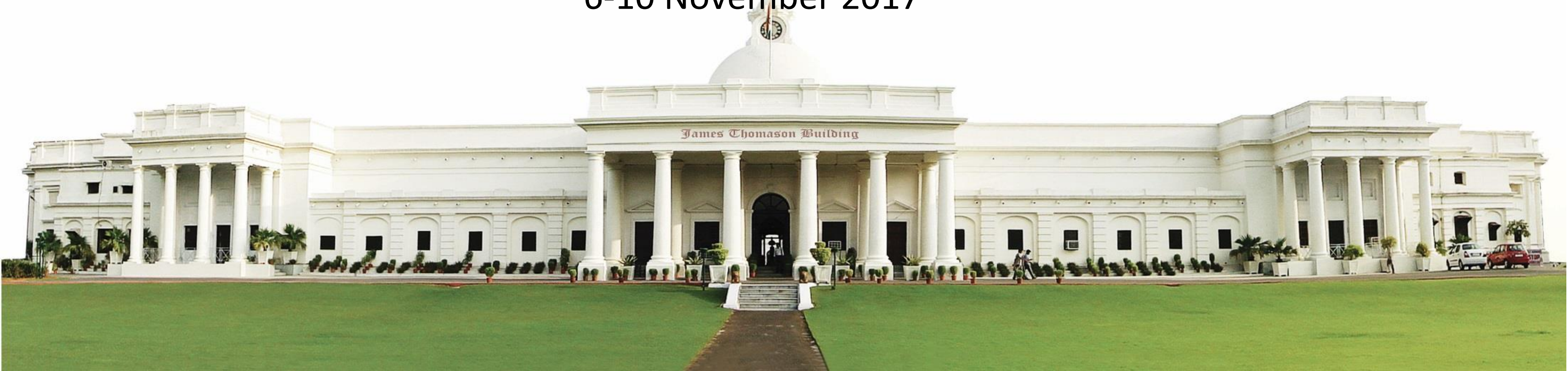
Nuclear Isomers in and around various Magic Nuclei

A. K. Jain

In collaboration with

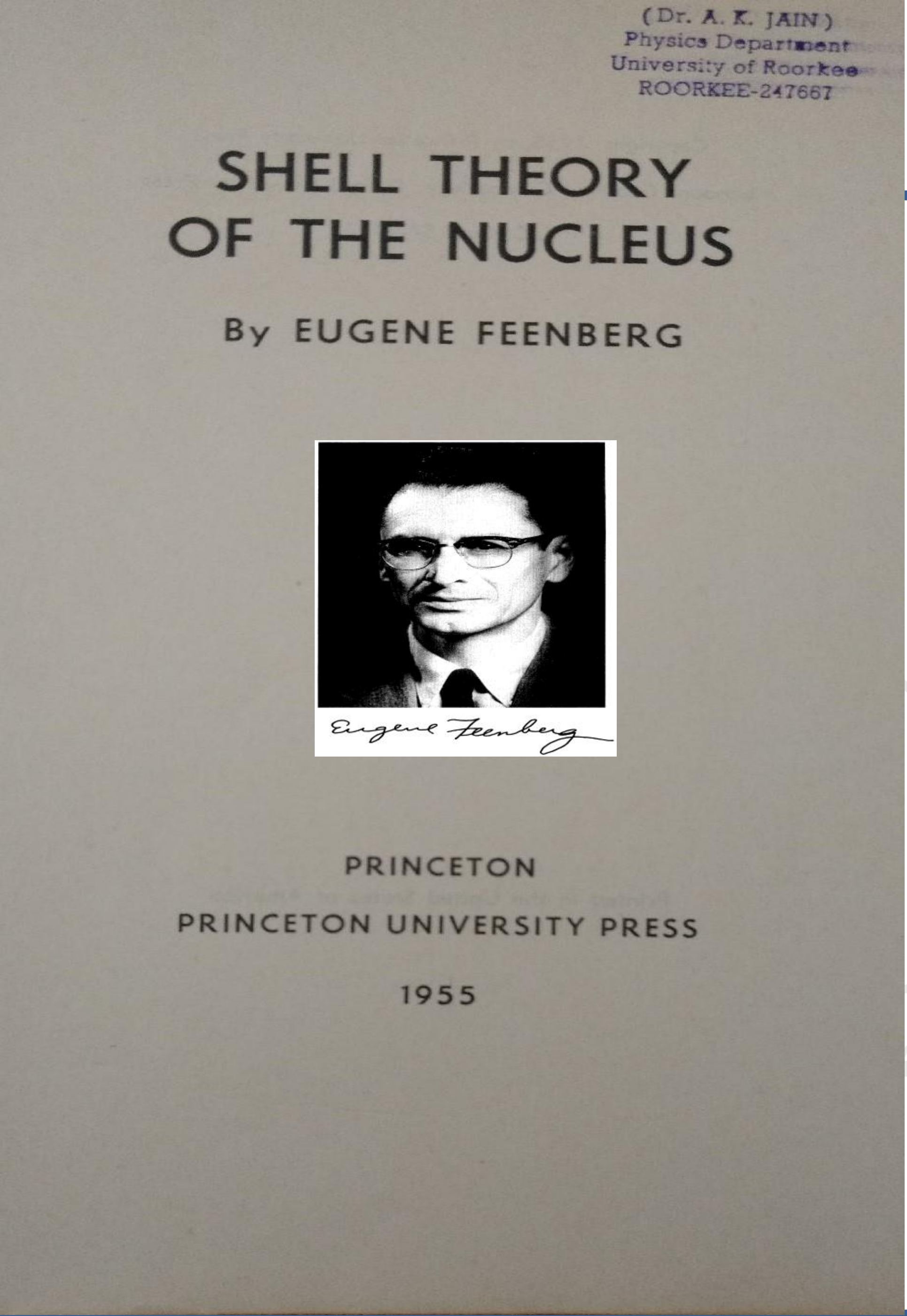
Bhoomika and Swati

International Conference on Shapes and Symmetries in Nuclei – Expt.
& Theory, CNRS, Paris, France
6-10 November 2017



Outline

- Nuclear Isomers – Feenberg’s contribution
- Chart of Isomers & Global systematics related to pairing
- Seniority Isomers
- Key Features of Seniority isomers
- BE2 systematics of Z=82 isotopes and N=126 Isotones
- Extension to multi-j scheme from pure-j scheme
- Selection rules in Multi-j scheme and the new type of seniority isomers which decay by odd-tensor transitions
- BE2 puzzle in the 2+ states of Sn isotopes
- g-factors in the 2+ and 10+ states of Sn isotopes
- BE3 trends of the 3- states in Sn isotopes



