

Ab Initio View of Emergent Symmetries, Shapes, and Collectivity

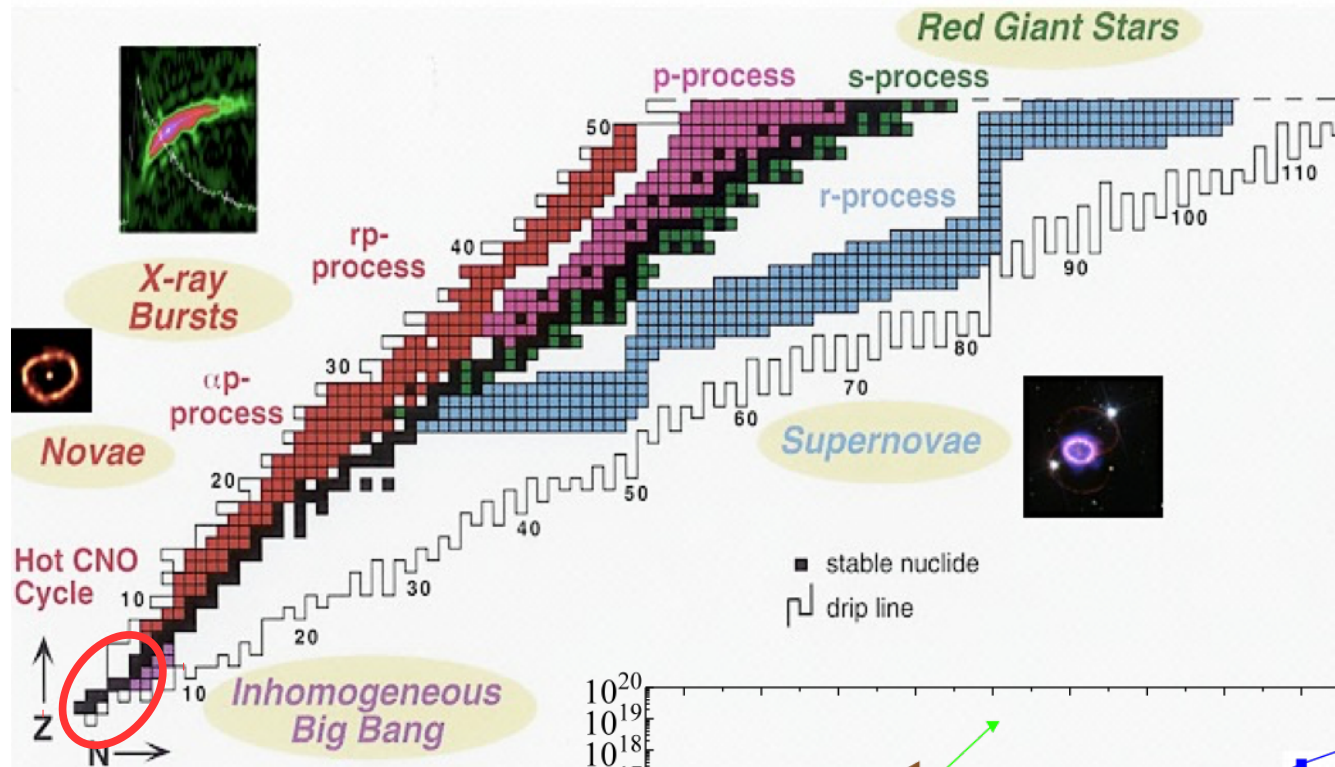
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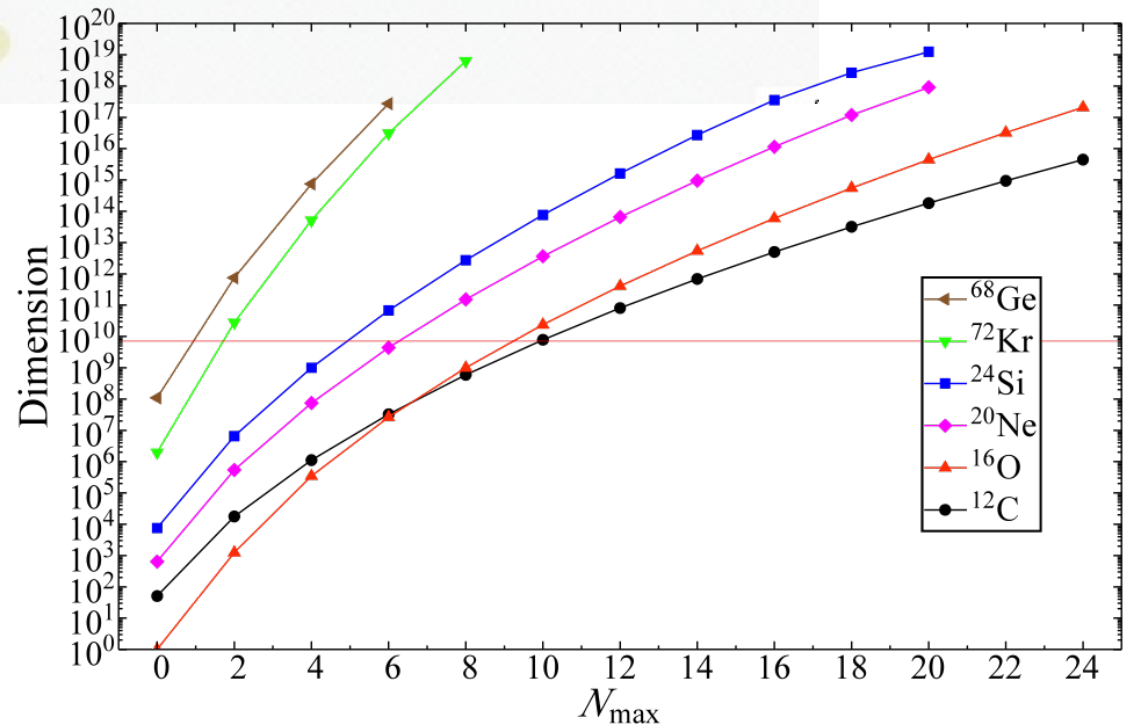
Motivation



No-core Shell Model

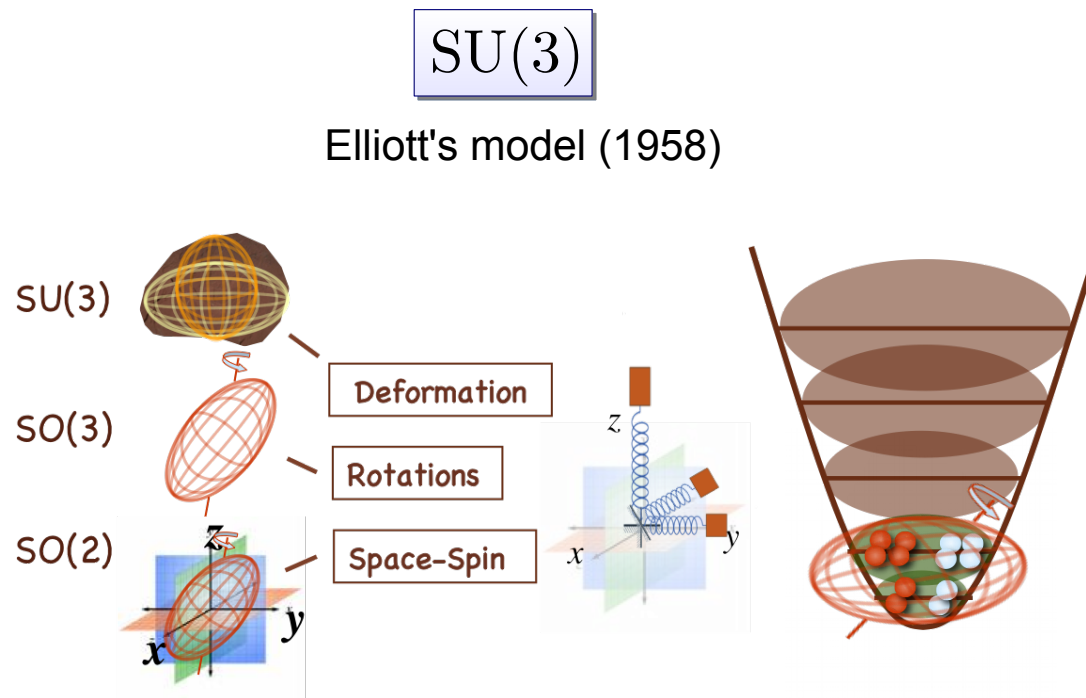
simple, versatile approach

- extremely large Hamiltonians needed
- study heavier nuclei
- describe collective and cluster states



Symmetry-Adapted No-core Shell Model

- **Key feature:** NCSM with basis organized according to symmetries of nuclear collective motion
- basis "designed" for description of nuclear collective dynamics and shapes
- model space refined to include dominant collective modes and shapes



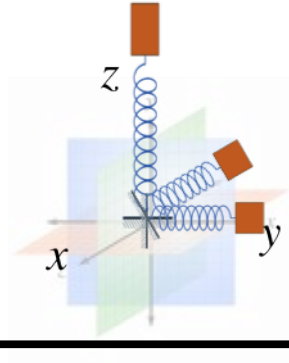
- nuclear deformations and rotations in a valence shell

- Developed formalism for ab initio NCSM computations in multi-shell $SU(3)$ scheme basis

Multi-shell SU(3) scheme Basis

Quantum numbers

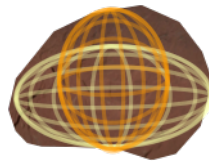
number of HO excitations



N

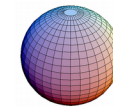
shape

SU(3)

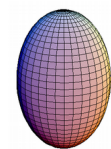


$(\lambda \mu)$

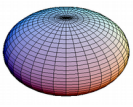
(00)



