Effective field theories for nuclear deformations and vibrations

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Shapes and Symmetries in Nuclei: from Experiment to Theory

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SciDAC Scientific Discovery through Advanced Computing

NUCLEI Nuclear Computational Low-Energy Initiative

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Menu

What's cooking in nuclear theory?

- Ideas from EFT and the renormalization group
- Ever-increasing computer power*

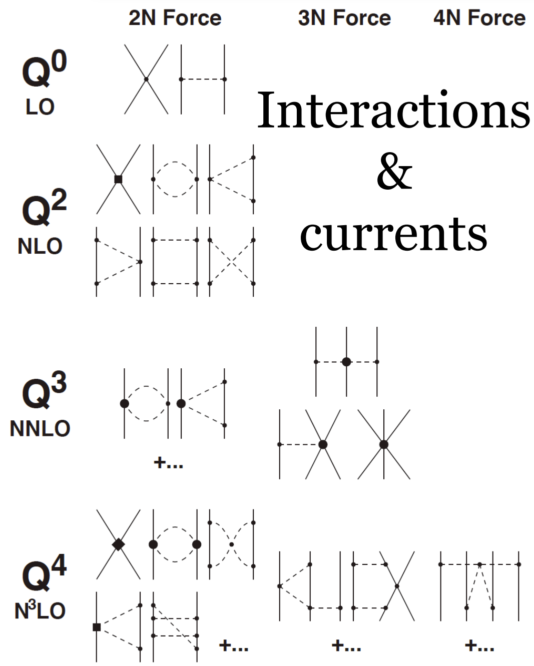
Today's special

1. Effective field theory (EFT) for nuclear vibrations
2. EFT for deformed nuclei

*Actually not true: ceiling is at about 10 MW✣

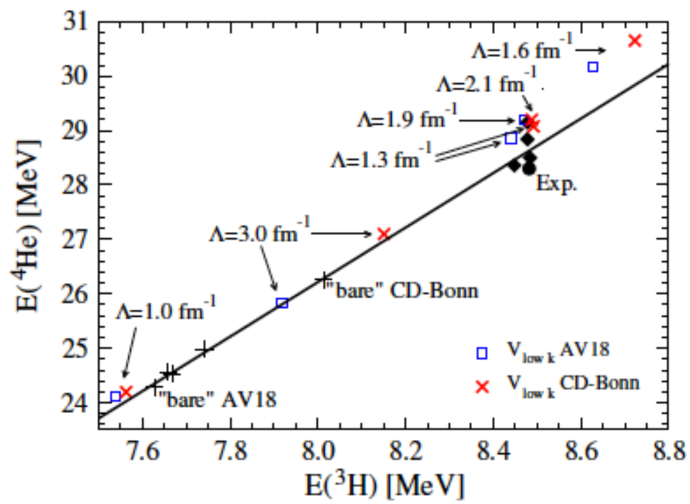
✣Would buy about 10^{27} bit erasures per second at room temperature (Landauer)