SHELL THEORY OF THE NUCLEUS

By EUGENE FEENBERG

Eugene Feenberg

PRINCETON
PRINCETON UNIVERSITY PRESS

1955

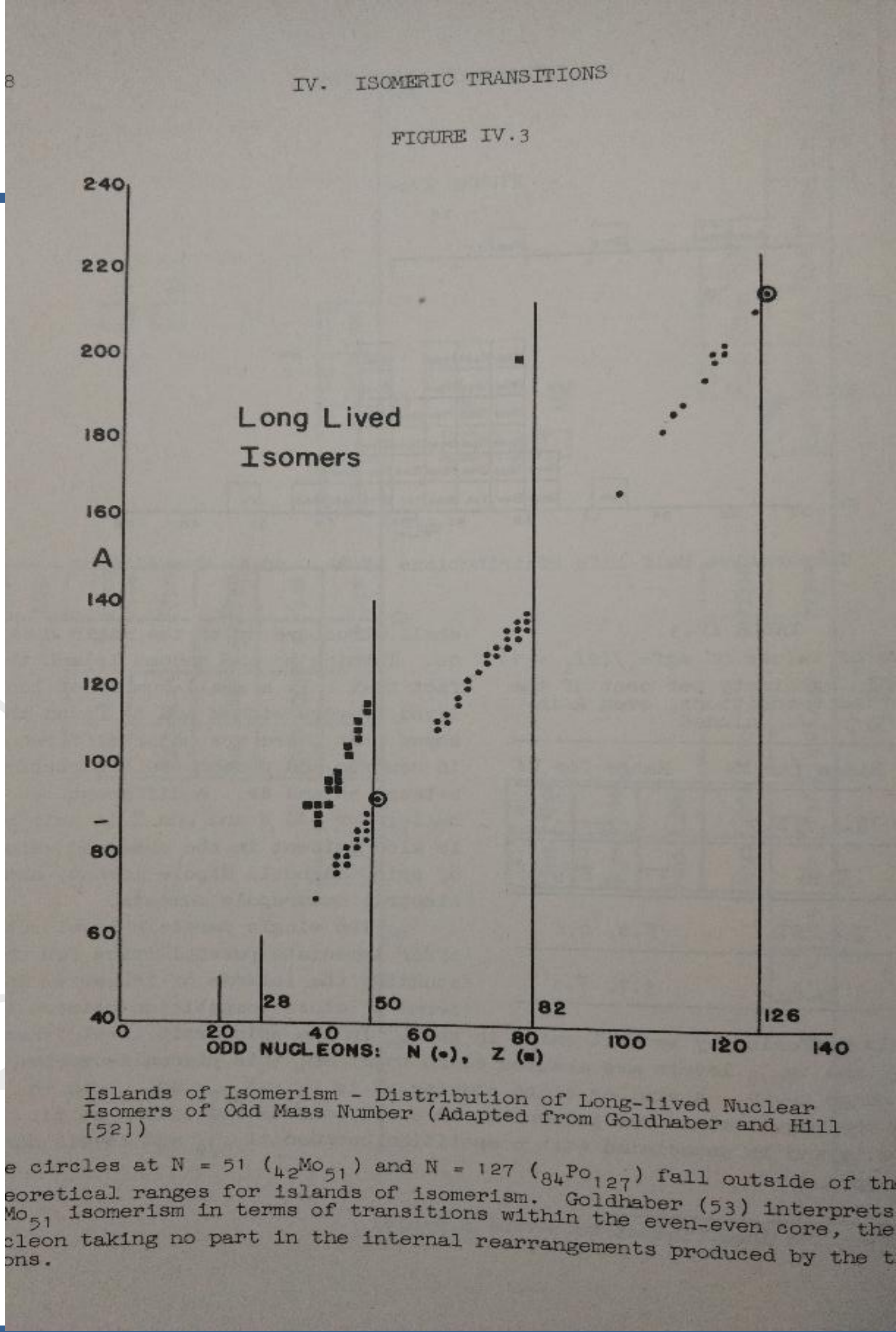


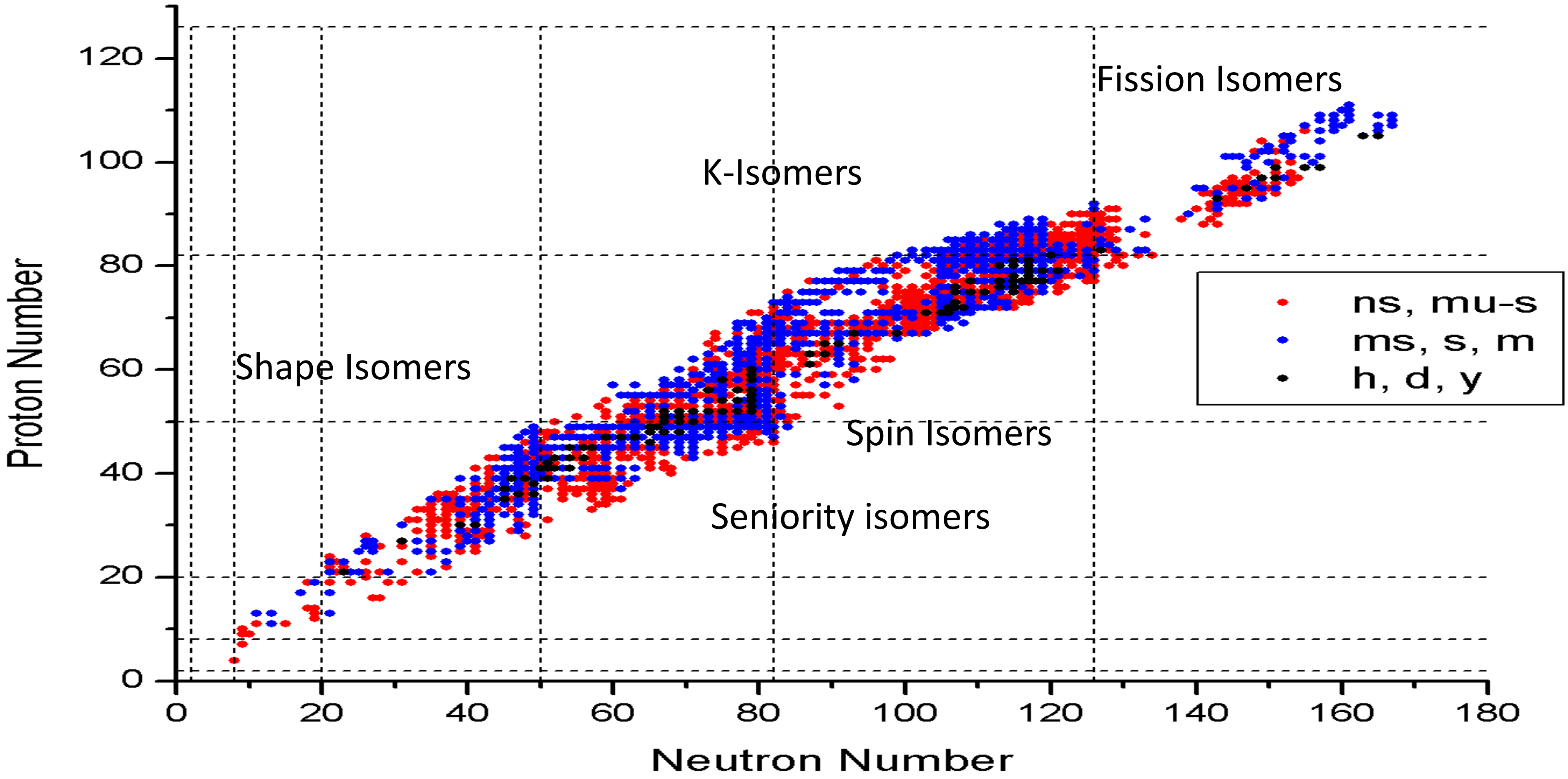
FIGURE IV.3

Islands of Isomerism - Distribution of Long-lived Nuclear Isomers of Odd Mass Number (Adapted from Goldhaber and Hill [52])

e circles at N = 51 ($_{42}$Mo$_{51}$) and N = 127 ($_{84}$Po$_{127}$) fall outside of th
eoretical ranges for islands of isomerism. Goldhaber (53) interprets
Mo$_{51}$ isomerism in terms of transitions within the even-even core, the
cleon taking no part in the internal rearrangements produced by the t
ons.

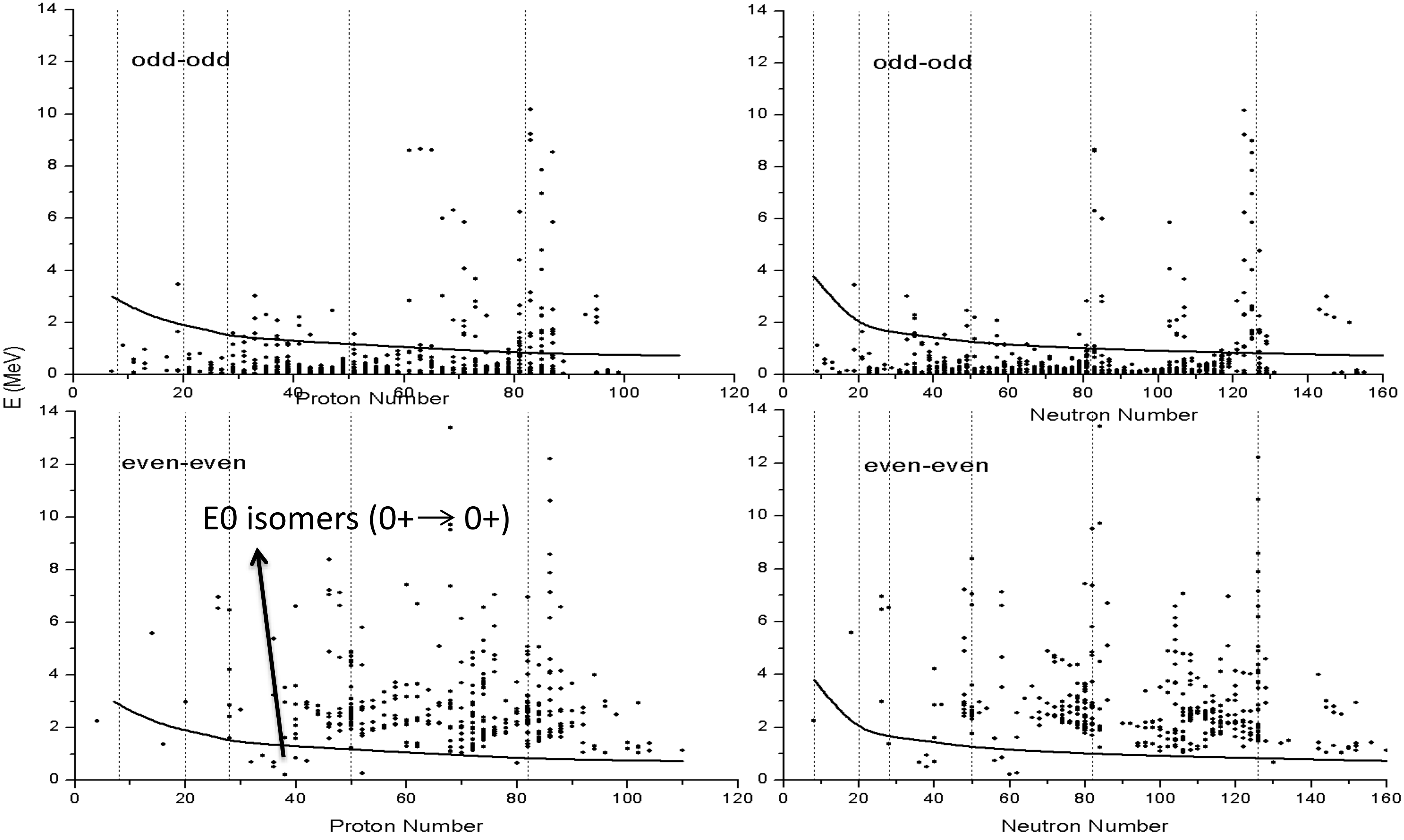
Chart of Nuclear Isomers

- Nuclear Isomers are longer lived excited states $\geq 10\ ns$.



“Atlas of Nuclear Isomers”, A. K. Jain, B. Maheshwari, S. Garg, M. Patial and B. Singh, Nuclear Data Sheets **128** (2015) 1-130.

