Effects of reflection-asymmetric shapes on nuclear collective and isomeric properties

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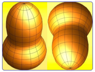


SSNET'17, Gif-sur-Yvette, 10 November 2017

Contents

Quadrupole-octupole core plus particle Hamiltonian

$$H = H_{qo} + H_{s.p.} + H_{pair} + H_{Coriol}$$

$$H_{qo} = -\frac{\hbar^2}{2B_2}\frac{\partial^2}{\partial\beta_2^2} - \frac{\hbar^2}{2B_3}\frac{\partial^2}{\partial\beta_3^2} + U(\beta_2, \beta_3, I)$$

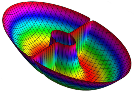
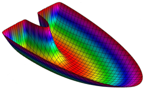
$$U(\beta_2, \beta_3, I) = \frac{1}{2}C_2{\beta_2}^2 + \frac{1}{2}C_3{\beta_3}^2 + \frac{d_0 + \hat{I}^2 - {\hat{I}_z}^2}{2\mathcal{J}(\beta_2, \beta_3)}$$

$$H_{Coriol} = -\frac{(\hat{I}_+\hat{j}_- + \hat{I}_-\hat{j}_+)}{2\mathcal{J}(\beta_2, \beta_3)}, \qquad \mathcal{J}(\beta_2, \beta_3) = (d_2\beta_2^2 + d_3\beta_3^2)$$

DSM: $H_{sp} = T + V_{ws}(\beta_2, \beta_3, ...) + V_{s.o.} + V_c$

DSM+BCS: $H_{qp} \equiv H_{sp} + H_{pair} \to \epsilon_{qp}^K = \sqrt{(E_{sp}^K - \lambda)^2 + \Delta^2}$

Coherent quadrupole-octupole mode (CQOM) in the even core

$U(\beta_2, \beta_3, I)$:   $(\beta_2 > 0)$

$$U(\beta_2, \beta_3, I) + \langle H^K_{\text{Coriol}} \rangle = \frac{1}{2} C_2 {\beta_2}^2 + \frac{1}{2} C_3 {\beta_3}^2 + \frac{\widetilde{X}(I,K)}{d_2\beta_2^2 + d_3\beta_3^2}$$

$$\widetilde{X}(I,K) = \tfrac{1}{2}[d_0 + I(I+1) - K^2 + 2\mathcal{J}(\beta_2,\beta_3) \cdot \langle H^K_{\text{Coriol}} \rangle]$$

CQOM: $\omega = \sqrt{C_2/B_2} = \sqrt{C_3/B_3} \equiv \sqrt{C/B}$

$$H_{\text{qo}} + H_{\text{Coriol}} : \quad \widetilde{E}^{\text{qo}}_{n,k}(I,K) = \hbar\omega \left[2n + 1 + \sqrt{k^2 + b\widetilde{X}(I,K)} \right]$$

Quadrupole-octupole (QO) vibration function of the core

$$\Phi^{\pi}_{n,k,I}(\eta,\phi)=\psi^{I}_{nk}(\eta)\varphi^{\pi}_{k}(\phi)$$

$$\beta_2=\sqrt{d/d_2}\eta\cos\phi\ ,\quad \beta_3=\sqrt{d/d_3}\eta\sin\phi\ ,\quad d=(d_2+d_3)/2$$

$$\psi^{I}_{nk}(\eta)=\sqrt{\frac{2c\Gamma(n+1)}{\Gamma(n+2s+1)}}e^{-c\eta^2/2}c^{s}\eta^{2s}L^{2s}_{n}(c\eta^2)$$

$$\begin{aligned}\varphi^{+}_{k}(\phi)&=\sqrt{2/\pi}\cos(k\phi)\ ,\quad k=1,3,5,\ldots\\ \varphi^{-}_{k}(\phi)&=\sqrt{2/\pi}\sin(k\phi)\ ,\quad k=2,4,6,\ldots\end{aligned}$$

[N. M. et al, Phys. Rev. C **73**, 044315 (2006); **76**, 034324 (2007)]

