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DECAY CHAINS OF FEW SUPERHEAVY NUCLEI WITH ATOMIC NUMBERS 119 AND 120

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OUTLINE

Theoretical Models for α- and cluster decay

Half-life, AKRA, UNIV, ASAF, semFIS

Theoretical Models for Spontaneous Fission

Inertia Tensor, Least action, Fission Dynamics

Results

α- and Cluster Decay, Spontaneous Fission

MACROSCOPIC-MICROSCOPIC MODEL

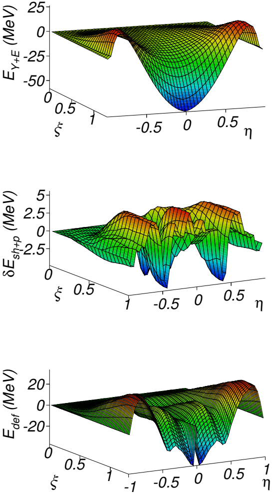
The asymmetric mass distributions of fission fragments and the spontaneously fissioning shape isomers, discovered by S.M. Polikanov *et al.* in 1962, could not be explained until 1967, when V.M. Strutinsky reported his macroscopic-microscopic method. He obtained a two hump potential barrier for heavy nuclei. Shape isomers occupied the second minimum.
He added to the phenomenological deformation energy, E_{def}, the shell plus pairing correction energy,

$$\delta E = \delta U + \delta P = (\delta U + \delta P)_p + (\delta U + \delta P)_n$$

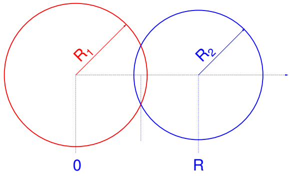
$$E_{def} = E_{LD} + \delta E$$

We use the Yukawa-plus-exponential model (Y+EM) to calculate $E_{def} = E_{Y+E}$, and R.A. Gherghescu's asymmetric two center shell model (ATCSM) Phys. Rev, C **67** (2003) 014309 to calculate δE. The BCS (Bardeen, Cooper, Schrieffer) system of two eqs. allows us to find the Fermi energy, λ, and the gap parameter, Δ, the pairing correction and the cranking inertia tensor needed to study Dynamics and calculate the half-life along the least action trajectory.

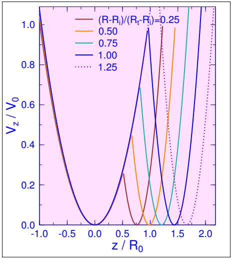
Potential Energy Surface of $^{294}120$



INTERSECTED SPHERES



Two intersected spheres. Volume conservation and R_2 = constant or $R_2 = f(R)$. One or two deformation parameters: separation distance R and R_2. Surface equation $\rho = \rho(z)$. Initial $R_i = R_0 - R_2$. Touching point $R_t = R_1 + R_2$. Normalized variable $x = (R - R_i)/(R_t - R_i)$



Example: $^{232}U \rightarrow\ ^{24}Ne +\ ^{208}Pb$

By assuming R_2 = constant we get one deformation parameter: the separation distance of the fragments, R.

HALF-LIFE

A parent nucleus AZ decays into a light fragment A_eZ_e and a heavy one A_dZ_d:

$$^{A}Z \rightarrow ^{A_e} Z_e + ^{A_d} Z_d$$

The half-life is calculated within quassiclassical WKB (Wentzel-Kramers- Brillouin) approximation:

$$T = [(h\ln 2)/(2E_v)]exp(K_{ov} + K_s)$$

with $B = \mu$ — the nuclear inertia (often taken as a reduced mass), $K = K_{ov} + K_s$ the action integral (for overlapping and separated fragments), and $E(R)$ is the total deformation energy. R_a, R_b are the turning points, defined by $E(R_a) - Q = E(R_b) - Q = 0$. Q is the released energy.

