SHAPES & SYMMETRIES IN NUCLEI: FROM EXPERIMENT TO THEORY

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SOME SPECTROSCOPIC PROPERTIES OF WELL-DEFORMED ODD NUCLEI IN THE RARE-EARTH REGION

Nurhafiza M. Nor & Meng-Hock Koh (UTM, Johor, Malaysia)

P. Quentin & L. Bonneau (CENBG, Bordeaux, France)

INTRODUCTION (I)

The present study is the first step, towards a microscopic description of time-odd nuclear states in (axially symmetrical) well-deformed nuclei encountered in

Odd, Odd-Odd, Even-Even (as in High-K isomeric states)

This description will make use of the particle number conserving
Highly Truncated Diagonalization Approach (HTDA)
A configuration mixing of particle - hole states over a vacuum based on the canonical basis of
an as good as possible self-consistent mean field Hamiltonian
using a delta or (separable in p) ersatz of gaussian residual force

INTRODUCTION (II)

To advance in this direction, we merely perform here
self-consistent blocked HF + BCS (with Cooper quasi-pairs)
calculations for well-deformed odd nuclei
in the rare earth region (and around)

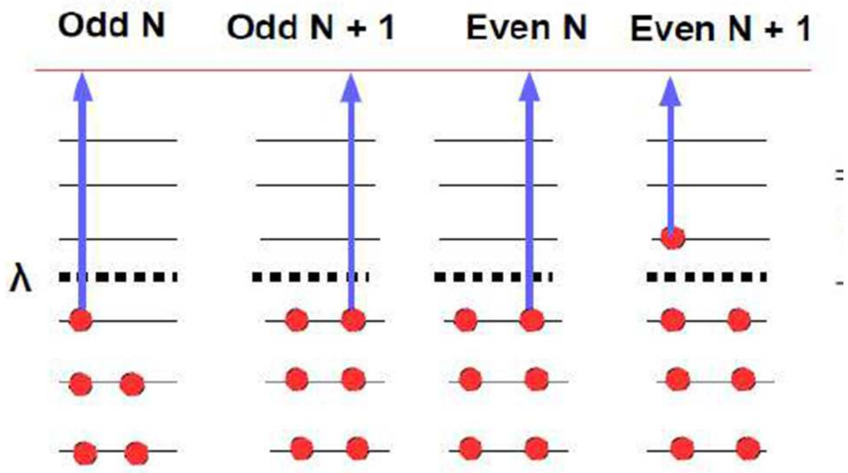
The Skyrme SIII interaction is used for p-h h-p channels
(in view of its well-established spectroscopic quality)
A crude seniority force is used
for the pp hh channels ($|T_z| = 1$ only)
A fit of the latter is made from explicit calculations
of odd even mass differences

PART 1. FITTING THE STRENGTH OF THE RESIDUAL INTERACTIONS (I)

The three-point mass differences

$$\Delta_n(N,Z) = \frac{(-1)^N}{2}\,[E(N+1,Z) - 2E(N,Z) + E(N-1,Z)]$$
$$= \frac{(-1)^N}{2}\,[S_n(N,Z) - S_n(N+1,Z)]$$
$$\Delta_p(N,Z) = \frac{(-1)^Z}{2}\,[E(N,Z+1) - 2E(N,Z) + E(N,Z-1)]$$
$$= \frac{(-1)^Z}{2}\,[S_p(N,Z) - S_p(N,Z+1)]$$

depend mostly on pair breaking energies around odd N / Z



The pairing matrix elements are parametrized as

$$\langle i\,\bar{i}\,|\tilde{v}_{residual}|\,j\,\bar{j}\rangle = \frac{G_q}{11 + N_a}$$

FITTING THE STRENGTH OF THE RESIDUAL INTERACTIONS (II)

To get the separation energies of e.g. the (N,Z) nucleus Hartree-Fock plus BCS calculations have been performed for both the (N,Z) and (N – 1,Z) or (N,Z) and (N,Z - 1) nuclei and not through some lowest qp energies in the (N,Z) nucleus as usually performed

This implies calculating odd-even or even-odd nuclei through self-consistent blocked Hartree-Fock plus BCS calculations where time-odd effects have been taken into account

SAMPLE OF NUCLEI FOR THE FIT

Condition 1:

For even-even nuclei, the ratio of

$$\frac{E_{4^+}}{E_{2^+}} > 3.28$$

Condition 2:

Experimental 3-points energy differences of odd nuclei

$$\Delta \quad > 0.50 \text{ MeV}$$

For the fit

Even Z-even N : 23

Odd Z-even N : 13

Even Z-Odd N : 17

Total : 53

N \ Z	62	63	64	65	66	67	68	69	70	71	72
110											Hf-182
109											Hf-181
108									Yb-178	Lu-179	Hf-180
107									Yb-177		Hf-179
106									Yb-176	Lu-177	Hf-178
105									Yb-175		Hf-177
104							Er-172	Tm-173	Yb-174		Hf-176
103							Er-171		Yb-173		
102			Gd-166	Tb-167	Dy-168	Ho-169	Er-170	Tm-171	Yb-172		
101			Gd-165		Dy-167		Er-179		Yb-171		
100			Gd-164	Tb-165	Dy-166	Ho-167	Er-168	Tm-169	Yb-170		
99			Gd-163		Dy-165						
98	Sm-160	Eu-161	Gd-162	Tb-163	Dy-164						
97	Sm-159		Gd-161		Dy-163						
96	Sm-158	Eu-159	Gd-160	Tb-161	Dy-162						
95	Sm-157										
94	Sm-156										

Focus of isomeric states studies to come

A two parameters fit (G_n, G_p) on two sets of data (Δ_n, Δ_p)

Independence of the results on G_p for reasonable values of G_n and similarly for G_p on G_n

(G_n, G_p in MeV and χ_n, χ_p in keV)

G_n	G_p	χ_p
14	15	186
15	15	187
16	15	191

G_n	χ_n	G_p
12	383	14
14	287	14
15	197	14
15.5	134	14
15.5	134	14
16	89	14
16.5	129	14
17	221	14
18	449	14

G_n	χ_n	G_p	χ_p
14	287	14	291
14		14.5	226
14		15	186
14		15.5	207
14		16	288
14		18	771

G_n = 16 MeV and G_p = 15 MeV

PART 2. MOMENTS OF INERTIA OF EVEN-EVEN NUCLEI (I)

QUESTION

How the above fit is consistent with a fit on quantities strongly varying with pairing correlations: Moments of inertia

CALCULATIONS

Use the non-self consistent ATDHFB approach of Inglis Belyaev

$$\mathscr{I}_{cr} = \sum_{kl}{}' \frac{|\langle k|j_+|l\rangle|^2}{E_k+E_l}(u_k v_l - u_l v_k)^2 + \frac{1}{2}\sum_{kl}{}'' \frac{|\langle k|j_+|\bar{l}\rangle|^2}{E_k+E_l}(u_k v_l - u_l v_k)^2,$$

Correct approximatelty for non-self-consistency the so-called Thouless-Valatin terms see E. Kh. Yuldasbhbaeva et al., Phys. Lett. B461, (1999)1 performed as in J. Libert et al., Phys. Rev. C60 (1999) 054301 by an enhancement factor of 1.32

MOMENTS OF INERTIA OF EVEN-EVEN NUCLEI (II)