Total core plus particle wave function

$$\Psi^{\pi,\pi^b}_{nkIMK}(\eta,\phi) = \frac{1}{2}\sqrt{\frac{2I+1}{16\pi^2}}\Phi^{\pi\cdot\pi^b}_{nkI}(\eta,\phi) \times \left[D^I_{M\,K}(\theta)\mathcal{F}^{(\pi^b)}_K + \pi\cdot\pi^b(-1)^{I+K}D^I_{M\,-K}(\theta)\mathcal{F}^{(\pi^b)}_{-K}\right]$$

$$\mathcal{F}_\Omega = \sum_{Nn_z\Lambda} C^{\Omega}_{Nn_z\Lambda}|Nn_z\Lambda\Omega\rangle \stackrel{\Omega=K}{=} \mathcal{F}^{(+)}_K + \mathcal{F}^{(-)}_K$$

$\mathcal{F}^{(\pi^b)}_K = \mathcal{F}^{(\pm)}_K \rightarrow$ projected s.p. wave function

$\pi^b = \pm$ experimental parity of the bandhead state

[N. M., S. Drenska, M. Strecker and W. Scheid, JPG **37**, 025103 (2010)]

Quasi parity-doublet spectrum from CQOM+DSM+BCS

$$E_{nk}(I^{\pi},K_b) = \epsilon_{\text{qp}}^{K_b} + \hbar\omega\left[2n+1+\sqrt{k^2+b\widetilde{X}(I^{\pi},K_b)}\right]$$

$$\begin{aligned}\widetilde{X}(I^{\pi},K_b) &= \frac{1}{2}\Bigg[d_0 + I(I+1) - K_b^2 + (-1)^{I+\frac{1}{2}}\left(I+\frac{1}{2}\right)a_{\frac{1}{2}}^{(\pi\pi^b)}\delta_{K_b,\frac{1}{2}} \\ &\quad - A\sum_{\substack{\nu\neq b \\ (K_\nu=K_b\pm 1,\frac{1}{2})}}\frac{\left[\widetilde{a}_{K_\nu K_b}^{(\pi\pi^b)}(I)\right]^2}{\epsilon^{K_\nu}-\epsilon^{K_b}}\Bigg]\end{aligned}$$

$\widetilde{a}_{K_\nu K_b}^{(\pi,\pi^b)}(I) \rightarrow$ Coriolis mixing ($\sim \langle \mathcal{F}_{K_{\nu'}}^{(\pi^b)}|\hat{j}_+|\mathcal{F}_{K_\nu}^{(\pi^b)}\rangle$ from DSM)

$a_{1/2}^{(\pi,\pi^b)} = \pi\pi_b a_{\frac{1}{2}-\frac{1}{2}}^{(\pi^b)} \rightarrow$ decoupling factor

[N. M., Phys. Scripta **T154**, 014017 (2013)]

Coriolis mixed core+particle wave function

$$\widetilde{\Psi}^{\pi,\pi^b}_{nkIMK_b} = \frac{1}{\widetilde{N}_{I\pi K_b}} \left[\Psi^{\pi,\pi^b}_{nkIMK_b} + A \sum_{\substack{\nu \neq b \\ (K_\nu = K_b \pm 1, \frac{1}{2})}} C^{I\pi}_{K_\nu K_b} \Psi^{\pi,\pi^b}_{nkIMK_\nu} \right]$$

$$C^{I\pi}_{K_\nu K_b} = \frac{\widetilde{a}^{(\pi\pi^b)}_{K_\nu K_b}(I)}{\epsilon^{K_\nu} - \epsilon^{K_b}}$$