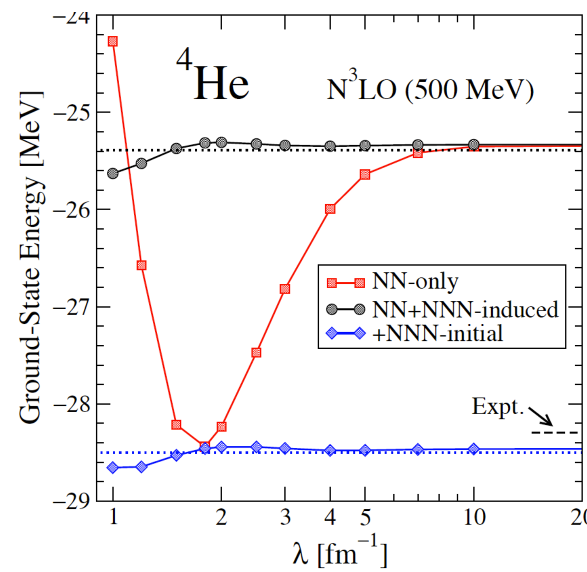
EFTs and RGs as tools



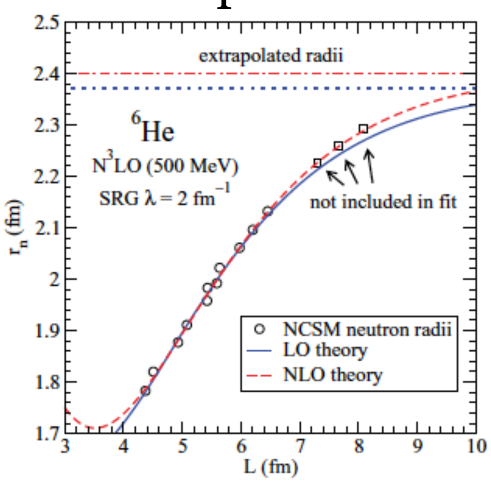
Interactions & currents

Scale and scheme dependences

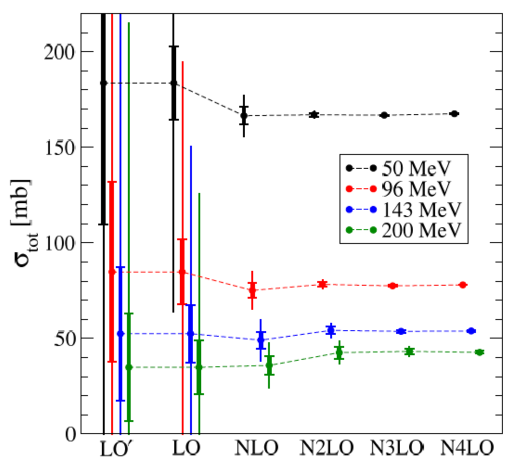


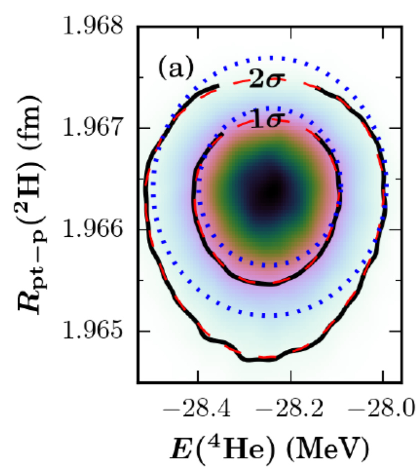


Extrapolations

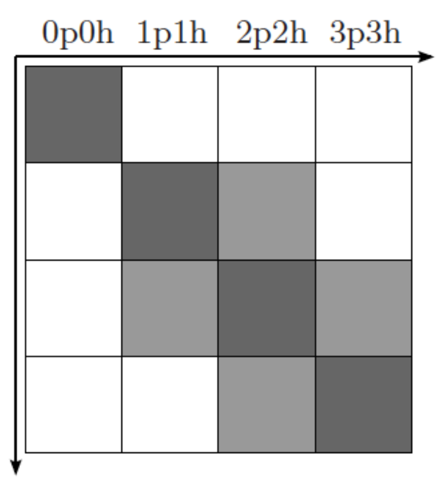


Uncertainty quantification





New methods



Energy scales and relevant degrees of freedom

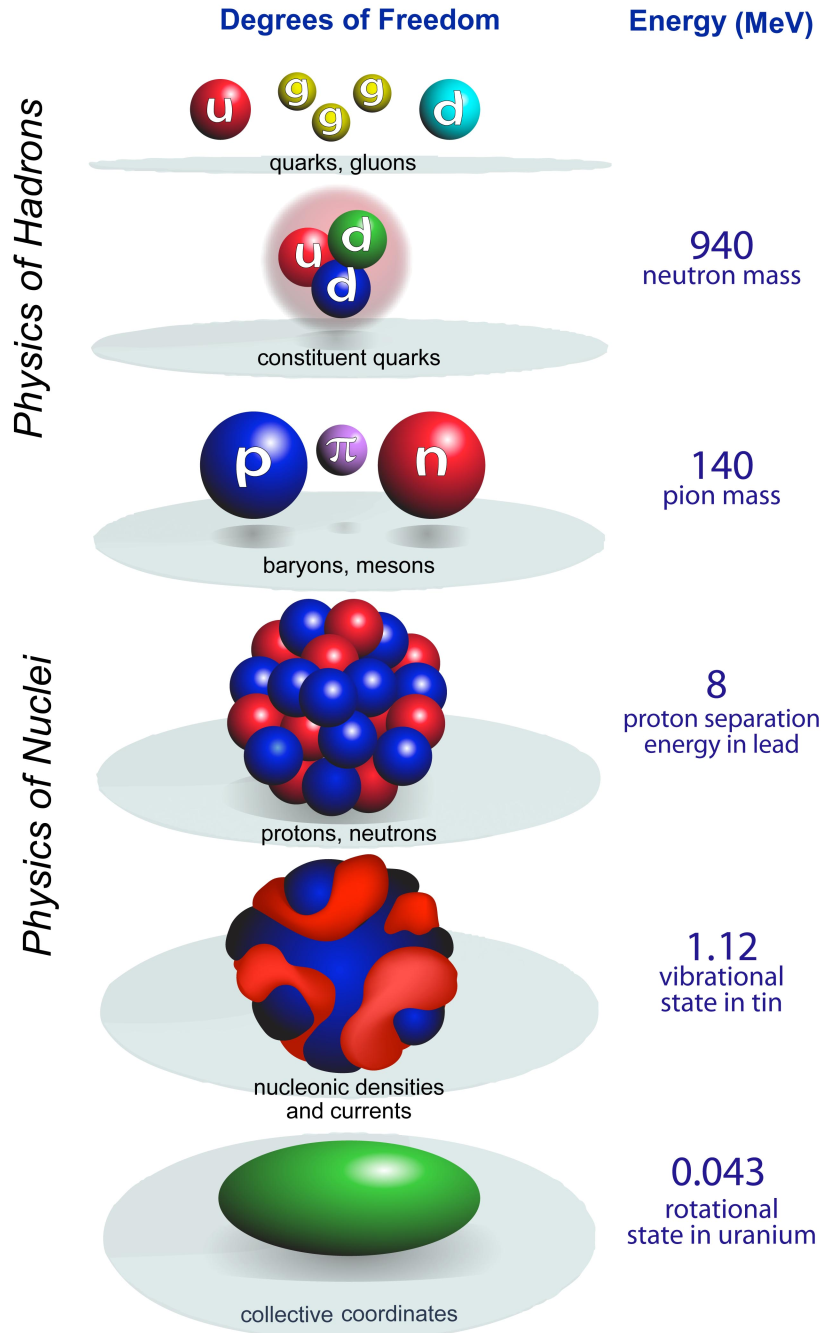


Fig.: Bertsch, Dean, Nazarewicz (2007)

Chiral EFT

Pion-less EFT

EFT for nuclear vibrations

EFT for deformed nuclei

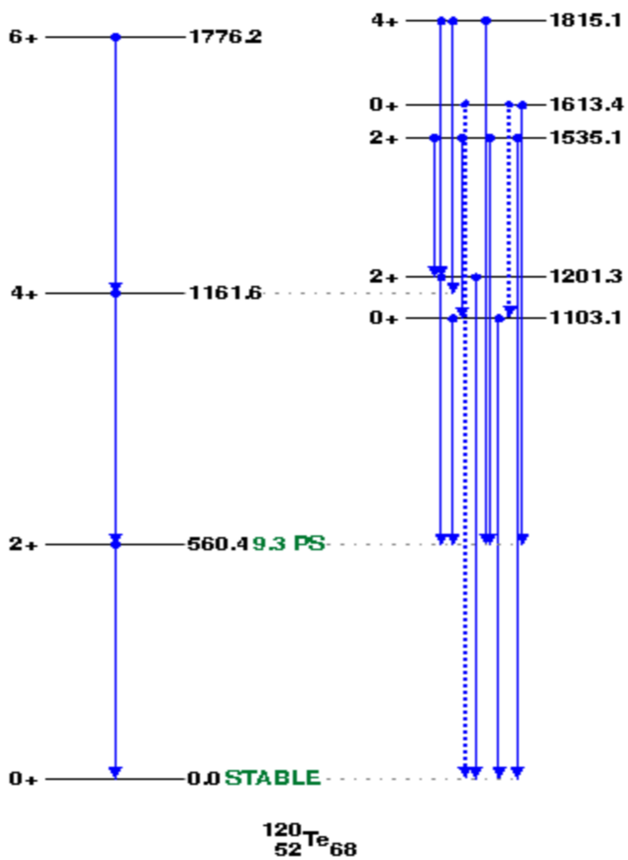
(not pictured: halo EFT)

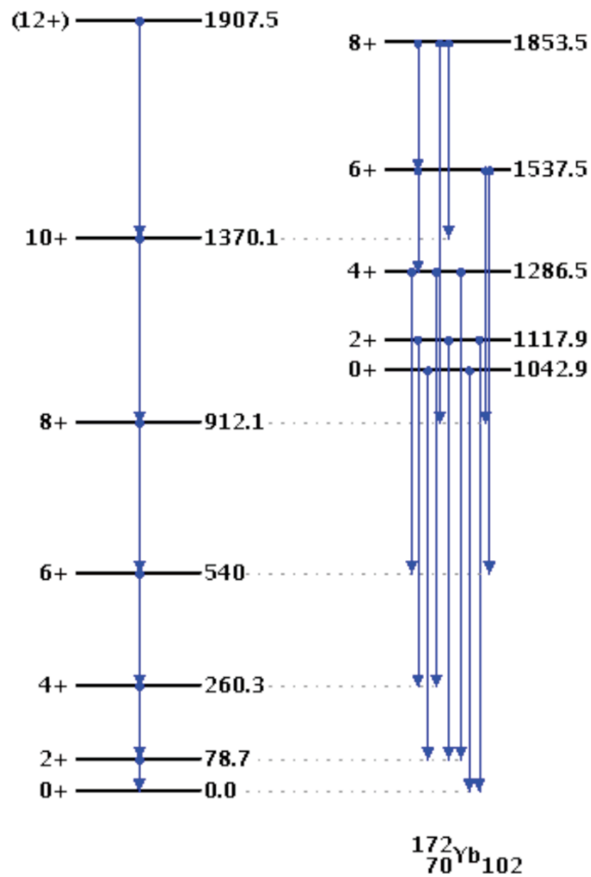
Key ingredients of an EFT

1. Identify symmetries and pattern of symmetry breaking
2. Identify relevant low-energy degrees of freedom
3. Identify the breakdown scale; develop power counting
4. Expand Hamiltonians and currents according to power counting
5. Adjust low-energy coefficients to data; make predictions; estimate/quantify uncertainties

