Non-equilibrium dynamics of the Heisenberg spin chain: Exact methods

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Outline

Work with the Budapest group (Márton Kormos, Márton Mestyán, Gábor Takács, Miklós Werner) and Lorenzo Piroli and Eric Vernier from SISSA.

- Introduction

- Overlaps, Quench action and TBA

- Loschmidt amplitude

- Open problems

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  Connection to statistical physics: equilibration, thermalization

- Thermalization in integrable models: The Generalized Gibbs Ensemble

$$\rho = \frac{1}{Z} e^{-\sum_j \beta_j Q_j}$$

- Exact calculations

- Comparison with experiments
  (the GGE has been measured already, *Science 348, p. 207*)

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- The model:

$$H = \sum_{j=1}^{L}(\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta(\sigma_j^z \sigma_{j+1}^z - 1))$$

But also higher spin or higher rank

- Time evolution from an initial state: $|\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle$

The initial state:

  - Ground state of some other Hamiltonian (quantum quench)
  - Any experimentally realizable state, prepared using simple rules
  - Examples:

$$|\Psi_0\rangle = |\text{Néel}\rangle \equiv \otimes_{i=1}^{L/2}|\uparrow\downarrow\rangle$$

or

$$|\Psi_0\rangle = |\text{Dimer}\rangle \equiv \otimes_{i=1}^{L/2}\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

  - Integrable initial states
  - Only translationally invariant cases!

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Objects of interest:

- Time-dependence of local observables

- Loschmidt amplitude (a.k.a. Return probability, Fidelity, etc)

$$G(t) = \langle \Psi_0 | e^{-iHt} | \Psi_0 \rangle$$

Objects of interest:

- Local observables, $\mathcal{O}(t) = \langle \sigma_1^a(t)\sigma_n^a(t)\rangle$, $n = 2, 3, \ldots$, $a = x, z$.

$$\langle \mathcal{O}(t)\rangle = \langle \Psi_0|e^{iHt}\mathcal{O}e^{-iHt}|\Psi_0\rangle$$
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- Overlaps $C_n = \langle \Psi_0|n\rangle$

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$$\lim_{t\to\infty} \langle \mathcal{O}(t)\rangle = \sum_n |C_n|^2 \langle n|\mathcal{O}|n\rangle$$

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Two important ingredients:

- Selecting the states that dominate the sum.

- Excited state correlations?
  $\langle n|\mathcal{O}|n\rangle =?$

  Factorized correlation functions, hidden Grasmannian structure, ...
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- In the TDL: States characterized by Bethe root densities

- If the exact overlaps $|C_n|^2$ are known: Quench Action method. Works in limited cases. Microscopic. Necessary for the full time evolution.

- Mean values of conserved charges Q_i Defining the charge density $q_i = Q_i/L$

$$q_i(t=0) = \langle \Psi_0|q_i|\Psi_0\rangle = q_i(t=\infty) = \langle n|q_i|n\rangle$$

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Bethe Ansatz

Coordinate Bethe Ansatz solution of the XXZ Hamiltonian

$$H = \sum_{j=1}^{L} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta(\sigma_j^z \sigma_{j+1}^z - 1)).$$

Defining $\Delta = \cosh(\eta)$ and

$$|\{\lambda\}_N\rangle = \sum_{y_1 < y_2 < \cdots < y_N} \phi_N(\{\lambda\}_N | y_1, \ldots, y_N) \sigma_{y_1}^- \ldots \sigma_{y_N}^- |0\rangle$$
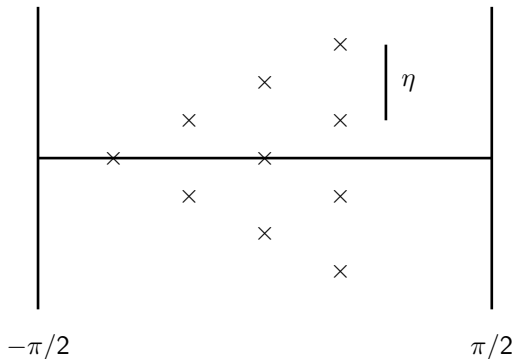
the wave function is

$$\phi_N(\{\lambda\}_N | \{y\}) =$$
$$= \sum_{P \in S_N} \left[\prod_{1 \le m < n \le N} \frac{\sin(\lambda_{P_m} - \lambda_{P_n} + i\eta)}{\sin(\lambda_{P_m} - \lambda_{P_n})} \right] \left[\prod_{l=1}^{N} \left(\frac{\sin(\lambda_{P_l} + i\eta/2)}{\sin(\lambda_{P_l} - i\eta/2)} \right)^{y_l} \right]$$