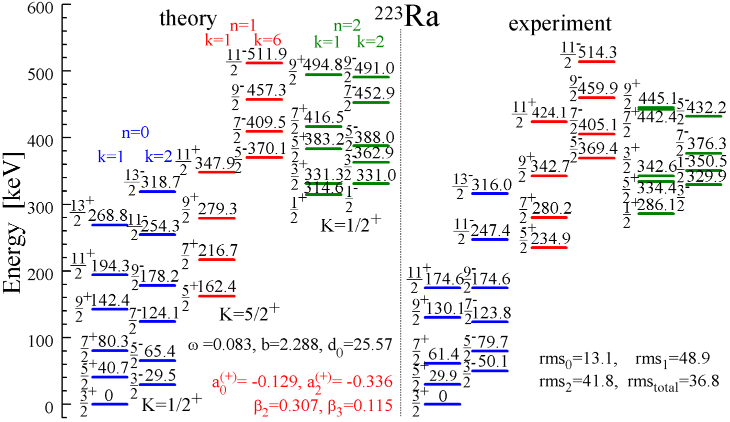
Reduced $E\lambda$ and M1 transition probabilities

$$B(E\lambda;\pi^{b_i}n_ik_iI_iK_i\to\pi^{b_f}n_fk_fI_fK_f) = \frac{1}{2I_i+1}\sum_{M_iM_f\mu}\left|\left\langle\widetilde{\Psi}^{\pi_f,\pi^{b_f}}_{n_fk_fI_fM_fK_f}\right|\hat{\mathcal{M}}_\mu(E\lambda)\left|\widetilde{\Psi}^{\pi_i,\pi^{b_i}}_{n_ik_iI_iM_iK_i}\right\rangle\right|^2$$

$$\langle\mathcal{F}^{(\pi^{b_f})}_{K_f}|\hat{M}1_z|\mathcal{F}^{(\pi^{b_i})}_{K_i}\rangle = \sqrt{\frac{3}{4\pi}}\mu_N\Big[(g_l-g_R)K_i\delta_{K_fK_i}\langle\mathcal{F}^{(\pi^{b_f})}_{K_f}|\mathcal{F}^{(\pi^{b_i})}_{K_i}\rangle + (g_s-g_l)\langle\mathcal{F}^{(\pi^{b_f})}_{K_f}|\hat{s}_z|\mathcal{F}^{(\pi^{b_i})}_{K_i}\rangle\Big]$$

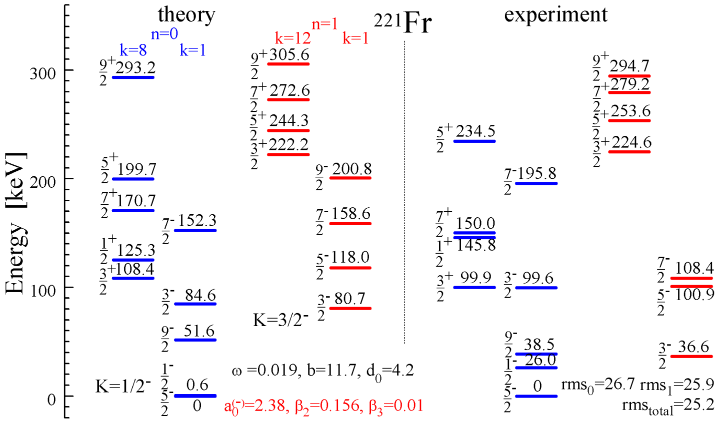
Coriolis K-mixed matrix elements $\Rightarrow$ permission of K-forbidden gamma transitions!

