Do quarks play explicit role in nuclear structure?

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Content

- Introduction
- The model: Strongly Correlated Quark Model (SCQM)
  - Nucleon Structure
  - Nucleus Structure
- Face Centered Cubic (FCC) lattice Model (*N.D. Cook)*
- Light Nuclear Isotopes
- Summary

Introduction

Idea: From symmetries of QCD to nuclear structure

- Symmetries in QCD
  - $SU(3)_c \times SU(2)_f \times SU(2)_S$
  - Chiral symmetry
  - Local gauge symmetry

Introduction (cont.)

QCD – fundamental theory of strong interactions

- **Constituents of hadrons** – **quarks** of different flavors carrying spin, charge, color.
  - **flavors: u, d, s, c, b, t**
  - **spin: ½**
  - **charge: ⅓ , ⅔**
  - **color: $\mathbf{SU(3)_{Color}}$** - *R, G, B*
- **Gauge fields – gluons** perform interactions between quarks.
- **Nucleons –** 3–quark (**u/d**), color-singlet systems
- **Mesons –** quark-antiquark systems

Introduction (cont.)

QCD is non-abelian theory → hard to derive the features of hadrons and nuclei from the first principles of QCD.

Hadronic processes with high Q^2

pQCD: $\alpha_S < 1$, $m_q \to 0$, chiral symmetry

Low energy hadron and nuclear physics

non-pQCD: $\alpha_S > 1$, $m_q \neq 0$, chiral symmetry breaking

- Low energy approx. of QCD
- QCD–inspired phenomenology
  - NR constituent quark models
  - Bag models
  - Chiral quark models
  - **Soliton** models

pQCD → low energy
Chiral Symmetry Breaking

- $m_q = 0$
- Chiral Symmetry

  $SU(2)_L \times SU(2)_R$ for $\psi_{L,R} = u, d$ – current quarks

- Chiral symmetry breaking ≡ quark or *chiral* condensate:

  $\langle \bar{\psi}\psi \rangle \simeq -(250\ \text{MeV})^3, \quad \psi = u,d$

- As a consequence massless valence quarks (u, d) acquire dynamical masses which we call constituent quarks

$$M_C \approx 350 - 400\ \text{MeV}$$

The Model:

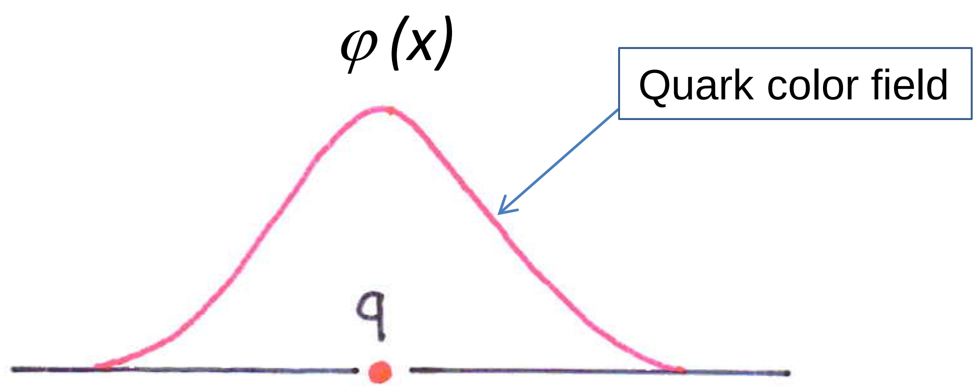
Strongly Correlated Quark Model of Hadron Structure

⇓

Strongly Correlated Quark Model of Nucleus Structure

Strongly Correlated Quark Model

(SCQM)



Vacuum polarization by color field of a single quark

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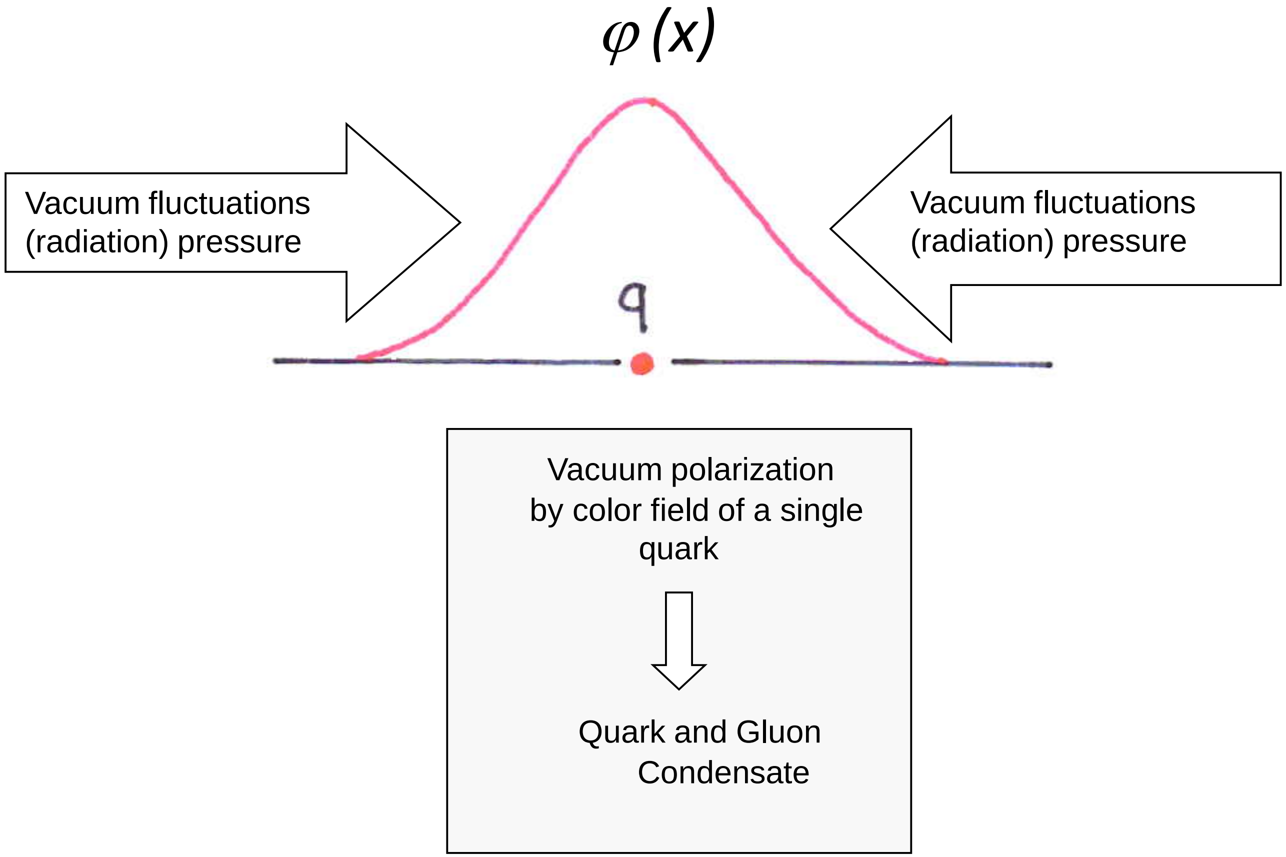
Quark and Gluon Condensate

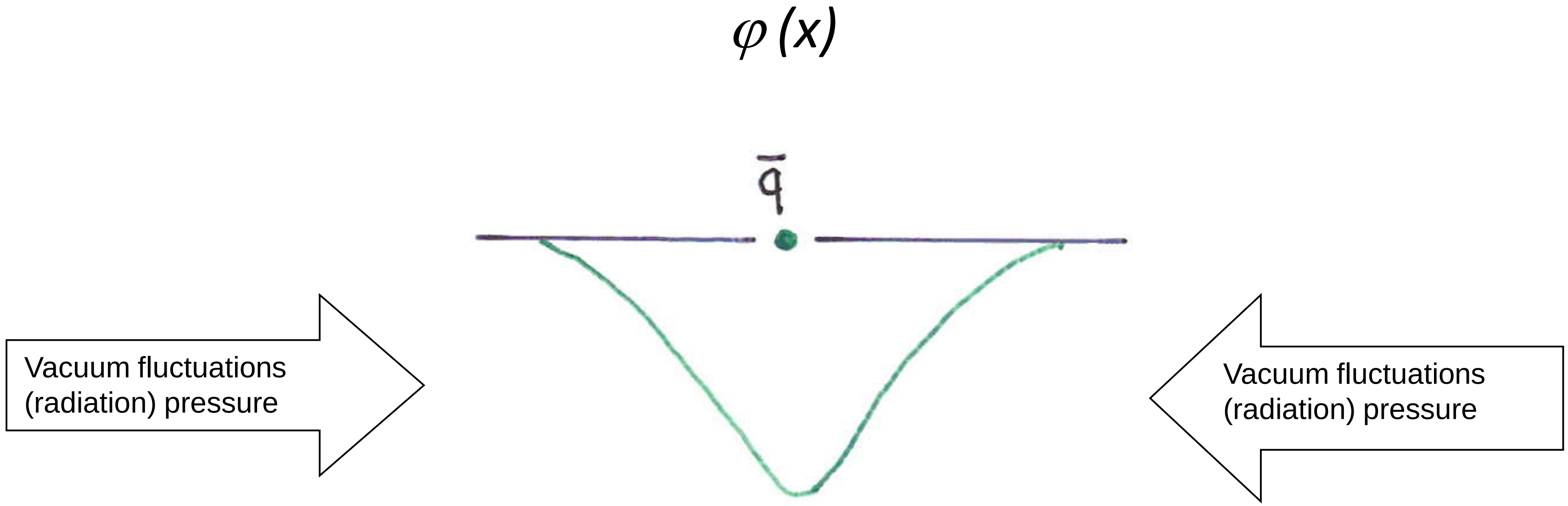
Strongly Correlated Quark Model

(SCQM)



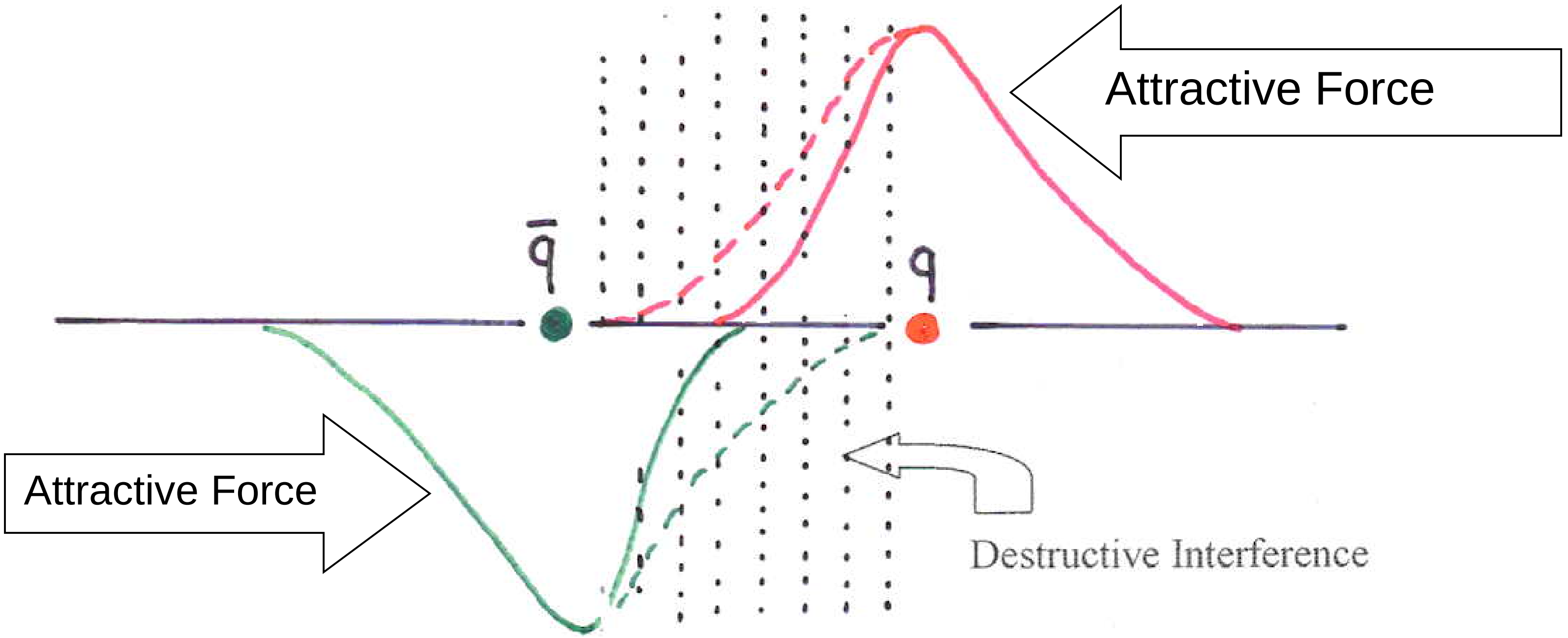
Strongly Correlated Quark Model

(SCQM)



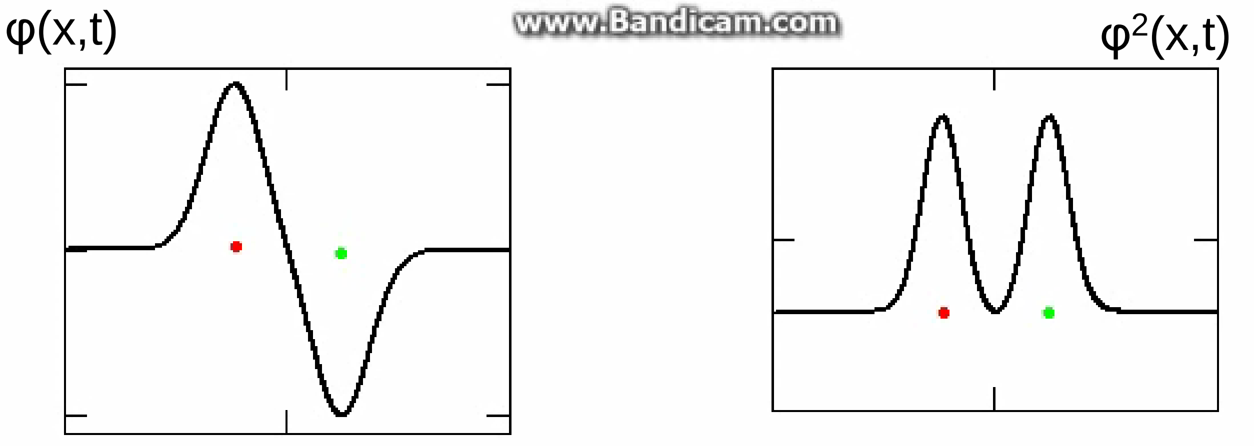
Strongly Correlated Quark Model

(SCQM)



Overlap of opposite color fields → attraction force between quark and antiquark

quark – antiquark pair
soliton – antisoliton pair



Constituent Quarks – Solitons

SCQM ≡ Breather Solution of
Sine- Gordon equation

$$\partial_\mu \partial^\mu \phi(x,t) + \sin\phi(x,t) = 0$$

Breather – oscillating soliton-antisoliton pair,
the periodic solution of SG:

$$\phi(x,t)_{s-as} = 4\tan^{-1}\left[\frac{\sinh\left(ut/\sqrt{1-u^2}\right)}{u\cosh\left(x/\sqrt{1-u^2}\right)}\right]$$

$$\varphi(x,t)_{s-as} = \frac{\partial\phi(x,t)_{s-as}}{\partial x}$$

is identical to our quark-antiquark system.