Even-even & odd-odd isomers (Even-A)



E0 isomers (0+ → 0+)

Seniority and Seniority Isomers

- Seniority was introduced by Racah (1943) in atomic context
- Adopted for nuclei in a similar fashion.
- Seniority quantum number (*v*) may be defined as the number of unpaired nucleons.
- A set of states diagonalizable by a short range pairing interaction give rise to good or, nearly good seniority states
- Specific selection rules emerge, which may lead to isomeric states in single closed shell nuclei

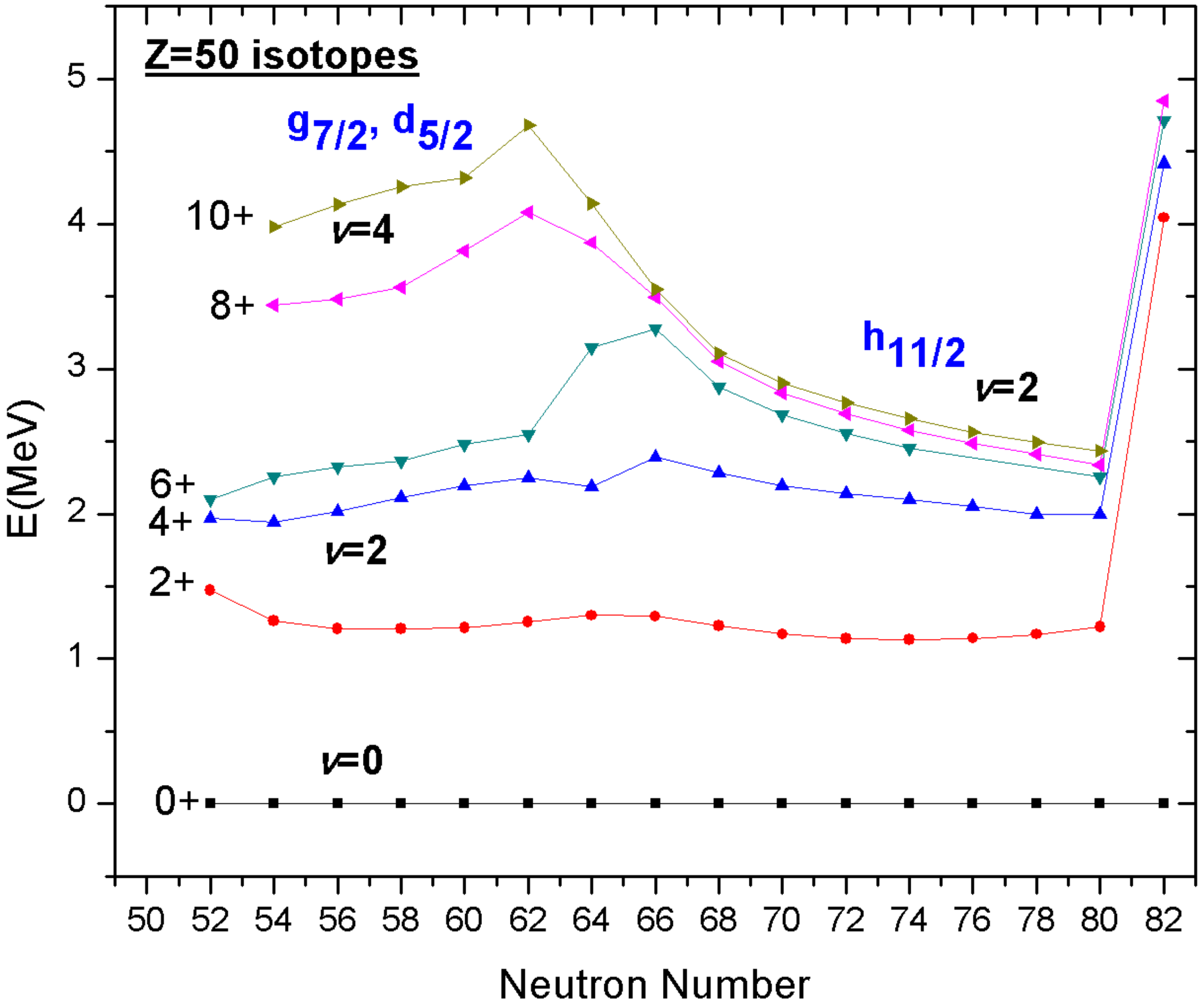
Ref.: Books of Talmi, Lawson, Griener and Maruhn, and Casten carry an excellent technical/general description of the seniority quantum number.

Key features

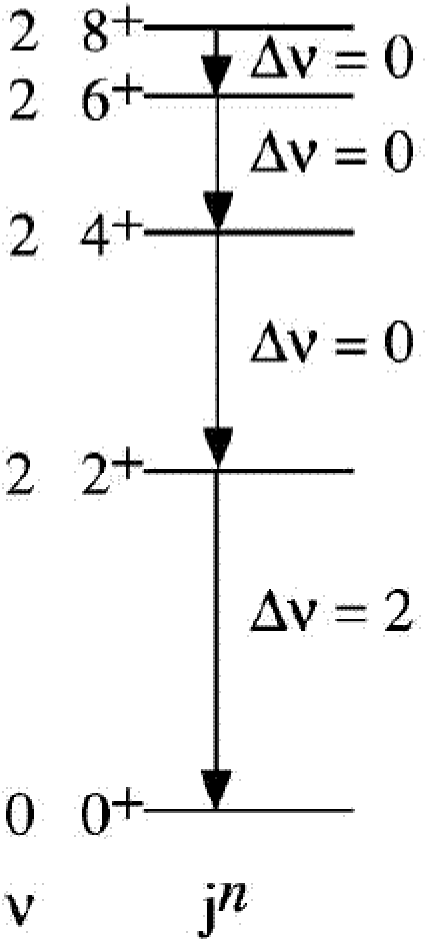
I. Talmi, Simple Models of Complex Nuclei. (Harwood, 1993).

- Particle number independent energy
- Constant pairing gap
- Specific selection rules and parabolic behavior of transition probabilities
- Any interaction between identical fermions in single-*j* shell exactly conserves seniority if $j \leq 7/2$.
- The seniority is nearly conserved up to j=11/2 in Sn-isomers after the mid-shell, where small seniority mixing may take place, but still the features of seniority persist.

Particle number independence of energy



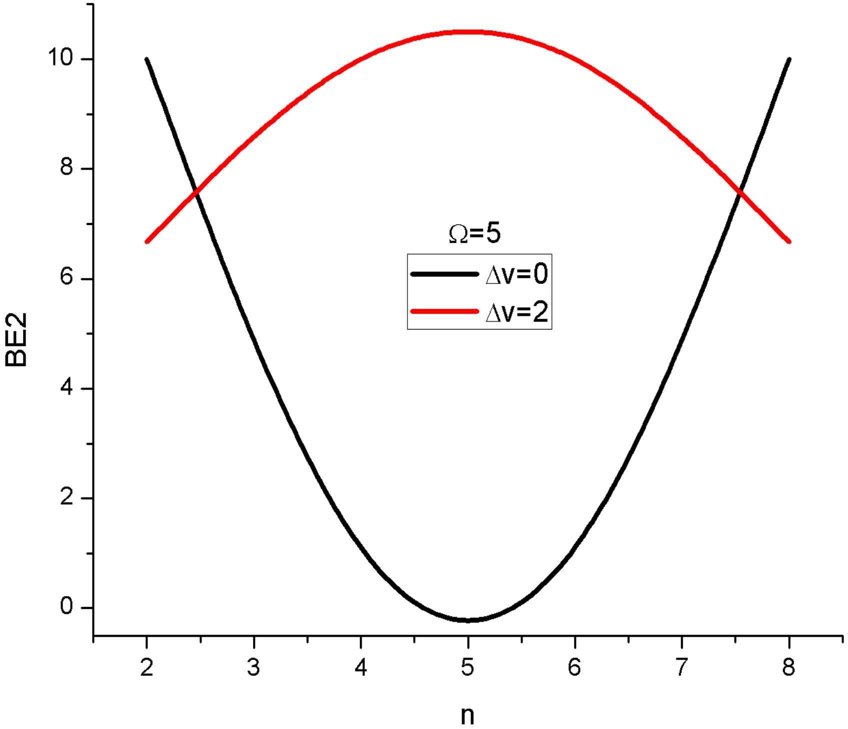
BE2 variation in single-j: Schematic only