CQOM-DSM-BCS description of QD spectrum in ^{223}Ra



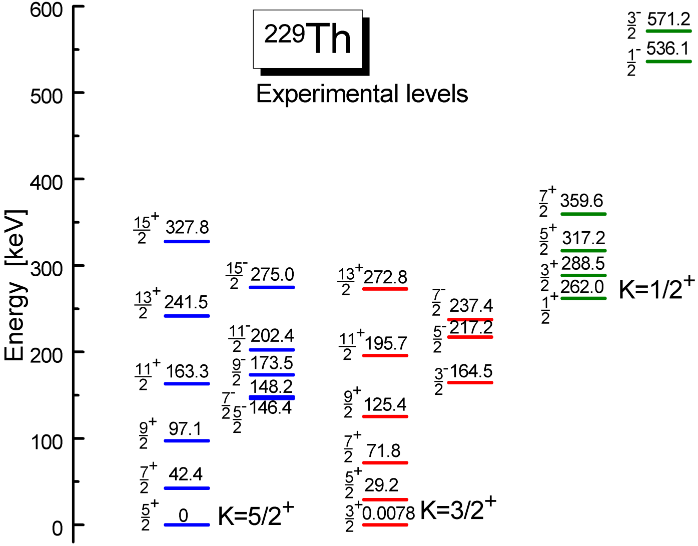
N. M., Phys. Scripta **T154**, 014017 (2013)

CQOM-DSM-BCS description of QD spectrum in ^{221}Fr

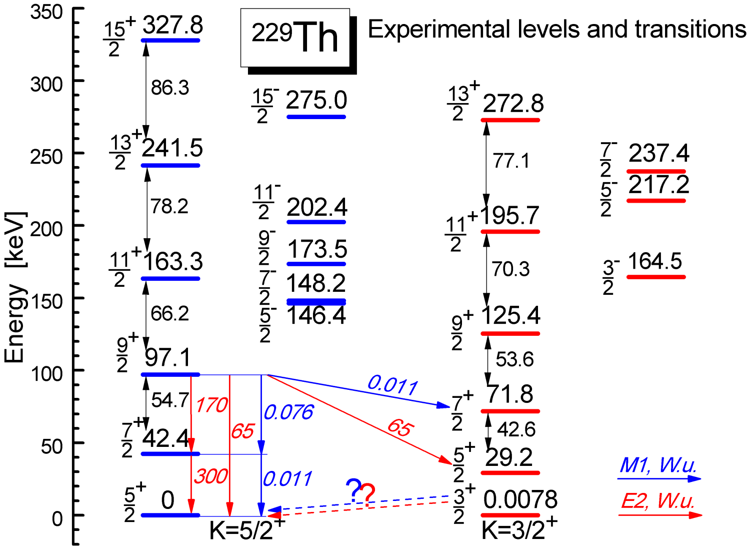


N. M., Phys. Scripta **T154**, 014017 (2013)

^{229}Th: experimental spectrum



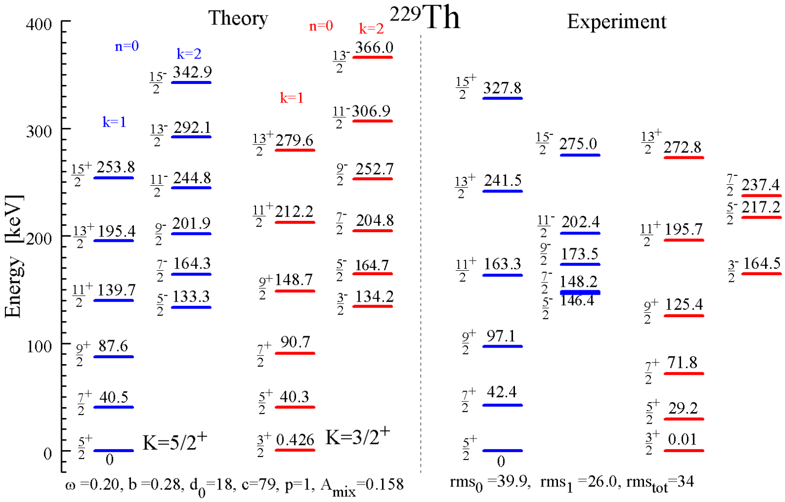
^{229}Th: Low-energy levels and transitions



