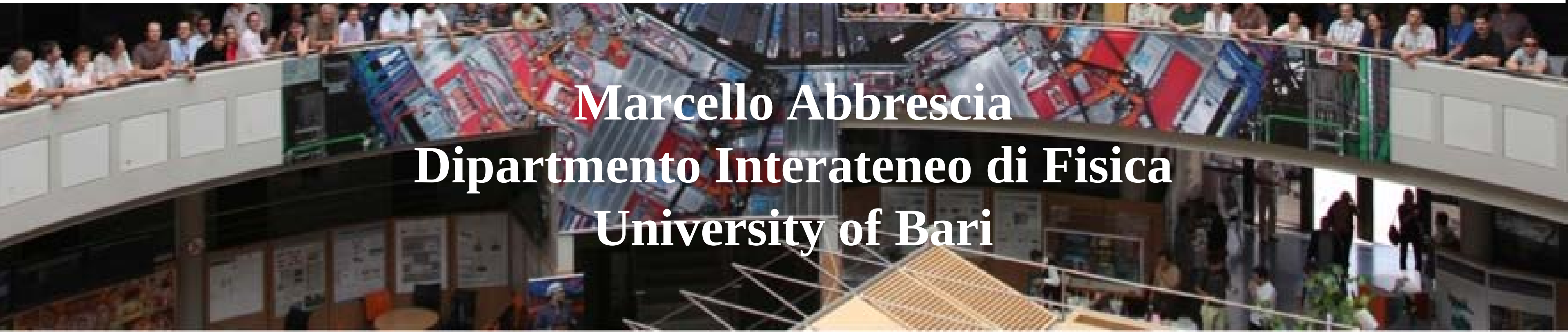
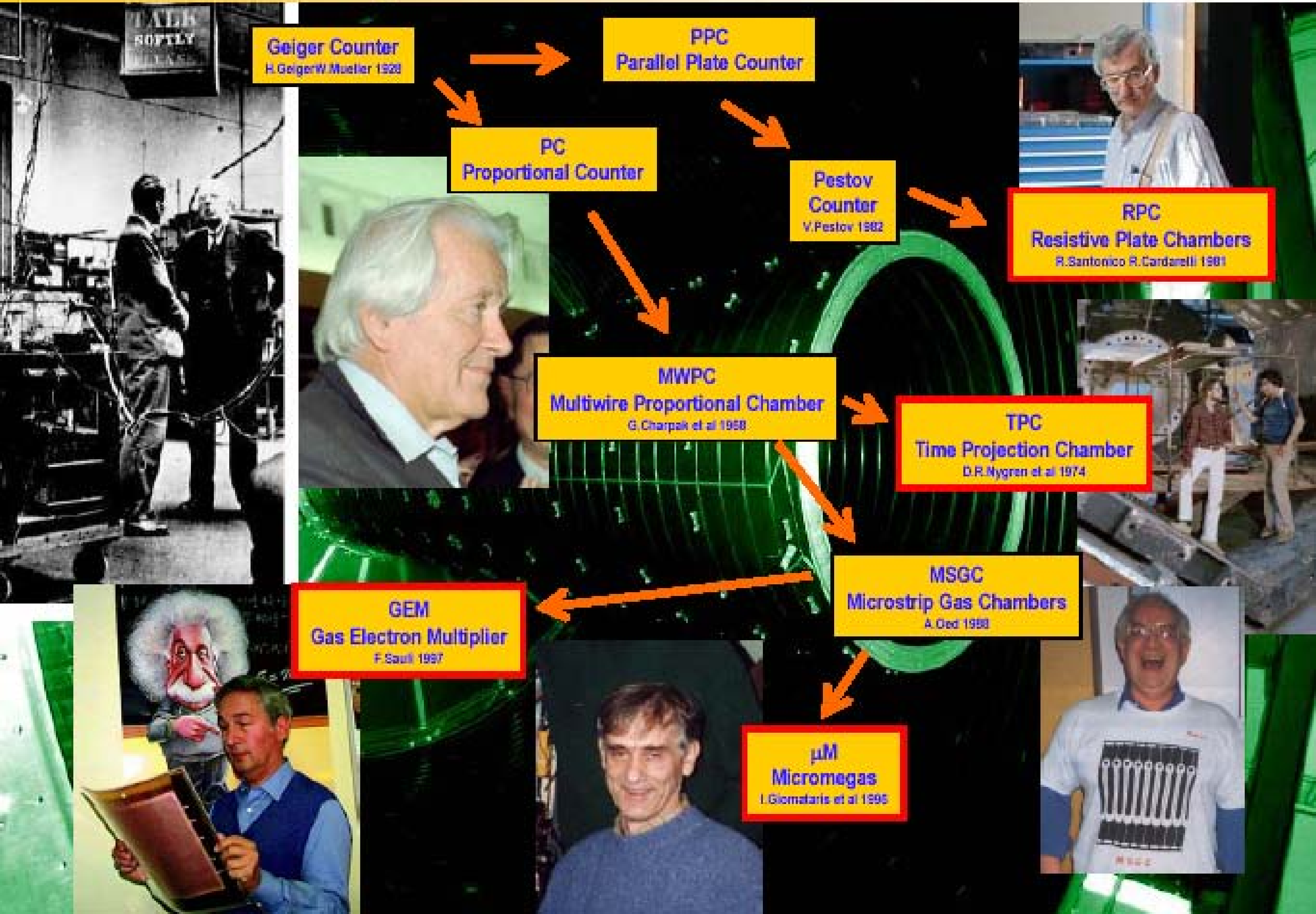
“Analytical” simulation of Resistive Plate Chambers



Marcello Abbrescia
Dipartmento Interateneo di Fisica
University of Bari

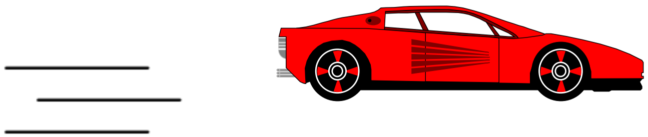
Gas Detector History



Few historical notes

- 1949: Keufell (+ Madanski and Pidd) build Parallel Plate Chambers (PPC): parallel metallic electrodes
- 1980s: Pestov developes Planar Spark Chambers: discharge localized thanks to the use of resistive materials (Parkhomchuk)

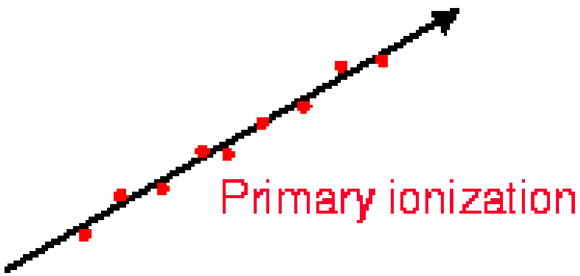
- 1981: Santonico developes Resistive Plate Chambers: **easy to build and use**
- 1992: high rate RPCs were developed: LHC experiments
- 1995: Williams develops multi-gap RPCs

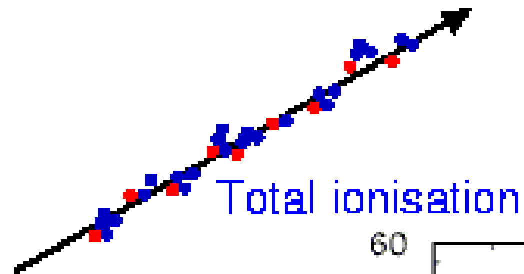


In 1990s RPC physics was poorly know: simulations and analytical models were an **essential tool** to progress in this field

Let us start from the basics

Primary electrons are generated from the interaction of ionizing particles with; quite often their kinetic energy Is enough to generate secondary electrons

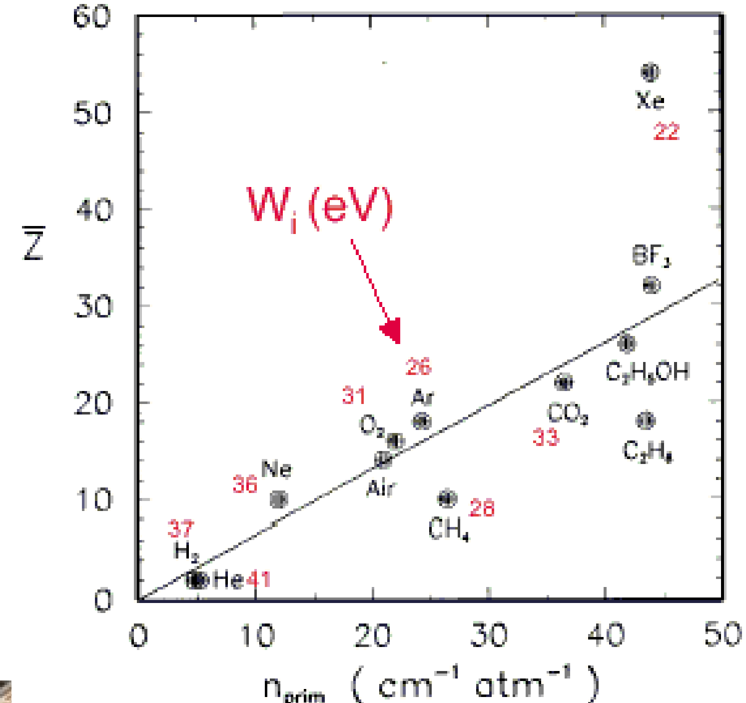




$$n_{total} \approx 3 \ldots 4 \cdot n_{primary}$$

$$n_{total} = \frac{\Delta E}{W_i}$$

n_{total}: total number of ion/electron pairs generated
ΔE: total energy loss
W_i: <energy loss>/pair

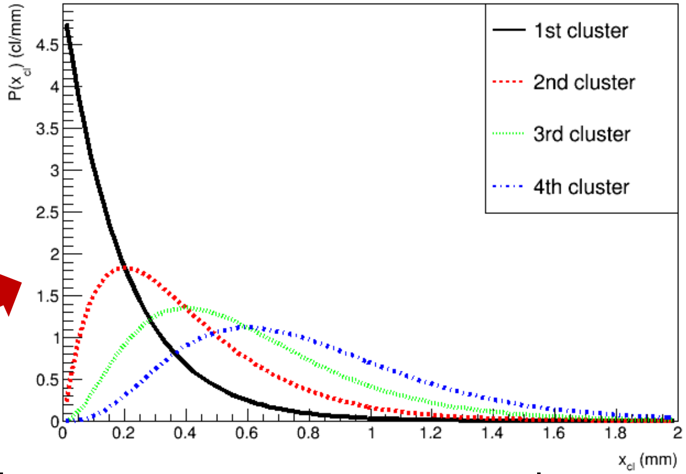


In RPC simulation the ionizing particle energy is typically assumed to be constant throughout the whole gas gap

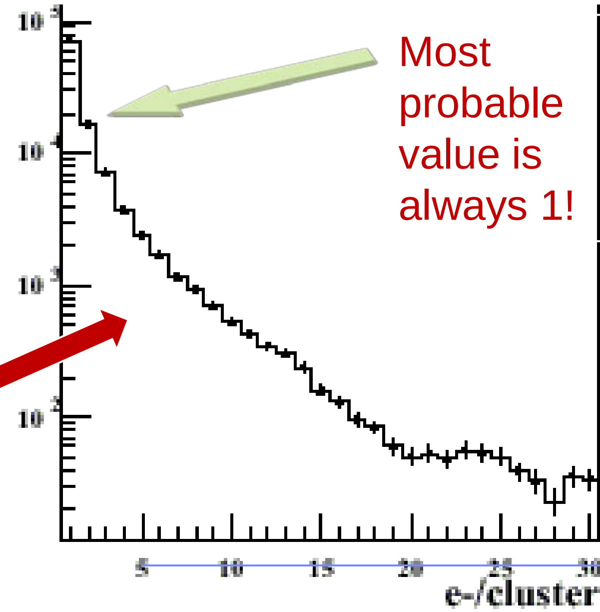
Very simple…

- Experimental data for # clusters/mm (usually indicated as λ) are available for many gas and gas mixtures
  - ✓Very good simulation programs do exist
  - ✓You take it from "literature"
  - ✓From Poisson statistics you can predict their position:

$$P_P^j(x_0^j = x) = \frac{\lambda}{(j-1)!}(x\lambda)^{j-1}e^{-x\lambda}$$



- Less experimental data available for cluster size
  - ✓Just for Ar, CO2, few hydrocarbons
  - ✓Very good simulation programs do exist
  - ✓Typically going $\approx \frac{1}{n^2}$



Cluster size in Ar (experimental)

All are stochastic variables

Number of primaries is distributed following a Poisson distribution

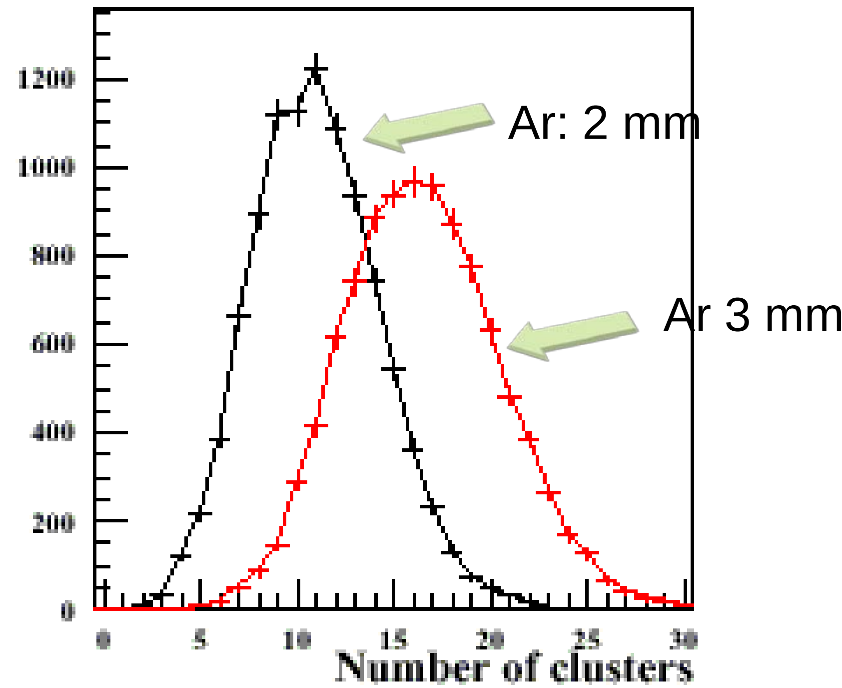
$$P_{cl}(n_{cl}=k)=\frac{(g\lambda)^k}{k!}e^{-g\lambda}$$