The Strongly Correlated Quark Model

Hamiltonian of the Quark – AntiQuark System

$$H = \frac{m_{\bar{q}}}{(1-\beta_{\bar{q}}^{\,2})^{1/2}} + \frac{m_q}{(1-\beta_q^{\,2})^{1/2}} + V_{\bar{q}q}(2x)$$

$m_{\bar{q}}$, m_q are the current masses of quarks,

$\beta = \beta(x)$ *– the velocity of the quark (antiquark),*

$V_{\bar{q}q}$ *is the quark–antiquark potential.*

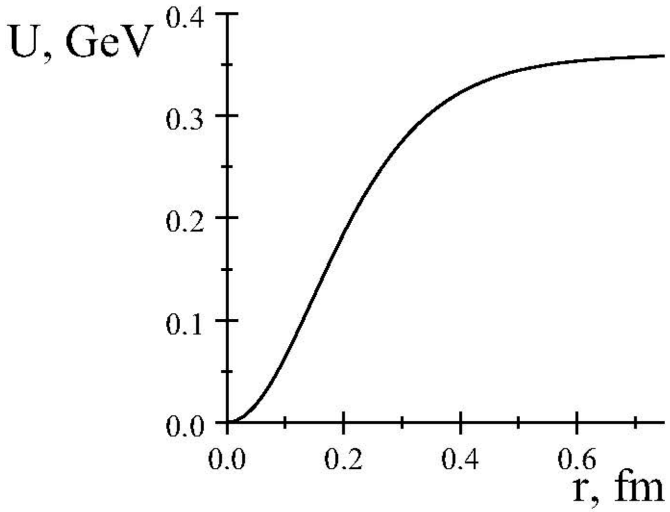
$$H = \left[\frac{m_{\bar{q}}}{(1-\beta_{\bar{q}}^{\,2})^{1/2}} + U(x)\right] + \left[\frac{m_q}{(1-\beta_q^{\,2})^{1/2}} + U(x)\right]$$

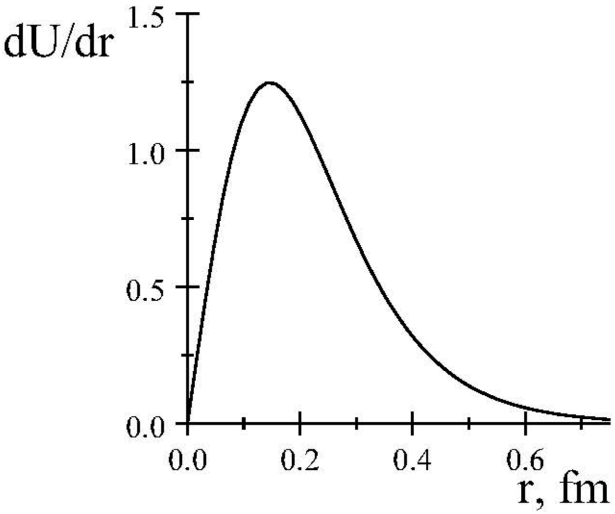
$U(x) = \frac{1}{2} V_{\bar{q}q}(2x) = m \tanh^2(ax)$ is the potential energy of a single quark.

Quark Potential

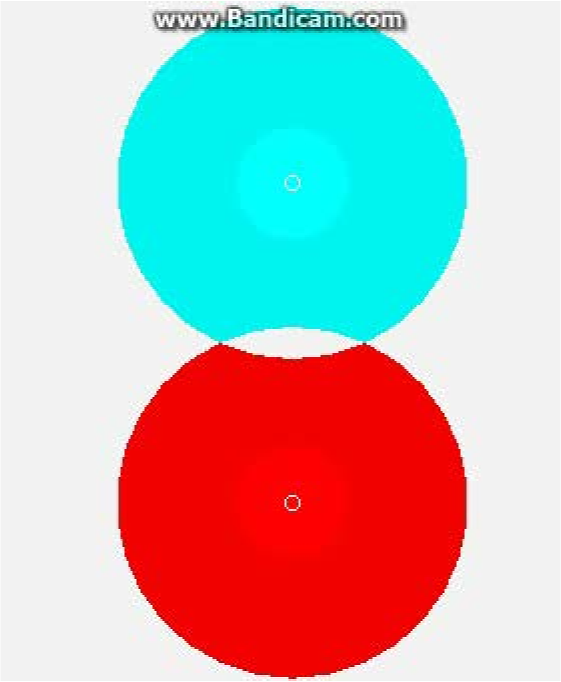
Force of quark-antiquark interaction

$U_q = 0.36\tanh^2(ax)$





quark–antiquark pair
meson



QCD: Exchange by gluons

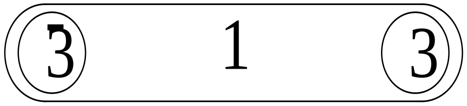
SCQM: Overlap of color fields

Generalization to the 3 – quark system (baryons)

$SU(3)_{Color}$

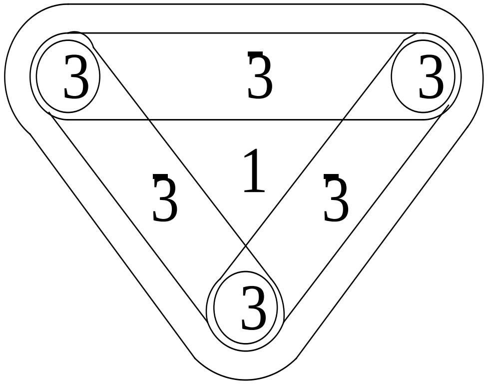
$$q \Rightarrow \mathbf{SU(3)} \Leftrightarrow \mathbf{RGB} \qquad \bar{q} \Rightarrow \mathbf{SU(\bar{3})} \Leftrightarrow \mathbf{CMY}$$

$$\bar{q}q \Rightarrow$$

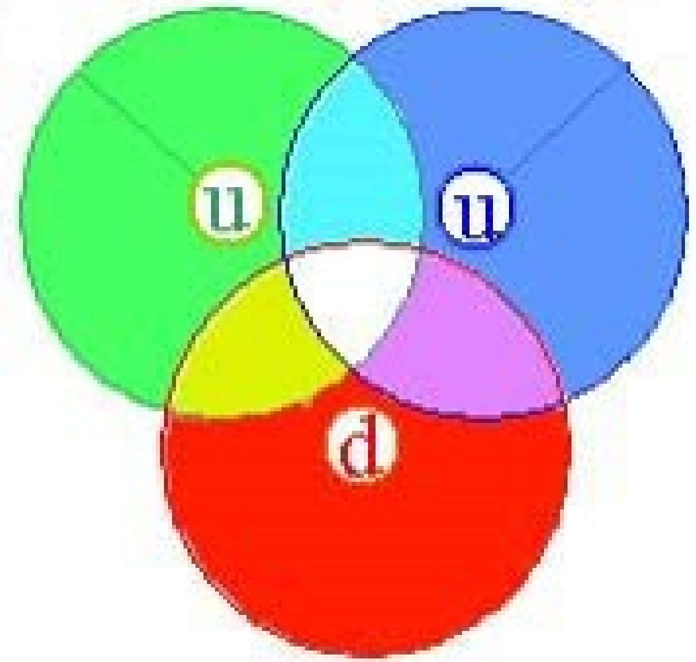


$$qq \rightarrow 3 \times 3 = 6 \oplus \bar{3} \quad \Rightarrow \quad \bar{q} \rightarrow qq$$

$$qqq \Rightarrow$$



Nucleon



SCQM $\Longrightarrow$ **The Local Gauge Invariance Principle**

Destructive Interference of color fields ≡ Phase rotation of the quark w.f. in color space:

$$\psi(x)_{Color} \to e^{ig\theta(x)}\psi(x)$$

Phase rotation in color space $\Longrightarrow$ quark dressing (undressing) ≡ the gauge transformation

$$A^{\mu}(x) \to A^{\mu}(x) + \partial^{\mu}\theta(x)$$

Therefore, during quark oscillation its

color charge

momentum

mass

are continuously varying functions of time.

Relation SCQM to QCD

Considering a single quark oscillating in the potential U(r) we reduce interaction of color quarks via **non-Abelian** fields to **E-M** analog:

$$A_a^{\mu}(x) \rightarrow A^{\mu}(x)$$

$$F_a^{\mu\nu} = \partial^{\mu} A_a^{\nu} - \partial^{\nu} A_a^{\mu} - \lambda f^{abc} A_b^{\mu} A_c^{\nu} \rightarrow F_{ch}^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

Parameters of SCQM for the Nucleon

1.Mass of Consituent Quark

$$M_{Q(\overline{Q})}(x_{\max}) = \frac{1}{3}\left(\frac{m_{\Delta} + m_{N}}{2}\right) \approx 360 MeV,$$

2.Amplitude of VQs oscillations : x_{max}=0.64 fm,

3.Constituent quark dimensions (parameters of gaussian distribution): $\sigma_{x,y}$=0.24 fm, σ_z =0.12 fm

Parameters 2 and 3 are derived from the calculations of Inelastic Overlap Function (IOF) σ_{tot} and $\bar{p}\,p$ in and *pp* – collisions.

Summary on Quarks in Hadrons

- Quark – quark interaction is due to **overlap** of their color fields
- Constituent quarks are identical to solitons.
- Quarks inside nucleons are strongly correlated;
- Hadronic matter distribution inside hadrons is fluctuating quantity;
- Nucleons are not spherically symmetric, **oblate objects**.

Quark Arrangement inside Nuclei

Strongly Correlated Quark Model

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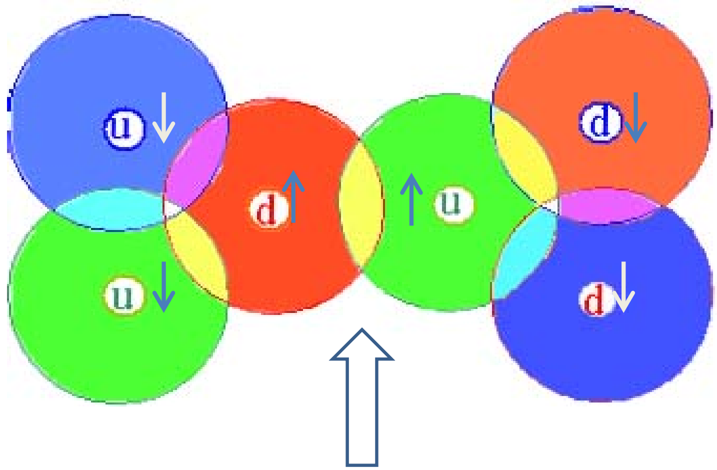
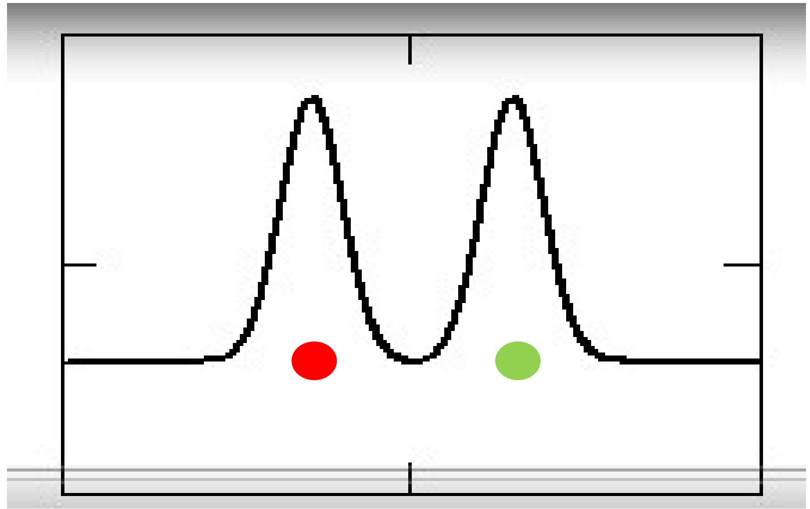
Crystal-like arrangement of Nuclear Structure

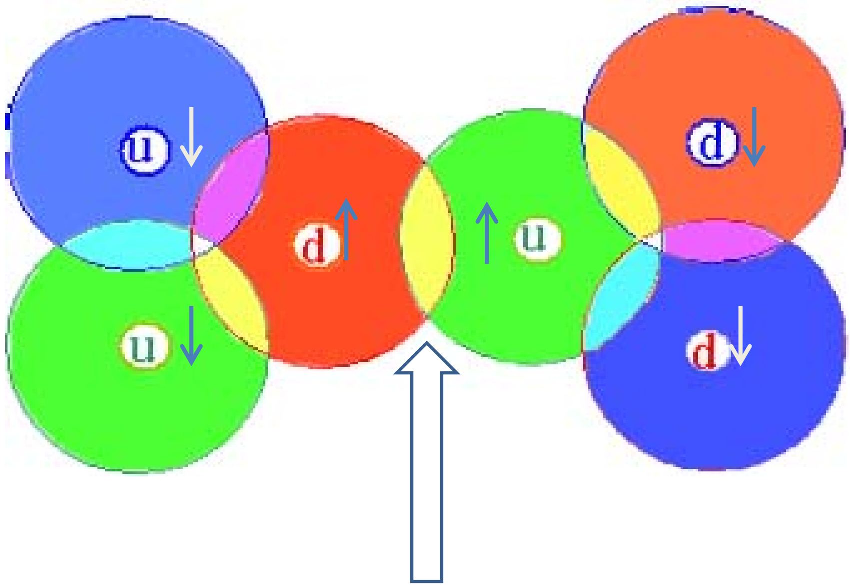
Two Nucleon System in SCQM



Interaction between nucleons is due to **overlap** of their quark color fields



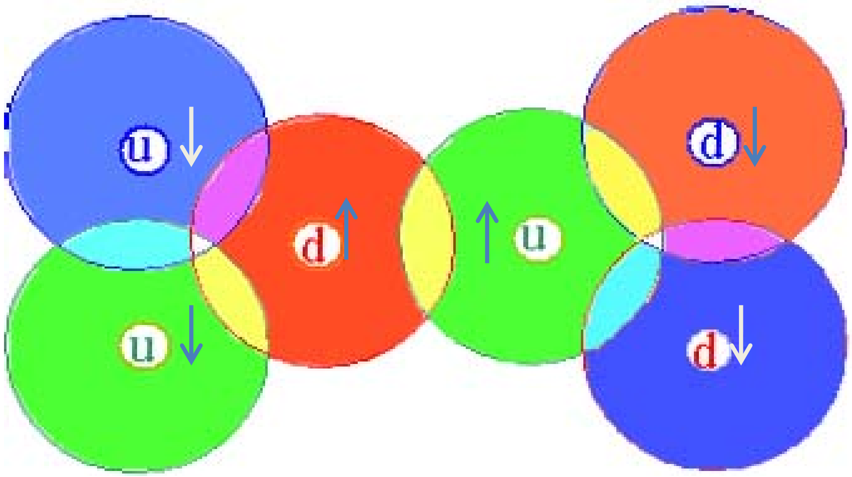
Two Nucleon System in SCQM



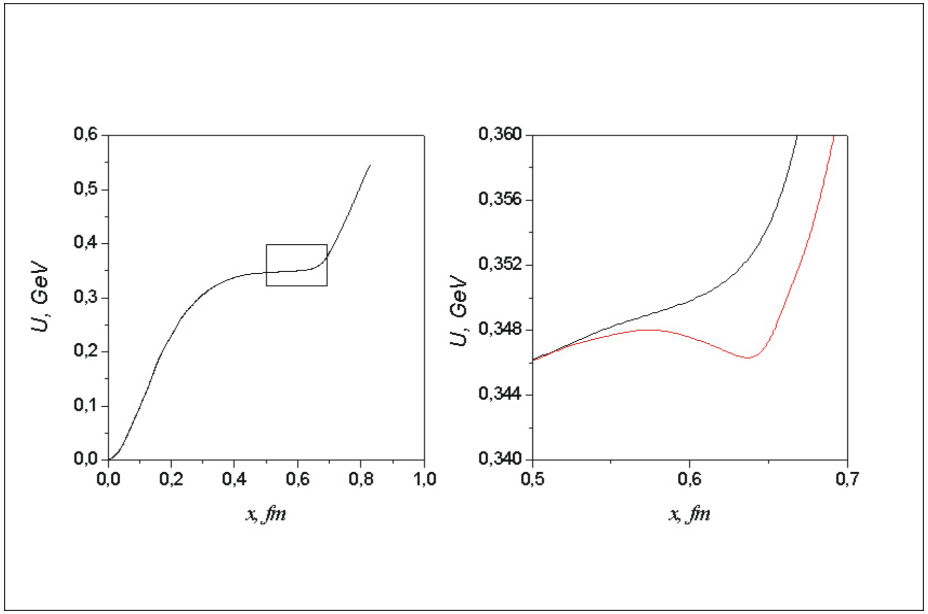
Selection rules for binding two quarks of neighboring nucleons at a junction:

- $SU(3)_{Color}$ – of different colors
- $SU(2)_{Flavor}$ – of different flavors
- $SU(2)_{Spin}$ – of parallel spins

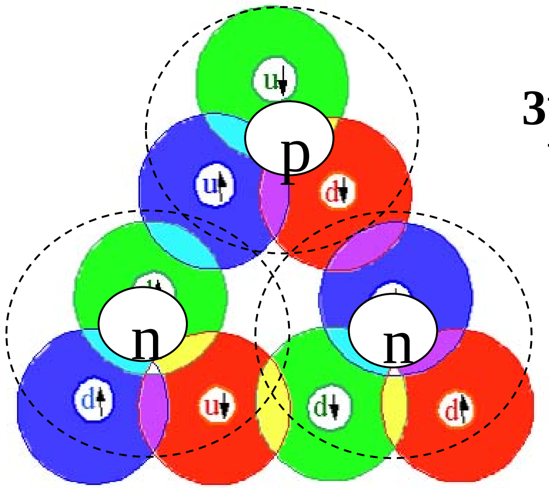
Two Nucleon System in SCQM



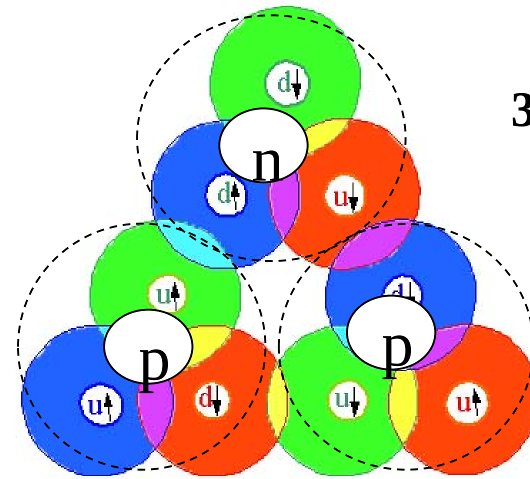
Quark Potential Inside Nuclei



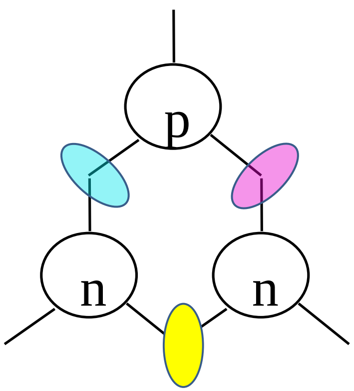
Three Nucleon Systems in SCQM



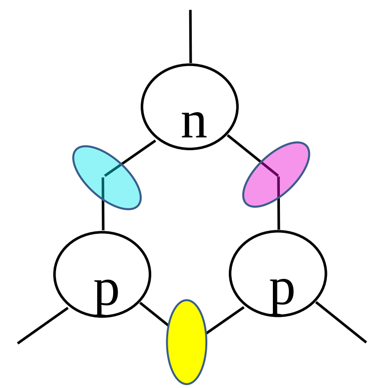
^3H



^3He



Summary color charge at 3 junctions = zero!



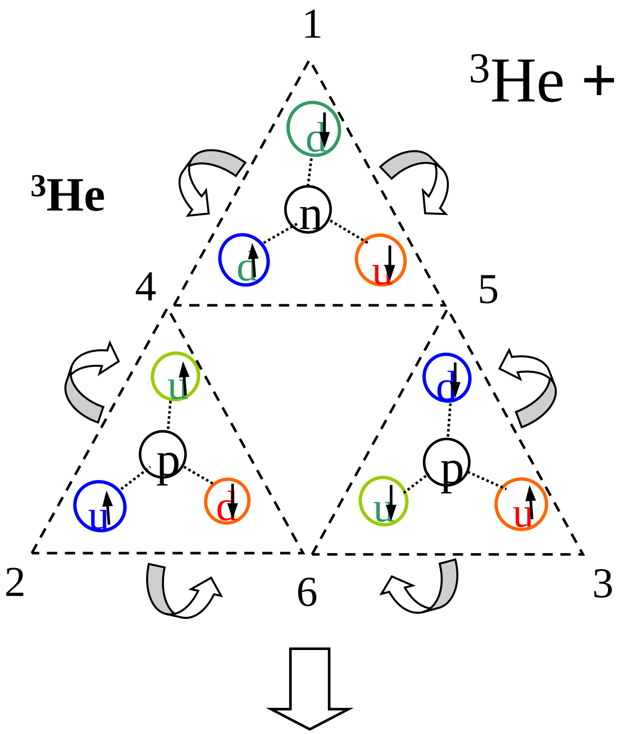
Quark loop formed by 3 nucleons → 3–body force

4-nucleon system: ^4He

n

^3He + neutron or ^3H + proton

^3He

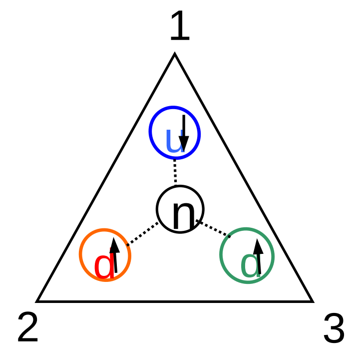


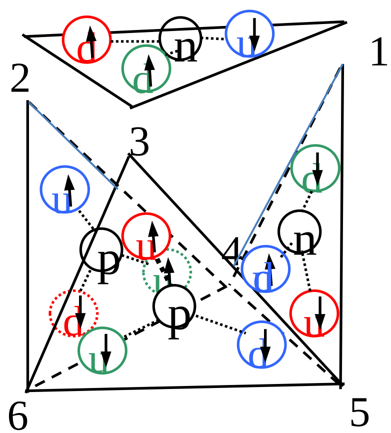
Junctures

1 ←→ 1

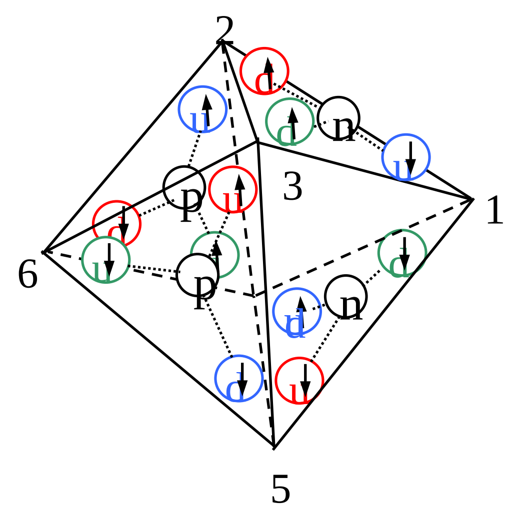
2 ←→ 2

3 ←→ 3





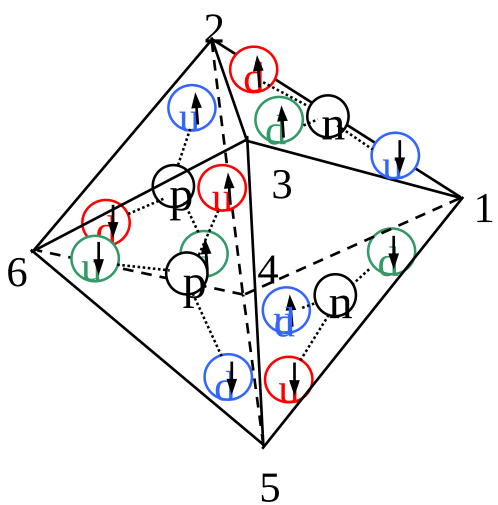
⇒ 4 – quark loops



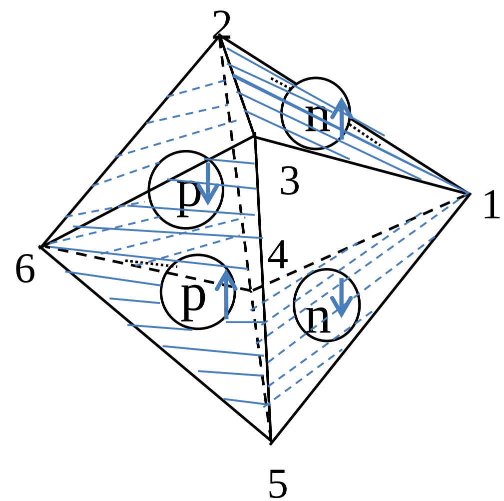
The closed shell n = 0, nucleus ^{4}He

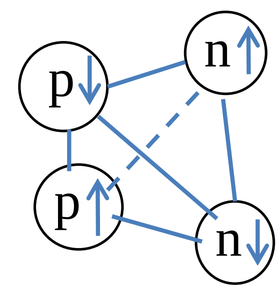
Antisymmetrization
12 quarks in SU(12) state
$SU(2)_F \times SU(2)_S \times SU(2)_C$

4 nucleons in **s-**state



Shell Closure





Octahedron with smooth edges and vertices – deformed sphere

Point-nucleon charge distributions of ^{3}**He** and ^{4}**He**

Hole inside ^{3}**He** and ^{4}**He**

I. Sick, PRC, vol. 15, No.4; LNP, vol. 87, p.236

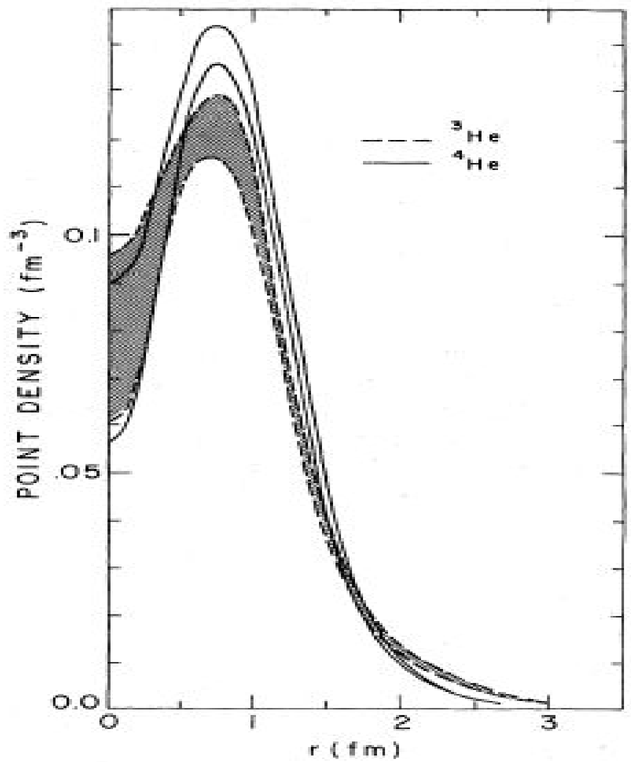


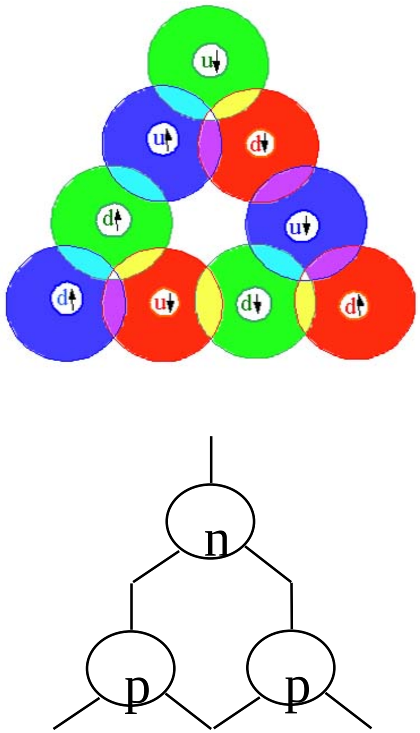
FIG. 15. Model-independent densities of pointlike protons in ^{3,4}He.

Binding Energy of Stable Nuclei Experiment

Nucleus	E_B, MeV per nucleon	Number of quark loops	Free quark ends	Nuclear forces
d	1.1	no	4	2-body
^3H	2.83	1	3	3-body
^3He	2.57	1	3	3-body
^4He	7.07	4	0	4-body