Details of the CQOM+DSM+BCS model calculations

- General: two quasi-doublets with identical QO oscillation quantum numbers $n = 0$, $k^+ = 1$, $k^- = 2$ built on 5/2[633] and 3/2[631] s.p. orbitals
- DSM: β_2 and β_3 determination $\rightarrow$ correct positions and mutual spacing of the 5/2[633] and 3/2[631] orbitals $\Rightarrow$ $\beta_2 = 0.240$ and $\beta_3 = 0.115$
- CQOM: parameters fits $\rightarrow$ ω, b, d_0 (for energy levels); c, p (transition probabilities); K-mixing constant A (energies and transitions)
- BCS: pairing constants tuning $\rightarrow$ $E(3/2^+) \sim 0.4$ keV
- Isomer energy adjustment: ω b, d_0 tuning $\Rightarrow$ $E(3/2^+) \sim 0.0078$ keV $\rightarrow$ rms deterioration $0.4 - 1.0$ keV

Theoretical and experimental quasi parity-doublet spectrum of ^{229}Th



Theoretical and experimental B(E2) and B(M1) transition values for ^{229}Th

N. M. and A. Pálffy, Phys. Rev. Lett. **118**, 212501 (2017)

Type/Mult	Transition	Th1[Th2] (W.u.)	Exp (W.u.)
E2	$7/2^+_{yrs} \to 5/2^+_{yrs}$	252 [267]	300 ($\pm$16)
E2	$9/2^+_{yrs} \to 5/2^+_{yrs}$	82 [85]	65 ($\pm$7)
E2	$9/2^+_{yrs} \to 7/2^+_{yrs}$	213 [224]	170 ($\pm$30)
E2	$9/2^+_{yrs} \to 5/2^+_{ex1}$	19.98 [17.37]	6.2 ($\pm$0.8)
E2	$\mathbf{3/2^+_{ex1} \to 5/2^+_{yrs}}$	27.04 [23.05]	?
M1	$7/2^+_{yrs} \to 5/2^+_{yrs}$	0.0093 [0.0085]	0.0110 ($\pm$0.0040)
M1	$9/2^+_{yrs} \to 7/2^+_{yrs}$	0.0178 [0.0157]	0.0076 ($\pm$0.0012)
M1	$9/2^+_{yrs} \to 7/2^+_{ex1}$	0.0151 [0.0130]	0.0117 ($\pm$0.0014)
M1	$\mathbf{3/2^+_{ex1} \to 5/2^+_{yrs}}$	0.0076 [0.0061]	?

Th1 $\to E(3/2^+) = 0.4263$ keV   Th2 $\to E(3/2^+) = 0.0078$ keV

Theoretical B(E2) and B(M1) transition values for ^{229}Th at different parameter sets

ω	b	d_0	c	p	A	$k_{yr}^{(-)}$	$k_{ex}^{(-)}$	rms_{yr}	rms_{ex}	rms_{tot}	$E_{ex}(\frac{3}{2}^+)$	$B(E2)$	$B(M1)$
0.2039	0.28	18	79	1.0	0.158	2	2	39.9	26.0	34	0.4263	27.04	0.0076
0.2361	0.28	33	89	1.0	0.141	2	2	41.2	26.4	35	0.0078	23.05	0.0061
0.0912	2.39	49	245	1.0	0.152	4	6	37.6	15.8	29	0.3556	25.80	0.0071
0.0635	4.51	45	321	1.0	0.144	6	8	36.4	12.4	28	0.0725	22.86	0.0063
0.0563	7.34	66	473	1.0	0.138	8	10	38.3	11.9	29	10^{-9}	21.31	0.0058

$\Rightarrow$ experimental transition probabilities for the $3/2^+$-isomer decay in ^{229}Th expected in the limits:

$B(E2)$=20–30 W.u.
$B(M1)$=0.006–0.008 W.u.