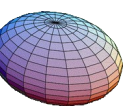
$(\lambda 0)$



(0μ)



$(\lambda \mu)$



rotation

SO(3)



L

total proton, total neutron and total intrinsic spins

$S_p S_n S$

total angular momentum

J

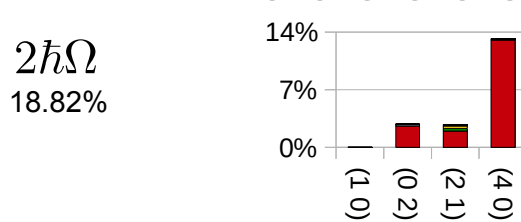
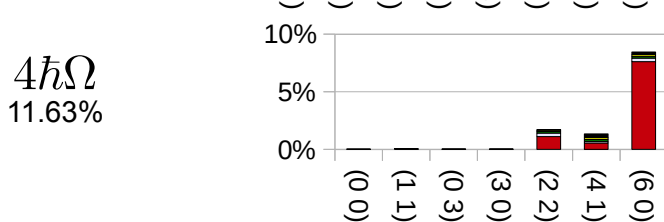
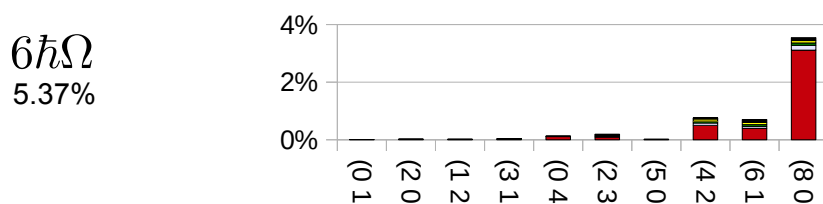
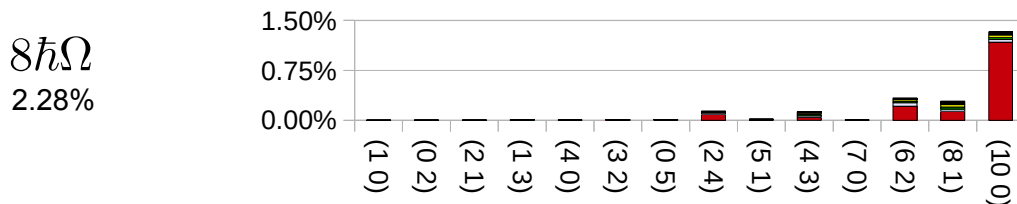
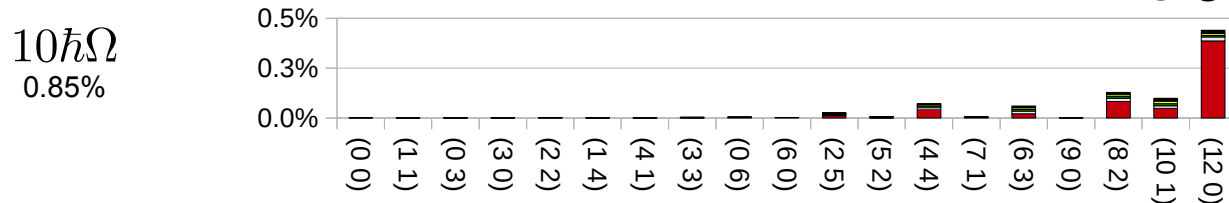
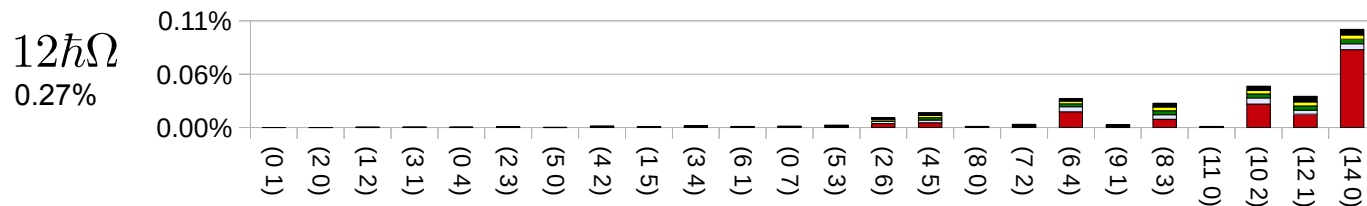
Emergence of Simple Patterns



$N_{\text{max}} = 12$

JISP16 + Vcoul

$\hbar\Omega = 20$ MeV



■ remaining Sp Sn S

■ Sp=1/2 Sn=3/2 S=2

■ Sp=3/2 Sn=1/2 S=2

■ Sp=3/2 Sn=3/2 S=3

■ Sp=1/2 Sn=1/2 S=1

~99% of ground state

Dominant deformations

$$\lambda + 2\mu = \lambda_0 + 2\mu_0 + N$$

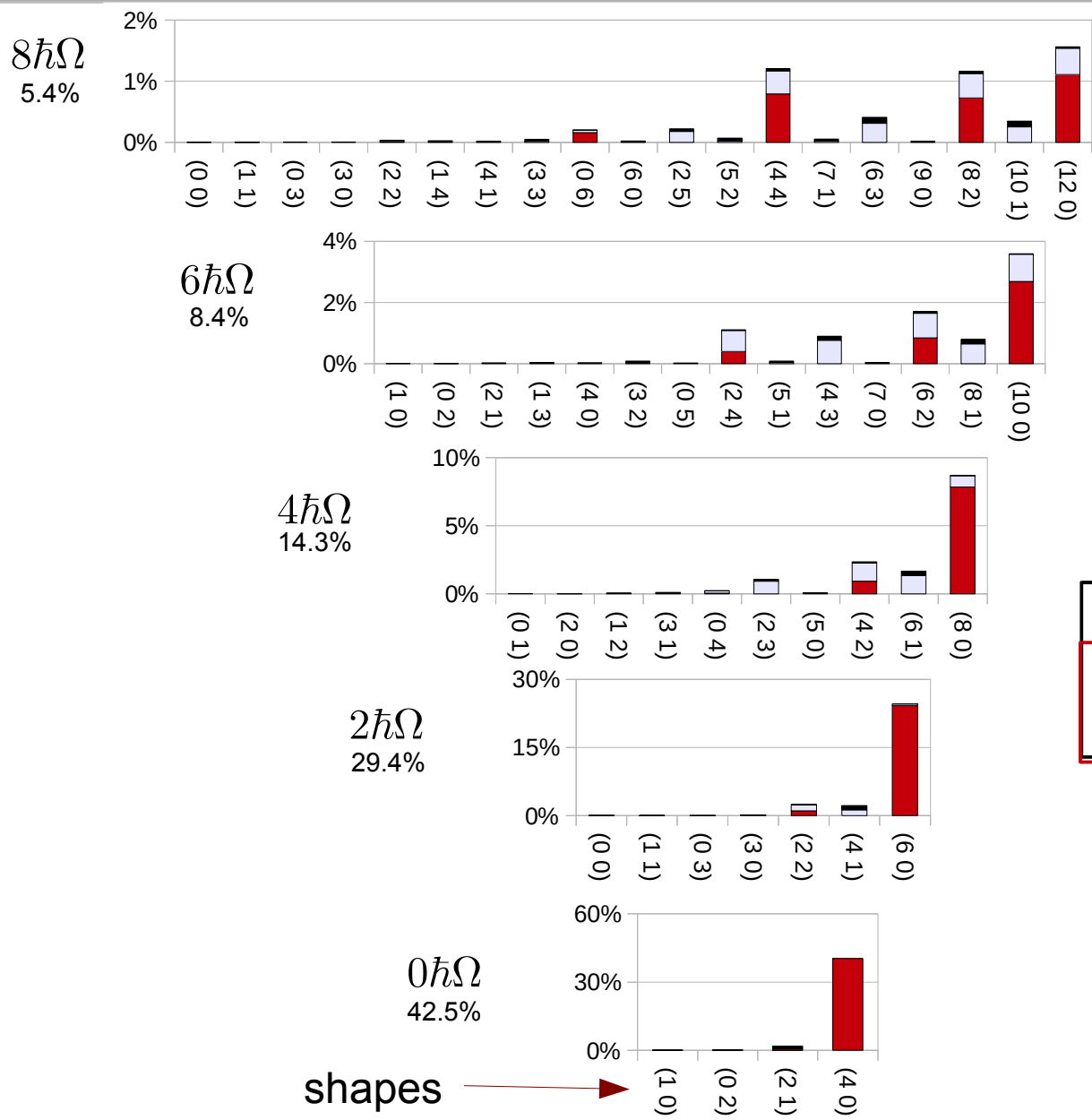


$$(\lambda_0 \mu_0) = (2 0)$$

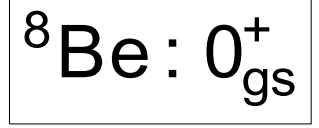
Probability (%)

shapes

Emergence of Simple Patterns



even-even nucleus



different interaction

N3LO + Vcoul

identical features ...