J=9/2

$$B(E2) \propto \left(\frac{\Omega - n}{\Omega - v}\right)^2, \Delta v = 0$$

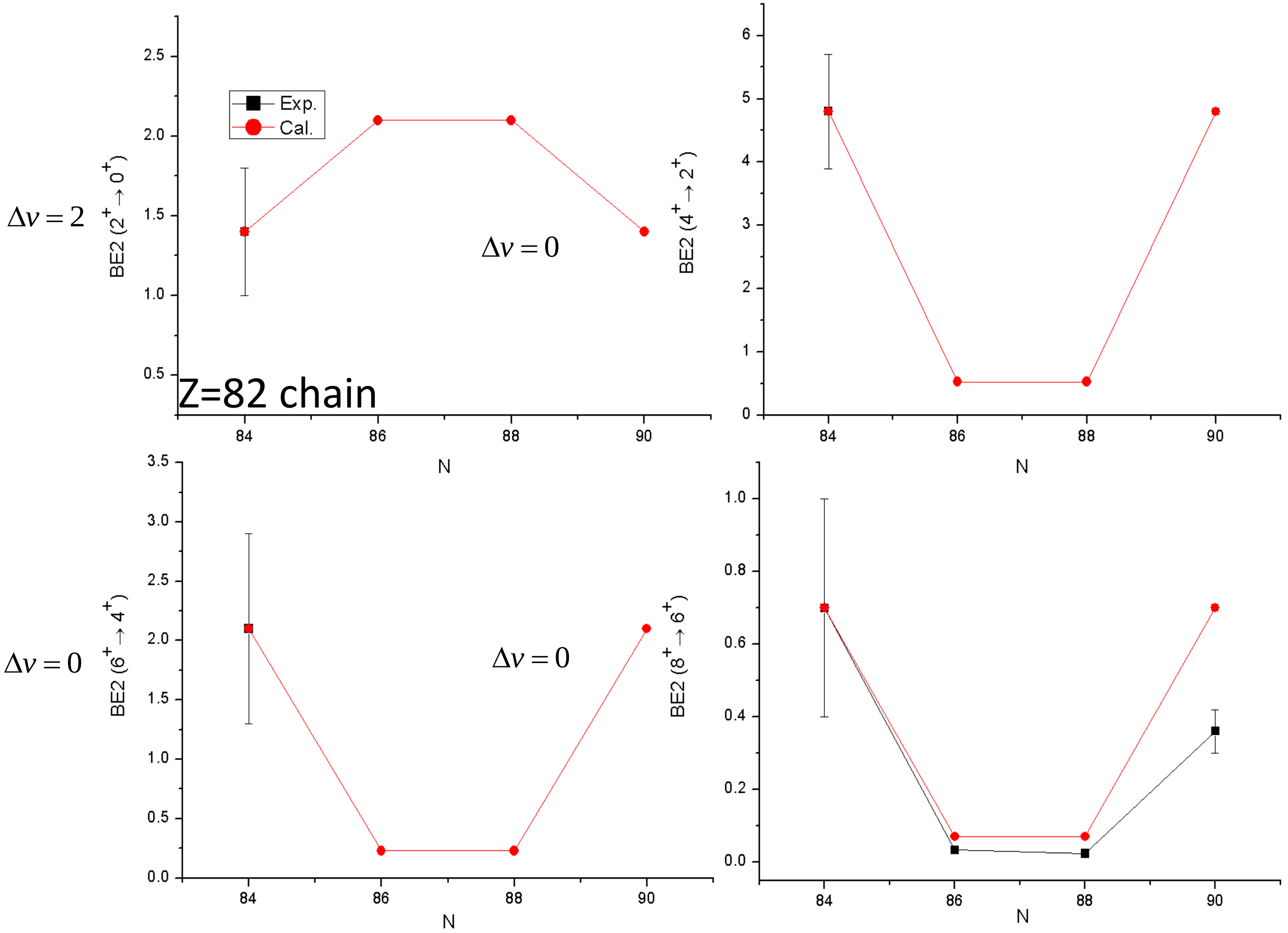
$$B(E2) \propto \frac{(n - v + 2)(2\Omega + 2 - n - v)}{2(2\Omega + 2 - 2v)}, \Delta v = 2$$



For schematic representation only.

B(E2)s in the even-even Z=82, 8+ isomers

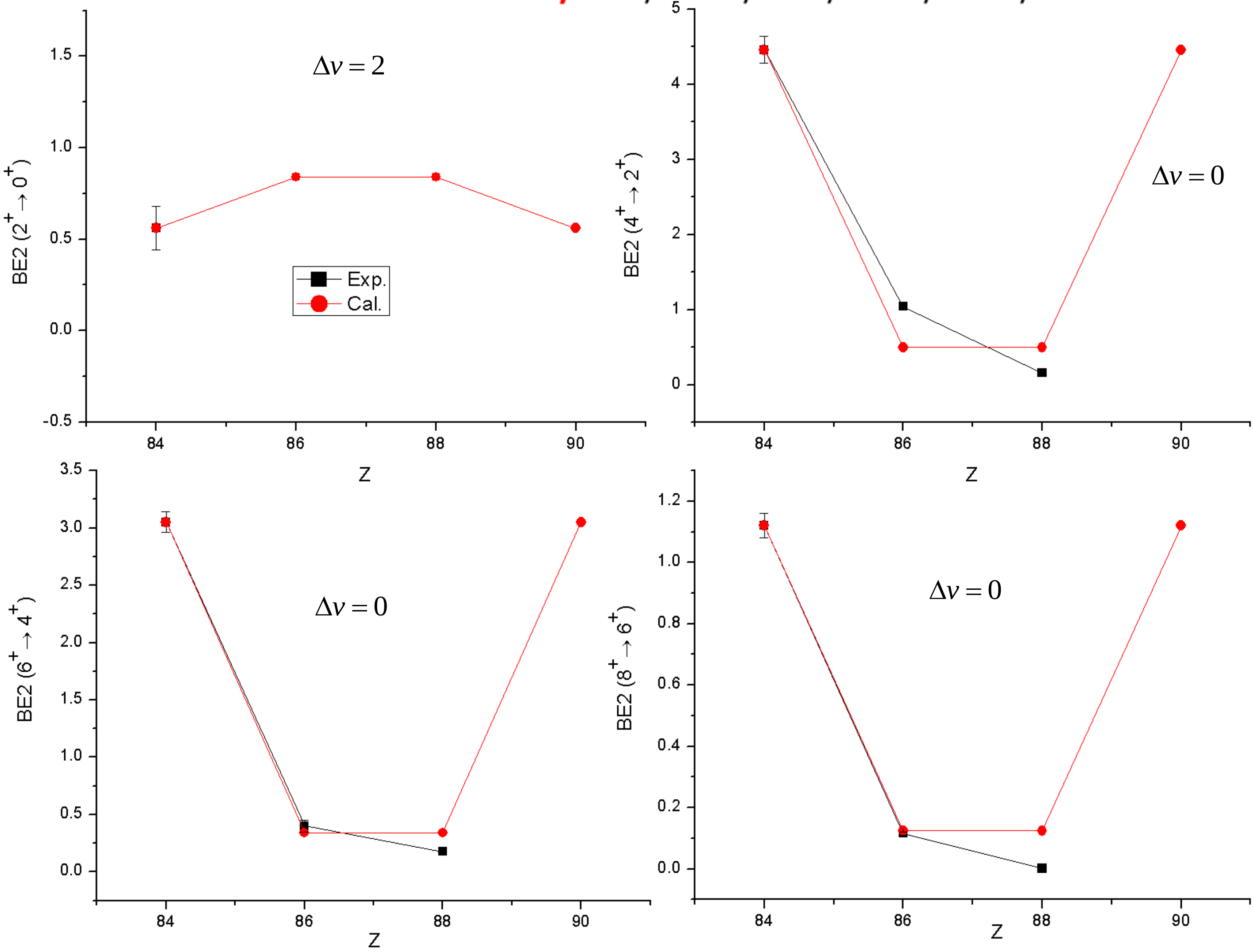
Neutrons ($\mathbf{g_{9/2}}$, $i_{11/2}$, $j_{15/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, $d_{3/2}$)



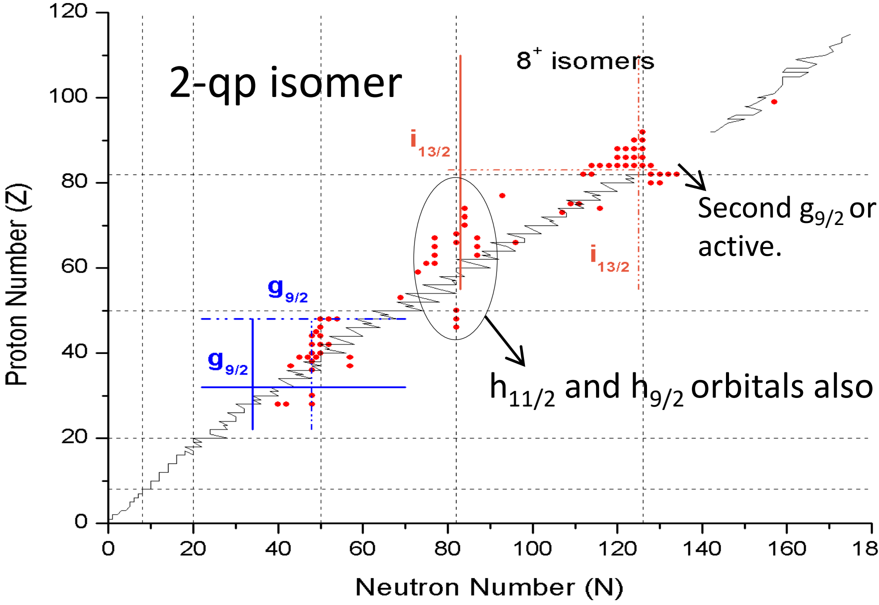
All BE2s are in Weisskopf Units

B(E2)s in the even-even N=126 chain, 8+ isomers

Protons (**$h_{9/2}$**,$f_{7/2}$,$i_{13/2}$,$f_{5/2}$,$p_{3/2}$,$p_{1/2}$)



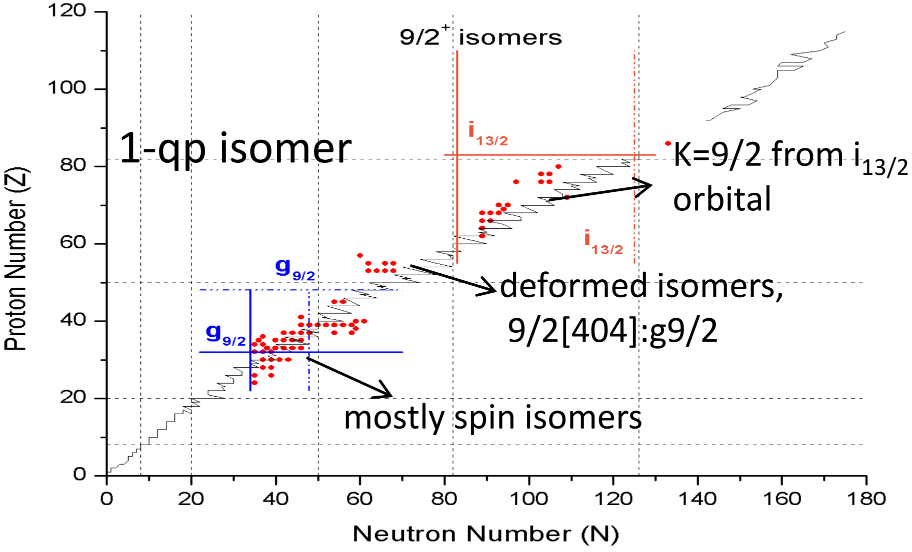
$g_{9/2}$: $9/2^+$, 8^+ and $21/2^+$ isomers



2-qp isomer

Second $g_{9/2}$ orbital active.

