

Matrix elements for Dark Matter searches

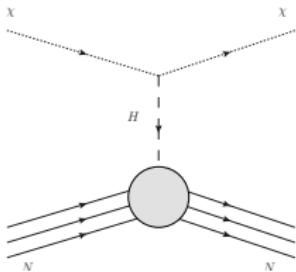
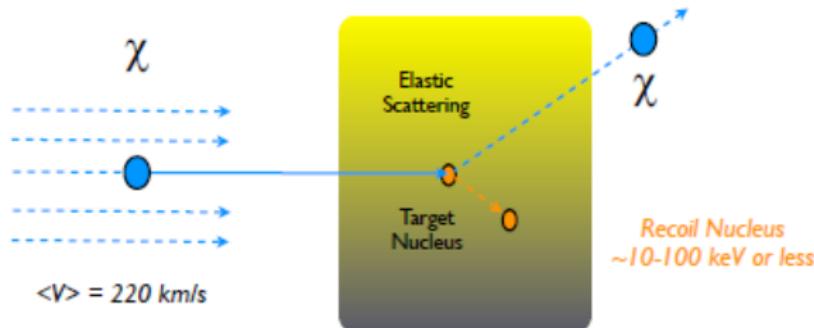
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Budapest-Marseille-Wuppertal collaboration (BMWc)
(Phys.Rev.Lett. 116 (2016) 172001 and in preparation)



Direct WIMP dark matter detection



$$\mathcal{L}_{q\chi} = \sum_q \lambda_q^\Gamma [\bar{q}\Gamma q][\bar{\chi}\Gamma\chi] \longrightarrow \mathcal{L}_{N\chi} = \lambda_N^\Gamma [\bar{N}\Gamma N][\bar{\chi}\Gamma\chi]$$

Quarks are confined within nucleons
→ nonperturbative QCD tool

WIMP-nucleus spin-independent cross section . . .

In low- E limit

$$\frac{d\sigma_{XZ}^{\text{SI}}}{dq^2} = \frac{1}{\pi v^2} [Zf_p + (A-Z)f_n]^2 |F_X(q^2)|^2$$

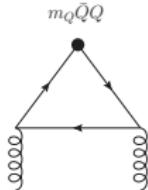
w/ $F_X(\vec{q} = 0) = 1$ nuclear FF and XN couplings ($N = p, n$)

$$\frac{f_N}{M_N} = \sum_{q=u,d,s} f_q^N \frac{\lambda_q}{m_q} + \sum_{Q=c,b,t} f_Q^N \frac{\lambda_Q}{m_Q}$$

such that ($f = u, \dots, t$ and $\langle N(\vec{p}')|N(\vec{p})\rangle = (2\pi)^3 \delta^{(3)}(\vec{p}' - \vec{p})$)

$$f_{ud}^N M_N = \sigma_{\pi N} = m_{ud} \langle N|\bar{u}u + \bar{d}d|N\rangle, \quad f_f^N M_N = \sigma_{fN} = m_f \langle N|\bar{f}f|N\rangle$$

For heavy $Q = c, b, t$ (Shifman et al '78)



$$\rightarrow m_Q \bar{Q} Q = -\frac{1}{3} \frac{\alpha_s}{4\pi} G^2 + O\left(\frac{\alpha_s^2 \mathcal{O}_6}{4m_Q^2}\right)$$

... and relevant hadronic matrix elements

Then obtain f_Q^N in terms of f_q^N through to $M_N = \langle N | \theta^\mu_\mu | N \rangle$, w/

$$\theta^\mu_\mu = (1 - \gamma_m(\alpha_s)) \left[\sum_{q=u,d,s} m_q \bar{q} q + \sum_{Q=c,b,t} m_Q \bar{Q} Q \right] + \frac{\beta(\alpha_s)}{2\alpha_s} G^2$$

Find, $Q = c, b, t$,

$$f_Q^N \equiv \frac{\langle N | m_Q \bar{Q} Q | N \rangle}{M_N} = \frac{2}{27} \left[1 - \sum_{q=u,d,s} f_q^N \right] (1 + O(\alpha_s)) + O(\alpha_s^2 \frac{\Lambda_{\text{QCD}}^2}{4m_Q^2})$$

since $4\pi\beta(\alpha_s) = -\beta_0\alpha_s^2 + O(\alpha_s^3)$ and $\beta_0 = 11 - \frac{2}{3}N_q - \frac{2}{3}N_Q$

- Still need to measure or compute nonperturbative QCD quantities $f_q^N M_N = \sigma_{qN} = m_q \langle N | \bar{q} q | N \rangle$, $q = u, d, s$
- Get f_Q^N up to $O(\alpha_s^2 (\Lambda_{\text{QCD}}/2m_c)^2) \sim 0.001$, i.e. $\sim 2\%$ of $f_Q^N \sim 0.07$

Sigma terms from phenomenology (ca. 2010)

Besides DM, important for: hadron spectrum, m_s/m_{ud} , πN and KN scattering, counting rates in Higgs search ...