- remaining Sp Sn S
- Sp=1 Sn=1 S=2
- Sp=0 Sn=0 S=0

~98% of the ground state

Dominant deformations

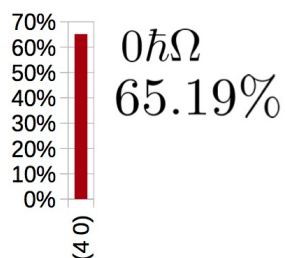
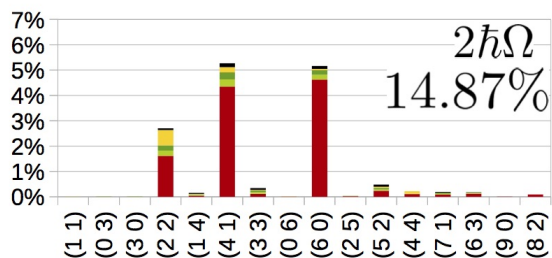
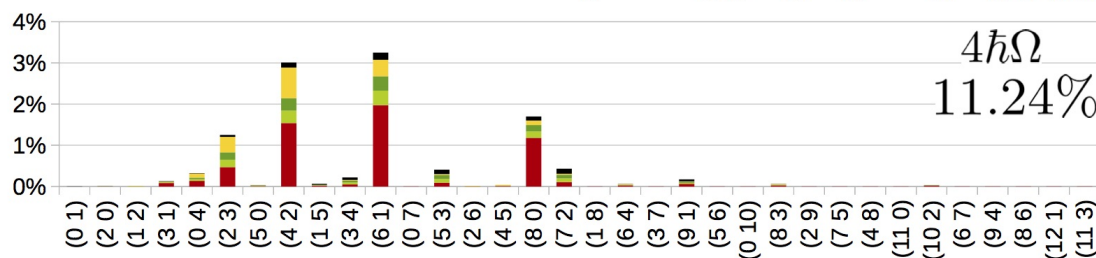
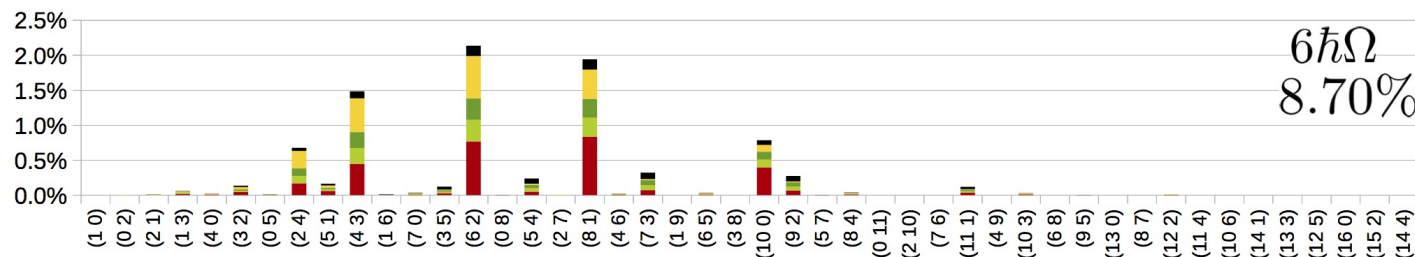
$$\lambda + 2\mu = \lambda_0 + 2\mu_0 + N$$

↑

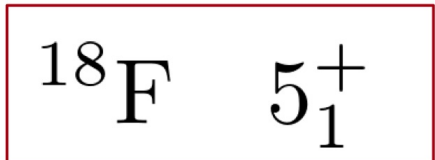
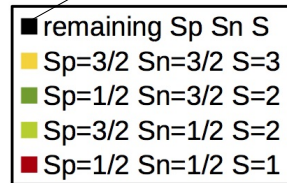
$$(\lambda_0 \mu_0) = (4 0)$$

Emergence of Simple Patterns

$N_{\max} = 6$
N2LO_{opt} + V_{coul}
 $\hbar\Omega = 20$ MeV

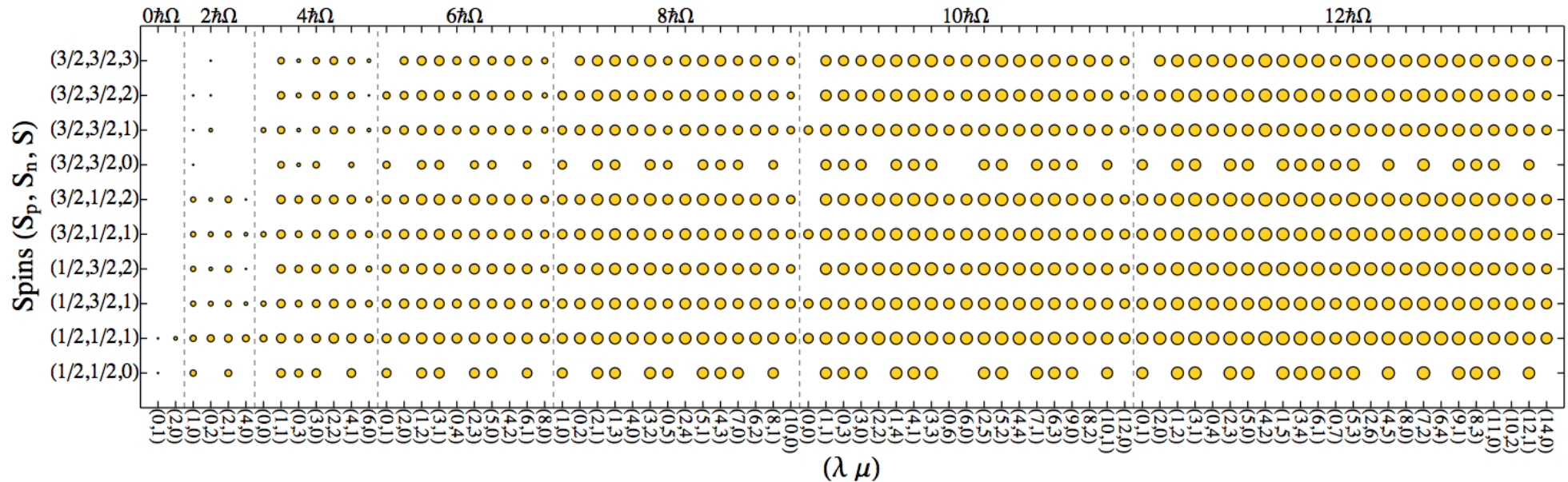


remaining $S_p S_n S$



NCSM model space in SU(3) scheme Basis

$${}^6\text{Li} : N_{\text{max}} = 12$$



Multi-shell SU(3) scheme basis enables truncations according to:

- (1) maximal number of total HO quanta N_{max}
- (2) intrinsic spins
- (3) shapes

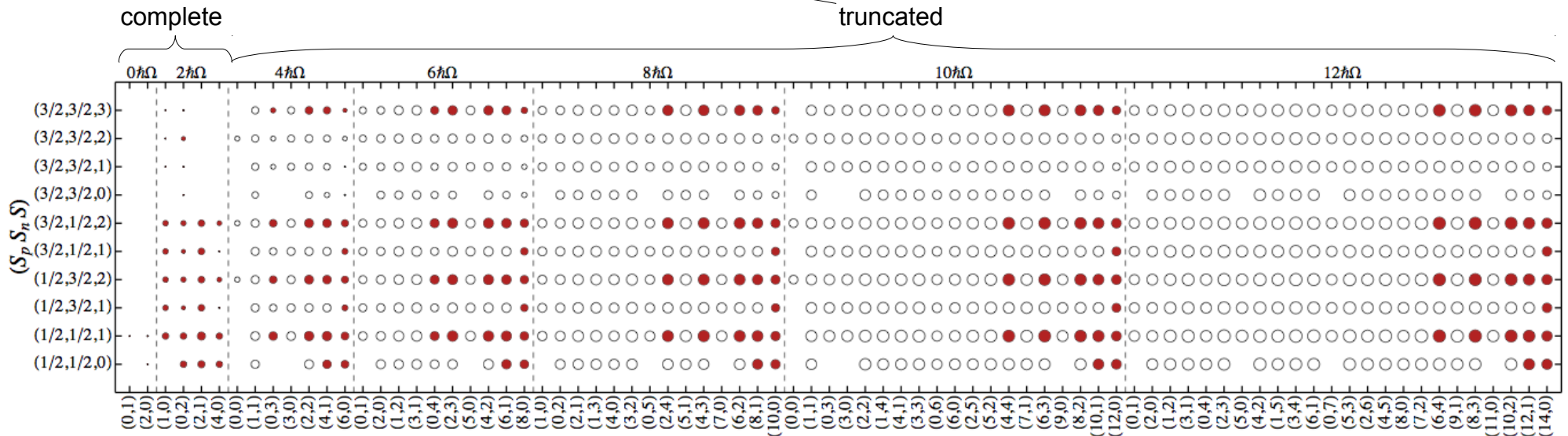
c.m. spurious states can be removed from each subspace of equivalent shapes and spins exactly

Symmetry-Guided Selection of Model Space

SU(3) and spin symmetry-guided truncation

- $\langle N'_{\max} \rangle 12$ complete space up to N'_{\max} and truncated beyond up to $N_{\max} = 12$