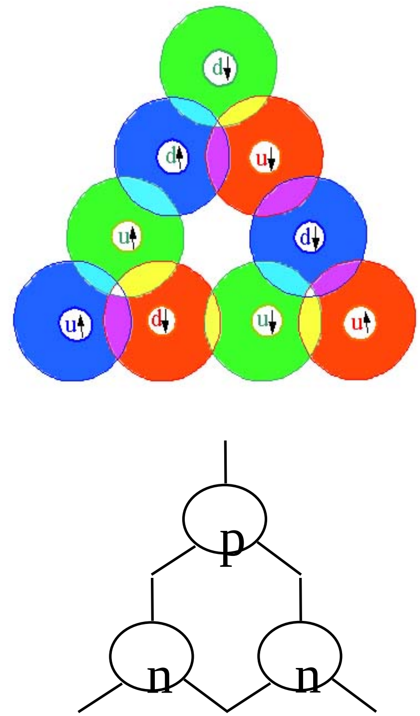
Building blocks in Nuclear Structure

^{3}H



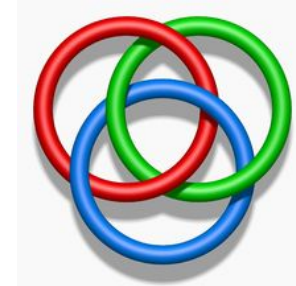
^{3}He – block

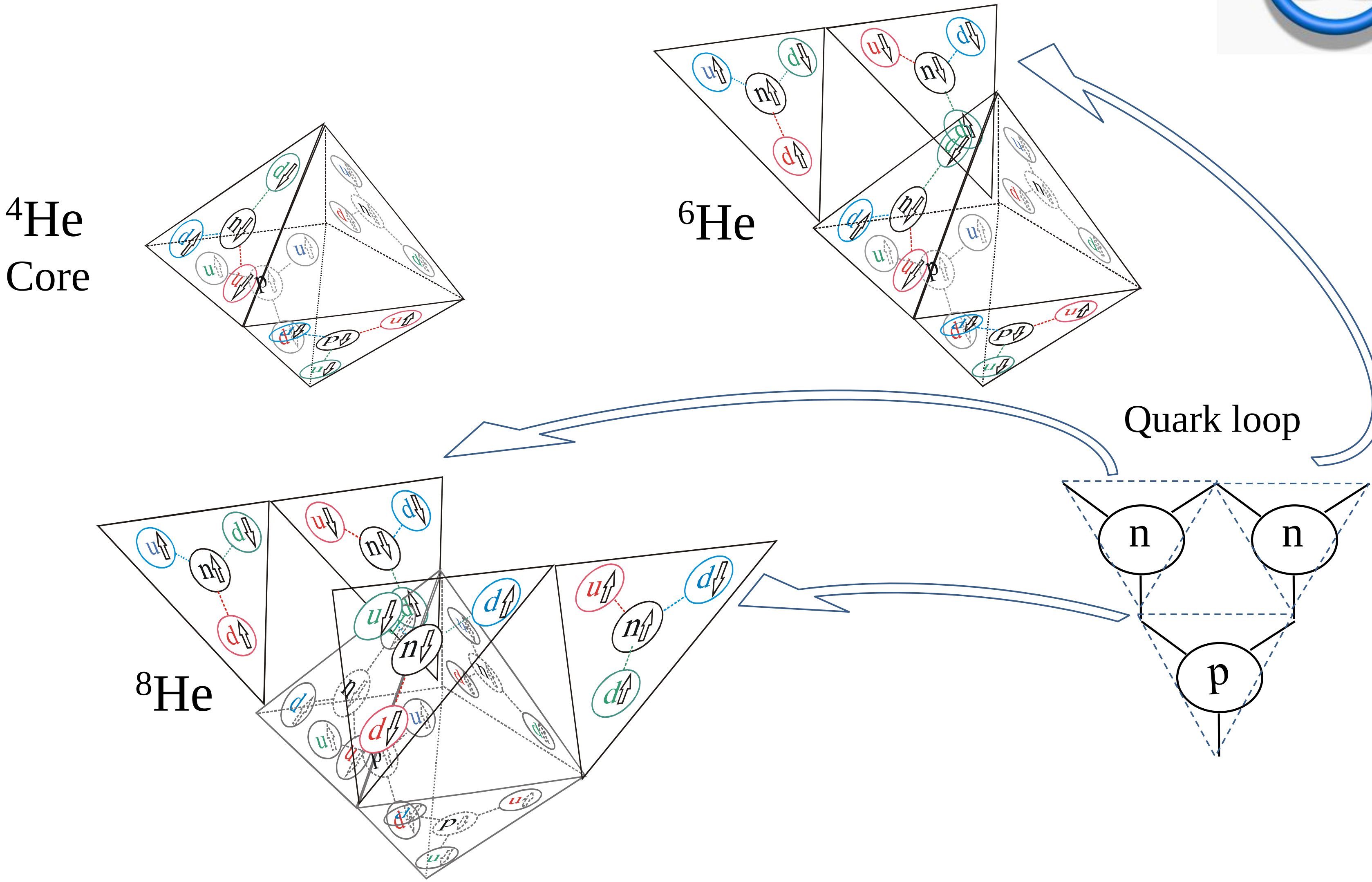
^{3}He



^{3}H – block

Helium Isotopes
Borromean Nuclei

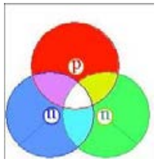


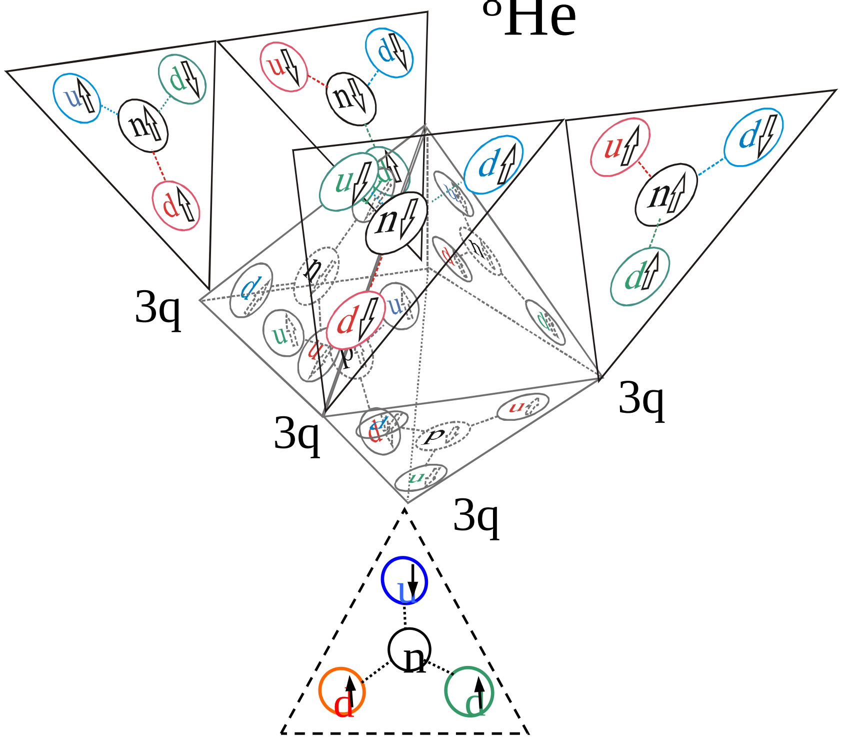


^{10}He – as a bound system does not exist!