Not measured directly in experiment

- ⇒ phenomenological determinations from precise relation to Born-subtracted, $I = 0$, πN amplitude at **unphysical** Cheng-Dashen point $t = 2M_\pi^2$, Σ (Brown et al '71)
- ⇒ extrapolation of data to CD point difficult
- ⇒ inconsistent results in the literature

Using Gasser et al '91 and Bernard et al '96 to get $\sigma_{\pi N}$ from Σ , then Borasoy et al '97 and $m_s/m_{ud} = 24.4(1.5)$ (Leutwyler '96) to get σ_{sN} from $\sigma_{\pi N}$, find

$$\begin{aligned}\sigma_{\pi N} &= m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle & \sigma_{sN} &= m_s \langle N | \bar{s}s | N \rangle \\ y_N &= 2 \langle N | \bar{s}s | N \rangle / \langle N | \bar{u}u + \bar{d}d | N \rangle\end{aligned}$$

Canonical result

$$\begin{aligned}\Sigma &= 60(7) \text{ MeV [12%]} \\ &\quad (\text{Gasser et al '91}) \\ \rightarrow \sigma_{\pi N} &= 45(7) \text{ MeV [16%]} \\ \rightarrow f_{udN} &= 0.048(8) [16\%] \\ \rightarrow y_N &= 0.20(20) [100\%] \\ \rightarrow f_{sN} &= 0.11(12) [105\%]\end{aligned}$$

← differ by $\times 1.4$ →
← differ by $\times 3$ →

More recent

$$\begin{aligned}\Sigma &= 79(6) \text{ MeV [8\%]} \\ &\quad (\text{Pavan et al '02}) \\ \rightarrow \sigma_{\pi N} &= 64(6) \text{ MeV [10\%]} \\ \rightarrow f_{udN} &= 0.068(7) [10\%] \\ \rightarrow y_N &= 0.44(12) [28\%] \\ \rightarrow f_{sN} &= 0.36(11) [31\%]\end{aligned}$$

Squared in SI cross section!

What is lattice QCD (LQCD)?

To describe ordinary matter, QCD requires ≥ 104 numbers at every point of spacetime

$\rightarrow \infty$ number of numbers in our continuous spacetime

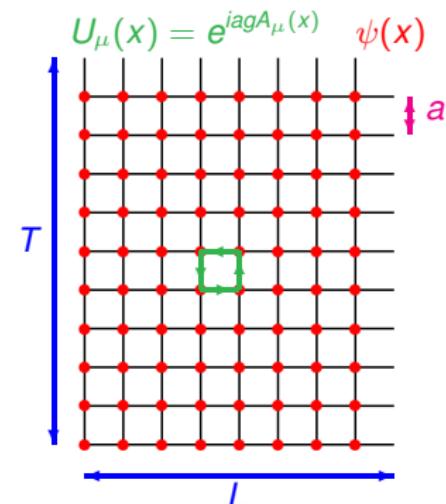
\rightarrow must temporarily “simplify” the theory to be able to calculate (regularization)

\Rightarrow Lattice gauge theory \rightarrow mathematically sound definition of NP QCD:

- UV (& IR) cutoff \rightarrow well defined path integral in Euclidean spacetime:

$$\begin{aligned}\langle O \rangle &= \int \mathcal{D}U \bar{\psi} \mathcal{D}\psi e^{-S_G - \int \bar{\psi} D[M] \psi} O[U, \psi, \bar{\psi}] \\ &= \int \mathcal{D}U e^{-S_G} \det(D[M]) O[U]_{\text{Wick}}\end{aligned}$$

- $\mathcal{D}U e^{-S_G} \det(D[M]) \geq 0$ & finite # of dofs
 \rightarrow evaluate numerically using stochastic methods



LQCD is QCD but only when $N_f \geq 2 + 1$, $m_q \rightarrow m_q^{\text{phys}}$, $a \rightarrow 0$, $L \rightarrow \infty$

HUGE conceptual and numerical challenge (integrate over $\sim 10^9$ real variables)

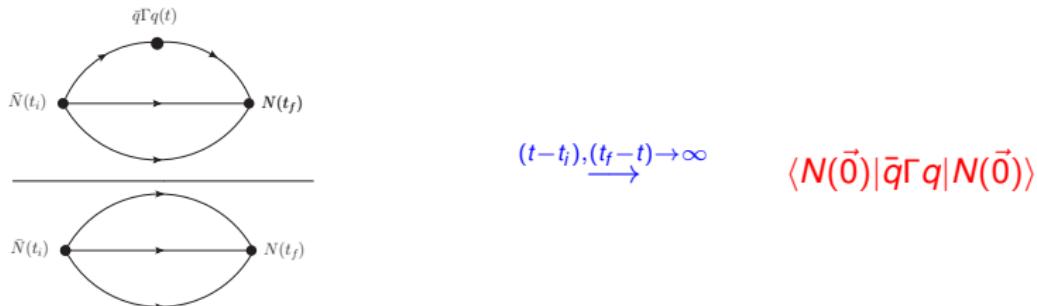
\Rightarrow very few calculations control all necessary limits