$h_{11/2}$ and $h_{9/2}$ orbitals also active.

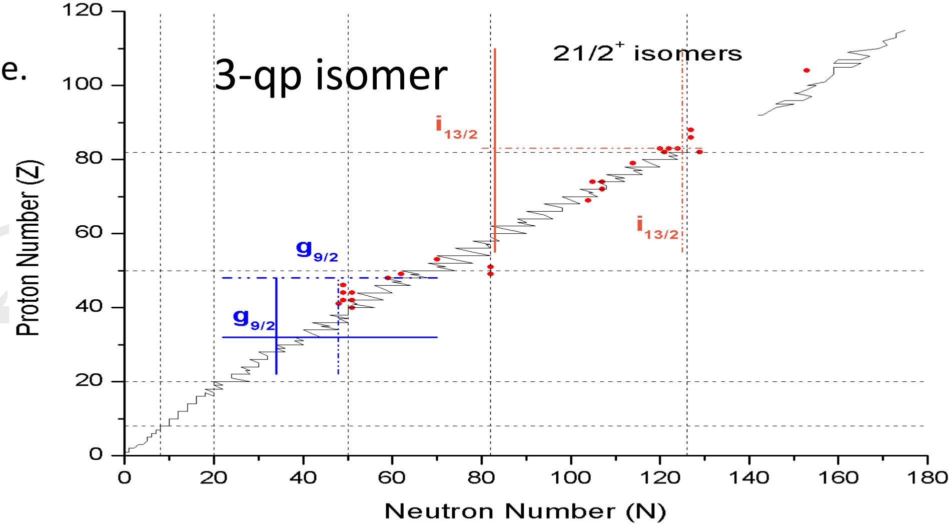


1-qp isomer

K=9/2 from $i_{13/2}$ orbital

deformed isomers, 9/2[404]:g9/2

mostly spin isomers



3-qp isomer

A. K. Jain and B. Maheshwari, https://www-nds.iaea.org/publications/indc/indc-ind-0048/

B(E2) values from Seniority scheme

$$B(EL) = \frac{1}{2J_i + 1} \left| \left(J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i \right\| J_i \right) \right|^2$$

In single-j case,

$$\boxed{\Omega = \frac{1}{2}(2j+1)}$$

For L even

seniority

$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^n v l' J_i \right\rangle = \left(\frac{\Omega - n}{\Omega - v} \right) \left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^v v l' J_i \right\rangle$$

$$\left\langle j^n v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^n, v \mp 2, l' J_i \right\rangle = \sqrt{\frac{(n - v + 2)(2\Omega + 2 - n - v)}{2(2\Omega + 2 - 2v)}} \left\langle j^v v l J_f \left\| \sum_i r_i^L Y^{(L)}(\theta_i, \phi_i) \right\| j^v, v \mp 2, l' J_i \right\rangle$$

It is easy to generalize these results for multi-j case by defining,

$$\tilde{j} = j \otimes j' \ldots \quad \boxed{\Omega = \frac{1}{2} \sum_j (2j+1)} \quad n = \sum_j n_j$$

Generalized seniority

$$B(E2) \propto \left(\frac{\Omega - n}{\Omega - v} \right)^2, \Delta v = 0$$

$$B(E2) \propto \frac{(n - v + 2)(2\Omega + 2 - n - v)}{2(2\Omega + 2 - 2v)}, \Delta v = 2$$

B(E2) relations valid for single-j, and multi-j cases!!

B. Maheshwari, A. K. Jain, Phys. Lett. B753, 122 (2016)

Selections rules (Extension to multi-j shell)

Single-j shell

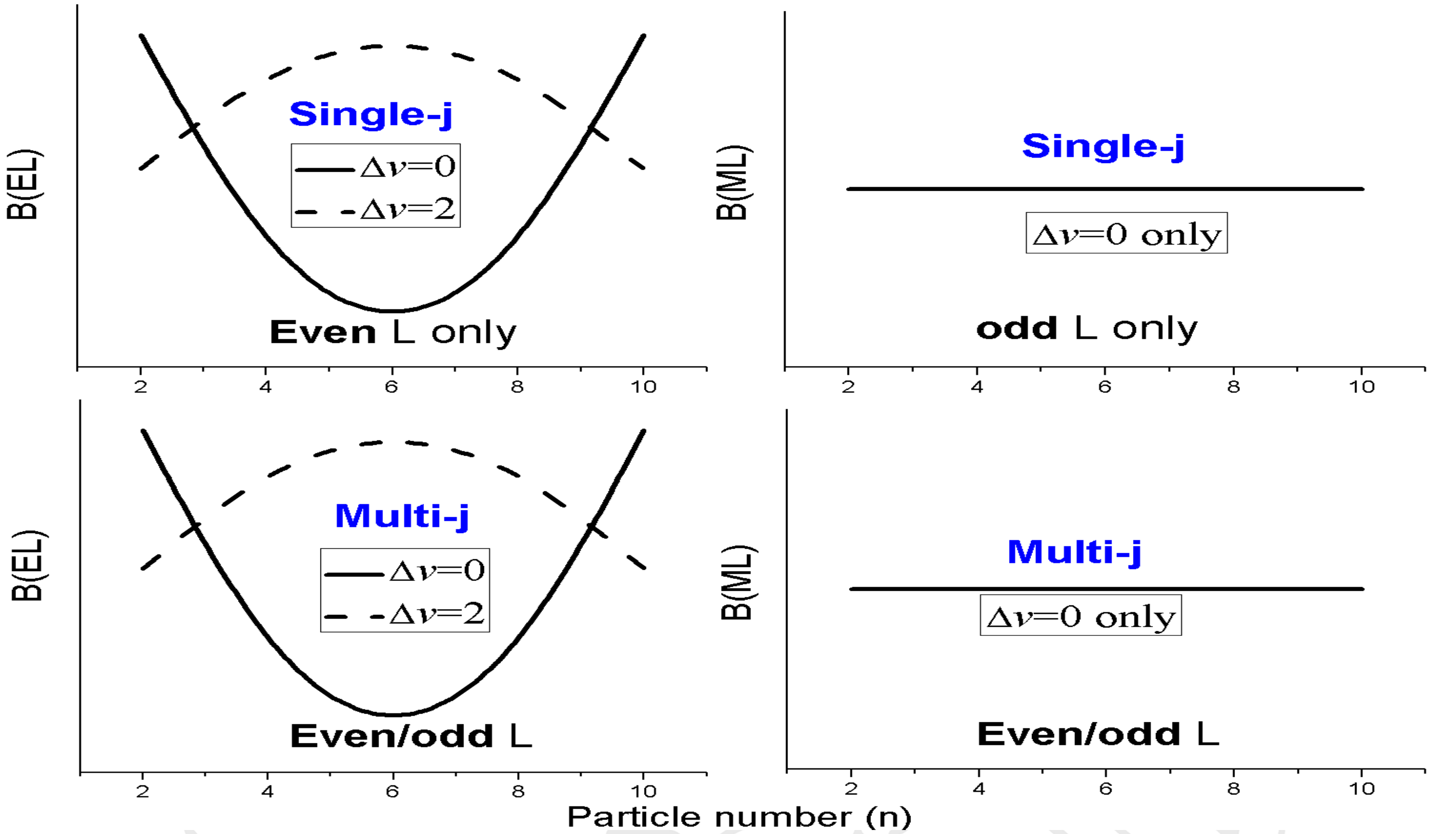
- What seniority says.
  - Odd L -> quasi-spin scalar -> Δv=0
  - Even L -> quasi-spin vector -> Δv=0, 2
- What EM decay says.
  - Odd L -> only magnetic transitions.
  - Even L -> only electric transitions.