Why? Only up to 3 quarks (RGB) can be linked at a junction

3q

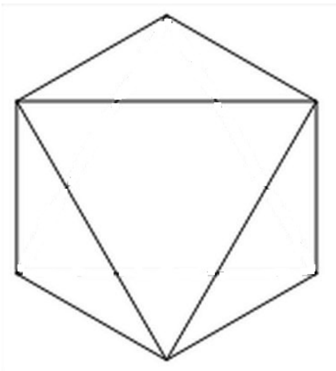




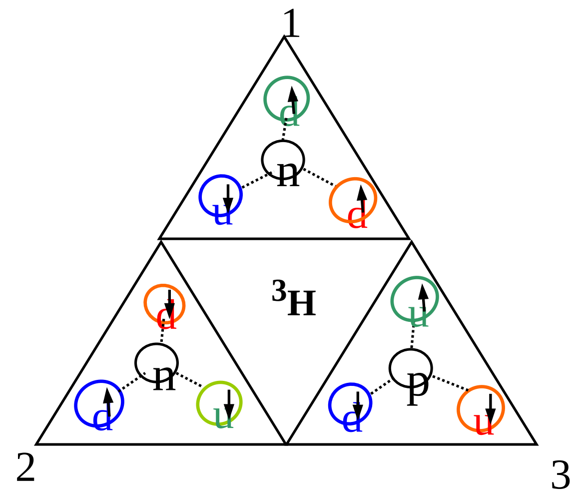
^{9}He- unbound, resonant state

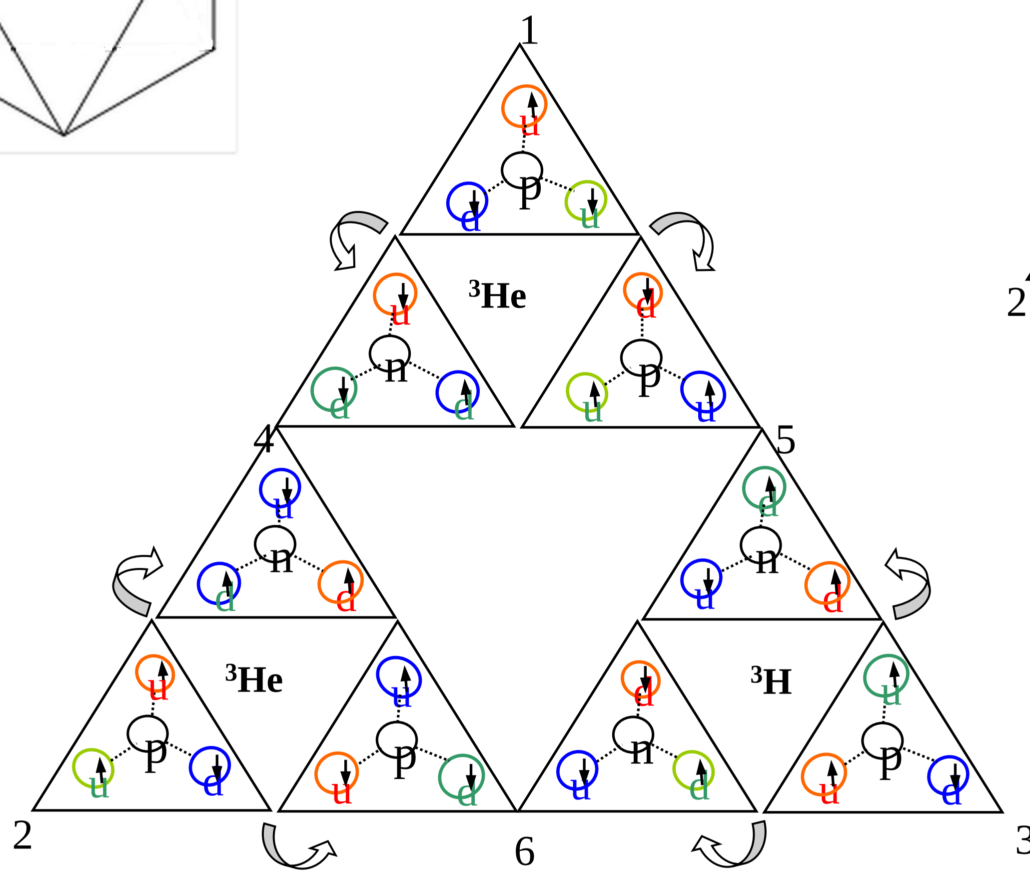
The closed shell n = 1, ^{16}O

Shell Closure

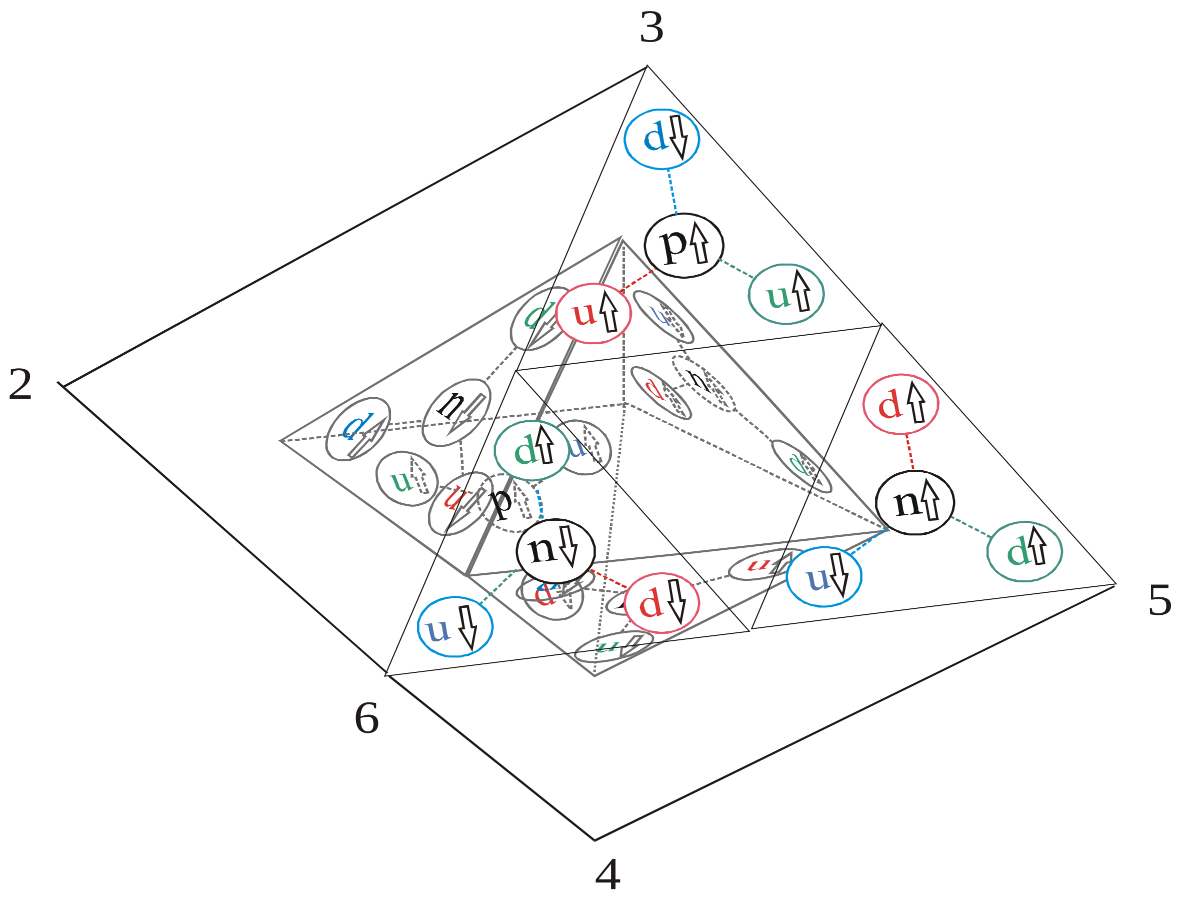


Face of ^{16}O octahedron

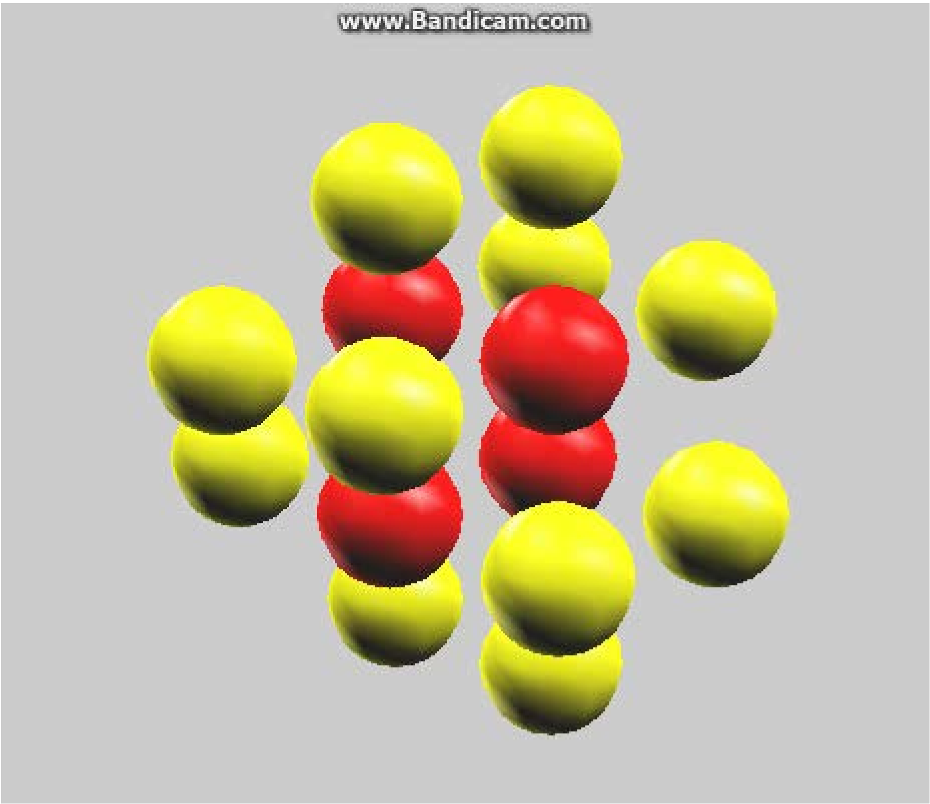




The closed shell n = 1, ^{16}O

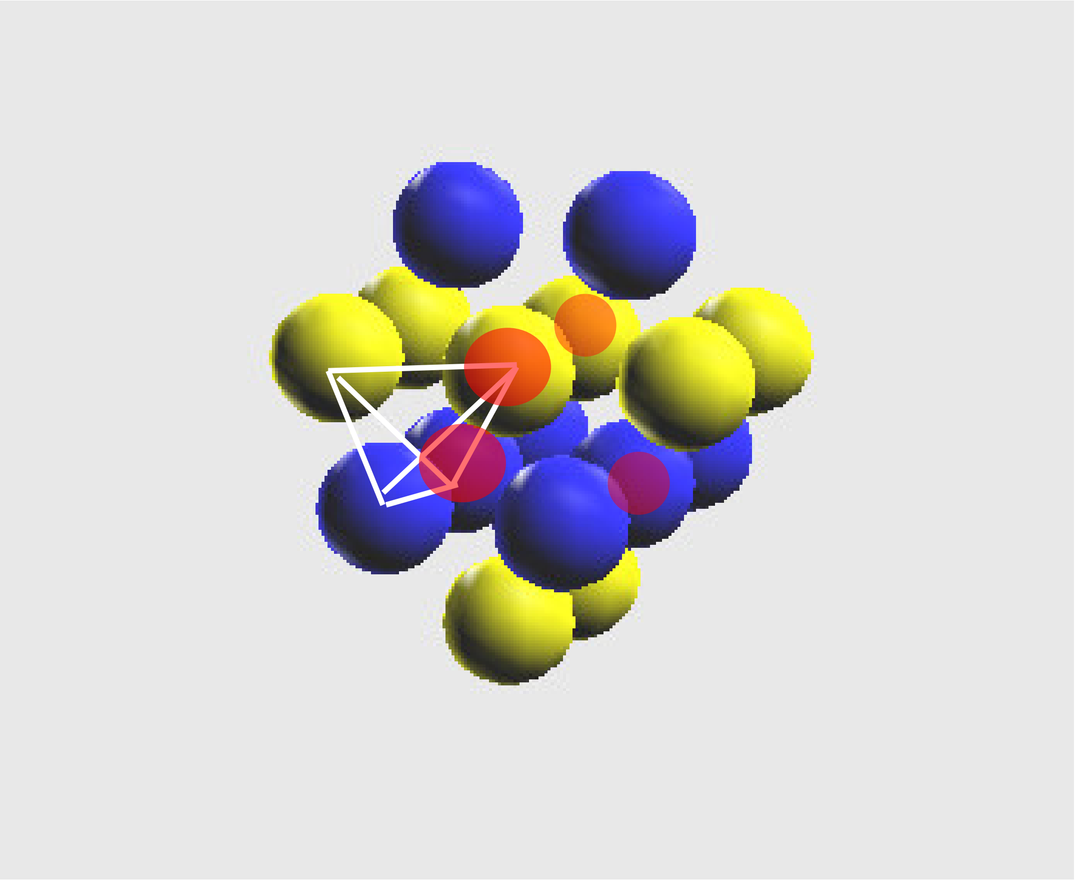


^{16}O



red-colored spheres – s-shell nucleons
yellow colored spheres – p-shell nucleons

Up to 6 virtual α-clusters in ^{16}O

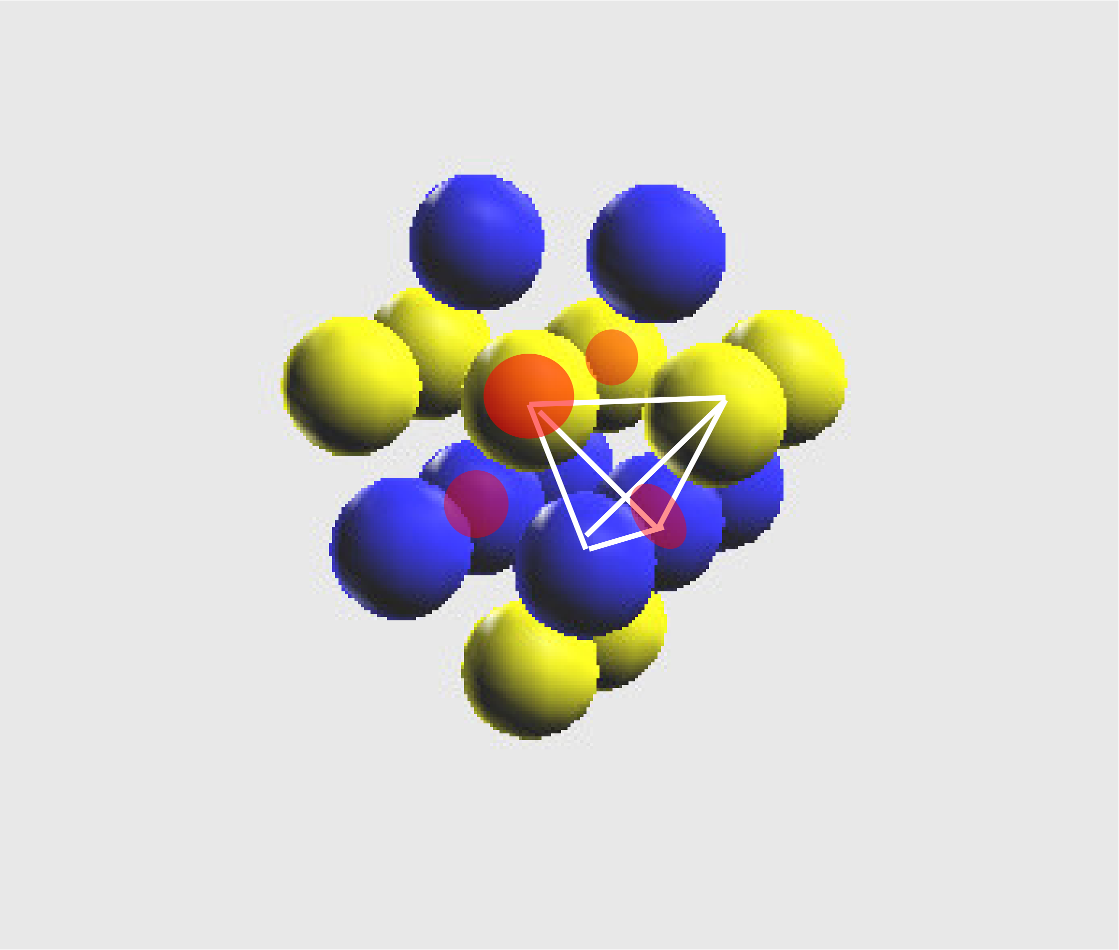


Marked by red spots are a nucleons exchanged by 2 neighboring virtual α-clusters

Blue-colored spheres – neutrons

Yellow-colored spheres - protons

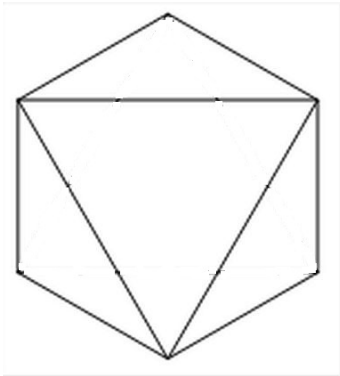
^{16}O



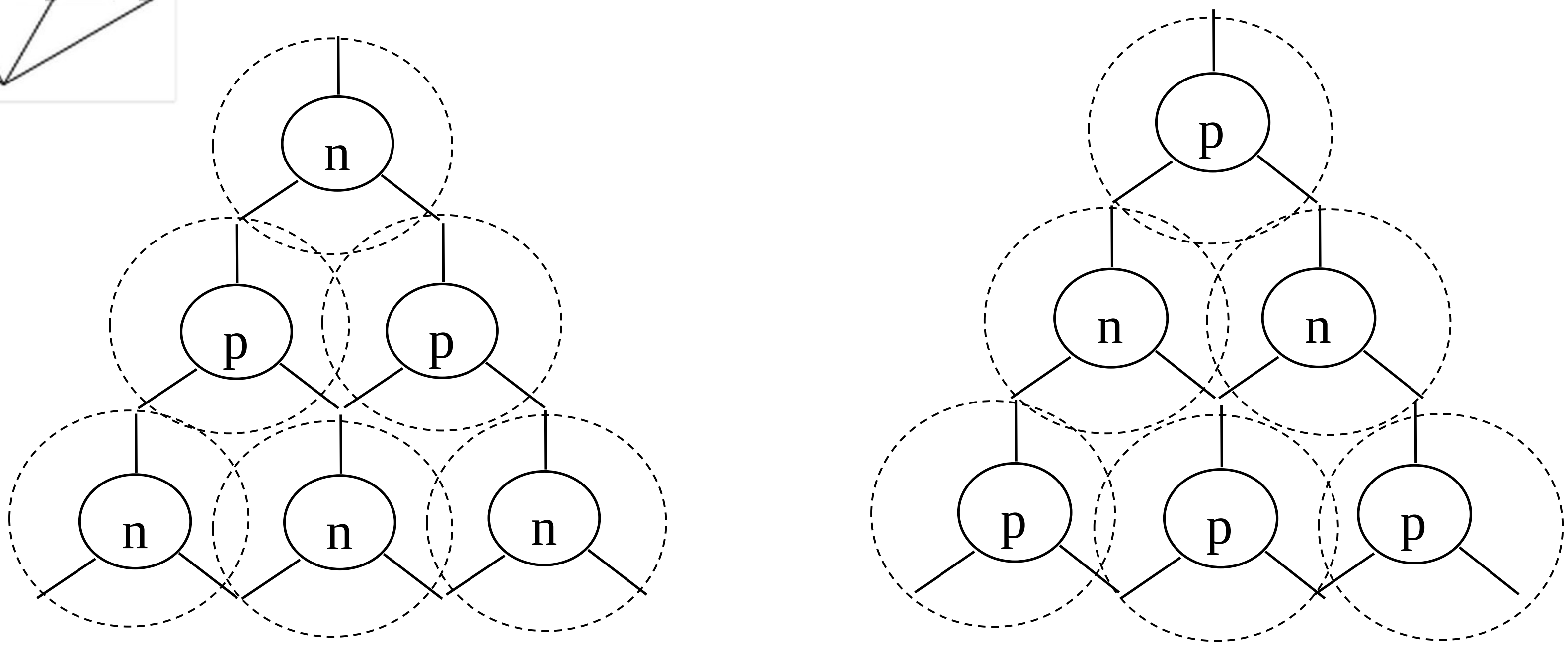
s-shell nucleons perform coupling between neighboring virtual α-clusters

The closed shell n = 2, ^{40}Ca

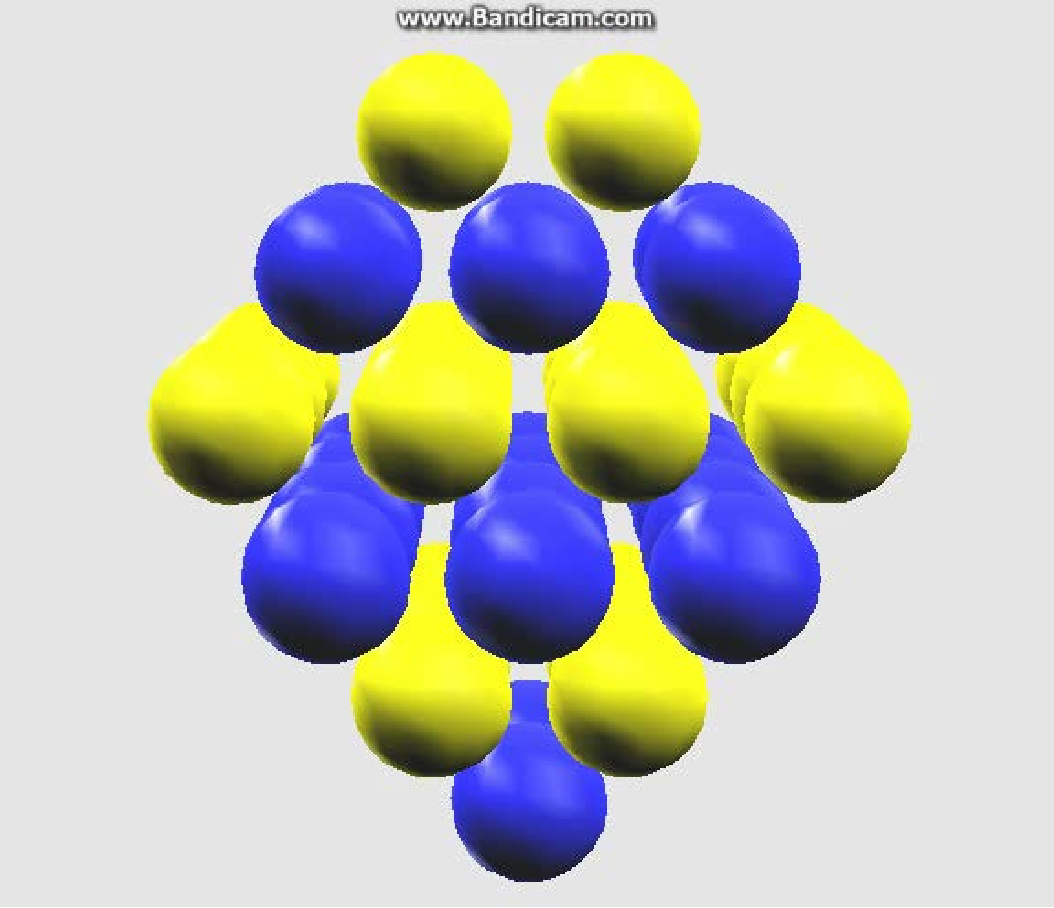
Shell Closure



Faces of ^{40}Ca octahedron



^{40}Ca



Blue-colored spheres – neutrons

Yellow-colored spheres - protons

Resume on nuclear symmetry

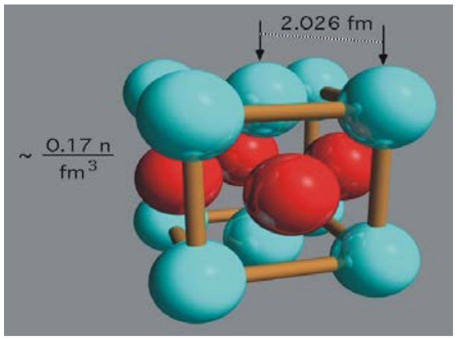
- Nucleon centers are located on the sites of face-centered cubic lattice.
- If we connect the nucleons positions by bonds the nuclei with a closure shells has a shape of tetrahedron (s-shell) and truncated tetrahadrons (p, d, f, …-shells).
- Nucleons are arranged in alternating (antiferromagnetic) spin, isospin layers.
- Nucleons in nuclei are arranged into virtual α-clusters and three-nucleon building blocks
- **SCQM is identical to FCC-lattice nuclear model of N.D. Cook!**

N.D. Cook and V. Dallacasa, Phys. Rev. C 36, 1883 (1987).