σ -terms from LQCD: matrix element (ME) method

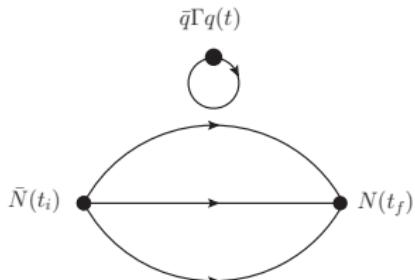
$$\sigma_{\pi N} = m_{ud} \langle N | \bar{u}u + \bar{d}d | N \rangle$$

$$\sigma_{sN} = m_s \langle N | \bar{s}s | N \rangle$$

Extract directly from time-dependence of 3-pt fns:



- ✓ Desired matrix element appears at leading order
- ✗ Must compute more noisy 3-pt fn
- ✗ Quark-disconnected contribution difficult, though $1/N_c$ suppressed
- ✗ $m_q \bar{q}q$ renormalization challenging (Wilson fermions)

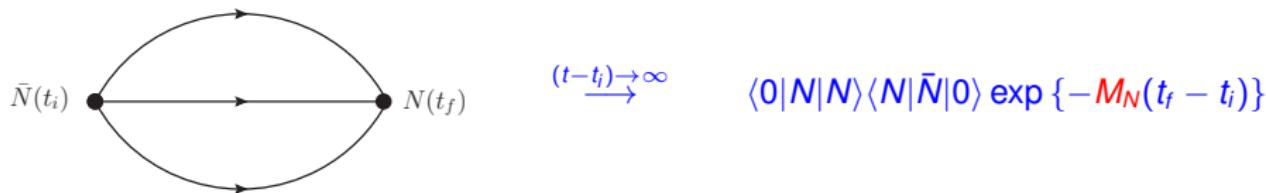


σ -terms from LQCD: Feynman-Hellmann (FH) method

Feynman-Hellmann theorem yields:

$$\langle N | m_q \bar{q} q | N \rangle = m_q \left. \frac{\partial M_N}{\partial m_q} \right|_{m_q^\Phi}$$

On lattice get M_N from time-dependence of 2pt-fn, e.g.:



- ✓ Only simpler and less noisy 2pt-fn is needed
- ✓ No difficult quark-disconnected contributions
- ✓ No difficult renormalization
- ✗ $\partial M_N / \partial m_q$ small for $q = [ud]$ and even smaller for $q = s, c, \dots$

Strategy of calculation

Objective:

- Determine slope of M_N wrt m_q , $q = u, d, s$, at physical point
 - ⇒ determine physical point, i.e. physical values m_q^Φ of m_q , $q = u, d, s, c$, and Λ_{QCD} in $a \rightarrow 0$ and $L \rightarrow \infty$ limit

Method:

- Perform many high-statistics simulations with various m_q around physical values, various $\beta \ni a \lesssim 0.1 \text{ fm}$ and various $N_L \ni L = aN_L \geq 6 \text{ fm}$
- For each compute M_π ($\rightarrow m_{ud}$), M_K ($\rightarrow m_s$), M_{D_s} ($\rightarrow m_c$) and $M_{\Omega/N}$ ($\rightarrow \Lambda_{\text{QCD}}$)
- Study simultaneously dependence of M_π, \dots on m_q 's, a and L
 - ⇒ for each simulation determine a , m_q^Φ 's such that M_π, \dots take their physical value in $a \rightarrow 0$ and $L \rightarrow \infty$ limit
- Read off $m_q(\partial M_N / \partial m_q)|_{m_q^\Phi}$, $q = [ud], s$, in $a \rightarrow 0$ and $L \rightarrow \infty$ limit
- NO $SU(3)$ hypotheses to get f_{ud}^N and $f_s^N \Rightarrow$ significantly improved accuracy