Bethe Ansatz

Bethe Ansatz equations:

$$\left(\frac{\sin(\lambda_j + i\eta/2)}{\sin(\lambda_j - i\eta/2)}\right)^L \prod_{k \neq j} \frac{\sin(\lambda_j - \lambda_k - i\eta)}{\sin(\lambda_j - \lambda_k + i\eta)} = 1$$

String solutions: $(\Delta > 1)$

In the thermodynamic limit: densities of roots: $\rho_{r,k}(\lambda)$

The number ΔN of k-strings with centers between λ and $\lambda + \Delta\lambda$:
$\Delta N = L\rho_{r,k}(\lambda)\Delta\lambda/2\pi$.

Densities of holes: $\rho_{h,k}(\lambda)$.

They satisfy

$$\rho_{r,k} + \rho_{h,k} = \delta_{k,1}d + d \star (\rho_{h,k-1} + \rho_{h,k+1}),$$

where

$$(f \star g)(u) = \int_{-\pi/2}^{\pi/2} \frac{d\omega}{2\pi} f(u - \omega)g(\omega).$$

$$d(u) = 1 + 2\sum_{n=1}^{\infty} \frac{\cos(2nu)}{\cosh(\eta n)}$$

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Overlaps

Quench Action method. How to compute the overlaps?

The simplest case: the Néel state

Consider a chain of length $L = 2N$. The overlaps are

$$\langle \text{Néel}|\{\lambda\}\rangle = \phi_N(\{\lambda\}_N|\{2, 4, 6, \ldots, 2N\}) =$$

$$= \sum_{P \in S_N} \left[\prod_{1 \leq m < n \leq N} \frac{\sin(\lambda_{P_m} - \lambda_{P_n} + i\eta)}{\sin(\lambda_{P_m} - \lambda_{P_n})} \right] \left[\prod_{l=1}^{N} \left(\frac{\sin(\lambda_{P_l} + i\eta/2)}{\sin(\lambda_{P_l} - i\eta/2)} \right)^{2l} \right]$$

How to sum it up?

Solution: relation to the six-vertex model

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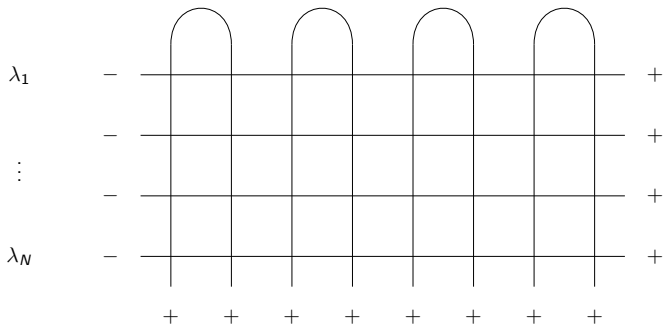
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The overlap is related to a partition function, which is known:

O. Tsuchiya, Determinant formula for the six-vertex model with reflecting end, arXiv:solv-int/9804010

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BP., J. Stat. Mech. (2014) P06011

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Analogy: $v(\lambda) \sim e^{-\beta E(\lambda)}$

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Analogy: $v(\lambda) \sim e^{-\beta E(\lambda)}$

M. Brockmann, J. De Nardis, B. Wouters, and J.-S. Caux, J. Phys. A: Math. Theor. 47 (2014) 145003

K. K. Kozlowski, B. P., J. Stat. Mech. (2012) P05021

$$v_{\text{Néel}}(\lambda) = \frac{\tan(\lambda + i\eta/2)\tan(\lambda - i\eta/2)}{4\sin^2(2\lambda)}, \quad \text{for} \quad |\{\lambda\}_N\rangle = \left|\{-\tilde{\lambda}, \tilde{\lambda}\}_{N/2}\right\rangle$$

$$C = \frac{\det G^+}{\det G^-}$$

Overlaps

Other states?