AKRA model

AKRA — from the name of the first author Akrawy D.T.
The *AKRA model* DTA and DNP, J. Phys. G **44** (2017) 105105 was obtained by adding 2 parameters (d and e) and the quantity $I = (N - Z)/A$ to the G. Royer formula published in 2010:

$$T_{1/2} = a + bA^{1/6}\sqrt{Z} + \frac{cZ}{\sqrt{Q_\alpha}} + dI + eI^2$$

with initial parameters
$a = -27.657; -28.408; -27.408, and - 24.763,$
$b = -0.966; -0.920; -1.038, and - 0.907,$ and
$c = 1.522; 1.519; 1.581, and 1.410$ for e-e, e-o, o-e, and o-o, respectively.

ASAF model

Analytical Super-Asymmetric Fission Model.
For *ASAF* $E(R) - Q = E(R) - E_{corr}] - Q$ with E_{corr} a correction energy.
The turning points of the WKB integral are:
$R_a = R_i + (R_t - R_i)[(E_v + E^*)/E_b^0]^{1/2}$ and
$R_b = R_t E_c\{1/2 + [1/4 + (Q + E_v + E^*)E_l/E_c^2]^{1/2}\}/(Q + E_v + E^*)$
where E^* is the excitation energy concentrated in the separation degree of freedom, $R_i = R_0 - R_e$ is the initial separation distance, $R_t = R_e + R_d$ is the touching point separation distance, $R_j = r_0 A_j^{1/3}$ ($j = 0, e, d;\ r_0 = 1.2249$ fm) are the radii of parent, emitted and daughter nuclei, and $E_b^0 = E_i - Q$ is the barrier height before correction.
The two terms of the action integral K, corresponding to the overlapping (K_{ov}) and separated (K_s) fragments, are calculated by analytical formulas (approximated for K_{ov} and exact for K_s).

UNIV model

Universal Formula is obtained from

$$\lambda = \ln 2/T = \nu S P_s$$

with ν — the frequency of assaults on the barrier per second, S is the preformation probability of the cluster at the nuclear surface, and P_s is the quantum penetrability of the external potential barrier.
Assuming $S = S(A_e)$ and $\nu(A_e, Z_e, A_d, Z_d) =$ constant, we get a straight line *universal curve* on a double logarithmic scale

$$\log T = -\log P_s - 22.169 + 0.598(A_e - 1)$$

semFIS model

Semi-emirical formula based on a fission theory of α-decay, was introduced by one of us (DNP) in 1980 to improve the behaviour in the neighborhood of the magic numbers of nucleons.

$$\log T = 0.43429 K_s \chi - 20.446$$

where

$$\begin{aligned} K_s &= 2.52956 Z_{da}[A_{da}/(AQ_\alpha)]^{1/2}[\arccos\sqrt{x} - \sqrt{x(1-x)}] \; ; \\ x &= 0.423 Q_\alpha (1.5874 + A_{da}^{1/3})/Z_{da} \end{aligned} \quad (1)$$

and the numerical coefficient χ, close to unity, is a second-order polynomial $\chi = B_1 + B_2 y + B_3 z + B_4 y^2 + B_5 yz + B_6 z^2$ in the reduced variables y and z, expressing the distance from the closest magic-plus-one neutron and proton numbers N_i and Z_i:

$y \equiv (N - N_i)/(N_{i+1} - N_i) \; ; \; N_i < N \leq N_{i+1}$ and

$z \equiv (Z - Z_i)/(Z_{i+1} - Z_i) \; ; \; Z_i < Z \leq Z_{i+1}$

with $N_i = ..., 51, 83, 127, 185, 229, ...$, $Z_i = ..., 29, 51, 83, 115, ...$, and $Z_{da} = Z - 2$, $A_{da} = A - 4$. The coefficients B_i are obtained by fit with experimental data, using a computer program making automatically the best fit, DNP, MI, DM, Comp. Phys. Comm. **25** (1982) 297.