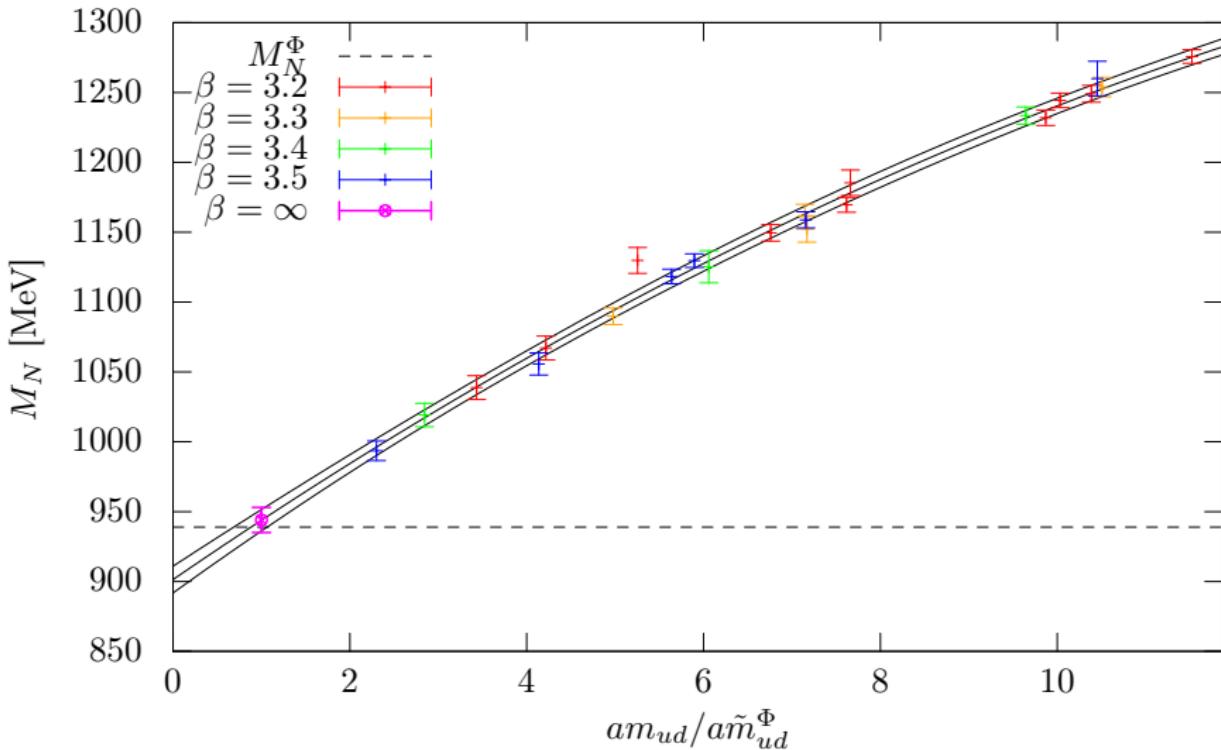
Lattice details

- $N_f = 1 + 1 + 1 + 1$
- 3HEX clover-improved Wilson fermions on tree-level improved Symanzik gluons
- 29 ensembles w/ total ~ 155000 trajectories
- ~ 500 measurements per configuration
- 4 $a \in [0.064, 0.102]$ fm;
- $M_\pi \in [195, 450]$ MeV w/ $LM_\pi > 4$