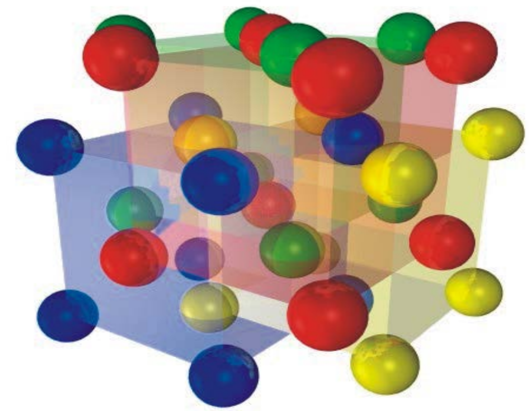
Face – Centered – Cubic Lattice Model (FCC)

(N. Cook, 1987)

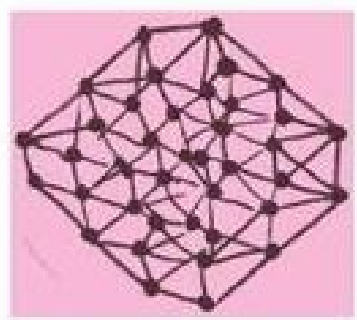
FCC unit



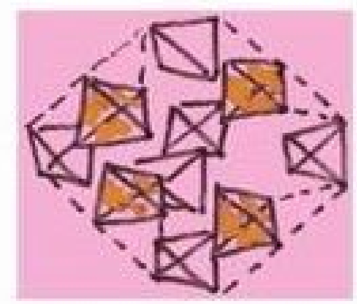
^{40}Ca



Shell model

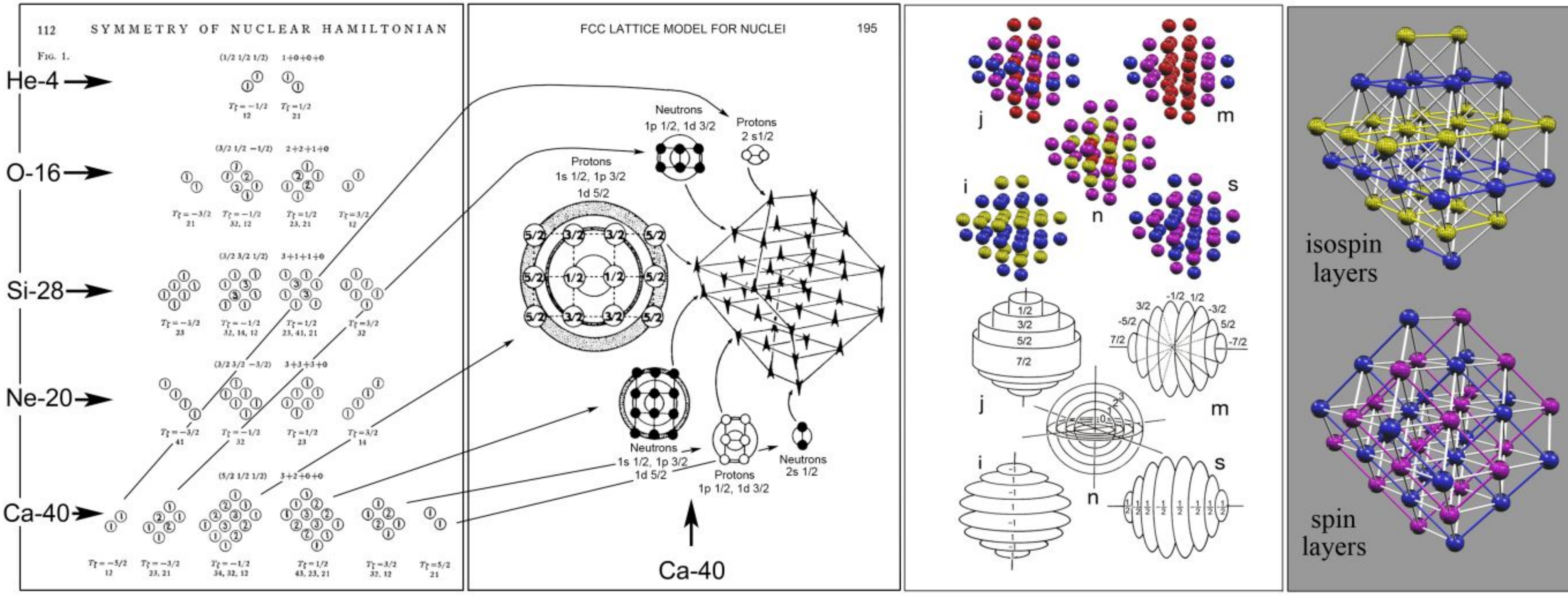


Liquid drop model



Cluster model

Face – Centered – Cubic Lattice Model (FCC)



(A) Wigner (1937) (B) Cook (1976) (C) NVS freeware (2010)

FCC Lattice Model

N.D. Cook and V. Dallacasa, Phys. Rev. C 36, 1883 (1987).

Particle in 3D box

$$-(h^2/2m)(d^2\Psi/dr^2) + V(r)\,\Psi(r) = E\,\Psi(r)$$

For harmonic oscillator potential cartesian coordinate system

$$E_N = h\omega_0(n_x + n_y + n_z + 3/2) = h\omega_0(N + 3/2)$$

$$N = 0, 1, 2, 3, \ldots$$

Different combinations of $\mathbf{n_x}$, $\mathbf{n_y}$ and $\mathbf{n_z}$ that give the same total **N** – value denote spatially distinct " degenerate" states, with the same energy.

If the origin of the coordinate system is taken as the center of the central tetrahedron, then the closure of each consecutive, symmetrical (x=y=z) geometrical shell in the lattice composes precisely the numbers of nucleons in the shells derived from the three-dimensional Schrodinger equation.

FCC Lattice Model

- The principal quantum number, **n**

  Assuming x, y and z coordinates of nucleons are odd – integers, define n – value of k-th nucleon as

$$n_{\text{nucleon(k)}} = (|x_{\text{nucleon(k)}}| + |y_{\text{nucleon(k)}}| + |z_{\text{nucleon(k)}}| - 3)/2$$

The first shell (s-shell, n=0) contains 4 nucleons with coordinates 111, -1-11, 1-1-1, -11-1.

The second shell (p-shell): 12 nucleons
31-1, 3-11, -311, -3-1-1, 1-31, -131, 13-1, -1-3-1, -113,
11-3, 1-13, -1-1-3
and so on…

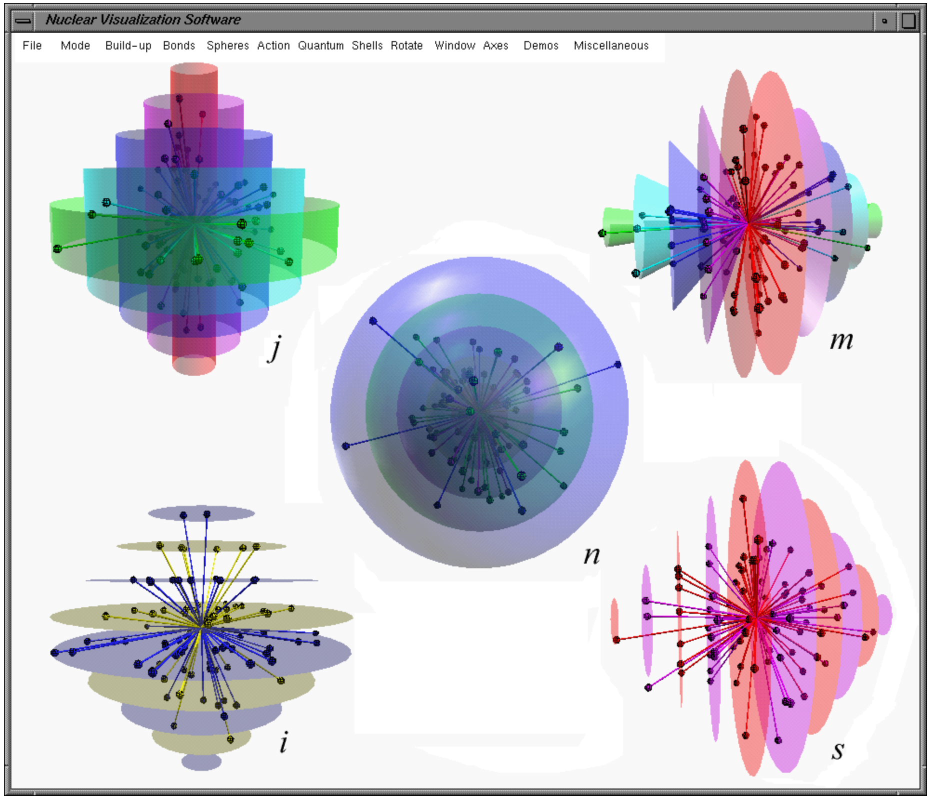
- Total angular momentum, **j**

$$j_{\text{nucleon(k)}} = (|x_{\text{nucleon(k)}}| + |y_{\text{nucleon(k)}}| - 1)/2$$

- Magnetic quantum number, **m**

$$m = |x|/2$$

FCC Lattice Model



$$\mathbf{n}=(x + y + z - 3)/2 = (r \sin\theta \cos\phi + r \sin\theta \sin\phi + r \cos\theta - 3) / 2$$

$$\mathbf{j} = l + s = (x + y - 1) / 2 = (r \sin\theta \cos\phi + r \sin\theta \sin\phi - 1) / 2$$

$$\mathbf{m} = x / 2 = (r \sin\theta \cos\phi) / 2$$

SCQM+FCC ←→ SM

Coincidence

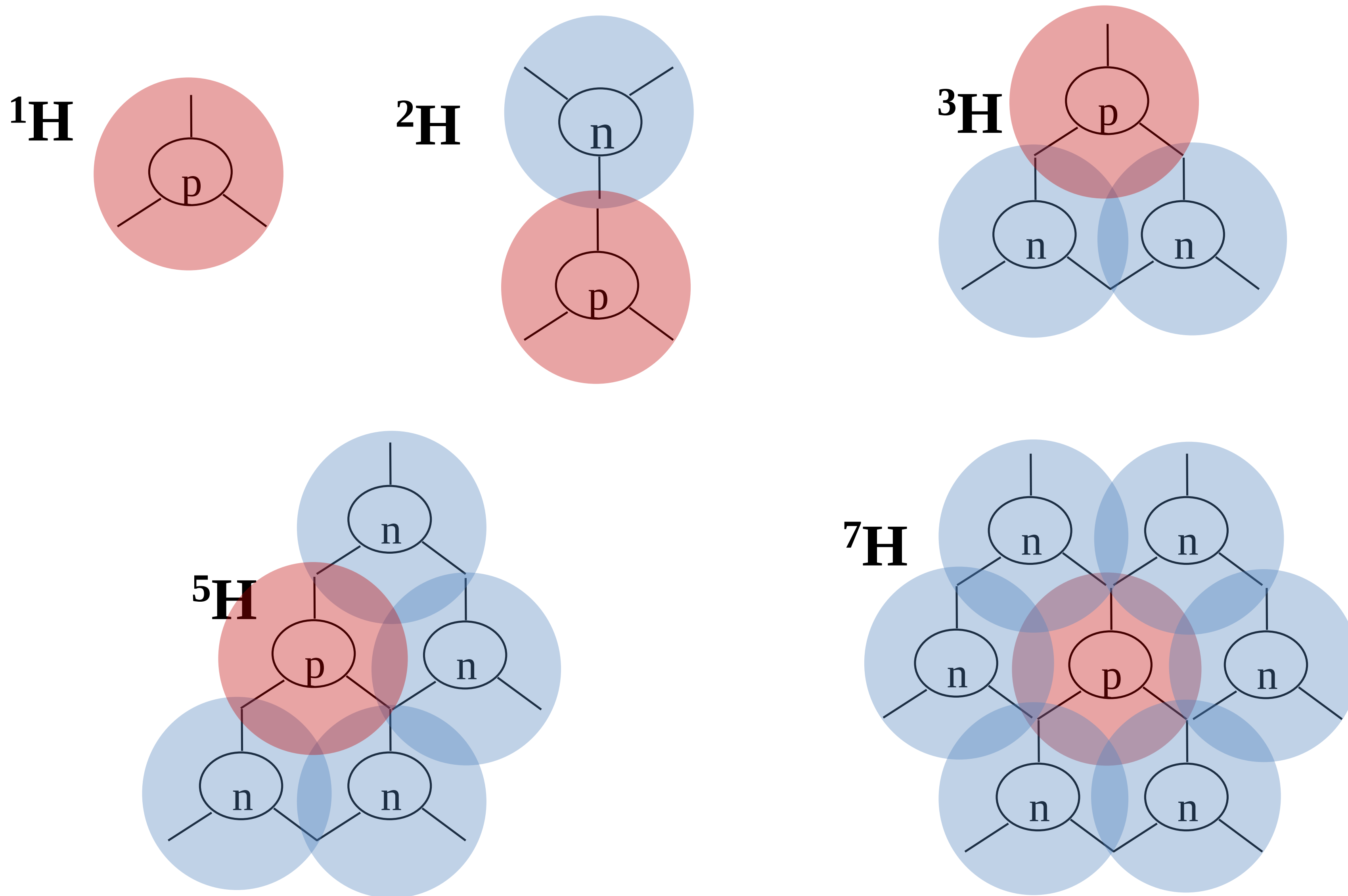
- In both cases a mean field composed of nucleon fields is created.
- Quantum numbers of both coincide up to medium nuclei.

Differences

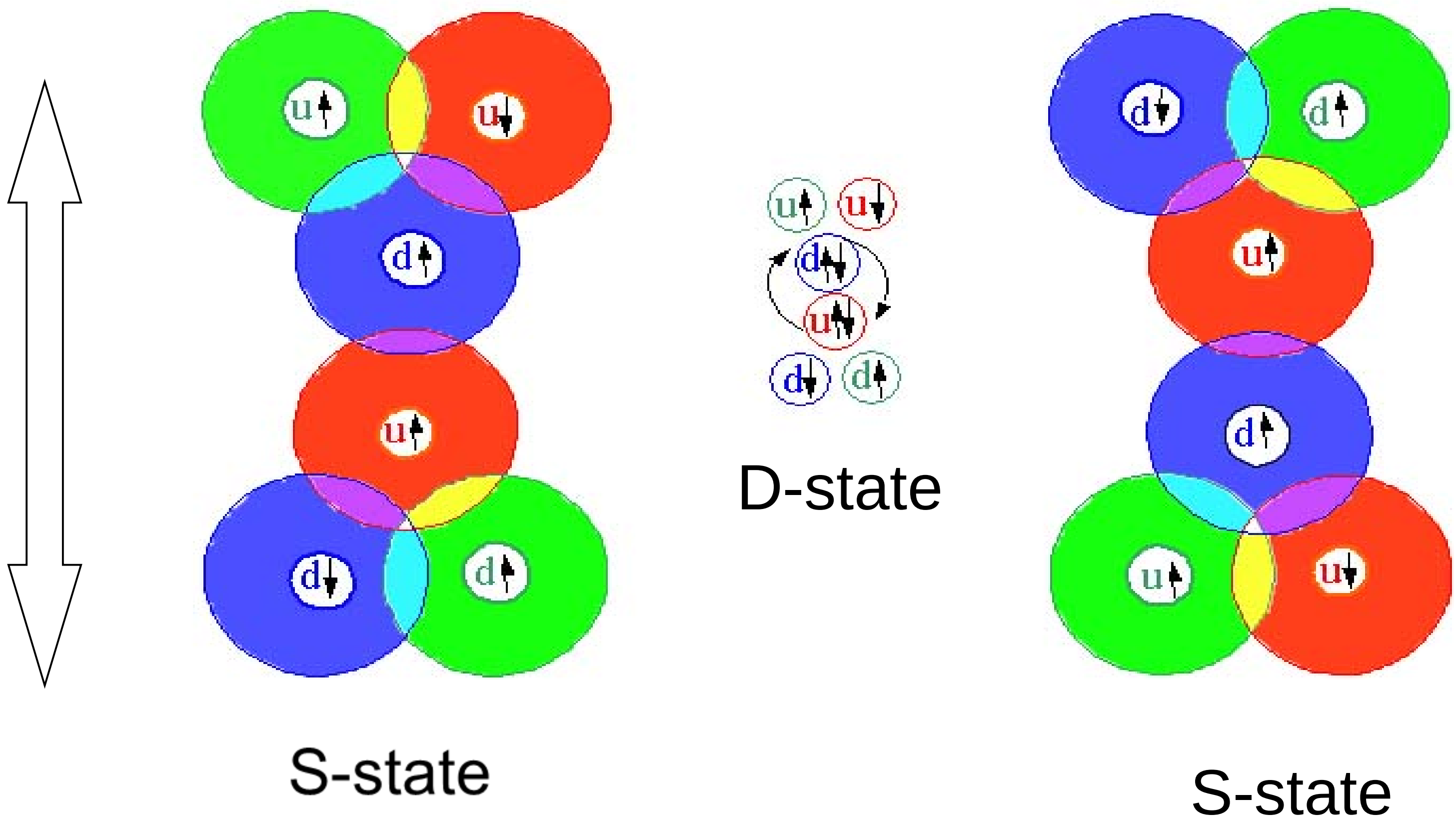
- **SM**: independent nucleons
- S**CQV+FCC**: nucleons are strongly correlated
- **SM**: shell closure nuclei are spherically symmetric
- **SCQM+FCC**: All nuclei are deformed