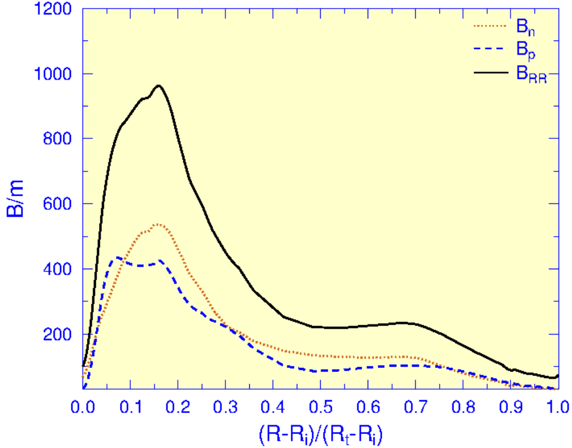
FISSION DYNAMICS

The inertia tensor, B, is calculated using the cranking model introduced by Inglis, M. Brack et al, Rev. Mod. Phys. **44** (1972) 320

$$B_{ij} = 2\hbar^2 \sum_{\nu\mu} \frac{\langle\nu|\partial H/\partial\beta_i|\mu\rangle\langle\mu|\partial H/\partial\beta_j|\nu\rangle}{(E_\nu + E_\mu)^3}(u_\nu v_\mu + u_\mu v_\nu)^2 + P_{ij}$$

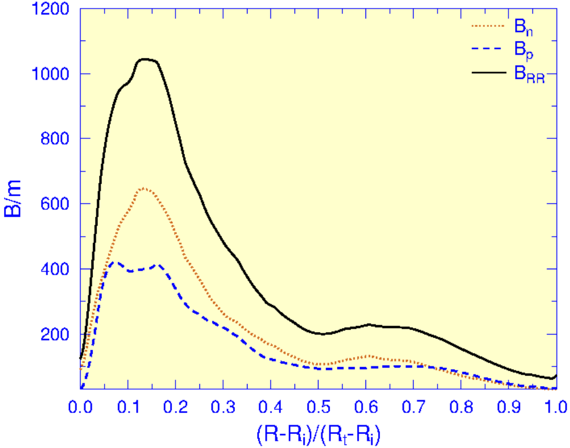
with H the single-particle Hamiltonian used to determine the energy levels and the wave functions $|\nu\rangle$, u_ν, v_ν are the BCS occupation probabilities, E_ν is the quasiparticle energy, and P_{ij} gives the contribution of the occupation number variation when the deformation is changed (terms including variation of the gap parameter, Δ, and Fermi energy, λ, $\partial\Delta/\partial\beta_i$ and $\partial\lambda/\partial\beta_i$).
In the present work, in order to get a first order estimation of the half-life, we use one deformation parameter, R, instead of 5 from RAG, Phys. Rev. C **67** (2003) 014309. Only the parameter B_{RR} is calculated; the dimensionless quantity B_{RR}/m, where m is the nucleon mass.

EXAMPLE of INERTIA for $^{298}120$



$$^{298}120 \rightarrow ^{136}Xe + ^{162}Dy$$

EXAMPLE of INERTIA for $^{296}120$



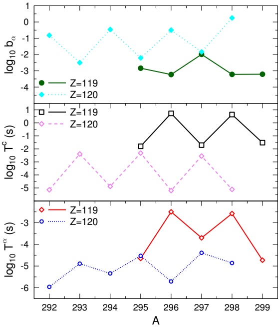
$$^{296}120 \rightarrow ^{134}Xe + ^{162}Dy$$

Prediction of heavy ion radioactivity

The New Encyclopaedia Britannica: "**Heavy-ion radioactivity.** *In 1980 A. Sandulescu, D.N. Poenaru, and W. Greiner described calculations indicating the possibility of a new type of decay of heavy nuclei intermediate between alpha decay and spontaneous fission. The first observation of heavy-ion radioactivity was that of a 30-MeV carbon-14 emission from radium-223 by H.J. Rose and G.A. Jones in 1984.*" The following cluster decay modes have been experimentally confirmed: ^{14}C, ^{20}O, ^{23}F, $^{22,24-26}$Ne, ^{28,30}Mg, ^{32,34}Si with half-lives in good agreement with predicted values within our analytical superasymmetric fission model.

http://www.britannica.com/EBchecked/topic/465998/

In figure and table we compare the half-lives for αD and CR, and the branching ratio relative to α decay $b_\alpha = \log_{10} T_\alpha(s) - \log_{10} T_c(s)$ for the examples we took.



