

# Testing naturalness through precision measurements

Work in progress

**Jérémie Bernon**

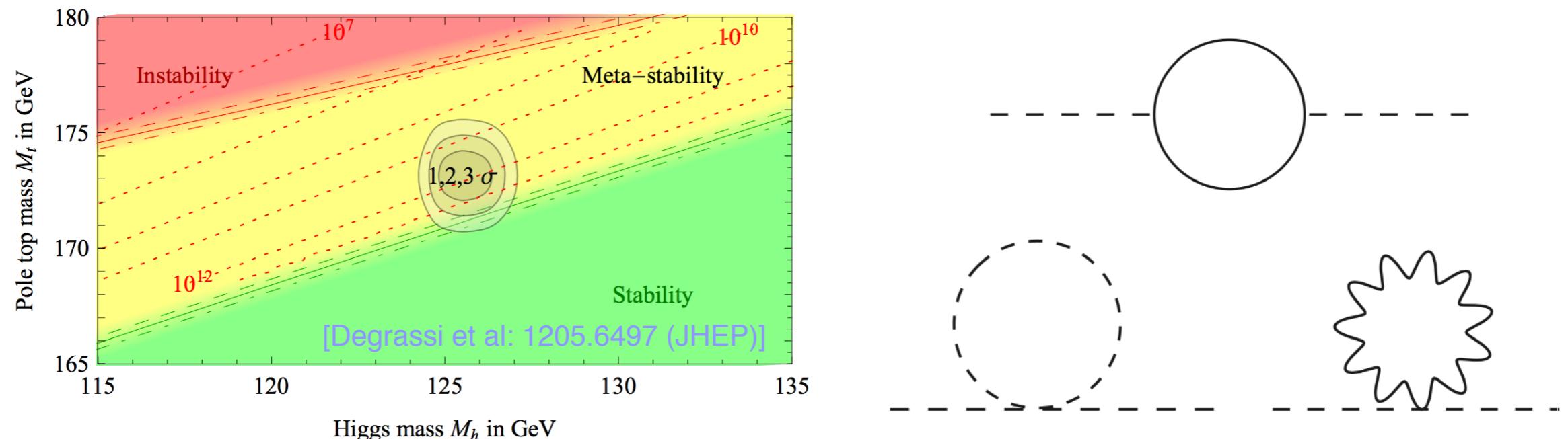
The Hong Kong University of Science and Technology

In collaboration with:

**Yun Jiang** (Niels Bohr Institute) and  
**Tao Liu** (The Hong Kong University of Science and Technology)

# Hierarchy problem

- The Standard Model is a consistent theory up to the Planck scale (metastable electroweak vacuum): no need to complete it



- Mass of a fundamental scalar is quadratically sensitive to the scale of New Physics  $\Lambda$ 
$$\delta m_h^2 \approx f(g)\Lambda^2$$
- No such scale in the SM alone, but due to gravity  $\Lambda \leq M_{Pl}$
- Fine tuning alleviated if  $f(g) \approx 0$  due to some symmetry or if the cut-off is actually much smaller than expected due to some geometrical setup

# Veltman-like conditions

- One-loop effective potential, in Landau gauge:

$$V_1(h) = \int \frac{d^4 k}{(2\pi)^4} \text{Str} \log(k^2 + |\mathcal{M}(h)|^2)$$

$$\text{Str}\mathcal{M}^2 = \sum_{j=0, \frac{1}{2}, 1} (-1)^{2j} (2j+1) \text{Tr}\mathcal{M}_j^2$$

- Expanding in terms of  $\mathcal{M}^2/k^2$ :

$$V_1(h) = \text{Str} \mathbf{1} \int \frac{d^4 k}{(2\pi)^4} \log k^2 + \text{Str} |\mathcal{M}(h)|^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} + \dots$$



Zero-point energy ( $\Lambda^4$ )



$\text{Str } \mathbf{1} = n_B - n_F$

Quadratic divergence ( $\Lambda^2$ )

Gauge invariant [Fukuda, Kugo: '75 (PRD)]

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Quadratic divergence ( $\Lambda^2$ )

Gauge invariant [Fukuda, Kugo: '75 (PRD)]

- Regularization of the Higgs 2-point function: [Einhorn, Jones: '92 (PRD)]

$$\boxed{\frac{1}{2} \frac{\partial^2}{\partial h \partial h} \text{Str}|\mathcal{M}(h)|^2 \Big|_{h=0} = 0}$$

Automatic in symmetry-based  
solutions to the hierarchy problem

Condition on dimensionless parameters

- In the SM:

$$\frac{\lambda}{2} + 3 \times \frac{\lambda}{6} + 6 \times \frac{g^2}{4} + 3 \times \frac{(g^2 + g'^2)}{4} - 12 \times \frac{y_t^2}{2} - \dots = 0$$

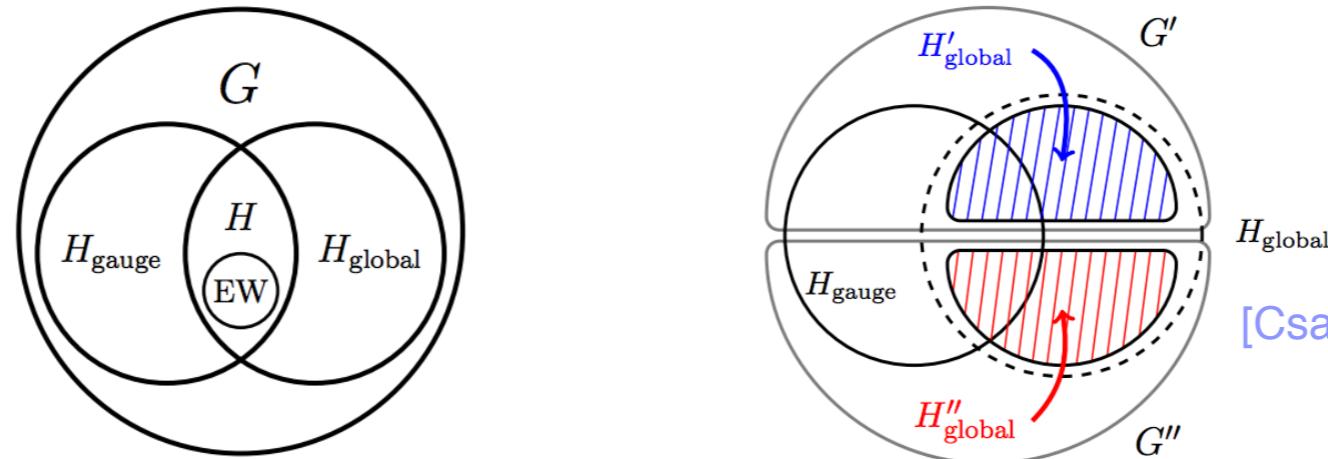
[Veltman: '80 (APP)]

# Naturalness from symmetries

Supersymmetry: extended space-time symmetry, protect the Higgs chirally

Global symmetry:

- Composite Higgs: strong dynamics spontaneously breaks an explicitly broken global symmetry → Higgs as a pseudo Nambu Goldstone boson [Kaplan, Georgi: '84 (PLB)]



- Little Higgs: collective symmetry breaking, i.e. global symmetry breaking communicated by the interplay of 2 different sectors → further protection of m<sub>h</sub>

[Arkani-Ahmed, Cohen, Georgi: '01 (PLB)]

Discrete symmetry:

- Twin Higgs: Z<sub>2</sub> symmetry relating two SM gauge group copies (twins), approximate global SU(4) symmetry of the Higgs sector broken → Higgs as a pseudo Nambu Goldstone boson

[Chacko, Goh, Harnik: '05 (PRL)]

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Supersymmetry: extended space-time symmetry, protect the Higgs chirally

Global symmetry:

- Composite symmetry -

Partner quantum #s	Global	SUSY
<b>QCD x EWK</b>	CHM, Little Higgs	MSSM
<b>Neutral x EWK</b>	Quirky Little Higgs	Folded SUSY
<b>Neutral x Neutral</b>	Twin Higgs	????