- At present exact overlaps are only known for

$$|\Psi_0\rangle = \prod \frac{|+-\rangle + \alpha|-+\rangle}{\sqrt{1 + |\alpha|^2}}$$

  They correspond to diagonal K-matrices in the boundary Algebraic Bethe Ansatz.

- Other two-site states probably possible
  (Tsushiya-determinant with off-diagonal K-matrices)

- MPS states encountered in the AdS/CFT literature

  M. de Leeuw, Ch. Kristjansen, K. Zarembo, JHEP08(2015)098
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  Closely related!

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  Closely related!

Thermodynamic Bethe Ansatz (TBA)

Compute

$$Z = \text{Tr } e^{-H/T} = e^{-fL/T}$$

in a Bethe Ansatz solvable model with

$$E = \sum_{j=1}^{N} e(\lambda_j) \quad \text{and} \quad e^{ip(\lambda_j)L} \prod_{k \neq j} S(\lambda_j - \lambda_k) = 1$$

In the TDL densities for Bethe roots and holes, satisfying

$$\rho_r + \rho_h = p' + \varphi \star \rho_r,$$

with $\varphi = -i d \log(S(\lambda))/d\lambda$.

Express the partition function as a functional integral

$$Z = \int \mathcal{D}\rho_r(\lambda) e^{-S[\rho_r]L}, \qquad S = \int \frac{d\lambda}{2\pi} e(\lambda)\rho_r(\lambda) + S_{YY}$$

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Minimization of the free energy functional gives

$$\varepsilon = \beta e + \varphi \star \log(1 + e^{-\varepsilon}) \qquad f = -p' \star \log(1 + e^{-\varepsilon}),$$

where the pseudo-energy is defined as $e^{-\varepsilon(u)} = \rho_r(u)/\rho_h(u)$.

What about quenches? If the overlap is of the form

$$|\langle \Psi_0 | \{\lambda\}_N \rangle|^2 = C \prod_{j=1}^{N} v(\lambda_j)$$

Analogy: $v(\lambda) \sim e^{-\beta E(\lambda)}$ gives the Quench Action

$$S_{QA} = \int \frac{d\lambda}{2\pi} \log(v(u)) \rho_r(\lambda) + \frac{1}{2} S_{YY}$$

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J-S. Caux, F. H. L. Essler, Phys. Rev. Lett. 110, 257203 (2013)

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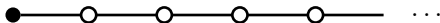J-S. Caux, F. H. L. Essler, Phys. Rev. Lett. 110, 257203 (2013)

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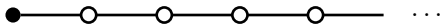*J-S. Caux, F. H. L. Essler, Phys. Rev. Lett. 110, 257203 (2013)*

Thermodynamic Bethe Ansatz (TBA)

XXZ spin chain for $\Delta > 1$: for every k-string, $k = 1 \ldots \infty$
$\rho_{r,k}(\lambda), \rho_{h,k}(\lambda)$ and $Y_k(\lambda) = \frac{\rho_{h,k}(\lambda)}{\rho_{r,k}(\lambda)}$

Finite temperature: $Z = \mathrm{Tr} e^{-sH}$, $\beta = 2\sinh(\eta)s$

$$\log(Y_j) = -\beta d\delta_{j,1} + d \star (\log(1 + Y_{j+1}) + \log(1 + Y_{j-1}))$$

$$d(u) = 1 + 2\sum_{n=1}^{\infty} \frac{\cos(2nu)}{\cosh(\eta n)}$$

Important relation, Y-system:

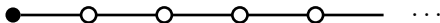$$Y_j(u + i\eta/2)Y_j(u - i\eta/2) = (1 + Y_{j-1}(u))(1 + Y_{j+1}(u))$$