Since 2011 we saw that **for $Z > 121$ cluster decay may compete with α decay: DNP, RAG, WG, Phys. Rev. Lett. 107 (2011) 062503.**

Parent	Emitted	Q_c(MEV)	$\log_{10} T_\alpha(s)$	b_α
^{295}119	^{87}Rb	312.61	-1.80	-2.84
^{296}119	^{88}Rb	312.13	0.73	-3.23
^{297}119	^{89}Rb	311.65	-1.71	-1.99
^{298}119	^{90}Rb	311.36	0.64	-3.22
^{299}119	^{91}Rb	310.63	-1.52	-3.21
^{292}120	^{88}Sr	323.83	-5.14	-0.82
^{293}120	^{88}Sr	323.43	-2.39	-2.50
^{294}120	^{88}Sr	323.02	-4.88	-0.46
^{295}120	^{89}Sr	322.62	-2.32	-2.21
^{296}120	^{90}Sr	322.38	-5.20	-0.50
^{297}120	^{90}Sr	322.08	-2.55	-1.84
^{298}120	^{90}Sr	321.73	-5.11	0.25

Parent	WAR	STA	XU	SAN	REN
^{295}119				9.52	9.98
^{296}119				12.15	8.93
^{297}119					0.79
^{298}119					-1.75
^{299}119				6.70	-11.38
^{292}120	12.15	0.14	16.00	8.65	26.18
^{293}120				11.72	21.49
^{294}120	15.82	1.23	16.22		24.45
^{295}120					18.31
^{296}120	19.57	2.89	15.7	7.99	19.26
^{297}120				10.60	11.93
^{298}120	24.62	4.28	14.5		10.70

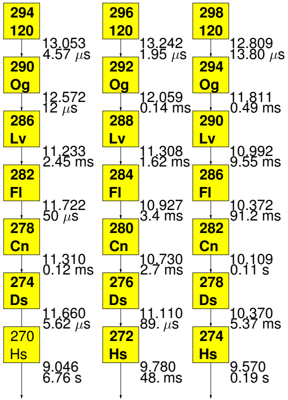
For ^{292}120 the half-lives may differ by 26 orders of magnitude!

WAR — M. Warda and J.L. Egido, Phys. Rev. C **86** (2012) 014322.
STA — A. Staszczak, A. Baran and W. Nazarewicz, Phys. Rev. C **87** (2013) 024320.
XU — Chang Xu, Zhongzhou Ren and Yanqing Guo, Phys. Rev. C **78** (2008) 044329.
SAN — K.P. Santhosh, R.K. Biju and S. Sahadevan, Nucl. Phys. A **832** (2010) 220-232.
REN — Zhongzhou Ren and Chang Xu, Nucl. Phys. A **759** (2005) 64-78.

Parent	Channel	E_b (MeV)	$\log_{10} T_f(s)$
$^{296}120$	$^{138}Ba + ^{158}Gd$	4.83	2.732
$^{296}120$	$^{136}Xe + ^{160}Dy$	4.76	1.583
$^{296}120$	$^{134}Xe + ^{162}Dy$	4.97	0.932
$^{298}120$	$^{138}Ba + ^{160}Gd$	7.22	10.76
$^{298}120$	$^{136}Xe + ^{162}Dy$	7.23	8.581
$^{298}120$	$^{134}Xe + ^{164}Dy$	5.14	1.250

THREE ALPHA DECAY CHAINS

To identify the superheavy nuclei $^{294,296,298}120$ one can use the decay chains from the figure, with calculated kinetic energy and half-lives



CONCLUSIONS

- We studied decay chains useful to guide experiments of producing superheavy nuclei $^{292,294,296,298}120$.
- For α- and cluster decay the half-lives are of the orders of aboout 10 μs with branching ratios $b_\alpha = -0.8, -0.4, -0.5, 0.25$! **It make sense to search for cluster decay modes: emission of ^{88,90}Sr with large kinetic energy, $Q > 300$ MeV.**
- Spontaneous fission half-lives are longer, at least with 6 orders of magnitude larger than for α-decay.
- There is a need to improve the accuracy of calculating spontaneous fission half-lives, because one can meet large dicrepancies from model to model, e.g. **26** orders of magnitude!