DATA

Should be closest to adiabaticy: the first 2^+ state

Avoid coupling with other modes: in very well deformed nuclei

Sample: 19 rare-earth nuclei

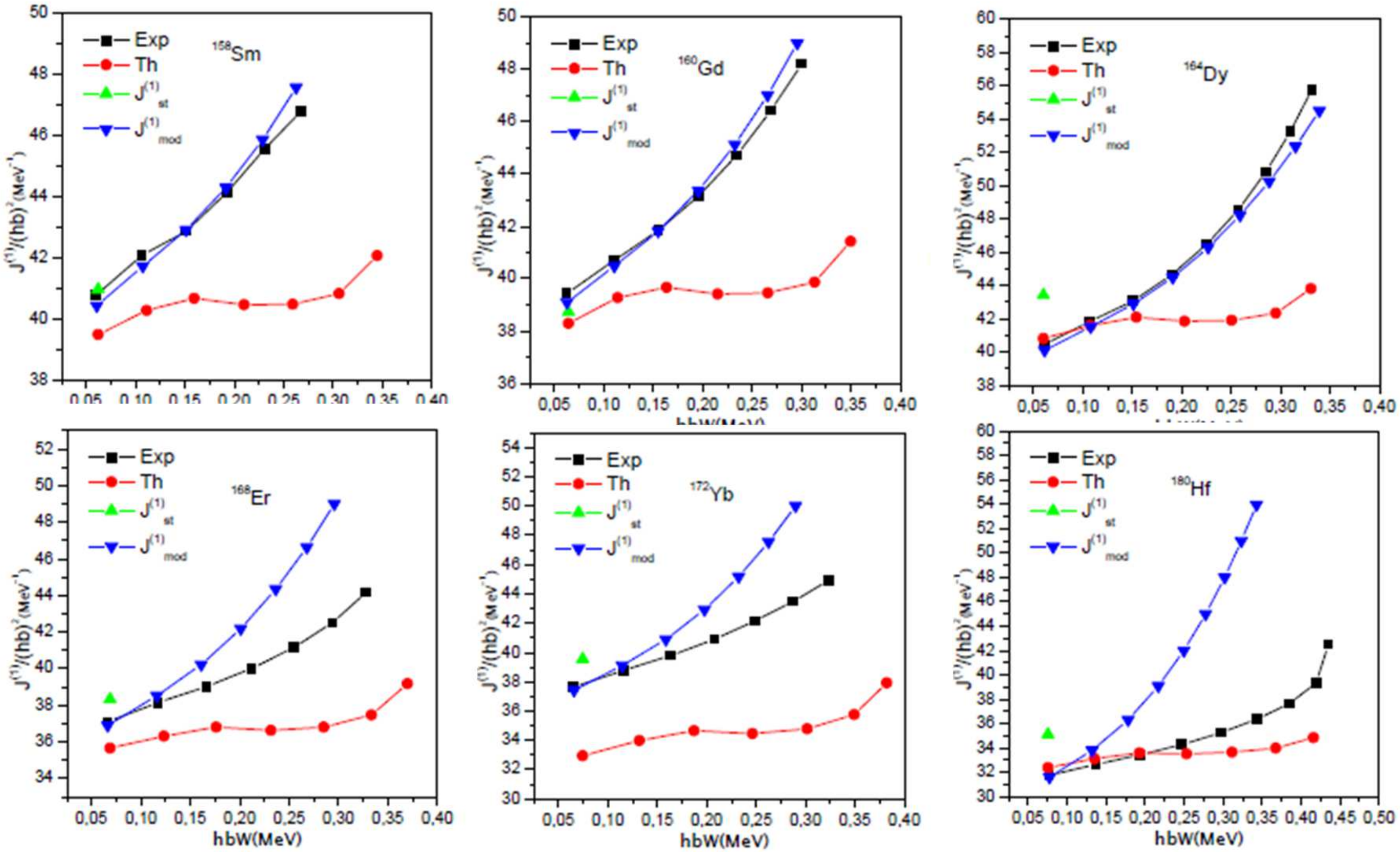
most of them with $E(4^+)/E(2^+)$ equal or larger than 3.3

SIDE TEST

Compare Moments of inertia obtained for the first 2^+ state in recent Bohr Hamiltonian calculations by the Algiers group (M. Rebhaoui, M. Imadalou, D.E. Medjadi, P. Q., to be pub.) corresponding to different but close pairing properties with the value obtained in static calculations at ground state