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Quench Action TBA

TBA using the overlaps of the form $\quad \langle \Psi_0 | \{ \lambda_N \} \rangle = C \times \prod_{j=1}^{N} v(\lambda_j)$

$$\log(Y_j) = d_j + d \star (\log(1 + Y_{j+1}) + \log(1 + Y_{j-1}))$$

with

$$d_j = -g_j + d \star (g_{j-1} + g_{j+1}), \text{ with } g_0 = 0.$$

where

$$g_j(\lambda) = -\sum_{k=1}^{j} \log \left(v(\lambda + i\eta(n+1-2k)/2) \right)$$

Y-system still holds!

$$Y_j(u + i\eta/2)Y_j(u - i\eta/2) = (1 + Y_{j-1}(u))(1 + Y_{j+1}(u))$$

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Quench Action TBA

In the simplest cases

$$v_{\text{Néel}}(\lambda) = \frac{\tan(\lambda + i\eta/2)\tan(\lambda - i\eta/2)}{4\sin^2(2\lambda)} \quad \text{and} \quad v_{\text{D}}(\lambda) = \frac{\sinh^4(\eta/2)\cot^2(\lambda)}{\sin(2\lambda + i\eta)\sin(2\lambda - i\eta)}$$

Correlators:

- Substitute the overlaps into the TBA
- Find $Y_j(u)$
- Find the root and hole densities from

$$\rho_{h,k}(1 + Y_k^{-1}(u)) = \delta_{k,1}d + d * (\rho_{h,k-1} + \rho_{h,k+1}),$$

- Find the local observabels using the theory of factorized correlators.
  M. Mestyán and BP, J. Stat. Mech. (2014) P09020

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Quenches in the XXZ chain

$\Delta = 3, \qquad |\Psi_0\rangle = |D\rangle, \qquad \langle \sigma_1^z \sigma_3^z \rangle (t)$
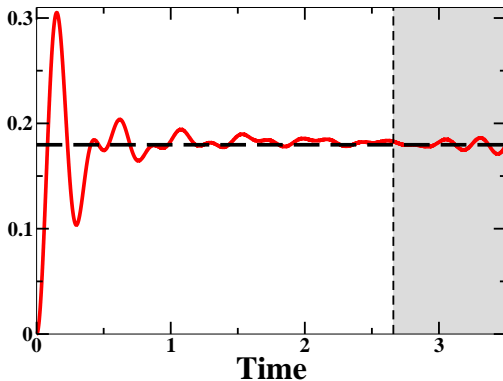
iTEBD simulation, by Miklós Werner

B. P., M. Mestyán, M. A. Werner, M. Kormos, G. Zaránd, G. Takács, Phys. Rev. Lett. 113 (2014) 117203

Exact solutions possible!

$$1 + Y_1(u) = (1 + \mathfrak{a}(u + i\eta/2))(1 + \mathfrak{a}^{-1}(u - i\eta/2))$$

with

$$\mathfrak{a}_{\text{Néel}}(u) = \frac{\sin(2u - i\eta)}{\sin(2u + i\eta)} \frac{\sin(u + i\eta)}{\sin(u - i\eta)} \quad \text{and} \quad \mathfrak{a}_{\text{D}}(u) = \frac{\sin(2u - i\eta)}{\sin(2u + i\eta)} \frac{\cos^2(u + i\eta)}{\cos^2(u - i\eta)}$$

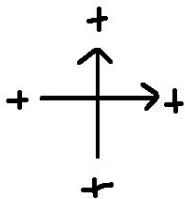Using fusion relations and T-system.

Higher Y_j from the Y-system.

M Brockmann, B Wouters, D Fioretto, J De Nardis, R Vlijm and J-S Caux,
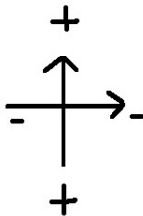*J. Stat. Mech. (2014) P12009*

Algebraic constructions

The R-matrix acting on $\mathbb{C}^2 \otimes \mathbb{C}^2$:

$$R(u) = \frac{1}{\sinh(u + \eta)} \begin{pmatrix} \sinh(u + \eta) & 0 & 0 & 0 \\ 0 & \sinh(u) & \sinh(\eta) & 0 \\ 0 & \sinh(\eta) & \sinh(u) & 0 \\ 0 & 0 & 0 & \sinh(u + \eta) \end{pmatrix}$$
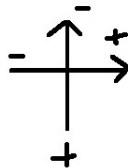
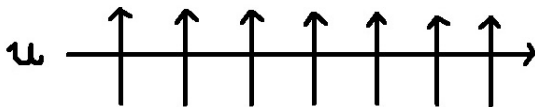Pictorial representation:

$\sinh(u + \eta)$　　　　　　$\sinh(u)$　　　　　　$\sinh(\eta)$

Algebraic constructions

The monodromy matrix:

$$T(u) = R_{M0}(u) \dots R_{10}(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

Graphically:

The transfer matrix:

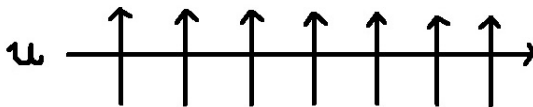$$t(u) = \mathrm{Tr}_0 \, T(u)$$

From Yang-Baxter:

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Conctructing the charges

- Ultra-local charges
  - $t(0) = U$ the translation operator
  - Defining

  $$Q_j = \left(\frac{d}{du} \right)^{j-1} \log t(u) \Bigg|_{u=0}$$

  Q_j is a sum of j-site operators
- Quasi-local charges from spin-s (fused) transfer matrices $t_s(u)$

  $$Q_{2s,j} = \left(\frac{d}{du} \right)^{j-1} \log t_s(u) \Bigg|_{u=0}$$

E. Ilievski, M. Medenjak, T. Prosen, L. Zadnik, J. Stat. Mech. (2016) 064008

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Constructing the charges

From charges to correlators:

- Initial states:

$$|\Psi_0\rangle \quad \rightarrow \quad Q_{s,j}$$

  M. Fagotti and F. H. L. Essler, J. Stat. Mech. (2013) P07012

- Root densities:

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  E. Ilievski, J. D. Nardis, B. Wouters, J-S. Caux, F. H. L. Essler, T. Prosen, Phys. Rev. Lett. 115, 157201 (2015)

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- **GGE = TBA**

- Canonical form?

$$\varrho_{GGE} = \frac{1}{Z} \exp\left(-\sum_{s=1}^{\infty} \int_{-\pi/2}^{\pi/2} du \ \mu_s(u)\hat{\rho}_s(u) \right)$$

E. Ilievski, E. Quinn, J-S. Caux, Phys. Rev. B, 95, 11, id.115128

$$\varrho_{GGE} = \lim_{N_s, N_d \to \infty} \frac{1}{Z} \exp\left(-\sum_{s=1}^{N_s} \sum_{j=1}^{N_d} \beta_{s,j} Q_{s,j} \right)$$

BP, E. Vernier, arXiv:1703.09516, to appear in JSTAT

Infinite resolution in rapidity space: Not possible with quasi-local operators

GGE

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E. Ilievski, E. Quinn, J-S. Caux, Phys. Rev. B, 95, 11, id.115128

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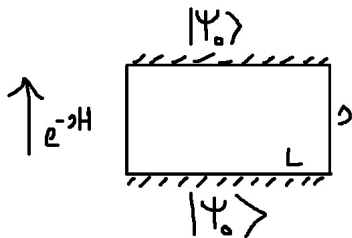
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The Loschmidt amplitude

$$G(s) = \langle \Psi_0 | e^{-sH} | \Psi_0 \rangle, \qquad s \in \mathbb{C}$$

Dynamical free energy:
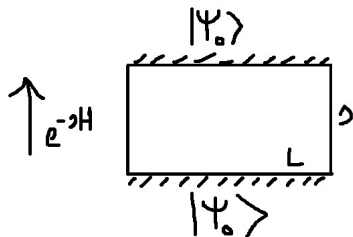
$$g(s) = -\lim_{L \to \infty} \frac{1}{L} \log G(s)$$