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THREE CENTER SHELL MODEL

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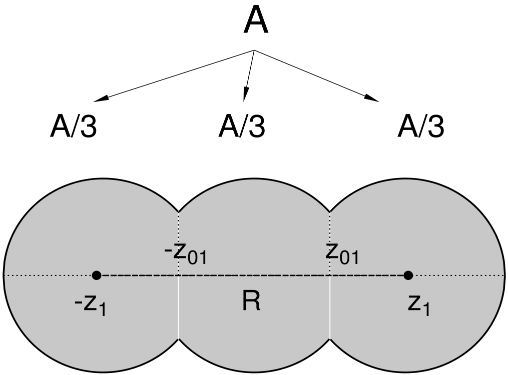
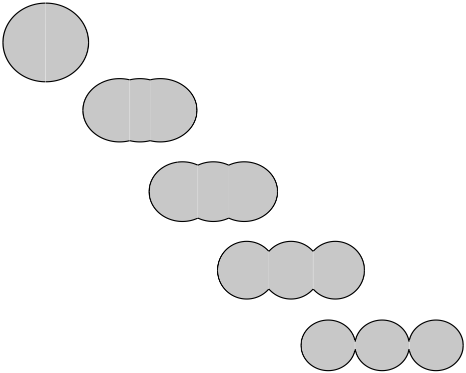
OUTLINE

- Three-center model
- Barriers for ternary fission

THREE CENTER SHELL MODEL



THREE CENTER SHELL MODEL- SHAPE EVOLUTION



THE THREE CENTER POTENTIAL

$$V(\rho, z, \phi) = V(\rho) + V_3(z)$$

where the ρ-axis term is:

$$V(\rho) = \frac{1}{2} m_0 \omega_\rho^2 \rho^2$$

and the z-axis term:

$$V_3(z) = \begin{cases} \frac{1}{2} m_o \omega_z^2 (z - z_1)^2 & , z > z_{01} \\ \frac{1}{2} m_o \omega_z^2 z^2 & , -z_{01} < z < z_{01} \\ \frac{1}{2} m_o \omega_z^2 (z + z_1)^2 & , z < -z_{01} \end{cases}$$

centered in the middle of every fragment.

THE THREE CENTER HAMILTONIAN

$$H = H_{3osc} + V_{\hat{l}\hat{s}} + V_{\hat{l}^2}$$

where

$$H_{3osc} = -\frac{\hbar^2}{2m_0}\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2}\right] + V(\rho) + V_3(z)$$

Total wave function:

$$\Psi(\rho, z, \phi) = \Phi_m(\phi) R_{n_\rho}^{|m|}(\rho) Z_\nu(z)$$

where

$$\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}} \exp(im\phi)$$

$$R_{n_\rho}^{|m|}(\rho) = \left(\frac{2\Gamma(n_\rho+1)\alpha_1^2}{\Gamma(n_\rho+|m|+1)}\right)^{\frac{1}{2}} \exp\left(-\frac{\alpha_1^2\rho^2}{2}\right)(\alpha_1^2\rho^2)^{\frac{|m|}{2}} L_{n_\rho}^{|m|}(\alpha_1^2\rho^2)$$

THREE CENTER WAVE FUNCTION

The ternary character of the process is given by the z - axis wave equation:

$$\left[\frac{\partial^2}{\partial z^2} + \frac{2m_0 E_z}{\hbar^2} - \frac{2m_0}{\hbar^2} V_3(z)\right] Z_\nu(z) = 0$$

where:

$$Z_\nu(z) = \begin{cases} C_{1n} \exp\left[-\frac{\alpha^2 (z-z_1)^2}{2}\right] \mathcal{H}_\nu[\alpha(z-z_1)] & , \quad z \geq z_{01} \\ C_{0n} \exp\left(-\frac{\alpha^2 z^2}{2}\right) [\mathcal{H}_\nu(z) + (-1)^n \mathcal{H}_\nu(-z)] & , \quad -z_{01} < z < z_{01} \\ (-1)^n C_{1n} \exp\left[-\frac{\alpha^2 (z+z_1)^2}{2}\right] \mathcal{H}_\nu[-\alpha(z+z_1)] & , \quad z \leq -z_{01} \end{cases}$$