Odd L,
Magnetic transitions, Δv=0
Even L,
Electric Transitions, Δv=0, 2

Multi-j shell

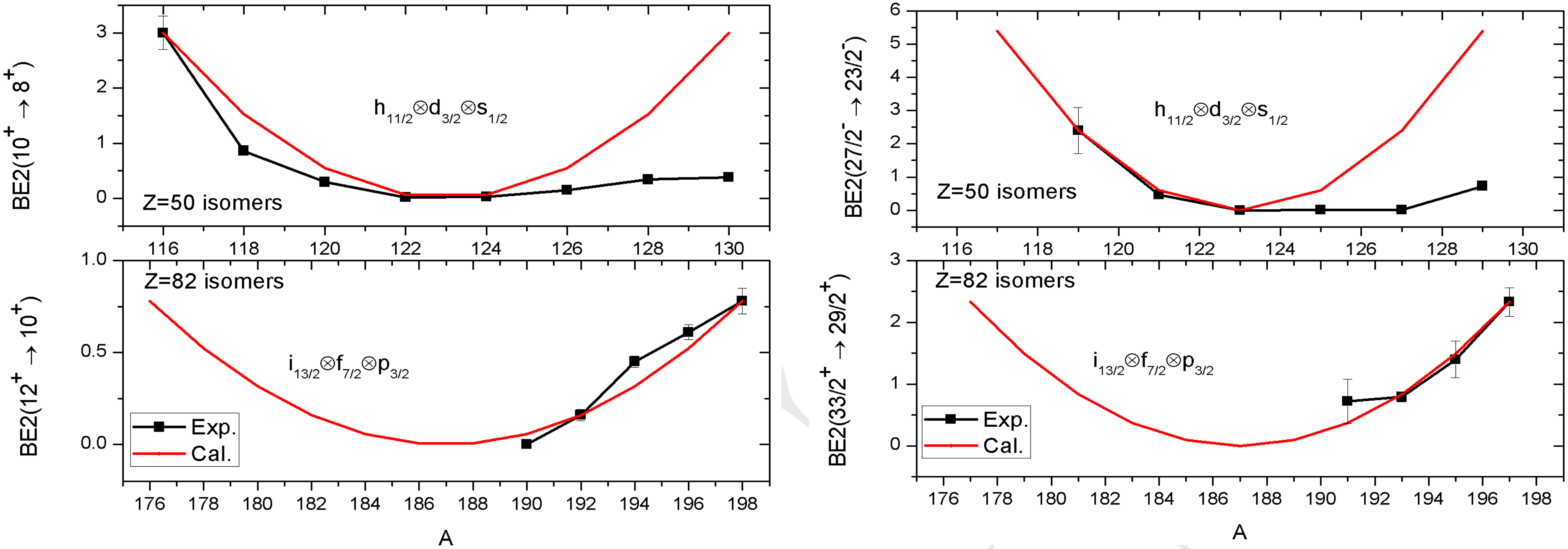
- What seniority says.
  - **Odd/Even L -> Magnetic transitions -> quasi-spin scalar -> Δv=0**
  - **Odd/Even L -> Electric transitions -> quasi-spin vector -> Δv=0, 2**
- What EM decay says.
  - Odd/Even L -> both electric/ magnetic transitions.

Comparison of single-j and multi-j shell



B. Maheshwari and A. K. Jain, Phys. Lett. B **753**, 122 (2016).
B. Maheshwari, A. K. Jain and B. Singh, Nucl. Phys. A **952**, 62 (2016).
A.K. Jain and B. Maheshwari, Nuclear Physics Review **34**, 73 (2017).
A.K. Jain and B. Maheshwari, Physica Scripta **92**, 074004 (2017).
B. Maheshwari, Swati and A. K. Jain, Pramana-Journal of Physics 89, 75 (2017).

BE2s of High-spin isomers in Sn and Pb isotopes



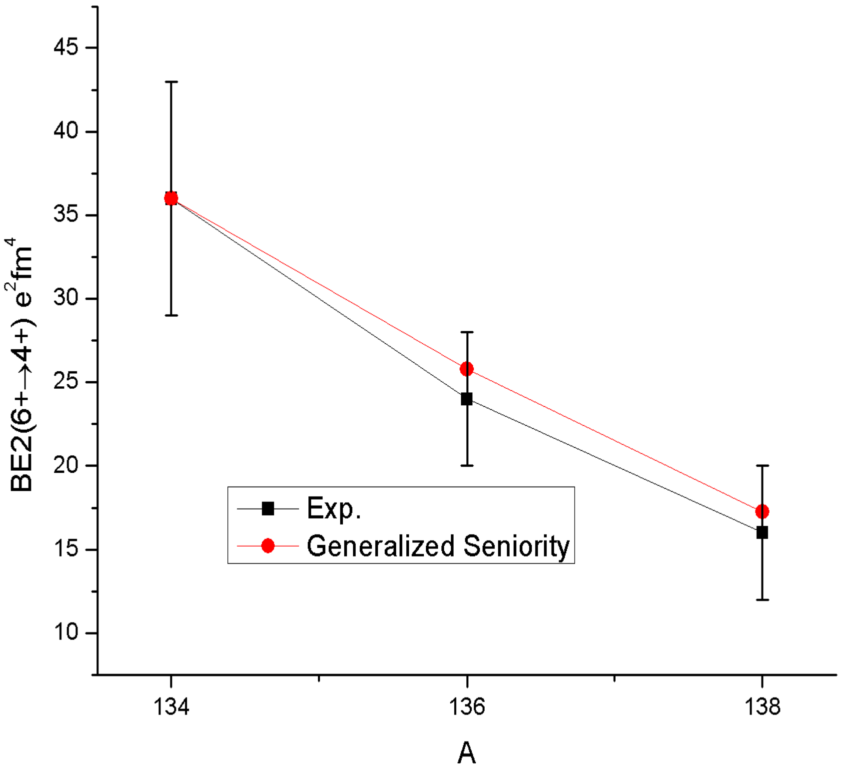
- This concludes that the 12+, 10+, 33/2+ and 27/2+ isomers, arising mainly from the intruder orbitals, also require the configuration mixing to explain the measured data.
- The right hand sides of Z=50 isomers which do not follow this generalized seniority scheme, may further need the inclusion of the non-degenerate multi-j orbits.

B(E2)s from generalized seniority in Z=50 & N=82 chains

Z=50

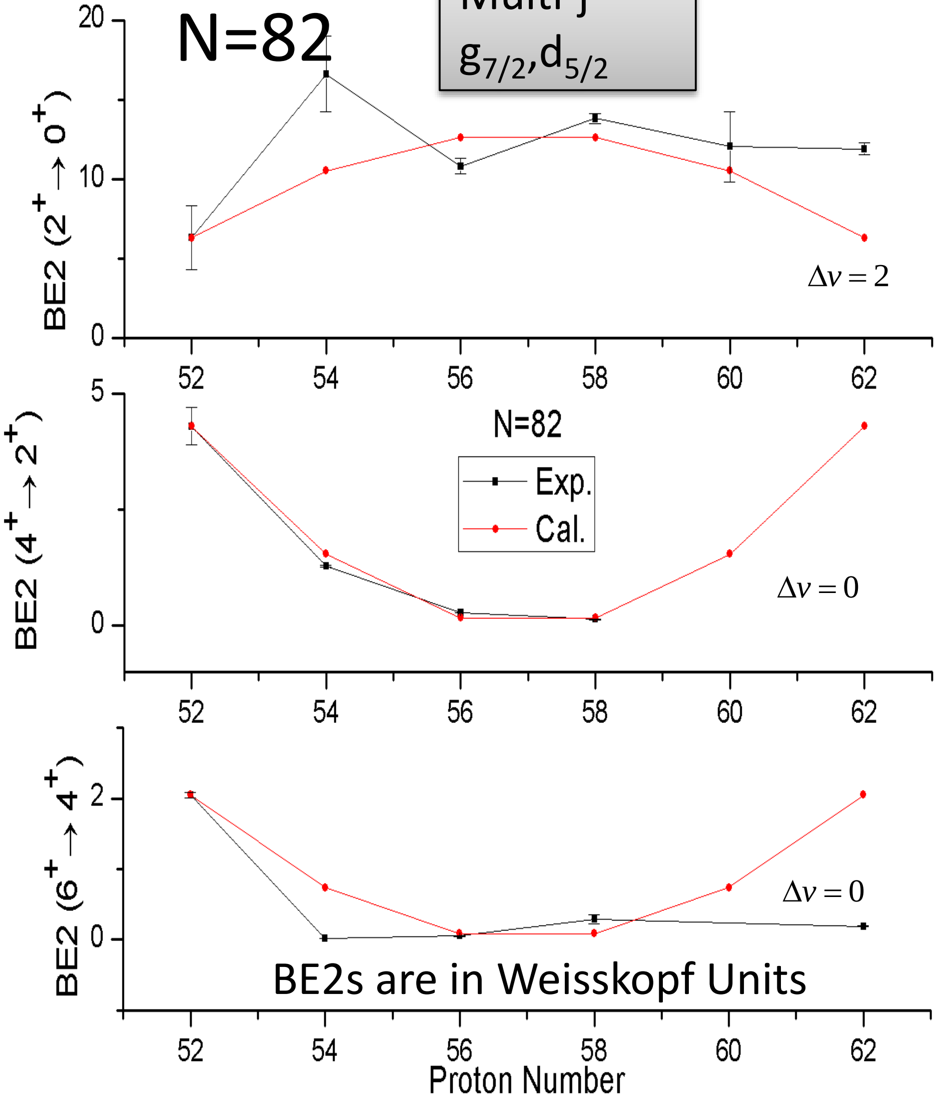
Multi-j
$f_{7/2}, p_{3/2}, p_{1/2}, f_{5/2}, h_{9/2}$