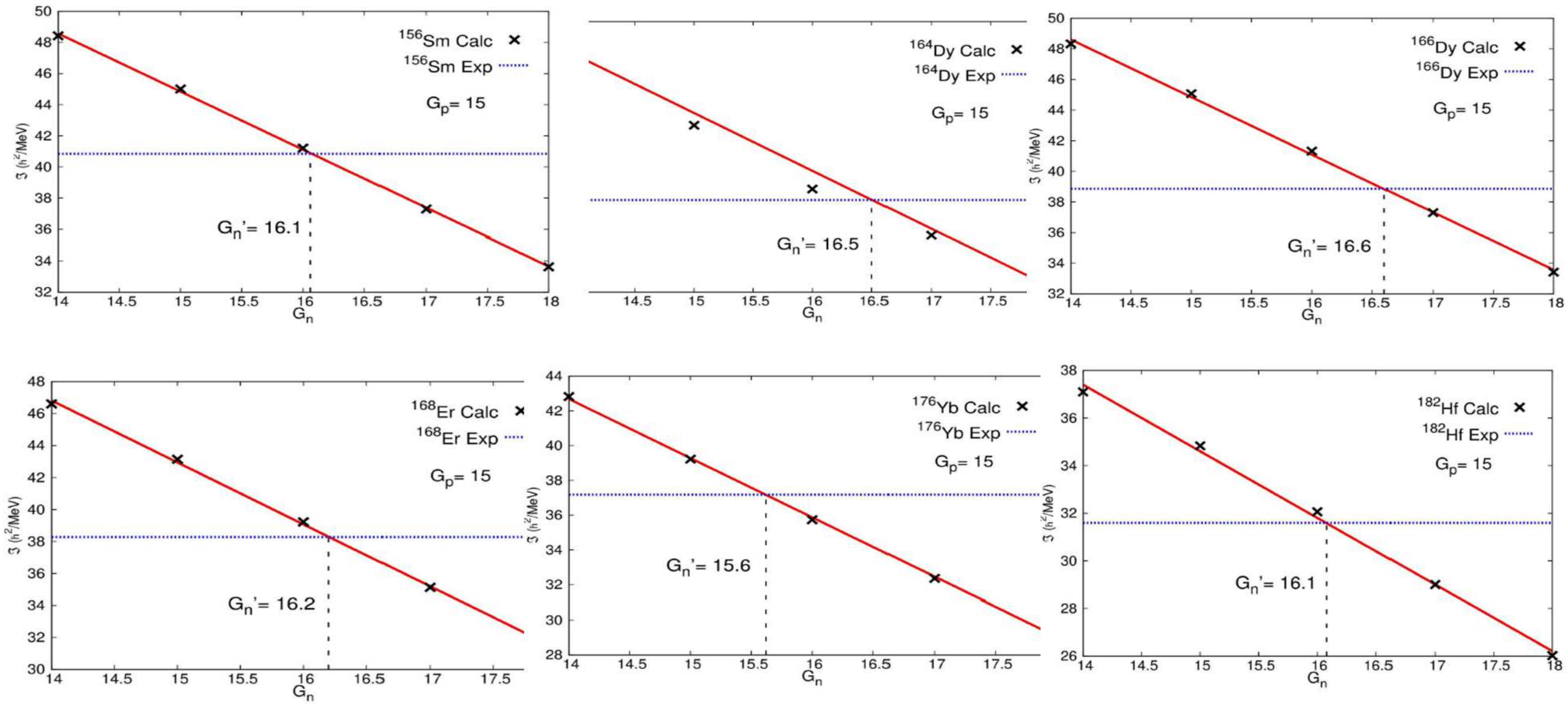
BOHR HAMILTONIAN RESULTS AS A CONSISTENCY TEST

M. Rebhaoui, M. Imadalou, D.E. Medjadi, P. Q.)



PAIRING STRENGTH FIT FROM MOMENTS OF INERTIA (I)

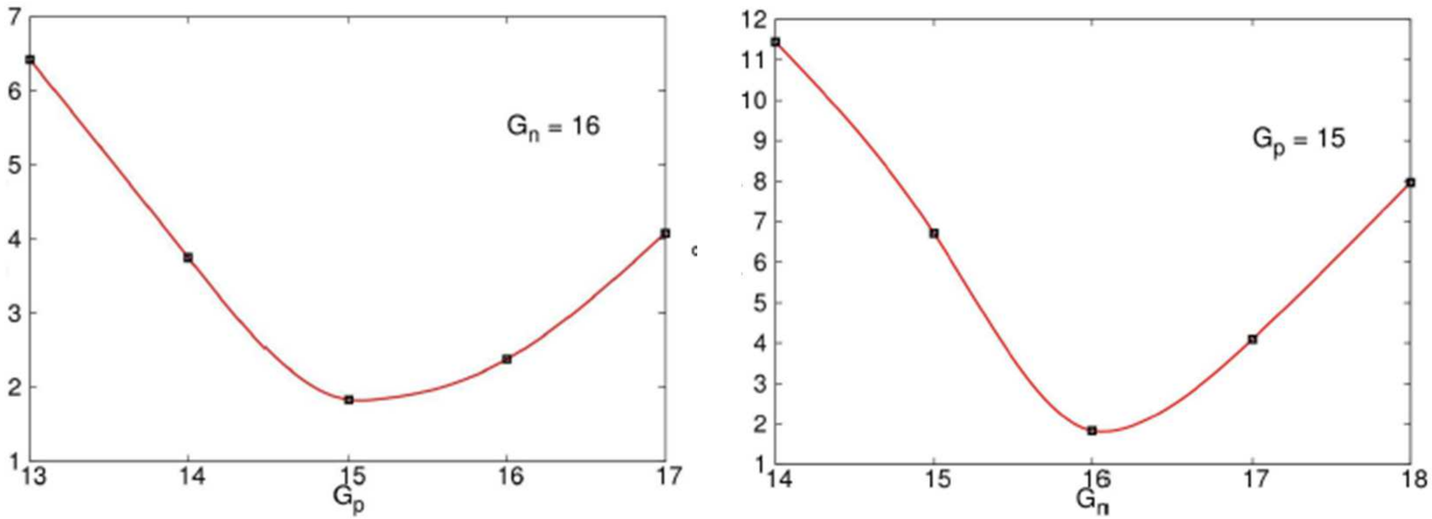
Some fits of moments of inertia on G_n (at fixed G_p)