$s \to it$: Return probability

Overlap-based evaluation:

$$G(s) = \sum_n |C_n|^2 e^{-sE_n}$$

In the $s \to 0$ limit: Back to the Quench Action

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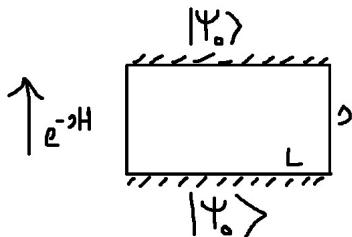
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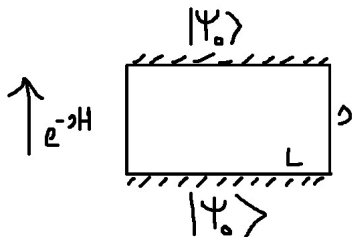$s \to it$: Return probability

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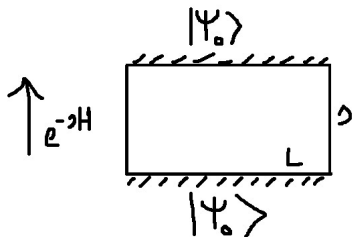
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$$\log(Y_j) = d_j - \beta d \delta_{j,1} + d \star (\log(1 + Y_{j+1}) + \log(1 + Y_{j-1}))$$

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where $\beta = 2\sinh(\eta)s$ and $\tilde{Y}_1(u)$ is the solution at $s = 0$.

Valid even in the cases where the overlaps are not known!

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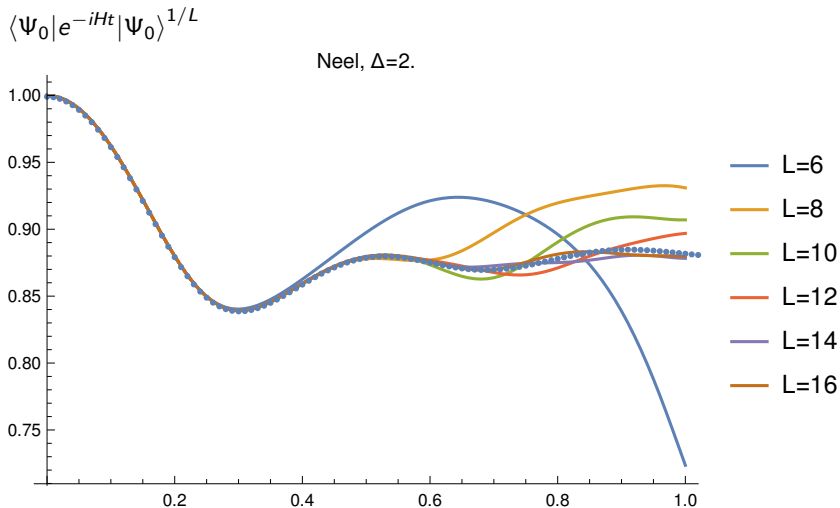
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$\langle \Psi_0 | e^{-iHt} | \Psi_0 \rangle^{1/L}$

Neel, Δ=2.

- L=6
- L=8
- L=10
- L=12
- L=14
- L=16

L. Piroli, BP, E. Vernier, J. Stat. Mech. (2017) 023106
[BP, J. Stat. Mech. (2013) P10028]

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Trotter decomposition:

$$e^{-sH} = \lim_{N \to \infty} \left(1 - \frac{sH}{N}\right)^N$$

We have

$$1 - \frac{sH}{N} = T(-\beta/2N)T(-\eta + \beta/2N) + \mathcal{O}(N^{-2})$$

and

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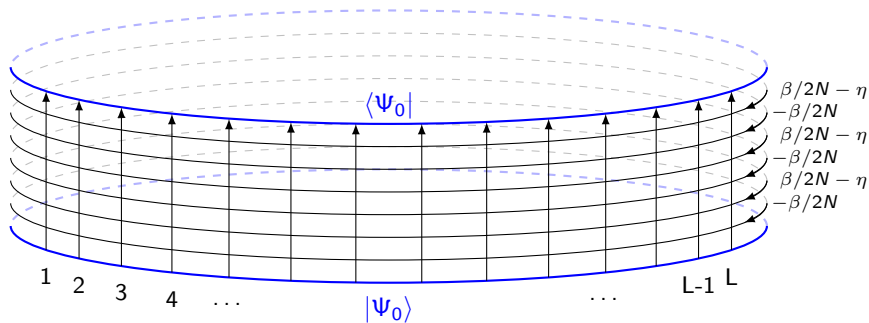$$e^{-sH} = \lim_{N \to \infty} \left(1 - \frac{sH}{N} \right)^N$$