Light Nuclear (bound) Isotopes

Bound Hydrogen Isotopes

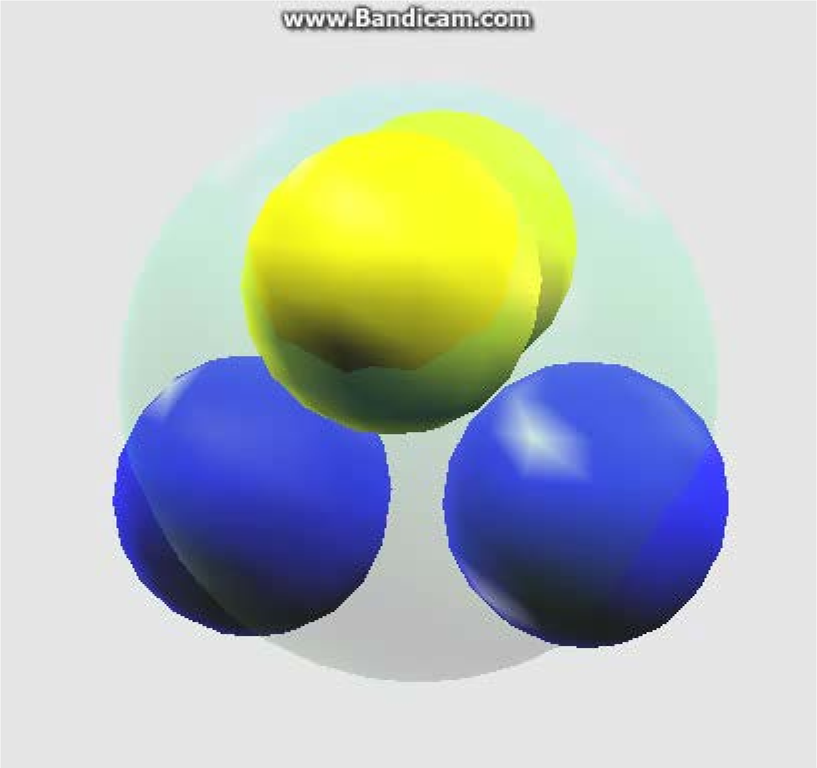


Deutron

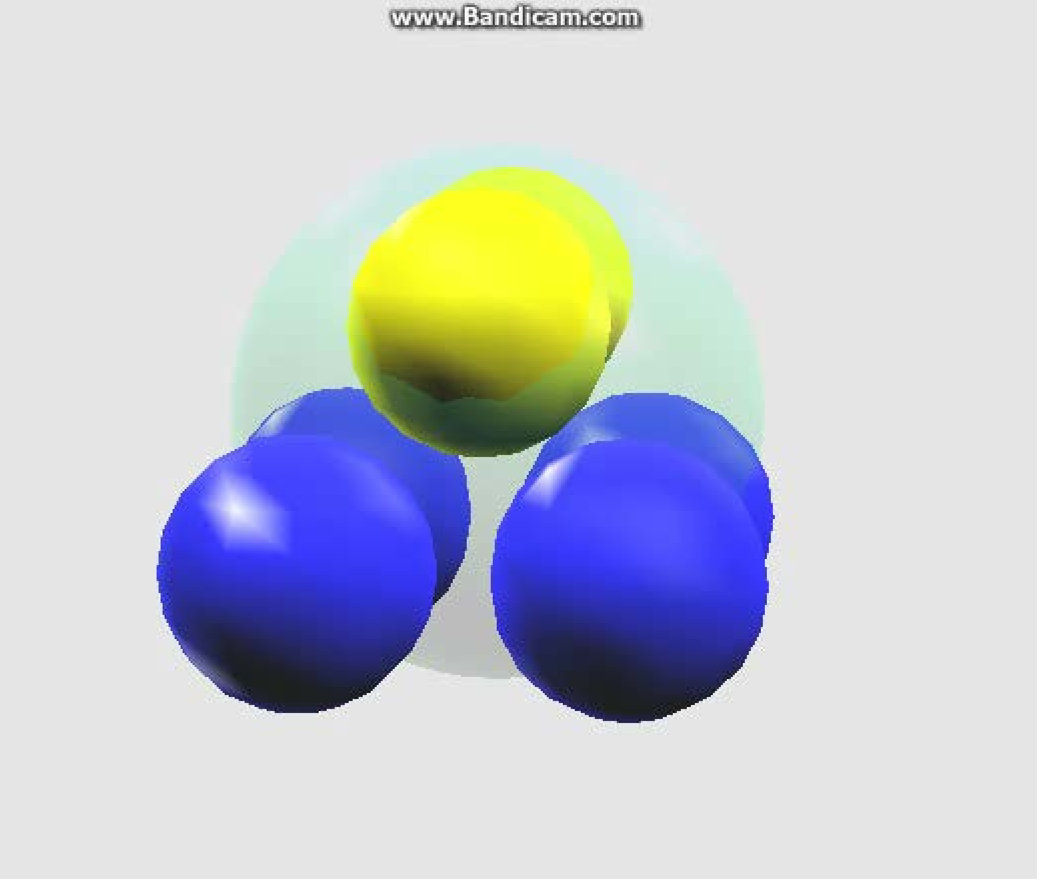


Oscillating proton-neutron system

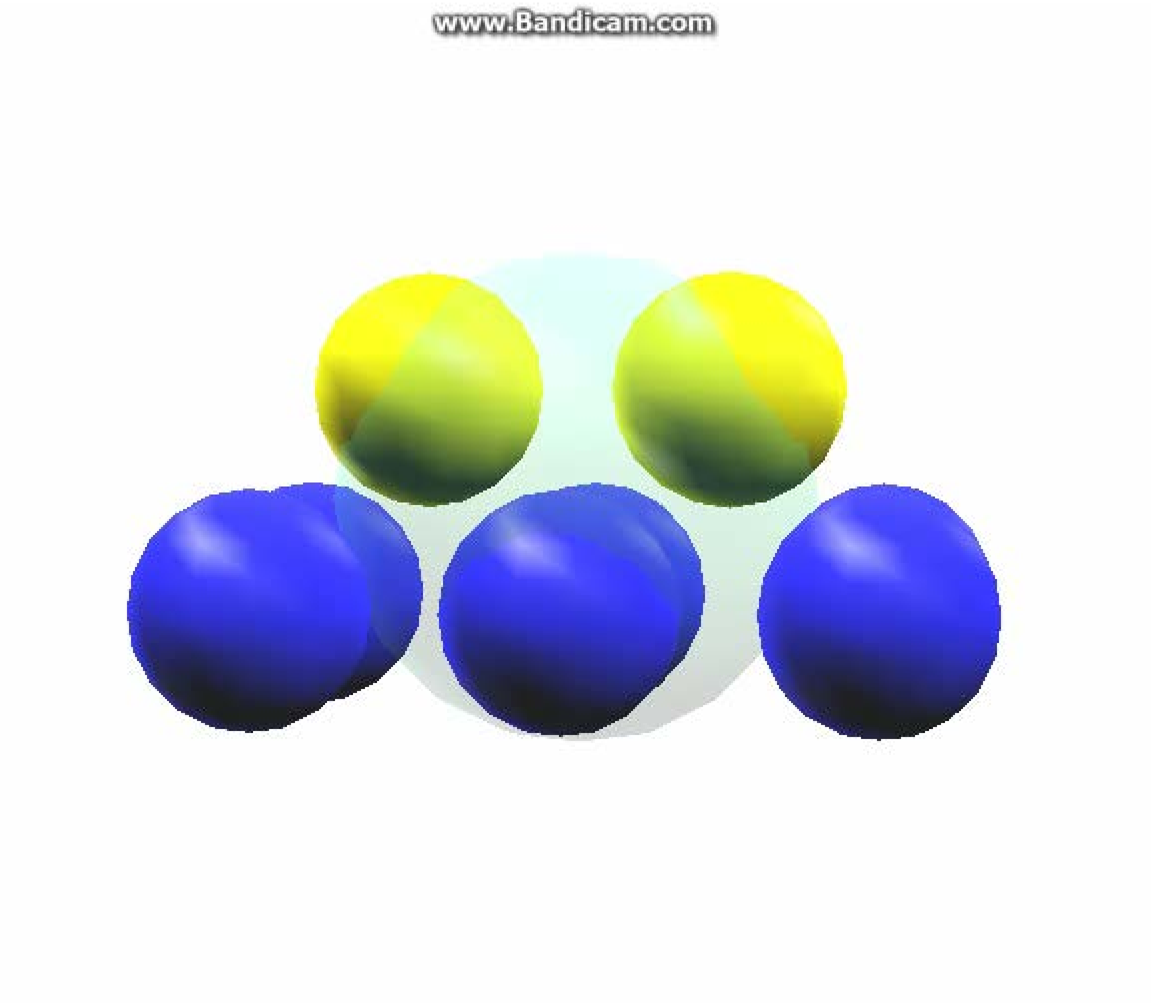
^4He



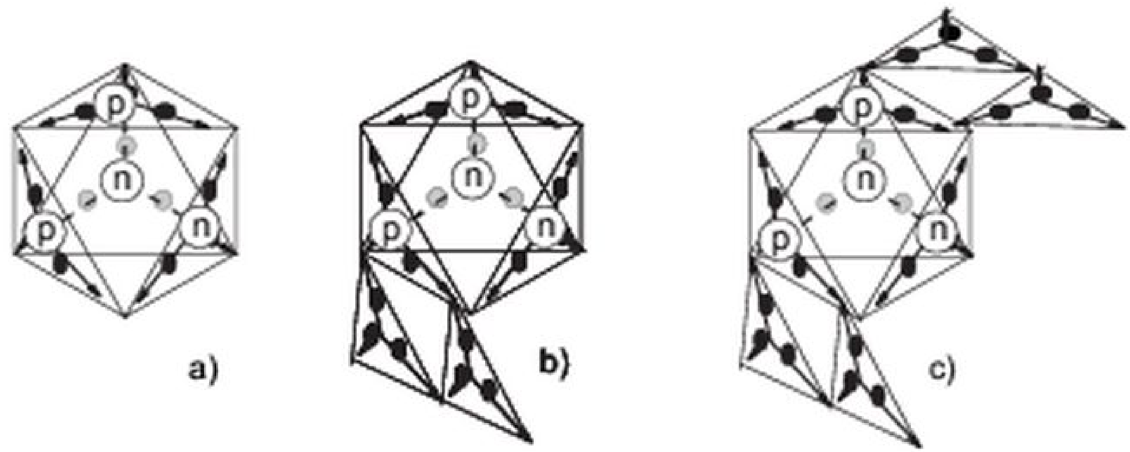
^{6}He, borromean



^{8}He, borromean



Bound Helium Isotopes



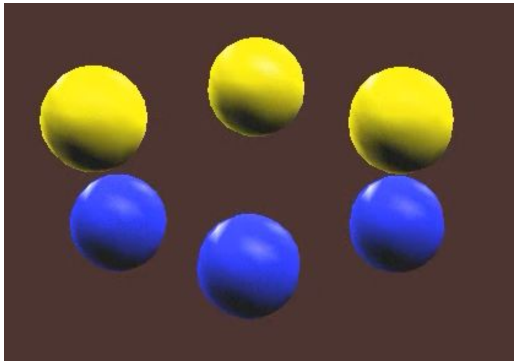
^4He ^6He ^8He

$$A_{bound} \leq 8,\ A_{total} \leq 9$$

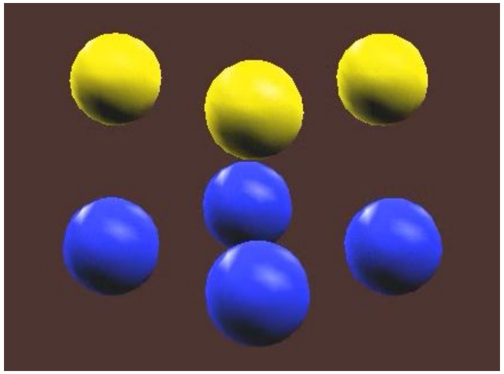
Lithium Isotopes

$A_{bound} \leq 11$

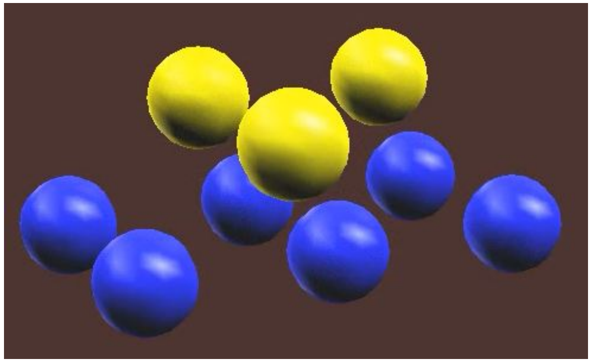
^{6}Li 1^{+}



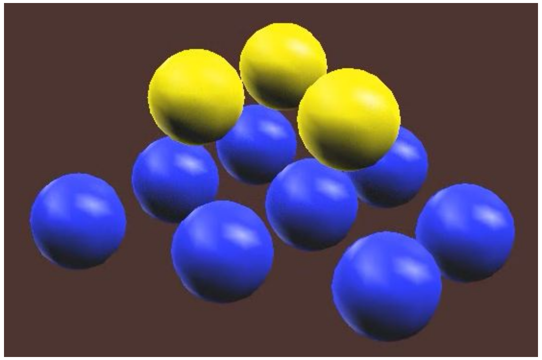
^{7}Li $3/2^{-}$



^{9}Li $3/2^{-}$



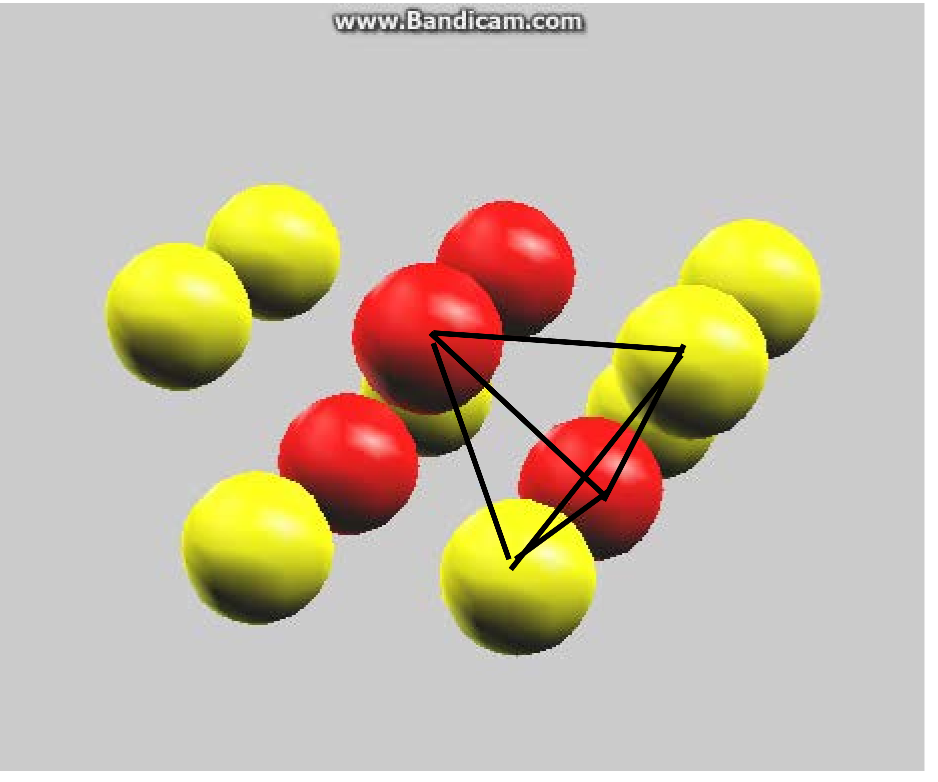
^{11}Li $3/2^{-}$



^{12}C

$A_{bound} \leq 22$

^{12}C (0+)