$\langle G_n \rangle$ = 16.2 MeV with a r.m.s. deviation of 0.3 MeV for 19 nuclei

PAIRING STRENGTH FIT
FROM MOMENTS OF INERTIA (II)

r.m.s. differences on moments of inertia ($\hbar^2$ MeV^{-1})



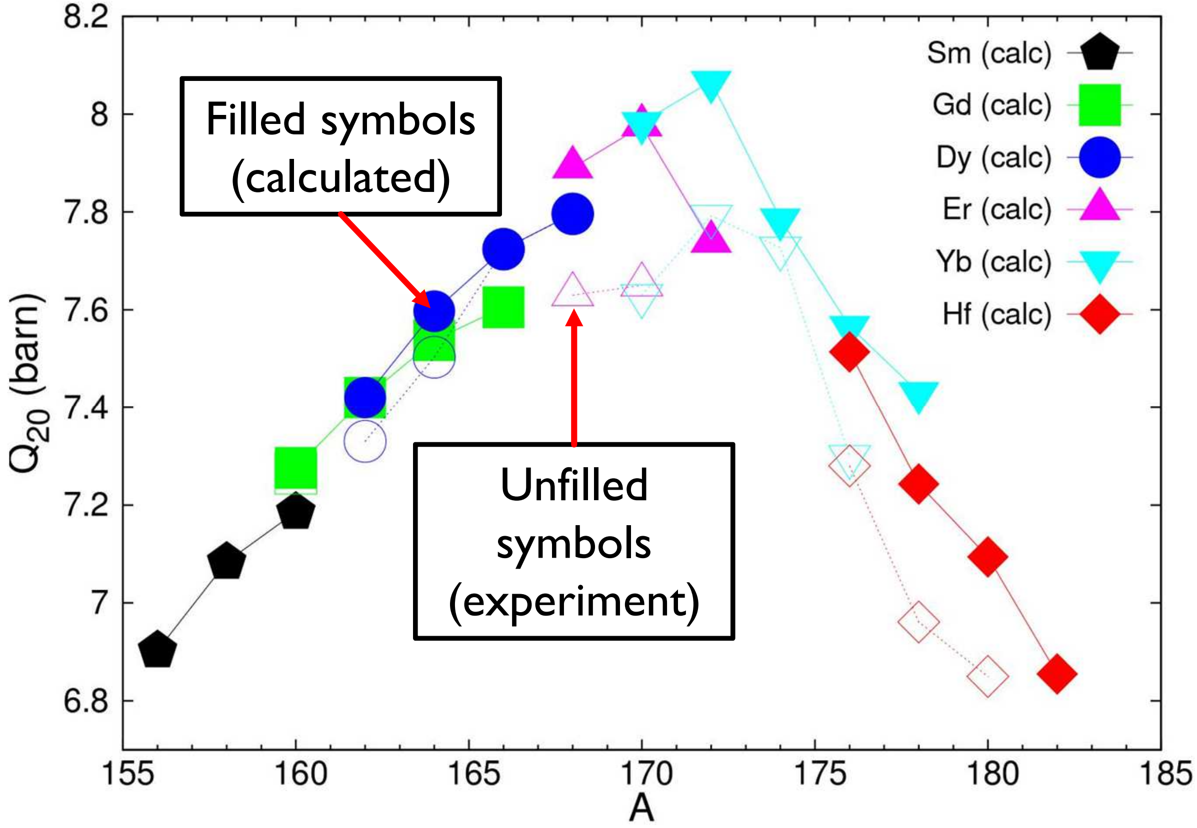
CONCLUSION

The two fits on Odd-Even Mass Differences and on Moments of inertia of very well-deformed nuclei yield consistent results