From David Curtin/Nathaniel Craig

[Arkani-Ahmed, Cohen, Georgi: '01 (PLB)]

Discrete symmetry:

- Twin Higgs:  $Z_2$  symmetry relating two SM gauge group copies (twins), approximate global  $SU(4)$  symmetry of the Higgs sector broken  $\rightarrow$  Higgs as a pseudo Nambu Goldstone boson

[Chacko, Goh, Harnik: '05 (PRL)]

# Naturalness parameter

- We define a naturalness parameter as a measure of the departure from the Veltman condition at low-energy, for a given New Physics model:

Veltman condition:

$$\frac{1}{2} \frac{\partial^2}{\partial h \partial h} \text{Str}|\mathcal{M}(h)|^2 \Big|_{h=0} = 0$$

$$\implies f(\text{BSM}) + g(\text{SM}) = 0$$

f, g: functions of dimensionless couplings

Naturalness parameter:

$$\mu \equiv -\frac{f(\text{BSM})}{g(\text{SM})}$$

$\mu = 1 \Leftrightarrow$  Cancellation of the quadratic divergence

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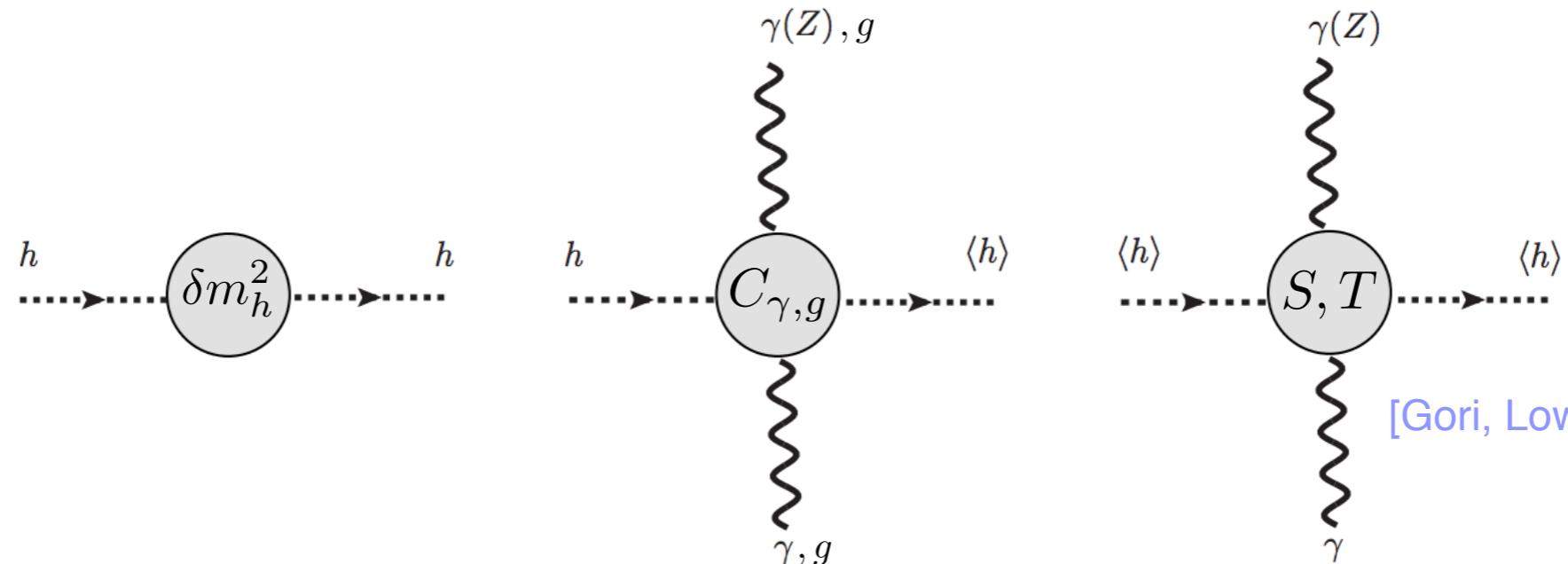
$$\mu \equiv -\frac{f(\text{BSM})}{g(\text{SM})}$$

$$\mu = 1 \Leftrightarrow \text{Cancellation of the quadratic divergence}$$

- Here we will only be interested in cancelling the top-quark SM contribution

Assuming the discovery of a top-partner-like state,  
how do we assess its contribution to the cancellation of the quadratic divergence ?  
Does it cancel fully the SM top-contribution, i.e.  $\mu=1$  ?

# Precision observables as a probe of naturalness



[Gori, Low: 1307.0496 (JHEP)]

- Goal: determine the naturalness parameter from the size of loop-induced Higgs couplings and electroweak Peskin-Takeuchi parameters
- More precisely, see e.g. Higgs low-energy theorems:

$$\mathcal{L}_{hgg} = \frac{\alpha_s}{16\pi} \frac{h}{v} G_{\mu\nu}^a G^{a\mu\nu} \sum_{i=S,F,V} b_i T(i) \frac{\partial}{\partial \log v} \log \det |\mathcal{M}_i|^2$$

Expect correlation between deviation of Higgs couplings and quadratic cancellation.

- For a given model, we can assess the sensitivity of future colliders to the naturalness parameter.

# A $t_R$ -like top partner

# A fermionic top-partner

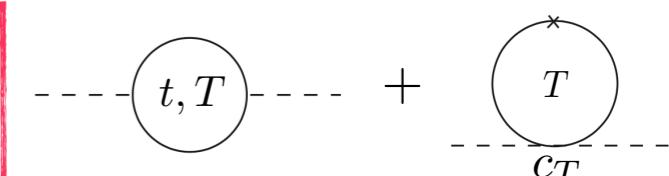
- Inspired by Little Higgs models, introduce a pair of vector-like quarks  $T_{L,R} \sim (3, 1, 2/3)$

$$\mathcal{L}_T = -\lambda_T \bar{Q}'_L \tilde{H} T'_R - y_t \bar{Q}'_L \tilde{H} t'_R - M \bar{T}'_L T'_R - \frac{c_T}{M} H^\dagger H \bar{T}'_L T'_R + h.c.$$

Primed fields: weak eigenstates

- Mass matrix:  $\mathcal{L}_T \supset -\begin{pmatrix} \bar{t}'_L & \bar{T}'_L \end{pmatrix} \begin{pmatrix} \frac{y_t v}{\sqrt{2}} & \frac{\lambda_T v}{\sqrt{2}} \\ 0 & M + \frac{c_T v^2}{2M} \end{pmatrix} \begin{pmatrix} t'_R \\ T'_R \end{pmatrix} + h.c$

- Veltman condition:  $|\lambda_T|^2 + 2c_T + |y_t|^2 = 0 \Rightarrow \mu \equiv -\frac{|\lambda_T|^2 + 2c_T}{|y_t|^2}$



- Fermionic top-partners achieve cancellation through dim-5 operator only,  $c_T < 0$

# A fermionic top-partner

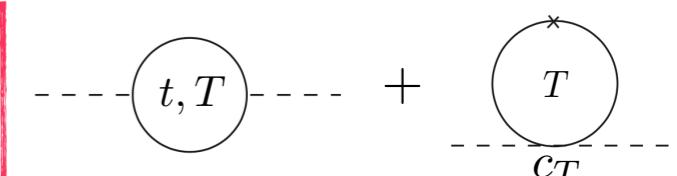
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- Fermionic top-partners achieve cancellation through dim-5 operator only,  $c_T < 0$

- Mass eigenstates:  $\begin{pmatrix} t_{L,R} \\ T_{L,R} \end{pmatrix} = \begin{pmatrix} c_{L,R} & s_{L,R} \\ -s_{L,R} & c_{L,R} \end{pmatrix} \begin{pmatrix} t'_{L,R} \\ T'_{L,R} \end{pmatrix} \quad \tan \theta_R = \frac{m_t}{M_T} \tan \theta_L$

$$M_T = M \left[ 1 + \frac{\lambda_T^2 v^2}{4M^2} + \frac{c_T v^2}{2M^2} + \mathcal{O}\left(\frac{v^4}{M^4}\right) \right]$$

$$m_t = \frac{y_t v}{\sqrt{2}} \left[ 1 - \frac{\lambda_T^2 v^2}{4M^2} + \mathcal{O}\left(\frac{v^4}{M^4}\right) \right]$$

# Higgs couplings and alternative naturalness parameter

- Higgs couplings:  $\mathcal{L}_{h\psi\psi} \supset -[C_{htt}h\bar{t}_L t_R + C_{hTT}h\bar{T}_L T_R] + h.c.$

$$C_{htt} = \frac{m_t}{v} \cos \theta_L \cos \theta_L + \frac{c_T v}{M} \sin \theta_L \sin \theta_R \approx \frac{y_t}{\sqrt{2}} + \mathcal{O}\left(\frac{v^2}{M^2}\right)$$

$$C_{hTT} = \frac{M_T}{v} \sin \theta_L \sin \theta_L + \frac{c_T v}{M} \cos \theta_L \cos \theta_R \approx \frac{v}{2M} \left( |\lambda_T|^2 + 2c_T + \mathcal{O}\left(\frac{v^2}{M^2}\right) \right)$$

- Naturalness parameter:  $\mu \equiv -\frac{|\lambda_T|^2 + 2c_T}{|y_t|^2} \implies \tilde{\mu} \equiv -\frac{M_T}{v} \frac{C_{hTT}}{|C_{htt}|^2} = \mu + \mathcal{O}\left(\frac{v^2}{M^2}\right)$

Expressed in terms of quantities directly measurable at colliders: top Yukawa and hTT production  
(insensitive to the sign of the naturalness parameter) [Chen, Hajer, Liu, Low, Zhang: [collider study on-going](#)]