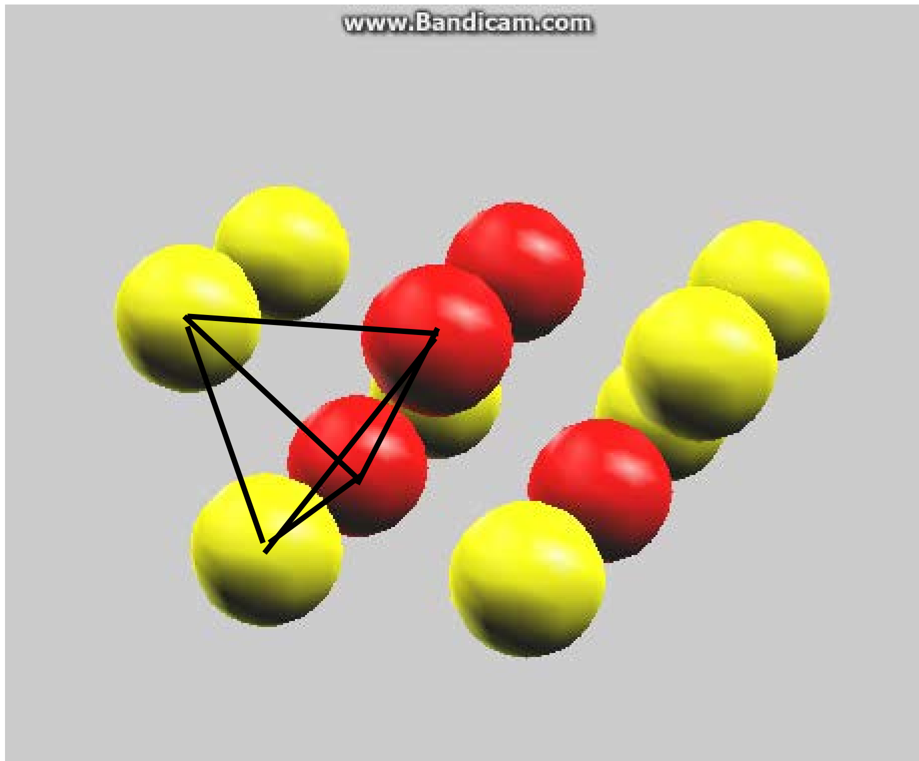


Up to 4 virtual α-clusters are in ^{12}C.
Nucleons of s-shell (red spheres) belonging neighboring virtual α-clusters perform coupling between them

^{12}C

$A_{bound} \leq 22$

^{12}C (0+)



Up to 4 virtual α-clusters are in ^{12}C.
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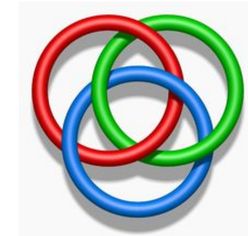
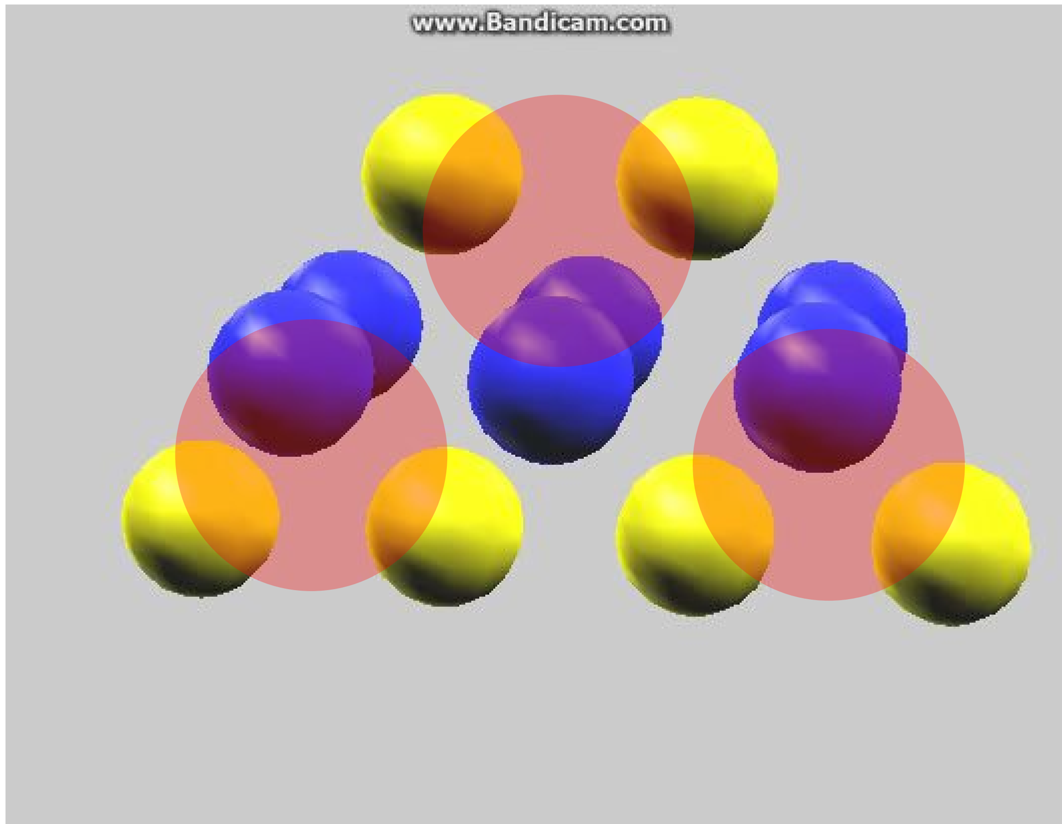
^{12}C

Hoyle state –Borromean nucleus



^{12}C (0+)

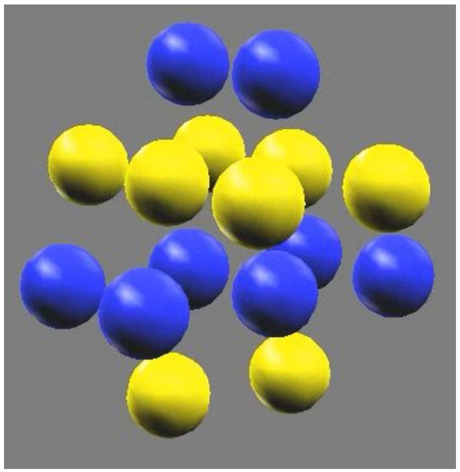


3 weakly bound (real) α-clusters coupled via 4 quark loops

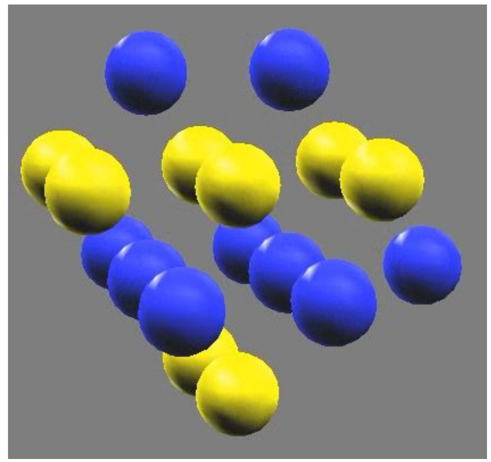
Oxigen Isotopes

$A_{bound} \leq 26$

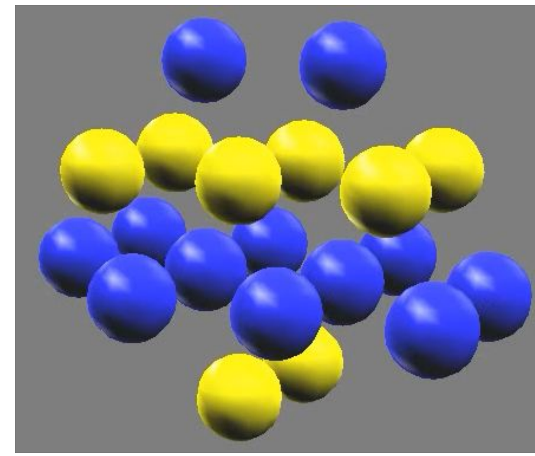
^{16}O



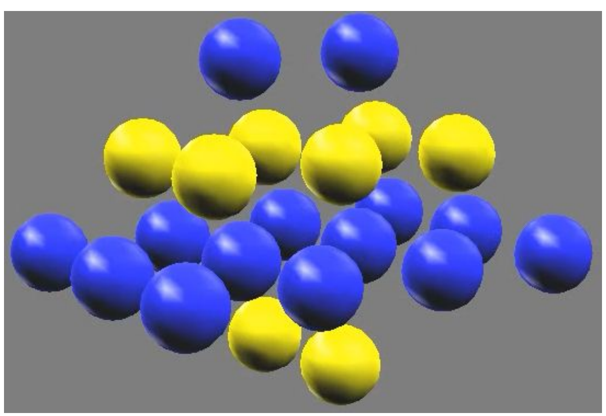
^{17}O



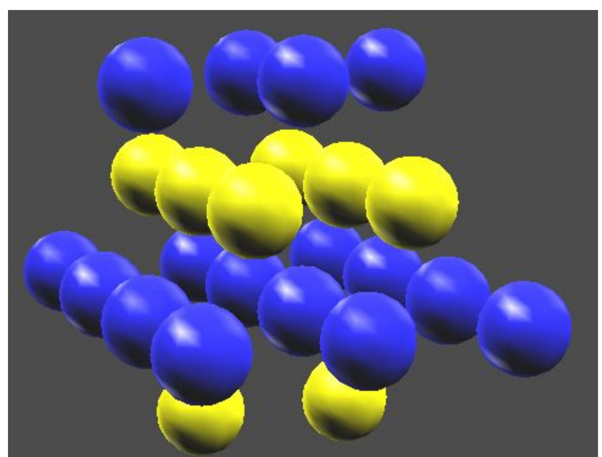
^{20}O



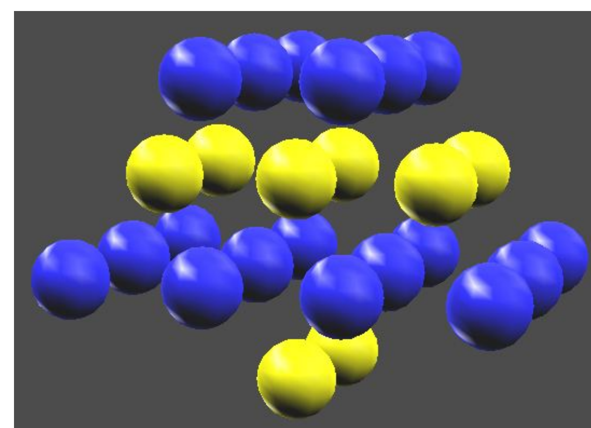
^{22}O



^{24}O

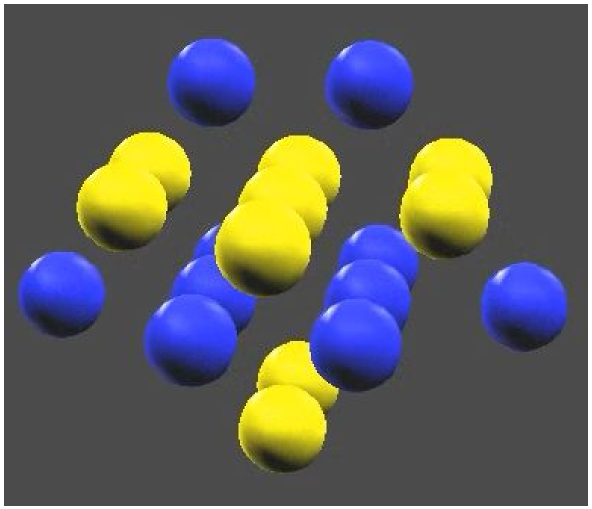


^{26}O

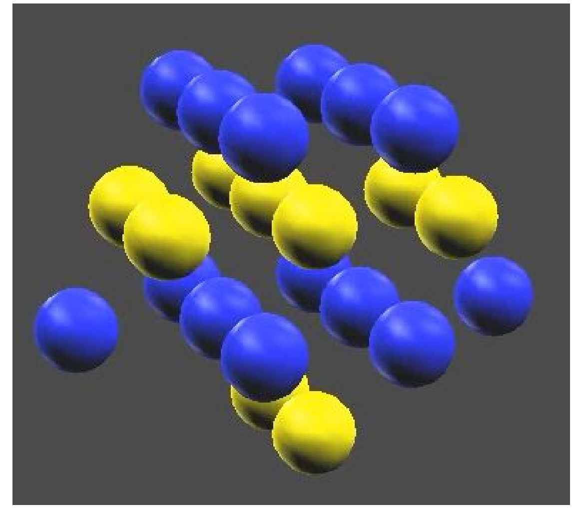


Fluorine Isotopes
$A_{bound} \leq 27$

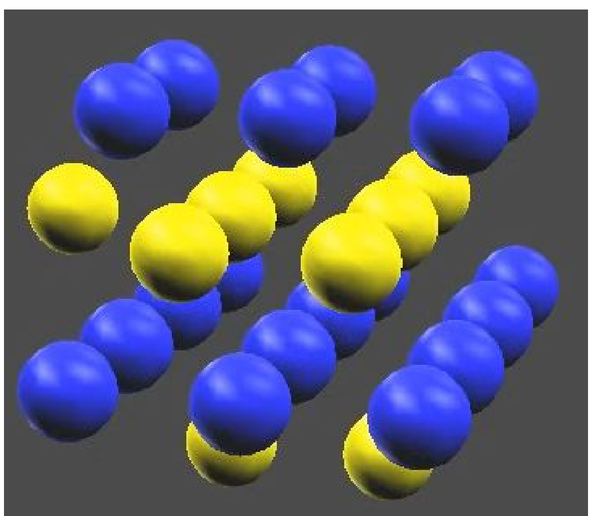
^{19}F



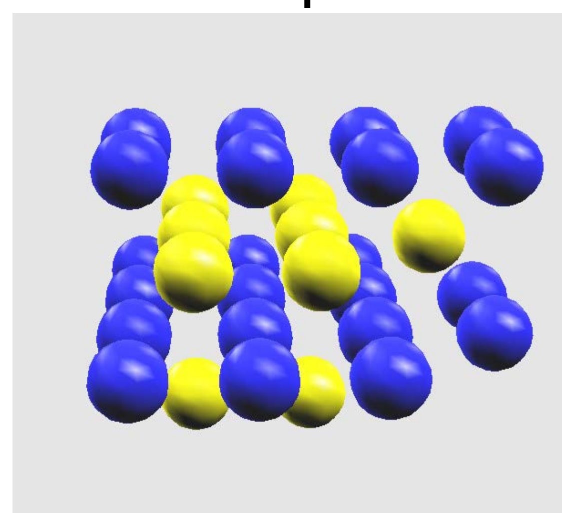
^{23}F



^{27}F



^{31}F



Summary

1. Quarks play an explicit role in formation of the nuclear structure.
2. Quarks and nucleons inside nuclei are correlated.
3. Quark loops are building blocks of nuclear binding.
4. **Nuclei possess crystal-like structure:**
   - Nucleon centers are arranged according to FCC lattice
   - Nucleon are non-linear standing waves at the nodes of FCC lattice
   - Even-even nuclei are composed of **virtual α-clusters**
   - Closed Shells = Octahedral Faces
5. ‘Halo’ nuclei **– fruits of quark-loop bindings**

Thank you!