(Practical note: the latter is much easier than the former)

PART 3.
EXAMPLES OF STATIC PROPERTIES

I. INTRINSIC QUADRUPOLE MOMENTS, EVEN-EVEN NUCLEI



DATA:
Intrinsic Quadrupole moments from B(E2, $2^+ \rightarrow 0^+$)

EXAMPLES OF STATIC PROPERTIES
2. MAGNETIC MOMENTS (NEAR ^{178}HF)

Nuclei	K^π	μ_{int}	g_R (Z/A)	μ_{theor}	μ_{exp}
^{177}Hf	$7/2^-$	0.88	0.242 (0.407)	1.07	0.7935 (7)
^{179}Hf	$9/2^+$	-1.00	0.427 (0.402)	-0.65	-0.6409 (13)
^{177}Lu	$7/2^+$	1.48	0.487 (0.401)	1.85	2.239 (7)
^{179}Ta	$7/2^+$	1.47	0.271 (0.408)	1.68	2.289 (9)

(All quantities in μ_N)

Core polarization effects are included
mostly affecting μ_{int} and much less g_R
For ^{179}Hf, L. Bonneau et al., Phys. Rev. C91 (2015) 054307
have estimated that the core polarization contribution
could be mocked up by a quenching factor of 0.734 for g_s

PART 4. BAND HEAD SPECTRA (FOR PARTICLE MODES ONLY)

Preliminary results* for the 4 odd nuclei neighbouring ^{178}Hf

*** preliminary: no rotational effects included, see below**

Restriction:

Only « seniority » 1 (1 broken pair) states in the calculations

- **comparison with data only credible for $E_{excitation}$ < Gap**
- **BCS over-emphasizes the pairing quenching due to blocking**
  Must use a particle number conserving approach (as HTDA) !

Comparison between HF+BCS and Nuclear energies:

- **Remove spurious rotational energy content (~ à la Lipkin)**
- **Dress the intrinsic solution for core (rotational) modes (~ à la Bohr-Mottelson)**

(At this stage Coriolis mixing is ignored excepted for the self coupling of |K| = 1/2 s.p. configurations)

BAND HEAD ENERGIES (PARTICLE MODES)

In Meng-Hock Koh et al. , Eur. Phys. J. A 52 (2016) 3
one has found including the above rotational effects,
that the band head energies are given by

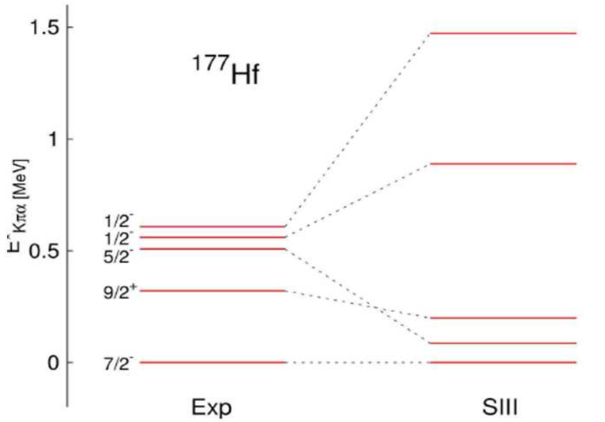
$$E_{K\pi\alpha} = \langle \Psi^{\alpha}_{K\pi} | \hat{H}_{\text{eff}} | \Psi^{\alpha}_{K\pi} \rangle + \frac{\hbar^2}{2\mathcal{J}} \left(2K - \delta_{K,\frac{1}{2}} a \right) - \frac{\langle \hat{\mathrm{J}}^2 \rangle_{\text{core}}}{2\mathcal{J}}$$