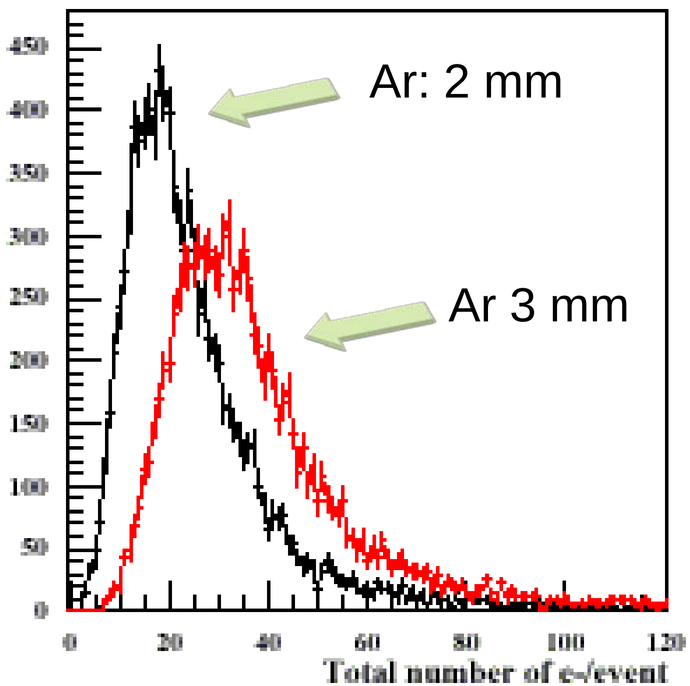
Maximum detection efficiency is therefore limited to:

$$\varepsilon_{\text{det}}=1-P(0)=1-e^{-\bar{n}}$$

For narrow gaps this can be significantly < 1:

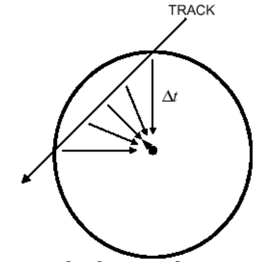
Ar: g= 0.8 mm → $n_{primary}$ ~ 2.3 → ε_{det} ~ 0.9

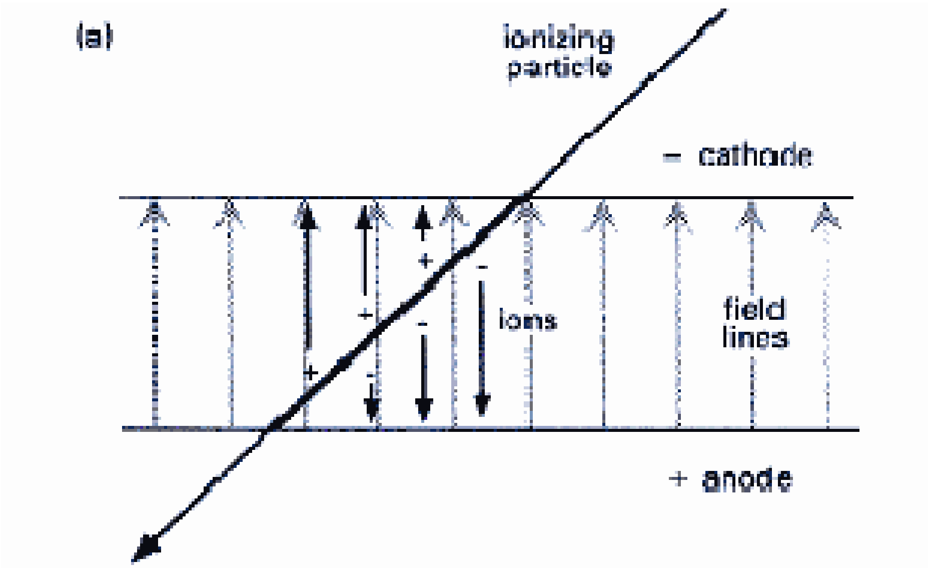


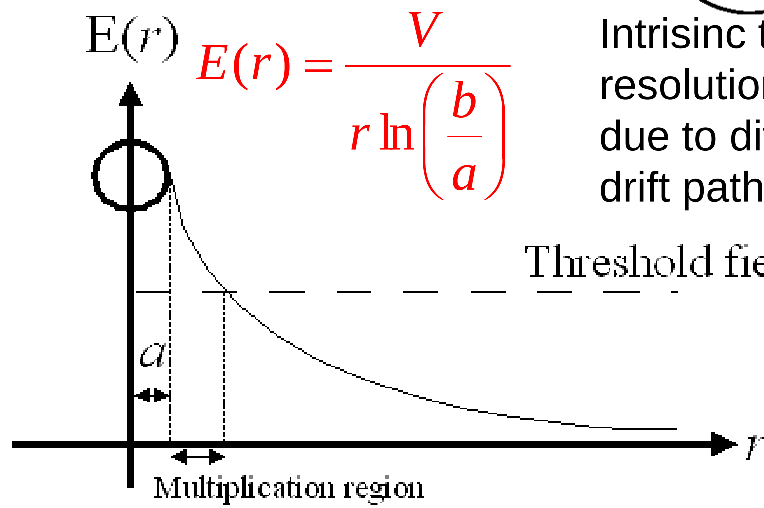


Total number of electrons is the convolution between Poisson distribution and cluster size

Avalanche multiplication







Intrisinc time resolution limit due to different drift paths

In an RPC the **whole gas volume** is suitable for avalanche developing
✓To obtain the electric field strenght suitable for multiplication, gas gap must be not larger than few mm

Avalanche (exponential) development

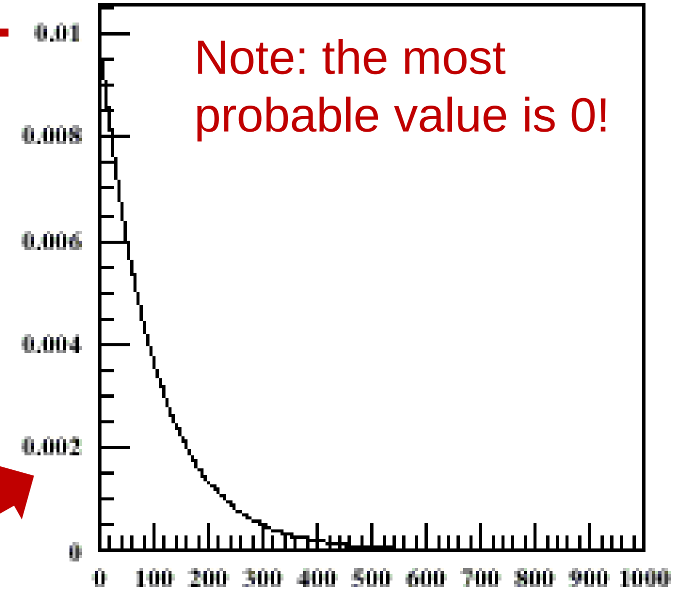
$$q(x) = \sum_{j=1}^{n_{cluster}} q_{el} n_j^0 M_j e^{\eta\left(x - x_j^0\right)}$$

η: 1st “effective” Townsend coefficient
x-x^0_j: distance covered by jth avalanche
M_j: we will see it in a moment

Avalanche fluctuations

Also the exponential avalanche growth is a stocastic process:

✓Probability to have at the end of the avalanche n electrons, where N is the number predicted by exponential growth:



➤Furry's law:

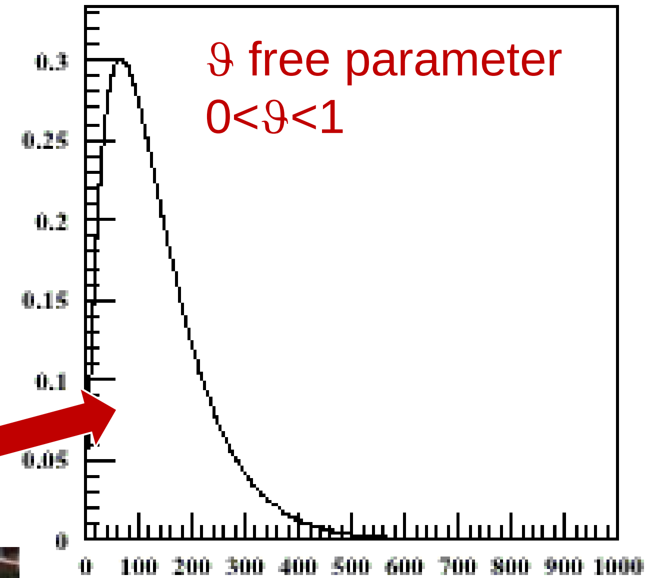
✓Valid for "low electric fields:"

$$P_F(n_{av}=n)=\frac{1}{N}e^{-\frac{n}{N}}$$

➤Polya distribution

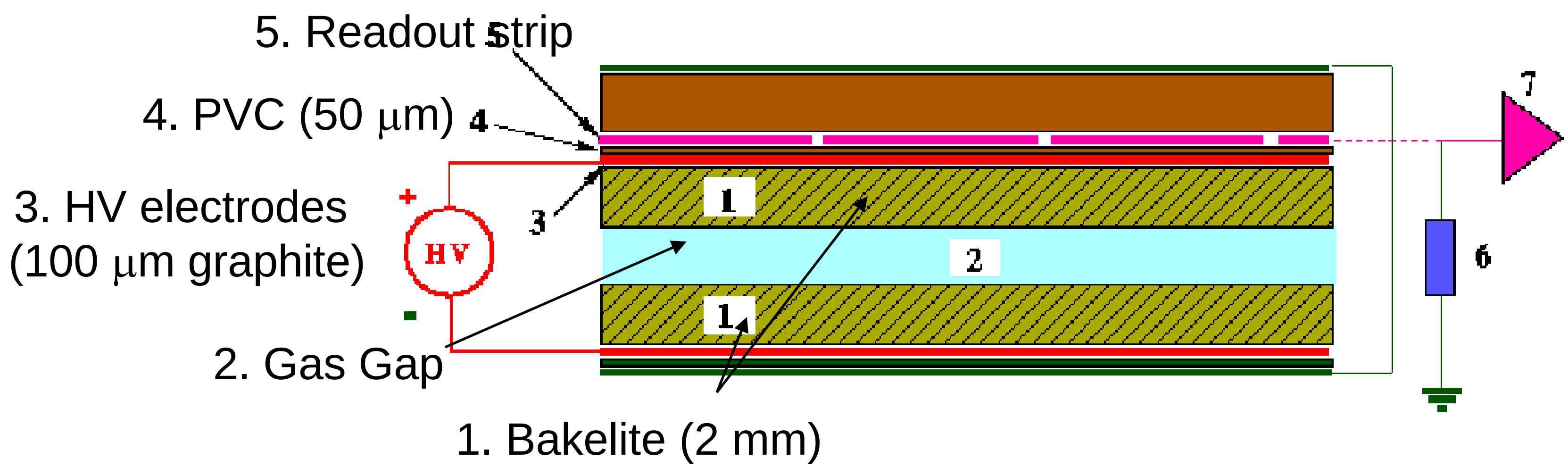
✓More suitable for higher electric fields (RPCs)

$$P_P(n_{av}=n)=\left[\frac{n}{N}(1+\vartheta)\right]^{\vartheta}e^{\left[-\frac{n}{N}(1+\vartheta)\right]}$$

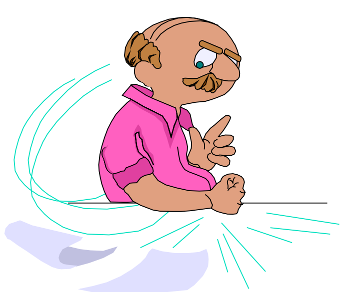


Signal generation in an RPC

In an RPC readout electrodes are completely separated from gas gap:



Electrons in the avalanche (or streamer) do NOT arrive onto the readout electrodes

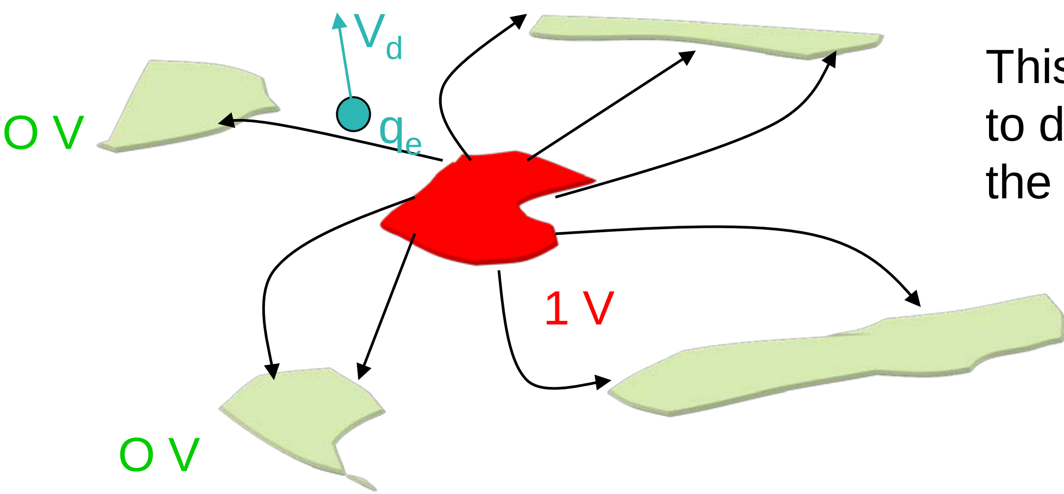


Signal is induced by the charges (electrons and ions) MOVEMENT in the gap

Popular way to speak about "charge collected on the strps" is VERY misleading

Signal induction

Signal induced on readout electrodes can be computed using the Ramo theorem: very basically you put 1 V on the readout electrode and 0 on all the others



This fictitious electric field has nothing to do with the electric field deriving from the applied voltage, and it is called:

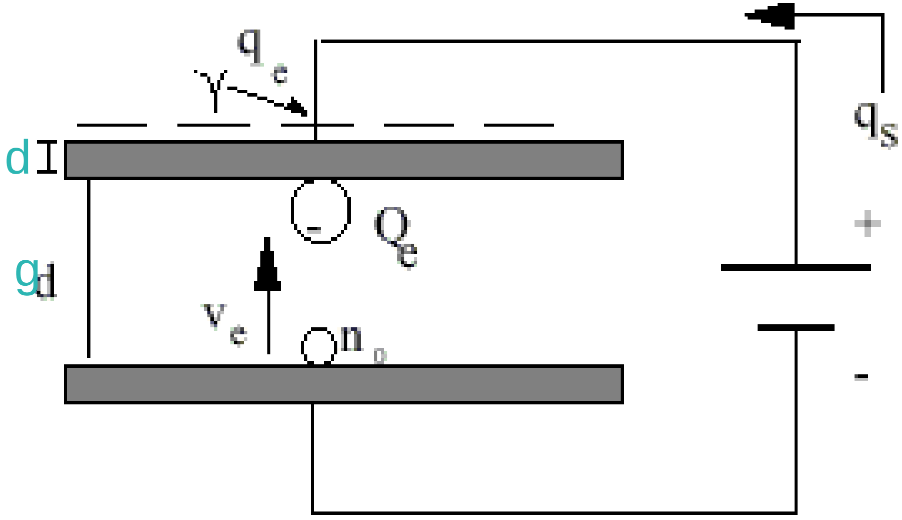
weighting field

$$i_{ind}(t) = -q_e E_w \cdot v_d$$

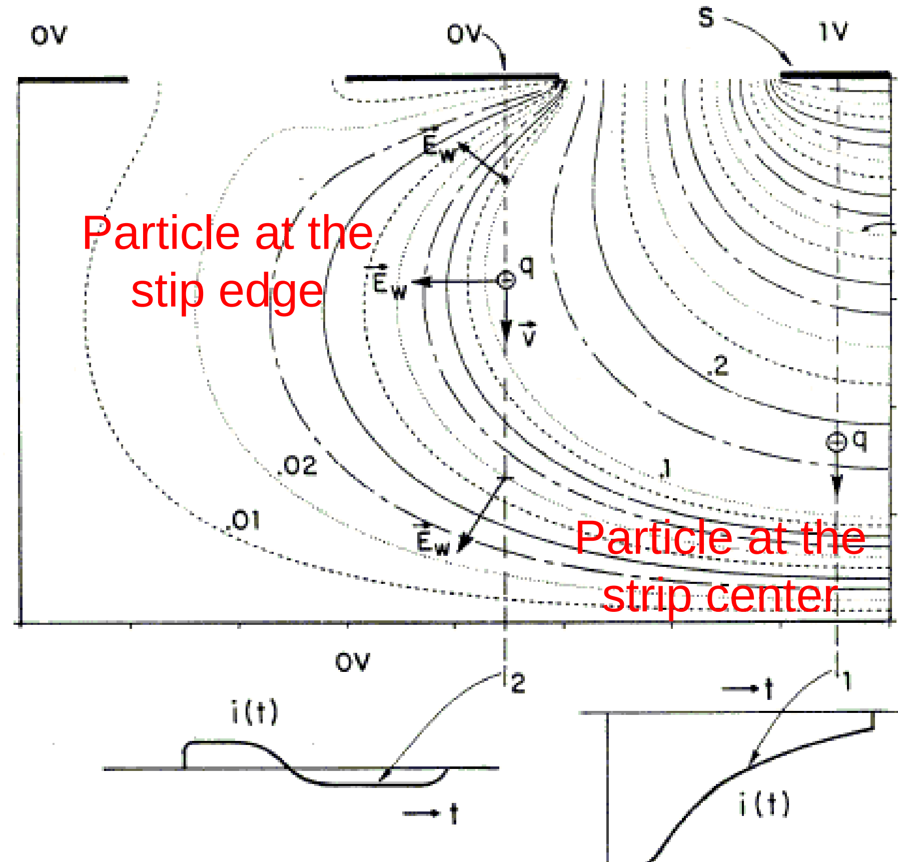
- q_e drifting charge
- E_w weighting field
- v_d drift velocity

$$q_{ind}(t) = \int i_{ind}(t)dt = -q_e \int_{P_1}^{P_2} E_w \cdot d\ell = q_e\left[V_w(P_1) - V_w(P_2)\right]$$

Weighting potential drop



During signal development (≈ few ns) bakelite or glass plates behave as perfect dielectrics



If the particle is not close to the strips edge, we can assume the weighting field to be constant across the gap:

$$\Delta V_w = \frac{\varepsilon_r g}{n_g \varepsilon_r g + (n_g + 1) d}$$

n_g: number of gaps
ε_r: electrode relative dielectric constant

Let us put all together

By using all the concepts reviewed up to now we end up with the following expressions, for the charge induced on readout electrodes (for charge spectra and efficiency).

$$q_{ind} = \frac{q_e}{\eta g} \Delta V_w \sum_{j=1}^{n_{cl}} n_0^j M_j \left[e^{\eta (g - x_0^j)} - 1 \right]$$

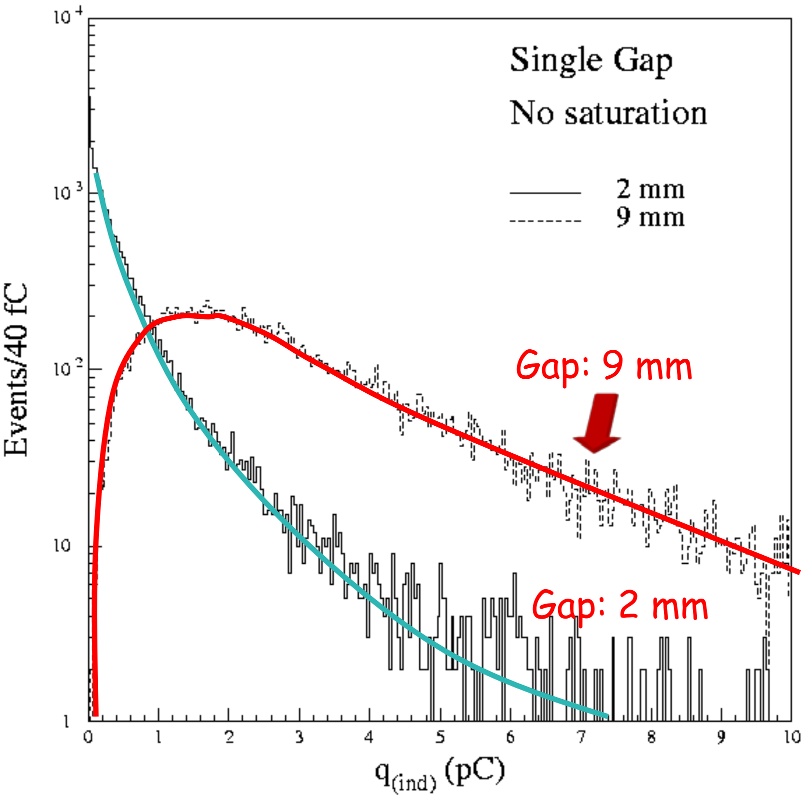
and for the current induced (signal time properties):

$$i_{ind}(t) = -\mathbf{v}_d \cdot \mathbf{E_w} q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}} n_0^j M_j$$

➢ In principle they contain all we need to know to explain experimental data and predict RPCs behaviour given their configuration (geometry, gas, operating voltage)

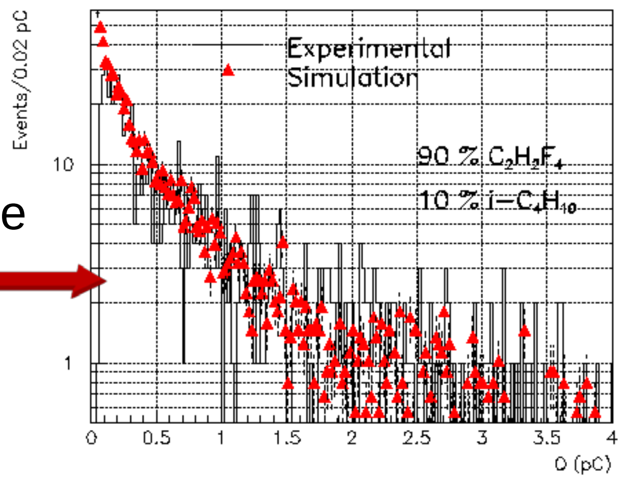
➢ Due to the fact that we have many stocasthic variables, only part of the calculations can be done analythically, for the rest you have to relay on Monte Carlo simulations

Predicted charge spectra

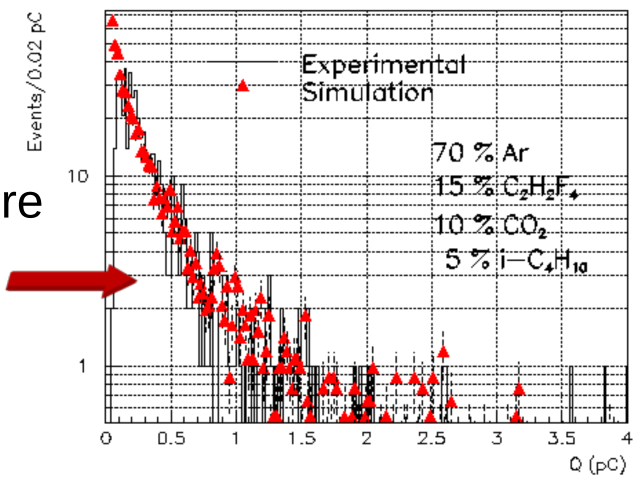


Comparison between Monte-Carlo predictions and experimental data

Freon rich mixture



Argon rich mixture

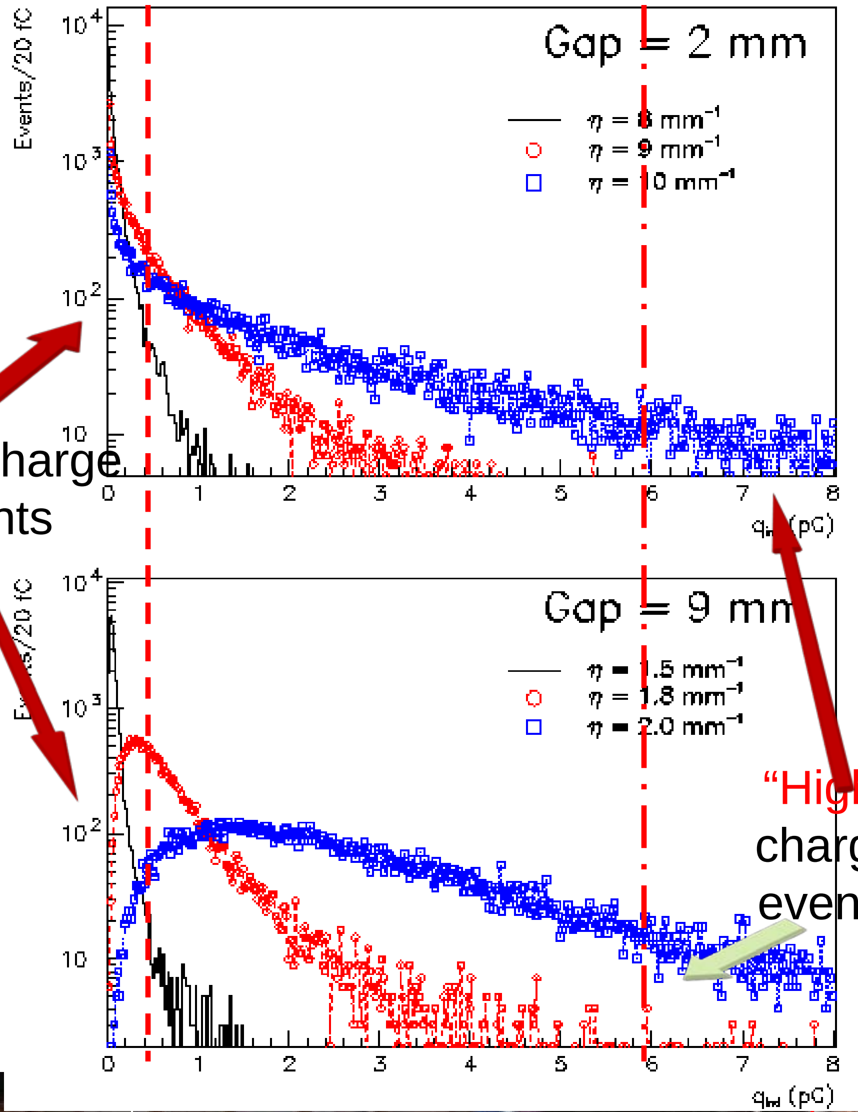


$$q_{ind} \propto Rq^{\frac{\lambda}{\eta}-1}$$

λ: primary cluster density (from 3 to 10 cl/mm)

η: 1st Townsend "effective" coefficient

Some considerations



"Low" charge events

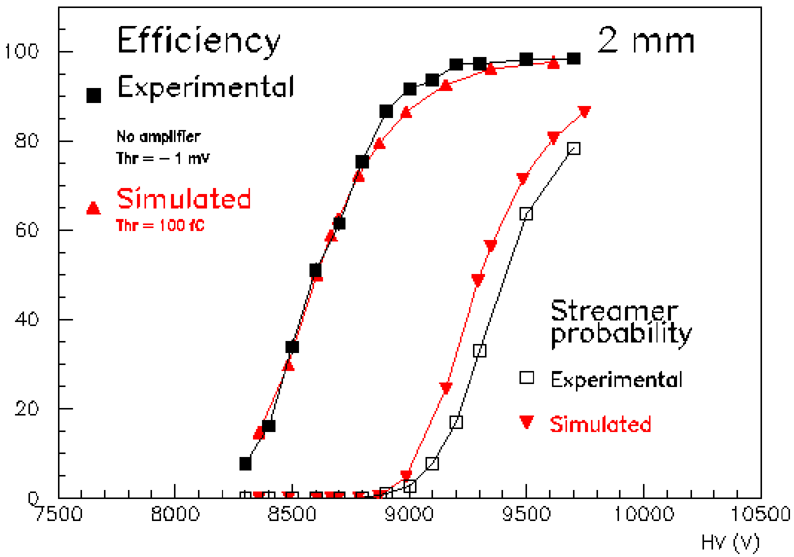
"High" charge events

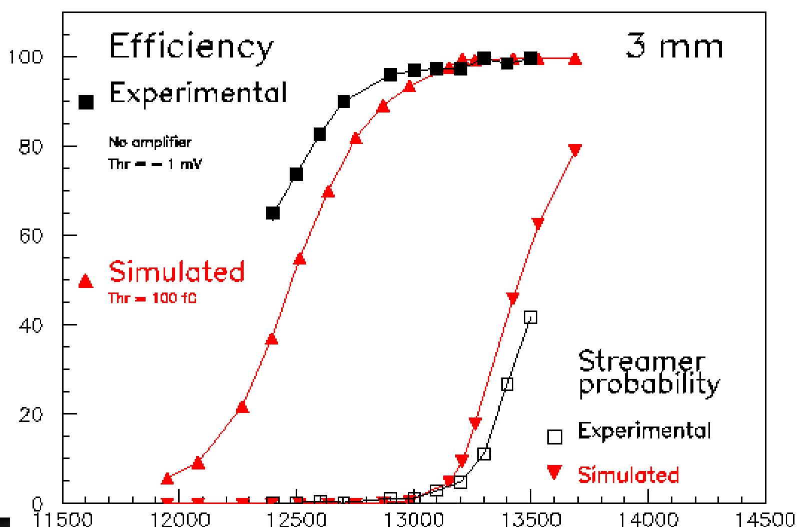
λ and η are bound by the fact that RPC total gain must be more or less constant. Typically ($\eta g \approx 18$)