Example: $\langle 2 \rangle 12$



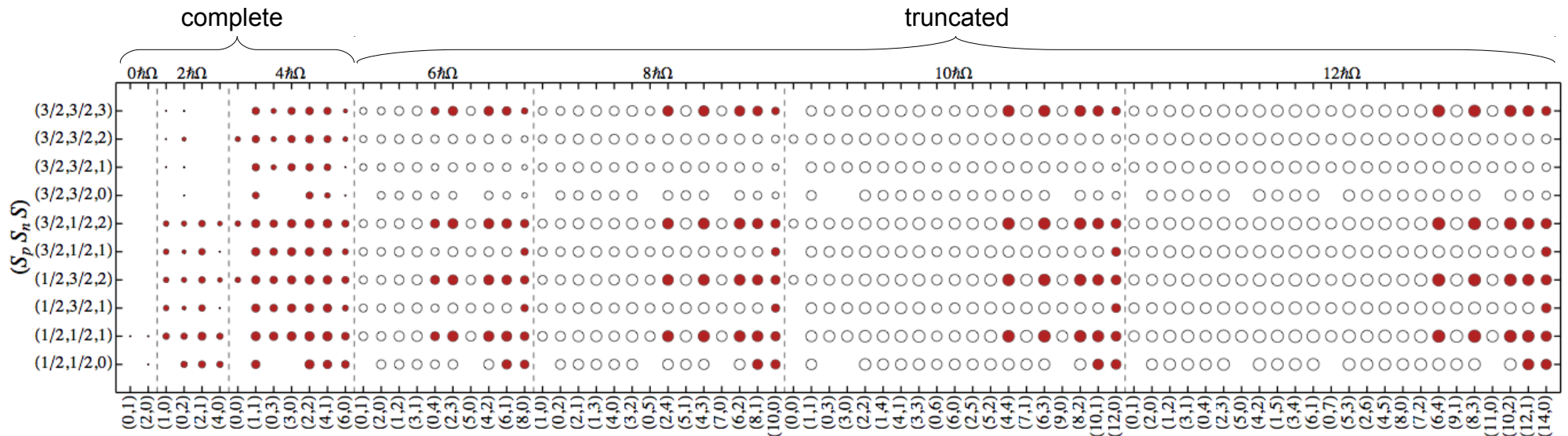
- Study convergence of SA-NCSM solutions for $\langle N'_{\max} \rangle \rightarrow 12$

Symmetry-Guided Selection of Model Space

SU(3) and spin symmetry-guided truncation

- $\langle N'_{\max} \rangle 12$ complete space up to N'_{\max} and truncated beyond up to $N_{\max} = 12$

Example: $\langle 4 \rangle 12$



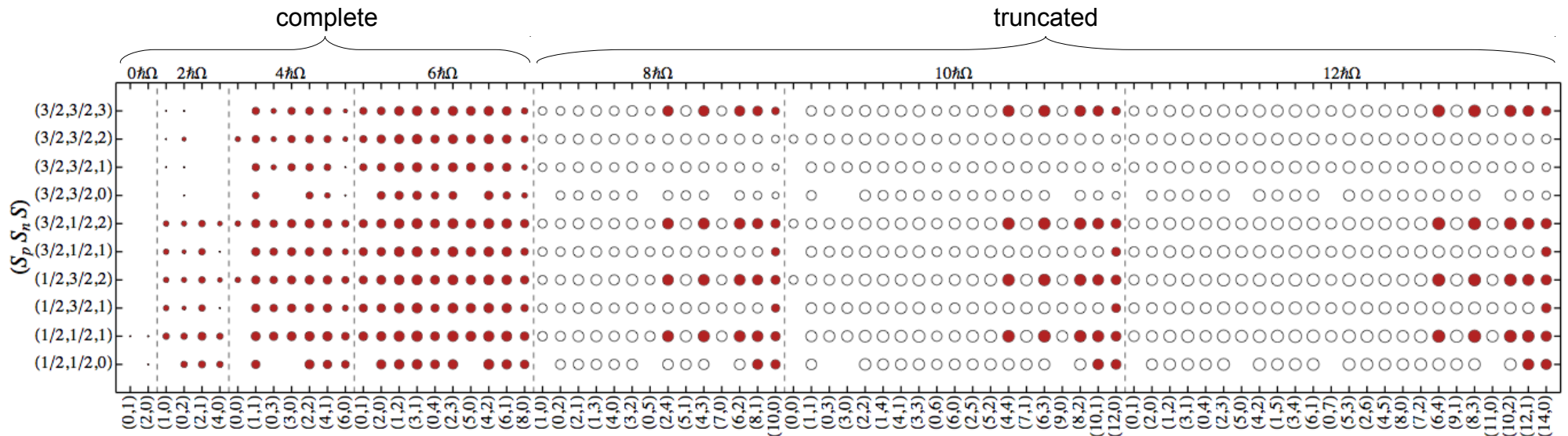
- Study convergence of SA-NCSM solutions for $\langle N'_{\max} \rangle \rightarrow 12$

Symmetry-Guided Selection of Model Space

SU(3) and spin symmetry-guided truncation

- $\langle N'_{\max} \rangle 12$ complete space up to N'_{\max} and truncated beyond up to $N_{\max} = 12$

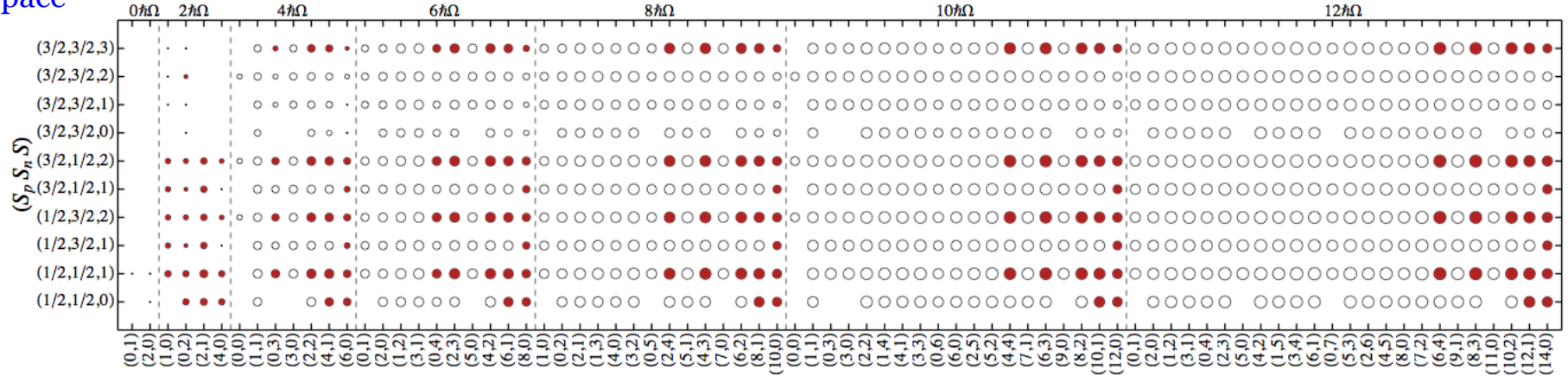
Example: $\langle 6 \rangle 12$



- Study convergence of SA-NCSM solutions for $\langle N'_{\max} \rangle \rightarrow 12$

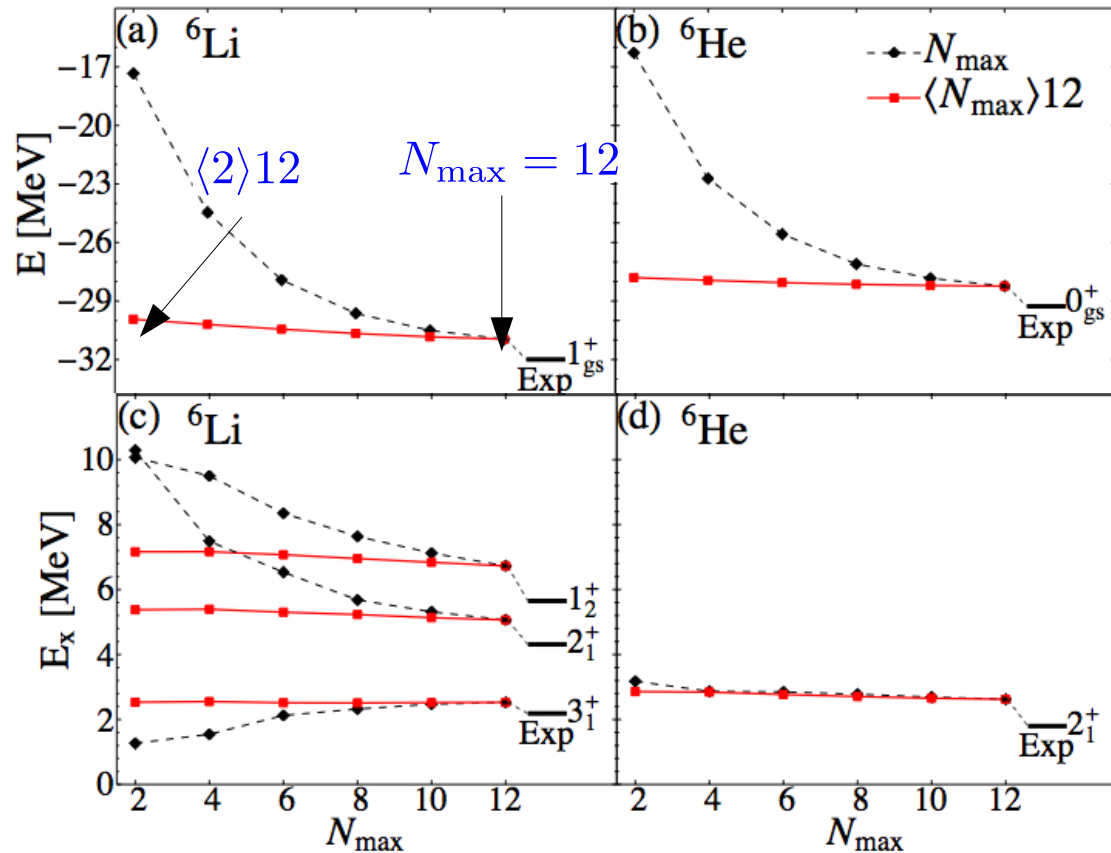
Symmetry-Guided Selection of Model Space