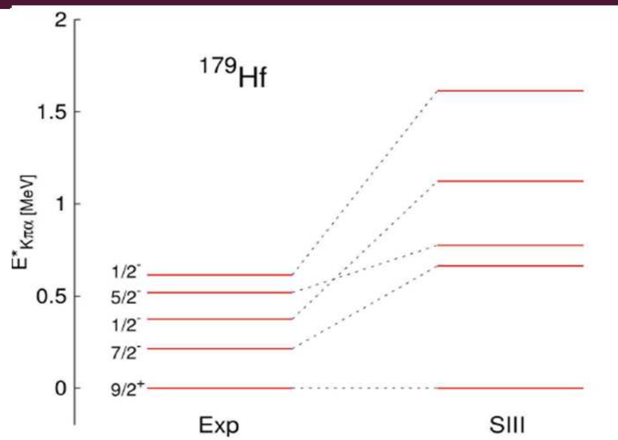
The above expectation value $\langle \hat{\mathrm{J}}^2 \rangle_{\text{core}}$ and the moment of inertia $\mathcal{J}$
are calculated for each polarized (configuration-dependent) core

They may often correspond thus to low pairing regimes
and thus may not be well described by BCS.
This is why here, we present non corrected values

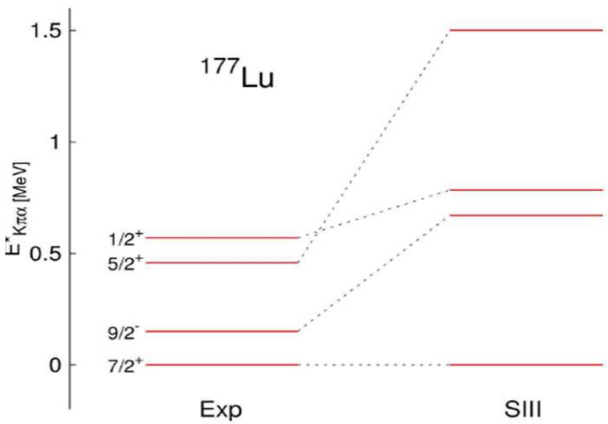
DATA from A.K. Jain et al., Rev. Mod. Phys. C 62 (1990) 2
Selecting states of particle nature and such that $\mathrm{E_{exc}} < \sim 0.5\ \mathrm{MeV}$

RESULTS

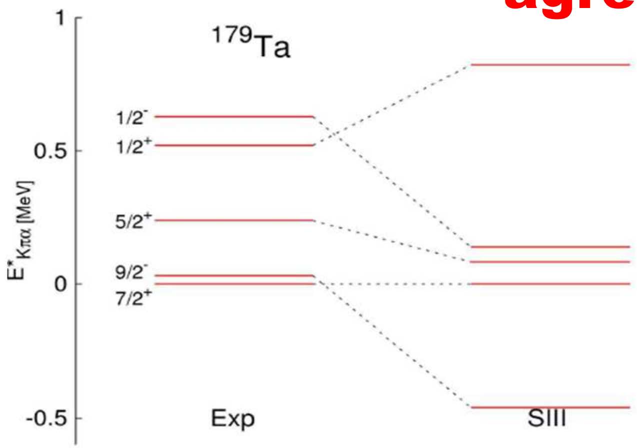




r.m.s. deviation 536 keV



Only a qualitative agreement



r.m.s. deviation 472 keV

CONCLUSIONS (I)

1) We probably have reached our goal of providing a good starting point for a pairing description better than HF+BCS or HFB necessary for T-odd (as e.g. HTDA)

2) We have shown that consistent fits of the pairing residual interaction over a whole deformation region are possible between odd-even mass differences and moments of inertia

3) We have obtained good reproductions of some moments (electric quadrupole and magnetic dipole)

4) Only qualitative agreement have been yielded yet for band head spectra
Some corrections strongly dependent on a good pairing, yet to be established, should be added

CONCLUSIONS (II)
PERSPECTIVES

To each our goal of describing T-odd systems
(of a non-vanishing seniority character)
especially high-K isomers which are our targets
moving to a particle number conserving approach is a must

We will do that through an efficient and tractable
kind of VAP approach
within the so-called HTDA framework

Residual interaction beyond the simplistic seniority force
will be considered for that purpose