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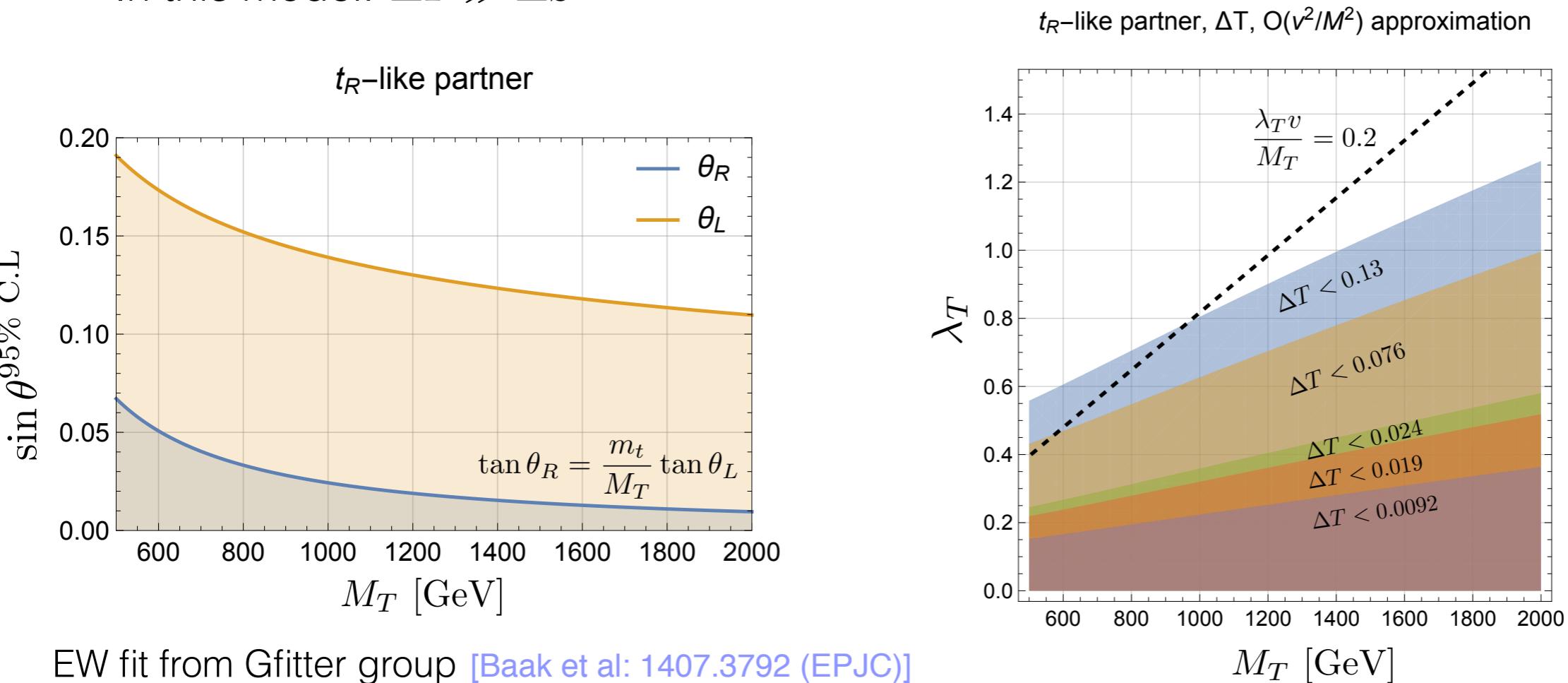
- Note that if the dim-5 operator were not present ( $c_T = 0$ ):

$$\frac{vC_{htt}}{m_t} + \frac{vC_{hTT}}{M_T} = 1$$

$\implies$  Approximatively no modification of the Higgs coupling to gluons.  
Only the presence of the dim-5 allows for sizeable deviation.

# Constraint on the mixing angle from S, T

- For  $M_T \gg m_t$  : 
$$\Delta T_F(\text{approx}) = T_{SM} s_L^2 \left[ -(1 + c_L^2) + s_L^2 r + 2c_L^2 \log(r) \right]$$
 
$$\Delta S_F(\text{approx}) = -\frac{N_C}{18\pi} s_L^2 \left[ 5c_L^2 + (1 - 3c_L^2) \log(r) \right].$$
  $r \equiv \frac{M_T^2}{m_t^2}$  [Dawson, Furlan: 1205.4733 (PRD)]
- Decoupling limit (SM limit) :  $r \gg 1, s_L \rightarrow 0 ; s_L = -\frac{\lambda_T v}{\sqrt{2}M} + \mathcal{O}(\frac{v^3}{M^3})$
- In this model:  $\Delta T \gg \Delta S$



Sensitivities from  
[Fan, Reece, Wang:  
1411.1054 (JHEP)]

# Higgs coupling to gluons

- Most sensitive loop-induced Higgs coupling:

$$\tilde{\mu} \equiv -\frac{M_T}{v} \frac{C_{hTT}}{|C_{htt}|^2}$$

$$C_g = 1 + \Delta C_g \equiv \sqrt{\frac{\Gamma(h \rightarrow gg)^{\text{BSM}}}{\Gamma(h \rightarrow gg)^{\text{SM}}}} = \left| \frac{\sum_{x=t,T} \frac{2C_{hxg}}{m_x} \mathcal{A}_{1/2}(m_x)}{\frac{2C_{htt}^{\text{SM}}}{m_t} \mathcal{A}_{1/2}(m_t)} \right|$$

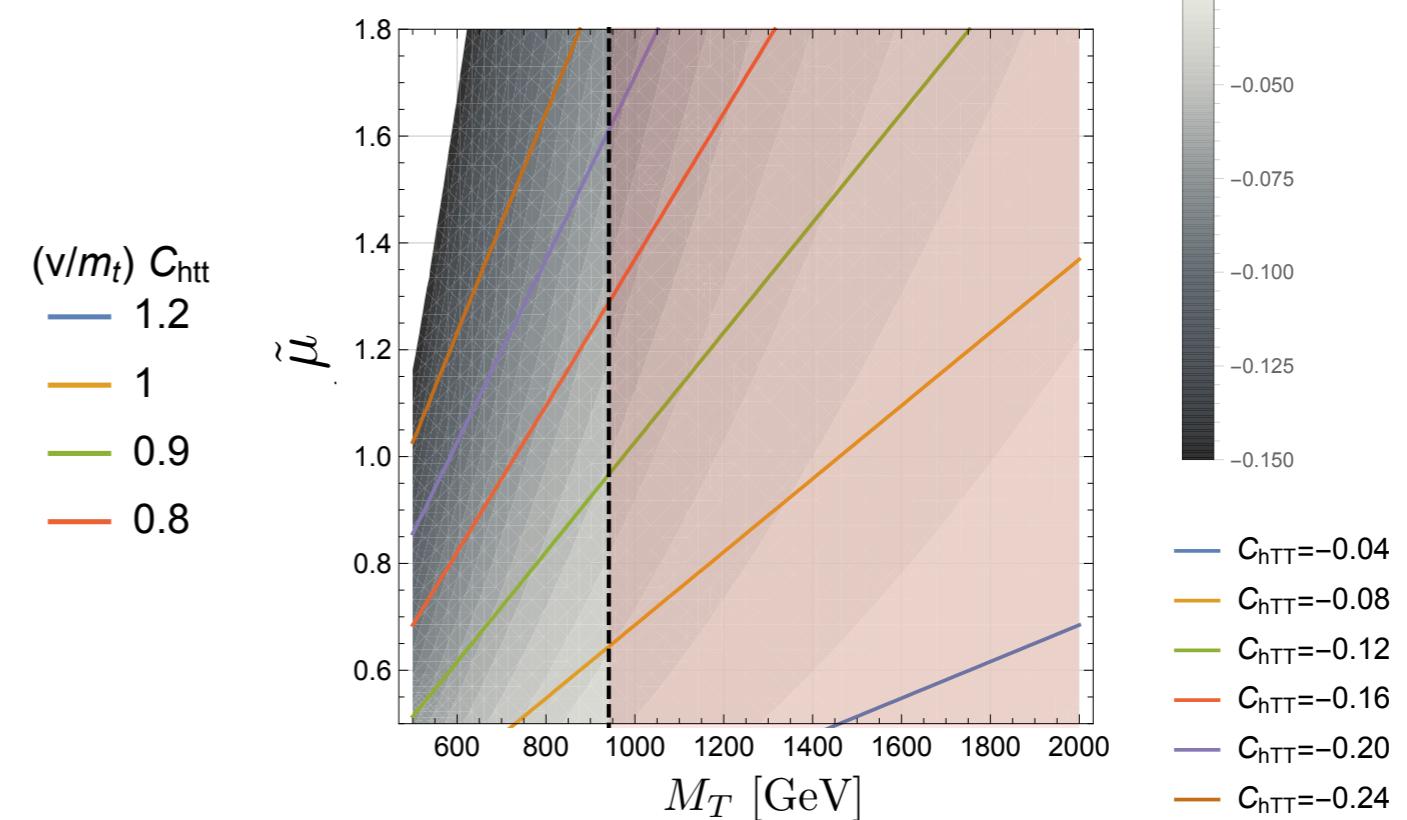
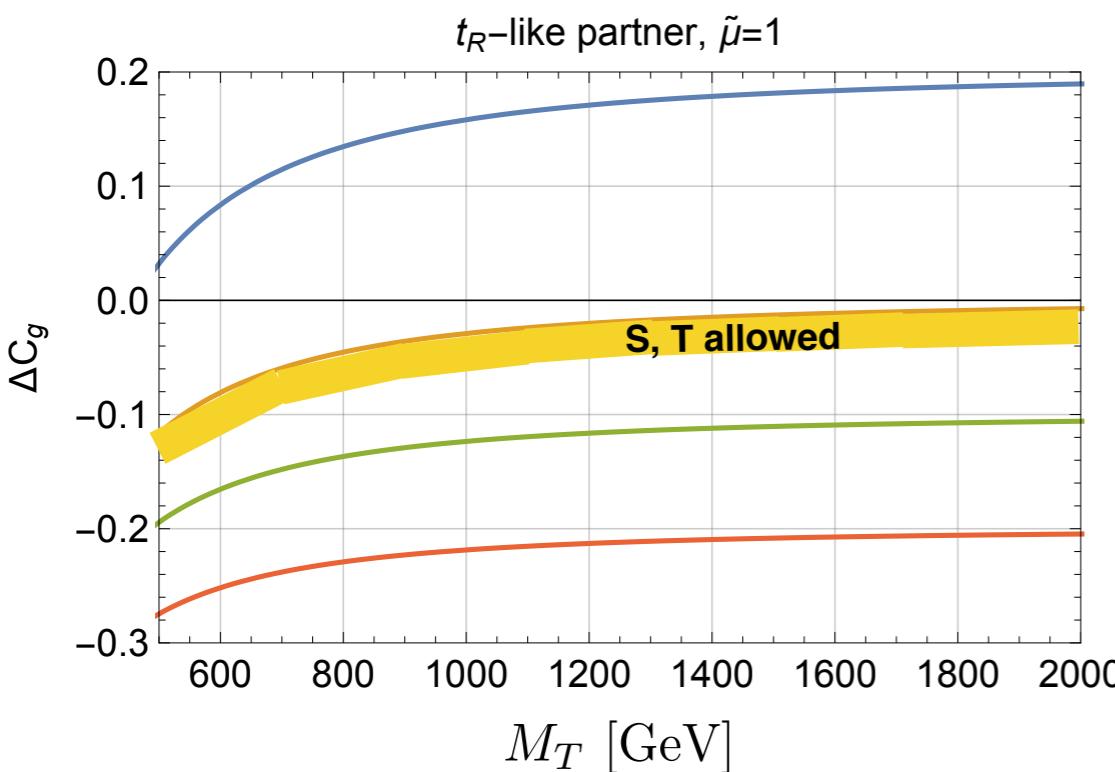
$$\textcircled{1} \quad m_t, M_T \gg m_h$$