Two paradigms: vibrations and rotations

Quadrupole degrees of freedom describe spins and parity of low-energy spectra

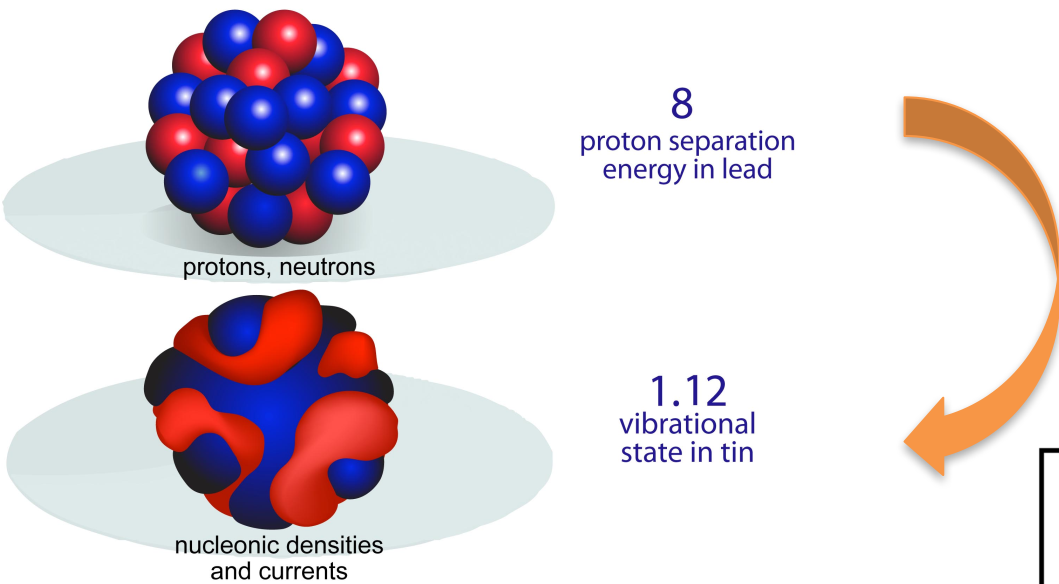




Nuclear vibration: EFT based on linear realization (Wigner / Weyl) of SO(3)

Nuclear rotation: emergent breaking of rotational symmetry of SO(3) → SO(2) for axial symmetry or SO(3) → 1 for triaxial nuclei; EFT based on nonlinear realization (Nambu-Goldstone) of SO(3)

EFT for nuclear vibrations

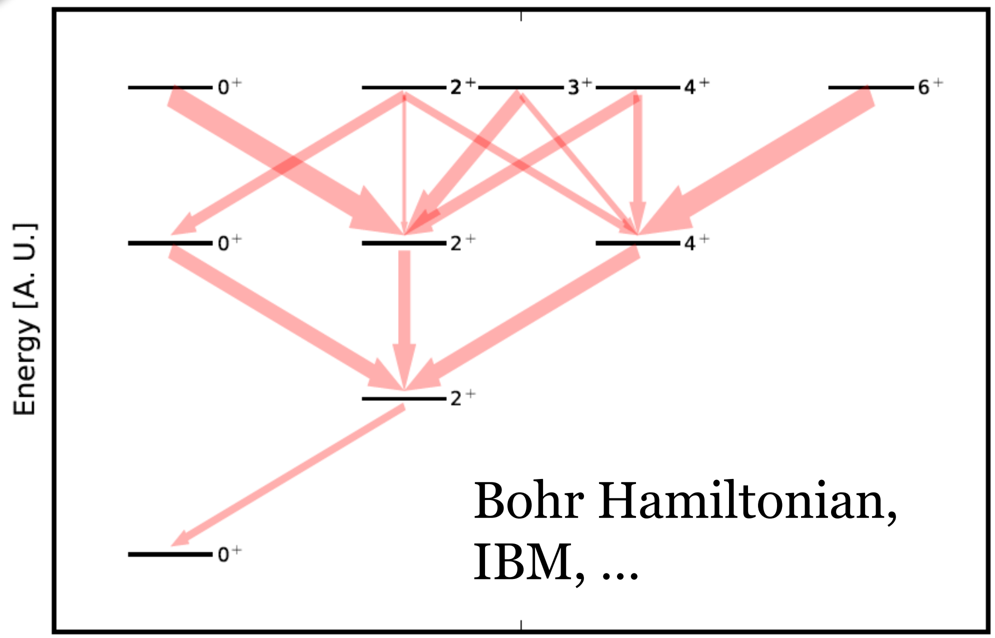


EFT for nuclear vibrations

[Coello Pérez & TP 2015, 2016]

Challenge: While spectra of certain nuclei appear to be harmonic, B(E2) transitions do not.

Garrett & Wood (2010): "Where are the quadrupole vibrations in atomic nuclei?"

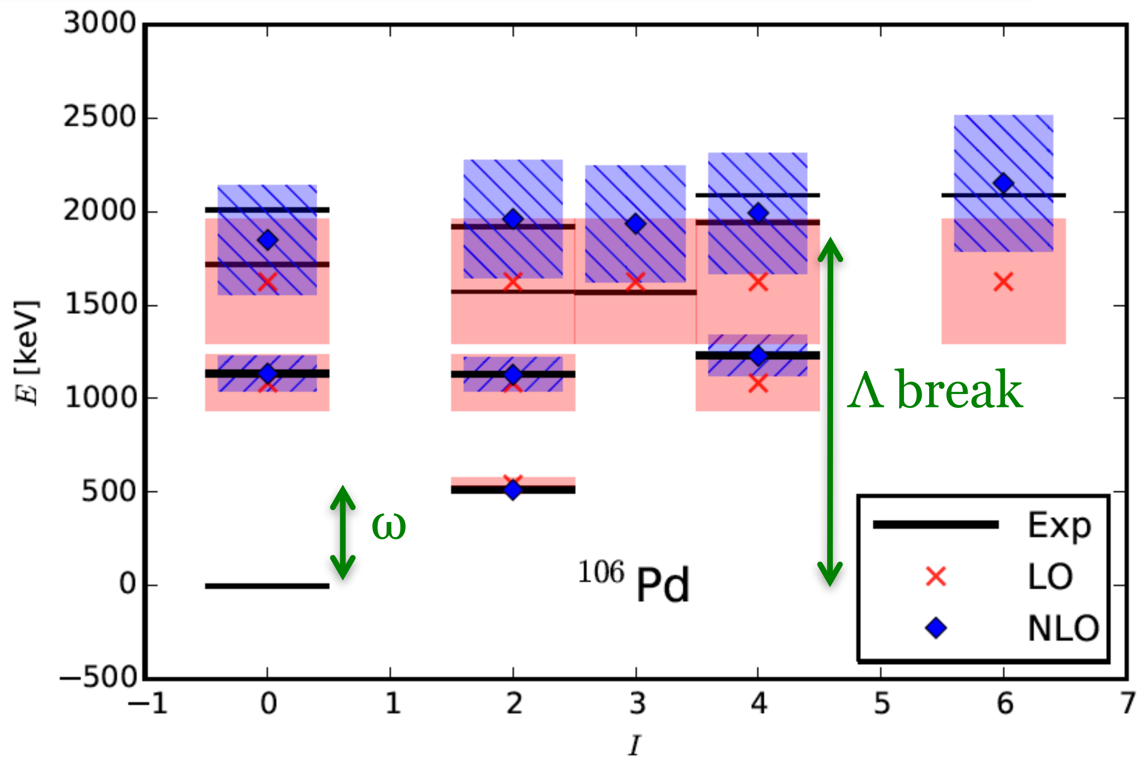


Spectrum and B(E2) transitions of the *harmonic* quadrupole oscillator

EFT for nuclear vibrations

EFT ingredients:

1. quadrupole degrees of freedom
2. breakdown scale around three-phonon levels
3. "small" expansion parameter: ratio of vibrational energy to breakdown scale: $\omega/\Lambda \approx 1/3$



- Uncertainties show 68% DOB intervals from truncating higher EFT orders [Cacciari & Houdeau (2011); Bagnaschi et al (2015); Furnstahl, Klco, Phillips & Wesolowski (2015)]
  - Expand observables according to power counting
  - Employ "naturalness" assumptions as log-normal priors in Bayes' theorem
  - Compute distribution function of uncertainties due to EFT truncation
  - Compute degree-of-believe (DOB) intervals.