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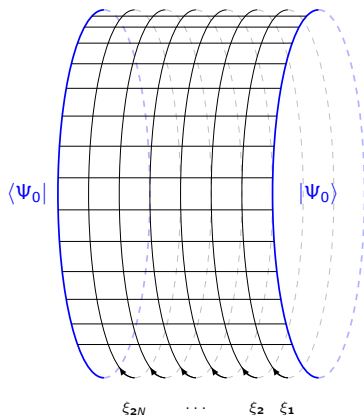$$1 - \frac{sH}{N} = T(-\beta/2N)\,T(-\eta + \beta/2N) + \mathcal{O}(N^{-2})$$

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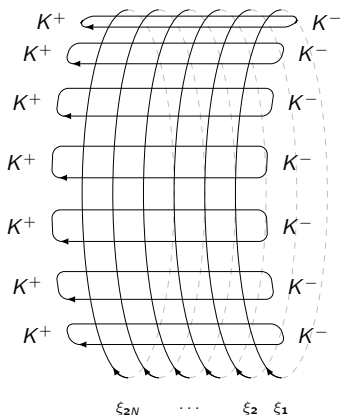
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$\langle \Psi_0 |$

$| \Psi_0 \rangle$

1 2 3 4 $\ldots$ $\ldots$ L-1 L

BP, J. Stat. Mech. (2013) P10028

The Loschmidt amplitude

Boundary transfer matrix:

$$\tau(u) = \text{Tr}_0\{K_+(u)T(u)K_-(u)\tilde{T}(u)\}$$

with $\tilde{T}(u) = \sigma_0^y T^{t_0}(-u)\sigma_0^y$.

Rewrite it as

$$\tau(u) = \langle v^+(u)|T(u) \otimes T(-u)|v^-(u)\rangle$$

where

$$|v^-(u)\rangle = -k_{12}^-(u)|\uparrow\uparrow\rangle + k_{11}^-(u)|\uparrow\downarrow\rangle - k_{22}^-(u)|\downarrow\uparrow\rangle + k_{21}^-(u)|\downarrow\downarrow\rangle$$
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Physical case: $|\Psi_0\rangle = \prod |v^+(0)\rangle = \prod |v^-(0)\rangle$.
For each two-site there is a corresponding K-matrix.

Diagonal K-matrices give states of the form $\frac{|\uparrow\downarrow\rangle + \alpha|\downarrow\uparrow\rangle}{\sqrt{1+|\alpha^2|}}$

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For real s: QTM is gapped, therefore

$$g(s) = -\lim_{L \to \infty} \log G(s) = -\frac{1}{2} \log(\Lambda_0)$$

Diagonalization of the boundary transfer matrix:

○ Diagonal K-matrices: Bethe Ansatz
Leads to boundary-NLIE, *BP 2013*

○ Off-diagonal K-matrices: Inhomogeneous T-Q equation

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$$\tau^{(n)}\left(u + \eta/2\right)\tau^{(n)}\left(u - \eta/2\right) = \tau^{(n-1)}(u)\tau^{(n+1)}(u) + \Phi_n(u)$$

Y-functions:

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Y-system:

$$y_j\left(u + \frac{\eta}{2}\right)y_j\left(u - \frac{\eta}{2}\right) = [1 + y_{j+1}(u)][1 + y_{j-1}(u)]$$

Identification:

$$y_j(u) = Y_j^{\text{TBA}}(u)$$

M. Takahashi and A. Klümper, J. Phys. A: Math. Gen. 34 L187

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Off-diagonal K-matrices: „overlap-TBA without overlaps''

Explicit calculation for „tilted Néel''

$$|N, \vartheta\rangle \equiv \otimes_{j=1}^{L/2} \left[\begin{pmatrix} \cos(\vartheta/2) \\ \sin(\vartheta/2) \end{pmatrix} \otimes \begin{pmatrix} -\sin(\vartheta/2) \\ \cos(\vartheta/2) \end{pmatrix} \right]$$

Root distribution give the correct charges!