Model space
 $\langle 2 \rangle_{12}$



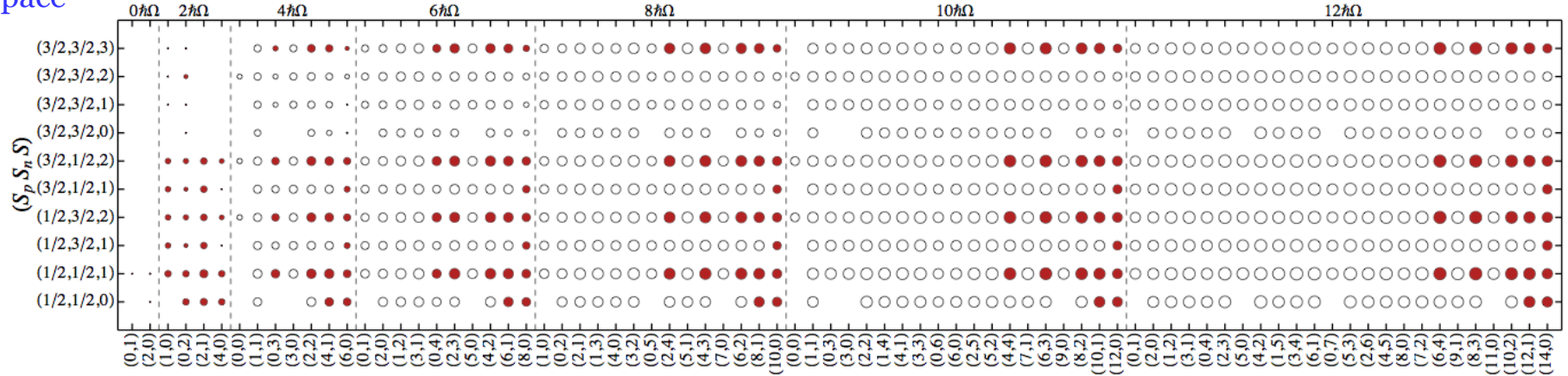
Interaction: JISP16+Vcoul

Selected shapes & spins crucial for binding and excitation energies

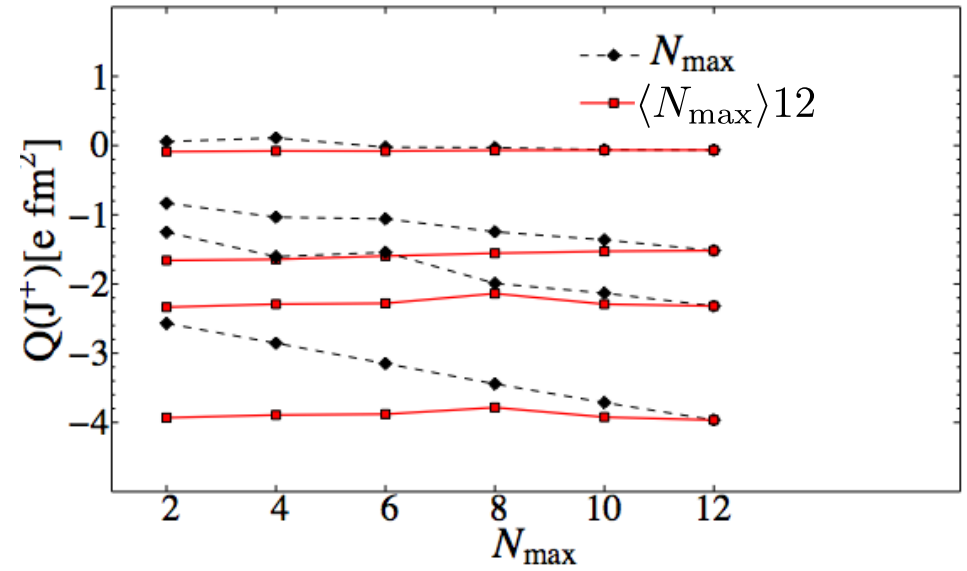
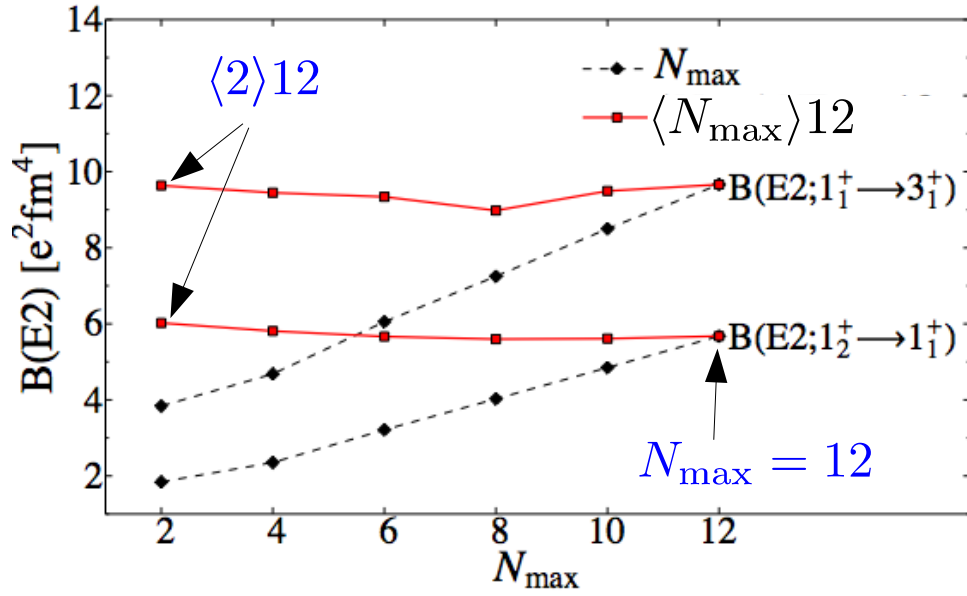


Symmetry-Guided Selection of Model Space

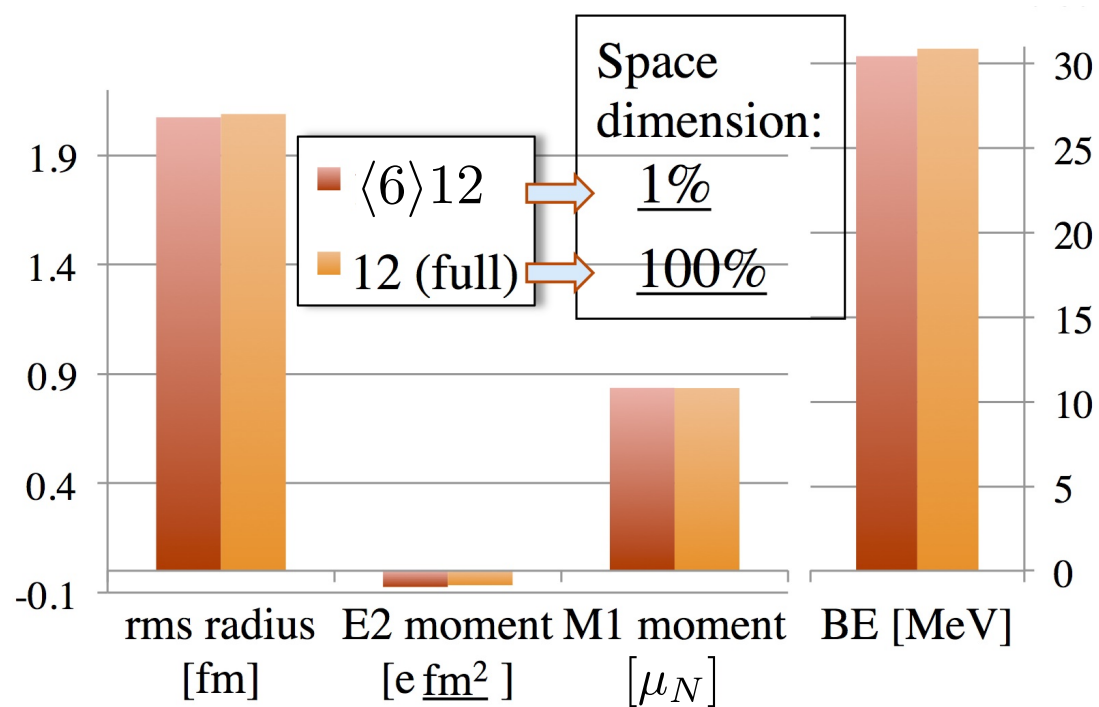
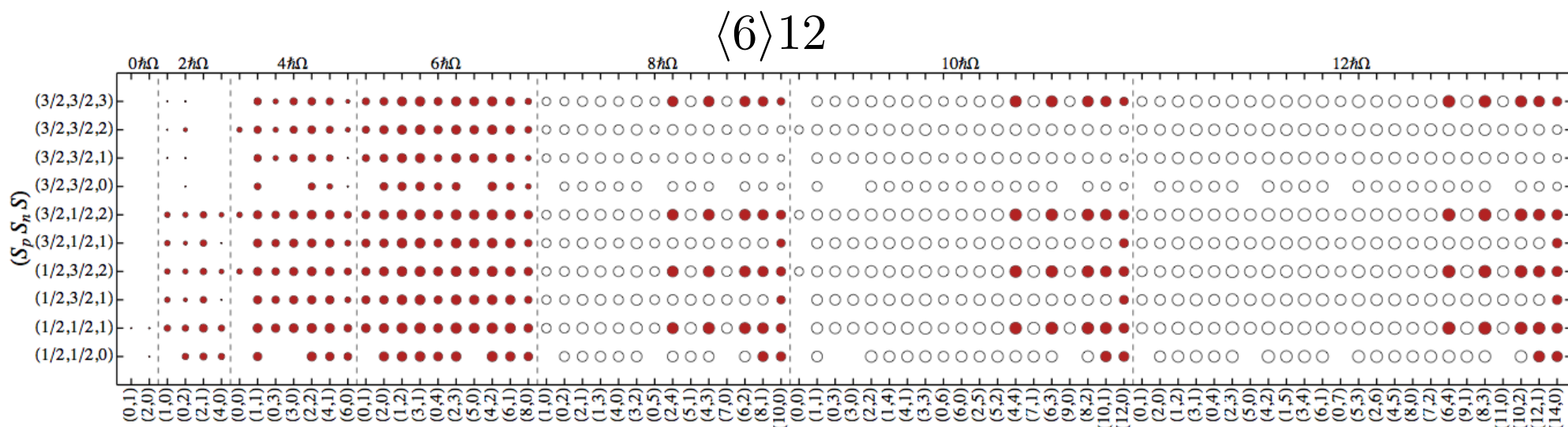
Model space
 $\langle 2 \rangle_{12}$