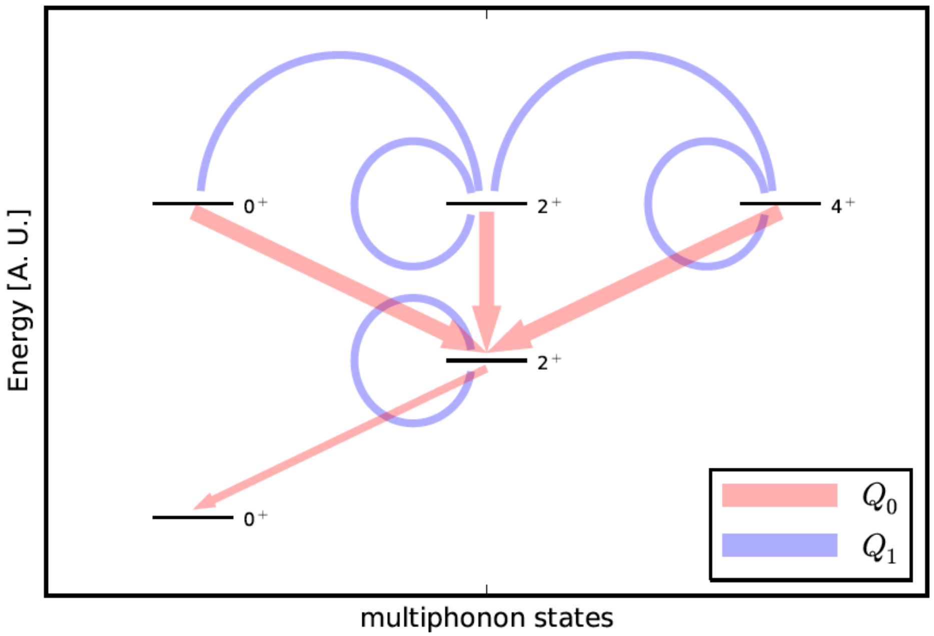
EFT result: sizeable quadrupole matrix elements are natural in size

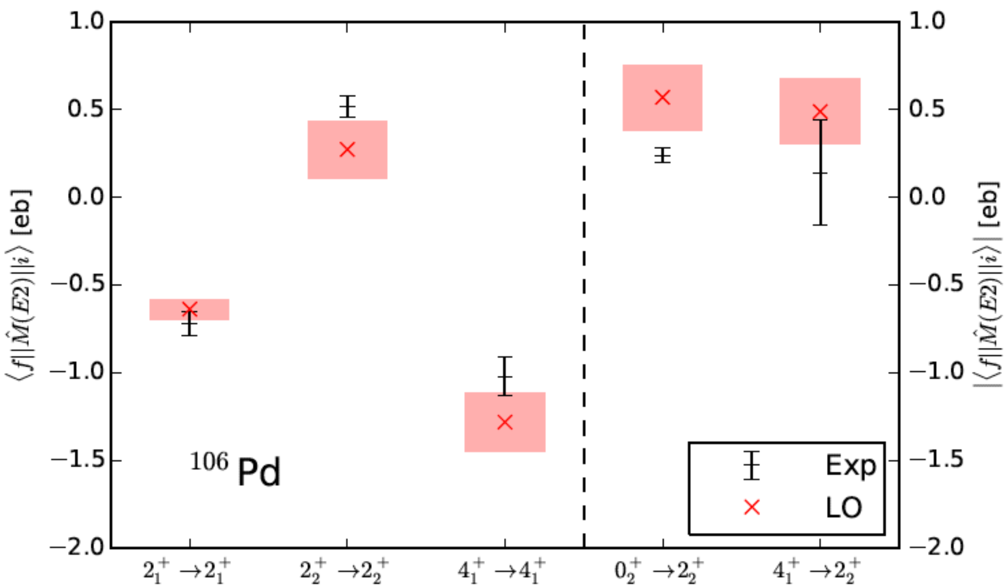
In the EFT, the quadrupole operator is also expanded:

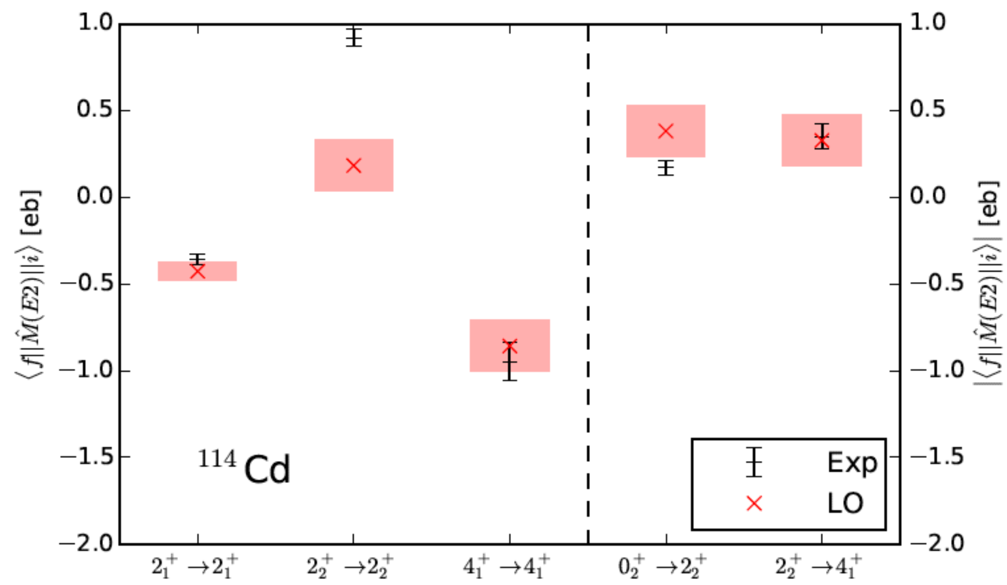
$$\hat{Q}_\mu = Q_0\left(d^\dagger_\mu + \tilde{d}_\mu\right) + Q_1\left(d^\dagger \times d^\dagger + \tilde{d}\times\tilde{d} + 2d^\dagger\times\tilde{d}\right)^{(2)}_\mu$$

Subleading corrections are sizable:

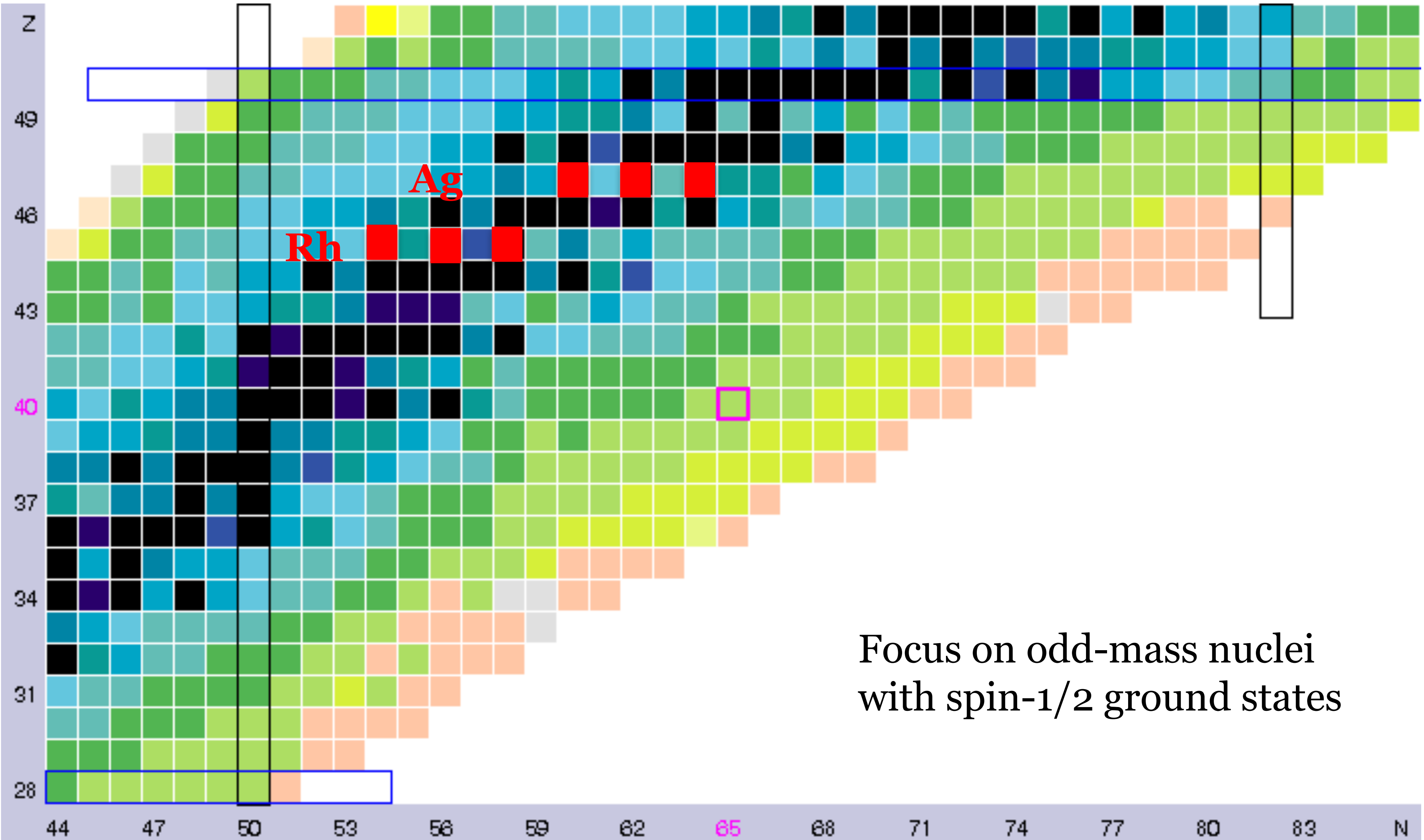
$$Q_1 \sim \left(\frac{\omega}{\Lambda}\right)^{1/2} Q_0$$