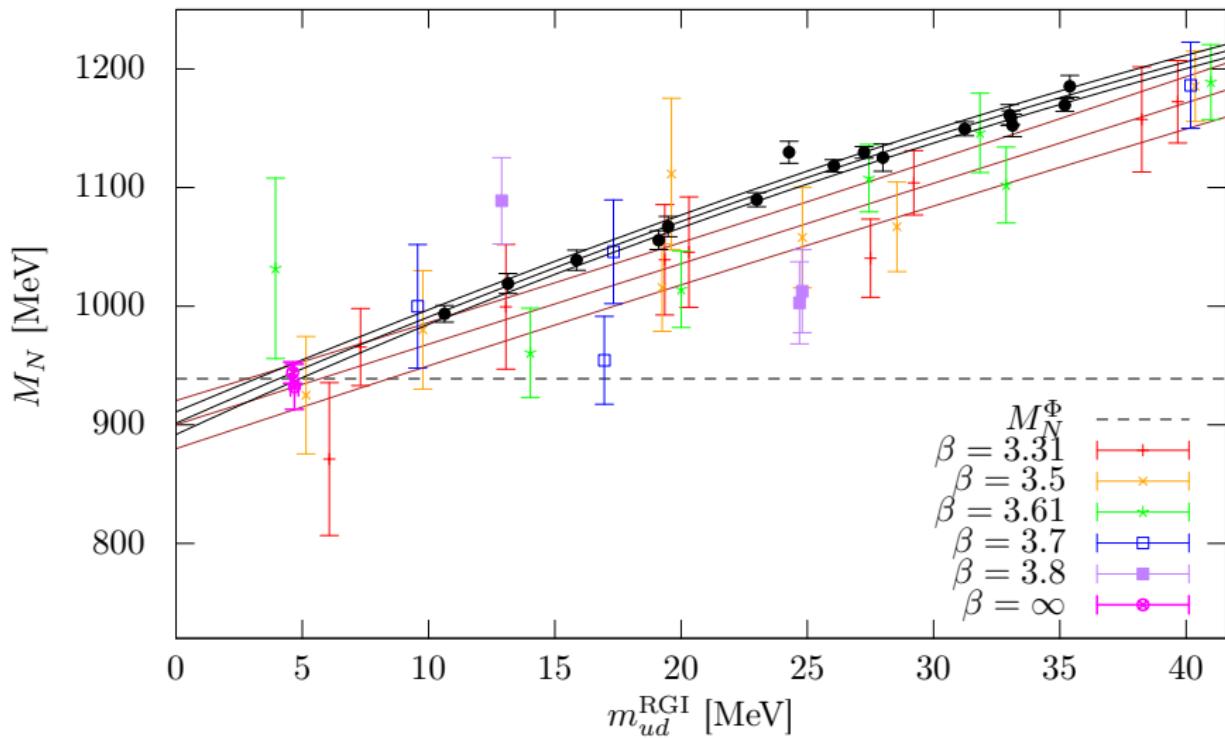
Improvements over BMWc, PRL '16

- ✓ Charm in sea
- ✓ $\gtrsim \times 100$ in statistics (reduced to $\gtrsim 35$ by later plateaux)
- ✓ $\gtrsim \times 2$ lever arm in m_s
- ✓ Like PRL '16 FH in terms of quark and not meson masses
- ✗ No physical m_{ud} , but small enough and know M_N from experiment

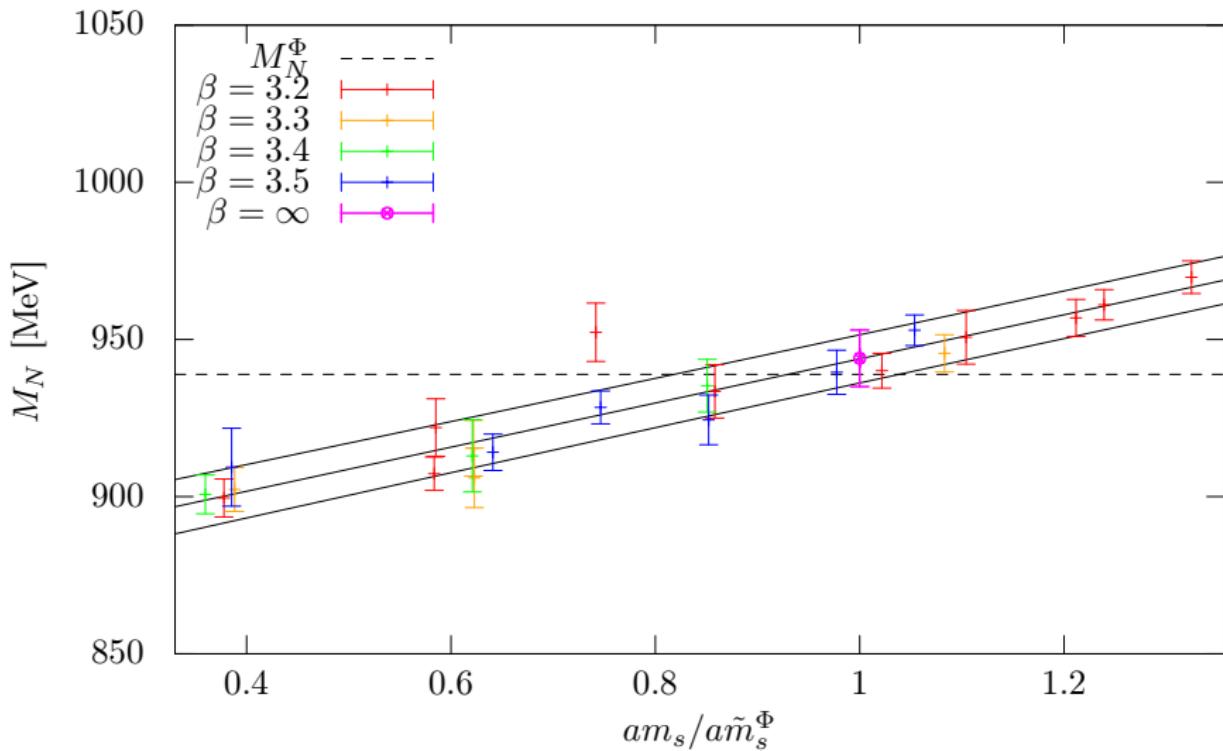
Example m_{ud} dependence of M_N (preliminary)