"Narrow" gap	"Wide" gap
"High" **E** field	"Low" **E** field
$\frac{\lambda}{\eta} < 1$	$\frac{\lambda}{\eta} > 1$
Monotonically decreasing spectrum	Spectrum going to zero close the origin

In a sense, the "narrow" gap case is the worst possible

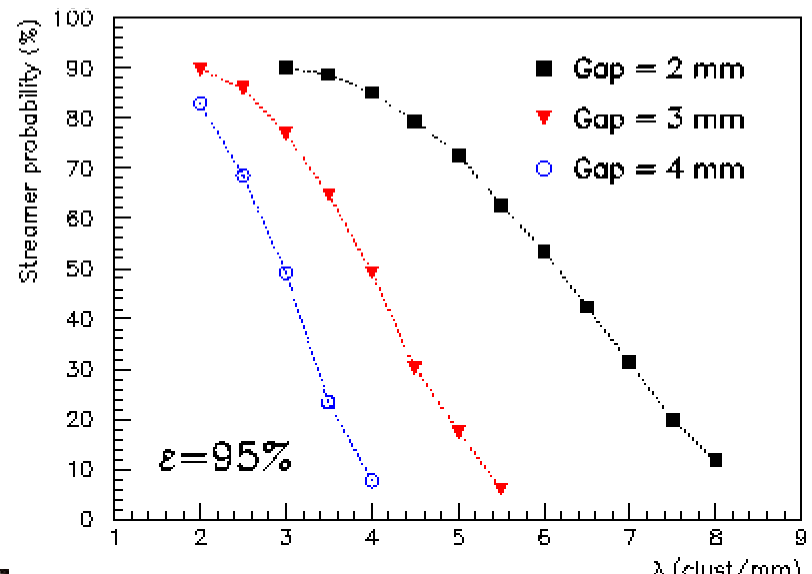
Some (other) considerations





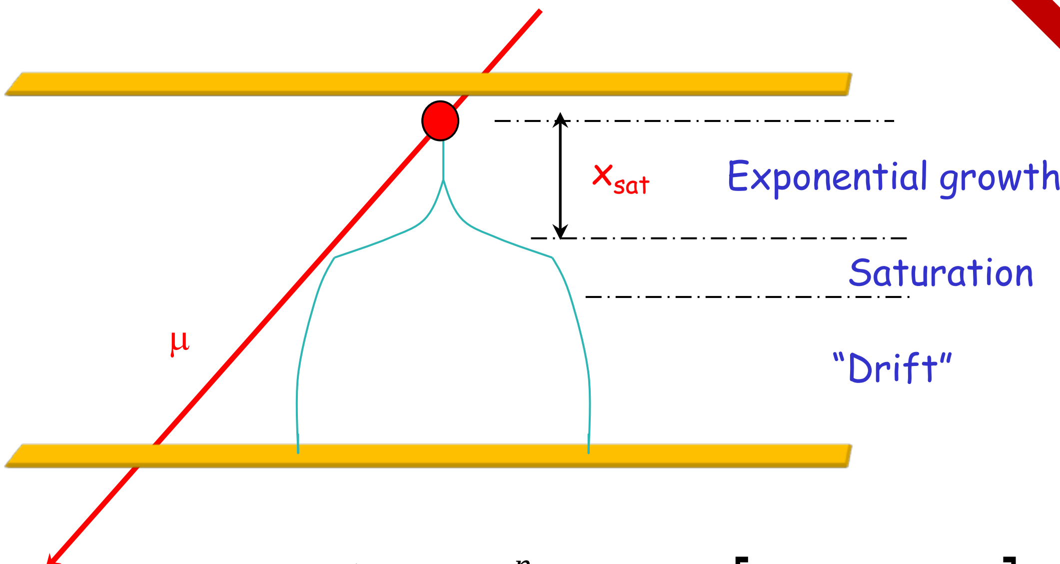
- ➢ Low charge events are related to detector efficiency
- ➢ High charge events are related to streamer probability
- ✓ By using the charge spectra, efficiency and streamer probability curves can be predicted.

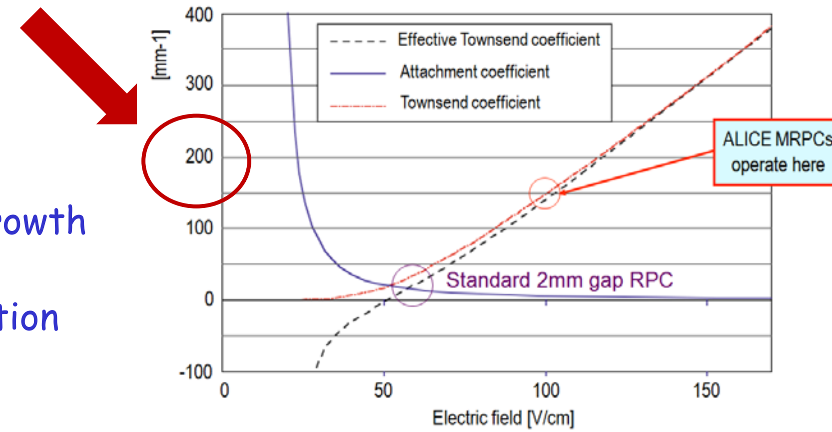
One deduction: high $\lambda \rightarrow$ low streamer probability



Saturation effects

➢They were invoked to explain high efficiency in very narrow (few hundreds microns) gap RPC avalanche electric field becomes comparable to applied electric field



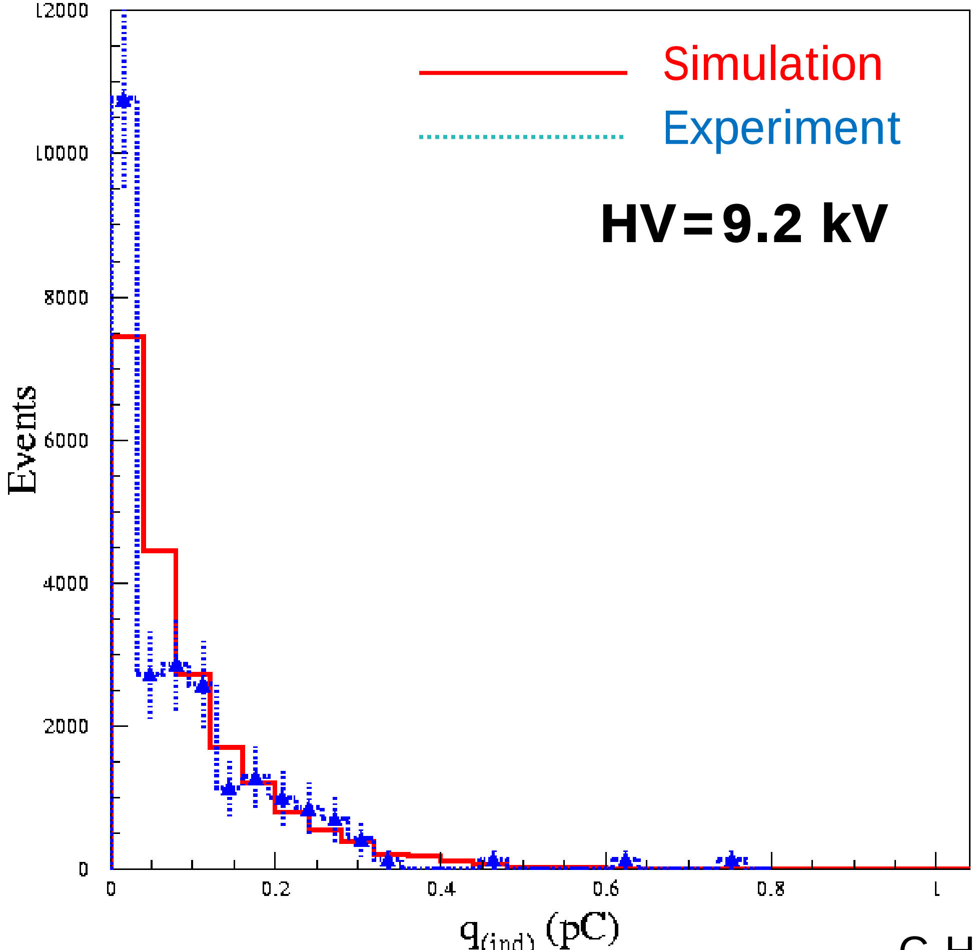


$$\underbrace{q_{ind} = \frac{\Delta V_w}{\eta g} q_e \sum_{j=1}^{n_{cl}} n_0^j M_j \left[e^{\eta \left(x_{sat}^j - x_0^j \right)} - 1 \right]}_{\text{Exponential growth}} + \underbrace{\sum_{j=1}^{n_{cl}} \Delta V_w M_j \frac{g - x^j{}_{sat}}{g} q_{sat}}_{\text{Saturation}}$$

Many models used to describe the saturation phase

When saturation becomes important

This is not a fit!



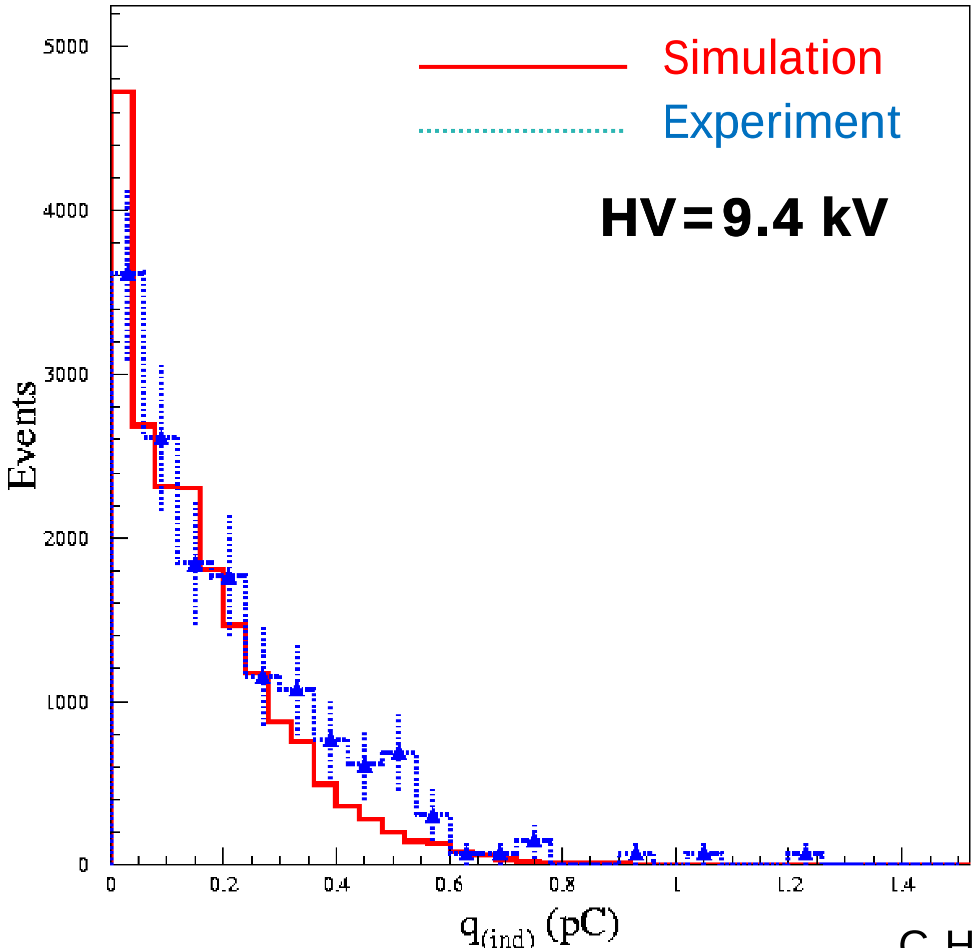
Gas mixture:
$C_2H_2F_4/C_4H_{10}$ 97/3 + SF_6 2%

Input for simulation: Colucci et al., NIM A 425 (1999) 84-91

Experimental data from Camarri et al., NIM A 414 (1998) 317-324

When saturation becomes important

This is not a fit!



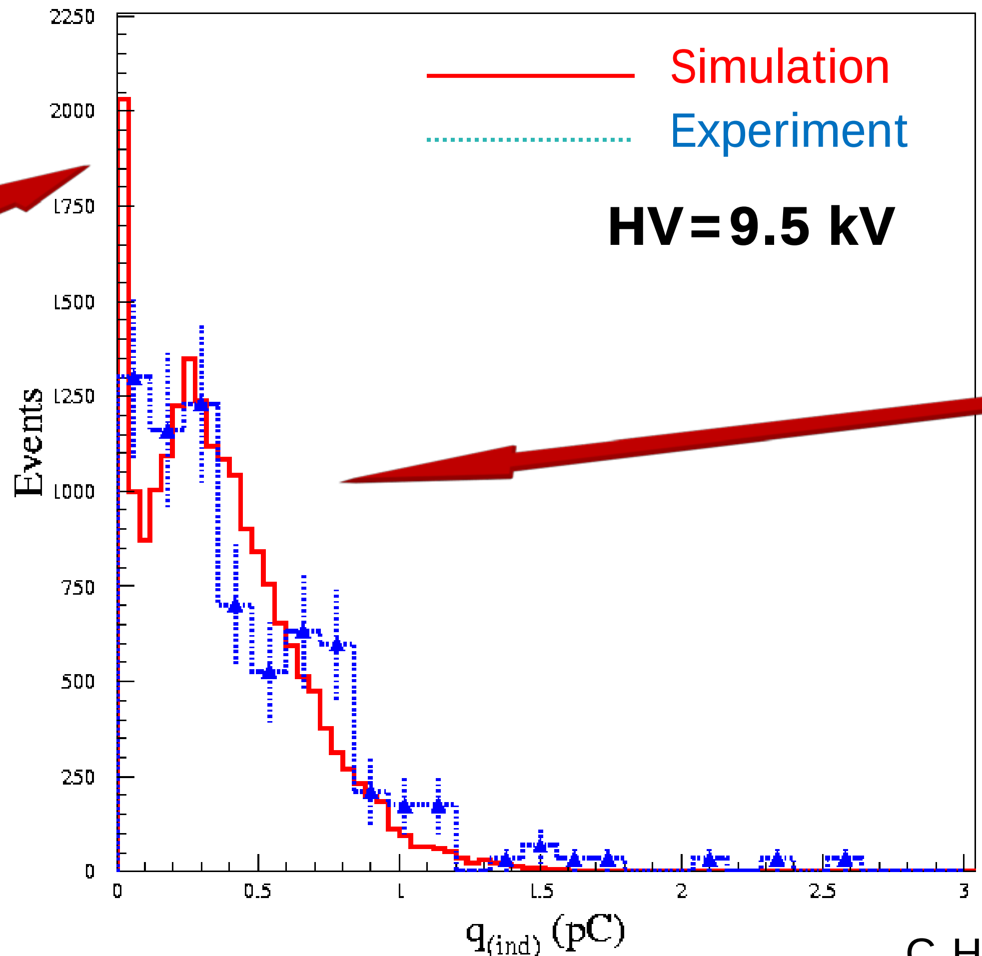
Gas mixture:
$C_2H_2F_4/C_4H_{10}$ 97/3 + SF_6 2%

Input for simulation: Colucci et al., NIM A 425 (1999) 84-91

Experimental data from Camarri et al., NIM A 414 (1998) 317-324

When saturation becomes important

This is not a fit!