Rhodium as a proton coupled to ruthenium
Silver as a proton (hole) coupled to palladium (cadmium)

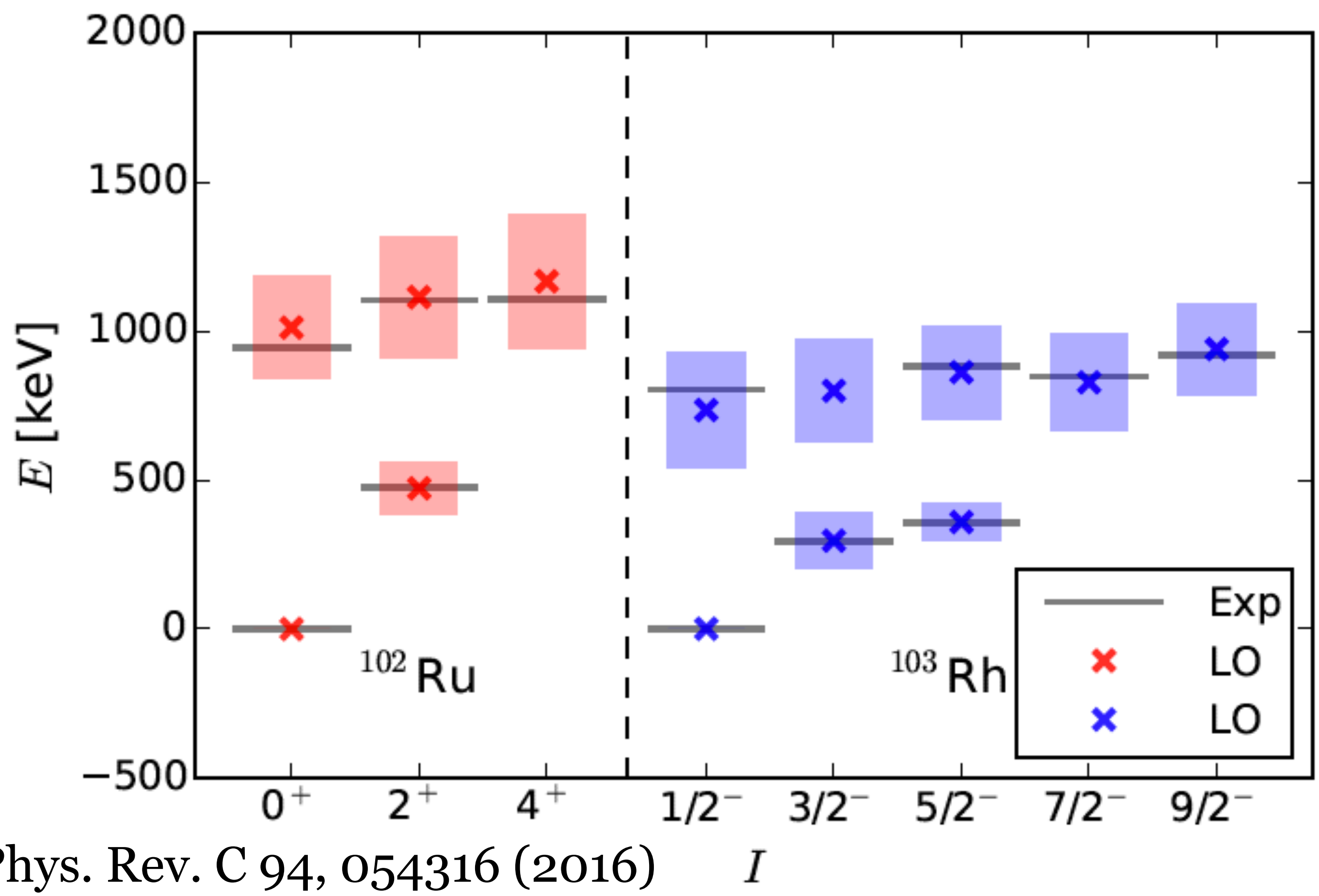


Focus on odd-mass nuclei
with spin-1/2 ground states

Fermion coupled to vibrating nucleus

Approach follows halo EFT [Bertulani, Hammer, van Kolck (2002); Higa, Hammer, van Kolck (2008); Hammer & Phillips (2011); Ryberg et al. (2014)], and particle-vibrator models [de Shalit (1961); Iachello & Scholten (1981); Vervier (1982);…]

Two new LECs enter at lowest interesting order



E. A. Coello Pérez & TP, Phys. Rev. C 94, 054316 (2016)

Static E2 moments (in eb)

Nucleus	I_i^π	$Q_{\rm exp}$	$Q_{\rm EFT}$
^{102}Ru	2_1^+	$-0.63(3)$	$-0.41(6)$
	2_2^+		$0.18(18)$
	4_1^+		$-0.82(14)$
^{103}Rh	$\frac{3}{2}{}_1^-$	$-0.3(2)$	$-0.29(7)$
	$\frac{5}{2}{}_1^-$	$-0.4(2)$	$-0.41(6)$
^{106}Pd	2_1^+	$-0.54(4)$	$-0.50(7)$
	2_2^+	$0.39(6)$	$0.21(20)$
	4_1^+	$-0.79(11)$	$-1.00(17)$
^{107}Ag	$\frac{3}{2}{}_1^-$		$-0.35(8)$
	$\frac{5}{2}{}_1^-$		$-0.50(7)$

Nucleus	I_i^π	$Q_{\rm exp}$	$Q_{\rm EFT}$
^{108}Pd	2_1^+	$-0.56(3)$	$-0.57(7)$
	2_2^+	$0.73(9)$	$0.24(20)$
	4_1^+	$-0.78(11)$	$-1.14(17)$
^{109}Ag	$\frac{3}{2}{}_1^-$	$-0.7(3)$	$-0.40(8)$
	$\frac{5}{2}{}_1^-$	$-0.3(3)$	$-0.57(6)$
^{110}Cd	2_1^+	$-0.39(3)$	$-0.57(7)$
	2_2^+		$0.24(17)$
	4_1^+		$-1.12(14)$
^{109}Ag	$\frac{3}{2}{}_1^-$	$-0.7(3)$	$-0.39(6)$
	$\frac{5}{2}{}_1^-$	$-0.3(3)$	$-0.56(6)$