$$\textcircled{2} \quad C_{htt} \simeq \frac{m_t}{v}$$

$$\approx \textcircled{1} \left| \frac{v}{m_t} C_{htt} \left( 1 - \tilde{\mu} C_{htt} \frac{v^2}{M_T^2} \right) \right| \approx \textcircled{2} \left| 1 - \tilde{\mu} \frac{m_t^2}{M_T^2} \right|$$

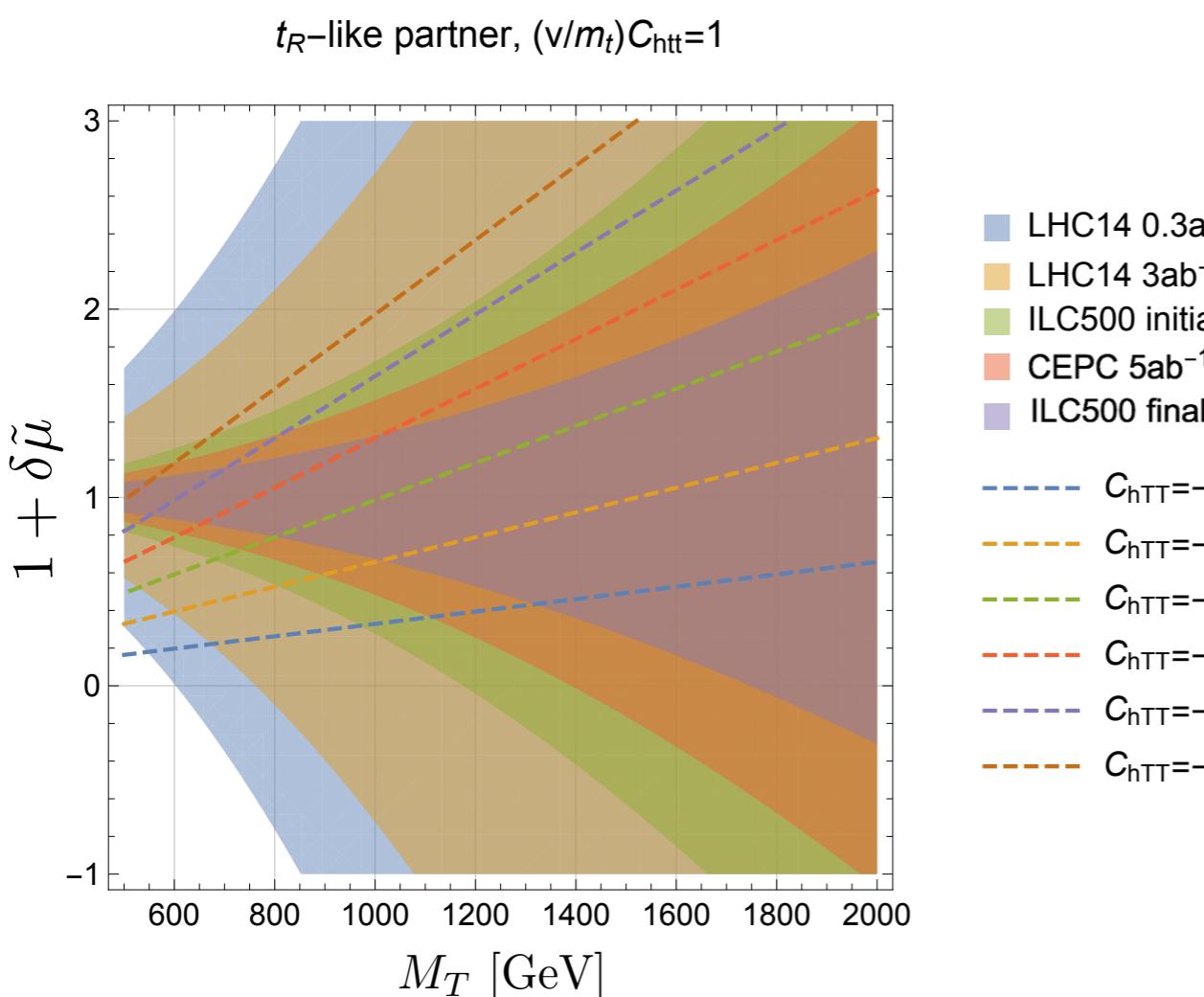
	<b>LHC14 0.3iab</b>	<b>LHC14 3iab</b>	<b>ILC500 0.5iab</b>	<b>ILC500 4iab</b>	<b>CEPC 5iab</b>
<b><math>\Delta C_g</math></b>	8%	5%	2.1%	0.95%	1.5%