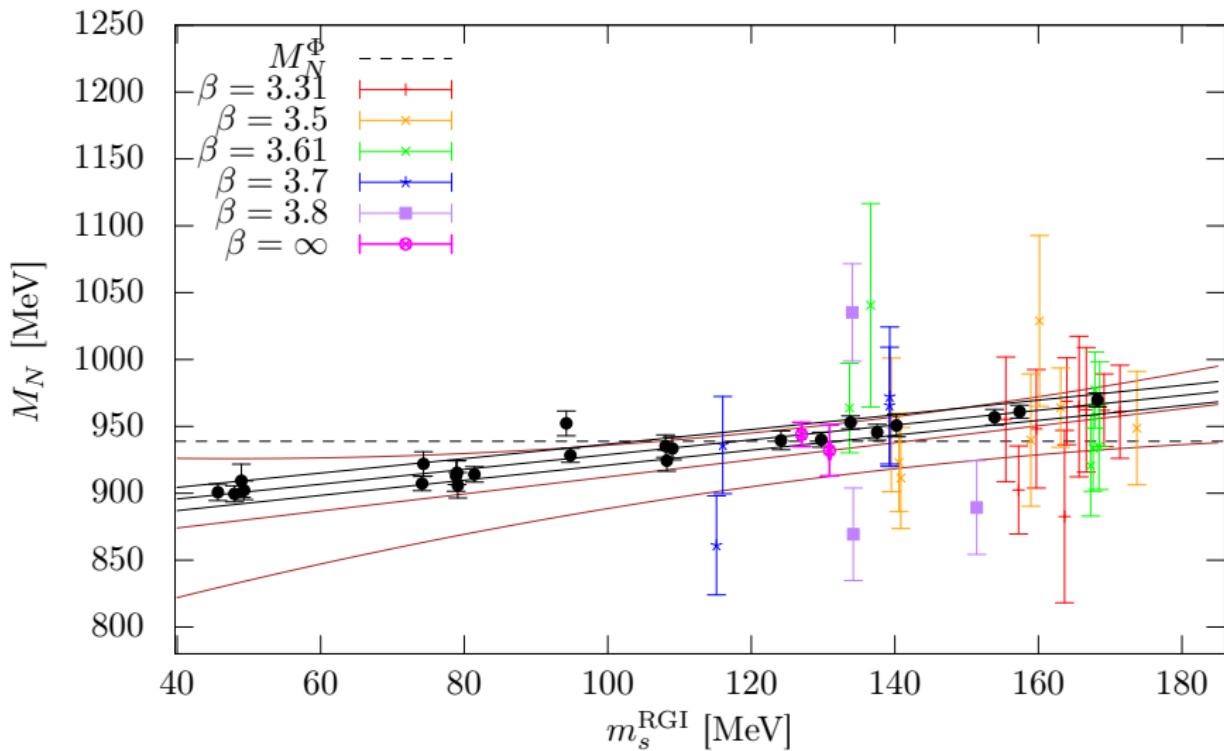
Comparison with BMWc, PRL '16 (preliminary)



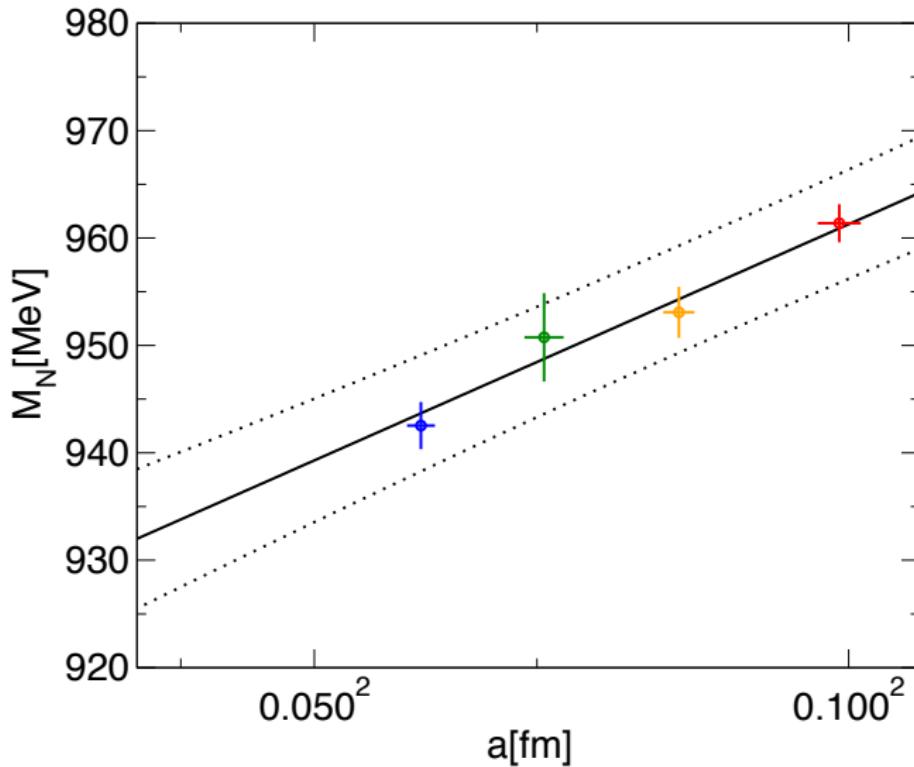
Example m_s dependence of M_N (preliminary)