Single LEC Q_1 fit to all data with EFT weighting. $\hat{Q}_\mu = Q_0(d_\mu^\dagger + \tilde{d}_\mu) + Q_1(d^\dagger \otimes \tilde{d})_\mu^{(2)}$

E. A. Coello Pérez & TP, Phys. Rev. C 94, 054316 (2016)

Magnetic moments: Relations between even-even and even-odd nuclei

Nucleus	I_i^π	$\mu_{\mathrm{exp}}(I_i^\pi)$	$\mu_{\mathrm{EFT}}(I_i^\pi)$
^{102}Ru	2_1^+	$0.85(3)^*$	$0.85(5)$
	2_2^+		$0.85(10)$
	4_1^+		$1.70(8)$
^{103}Rh	$\frac{1}{2}_1$	-0.088^*	-0.088
	$\frac{3}{2}_1$	$0.77(7)$	$0.81(5)$
	$\frac{5}{2}_1$	$1.08(4)$	$0.76(5)$
	$\frac{7}{2}_1$	$2.0(6)$	$1.7(1)$
	$\frac{9}{2}_1$	$2.8(5)$	$1.6(1)$

Nucleus	I_i^π	$\mu_{\mathrm{exp}}(I_i^\pi)$	$\mu_{\mathrm{EFT}}(I_i^\pi)$
^{106}Pd	2_1^+	$0.79(2)^*$	$0.79(5)$
	2_2^+	$0.71(10)$	$0.79(10)$
	4_1^+	$1.8(4)$	$1.58(8)$
^{107}Ag	$\frac{1}{2}_1$	-0.11^*	-0.11
	$\frac{3}{2}_1$	$0.98(9)$	$0.78(5)$
	$\frac{5}{2}_1$	$1.02(9)$	$0.68(4)$
	$\frac{7}{2}_1$		$1.6(1)$
	$\frac{9}{2}_1$		$1.5(1)$

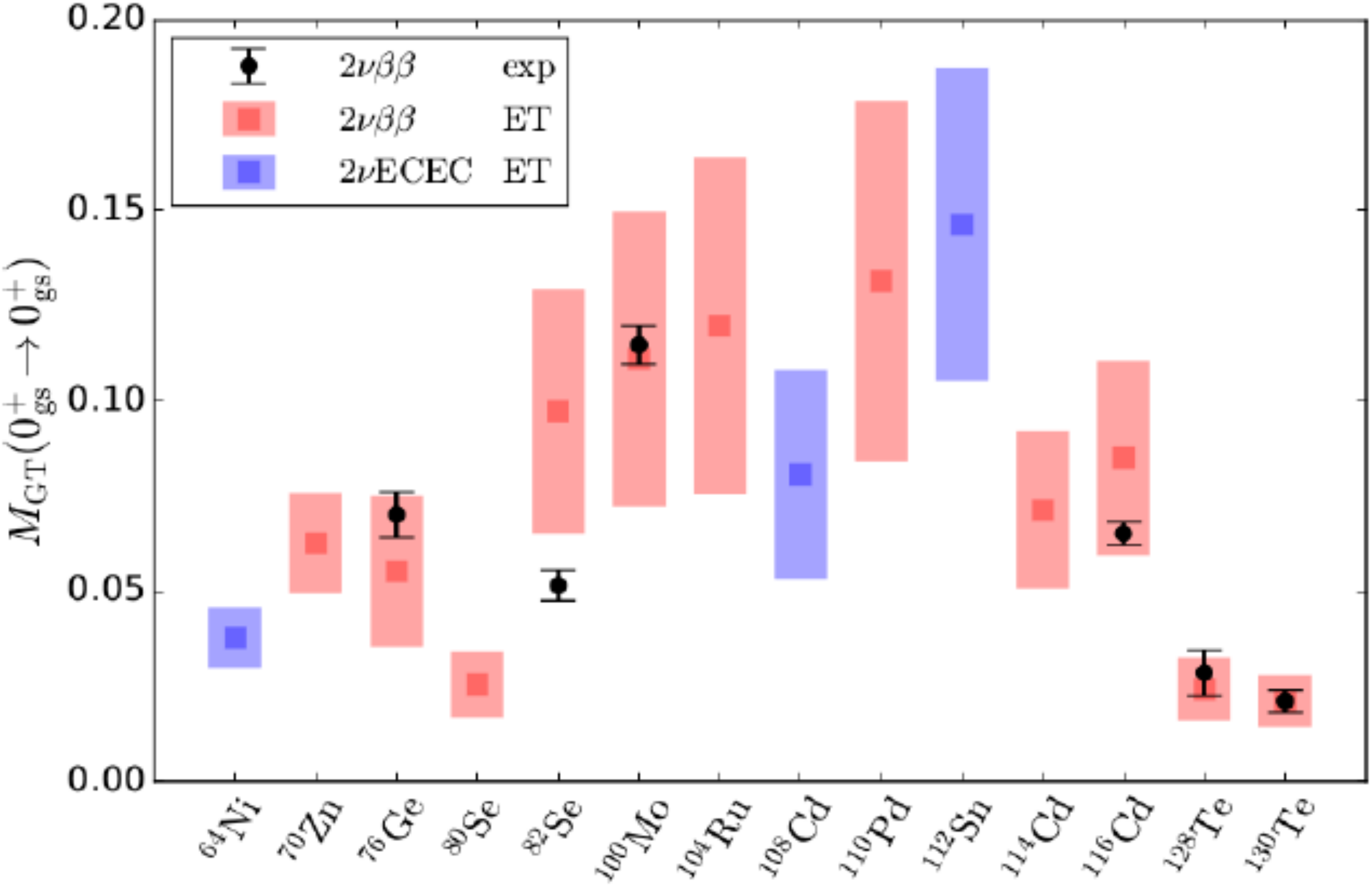
Results in nuclear magnetons.

At LO, one new LEC enters to describe the magnetic moments in the odd-mass neighbor $\hat{\mu}_\mu = \mu_d \hat{\mathbf{J}}_\mu + \mu_a \hat{\mathbf{j}}_\mu$