Phys. Rev. Lett. **118**, 212501 (2017)

DSM+BCS analysis of 2qp energies and magnetic moments

Two-quasiparticle energies:

$$E_{2qp}^{K\pi} = E_{1qp}^{\Omega_1\pi_1} + E_{1qp}^{\Omega_2\pi_2} \ , \qquad E_{1qp}^{\Omega\pi} = \sqrt{(E_{sp}^{\Omega\pi} - \lambda)^2 + \Delta^2}$$

Magnetic moment of the 2qp configuration

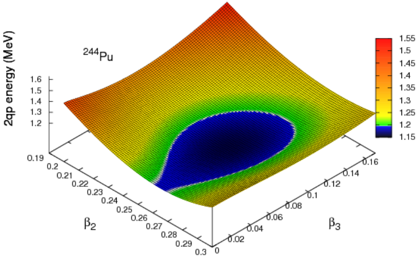
$$\mu = \mu_N \left[g_R \frac{I(I+1) - K^2}{I+1} + g_K \frac{K^2}{I+1} \right]$$

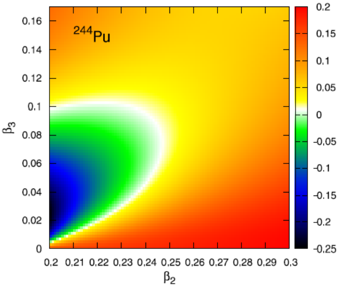
$\mu_N = e\hbar/(2mc)$, $g_R = Z/A$

$$g_K = \frac{1}{K} \sum_{n=1,2} \langle \mathcal{F}_{\Omega_n} | g_s \cdot \Sigma + g_l \cdot \Lambda | \mathcal{F}_{\Omega_n} \rangle$$

$\Sigma = \Omega \mp \Lambda$, $g_s = 0.6 g_s^{free}$

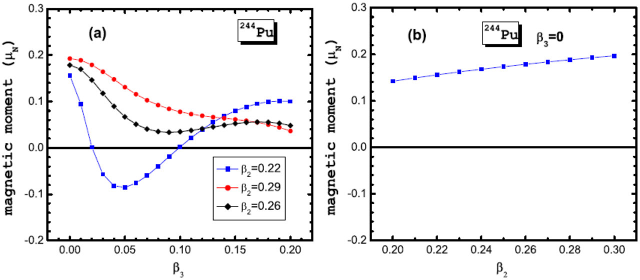
Two-quasiparticle energy and magnetic moment for the $K^{\pi} = 8^{-}$ $\{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration in ^{244}Pu



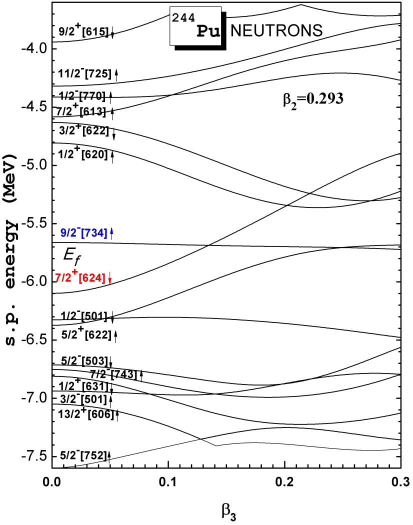


[P. M. Walker and N. Minkov, Phys. Lett. B **694**, 119-122 (2010)]

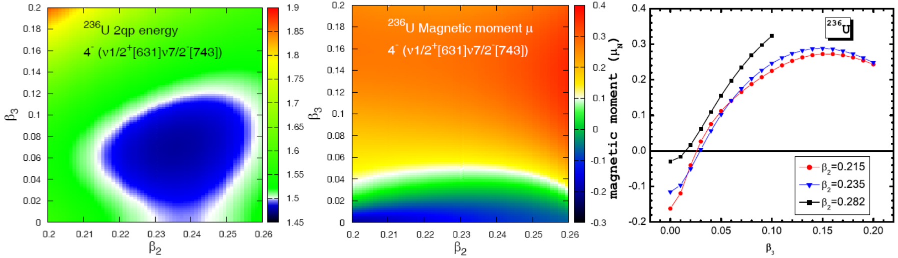
Magnetic moment in the $K^\pi = 8^-$, $\{\nu 7/2[624] \otimes \nu 9/2[734]\}$, state of ^{244}Pu



Neutron s.p. levels in ^{244}Pu ($K^{\pi} = 8^{-}\{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration)