Comparison with BMWc, PRL '16 (preliminary)



Example continuum extrapolation of M_N (preliminary)



New method for obtaining $f_{u/d}^{p/n}$

[BMWc, PRL 116 (2016)]

- Input: f_{ud}^N and $\Delta_{\text{QCD}} M_N = M_n - M_p$ (from BMWc, Science '15)
- SU(2) relations w/ $\delta m = m_d - m_u$

$$H = H_{\text{iso}} + H_{\delta m}, \quad H_{\delta m} = \frac{\delta m}{2} \int d^3x (\bar{d}d - \bar{u}u)$$

$$\Delta_{\text{QCD}} M_N = \delta m \langle p | \bar{u}u - \bar{d}d | p \rangle$$

lead to, w/ $r = m_u/m_d$,

$$f_u^{p/n} = \left(\frac{r}{1+r} \right) f_{ud}^N \pm \frac{1}{2} \left(\frac{r}{1-r} \right) \frac{\Delta_{\text{QCD}} M_N}{M_N}$$

$$f_d^{p/n} = \left(\frac{1}{1+r} \right) f_{ud}^N \mp \frac{1}{2} \left(\frac{1}{1-r} \right) \frac{\Delta_{\text{QCD}} M_N}{M_N}$$

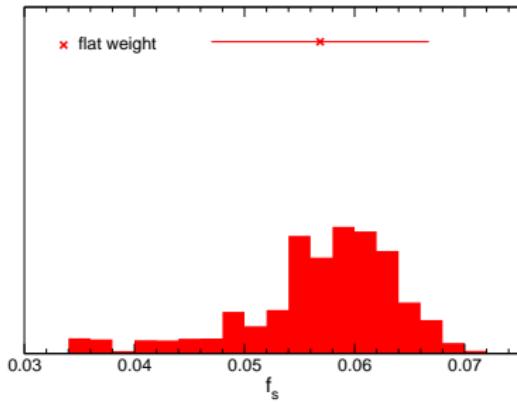
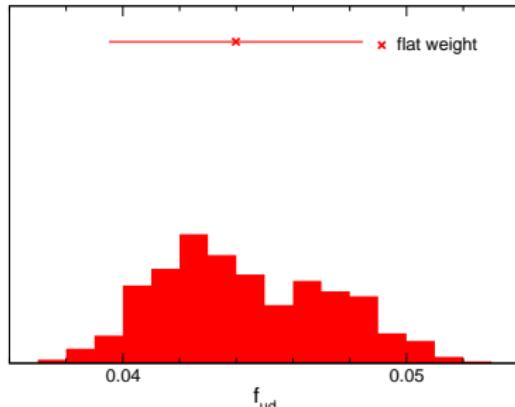
- Huge improvement on usual SU(3)-flavor approach

systematic: $\left(\frac{m_s - m_{ud}}{\Lambda_{\text{QCD}}} \right)^2 \approx 10\% \rightarrow \left(\frac{m_d - m_u}{\Lambda_{\text{QCD}}} \right)^2 \approx 0.01\% .$

Systematic error assessment (preliminary)

Estimated using extended frequentist approach (BMWc, Science '08, Science '15)

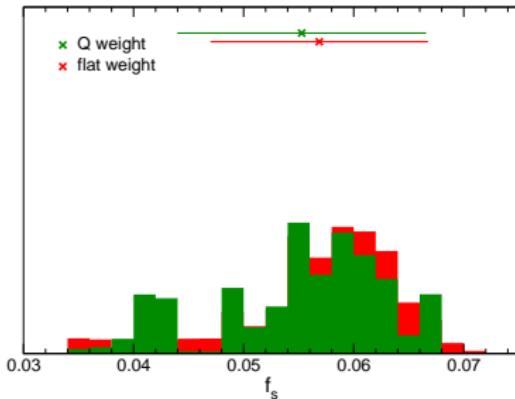
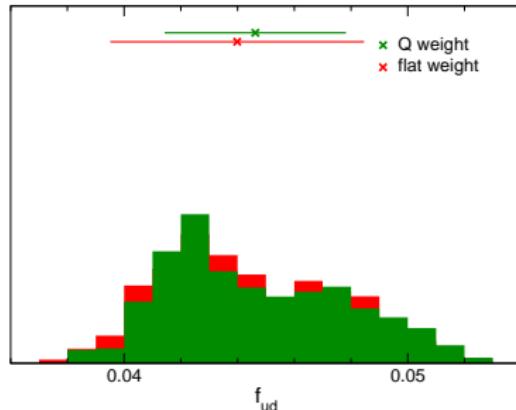
- Excited state contamination: 2 time intervals ($t_{\min} = 1.3$ or 1.4 fm)
 - Mass interpolation/extrapolation errors
 - $M_\pi \leq 380$ MeV & 480 MeV
 - different m_q dependences (polynomials, Padés, χ PT)
 - continuum extrapolation: $O(\alpha_S a)$ vs $O(a^2)$
- ⇒ $O(10k)$ analyses