Selected SU(3) & spins are crucial for E2 transitions and quadrupole moments



Symmetry-Guided Selection of Model Space

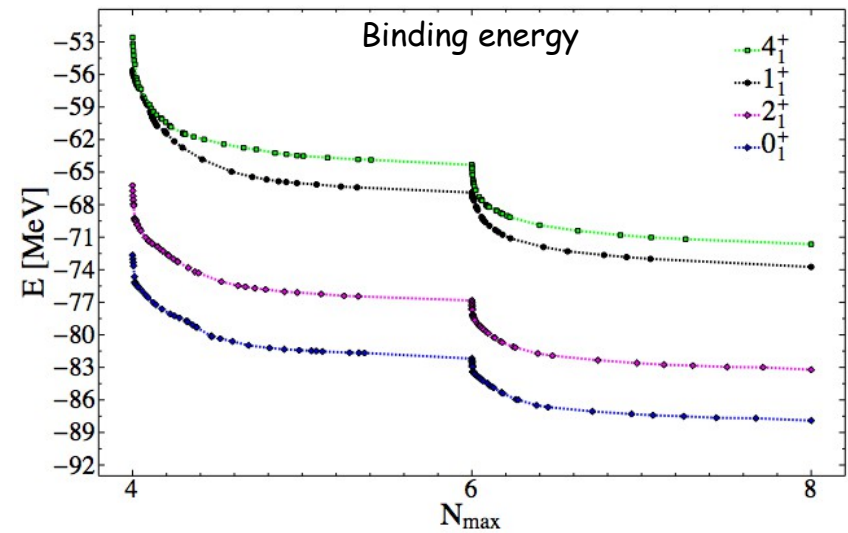
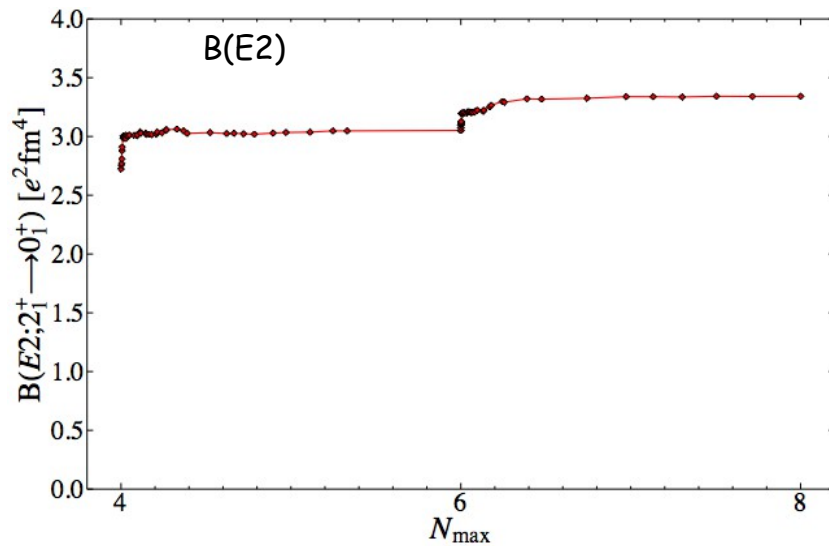


Testing symmetry-guided selection in ^{12}C

- Selecting SU(3)xSU(2) subspaces:

$$N_{\max} \quad \longrightarrow \quad N_{\max} + 2$$
$$(\lambda \mu) S_p S_n S \quad \longrightarrow \quad (\lambda + 2 \mu) S_p S_n S$$

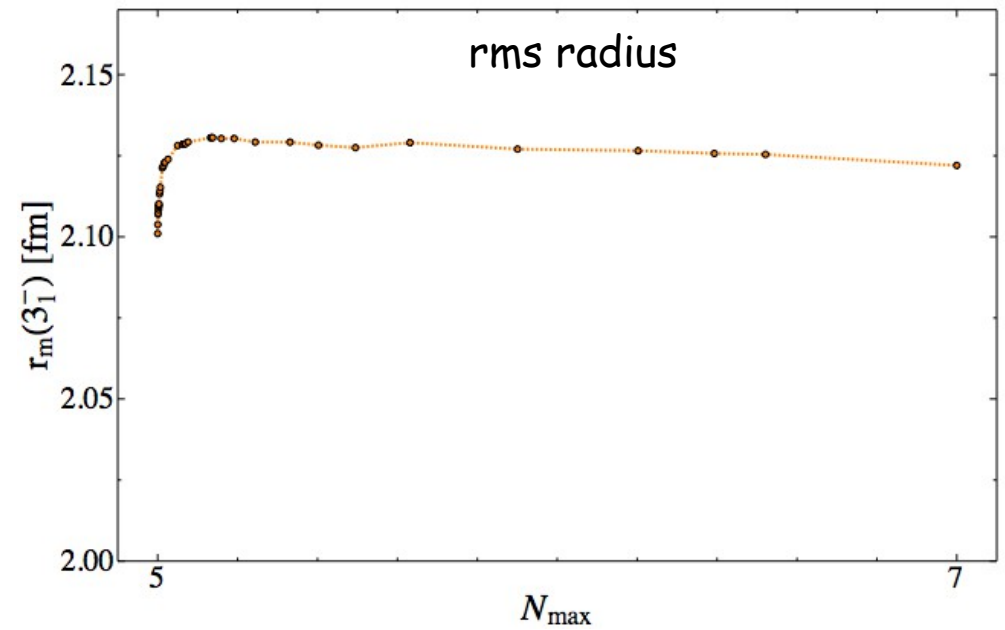
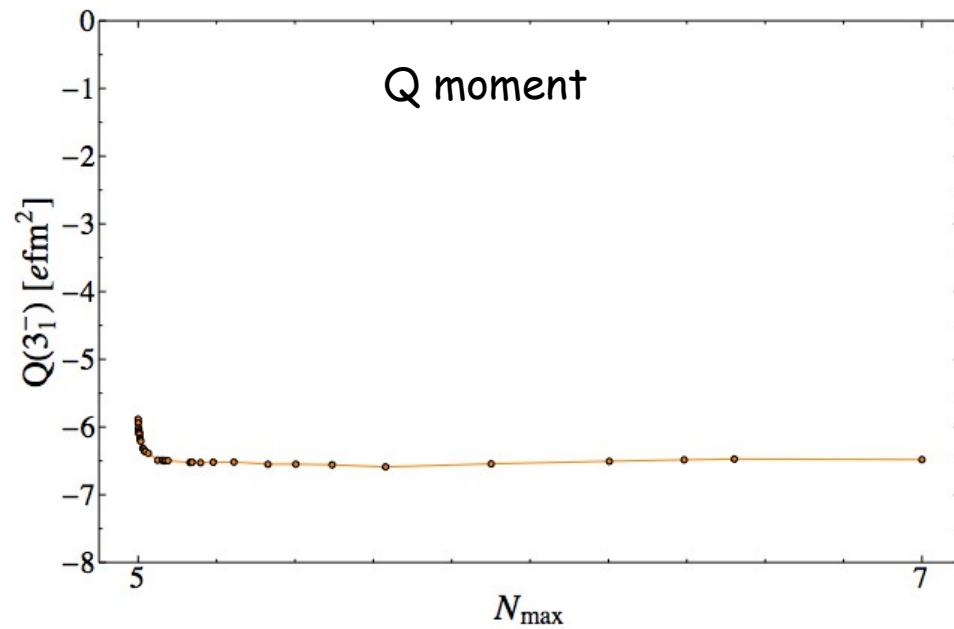
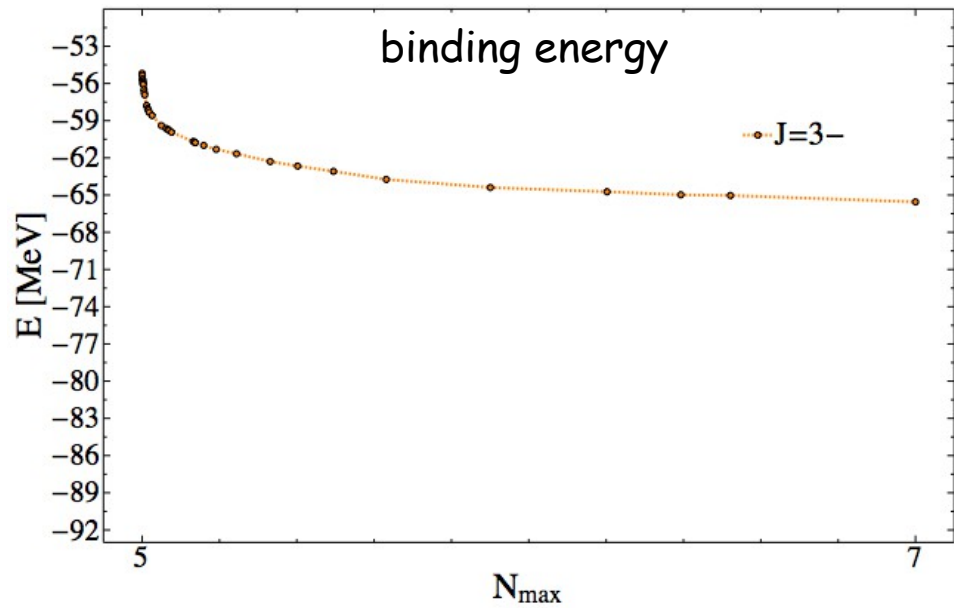
- Study convergence pattern of <4>6 and <6>8 model spaces: 0_1^+ , 1_1^+ , 2_1^+ , 4_1^+