$t_R$ -like partner,  $(v/m_t)C_{htt}=0.98$

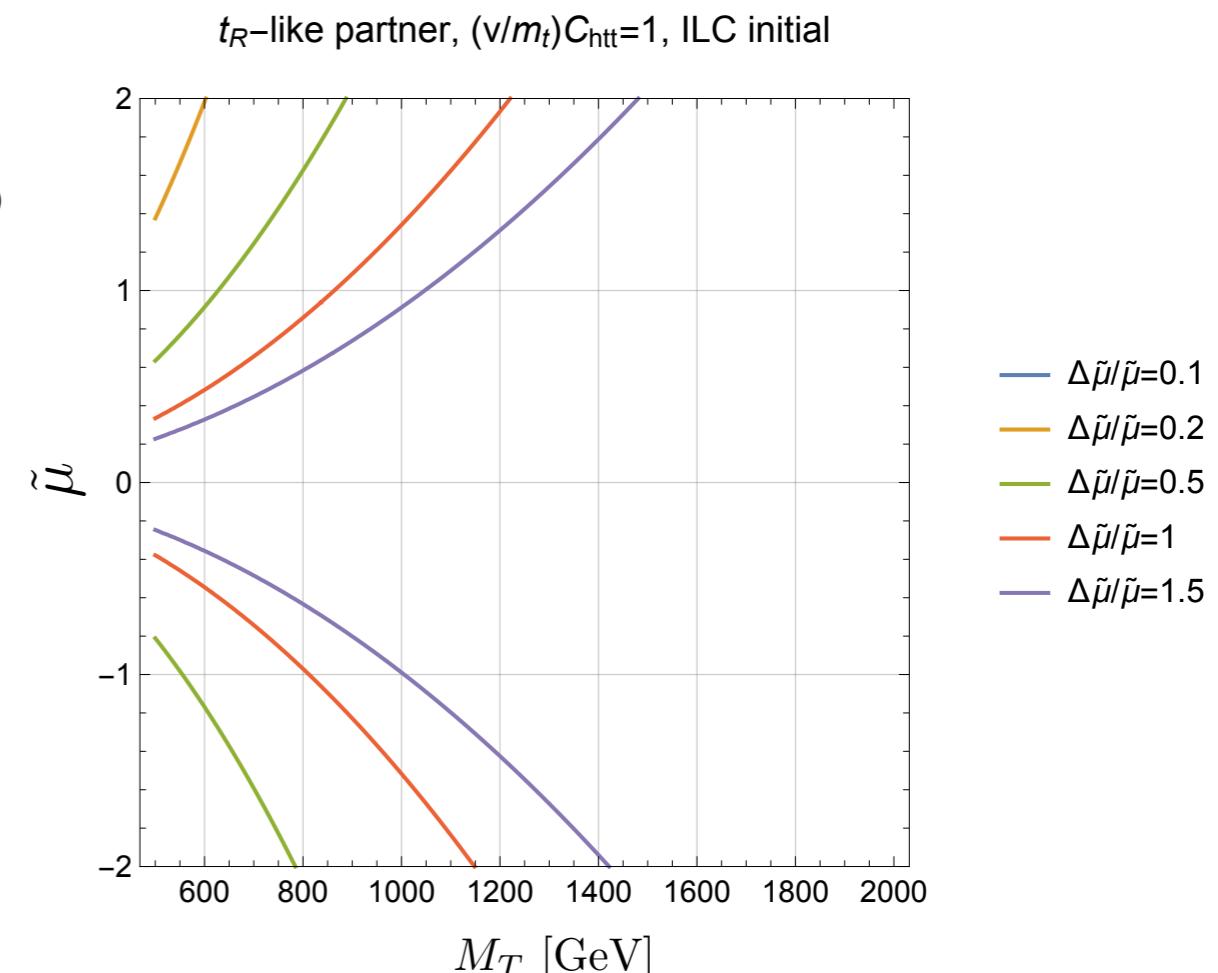


# Exclusion of non-natural models and measurement of $\tilde{\mu}$

Assuming that  $\tilde{\mu} = 1$ , how well can future colliders exclude non-natural models ?

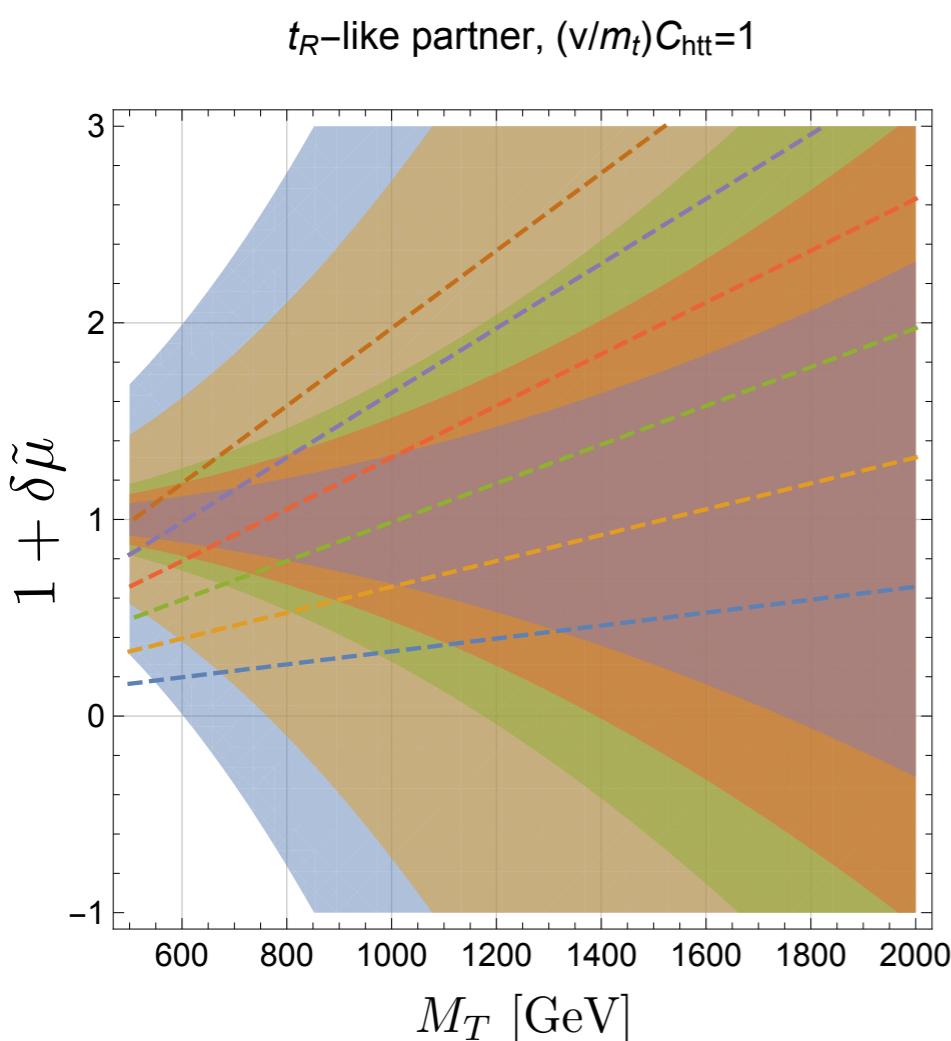


Precision on the measurement of  $\tilde{\mu}$ , given expected precision on gluon coupling and top Yukawa

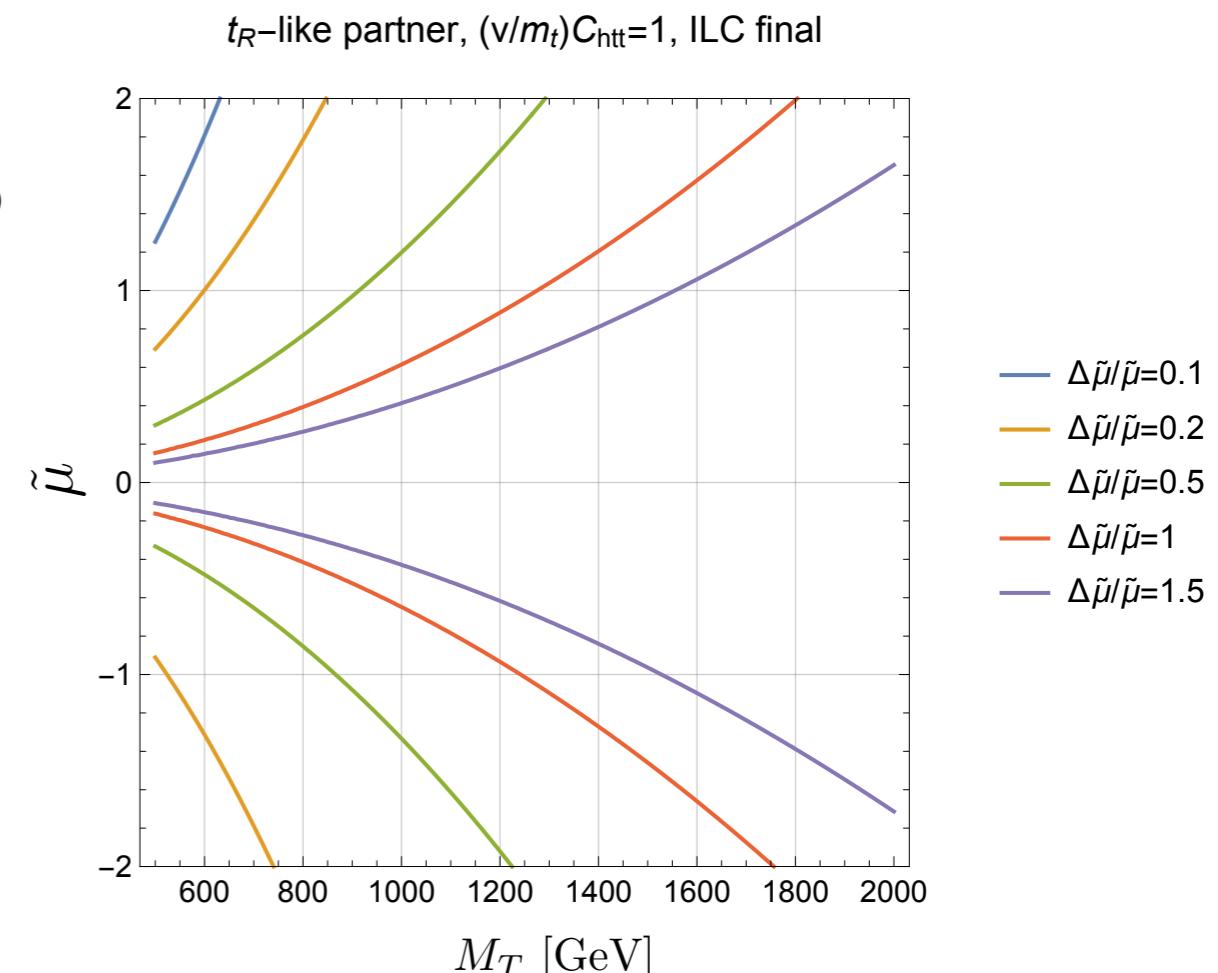


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# Real singlet scalar

# A few considerations

$$\mathcal{L}_S = \frac{1}{2} (\partial_\mu S)^2 - \frac{1}{2} M_S^2 S^2 - \Lambda_S S |H|^2 - \frac{1}{2} \kappa S^2 |H|^2 - \frac{1}{3!} \mu_S S^3 - \frac{1}{4!} \lambda_S S^4$$