Inefficiency peak

Saturation broad peak

Gas mixture: $C_2H_2F_4/C_4H_{10}$ 97/3 + SF_6 2%

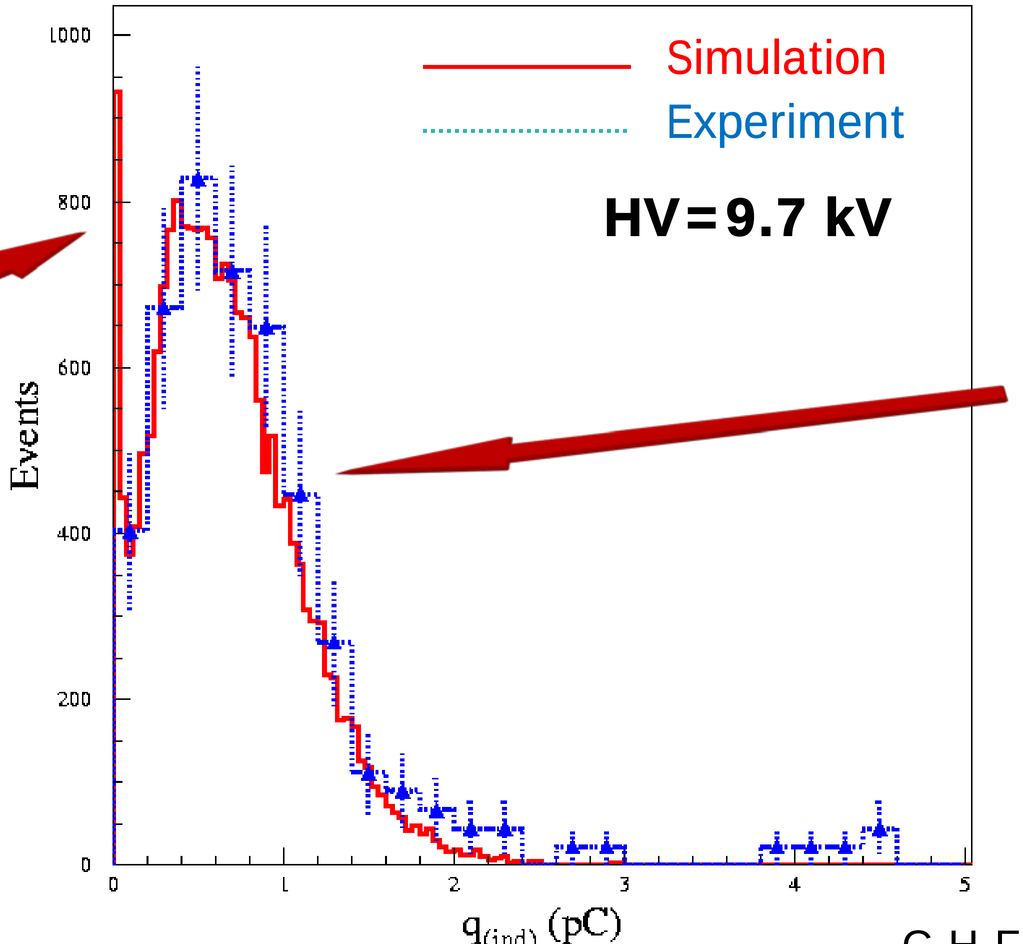
Input for simulation: Colucci et al., NIM A 425 (1999) 84-91

Experimental data from Camarri et al., NIM A 414 (1998) 317-324

When saturation becomes important

This is not a fit!

Inefficiency peak



Saturation broad peak

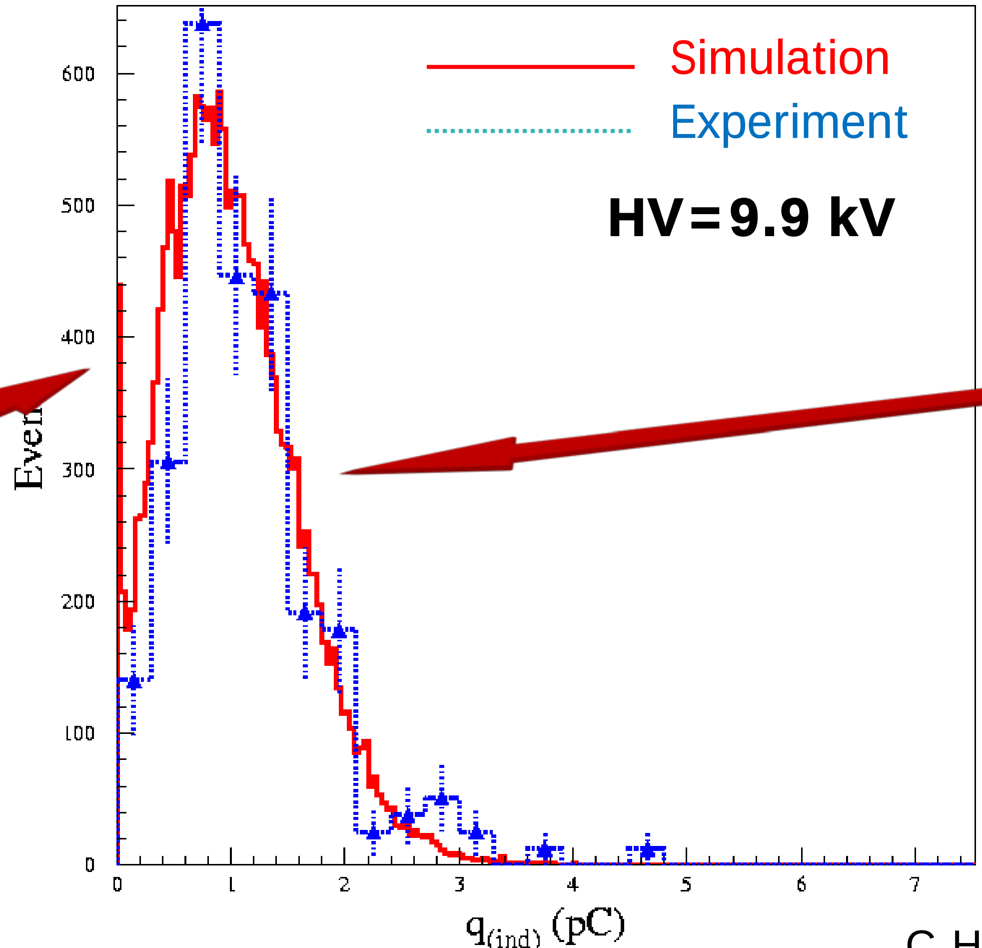
Gas mixture:
$C_2H_2F_4/C_4H_{10}$ 97/3 + SF_6 2%

Input for simulation: Colucci et al., NIM A 425 (1999) 84-91

Experimental data from Camarri et al., NIM A 414 (1998) 317-324

When saturation becomes important

This is not a fit!



Inefficiency peak

Saturation broad peak

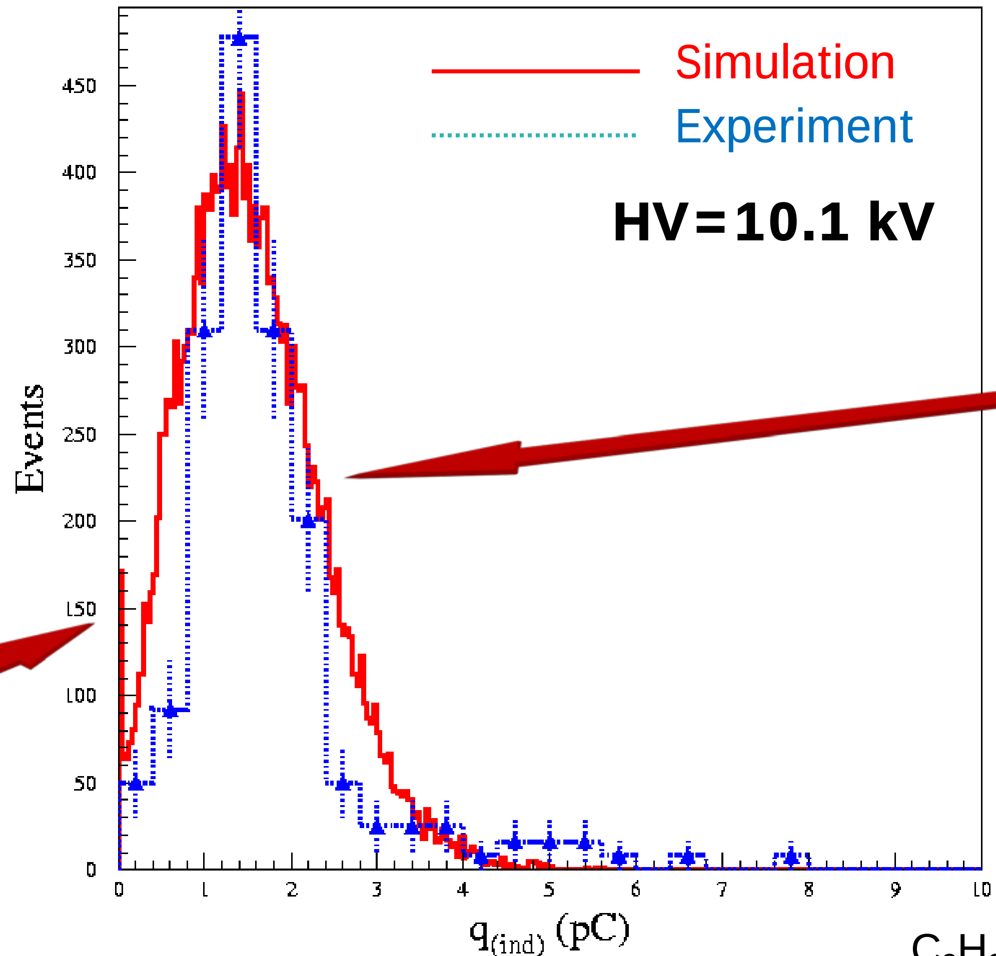
Gas mixture:
$C_2H_2F_4/C_4H_{10}$ 97/3 + SF_6 2%

Input for simulation: Colucci et al., NIM A 425 (1999) 84-91

Experimental data from Camarri et al., NIM A 414 (1998) 317-324

When saturation becomes important

This is not a fit!



Saturation broad peak

Inefficiency peak

Gas mixture:
$C_2H_2F_4/C_4H_{10}$ 97/3 + SF_6 2%

Input for simulation: Colucci et al., NIM A 425 (1999) 84-91

Experimental data from Camarri et al., NIM A 414 (1998) 317-324

The issue of rate capability

Let us start from one of prof. Santonico's presentations:
(Second ECFA workshop on HL-LHC, Aix-les-Bains, 21-23 Oct. 2014)

In the static limit the voltage applied to the chamber ΔV_{appl} is entirerly transferred to the gas; but, for a working current *i*, part of this voltage is needed to drive the current in the electrodes

$$\Delta V_{gap} = \Delta V_{appl} - RI = \Delta V_{appl} - \Delta V_{el}$$

With Φ =counts/surf. the voltage transferred to the electrodes can be written as:

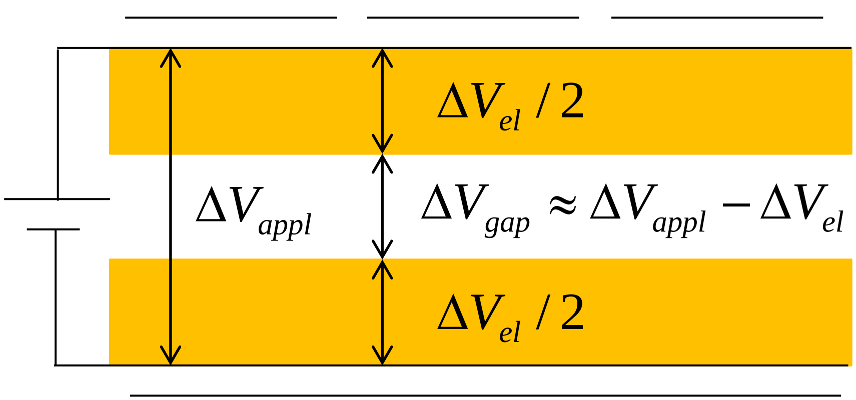
$$\Delta V_{el} = \rho\, d\, \Phi <Q>$$

Charge/count

Electrode resistivity

Electrode total thickness

"A high rate requires to keep ΔV_{el} at a negligible value wrt. ΔV_{gap} even under heavy irradiation"



Comparison with data

Essentially the same approach used in:

G. Carboni et al. *A model for RPC detectors operated at high rate*, NIM A 498(2003), 135-142

" The current drawn by a detector exposed to a particle flux Φ (particles/s) is:

$$I = \Phi\, q = \Phi\, G\, q_i$$

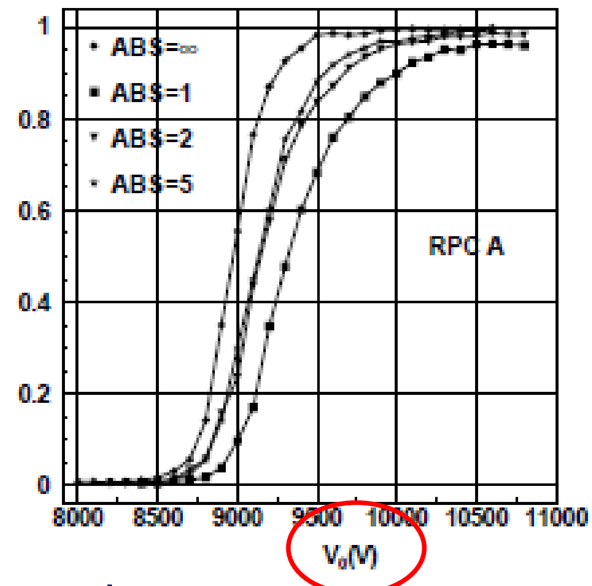
where q_i is the ionization charge and G is the gain."

