJ} \mathrm{Re}[M_{KI'} y^{I' I p}] f^{(1,0,0)}_{FFV}(m_I^2, m_K^2; m_J^2, m_a^2) \\
&+ \frac{1}{2} g^{aJ}_{I} \overline{g}^{I}_{bJ} g^{abp} f^{(0,0,1)}_{FFV}(m_I^2, m_J^2; m_a^2, m_b^2), \\
\overline{T}^{p}_{\overline{FF}V} =& g^{aJ}_{I} g^{aJ'}_{I'} \mathrm{Re}[y^{II'p} M^{*}_{JJ'}] \big[f_{\overline{FF}V}(m_I^2, m_J^2, m_a^2) + M_I^2 f^{(1,0,0)}_{\overline{FF}V}(m_I^2, m_{I'}^2; m_J^2, m_a^2) \big] \\
&+ g^{aJ}_{I} g^{aJ'}_{I'} \mathrm{Re}[M^{IK'} M^{KI'} M^{*}_{JJ'} y_{KK'p}] f^{(1,0,0)}_{\overline{FF}V}(m_I^2, m_{I'}^2; m_J^2, m_a^2) \\
&+ \frac{1}{2} g^{aJ}_{I} g^{bJ'}_{I'} g^{abp} M^{II'} M^{*}_{JJ'} f^{(0,0,1)}_{\overline{FF}V}(m_I^2, m_J^2; m_a^2, m_b^2), \\
\overline{T}^{p}_{\mathrm{gauge}} =& \frac{1}{4} g^{abc} g^{dbc} g^{adp} f^{(1,0,0)}_{\mathrm{gauge}}(m_a^2, m_d^2; m_b^2, m_c^2).
\end{aligned}
$$