- Veltman condition:  $\kappa - 12|y_t|^2 = 0 \implies \mu \equiv \frac{\kappa}{12|y_t|^2}$
- It turns out that integrating out  $S$  provides nice handle on the naturalness parameter:

$$\mathcal{L}_S^{\text{1loop}} \supset \frac{\kappa^2}{12M_S^2} \frac{1}{16\pi^2} \times \frac{1}{2} \partial_\mu |H|^2 \partial^\mu |H|^2 \supset \delta Z \times \frac{1}{2} \partial_\mu h \partial^\mu h \quad \boxed{\delta Z \equiv \frac{v^2}{M_S^2} \frac{|y_t|^4 \mu^2}{16\pi^2}}$$

- Canonical renormalisation of the Higgs field  $\rightarrow$  Higgs couplings to SM particles rescaled correspondingly, in particular:

$$\frac{\sigma_{Zh}}{\sigma_{Zh}^{\text{SM}}} = 1 - \delta Z$$

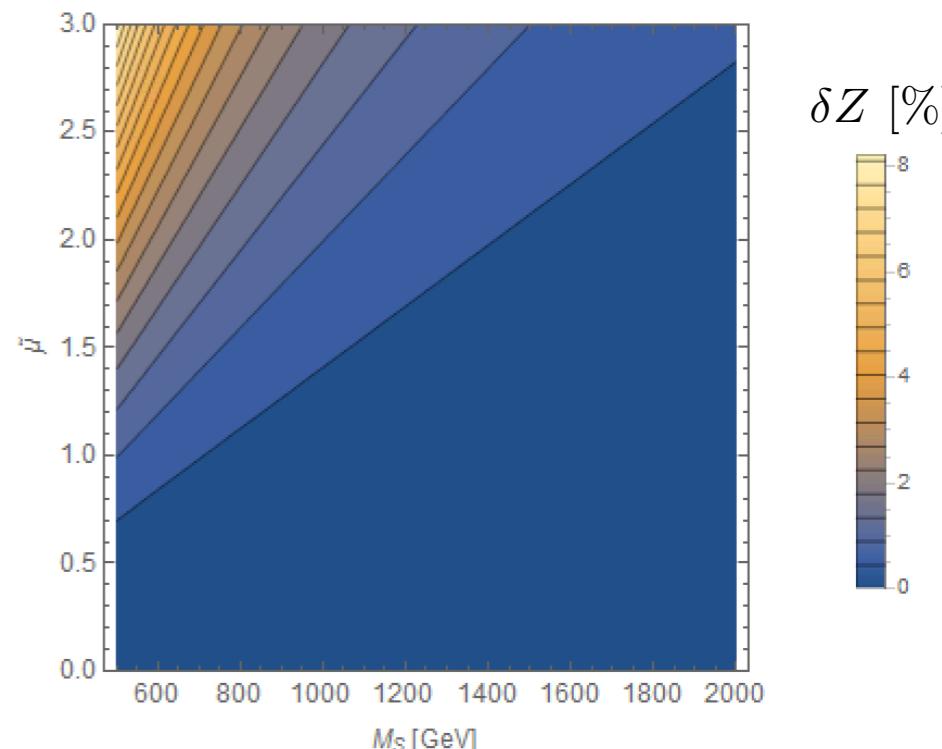
Zh cross-section at  
ee colliders

[Craig, Englert, McCullough:  
1305.5251 (PRL)]

$$\mu = 1 - \delta Z$$

(global Higgs signal strength)

- But also Higgs trilinear coupling, T parameter...



# Conclusions

# Conclusions

Discovery of a top-partner-like state is **a first step** towards deeper comprehension of electroweak physics

One should make sure that this state is indeed a top-partner by verifying the **required Veltman relation**

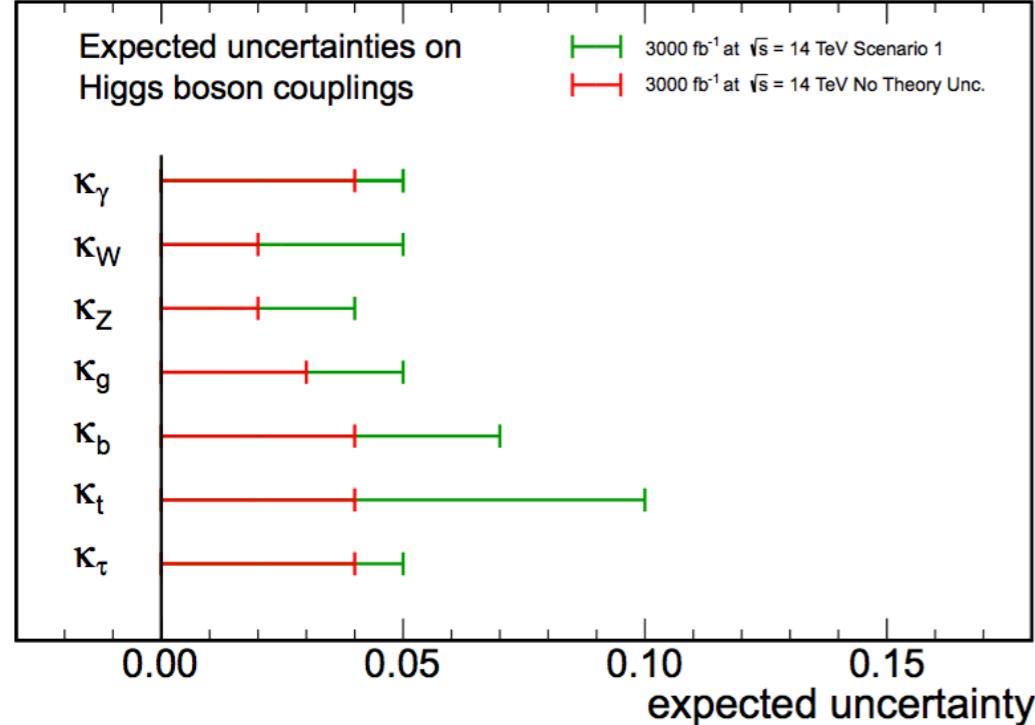
**Precision observables** provide interesting probes of the naturalness parameter in the **post-LHC era**, complementary to **direct production at colliders**

Perspective: real scalar singlet, two colored scalars (stop-like) ...

# Backup

# Higgs coupling projections

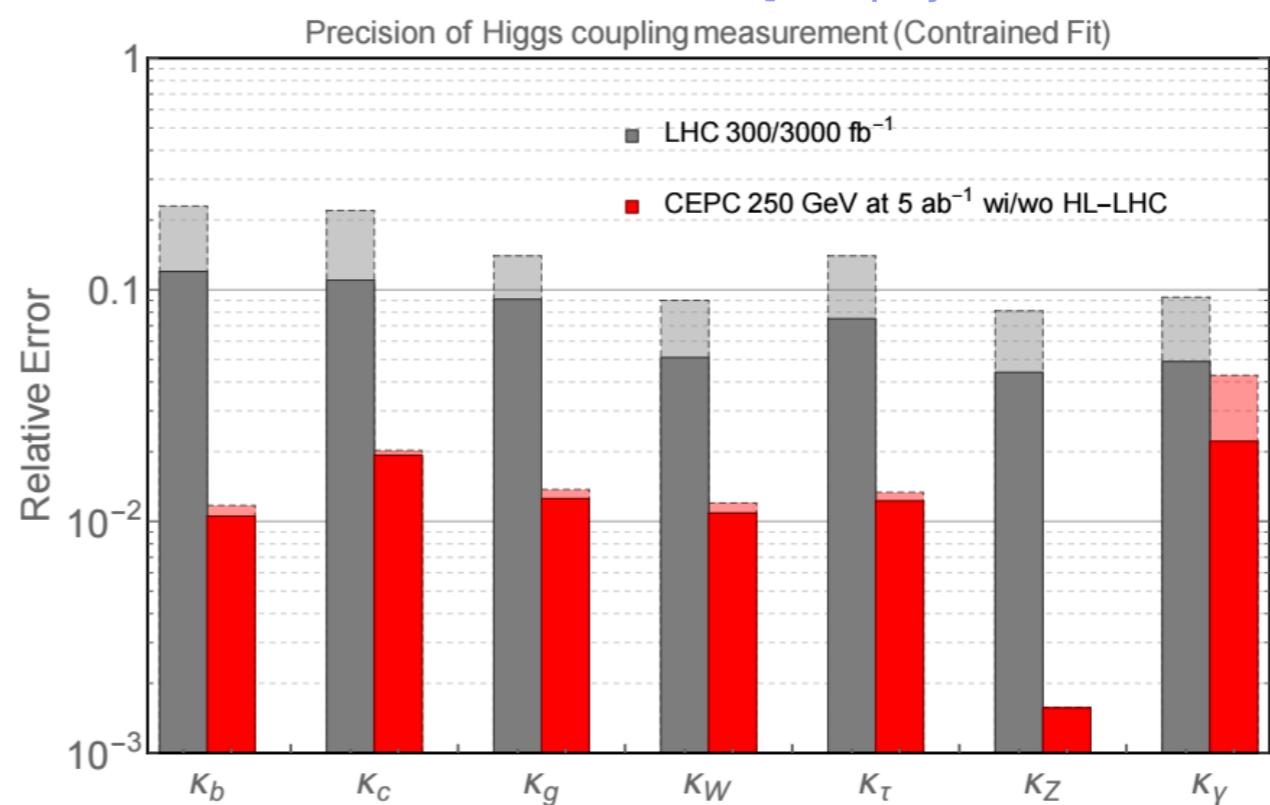
CMS Projection



[CMS projected performance: 1307.7135]