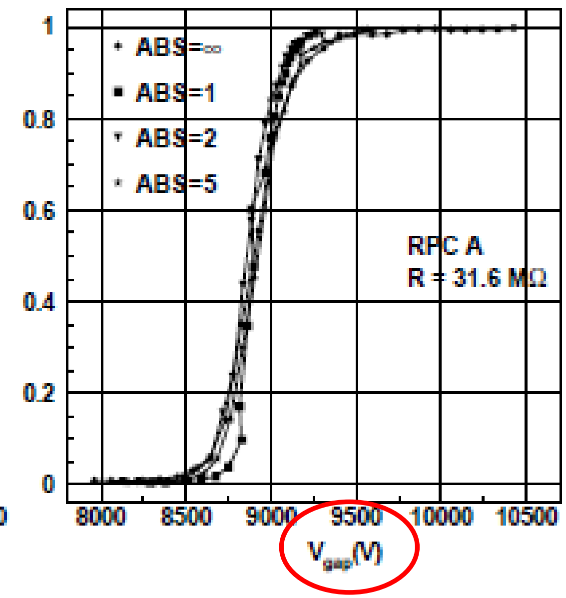
"In a given detector the electric field, the gain and the current are uniquely determined by ΔV_{gap}, where

$$\Delta V_{gap} = V_0 - IR$$

And R is the total electrode resistance, given by:

$$R = 2\rho\frac{d}{S}$$





Basically the application of the Ohm's law

What we learn from that formula

It is a first (rough) approximation of complex processes.

$$\boxed{\Delta V_{el} = \rho\ d\ \Phi\ \langle Q \rangle}$$

At first order:

- ✓Electrode resistivity does influence rate capability;
- ✓Electrode thickness does influence rate capability;
- ✓Gap thickness does not seem to play any role.

Later on…

There is not a direct way to compute which is the effect of a reduction on $\Delta V_{el} = \Delta V_{appl} - \Delta V_{gap}$ on the rate capability.

✓Bakelite thickness can account for a 25-50% reduction on ΔV_{el}

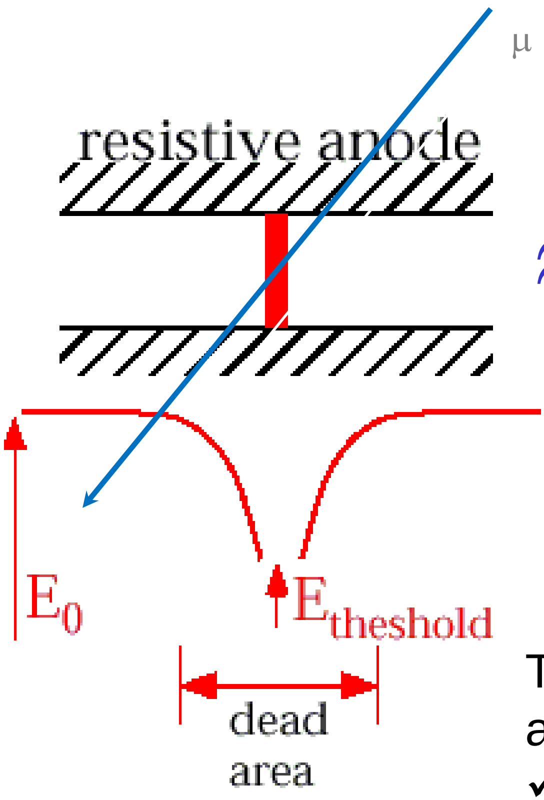
(Ex. from 2→1.5-1 mm)

✓Bakelite resistivity can account a 10 (or more) factor on ΔV_{el}

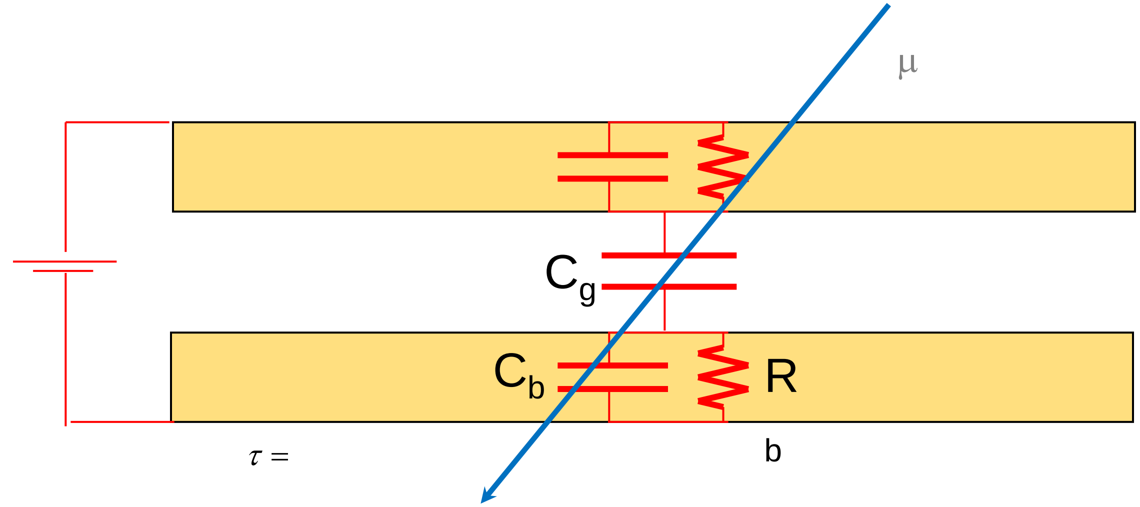
(Ex. From $5 \times 10^{10} \rightarrow 5 \times 10^{9}\ \Omega$cm)

→ Electrode thickness seems to play a second order role.

Let us move to a dynamic model



≈



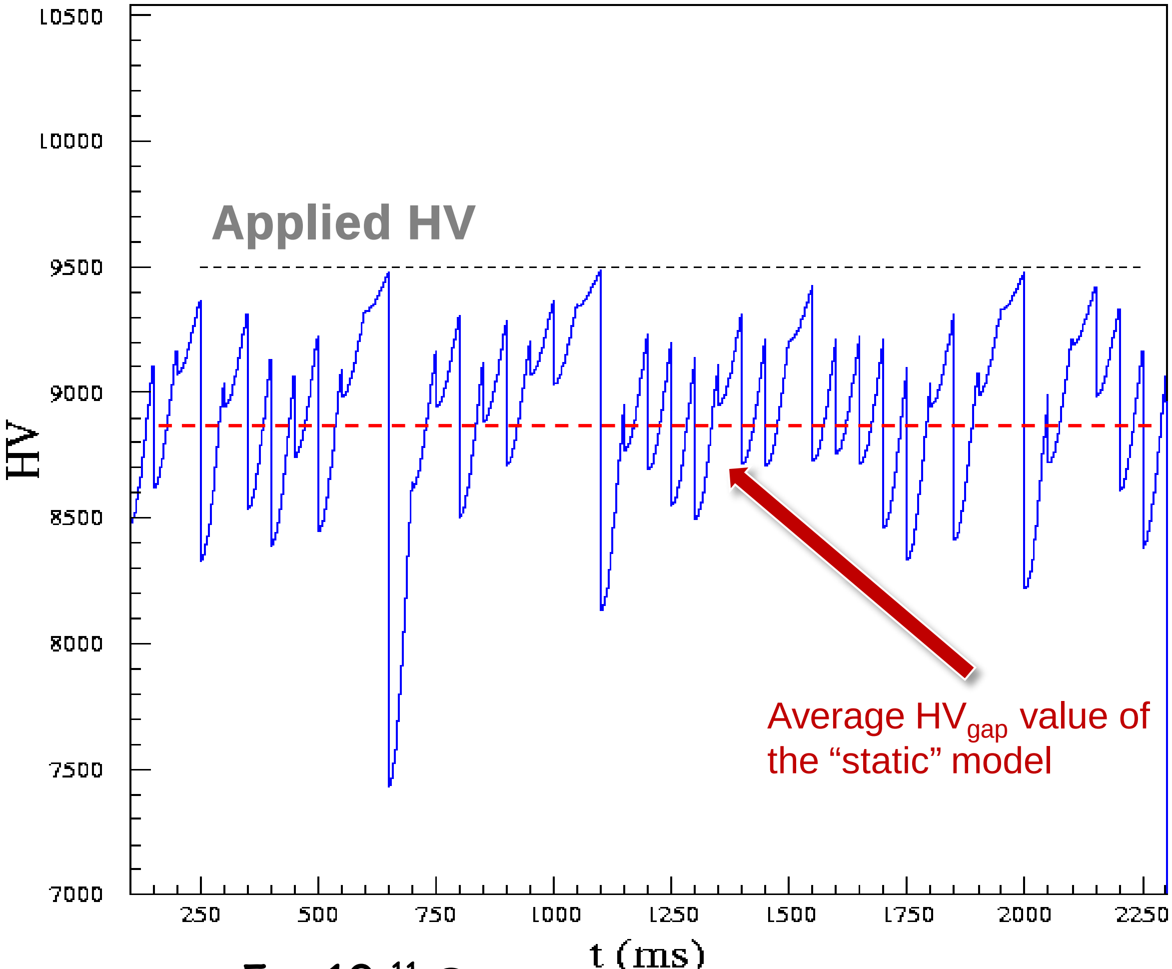
$$\tau = 2R_b(2C_b + C_g) = 2\rho_b\varepsilon_0(2\varepsilon_r + \frac{d}{g})$$

Time constant τ depending only on material characteristics and not on “cell” dimensions

✓Also capacitive effects taken into account

A few numbers:
- typical avalanche radius: 100 μm
- typical avalanche charge: 1 pC
- typical external charge contained in 100 μm: 10 pC

And what REALLY happens



$$i_{ind}(t) = -\mathbf{v}_d \cdot \mathbf{E_w} q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}} n_0^j M_j$$

We assume that the voltage on the electrodes goes back to HV_{appl} following an exponetial law

$$\begin{cases} HV(t) = HV_{ext}\left(1 - e^{-t/\tau}\right) \\ \tau = 1500 \text{ ms} \end{cases}$$

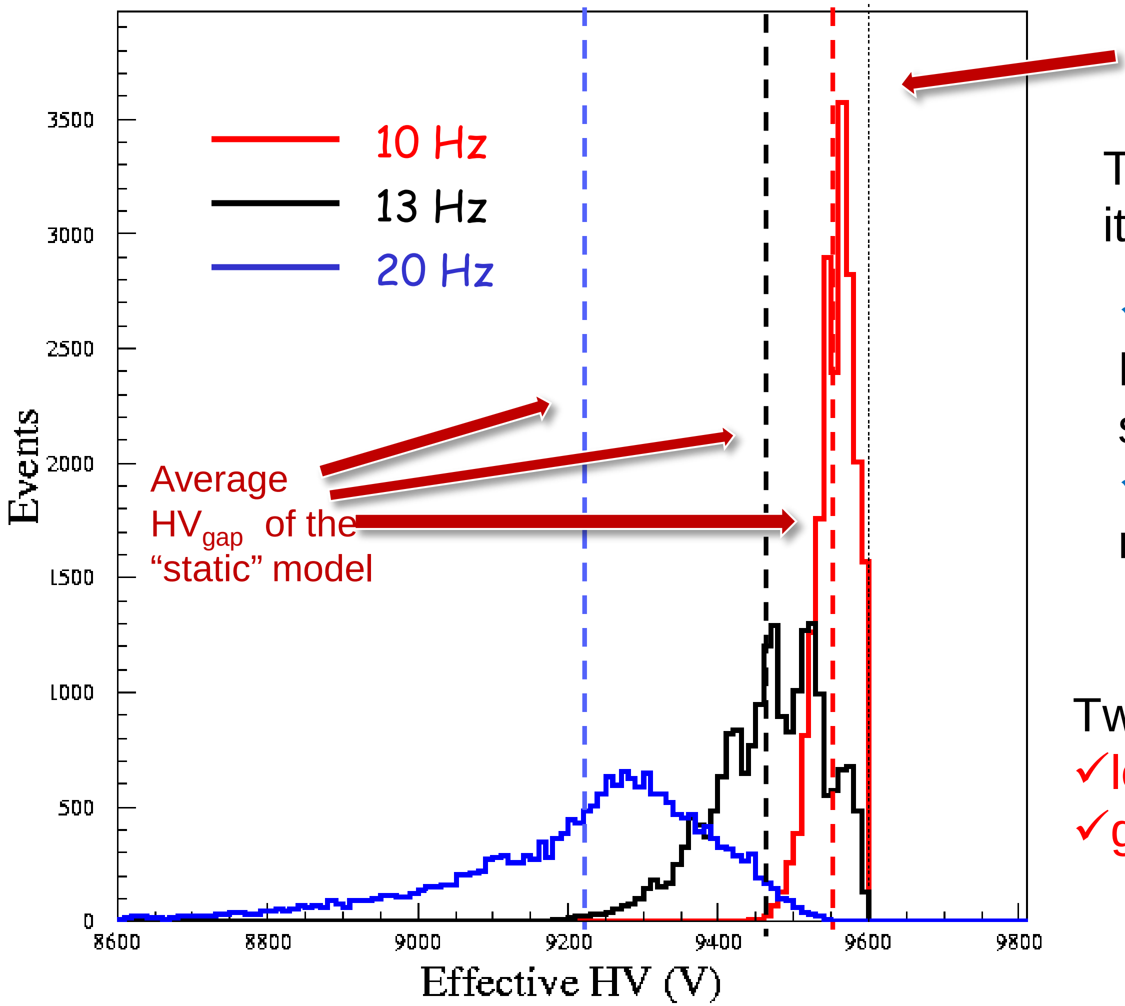
Note that “big” pulses come only after that HV_{appl} has been restored, and they are followed by “small” pulses

$\rho \approx 5 \times 10^{11}\ \Omega\text{cm}$

Cell area = 1 mm^2

Complex “feedback” mechanisms…

Some of the differences



The effective HV diminishes and its distribution is broader.