$\Delta \nu = 0$



N=82

Multi-j
$g_{7/2}, d_{5/2}$



BE2s are in Weisskopf Units

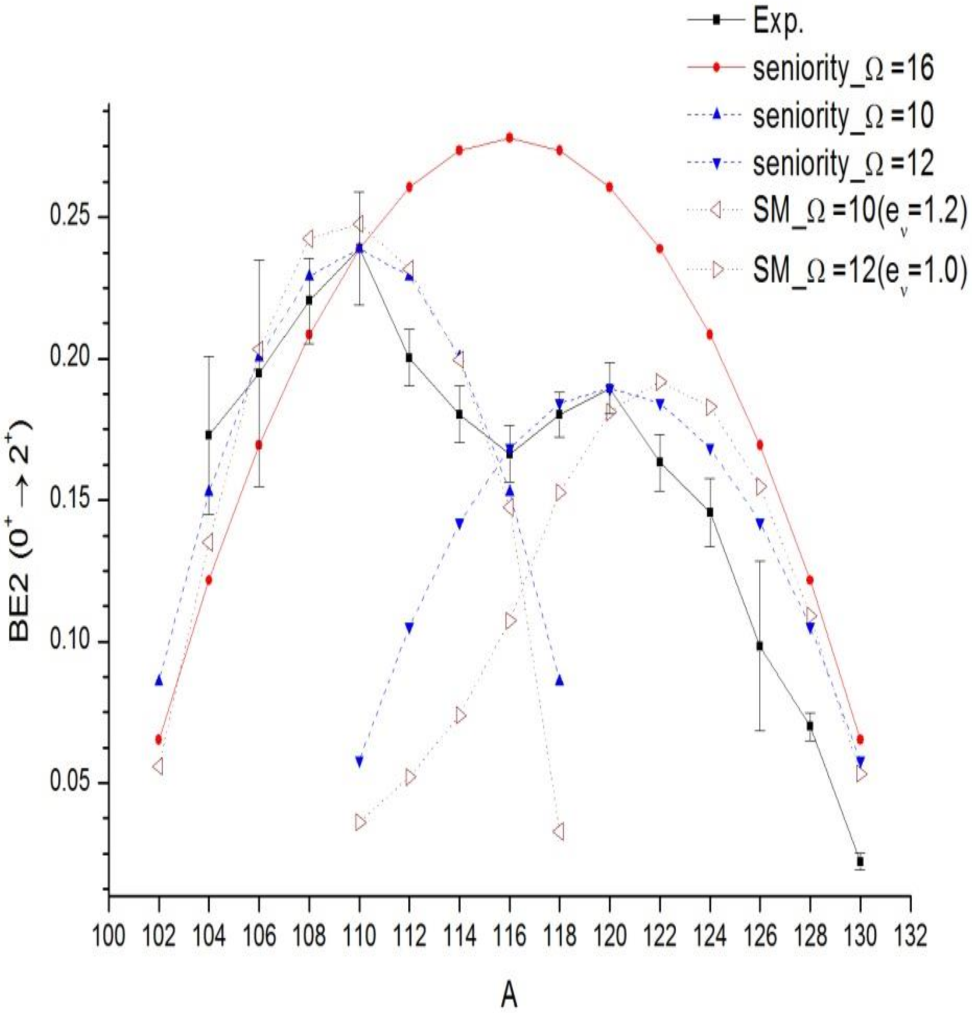
B. Maheshwari, A. K. Jain, Nucl. Phys. Rev. 34,73, 2017

The first excited 2+ states in Sn isotopes

- Expected: a single parabolic trend coming from the generalized seniority.
- Reality: A minima in the middle of the valence space in recent measurements.
- Reason: The different sets of orbits involved before and after midshell, as also discussed by Morales et al.

I. O. Morales, P. Van Isacker and I. Talmi, Phys. Lett. B **703**, 606-608 (2011).

BM, AKJ & BS, Nucl. Phys. A952,62,2016

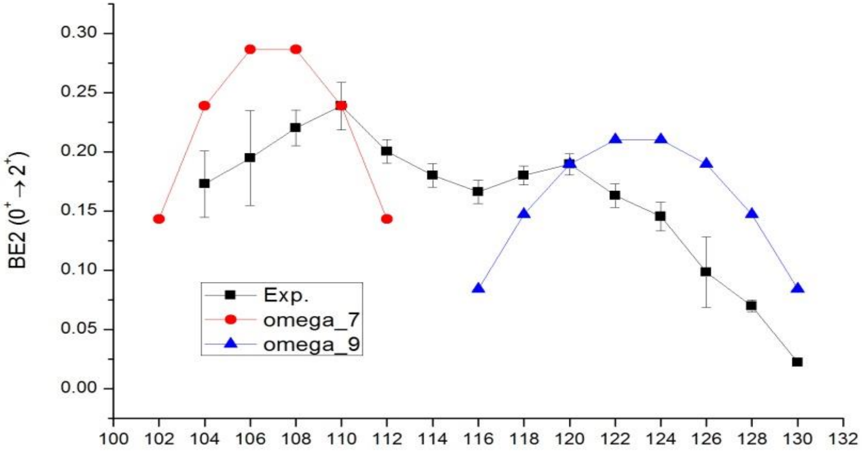


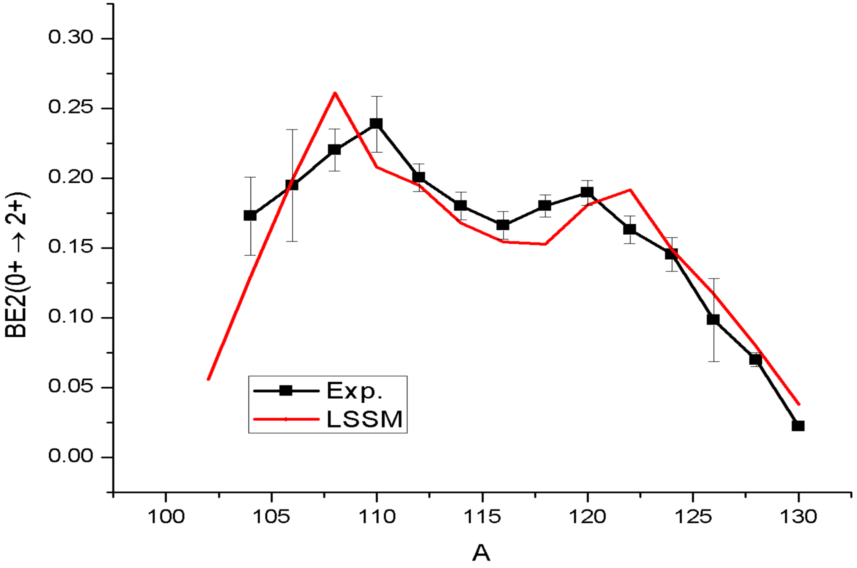
Seniority inspired LSSM

- Failure: if we use omega 7 and 9, which comes from the $g_{7/2}$, $d_{5/2}$ and $h_{11/2}$, $d_{3/2}$, $s_{1/2}$ configuration mixing sets.

- Success: Seniority inspired and truncated LSSM explains the measured data quite well, particularly around the middle, where dimensions become quite large.

- $^{102-108,124-130}$Sn have been treated in open space.





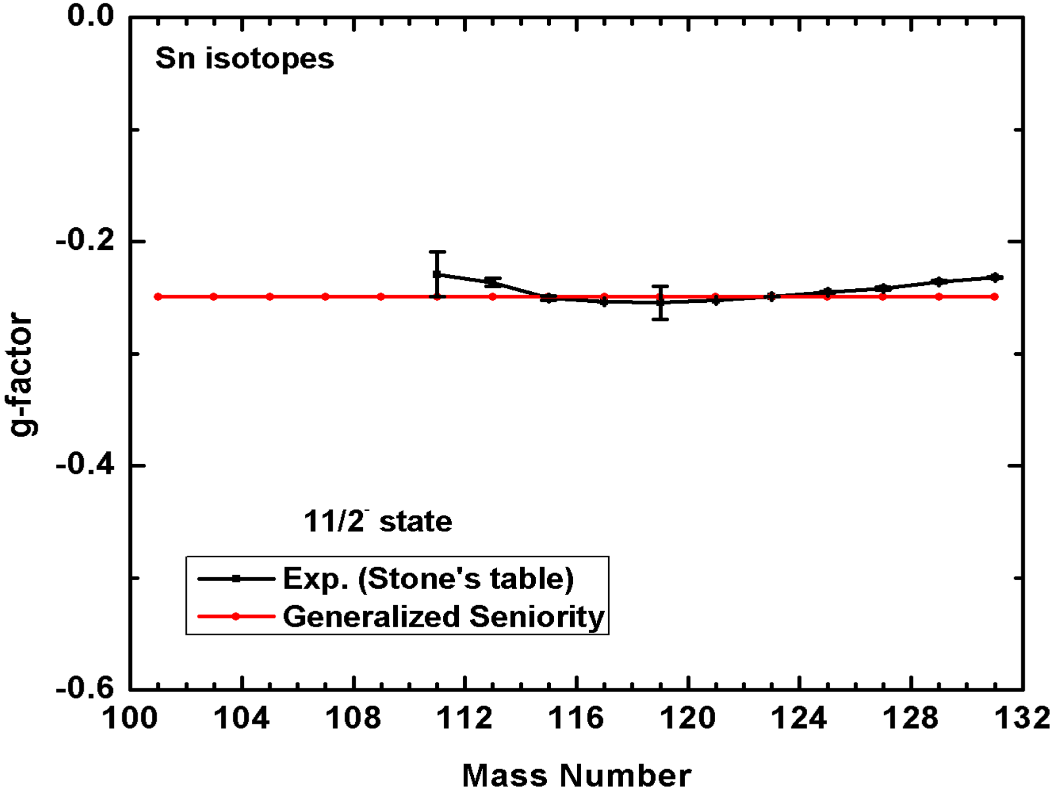
g-factor

The magnetic moments and g-factor must also show a particle number independent behavior.

The magnetic moment of identical nucleons in pure j^n configuration is given by (Talmi's book)

$$\mu = \sum g\, j_i = g \sum j_i$$
$$= g\, J$$

Hence, g-factors of all states of j^n configuration of identical nucleons should be equal to the g-factor of a single-j nucleon.