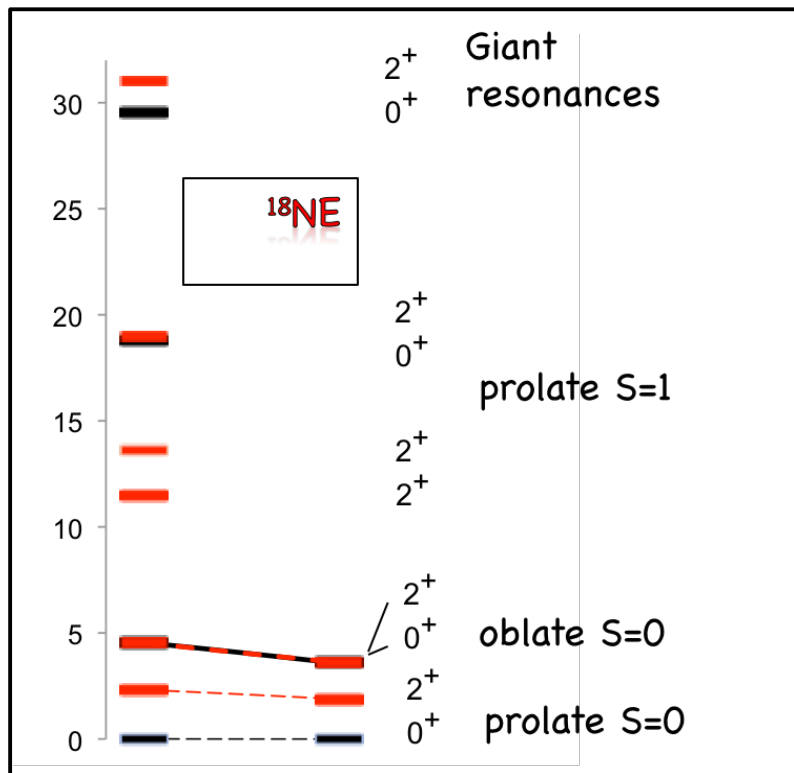
- Similar step-like pattern for quadrupole moments and rms radii

Testing symmetry-guided selection in ^{12}C

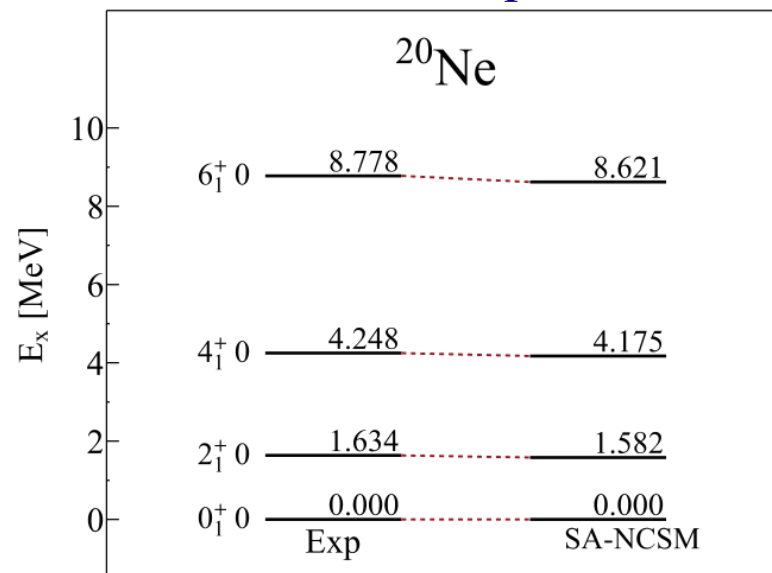
■ Study convergence pattern of <5>7 model spaces: 3_1^-



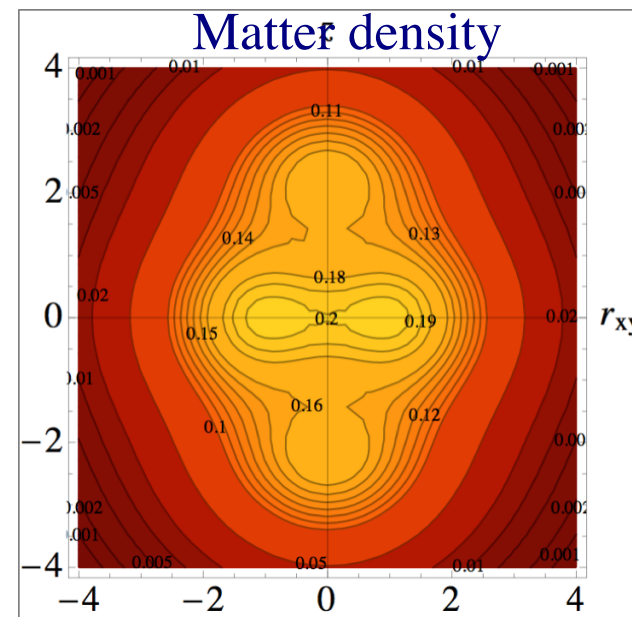
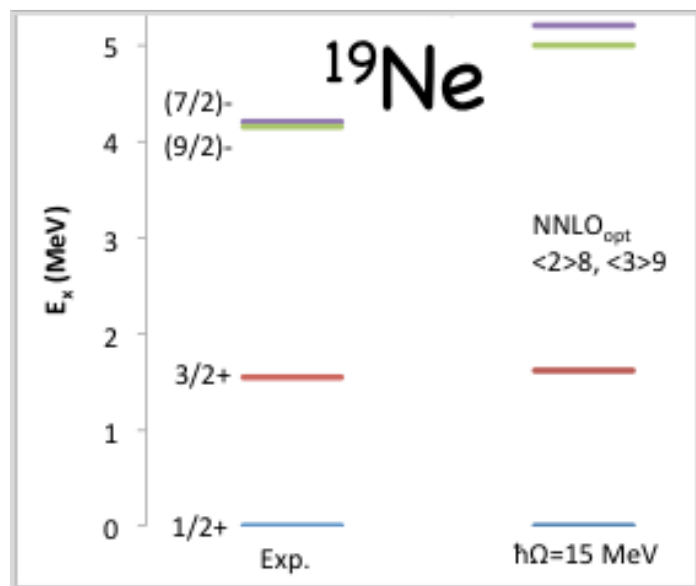
SA-NCSM: reaching towards medium mass nuclei



Excitation Spectrum



SA-NCSM: 50 million states
Complete space: 4000 billion states



Symmetry of microscopic nuclear collective motion

- Observed simple patterns of shapes and intrinsic spins -- signature of higher symmetry?

Symplectic model

$$Sp(3, \mathbb{R}) \supset SU(3)$$

- Microscopic realization and generalization of Bohr-Mottelson model

$Sp(3, \mathbb{R})$ symmetry preserving operators

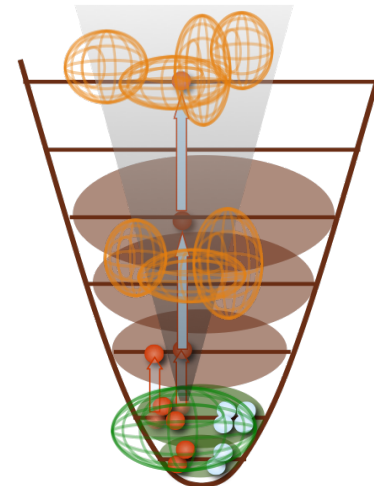
Kinetic energy
Harmonic oscillator potential
Orbital angular momentum
Monopole & Quadrupole momentum

- Symplectic states - nuclear collective modes in shell model

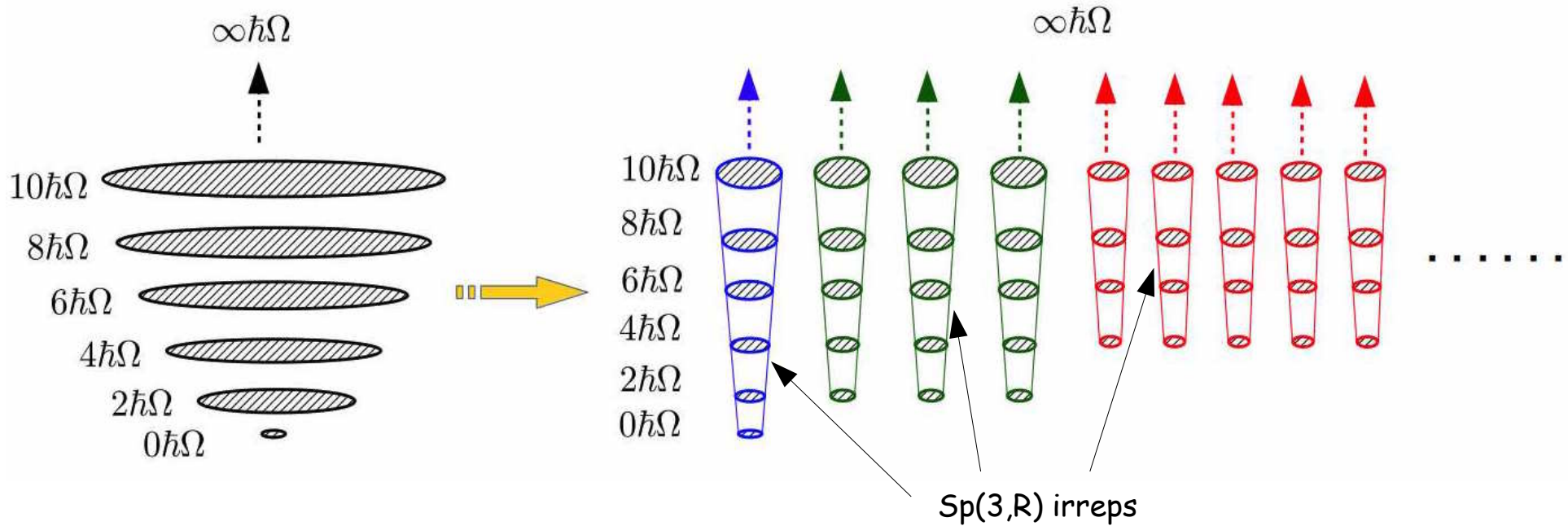
Quadrupole and monopole vibrations and deformations