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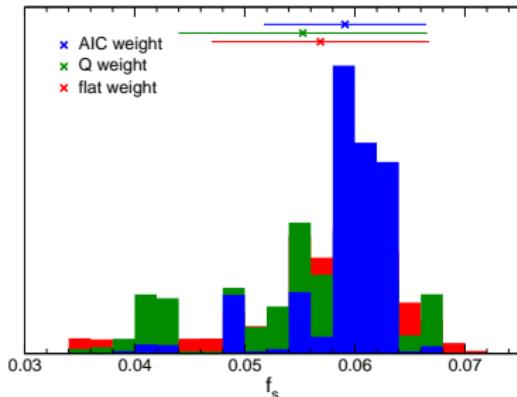
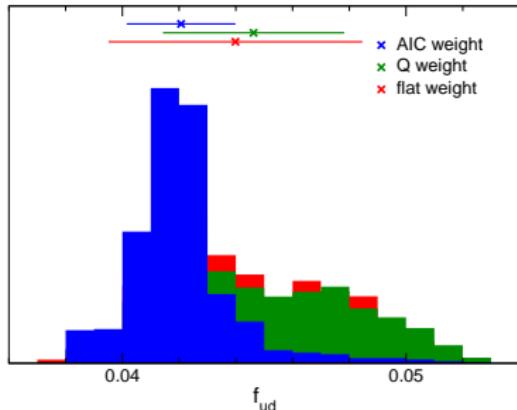
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Preliminary results

- Checks

- $Z_s(\beta) \sim 0.7$ and expected β -dependence
- Compare

$$M_N = 937(11)(4) \text{ MeV} \quad m_s/m_{ud} = 26.2(2)(6)$$

w/ $M_N = 939.$ MeV (PDG '16) and $m_s/m_{ud} = 27.53(20)(08)$ (BMWc, JHEP 08 (2011))

- Predictions

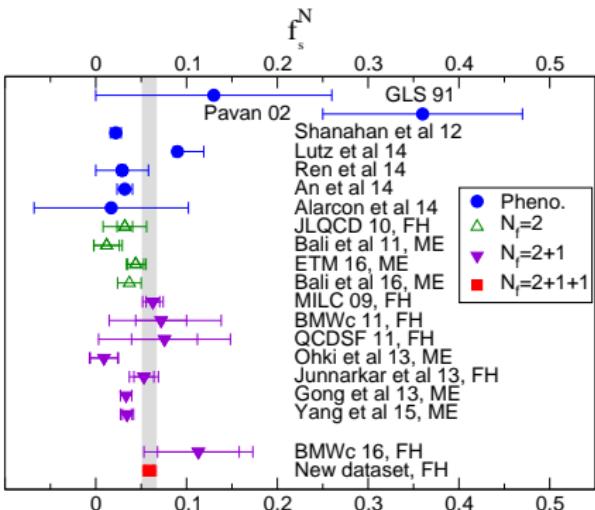
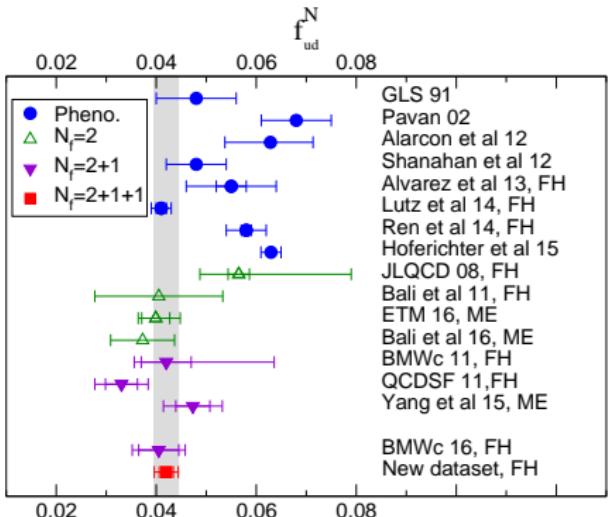
$$f_{ud}^N = 0.0420(14)(19) \quad f_s^N = 0.0591(58)(45)$$

$$f_u^p = 0.0144(7)(8) \quad f_d^p = 0.0263(11)(14)$$

$$f_u^n = 0.0121(5)(7) \quad f_d^n = 0.0313(10)(14)$$

$$f_Q^N = 0.0666(5)(3) \times (1 + O(\alpha_s))$$

Comparison



Conclusion

- Scalar quark contents have of p & n have been computed with full control over all sources of uncertainties
- f_{ud}^N will soon known to better than 10% & f_s^N to better than 15%
- Hadronic ME are no longer the dominant source of uncertainty in DM direct detection rate predictions ...
- ... or in the determination of WIMP couplings from possible DM signals