E. A. Coello Pérez & TP, Phys. Rev. C 94, 054316 (2016)

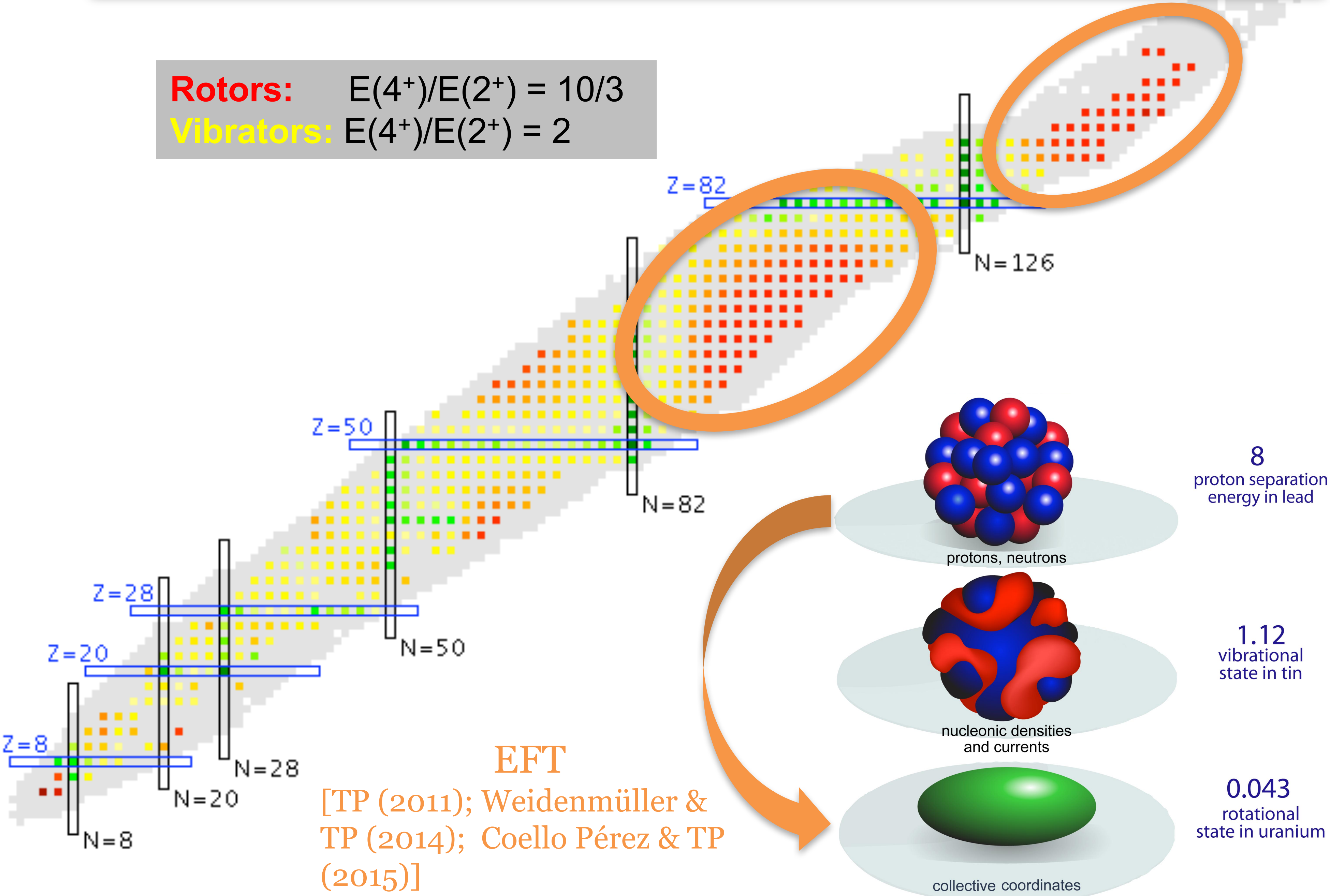
Double-beta decay: EFT results with low-energy coefficients fit to GT transitions

[Coello Perez, Menendez & Schwenk, arXiv:1708.06140]



EFT for deformed nuclei: rotations

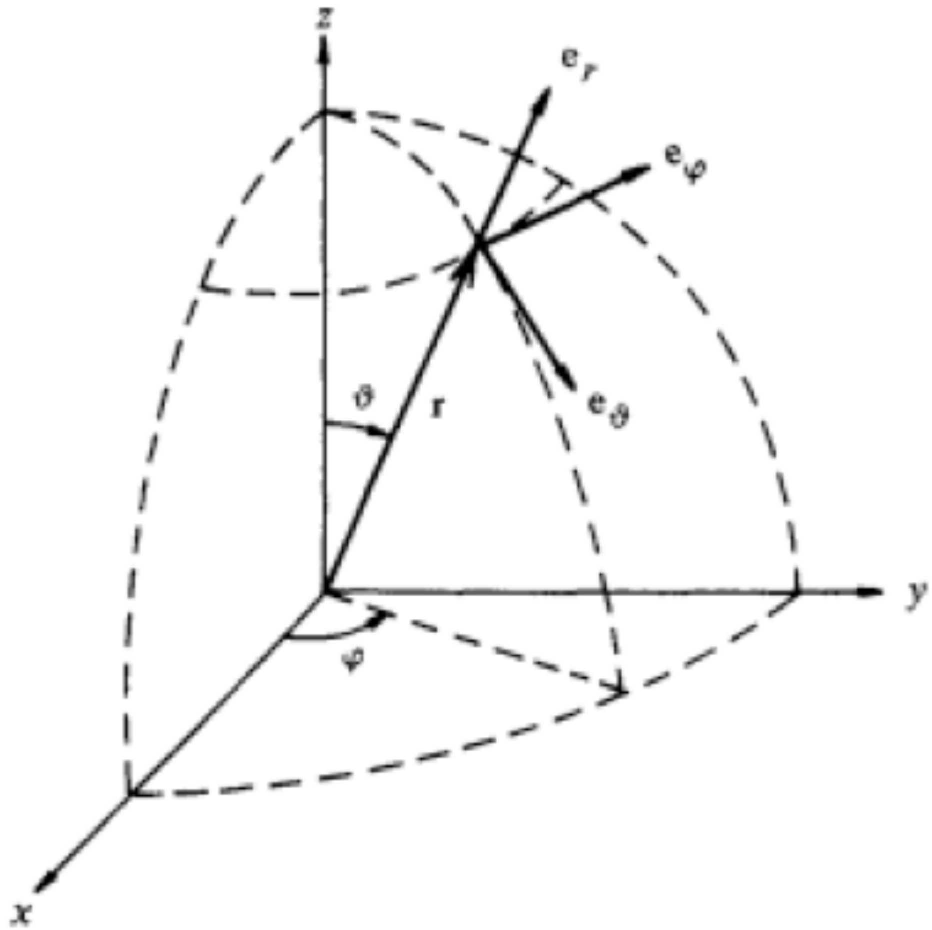
Rotors: $E(4^+)/E(2^+) = 10/3$
Vibrators: $E(4^+)/E(2^+) = 2$



EFT
[TP (2011); Weidenmüller & TP (2014); Coello Pérez & TP (2015)]

Rotors: Nonlinear realization of rotational symmetry
[follows Weinberg 1967; Coleman, Callan, Wess & Zumino 1969]

Spontaneous breaking of rotational symmetry: Nambu-Goldstone modes parameterize the coset SO(3)/SO(2) ~ S^2, i.e. the two sphere



$$\vec{n}(\theta,\phi) = \begin{pmatrix} \cos\phi\sin\theta \\ \sin\phi\sin\theta \\ \cos\theta \end{pmatrix}$$

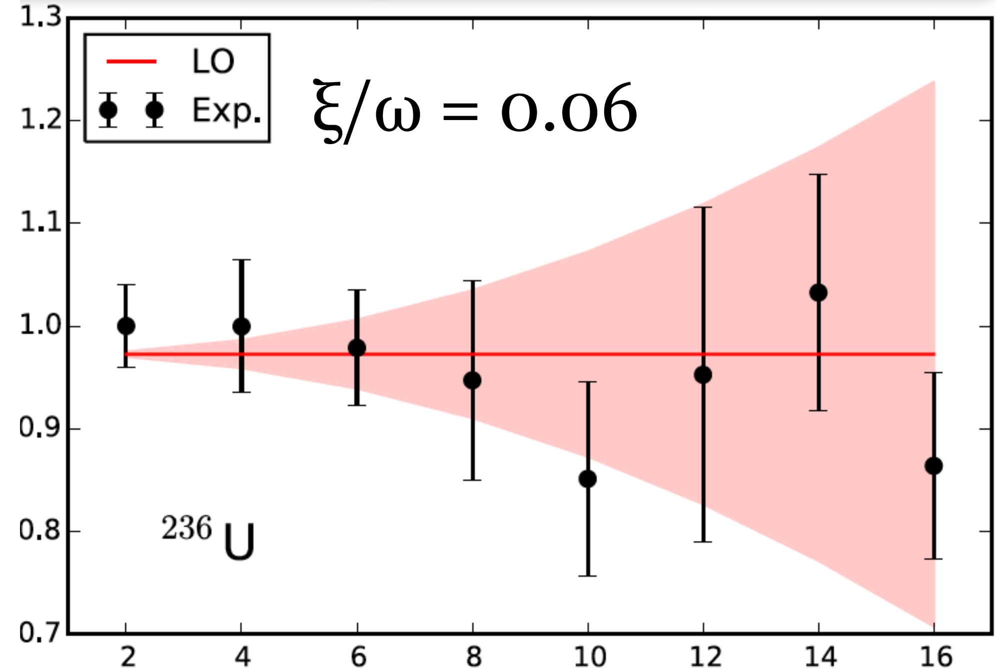
Comments:

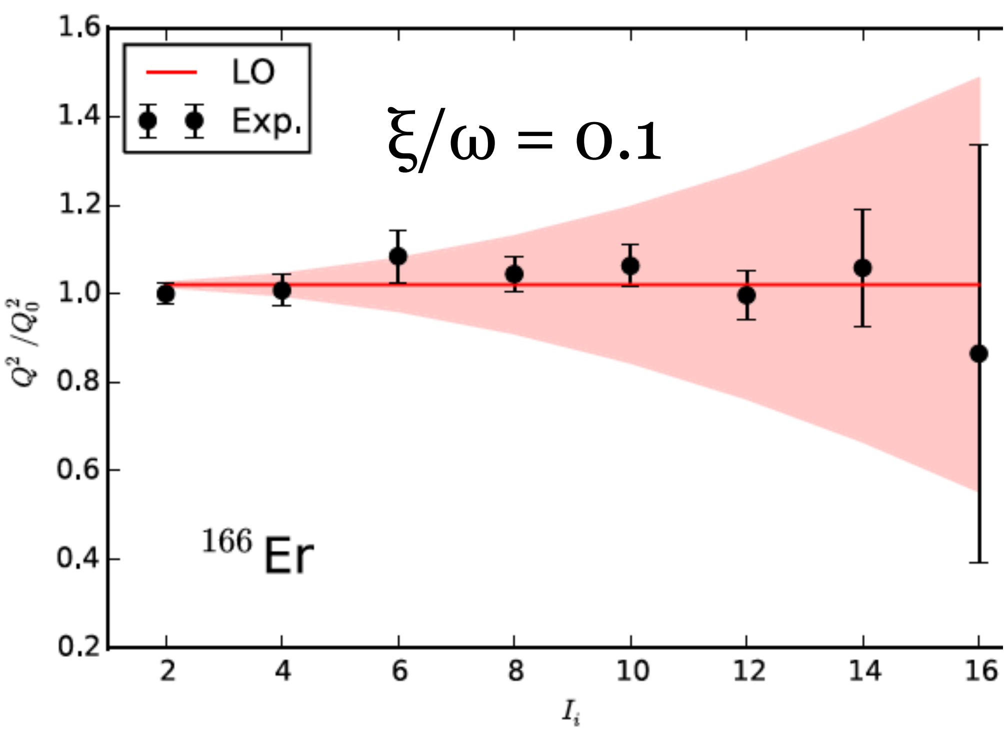
- Further degrees of freedom in the tangential plane can be added to the tangential plane
- Addition of monopole field yields nuclei with nonzero ground-state spins

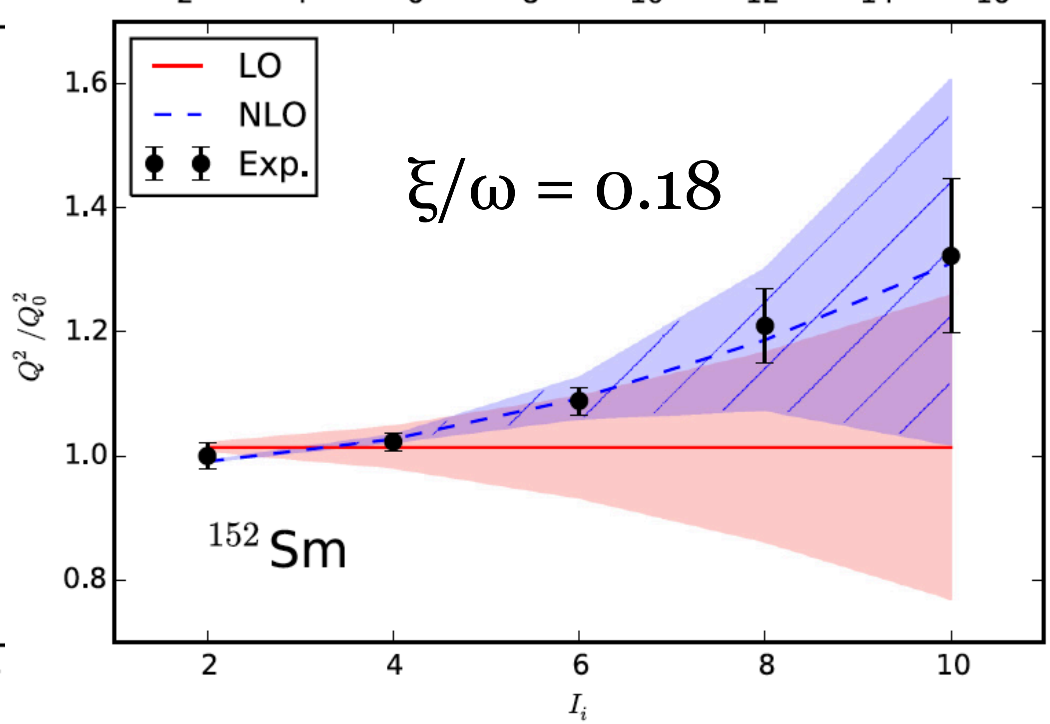
Axial:	SO(3)→SO(2)	TP & Weidenmüller; Phys. Scr. 91 (2016) 053004]
Tri-axial:	SO(3)→ I:	Chen, Kaiser, Meißner, Meng, EPJ A 53, 204 (2017)

EFT works well for a wide range of rotors

Bohr & Mottelson (1975):
"The accuracy of the present measurements of E2-matrix elements in the ground-state bands of even even nuclei is in most cases barely sufficient to detect deviations from the leading-order intensity relations."







Challenge: weak interband transitions (example: ^{154}Sm)

$i \to f$	$B(E2)_{\mathrm{exp}}$	$B(E2)_{\mathrm{ET}}$	$B(E2)_{\mathrm{CBS}}$	$B(E2)_{\mathrm{BH}}$
$2_g^+ \to 0_g^+$	0.863 (5)	0.863[a]	0.853	0.863
$4_g^+ \to 2_g^+$	1.201 (29)	1.233 (9)	1.231	1.234
$6_g^+ \to 4_g^+$	1.417 (39)	1.358 (23)	1.378	1.355
$8_g^+ \to 6_g^+$	1.564 (83)	1.421 (43)	1.471	1.424
$2_\gamma^+ \to 0_g^+$	0.0093 (10)	0.0110 (28)		0.0492
$2_\gamma^+ \to 2_g^+$	0.0157 (15)	0.0157[a]		0.0703
$2_\gamma^+ \to 4_g^+$	0.0018 (2)	0.0008 (2)		0.0050
$2_\beta^+ \to 0_g^+$	0.0016 (2)	0.0025 (6)	0.0024	0.0319
$2_\beta^+ \to 2_g^+$	0.0035 (4)	0.0035[a]	0.0069	0.0456
$2_\beta^+ \to 4_g^+$	0.0065 (7)	0.0063 (16)	0.0348	0.0821

[a]Values employed to adjust the LECs of the effective theory.

In-band transitions [in e^2b^2] are LO, inter-band transitions are NLO. Effective theory is more complicated than Bohr Hamiltonian both in Hamiltonian and E2 transition operator. EFT correctly predicts strengths of inter-band transitions with natural LECs.

[E. A. Coello Pérez and TP, Phys. Rev. C 92, 014323 (2015)]

Summary

- EFT for nuclear vibrations
  - Anharmonic vibrations consistent with data within uncertainties
  - Sizable quadrupole moments and transitions where models yield null result
  - Predictions for M1 and E2 moments and transitions
- EFT for deformed nuclei
  - LO recovers Bohr Hamiltonian
  - EFT explains weak interband transitions

Take-home message:

1. Systematic expansion of Hamiltonian and transition operators according to a power counting.
2. **All** collective models have a breakdown scale: ignore at own risk …
3. Uncertainty quantification (or at least estimates) exploit breakdown scale