$$1 + Y_1(\lambda) = \frac{\mathcal{N}(\lambda + i\eta/2)\mathcal{N}(\lambda - i\eta/2)}{\chi(\lambda)}$$

with

$$\mathcal{N}(\lambda) = \left[-2\cosh^2(\zeta)\cosh(2\eta) + \cosh(2\zeta)(2\cos(2\lambda) - 1) + 4\cosh(\eta)\sin^2(\lambda) + \cos(4\lambda) \right]$$

$$\chi(\lambda) = 16 \frac{\sin(2\lambda + 2i\eta)\sin(2\lambda - 2i\eta)}{\sin(2\lambda + i\eta)\sin(2\lambda - i\eta)} \left(\sin(\lambda + i\eta/2)\sin(\lambda - i\eta/2)\cos(\lambda - i\zeta)\cos(\lambda + i\zeta) \right)^2$$

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Exact solution from:

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For $s = 0$ we have

$$\langle \Psi_0 | e^{-sH} | \Psi_0 \rangle \to \langle \Psi_0 | \Psi_0 \rangle$$

The boundary QTM with no columns:

$$T_1(u) = \langle v_+(iu) | v_-(iu) \rangle = \mathrm{Tr}\left(K_+(u) K_-(u) \right)$$

Reproduces all previous exact solutions, and provides new ones for off-diagonal K-matrices

The Loschmidt amplitude

Exact solution from:

$$1 + Y_1(u) = \frac{T_1(u + \eta/2)\,T_1(u - \eta/2)}{\Phi_1(u)}$$

For $s = 0$ we have

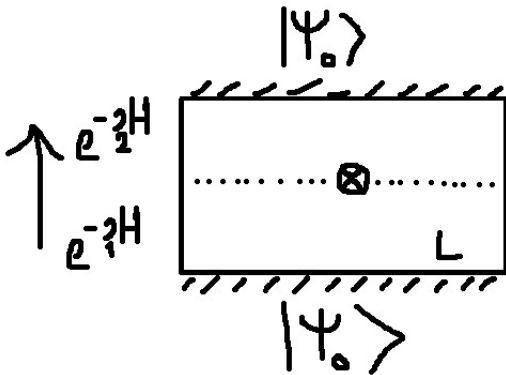$$\langle \Psi_0 | e^{-sH} | \Psi_0 \rangle \to \langle \Psi_0 | \Psi_0 \rangle$$

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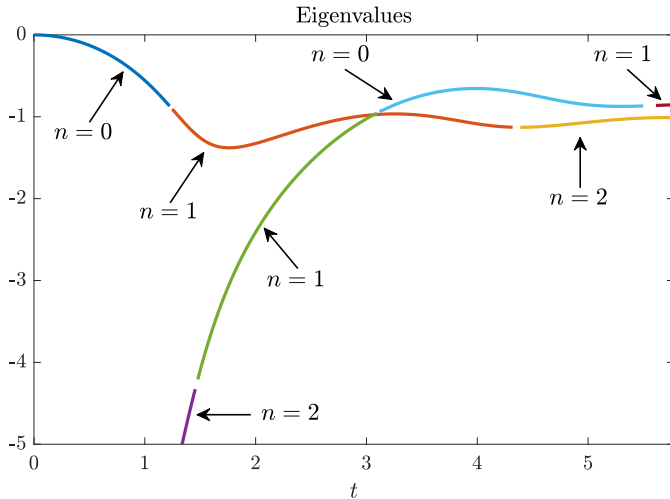Reproduces all previous exact solutions, and provides new ones for off-diagonal K-matrices

$$\mathcal{O}(t) = \langle \Psi_0 | e^{-s_2 H} \mathcal{O} e^{-s_1 H} | \Psi_0 \rangle$$

$|\Psi_0\rangle = |N\rangle, \Delta = 0.5$

Eigenvalues

- Non-analyticities in general

- What are integrable global quenches?
  Precise relations between boundaries and integrable initial states.
  Ghoshal-Zamolodchikov for lattice systems.
  Upcoming!

- Integrable MPS states (from AdS/CFT)

- Correlation functions!

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Open problems

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Thank you for the attention!