N.J. Stone, INDC(NDS)-0658, Feb. 2014
www-nds.iaea.org/publications

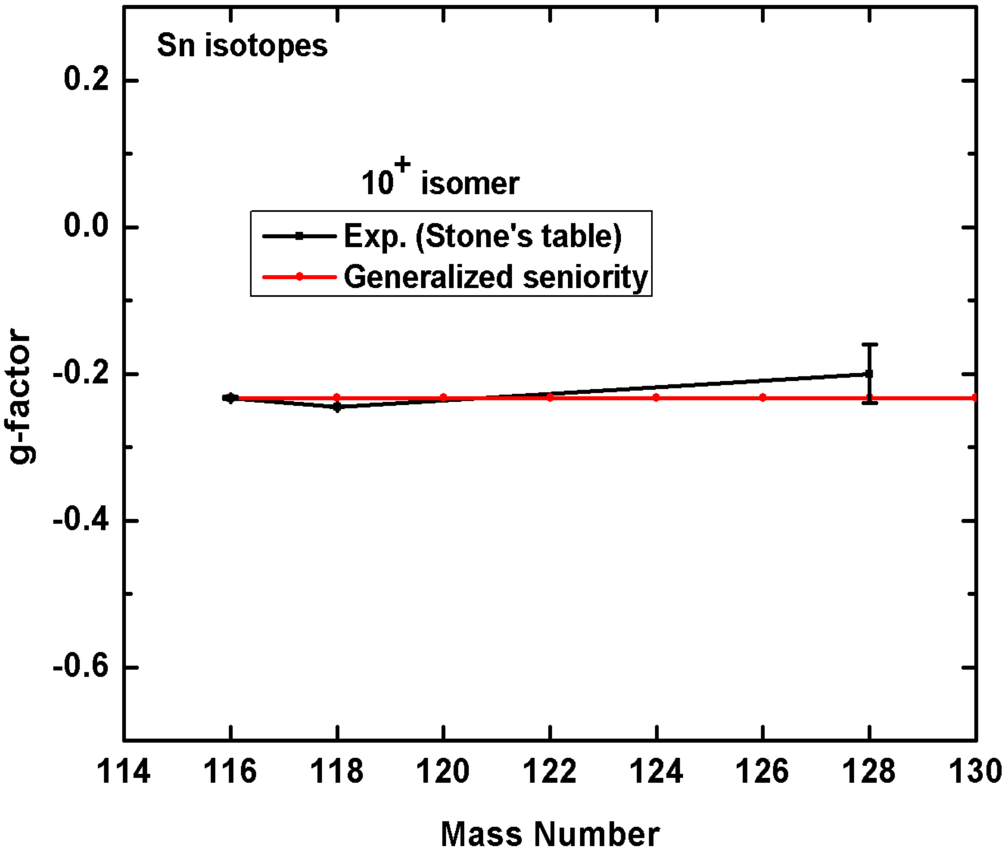
g-factor trend for 10^+ state in Sn isotopes

The magnetic moment of identical nucleons in the mixed configuration $\tilde{j}^n$ is given by

$$\mu = g\sum_{i=1}^{n}\tilde{j} = gJ$$

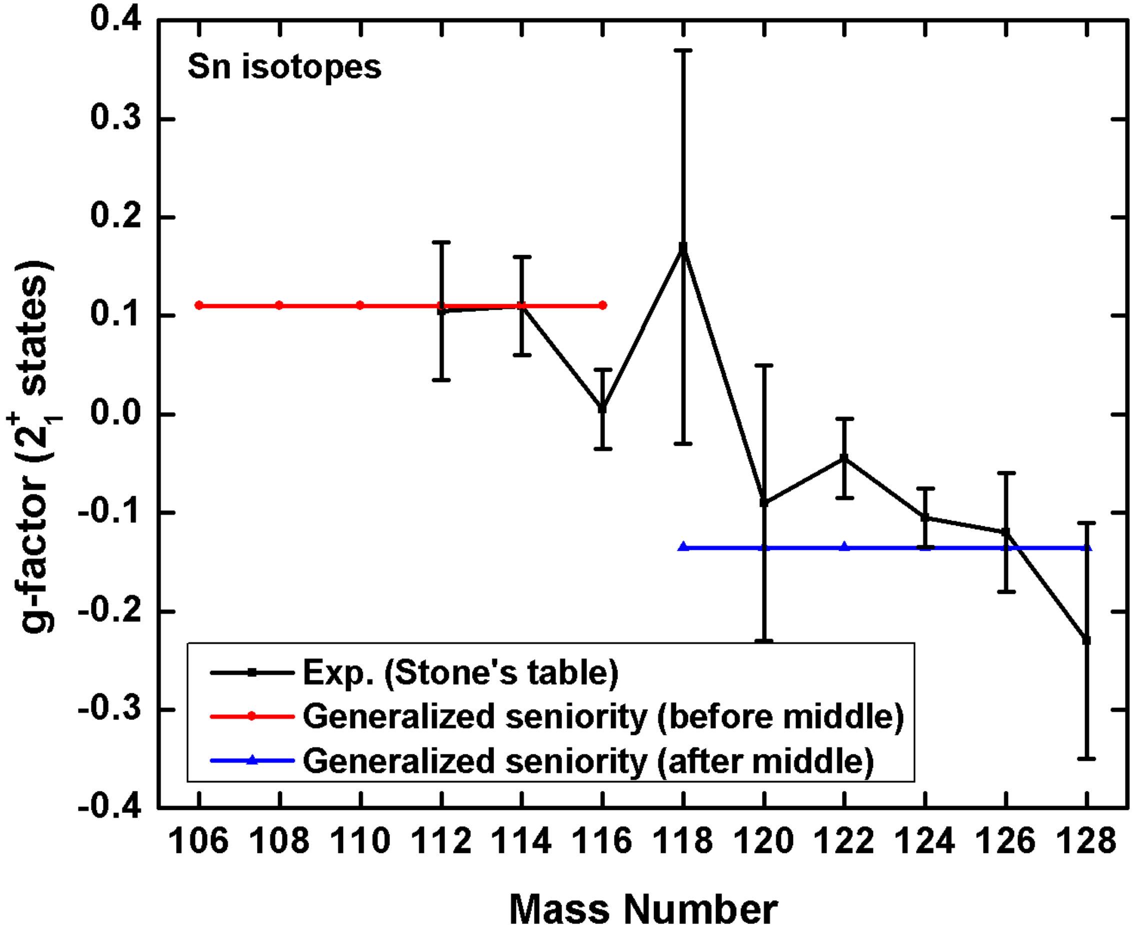
Hence, g-factors of all 10^+ states of configuration $\tilde{j}^n$ of identical nucleons should be equal to the g-factor of a single nucleon.

However, the data on $27/2^-$ isomer is not available till date. We expect and predict the g-factor of this state to be of the same order as for 10^+ state, since both follow same configuration mixing as shown in B(E2) results.



Points out the need of experiments in this direction!

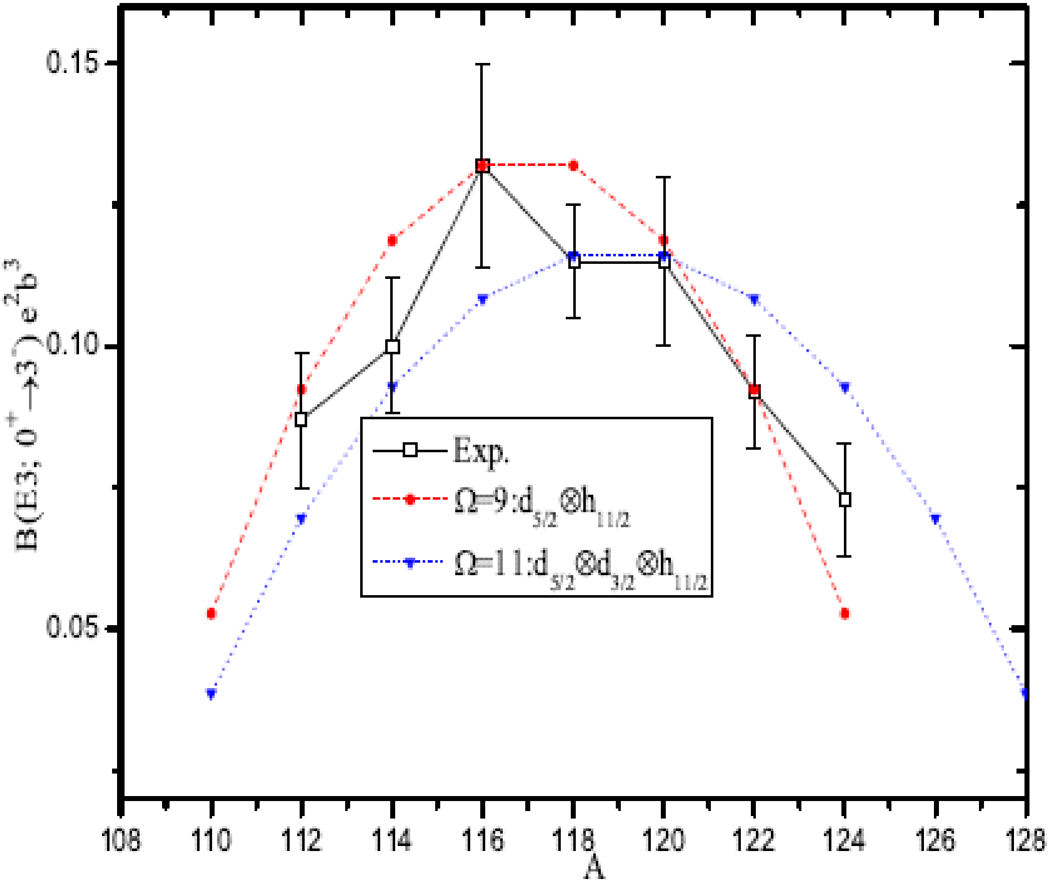
g-factor of first excited 2^+ states in Sn isotopes



Two parabolas in B(E2) curve highlights the difference in the configuration before and after the middle for the generation of the 2+ states in Sn isotopes.

Therefore, g-factor before and after the middle are expected to be different.

BE3 in Yrast 3- states in Sn isotopes



d5/2,h11/2 together explain the data very well.

Suggests an octupole character for the 3- states.

This interpretation is supported by the shell model calculations.

Pramana-J.Phys. 89, 75 (2017)

Summary

- We have presented a view of the seniority nuclear isomers in various semi-magic chains of isotopes.
- Their various properties and unique features have been studied.
- Examples from Z=50, Z=82, N=126 chains have been presented.
- Both seniority conserving and seniority changing transitions can be understood very well.
- Many predictions are possible based on this scheme.

Thank You