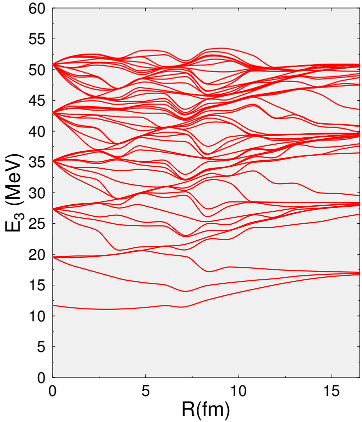
SPIN-ORBIT *ls* AND *l*^{2} OPERATORS

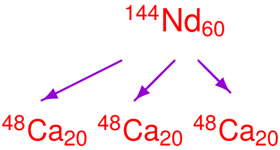
$$V_{ls} = \begin{cases} -\left\{\frac{\hbar}{m_0\omega_{01}}\kappa_1(\rho,z),(\nabla V(z>z_{01})\times p)s\right\} & , A_1 - region \\ -\left\{\frac{\hbar}{m_0\omega_{02}}\kappa_2(\rho,z),(\nabla V(-z_{01}<z<z_{01})\times p)s\right\} & , A_2 - region \\ -\left\{\frac{\hbar}{m_0\omega_{03}}\kappa_3(\rho,z),(\nabla V(z<-z01)\times p)s\right\} & , A_3 - region \end{cases}$$

and

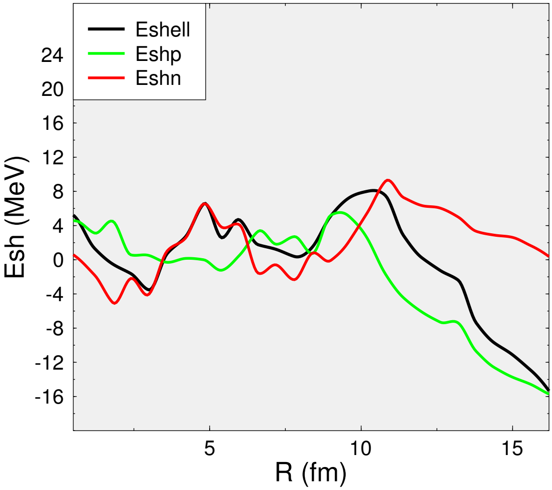
$$V_{l^2} = \begin{cases} -\left\{\frac{\hbar}{m_0^2\omega_{01}^3}\kappa_1\mu_1(\rho,z),(\nabla V(z>z_{01})\times p)^2\right\} & , A_1 - region \\ -\left\{\frac{\hbar}{m_0^2\omega_{02}^3}\kappa_2\mu_2(\rho,z),(\nabla V(-z_{01}<z<z_{01})\times p)^2\right\} & , A_2 - region \\ -\left\{\frac{\hbar}{m_0^2\omega_{03}^3}\kappa_3\mu_3(\rho,z),(\nabla V(z<-z_{01})\times p)^2\right\} & , A_3 - region \end{cases}$$

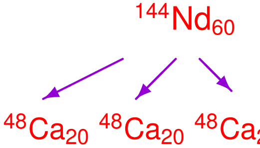
THREE CENTER LEVEL SCHEME FOR ^{144}Nd





SHELL CORRECTIONS FOR ^{144}Nd





CONCLUSIONS

- The three-center shell model describes the transition of the parent neutron and proton level scheme to the three partially overlapped and finally separated fragment level schemes.
- Minima in the shell correction energy could lower the macroscopic barrier and decide which parent nucleus can be chosen as favourable for ternary fission studies.
- Minima in the shell correction calculated with the three-center shell model level scheme could influence the stability of an elongated, linear 3-body type system.
- Total minima obtained within the tripartition of an initial level scheme can emphasize the existence of quasi-stable deformed isomeric states due to the formation of partially overlapped three shell closures of an elongated, linear 3-body type system.

THANK YOU