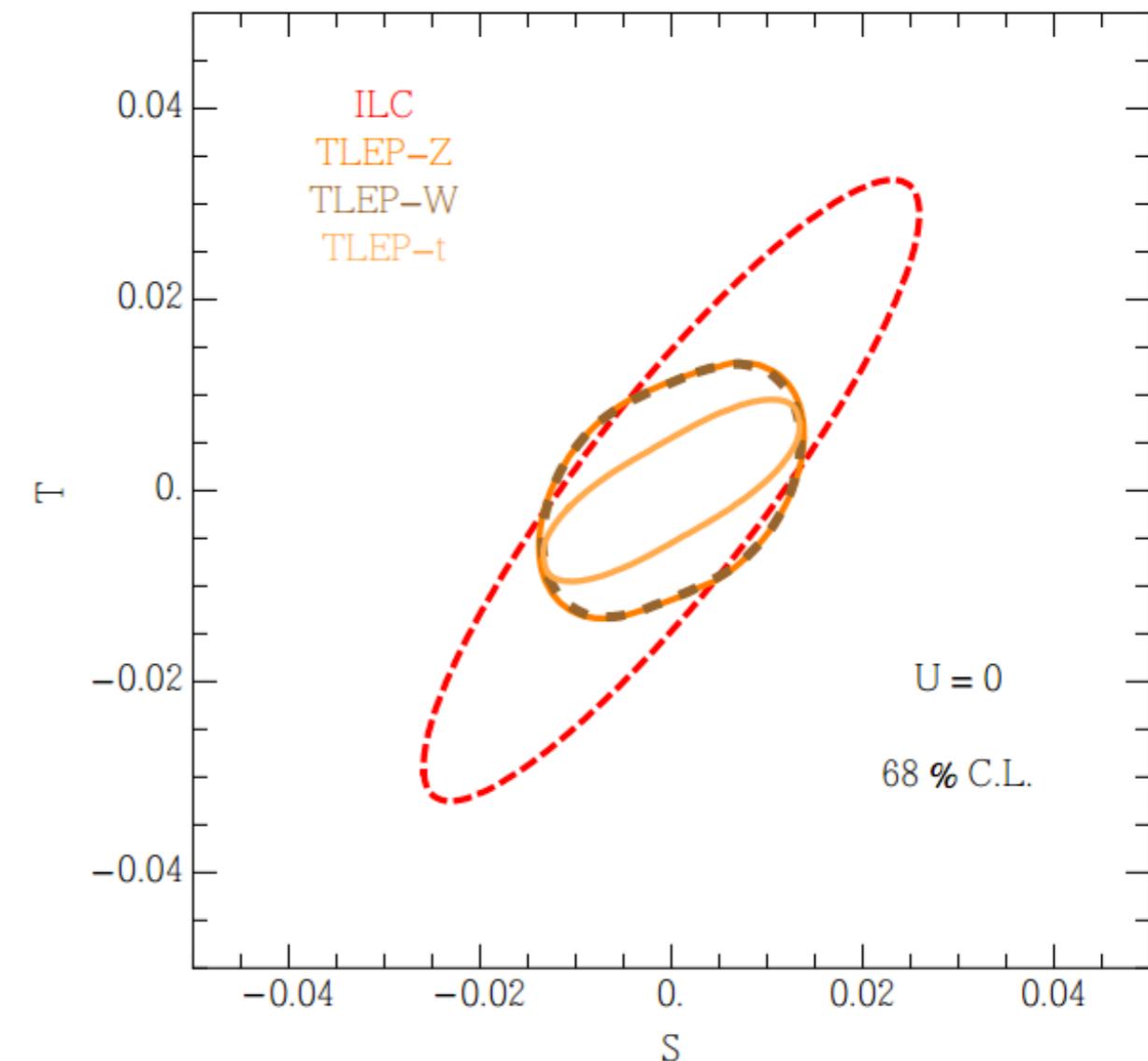
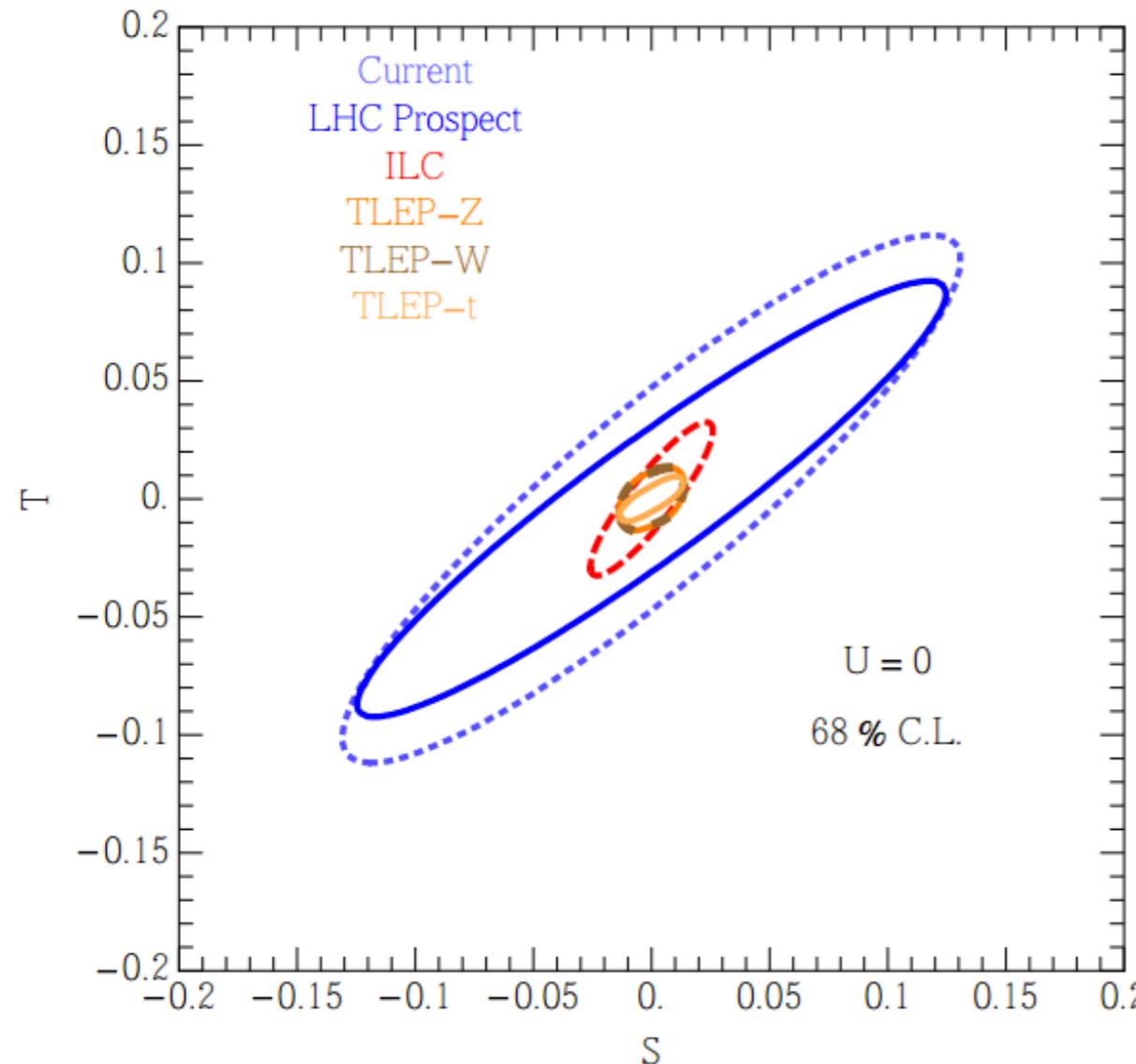
Topic	Parameter	Initial Phase	Full Data Set	%
Higgs	$g(hZZ)$	0.37	0.2	%
	$g(hWW)$	0.51	0.24	%
	$g(hb\bar{b})$	1.1	0.49	%
	$g(hgg)$	2.1	0.95	%
	$g(h\gamma\gamma)$	7.7	3.4	%
	$g(h\tau\tau), g(\mu\mu)$	1.5	0.73	%
	$g(hcc\bar{c}), g(htt\bar{t})$	2.5	1.1	%
	$\Gamma_{tot}$	1.8	0.96	%