✓Reduction off the effective HV correctly foreseen by the static (ohmic) model)

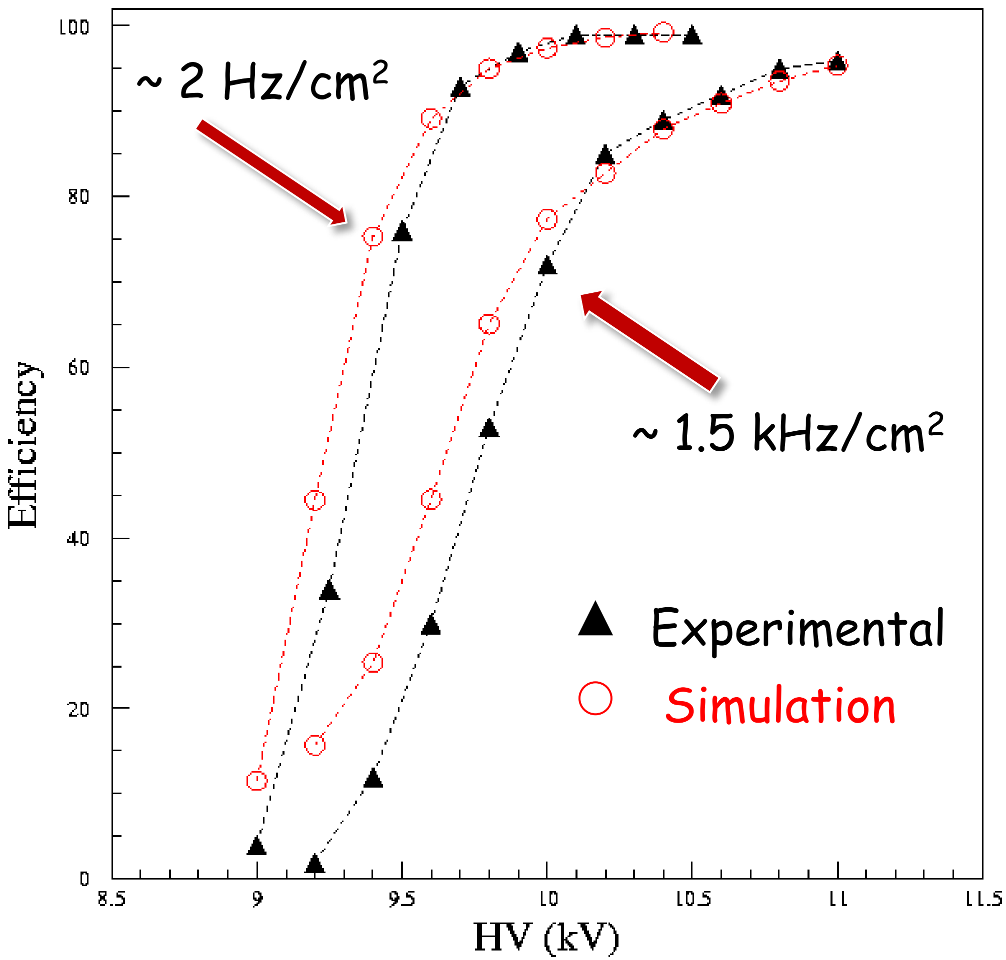
✓Spread of the effective HV not foreseen at all

Two consequences:

✓lower HV at high rate

✓greater HV variations at high rate

Efficiency: comparison with data



Data from G. Aielli et al., NIM A 478(2002) 271-276

Note that there also exists an approximated formula for efficiency:

$$\varepsilon = 1 - e^{-\lambda\left[g - \frac{1}{\eta}\ln\left(\frac{q_{thr}}{A}+1\right)\right]}$$

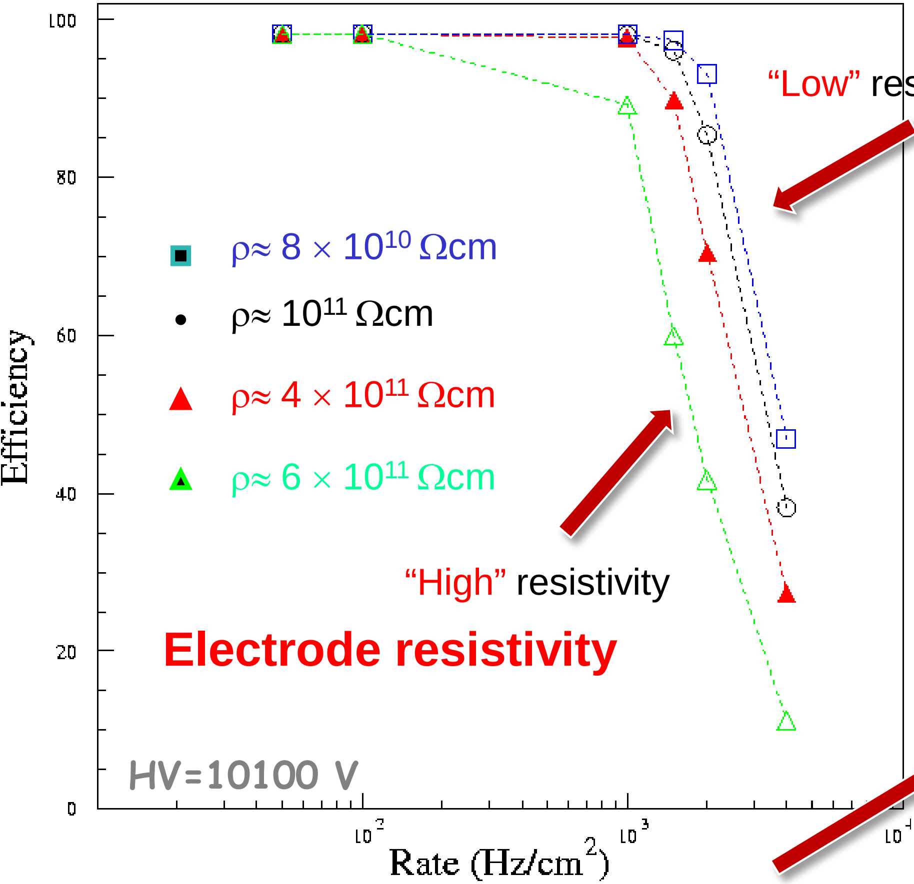
$$A = \frac{q_e \Delta V_w M n_0}{\eta g}$$

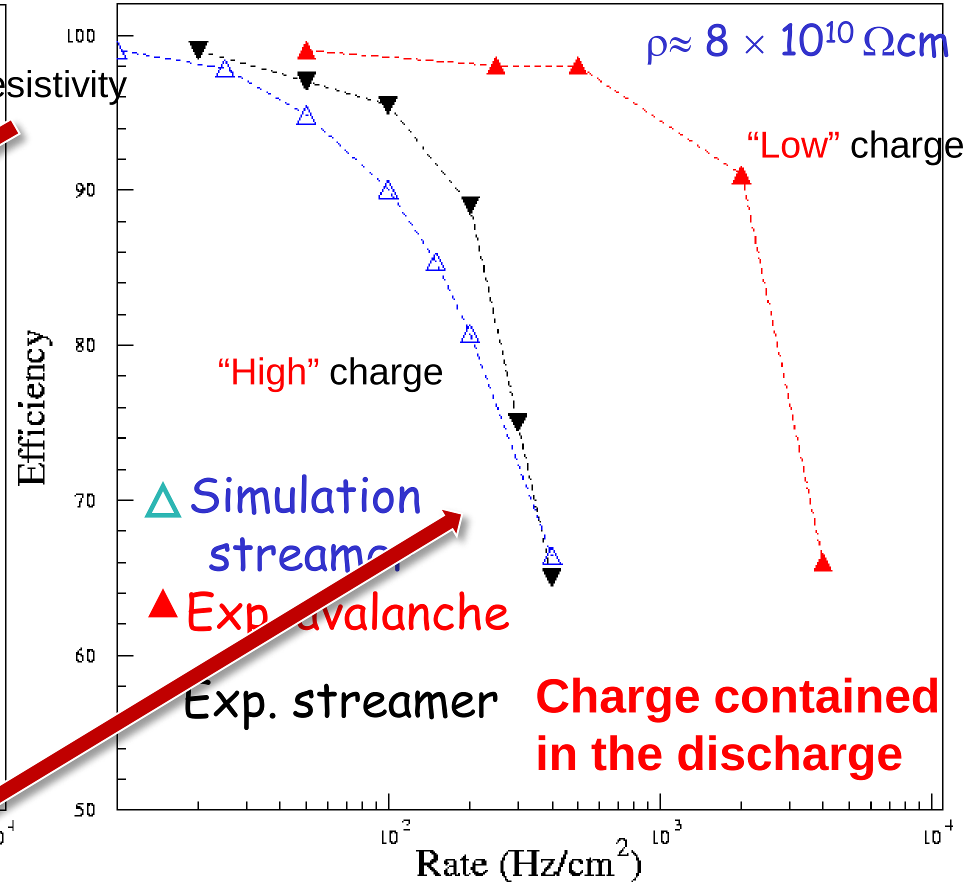
Basic aspects reproduced:

✓Plateau efficiency reduced at high rate
✓Shift of the efficiency curves
✓Change in slope of the efficiency curves

Not so immediate (if possible) to reproduce the same effect with the static model

Rate capability dependances



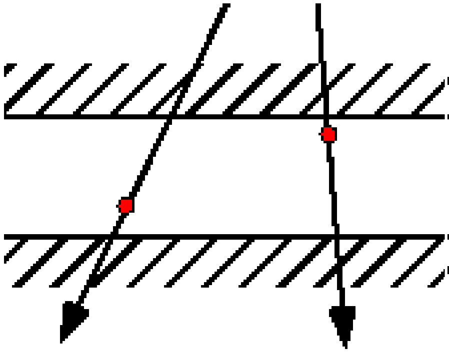


Here we find the other important aspect related to the charge travelling in the gap and the importance to reduce it

Data from R. Arnaldi et al., NIM A 456(2000) 73-76

✓But this implies a signal reduction as well!

Time resolution in RPCs



What is the origin of signal time fluctuations in an RPC?
There are **very good analytic studies (Mangiarotti et al.),** here just a few hints will be given from a MC point of view.

Only present at high rate
✓Fluctuations of η
✓Fluctuations of v_d

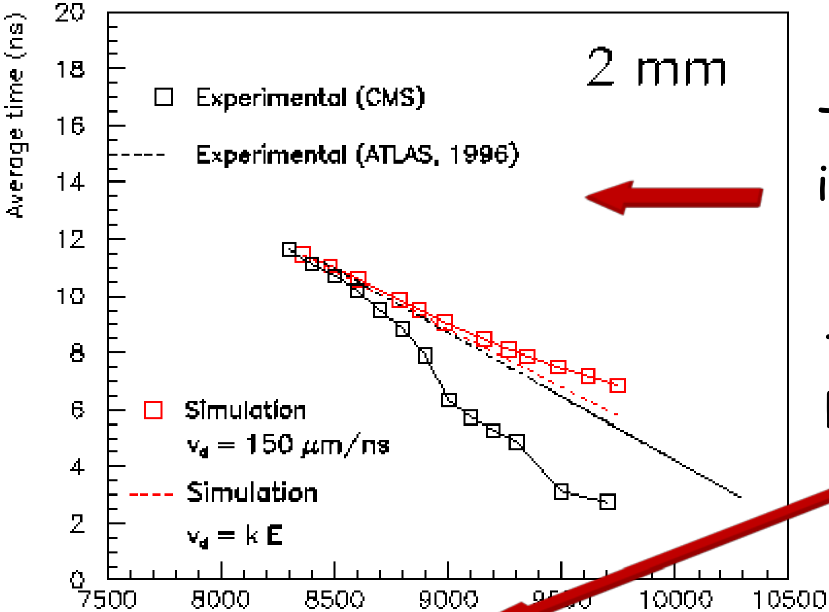
$$v_{out}(t) = i_{ind}(t) R = \left(-\mathbf{v}_d \cdot \mathbf{E_w} q_e e^{\eta v_d t} \sum_{j=1}^{n_{cl}} n_0^j M_j \right) R > v_{discr}$$

Present at low rate

- ✓ Total number of clusters
- ✓ Total number of electrons in each cluster
- ✓ Fluctuations superimposed on the exponential growth

All the cluster in the gap **contribute at the same manner** to the signal
Fluctuation are not (directly) related to the particle transit time in the gas gap