Rotational dynamics: rigid rotor to irrotational flow

Clusterization



Structure of NCSM model space in $Sp(3,R)$ basis



■ $Sp(3,R)$ basis states

- eigenstates of harmonic oscillator
- free of center-of mass spurious excitations
- can be efficiently expanded in multi-shell $SU(3)$ basis

Emerging symplectic symmetry

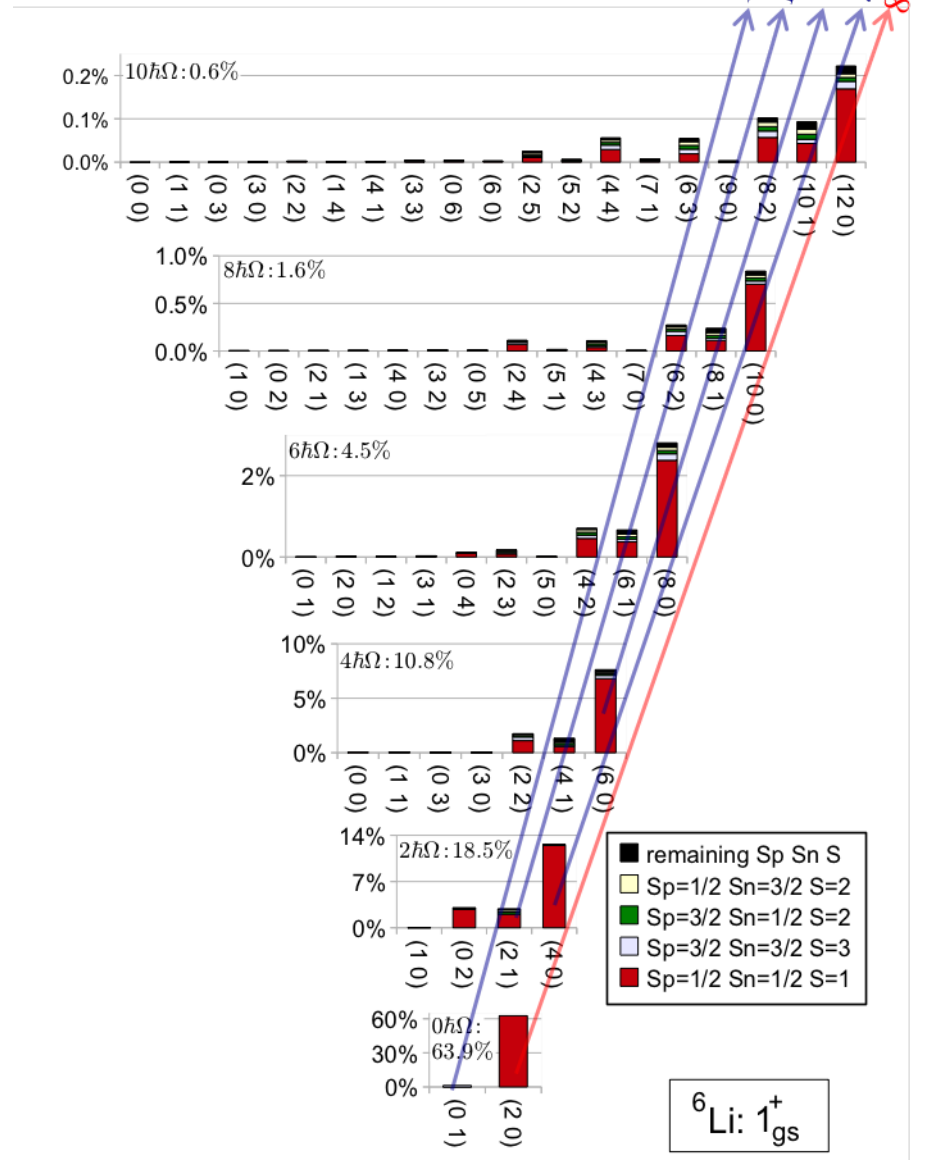
■ ${}^6\text{Li}$ -- Generated complete $N_{\text{max}}=12$ model space in $\text{Sp}(3, \mathbb{R})$ basis

■ ${}^{20}\text{Ne}$ -- Generated $\langle 4 \rangle 8$ model space in $\text{Sp}(3, \mathbb{R})$ basis

■ wave functions obtained with SA-NCSM using
(a) JISP16 (b) N2LOopt (c) N3LO interactions

1.4%
1.2%
2.0%
2.9%
79.4%
N3LO

JISP16



Emerging symplectic symmetry

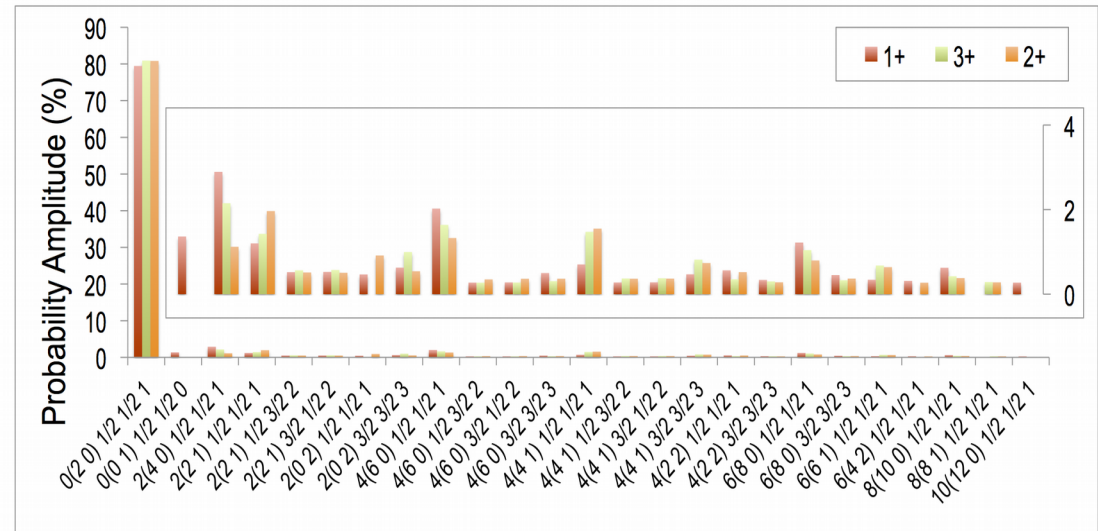
${}^6\text{Li}$

$1_1^+, 1_2^+, 2_1^+, 3_1^+, 3_2^+$

$1_3^+, \dots, 1_{10}^+$

$2_2^+, \dots, 2_{10}^+$

$3_3^+, \dots, 3_{10}^+$

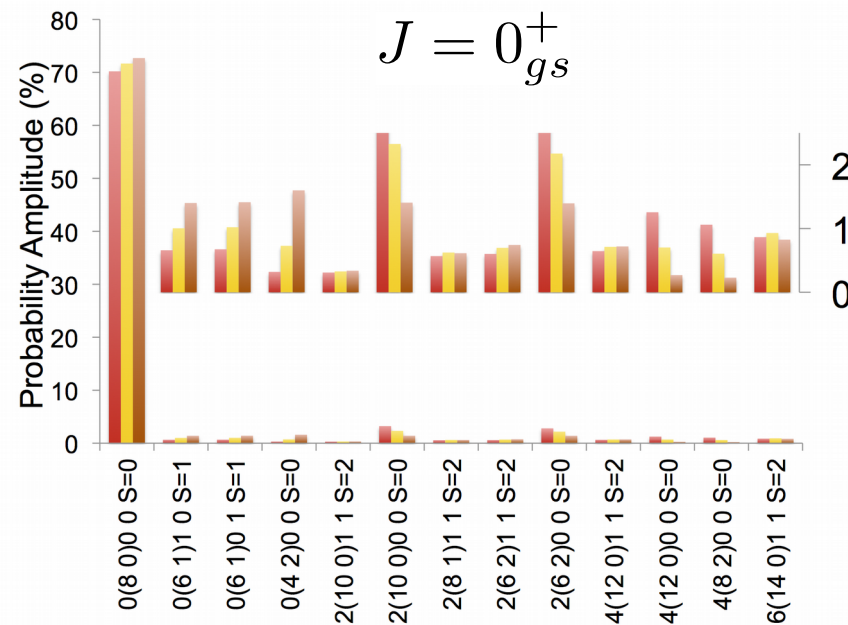


${}^{20}\text{Ne}$

■ N3LO @ 12MeV, 15 MeV, 20 MeV

$0_{gs}^+, 2_1^+, 4_1^+, 6_1^+$

$0_2^+, \dots, 0_{20}^+$



■ wave functions typically dominated (60%-70%) by one or two symplectic irreps

■ 10%-15% due to additional ~20 symplectic irreps.

Summary

- Simple patterns of shapes emerge from first principles studies of p- and sd-shell nuclei
- Provide physically relevant model spaces for ab initio modeling of nuclear structure
- Symplectic symmetry of nuclear collective motion emerges in ab initio results
- All codes publicly available <https://sourceforge.net/p/lisu3shell/>

Collaborators

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