[ILC physics case: 1506.05992]



[CEPC pre-CDR, Volume 1]

# S, T projections



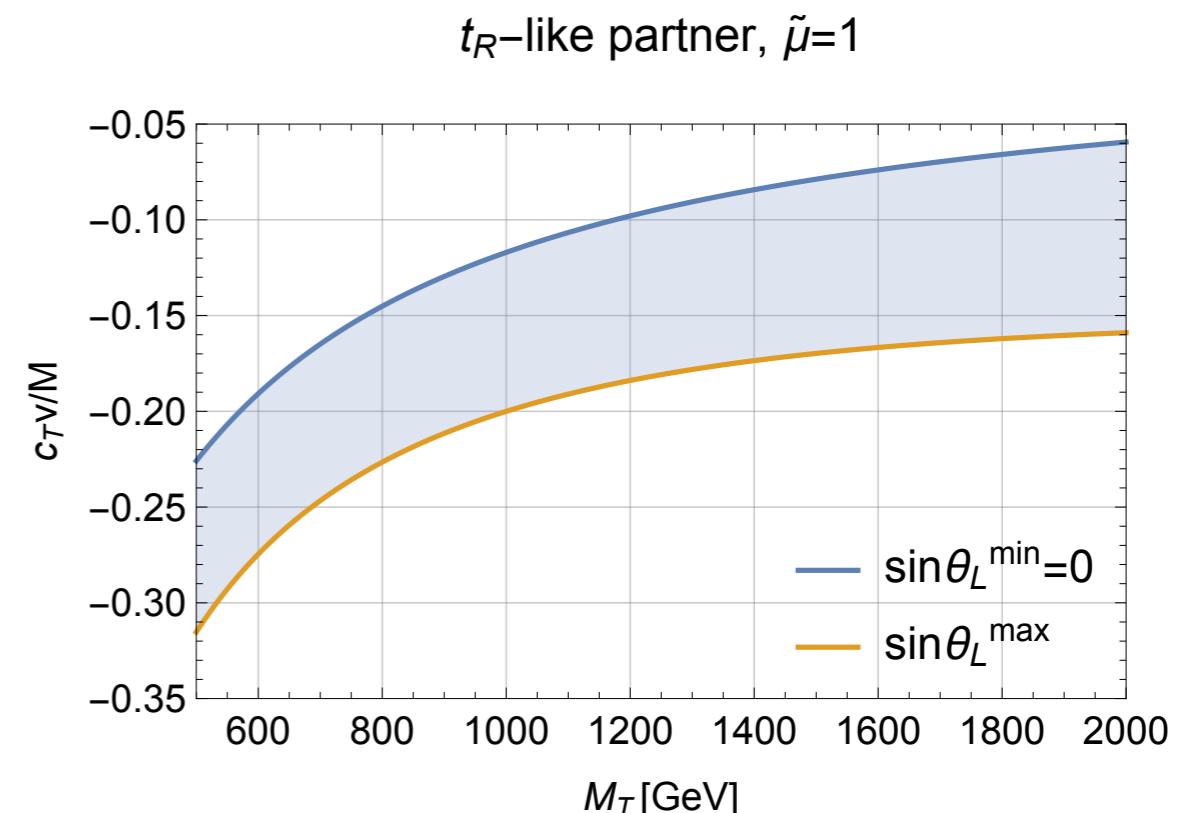
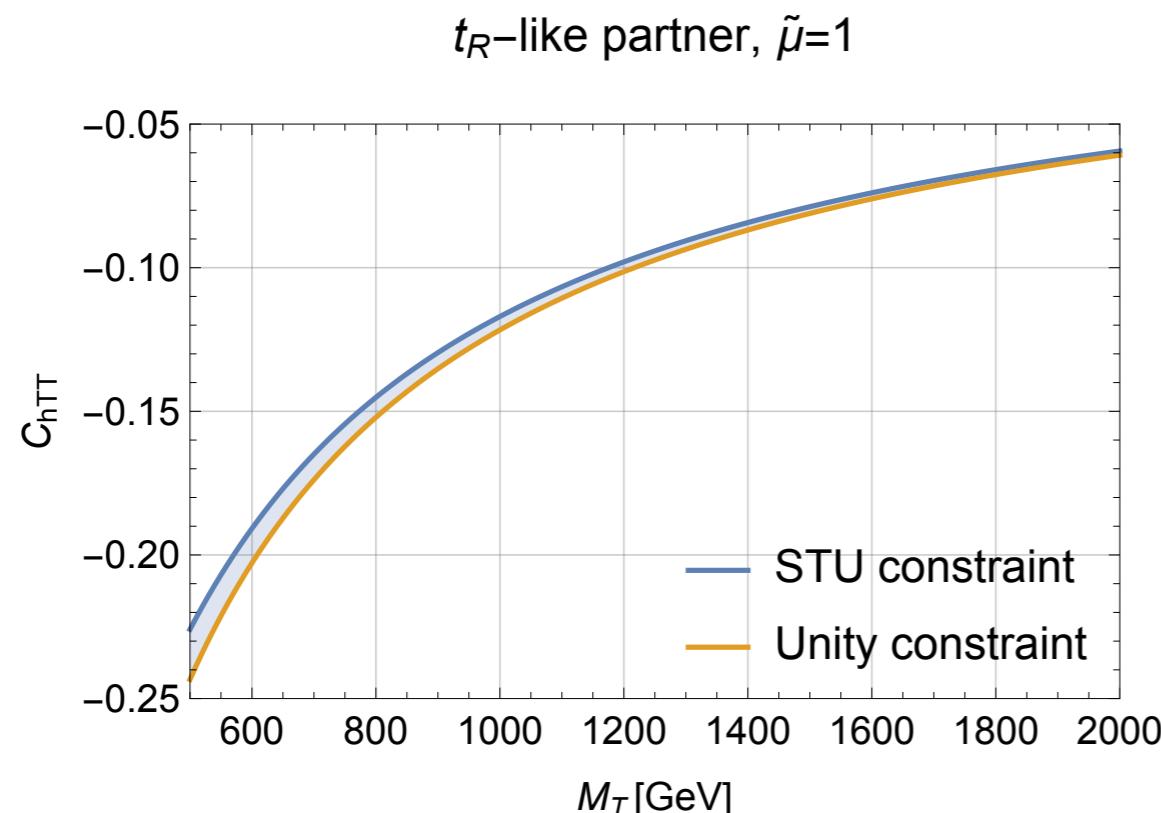
[Fan, Reece, Wang: 1411.1054 (JHEP)]

# Natural model in the light of S, T constraints

- In light of the constraint on the mixing angle, what if  $\tilde{\mu} = 1$  ?
- Link between mixing angle and naturalness parameter:

$$\tilde{\mu} \equiv -\frac{M_T}{v} \frac{C_{hTT}}{|C_{htt}|^2}$$

$$C_{htt} = \underbrace{\frac{m_t}{v} \cos \theta_L \cos \theta_L}_{\frac{y_t}{\sqrt{2}} + \mathcal{O}(\frac{v^2}{M^2})} + \underbrace{\frac{c_T v}{M} \sin \theta_L \sin \theta_R}_{\mathcal{O}(\frac{v^4}{M^4})} \implies c_L^2 \simeq \frac{v}{m_t} C_{htt} = \frac{v}{m_t} \sqrt{\frac{M_T}{v}} \sqrt{-C_{hTT}} \leq 1$$



# Formulae

$$\tan(2\theta_L) = \frac{-2\sqrt{2}vM(2M^2 + c_T v^2)\lambda_T}{4M^4 + c_T^2 v^4 + 2M^2 v^2(2c_T - |y_T|^2 - |\lambda_T|^2)}$$

$$\tilde{\mu} = \mu + (3\mu - 2) \frac{\lambda_T v^2}{2M^2} \left( \lambda_T + \frac{2\tilde{c}_T}{y_t} \right) - c_T^2 \frac{v^2}{y_t^2 M^2} + \mathcal{O}\left(\frac{v^4}{M^4}\right)$$

$$\frac{\alpha S}{4s_W^2 c_W^2} = -\frac{\Pi'_{3B}(0)}{s_W c_W} = \Pi'_{ZZ}(0) - \Pi'_{\gamma\gamma}(0) - \frac{c_W^2 - s_W^2}{s_W c_W} \Pi'_{Z\gamma}(0),$$

$$\alpha T = \frac{1}{M_W^2} [\Pi_{WW}(0) - \Pi_{33}(0)] = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} .$$

$$M_T(\text{Unitarity Bound}) \lesssim \frac{550 \text{ GeV}}{s_L^2} .$$