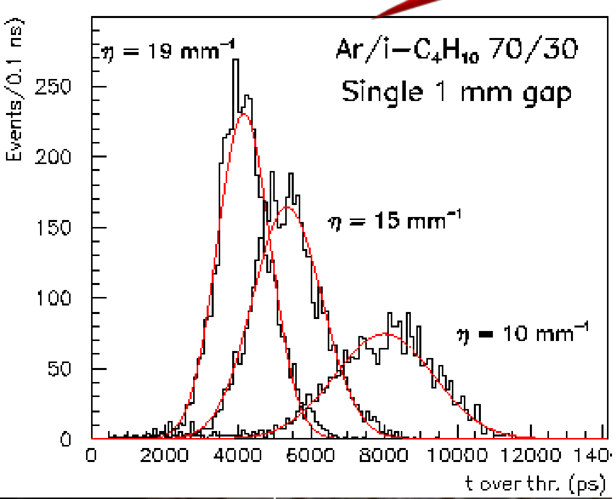
Some (obvious) results

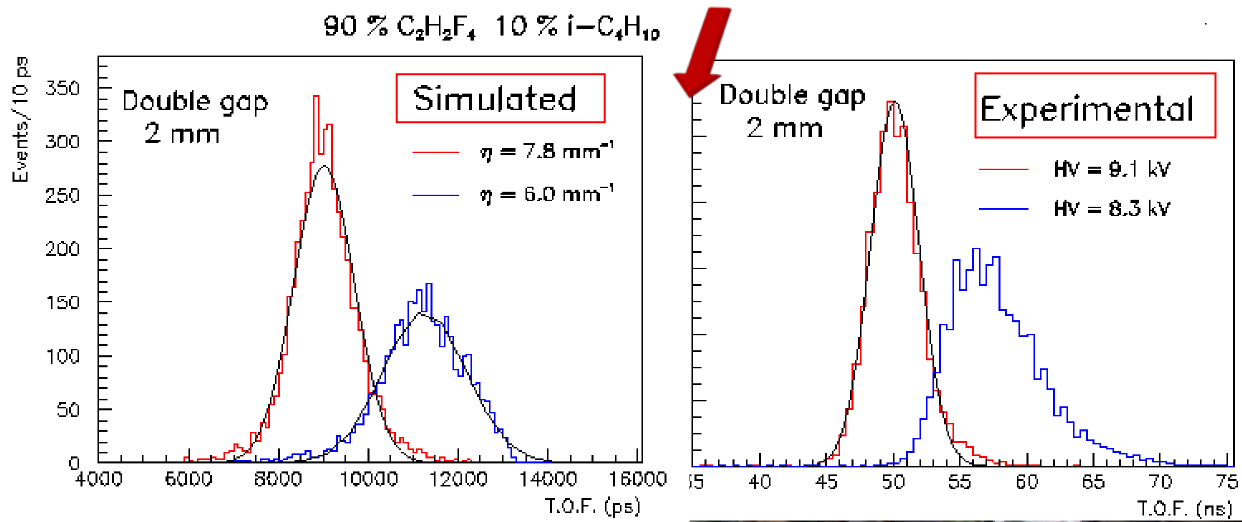


Time delay decreases with an HV increase

Time resolution becomes better with an HV increase

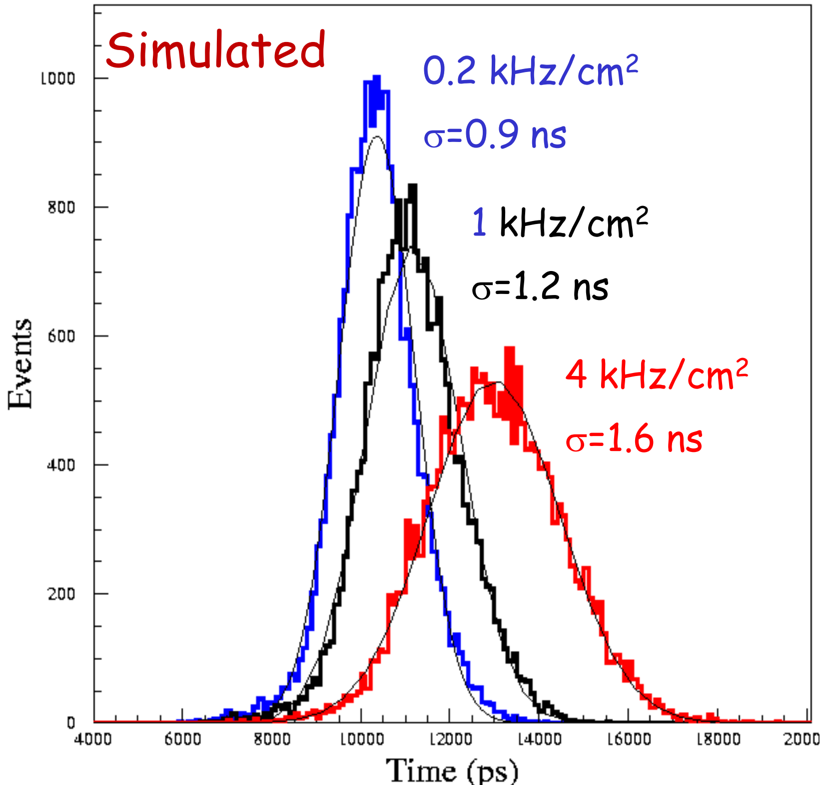
A double gap RPC has a better resolution with respect to a single gap RPC

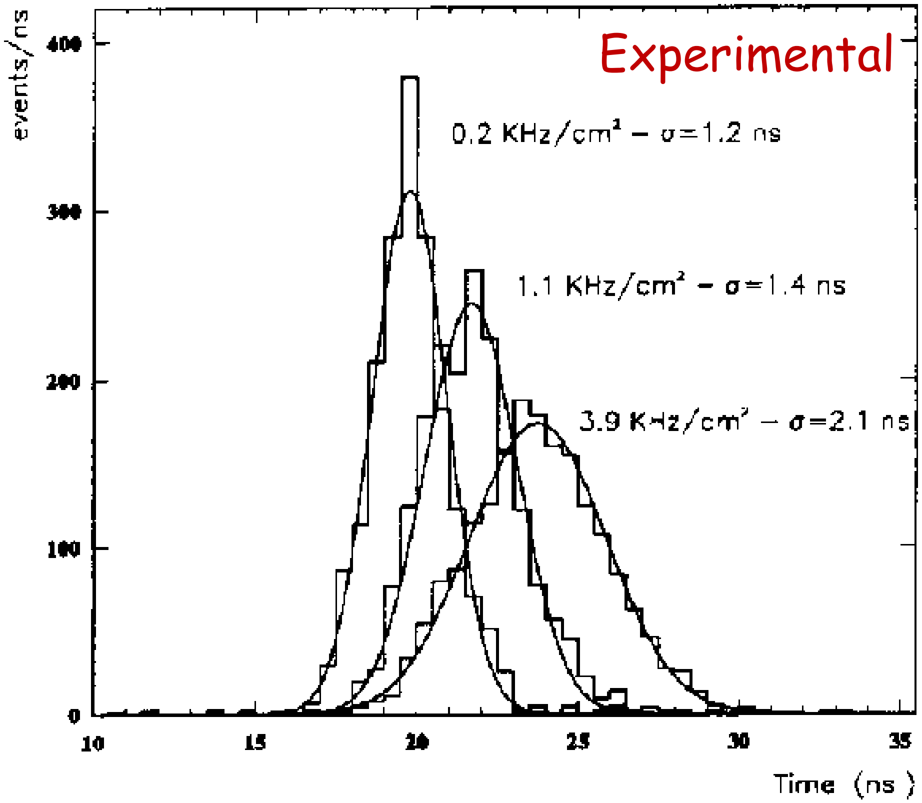




Time resolution at high rate

Data from C. Bacci et al., NIM A 352(1995) 552-556





General behaviour well reproduced:

✓Simulated time resolutions slightly less than experimental

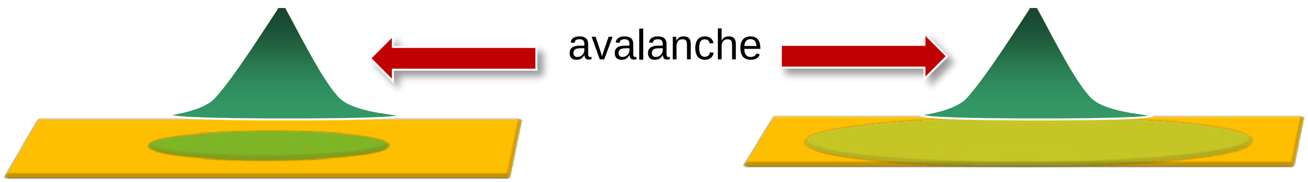
✓"Instrumental" effects not included

Note that all these aspects related to time resolution cannot be accounted for (or are very difficult to account for) in the static model

Reality is more complex

The avalanche charge arriving onto the electrodes surfaces spreads more or less depending on ration between surface resistivity ρ_{sur} and volume resistivity ρ_{vol}

✓It has a direct influence on the amount of local voltage drop in the gap

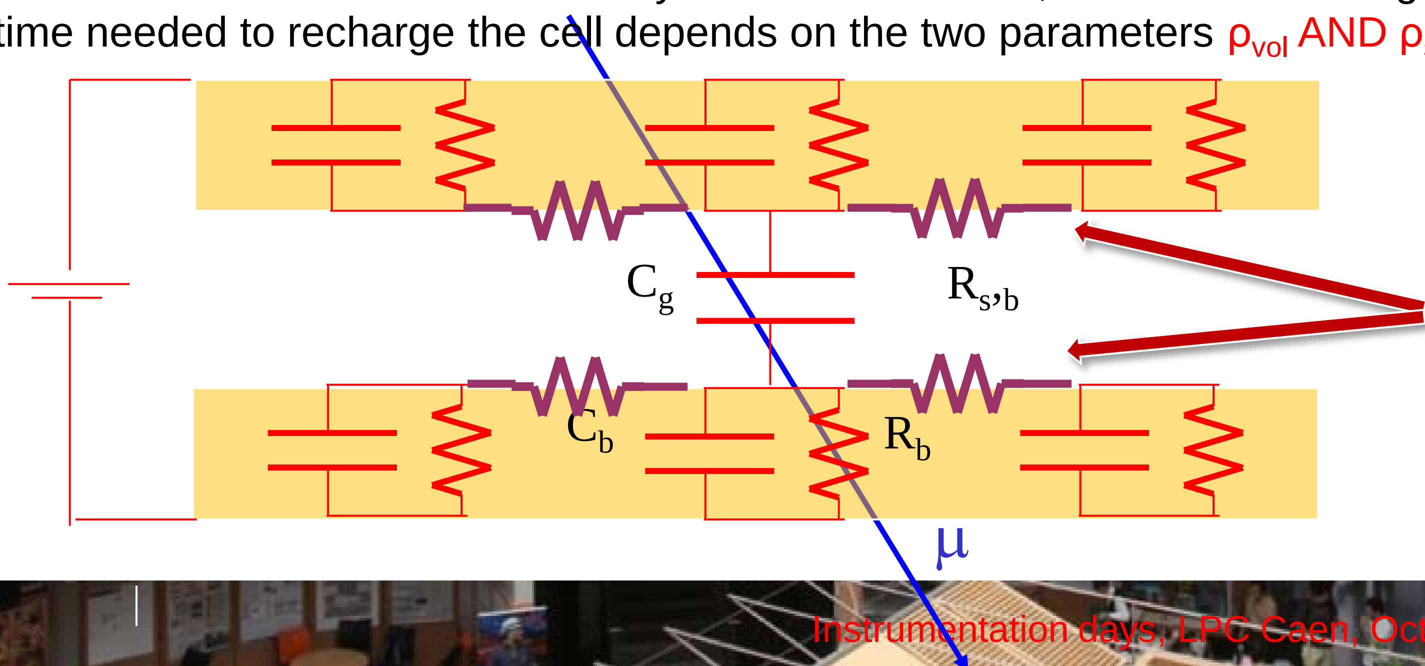


$\frac{\rho_{sur}}{\rho_{vol}}$ "large"

$\frac{\rho_{sur}}{\rho_{vol}}$ "small"

The result is that the current flows not only in the "central" cell, but also in the neighbouring ones.

➢The time needed to recharge the cell depends on the two parameters ρ_{vol} AND ρ_{sur}



(Some) Conclusions

- Detector physics is an exciting item!
  - It is intrinsically interesting from the theoretical point of view
  - It has an interest –historically very important- from the practical point of view
- What could appear a simple problem has many interesting perspectives to be taken into consideration
- Space charge, avalanche to streamer transition, and high rate behaviour are complex issues to worth to be studied in details
- Amazingly, the most complete and interesting pictures are still to be thoroughly investigated

More calculations and considerations are welcome!


Thank you!

About gap thickness

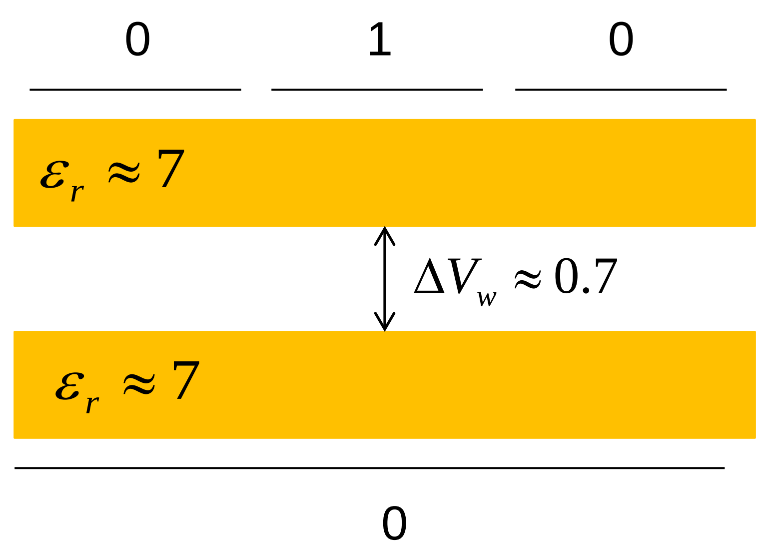
Saturation term

$$q_{ind} = \frac{\Delta V_w}{\eta g} \overbrace{q_e \sum_{j=1}^{n_{cl}} n_0^j M_j \left[e^{\eta\left(x_{sat}^j - x_0^j\right)}}^{Q_{gap}} - 1\right] + \sum_{j=1}^{n_{cl}} \Delta V_w M_j \frac{g - x^j{}_{sat}}{g} q_{sat}$$

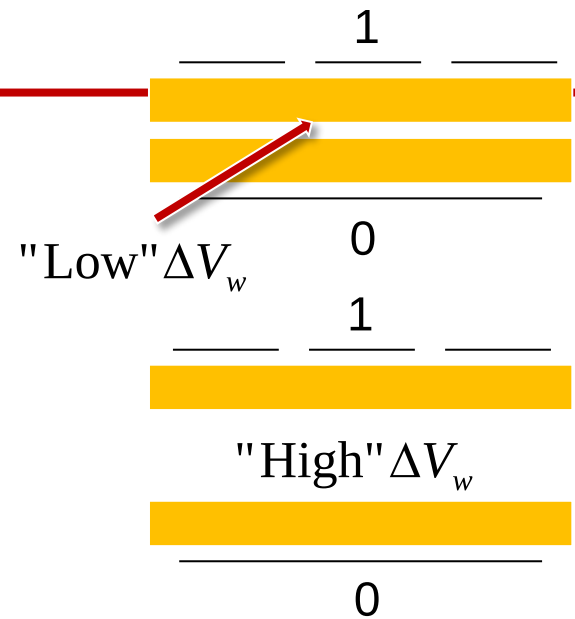
$$\Delta V_w = \frac{\varepsilon_r g}{n_g \varepsilon_r g + (n_g + 1)d}$$

n_g= number of gaps

The trick to increase rate capability is to increase q_{ind} keeping Q_{gap} constant, while ηg stays (roughly)constant.



About gap thickness:



✓The point is that if you reduce the gap thickness only the shielding electrostatic effect of the bakelite plates increases in proportion

- ➢q_{ind} is reduced (keeping Q_{gap} constant)
- ➢Rate capabity is reduced

✓The voltage drop in the gap related to the weighting field should be as high as possible

✓If you reduce the electrode thickness at the same time, the two effects cancel out

➢Experimental data show that wider gaps show higher rate capability (also for other reasons)

✓It would be "strange" that the 2 mm gap is the minimum