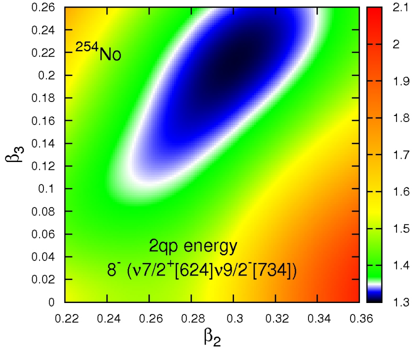


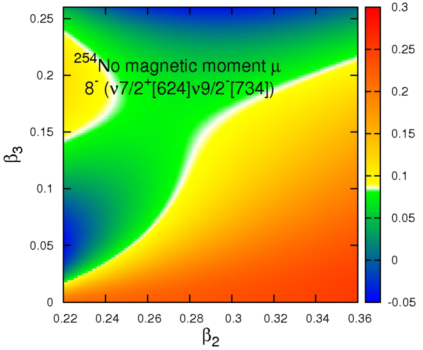
Two-quasiparticle energy and magnetic moment for the $K^{\pi} = 4^{-}$ $\{\nu 1/2[631] \otimes \nu 7/2[743]\}$ configuration in ^{236}U



[N. Minkov and P. M. Walker, Eur. Phys. Journal A, **48**: 80 (2012)]

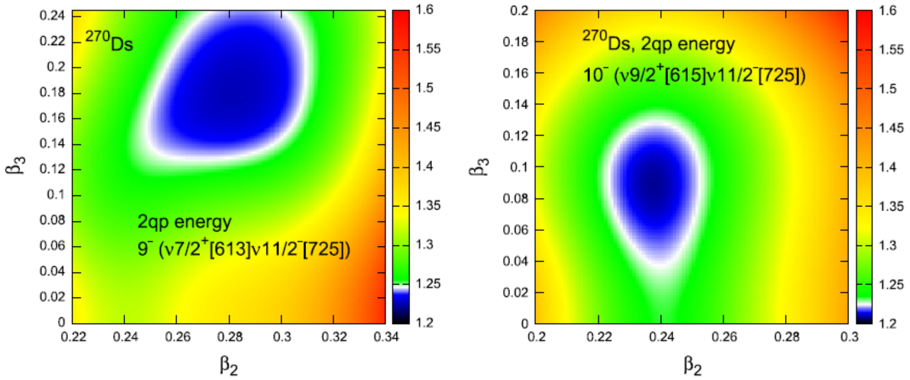
Two-quasiparticle energy and magnetic moment for the $K^{\pi} = 8^{-}$ $\{\nu 7/2[624] \otimes \nu 9/2[734]\}$ configuration in ^{254}No





[N. Minkov and P. M. Walker, Phys. Scripta **89**, 054021 (2014)]

2qp energy and magnetic moments for $K^{\pi} = 9^{-}, 10^{-}$ isomers in ^{270}Ds



[N. Minkov and P. M. Walker, Phys. Scripta **89**, 054021 (2014)]

SUMMARY

- Model: collective CQOM plus microscopic DSM+BCS with fully microscopically treated Coriolis interaction - E/M transitions with K-mixing.
- Applications: quasi-parity doublet spectra in odd-mass nuclei (^{223}Ra, ^{221}Fr).
- ^{229}Th: complete nuclear-structure-model calculation for the low-lying spectrum including the 7.8 eV isomer.
- ^{229m}Th interpretation: a bandhead of an excited parity quasi-doublet, built on 3/2[631] q.p. state coupled to a collective quadrupole-octupole vibration-rotation mode - remarkably fine interplay between all involved modes!
- ^{229m}Th decay: predicted B(E2), B(M1) values for $3/2^+_{isom} \rightarrow 5/2^+_{gs}$ and available data on other transition rates.
- DSM+BCS for high K-isomers: strong dependence of the magnetic moments on the octupole deformation and isomer energy minima at non-zero octupole deformation.

N. Minkov, Gif-sur-Yvette, 7 October 2017